Size effect in splitting diagonal cubes

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ABSTRACT

The experimental investigations on fracture mechanics of cement-based materials until 1970s indicated that classical linear elastic fracture mechanics (LEFM) is invalid for quasi-brittle materials such as concrete. This inapplicability of LEFM is due to existence of an inelastic zone with large scale and full cracks in front of the main crack tip in concrete. This so-called fracture process zone (FPZ) is ignored by LEFM. Consequently, several investigators have developed non-linear fracture mechanics approaches to describe failure of concrete/reinforced concrete structures. Deterministic size effect laws among these non-linear approaches, for instance size effect law (SEL) by Bazant (1984), suggest that size effect on strength is primarily related to a relatively large FPZ in concrete. One of the main requirements in this law is the need to test samples, which are geometrically similar and made of the same material, and which must provide a minimum size range=1:4.

The split-tension test has been used to indirectly test the tensile strength of quasi-brittle materials such as concrete and rock. Recently, concrete splitting specimens have been commonly used in concrete fracture because they have certain advantages, such as compactness and lightness, compared to beams. However, the number of theoretical and experimental studies with diagonal split-tension specimens, to which compressive forces are applied along two opposite edges, is limited.

In this study, two series of concrete diagonal cube specimens of different size (size range 1:4) were tested by splitting loading. The concrete mixes with the maximum aggregate size=8 mm were designed as the gap-graded aggregate and the continuously graded aggregate. The ultimate loads obtained from the test results were analysed via Bazant's SEL. Consequently, it was observed from the analysis based on SEL that the concrete with gap-graded aggregate is the more ductile material than the concrete with continuously graded aggregate.

INTRODUCTION

Recently, concrete splitting specimens have been commonly used in concrete fracture because they have certain advantages, such as compactness and lightness, compared to beams. Additionally, cubical and cylindrical test specimens have the following advantages [1-3].

a) These specimens are easy to handle, and there is no risk of breaking them during handling.
b) The same moulds can be used to cast specimens for both fracture and strength tests.
c) In determining the fracture parameters of cement-based materials, the contribution of the weight of the specimen can be ignored, unlike notched beams.

Tests in which cubical and cylindrical specimens are used to study concrete fracture can be classified as wedge-splitting tests and split-tension tests. Wedge-splitting tests have been performed on cubical and cylindrical specimens with an edge notch. The split-tension test has been used to indirectly test the tensile strength of quasi-brittle materials such as concrete and rock [4] and to study concrete fracture over the last decade [1-3].

Several design codes [5] have used the square-root formula, which does not account for size effect, for determining tensile strength capacity of concrete. However, it is well known that strength of concrete structures generally tends to decrease with increasing structure size. Size effect in concrete/reinforced concrete structures can be well explained by fracture mechanics. The effect of specimen size on strength of concrete has been investigated by means of size effect law (SEL) by previous investigators [6, 7].

In this study, two series of concrete diagonal cube specimens of different size (size range 1:4) were tested by splitting loading. Cubes to which compressive forces are applied along two opposite edges are called diagonal splitting cubes, and cubes to which compressive forces are applied along parallel midlines of the edges are called splitting cubes [4]. The concrete mixes with the maximum aggregate diameter=8 mm were designed as the gap-graded aggregate and the continuously graded aggregate. The peak loads obtained from the test results were analysed by using Bazant's SEL. Consequently, the results of this study indicate that the concrete with gap-graded aggregate is the more ductile material than the concrete with continuously graded aggregate.

A HISTORICAL OVERVIEW ON THE SPLITTING TESTS

The cylindrical and cubical split-tension specimens have been commonly used to determine the tensile strength of materials such as concrete and rock. The split-cylinder test is also called the Brazilian split test and it was first proposed by Carneiro and Barcellos [8] in 1949. This test was also applied successfully on cubes by Nilsson [9] in 1961. As shown in Fig. 2a, the split-tension specimen is placed between the platens of test machine and the load is applied until failure, which is occurred by splitting along the vertical diameter due to the lateral tensile stress distribution [4]. According to the elasticity theory [10], the nominal tensile strength of the split tension specimens is defined as

$$\pm_{Nc} = \frac{2P_c}{\circledR\, bd} \tag{1}$$

where P_c is the ultimate load, b is the specimen width and d is the specimen depth. However, Equations 13 is only valid for the concentrated loading condition as shown in Figs. 2a and 2b. In practice, the applied load is distributed to a finite width (*2t*) on the specimens by means of soft materials such as plywood and hard cardboard, as indicated in Fig 2c. Tang et al. [11] investigated the effect of the distributed load in both three point bending beams and split-tension cylinder. It was concluded that the nominal strength decreases as increasing width of the distributed load in split-tension cylinder, while this effect is no significant in the bending specimens. According to Tang [12], the maximum tensile stress value of the unnotched cylinder specimens at the plane of loading can be calculated from

$$\pm_{\max} = \frac{2P}{\circledR\, bd}\left(1 - \quad {}^{2}\right)^{3/2} \tag{2}$$

in which P is the total compressive load and $\quad = 2t/d$ is the ratio of the distributed-load width to the specimen depth, as depicted in Fig. 1.

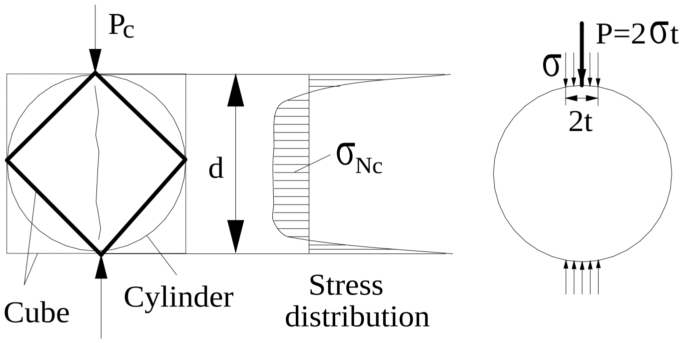
Rocco et al. [13] studied on both the cylinder and the cube specimens and proposed that the maximum tensile stress can be calculated for the unnotched cube specimens at the plane of loading as follows:

$$\pm_{\max} = \frac{2P}{\circledR\, bd}\left[\left(1 - \quad {}^{2}\right)^{5/3} - 0.0115\right] \tag{3}$$

By means of the cohesive crack model, Rocco et al. [13] simulated the Brazilian test by considering the effect of the specimen size, the specimen shape (Cubical/Cylindrical), and distributed-load width. They suggested closed-form analytical formulas including size-depended response for estimating the split-tensile strength of the cube and cylinder specimens. Besides, the investigators performed a series of experimental studies and it was concluded that the developed formulas were in a good agreement with the experimental results.

Similarly, using numerical techniques, the following formula was derived for the maximum tensile strength of the un-notched diagonal cubes by Ince [3], according to boundary element method analysis:

$$\pm_{\max} = \frac{2P}{\circledR\, bd}\left(\frac{1}{0.931+38.931\ \ ^{4.778}}\right) \tag{4}$$



a) **b)**

Figure 1 Split-tension specimens a) Stress distribution b) distributed load

SIZE EFFECT IN CONCRETE AND REINFORCED CONCRETE

In general, the change of a structural property when the size of a structure changes is known as a size effect related to this property. In other words, if geometrically similar specimens do not behave similarly for different sizes, this is called a size effect. It is well known that structures become more brittle as their size increases, but the classical procedure uses working stresses which are the same in design. Size effects occur in concrete in any loading conditions. Most significant cases occur in tensile or shear loading.

Kani [14] was one of the first to demonstrate the size effect in concrete structures. It has been shown that the shear strength of similar concrete beams decreases with increasing beam depth. Due to the fracture in a structural element being driven by stored elastic energy released from the whole structure, this size effect can be well interpreted by fracture mechanics. The fact that the strength of brittle materials is affected by the presence of imperfections is first suggested by Griffith [15], who is the founder of linear elastic fracture mechanics (LEFM). Due to his conclusion, it can be expected that the value of the ultimate strength will depend upon the size of specimens. As specimen size increases the strength is expected to be decreased since the probability of presence of weak links is increased. Traditionally, the size effect in fracture of concrete structural elements has been explained as Weibull's theory [16]. He showed that if tensile tests are performed on two geometrically similar specimens with different volumes, the corresponding ultimate strengths are different. Weibull's approach has been widely used for estimating safety factor of materials. In the early 1980s, it is realized that neither LEFM nor Weibull's approach were adequate for predicting size effect in cementitious materials [17]. For this reason, several investigators have developed deterministic size effect theory based on non-linear fracture mechanics.

Besides the statistical based size effect, the second size effect referred to as the fracture-type size effect in concrete fracture has been described by Bazant [17]. This is referred to as size effect law (SEL) which has been shown to agree well with test data.

Size effect in concrete/reinforced concrete behaviour has been extensively studied both experimental and theoretically with notable success in researching this problem [6, 7]. Bazant derived SEL, by considering the energy balance at crack propagation and dimensional analysis of geometrically similar specimens. The so-called SEL is expressed as

$$\pm_N = B\pm_0\left[1+\frac{d}{d_0}\right]^{-1/2} \tag{5}$$

where $\pm_N$ presents the nominal strength at failure and expressed as $\pm_N = c_n P_u / bd$, where d is characteristic dimension of the specimen, chosen to coincide specimen cross-sectional depth, b is specimen thickness, and c_n is a constant of load type. $\pm_0$ is referred to as the strength parameter and B and d_0 are empirical constants which can be determined by curve fitting to the test results of geometrically similar specimens. This is the only approach which expresses the size effect in concrete specimens based on fracture mechanics. SEL has been derived based on the following assumptions, the potential energy released during the fracture is proportional to the crack length (a), to the area of the cracking zone, width of the front of cracking zone ($ª_0 d_{max}$) is constant, where $ª_0$ is an empirical constant and d_{max} is the maximum aggregate size.

SEL of Bazant is illustrated in Fig 4. For small test specimens there is no size effect due to the strength at failure being proportional to the material strength. This case corresponds to strength criterion and is presented by the horizontal line in Fig 4. In the large test specimens, this presents the maximum possible size effect. The material strength at failure is proportional to a characteristic dimension and corresponds to classical linear elastic fracture mechanics which is presented by the inclined line with slope -1/2 in Fig 4. The intersection of the two asymptotes corresponds to $d=d_0$ and is called the transitional size. The results of most concrete test specimens in existing experimental studies lie in the transition zone between these extreme cases. Contrary to LEFM, size effect in Weibull-type statistical approach is characterized a straight line with slope -1/6, as shown in Fig. 4 [16].

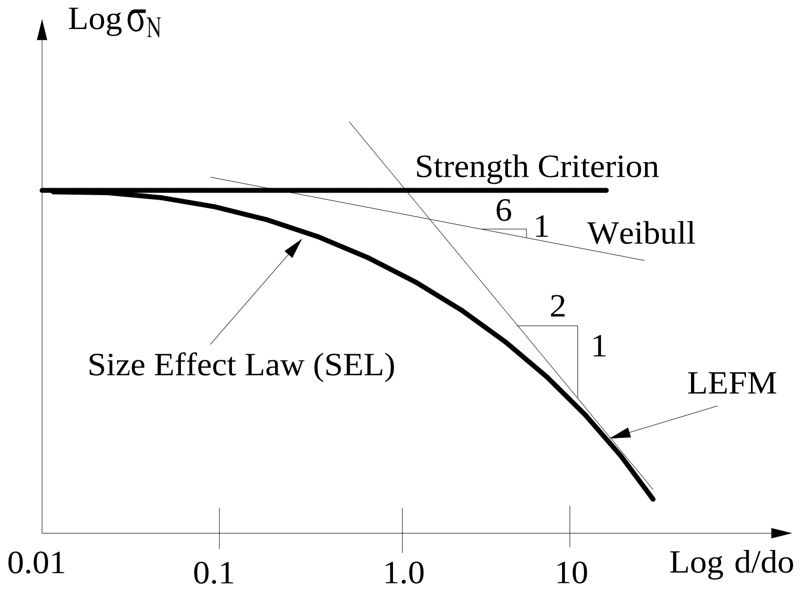


Figure 2 Bazant’s Size Effect Law (SEL).

EXPERIMENTAL PROGRAM

The test specimen was a cube. To determine size effect, specimens with cube sides *d*= 50, 100 and 200 mm were tested. All specimens in each series were cast from the same batch of concrete. Three identical cube specimens with 150 mm edge length were also cast from each batch of concrete to determine compressive strength of concrete. The maximum sand grain size was 4 mm maximum aggregate size=8 mm for each batch. Mineralogically, the aggregate consisted of river sand. The aggregate and sand were air-dried prior to mixing. CEM I 42.5 N cement was used in all mixes. Its specific gravity and 28-day compressive strength were 3.06 and 50.3 MPa, respectively. All specimens and identical cubes were removed from the mold after 1 day and were subsequently cured at approximately 20 °C in water until testing at 28 days.

The compression tests and the splitting-tension tests were performed using a digital compression machine with a capacity of 2000 kN. The plywood loading strips, with thicknesses of 3 mm and lengths 10 mm greater than the specimen width, were used in the splitting tests. They were attached on the specimens in the correct positions. The specimens were loaded monotonically until final failure and care was taken to apply a constant loading rate. A smooth bearing head of stainless steel with a chevron notch was used in the diagonal splitting tests, as detailed in Fig. 3a. The steel plates did not indicate any flexural or other type of deformation after testing. Typically, approximately 2 min (± 30 sec) elapsed before the maximum load capacity for each specimen was reached. Identical cubes were tested at an age similar to the other specimens.

TEST RESULTS AND ANALYSIS

The compressive strengths of the mixes f_{cc} are 43.1 MPa and 49.0 MPa for continuously graded and the gap-graded concrete, respectively. Table 1 summarizes the cube size *h*, the characteristic dimension d, the observed failure load P_k and the nominal strength according to Eq. (1) $\pm_N$ for each of the 18 specimens tested. Typically, it observed that as the load is gradually increased, the first crack occurred along the center line in the vertical direction (Fig. 1). When the maximum load is reached, two diamond-shaped wedges under the bearing plates were formed in the diagonal cube test (Fig. 3b). Similarly, in the cube test, wedges under the bearing plates were formed. Split-cube specimens, the similar rupture modes were also found in the study of Rocco et al. [13] and in during the tests by Ince [3].

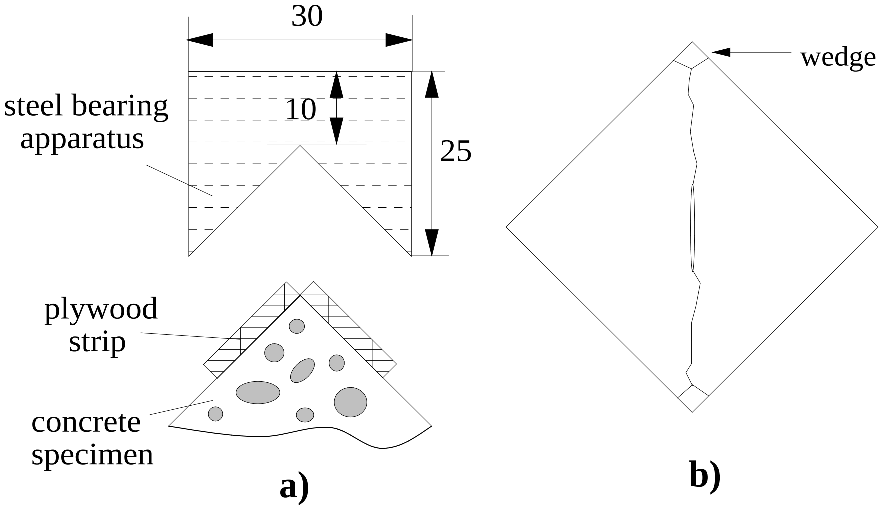


Figure 3 a) Test detail in the cube tests for d=100 mm b) crack pattern

Table 1 Experimental results

h (mm)	d (mm)	Continuously		Gap-Graded	
		P_k (kN)	σ_N (MPa)	P_k (kN)	σ_N (MPa)
50	70.7	22.3	4.015	29.0	5.222
50	70.7	25.1	4.520	23.7	4.267
50	70.7	26.5	4.772	38.2	6.878
100	141.4	75.3	3.390	79.3	3.570
100	141.4	70.7	3.183	87.1	3.921
100	141.4	82.8	3.727	82.8	3.727
200	282.8	220.5	2.481	286.7	3.227
200	282.8	260.0	2.926	314.0	3.534
200	282.8	201.8	2.271	272.6	3.068

For diagonal split-tension cube specimens, empirical constants in Eq. (5) can be calculated as $B = 1\big/\sqrt{C}$ and $d_0 = C/A$ from the linear regression made on $y=Ax+C$ with $y = 1\big/\pm_N^2$, $x = d$. Fig. 4a and Fig. 4b show results of the linear regression analysis, and the size effect law analysis in the bilogarithmic plane, respectively. The determination coefficients R^2 are also given in Fig. 4a for each batch.

CONCLUSIONS

Recently, split-tension specimens such as cylinders and cubes have been commonly used to determine the tensile strength of cement-based materials. Diagonal split-tension cubes have been used to determine the split-tension strength of concrete for continuously-graded and gap-graded mixes in this article.

The maximum loads obtained from the test results were analysed by SEL. Consequently, it was emphasized that the concrete with gap-graded aggregate is the more ductile material than the concrete with continuously graded aggregate.

The experimental studies have shown that fracture behavior of concrete is particularly influenced by the four material parameters: compressive strength, maximum aggregate size, water-cement ratio, and aggregate type [18-21]. It is noted that fracture resistance of concrete can also be affected by other material parameters such as type of cement and curing conditions, etc. However, this study revealed that aggregate gradation of mix is the one of the important parameters for concrete fracture.

When splitting specimens are produced with molds of the same size, the uncracked ligament length of the diagonal splitting cube specimens is √2 times greater than the lengths of other splitting specimens with the same size. This is another advantage of diagonal splitting cubes, and it is especially useful when studying the size effect.

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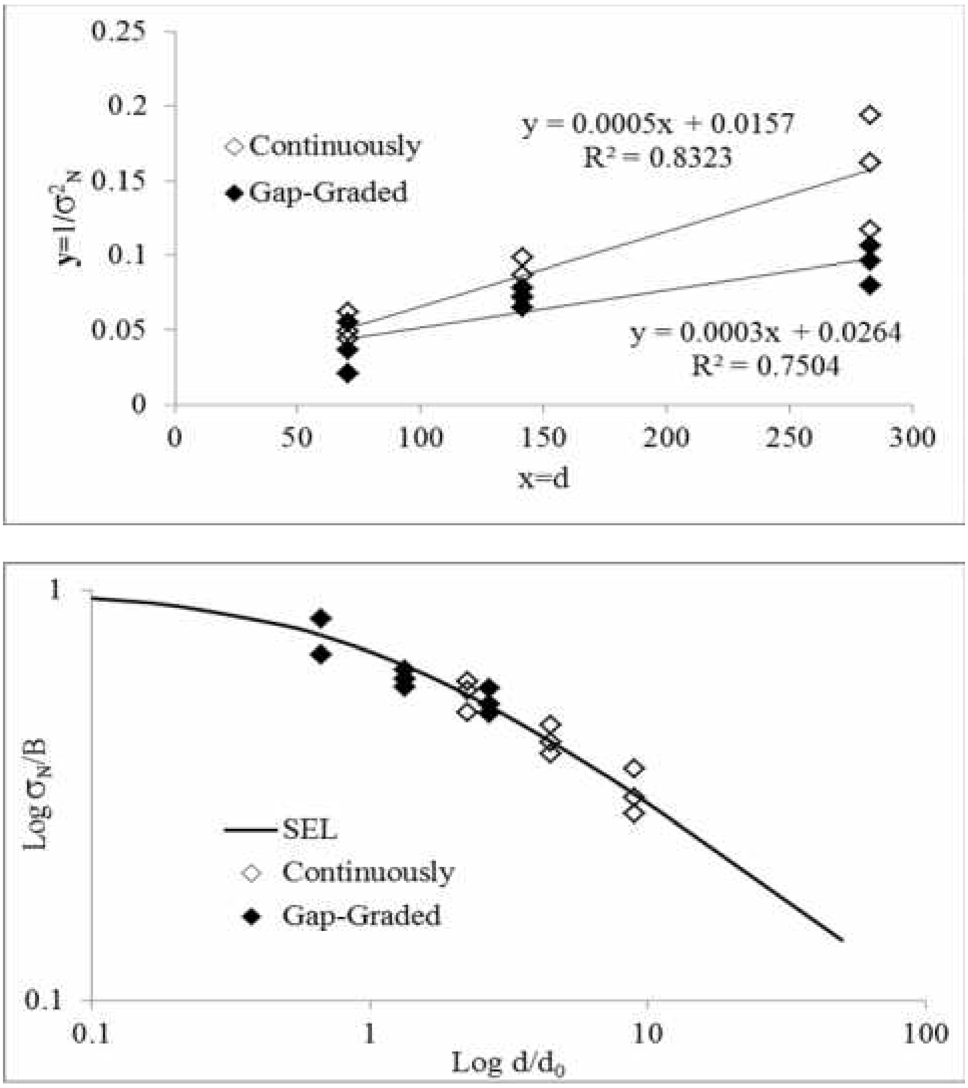

Figure 4 Application of Bazant’s SEL to diagonal split-tension samples

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