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1-ICAUD

On the Flexibility of Deployable Dome Structures and their Application in Architecture

Dimitrije Nikolic, Radovan Stulic, Predrag Sidjanin

(Dimitrije Nikolic, Master student, Faculty of Technical Sciences, Trg D. Obradovica 6, Novi Sad, Serbia, ofikon@sbb.rs)
(Radovan Stulic, Full Professor, Faculty of Technical Sciences, Trg D. Obradovica 6, Novi Sad, Serbia, sdrulias@eunet.rs)
(Predrag Sidjanin, Full Professor, Faculty of Technical Sciences, Trg D. Obradovica 6, Novi Sad, Serbia, mp_art@eunet.rs)

1 ABSTRACT

In this paper we discuss flexibility and applicability of deployable dome structures in contemporary architecture. A deployable dome is a spatial structure derived from appropriately connected planar polygonal panels. Nowadays, deployable domes are little researched comparing to other deployable structures, such as pseudo-cylinders, for example. When designing these configurations, either changeable supports' spans or strain of some structural elements might occur, depending on the underlying geometrical analysis. Such occurrences, usually being undesirable, can be avoided by the adequate geometrical solution which can also satisfy various sizes of the structures. Therefore, they could answer the purpose both in architectural and urban design. Thus, choosing suitable dimension of the structure, numerous applications can be obtained, starting from street furniture to large scale convertible structures for covering open spaces.

2 INTRODUCTION

In contemporary architectural practice, striving for multi-functionality of buildings, as well as of their composing elements has increasingly become present. The possibility of their rational production satisfying low energy consummation and non-complex manufacturing process brings them into highly desirable systems. As a result,complex structural systems formed of numerous simple elements are being obtained and the major problem that appears is the generation of proper structural geometry.

Deployable structural systems could satisfy these requirements and are suitable for constructing an envelope of various spaces. However, such configurations are still not either fully researched or adequately applied in architectural design, although their fundamental property is polyvalence, i.e. mobility and shape transformability. More rational use of public spaces may be achieved by the application of deployable structures, increasing their adaptability and attractiveness. In that sense their duality provides adaptability to various functional purposes, environmental conditions, as well as the aesthetic expression of an architect.

Two often used forms in architecture as a particular class among deployable structures are presented in this paper: a cylinder and a sphere, i.e. vault and dome. The results may be generalized, in order to become a common practice in architectural and urban design.

Researchof deployable, i.e. folding systems is based on the principles of origami – the traditional Japanese art, that is, the skill of paper folding. In that sense each of them represent “apiecewise linear developable surface that can realize a deployment mechanism if its facets and foldlines are substituted with rigid panels and hinges, respectively” (Tachi, 2010).

One of the main problems in the field of deployable structures is constructing a cylinderwhich is deployable in axial direction. The resulting geometrical form is cylindrical in macroscopic sense and is concave polyhedral in microscopic sense. It could be generated from developable double corrugation surface called pseudo-cylindrical concave polyhedral shell (Miura, 1969).

Pseudo-cylinders derived in this way are stable structures and are not deployable without elastic deformations (A. Sogame, H. Furuya, 2000, Hoberman, 1993). Guest and Pellegrino (1994) designed a multi-stable pseudo-cylindrical structure with zero strain at fully folded and fully deployed position. You and Cole (2006) introduced non-developable pattern into the previous model, providing the decrease of strain in structure’s interpositions and they showed that by increasingthe pseudo-cylinder sides’ number, the decrease of strain value appears.

On the other hand Sill and Ruckert (2006) proposed a foldable dome with serial extensibility and a single degree of freedom. However, the disadvantage of this solution is the fact that the constant span appears only in certain range of opening angles. After having thoroughly examined the above mentioned propositions of both cylindrical and spherical retractable structures, we have noticed that the particular geometric modeling can ameliorate the solutions of these problems.

3 ON GEOMETRIC MODELING OF DEPLOYABLE PSEUDO-CILINDERS

The initial form of a pseudo-cylinder is an *n*-sided right regular prism which suffers certain modifications in order that deployability becomes possible. The split of the panels on the edges of the neighboring sides ofthe prism appears during the folding, as it is shown in the Fig. 1 (please note that one side of the prism is coloured in red). Therefore, one of the issues is to preserve a non-split pseudo-cylindrical form during the deployment. On the bases of detailed geometrical analysis, the shape of the elements – panels which provides the preservation of the angles between neighboring sides has been defined. These panels, being a set of planar elements, are herewith named a *beak*, and they can be considered from two points of view: (a) strain appearance in the elements and (b) angle variation.

In the first case it is assumed that the pseudo-cylindrical structure is closed (non-split) so that the angles between neighboring sides remain constant during the deployment process. As it was mentioned before, certain deformations wereinevitable. In the second case when the adopted structure is open allowing the variation of the angles, the displacement of structure's supports is caused. According to the performed analysis it is concluded that by increasingthe sides' number, i.e. increasing the angle between them, as well as by choosing the appropriate geometry of the beak, it is possible to decrease either strain value or deviation from initial angle to small value ($< 1\%$).

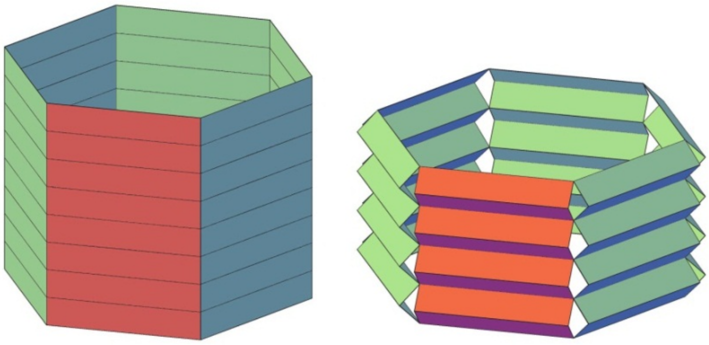


Fig. 1: Displacement of panels on the corners of deployable pseudo-cylinder.

This analysis is based on ideal non-materialized geometrical model, while in real conditions it is necessary to apply finite thickness materials, in order to produce structures that are load bearing. Therefore, the problem of the overlapping of panels of the beak and corresponding sides is imposed. In case of rigid panels, greater thickness is desirable to have beaks which, in folded position, have no more than two overlapped panels. Depending on applied material there are two types of systems: (a) flexible or (b) rigid. As for flexible systems,either membranes or translucent synthetic materials are usually applied, but the structure has poor stiffness and thus demands certain supportable subconstruction. However, these are light structures with small volume in folded position, providing easier transportation and assembling. On the other hand, rigid systems have fewer degrees of freedom and can produce larger self-supporting structures.Various types of beaks which can be applied, dependent on the required form of structure or chosen materialization, are presented in Table 1 with basic information.

In Table 1 used marks of beak type represent:

BPC: Beak of pseudo-cylinder
DBPC: Double beak of pseudo-cylinder
BPCi: Inverse beak of pseudo-cylinder
BPCis: Semi-inverse beak of pseudo-cylinder
DBPCi: Inverse double beak of pseudo-cylinder
B90: Beak for preservation of the right angle
B90i: Iverse beak for preservation of the right angle
B90is: Semi-inverse beak for preservation of the right angle
B90iFS: Inverse beak for preservation of the right angle with wixed supports

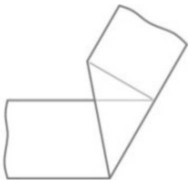
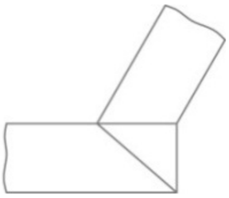
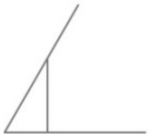
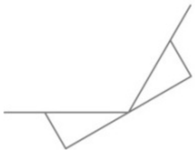
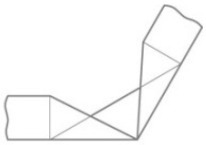
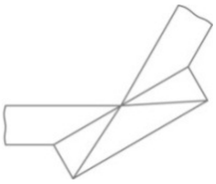
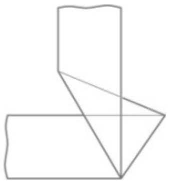
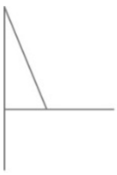
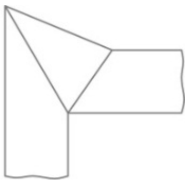
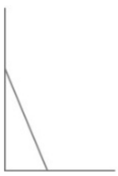
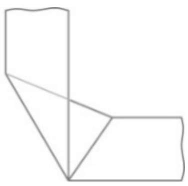
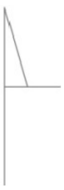
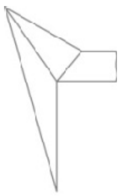
Beak family	Beak type mark	Folded position	Unfolded position	Application range	Number of overlapped panels
BPC	BPC			$90 < \varphi_0 < 180°$	6
	BPCi			$90 < \varphi_0 < 180°$	2
	BPCis			$0 < \varphi_0 < 90°$	4
DBPC	DBPC			$0 < \varphi_0 < 180°$	6
	DBPCi			$0 < \varphi_0 < 180°$	2
B90	B90			$\varphi_0 = 90°$	6
	B90i			$\varphi_0 = 90°$	2
	B90is			$\varphi_0 = 90°$	4
	B90iFS			$\varphi_0 = 90°$	2

Table 1: Various shapes of beaks.

4 THE PROPOSITIONS OF FORMS OF DEPLOYABLE STRUCTURES

In the sequel, different types of deployable structures that can find their purpose in architectural and urban design, based on the above mentioned principles, will be shown. A vaulted form can be obtained by dividing a pseudo-cylinder into halves. In the same manner, a domical structure can be generated out of pseudo-spherical forms.[1]

4.1 Pseudo-cylindrical structures

A pseudo-cylindrical form can be generated in the following ways:

(a) for $n = 4$: axial expanding of a rectangular profile (or its part)[2] (Fig. 2-a),
(b) for $n = 2k,\ k > 2$: axial expanding of a polygonal profile (or its part) (Fig. 2-b),
(c) revolving of a rectangular profile (or its part) (Fig. 2-c).

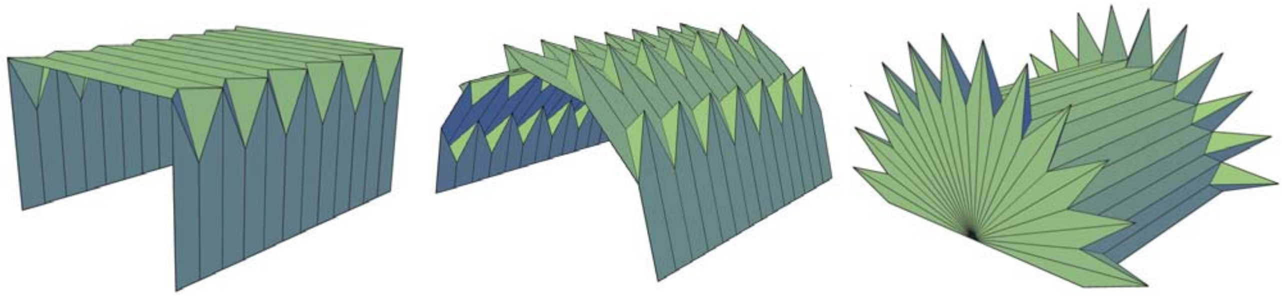


Fig. 2: Pseudo-cylindrical structures: (a) rectangular box (b) polygonal vault (axial expanding), (c) polygonal vault (revolving).

In the first case, in order to preserve the angles[3] between sides of pseudo-cylindrical, i.e. rectangular box structure, it is possible to apply various types of beaks, according to chosen materialization or desirable deployment procedure. As for thin panels, it is appropriate to apply beaks with more than two panels overlapped: *B90*, *B90is* or *DBPC*. However, as for panels with higher rigidity and thickness, it is necessary to apply inverse beaks, i.e. those with only two panels that are overlapped: *B90i* or *DBPCi*. In the second case, appropriate beaks for thin panels are *BPC*, *DBPC* or *BPCis*, whereas for the thick rigid panels the use of *BPCi* or *DBPCi* is adequate. Considering the fixed supports in third case, the application of *B90iFS* is suitable.

4.1.1 Supports displacement of a deployable rectangular box structure

From the Fig. 2-a it may be seen that a part of rectangular pseudo-cylindrical structure is assembled from lateral vertical walls and horizontal ceiling. For the preservation of the right angle between walls and ceiling *BPCis* is applied.

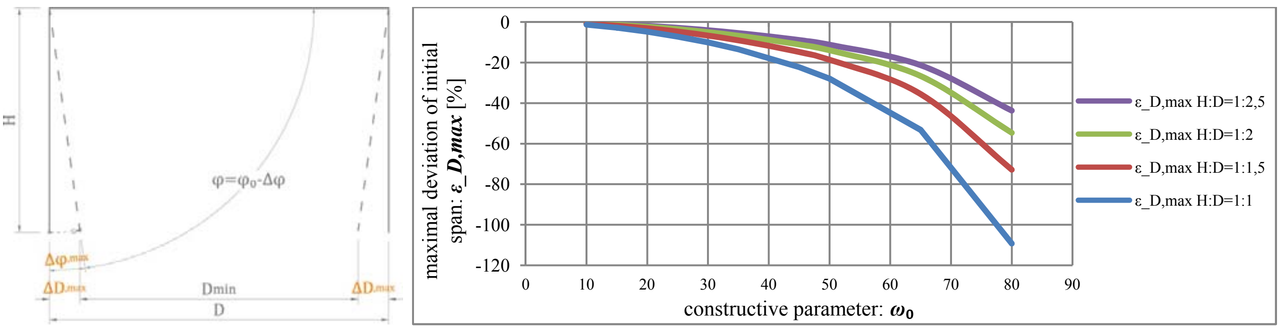


Fig. 3: Rectangular box structure: (a) schematic view of supports displacement, (b) diagram of $\varepsilon_{D,max}$ for different ratio H/D.

Maximal deviation from the right angle, during deployment, $\Delta\varphi_{max}$, depends on type of the applied beak. This implies the supports displacement, ΔD, which depends on walls height, H, and it is schematically showed in the Fig. 3-a. From the same figure it may be inferred:

$$\Delta D_{max} = H \cdot \sin \Delta\varphi_{max}$$

[1] Pseudo-sphere is spatial form which is spherical in macroscopic sense and polyhedron in microscopic sense.
[2] where n denotes the number of sides
[3] In the sense that the angle remains approximately constant during the deployment.

Since $D_{min} = D - 2 \cdot \Delta D_{max}$ it may be concluded that maximal deviation of initial span in percentage is:

$$\varepsilon_{D,max} = \frac{D_{min} - D}{D} \cdot 100 \ [\%].$$

Diagram of $\varepsilon_{D,max}$ for different ratio H/D is shown in the Fig. 3-b. The length of complete structure is equal to the sum of all segments (n_s - number of segments), i.e. $L = 2 \cdot b \cdot n_s$.

4.1.2 Supports displacement of a deployable polygonal vaulted structure

As for polygonal vaulted structure there are two possible ways of its forming:

(a) the sides' number of pseudo-cylinder[4] is divisible by 4,
(b) the sides' number is not divisible by 4.

In order to obtain symmetrical vaulted forms, only pseudo-cylindrical structures with even sides' number[5] will be taken into consideration. Between every two neighboring sides appears the deviation of initial angle and it causes the supports displacement which, thus, represents the sum of all deviations. Schematic view of these displacements is presented in the Fig. 4-a.

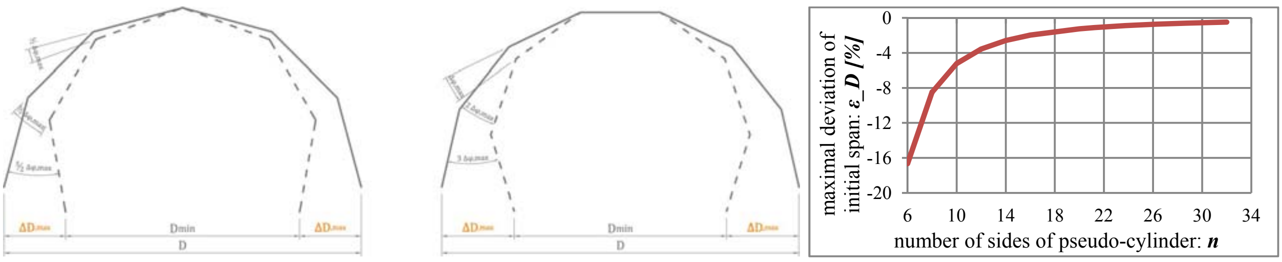


Fig. 4: Vaulted structure: (a) schematic view of supports displacement, (b) diagram of $\varepsilon_{D,max}$ for different values of n.

As for the previous case, it is possible to define the deviation of initial span during deployment. It may be inferred from the Fig. 4-b that in the case of larger number of pseudo-cylinder sides, the maximal deviation of initial span, $\varepsilon_{D,max}$, reduces to minor value.

4.2 Pseudo-spherical structures

The principles demonstrated on the examples with pseudo-cylinders can be applied for creating pseudo-spherical structures. Namely, a pseudo-spherical form can be generated by multiplying parts of a pseudo-cylinder which, in that way represent segments of a pseudo-spherical structure (Fig. 5-a).

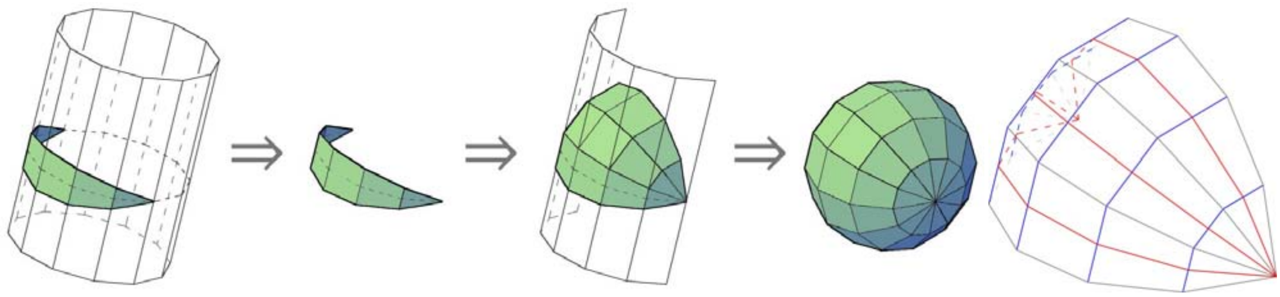


Fig. 5: (a) Generating a pseudo-sphere by multiplying parts of pseudo-cylinder, (b) foldlines (red) and beaks positions (blue).

Two major input parameters are number of sides in ground plan of dome, n_{xy}, and number of sides in vertical section, n_{yz}. By variation of diameter dimension, i.e. size of a dome, the application of this kind of structure is possible for different purposes. As it can be seen in the Fig. 5-a, the value of n_{xy} and n_{yz} is twelve.

The folding is produced along the edges marked by red lines (Fig. 5-b), while along the blue marked edges it is necessary to put beaks in order to produce deployability. In the case of panels of smaller rigidity, *BPC* or *DBPC* can be applied, while as for panels with greater rigidity it is necessary to apply inverse beaks: *BPCi* or *DBPCi*.

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[4]Sides of pseudo-cylinder form a regular polygon.
[5]It is not necessary to take a half of a polygon, but because of simplicity only this case is presented.

Two ways of beak arranging are possible: (a) in the same direction, which results in asymmetrical pseudo-sphere (Fig. 6-a) and (b) in opposite direction, which results in symmetrical pseudo-sphere (Fig. 6-b).

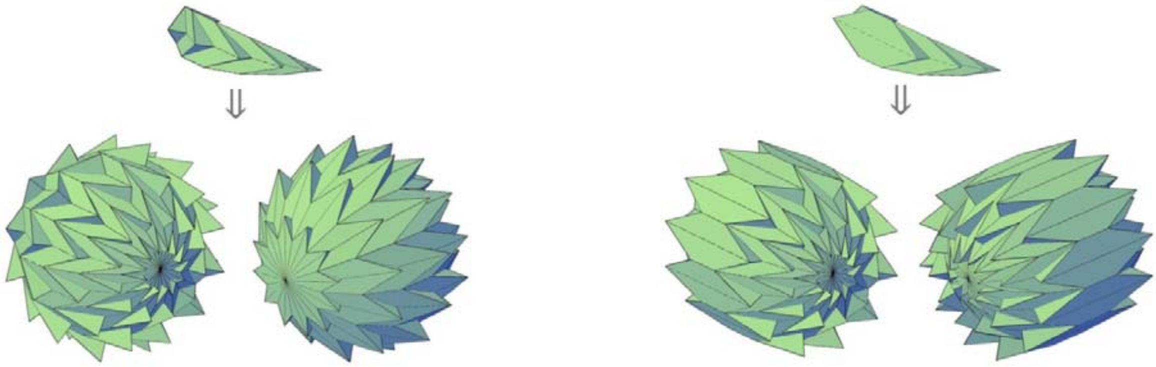


Fig. 6: (a) Asymmetrical pseudo-sphere and (b) symmetrical pseudo-sphere.

It should be noticed that forming of symmetrical pseudo-sphere requires that value n_{xy} is an even number indivisible by four. Phases of deployment of a symmetrical semi-dome with applied beaks' type *BPCi* and choose values $n_{xy} = 14$ and $n_{yz} = 12$ are shown in Fig. 7.

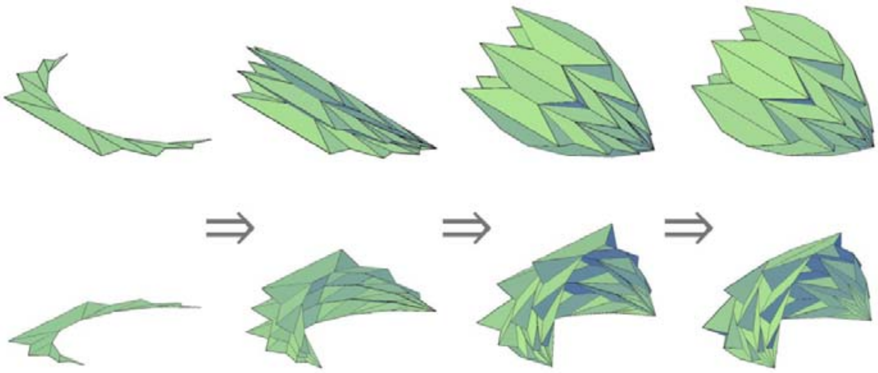


Fig. 7: Phases of deployment of asymmetrical semi-dome.

5 SELECTION OF CHARACTERISTIC PARTS OF DEPLOYABLE PSEUDO-SPHERE

Appropriate choice of part of a pseudo-sphere and the deployable axis' orientation enables creating of various forms which can be applied in architectural and urban design. Fig. 8-a shows the forming of asymmetrical dome and semi-dome with *BPC* type of beaks applied on them and chosen values $n_{xy} = 12$ and $n_{yz} = 12$. Axis of rotation is marked red. It is also possible to double certain parts of pseudo-sphere by placing one part across other, with opposite folding direction (Fig 8-b and Fig 8-c).

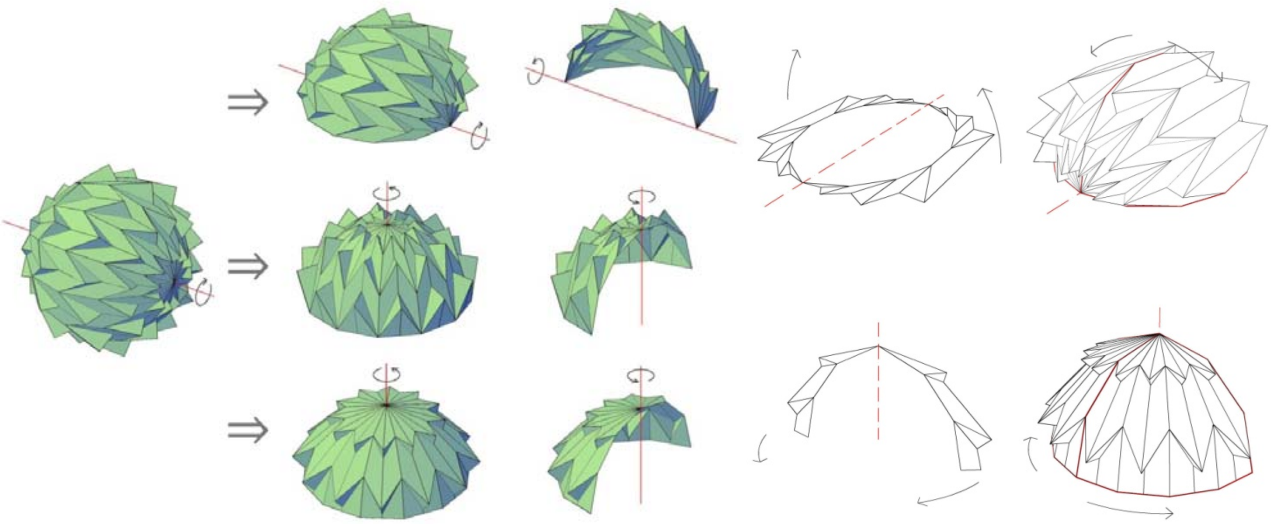
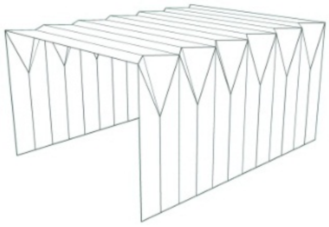
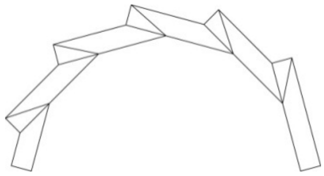
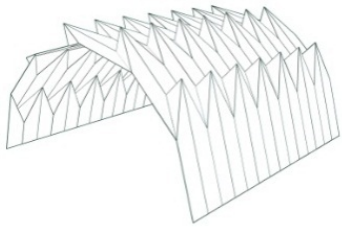
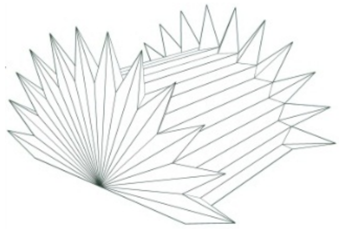
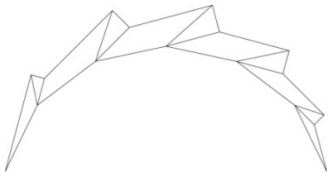
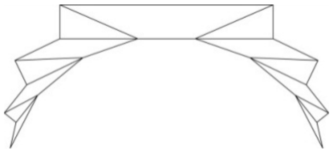
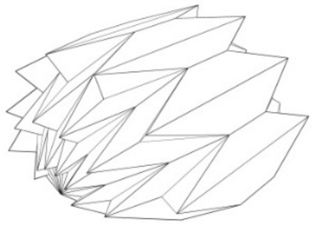
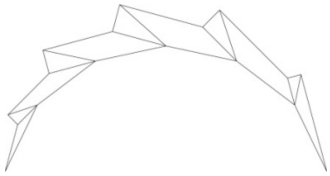
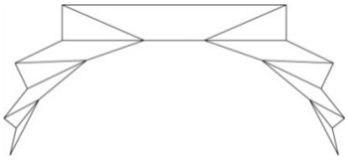
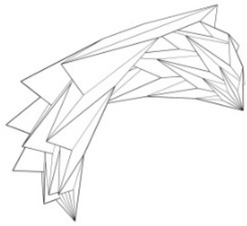


Fig. 8: (a) Proposition of forms generated from pseudo-sphere, (b) doubling of semi-dome, (c) doubling of quarter-dome.

Characteristic examples of deployable structural system are shown in Table 2.

Shape	Motion	Mark	Orthogonal view of folded position	Perspective view of unfolded position
Pseudo-cylindrical	Sliding	SPCs4		
		SPCsn		
	Revolving	SPCrn		
Pseudo-spherical	Revolving around horizontal axis	SPSa		
		SPSs		
		QPSa		
		QPSs		

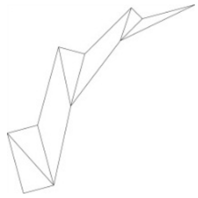
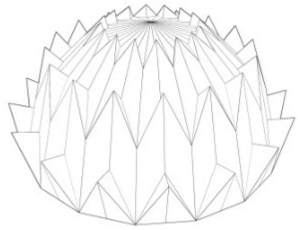
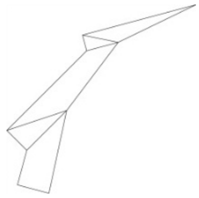
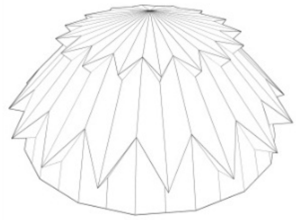
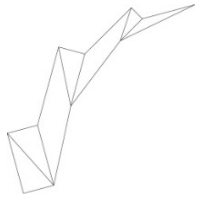
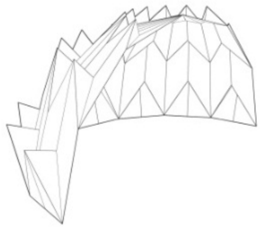
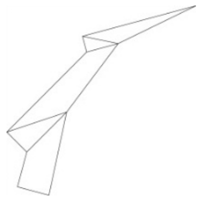
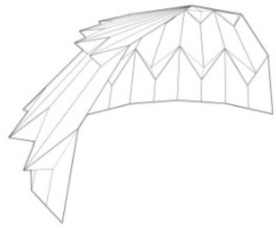
Pseudo- spherical	Revolving around vertical axis	SPSu		
		SPSd		
		QPSu		
		QPSd		

Table 2: Recapitulation of characteristic examples of deployable vaulted and domed structural systems.

In Table 2 used marks represent:

- SPCs4: Semi-pseudo-cylinder Sliding (four sided)
- SPCsn: Semi-pseudo-cylinder Sliding (*n*-sided)
- SPCrn: Semi-pseudo-cylinder Rotational (*n*-sided)
- SPSa: Semi-pseudo-sphere asymmetrical
- SPSs: Semi-pseudo-sphere symmetrical
- QPSa: Quarter-pseudo-sphere asymmetrical
- QPSs: Quarter-pseudo-sphere symmetrical
- SPSu: Semi-pseudo-sphere with beaks orientated up
- SPSd: Semi-pseudo-sphere with beaks orientated down
- QPSu: Quarter-pseudo-sphere with beaks orientated up
- QPSd: Quarter-pseudo-sphere with beaks orientated down

6 CAPPABILITY OF APPLICATION OF DEPLOYABLE SYSTEMS IN ARCHITECTURE

The application of vaulted and domed structures is widely spread in architecture. On the basis of material presented in previous chapter it is possible to talk about capabilities of deployable structures that form these shapes. Namely, regarding their movable and transformable character, their role can be dual, and only few examples will be shown.

These structures, in fully folded position, are suitable for various types of space covering. In this sense, being independent structures, they could be applied for covering public spaces in urban environment, but also they can find their purpose in deployable public umbrella, parasol or even different types of stand. However, the greatest application they can find is in the convertible roofs (see Fig. 8-b). The proposition of deployable public eaves (shelter) is presented in the Fig. 9-a. This approach enables multifunctionality and more effective use of given public space, considering theneedless of transportation and repetitive assembling. It is possible to construct this structure in the way that, in folded position, top layer is on the ground level or it can be put on the existing base which enables their eventual removing. Regarding to the rigidity of structure, there is only one degree of freedom, so the deployment could be activated by a single actuator, e.g. pneumatic or hydraulic cylinder which provides movement of the system in the whole.

Fig. 9: (a) Proposition of deployable semi-dome situated in the public space, (b) multiplication of urban pavilions.

Deployable dome could be a building in the whole, e.g. architectural pavilion. Regarding the shape transformation, they could be used for closing of commercial facilities in city center, for the purpose of winter garden. In the similar way they can be applied for swimming pools or other sport courts which requires changeable environment. By placing the rotation axis in vertical position deployable structures could find their purpose in space dividing, e.g. as folding door (see Fig. 8-c).

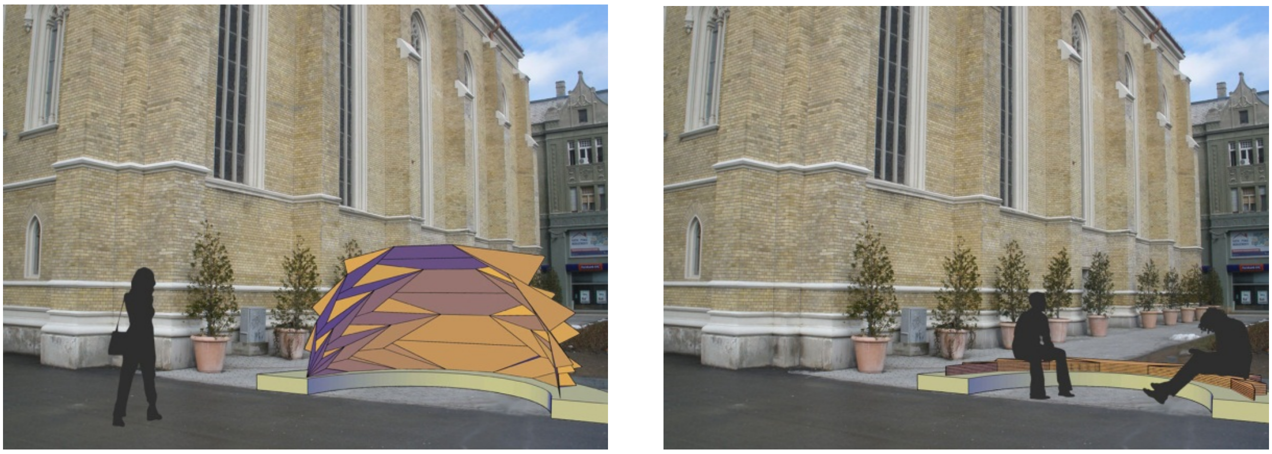


Fig. 10: Urban pavilion in (a) unfolded and (b) folded position.

In fully folded position, considering certain thickness of panels in real conditions, deployable systems can be applied in designing urban furniture, e.g. bench (Fig. 10). Regarding the small thickness of panels in the case of small spans, in folded position their total height (thickness) is not enough for the purpose of the bench, so it is necessary to provide additional pedestal. These structures have modular character, thus by their multiplying in the public space it is possible to achieve an attractive urban identity (Fig. 9-b).

7 FINAL REMARKS AND CONCLUSION

A proposition of deployable vaulted and domed structures is presented in this paper. A detailed and practical introduction to this topic is carried out. On the basis of detailed geometrical analysis the shape of the elements which enables deployment of these structures are obtained. Several problems that occur in the construction process are pointed out, and they are classified in two main cases. In the first case the assumption that the supports remain fixed during deployment is introduced, causing the deformations of some structural elements. On the basis of geometric analysis, the shape of the structure in which these deformations are reduced to small values is generated. In the second case, the supports being non-fixed allow a structure free of strain. The geometric analysis demonstrates that, by increasing the number of elements forming pseudo-sphere, the supports span is decreased, so it can be reduced to neglected magnitudes. Since large spans demand the use of rigid structural elements, as well as the materials capable to be loaded, it is emphasized that the application of the second solution is appropriate in real conditions.

By choosing adequate part of deployable pseudo-cylinder and pseudo-sphere,various formsof deployable vaulted and domed structures are obtained. Furthermore, by variation of the size of complete structure, either dimensions of the elements that form the structure, their adaption to various purposes is possible, starting from street furniture to large scale structures for covering public spaces. Several examples of the application of deployable domes are presented. First example has the purpose of the eaves of urban stage for public events, which is more rational comparing to other usual assembling systems, but also contributes to the attractiveness of urban space. The second application represents a deployable street umbrella which, in folded position, has the function of a bench. By multiplying these structures in the urban environment the special identity can be achieved. Furthermore, a deployable dome can be applied as the convertible roof structure or as a building in a whole.

Further research may be concentrated to generalizationof the geometry of deployable pseudo-spheres presented in this paper to deployable toroid. Nevertheless, the necessity of the experimental research and materialization is emphasized, in order to achieve the most rational and the most effective solution.

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