CHAOTIC BEHAVIOUR OF DAILY RAINFALL DATA IN THRACE (TURKEY)

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Abstract

Rainfall data are more sensitive to temporal and spatial variations than other climatic variables. In this study rainfall data from four different regions of Thrace, Edirne, Kırklareli, Tekirdağ and Florya (İstanbul) which observed over a period of 37 years (1970 –2006) are investigated via nonlinear time series analysis methods to understand the underlying dynamics of rainfall mechanism. The rainfall regime of Thrace can be characterized as partly Mediterranean and partly Black sea rainfall regime. The method of phase-space reconstruction is used to construct multidimensional phase – space from the scalar rainfall data. A calculated positive maximal Lyapunov exponent for each data set indicates a possible chaotic behaviour in the rainfall process.

Keywords: *Thrace. Rainfall, Chaos, Nonlinear time series, Lyapunov exponent*

Introduction

The study area is Thrace peninsula which located in the Northwest part of Turkey. Different rainfall regimes can be observed in the region (Black sea and Mediterranean rainfall regimes) (İkiel, 2005). The meteorological stations Edirne, Tekirdağ, Kırklareli and Florya (İstanbul) have been chosen to reflect this variety of regimes. The data sets are provided by Turkish State Meteorological Service and the observation period is over 37 years (1970 – 2006).

The underlying dynamics of rainfall mechanism is very complex and depends on the temporal and spatial variations. Natural systems such as rainfall are generally investigated via stochastic approaches. The visual inspection of the rainfall versus time plot shows some interesting properties of the system. A periodic and an eruptive behaviour can be considered as clues of chaotic dynamics. Nonlinear time series analysis tools are used to unravel the behaviour quantitatively. The results show that the rainfall can be considered as a chaotic system. Also some other studies (Sivakumar et al, 1999, Sivakumar, 2001) reported possible low dimensional deterministic chaos in the rainfall systems.

Data and Method

The rainfall data observed is a sequence of scalar measurements and can not represent the multidimensional phase space of the underlying dynamics. Using the method of delays, a multidimensional representation of phase space can be constructed from the invariant time series (Takens et al, 1981.). The *m* dimensional phase space vectors can be reconstructed from a one dimensional scalar time series, s_n, where n=1,2,..., N, according to

$$\mathbf{S}_n=(s_{n-(m-1)\tau}, s_{n-(m-2)\tau},\ldots, s_n)$$

Here m is called the embedding dimension and τ is the delay time. The reconstruction is valid only if the embedding dimension *m* is large enough.

There are two widely used approaches, autocorrelation function (Holzfuss et al, 1986) and mutual information method (Fraser et al, 1986.), to obtain a reasonable delay time. The mutual information of a given time series is calculated with

$$\text{MI}=-\sum_{ij} p_{ij}(\tau) ln \frac{p_{ij}(\tau)}{p_i p_j}$$

where p_i is the probability of finding a value in the i-th interval and $p_{ij}(\tau)$ is the joint probability of finding a value first in the i-th interval , and then in the j-th interval τ time later. Mutual information calculated for different values of τ gives a measurement of information shared between p_i and p_j. First local minimum of the mutual information gives a good estimation for the delay time.

The autocorrelation function is a measurement of the linear correlations between the succesive elements of a time series. The autocorrelation function is given by

$$C(\tau)=\frac{\frac{1}{N}\sum_{m-1}^{N}[s_{m+\tau}-\bar{s}\,][s_m-\bar{s}\,]}{\frac{1}{N}\sum_{m-1}^{N}[s_m-\bar{s}\,]^2}$$

where $\bar{s}$ is the mean value of the scalar measurements. The first zero crossing of autocorrelation function is used as an estimation of the delay time value.

False nearest neighbours' method (Kennel et al., 1992) is used to estimate a sufficient embedding dimension *m*. The method is based on the idea that if the phase space is embeded in a dimension that is smaller than the sufficient embedding dimension, some of the phase space points will fall into the neighborhoods of other points to which they would not be their neighbors in higher dimensions.

The existence of a positive maximal Lyapunov exponent is a very strong evidence for a chaotic behaviour. Lyapunov exponent can be defined as a measure of divergence between the nearby trajectories in the phase space. The stretching factor method (Eckmann et al., 1986) is used to calculate the Lyapunov exponent.

$$S(\Delta t) = \frac{1}{N}\sum_{n_0=1}^{N} ln\left[\frac{1}{|U_{s_0}|}\sum_{s\in u(s_0)}\left|s_{n_0+\Delta t} - s_{n+\Delta t}\right|\right]$$

Here s_{n_0} is the reference embedding vector. U_{s_0} denotes all the neighbours of the reference vector with distance smaller than a chosen ε value. The slope of the linear increase in the plot of S (Δt) versus Δt is estimated as the maximal Lyapunov exponent (Kantz, 1994).

Results

The daily rainfall data of four different locations are investigated. The data are observed over a period of 37 years (1970 –2006). Locations are choosen to represent the different rainfall regimes over Thrace region. Figure 1 shows the daily rainfall plots of the cities Edirne, Tekirdağ, Kırklareli and Florya (İstanbul). A visual inspection of the plots shows a periodic and a historical behaviour which suggest a chaotic behaviour.

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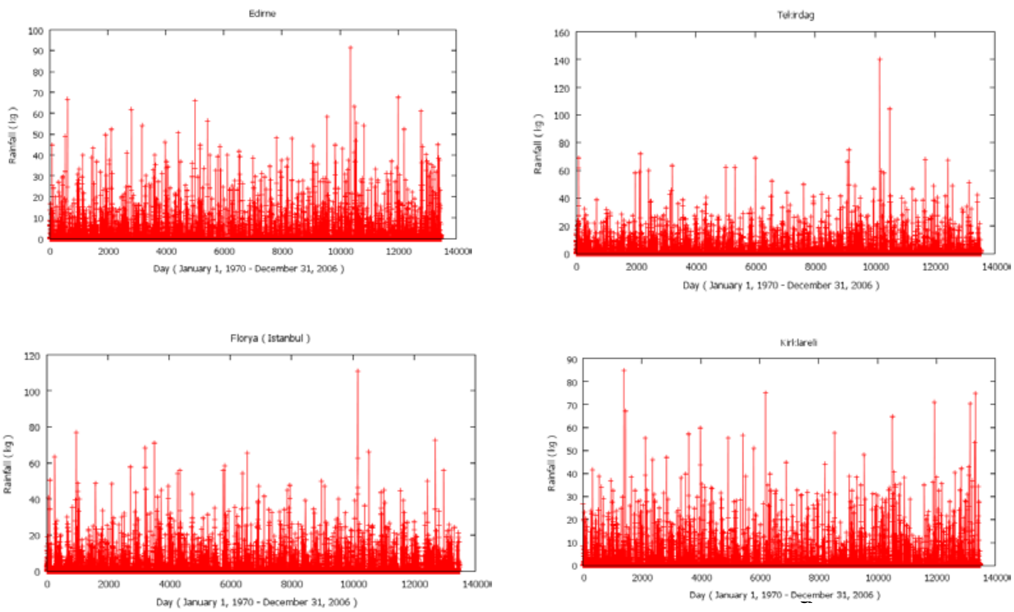

Figure1. Graphics of daily rainfall data for (a) Edirne, (b) Tekirdağ , (c) Florya, (d) Kırklareli.

TISEAN software package (Hegger et al., 1999) is used to apply the phase space reconstruction method. Some of the calculations are repeated with t series Chaos package in R. The results are shown in table 1.

Table1. Results of the daily rainfall data analysis

M. stations	Mutual Inf.	Autocorr. Func	Embeding Dim.	Lyapunov exp.
Edirne	6	19	8	0.007+-0.0015
Tekirdağ	5	29	8	0.008+-0.00066
Kırklareli	6	18	8	0.007+-0.001
Florya	4	15	8	0.023+-0.001

Figure 2 shows the average mutual information plots of the four locations. The delay time values obtained from first minimum of the average mutual information are nearly the same. The delay time values estimated from the first zero crossing of the autocorrelation function (Figure 3) also have nearly the same values. The values obtained from the autocorrelation function are approximately 4 times bigger than the values obtained from the mutual information. The delay time values estimated by the mutual information method are preferred for the further calculations due to its ability to measure the nonlinear correlations.

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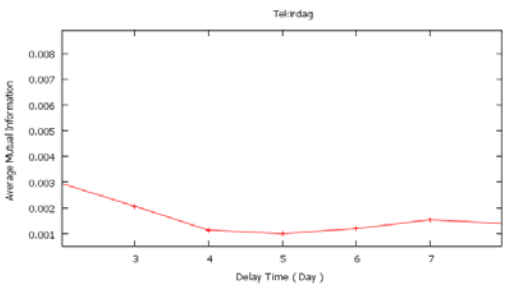

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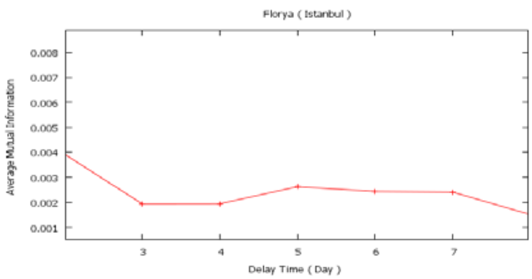

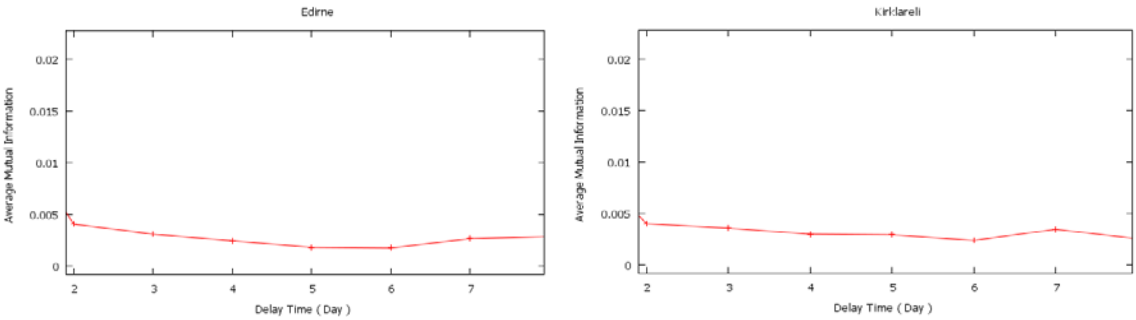

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Figure 2. Average mutual information versus delay time plots for (a) Edirne, (b) Tekirdağ , (c) Florya, (d) Kırklareli.

The embedding dimensions estimated from the false nearest neighbours method are same for all the locations. Figure 4 shows the graph of ratio of false nearest neighbors versus embedding dimension. The grahs show smiliar behaviour. Although the ratio of false neighbors does not fall to zero, it tends to become constant after 8. This effect can be a result of the noise presence in the data. Another possible reason is the zeros. Nearly %70 of the values is zero. This characteristic of the measurements can affect the algorithm which can lead to an underestimation of embedding dimension.

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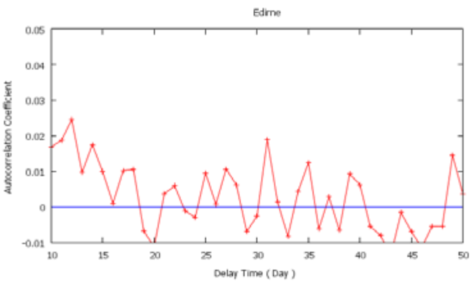

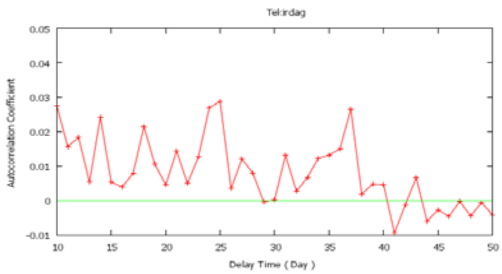

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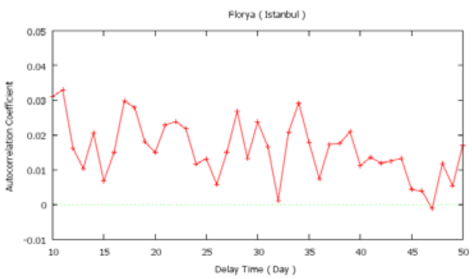

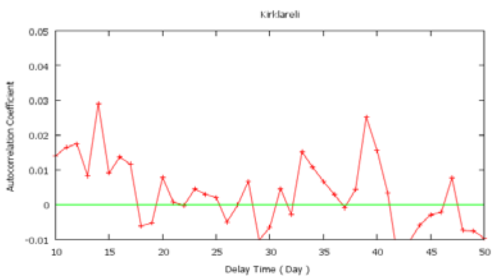

Figure3. Autocorrelation coefficient versus delay time plots for (a) Edirne, (b) Tekirdağ, (c) Florya, (d) Kırklareli.

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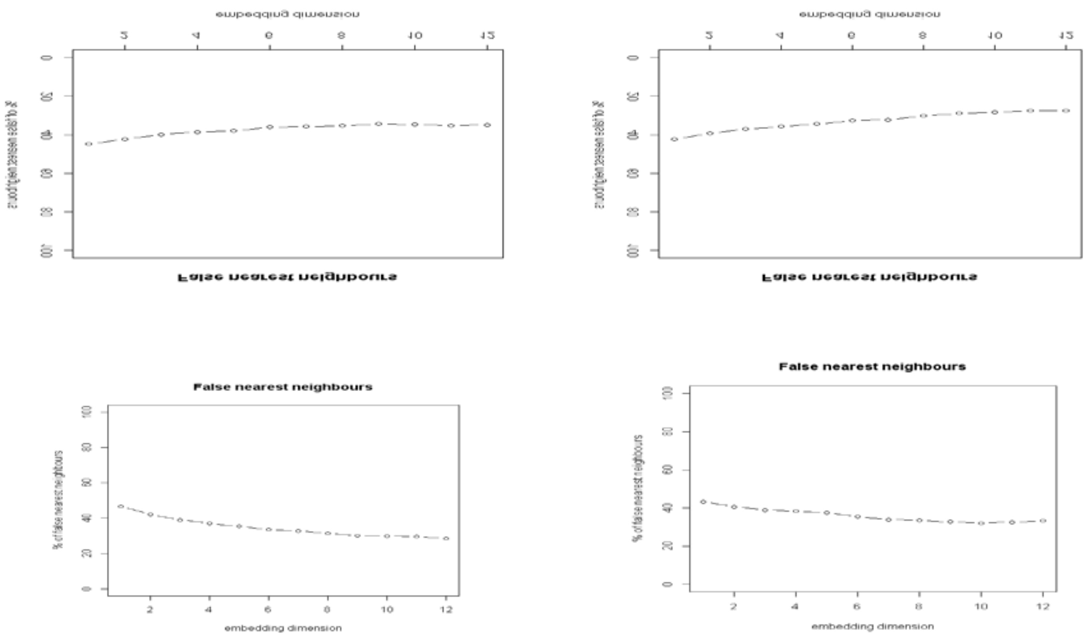

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Figure4. Graphics of ratio of false nearest neighbours versus embedding dimension

In figure 5 robust linear increasing parts of the strecthing factor graphs are shown. The maximal Lyapunov exponents are calculated from the slopes of the lines fitted to the linear increasing parts. The fluctuations in the graphs are due to the noise and folding and unfolding mechanism of the attractor.

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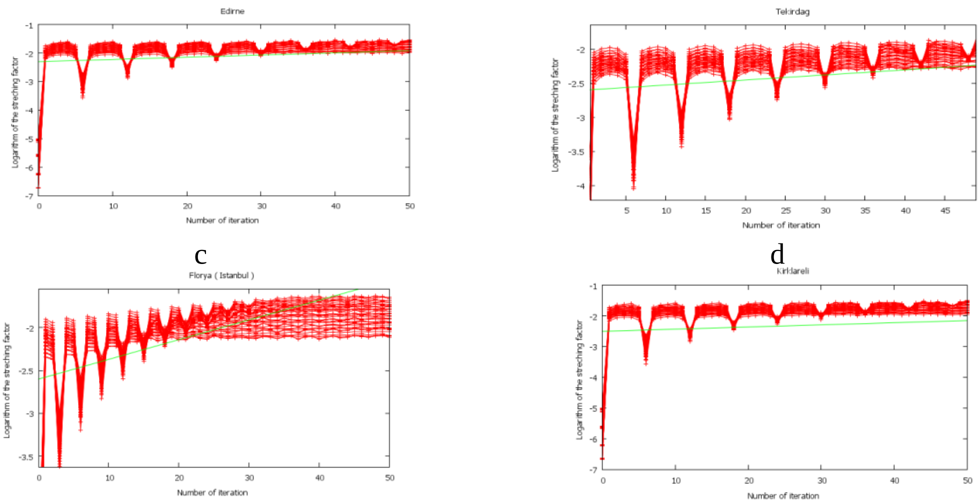

Figure 5. Logarithm of stretching factor versus number of iterations plots for (a) Edirne, (b) Tekirdağ , (c) Florya, (d) Kırklareli .

Conclusion

Natural systems exhibit complex behaviour and their data are generally considered as stochastic. In this study nonlinear time series analysis techniques are applied to the four different rainfall data observed over 37 years (1970 –2006) at Edirne, Kırklareli, Tekirdağ and Florya (İstanbul). The delay time values obtained from the autocorrelation function (19, 29, 18, and 15) and the average mutual information (6, 5, 6, and 4) are different. This can be a result of two different mechanisms underlying the rainfall phenomenon or simply because of the different nature of the two approaches. The mutual information method is sensitive to the nonlinear dependence in the time series. On the other hand autocorrelation function is a linear tool and measures the memory of the measurements.

The minimum embedding dimension obtained from the false nearest neighbours' method is 8 for all the data. An explanation for the behaviour of the graph of false nearest neighbors can be the noise in the data.

The maximal positive Lyapunov exponents are obtained for four locations. Edirne, Kırklareli and Tekirdağ have nearly the same values (~ 0.007). Florya has a bigger exponent value (0.023). These results can be regarded as an indication of possible chaotic behaviour of daily rainfall data in Thrace.

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