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Experimental Investigation of the Dip Effect in Rectangular Channel

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ABSTRACT

Two different data sets (LDA, propeller) were used to observe the occurrence of the dip effect in rectangular open channel flow for aspect ratios $1.5 \leq B/H \leq 12$. The measured and calculated velocity distributions are compared and three different situations are detected, depending on the magnitude of the aspect ratio (B/H). When $B/H \geq 5$ no dip effect was observed and the measured distributions along the cross section were in very good agreement along whole depth with the distributions as calculated by the entropy equation. When $B/H < 2$ the calculated distributions diverge from the measured ones for all vertical lines along the cross section due to the effects of free surface and sidewall. The dip effect for $2 \leq B/H < 5$ over the cross section is found only for verticals near the sidewall, where equation applicability is restricted. Herein, empirical correction for the entropy velocity distribution is proposed to account for dip effect. For validation propeller measurements were used. Relative velocity errors by adjusted equation were found to be only 32% of that by the original entropy equation for the near-surface ($z/H \geq 0.75$) region.

Keywords: Open channel flow, Dip effect, Velocity distribution, Entropy equation

INTRODUCTION

Determination of the velocity distributions over the cross-section is necessary in open channel design in order to calculate such important quantities as shear stress distributions, energy loss, sediment discharge, and turbidity. The pioneering works of Prandtl and von Karman originally gave expressions depicting the velocity distributions in circular pipes and over flat plates. In those studies, a linear velocity distribution was found to prevail in the laminar sub-layer close to the solid boundary in which the viscous forces were dominant whereas, away from the boundary where the turbulent effects became superior, the velocity distributions were observed to obey such laws as the velocity defect law or the law of the wall.

Chiu and Chiou [1] have developed a two dimensional mathematical model of flow in open channels which does not require velocity data. They asserted that the model has answered some of the major questions that existed in open channel hydraulics. Equations based on the entropy concept have been derived for describing the two-dimensional velocity distribution in an open channel cross section by Chiu [2, 3 and 4]. The Entropy function contained a parameter (M), which has been found to be useful as an index for characterizing and comparing various patterns of velocity distribution, and states of open channel flow systems. Chiu and Tung [5] also studied the location of the maximum velocity as a function of M. They also analyzed a large quantity of laboratory and field data and concluded that M is stable and constant at a channel section, invariant with time and discharge.

Xia [6] found the relation of the maximum velocity to the cross-sectional mean velocity on different straight reaches in the Mississippi River to be linear. Araujo and Chaudhry [7] found that the entropy model performs better than the logarithmic model, not only in general terms but also in practically all flow regions, especially those near the channel bed. Moramarco et al. [8] investigated the mean velocity in natural channels based on Chiu's velocity distribution equation. They proposed a simple method for estimating the velocity profiles at a river section. Ardıçlıoğlu et al. [9] investigated that applicability of logarithmic and entropy-based velocity distribution equations for rough-surface open-channel flow, with parameter M calculated with the original Chiu's formulation.

Owing to the presence of a free surface and to the friction along the channel wall, the velocities in a channel are not uniformly distributed in the channel section. The measured maximum velocity in ordinary channels usually appears to occur below the free surface at a distance of 0.05 to 0.25 of the depth; the closer to the banks, the deeper it is Chow [10]. This phenomenon, known as the 'velocity-dip', has been investigated both experimentally and theoretically in this paper.

Sarma et al. [11] studied velocity distributions in a smooth rectangular channel by dividing the channel into four regions. They suggested that the Froude number and aspect ratio might affect the magnitude of the dip and maximum velocity in the vertical velocity profile, and for the lateral velocity distribution in the outer region of the sidewall away from the bed. Tominaga and Nezu [12] investigated the 3-D turbulent structure of compound open channel flows experimentally and observed the maximum velocity tends to appear below the free surface even in the central region of the main channel. Nezu et al. [13] took velocity measurements in two different natural channels using three-component electromagnetic flow meters and observed that multicellular secondary currents were evident in a wide river, whereas large-scale free surface secondary currents associated with velocity dip phenomena were generated in a narrow river. Kırkgöz and Ardıçlıoğlu [14] used a Laser Doppler Anemometer to measure velocities in smooth open channel flow. They observed that, because of the secondary currents generated by the combined action of the bed and sidewall, the dip effect increases steadily toward the sidewall.

The objectives of this paper are to: 1) experimentally evaluate the velocity profiles at different distances from the centre vertical, and to investigate what controls the dip effect; and 2) to assess the applicability of the entropy equation near the flow surface.

EXPERIMENTS

In this study two different data sets are used. One of them used a Laser Doppler Anemometer (LDA) from Ardıçlıoğlu [15] at the Cukurova University in Turkey (KA1-12), and the second set of measurements used a propeller meter in the laboratory channel at Erciyes University, again by Ardıçlıoğlu (P1-6). The first group of data (LDA) was also used for calibration of the entropy equation parameters, whereas the second group (propeller) was used to validate the proposed empirical equation, concerning the dip effect. The flow characteristics are summarized in Table 1. As shown in Table 1 Q is the discharge, H is the flow depth, B/H is the aspect ratio, Fr ($=\bar{u}/\sqrt{gH}$) is the Froude number, Re ($=4\bar{u}R/\upsilon$) is the Reynolds number, R is the hydraulic radius, $\bar{u}$ is the mean velocity was calculated by integrating point velocity measurements over the cross-section and u_{max} is the maximum velocity of cross section. Point velocity measurements were taken different vertical lines as shown in Figure 1. Because of the symmetry, the velocity measurements were taken only on one side of the cross-section.

Table 1 Flow characteristics for measurements

Test	Q	H	B/H	Fr	Re	$\overline{u}$	u_{max}
No	(m^3 /s)	(mm)	(-)	(-)	(-)	(m/s)	(m/s)
1	2	3	4	5	6	7	8
KA1	0.0032	50	6.00	0.22	2.05E+04	0.156	0.192
KA2	0.0032	25	12.00	0.70	2.59E+04	0.345	0.420
KA3	0.0060	75	4.00	0.37	5.57E+04	0.318	0.362
KA4	0.0060	60	5.00	0.55	6.38E+04	0.425	0.493
KA5	0.0060	45	6.70	0.67	5.36E+04	0.442	0.535
KA6	0.0100	100	3.00	0.36	7.40E+04	0.352	0.389
KA7	0.0100	60	5.00	0.75	8.68E+04	0.578	0.654
KA8	0.0145	200	1.50	0.17	7.33E+04	0.244	0.280
KA9	0.0145	80	3.75	0.76	1.22E+05	0.670	0.776
KA10	0.0195	150	2.00	0.38	1.21E+05	0.459	0.520
KA11	0.0195	120	2.50	0.55	1.40E+05	0.598	0.675
KA12	0.0195	100	3.00	0.69	1.43E+05	0.679	0.760
P1	0.0058	97	6.20	0.10	2.57E+04	0.100	0.130
P2	0.0141	127	4.70	0.17	5.79E+04	0.185	0.217
P3	0.0156	120	5.00	0.20	6.53E+04	0.217	0.290
P4	0.0213	144	4.20	0.21	8.40E+04	0.246	0.291
P5	0.0286	165	3.60	0.23	1.08E+05	0.289	0.340
P6	0.0356	215	2.80	0.19	1.21E+05	0.276	0.313

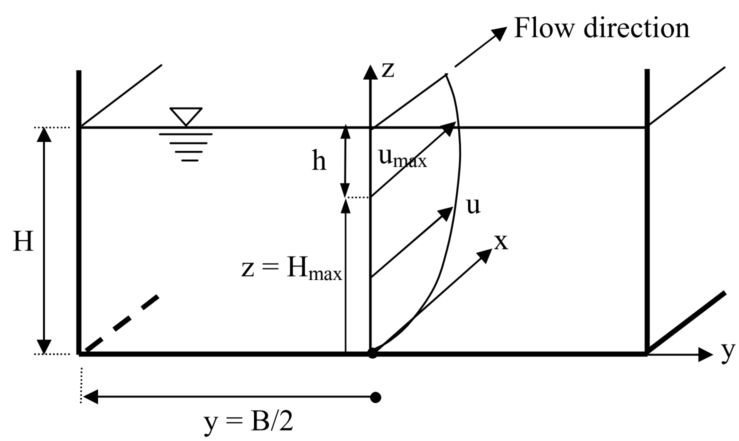

Figure 1 Definition sketch showing notation for velocity distribution

ENTROPY EQUATION FOR VELOCITY DISTRIBUTION IN OPEN CHANNELS

By probabilistic formulation and entropy maximization, Chiu [4] derived a two-dimensional velocity distribution in the form

$$u = \frac{u_{max}}{M} \ln\left[1 + \left(e^{M} - 1\right)\frac{\xi - \xi_0}{\xi_{max} - \xi_0}\right] \tag{1}$$

where u=velocity in the longitudinal direction; $(\xi - \xi_0)/(\xi_{max} - \xi_0)$ represents the cumulative probability distribution function, in which $\xi(y, z)$ is the curvilinear coordinate associated with the isovels; $\xi_{max} = \xi$ at the point where u_{max} occurs; $\xi_0 = \xi$ at the channel bed where u=0. On the vertical axis, called "z axis" hereafter, on which the maximum velocity u_{max} occurs, ξ may be expressed (Chiu and Said [16]) as a function of z as

$$\xi = \frac{z}{H - h}\exp\left(1 - \frac{z}{H - h}\right) = \frac{\frac{z}{H}}{1 - \frac{h}{H}}\exp\left(1 - \frac{\frac{z}{H}}{1 - \frac{h}{H}}\right) \tag{2}$$

in which z = vertical distance from the bed. In Eq. 1 M is the entropy parameter and is defined as a function of the ratio between the mean $\bar{u}$ and maximum velocities u_{max}

$$\frac{\bar{u}}{u_{max}} = \phi = \frac{e^{M}}{e^{M} - 1} - \frac{1}{M} \tag{3}$$

Moramarco et al. [8] adjusted Eq. 1 to vertical profiles, generating Eq. 4, which can be applied to any vertical, given u_{max} of that specific vertical.

$$u_i = \frac{u_{max_i}}{M_i} \ln\left[1 + \left(e^{M_i} - 1\right)\frac{z}{H_i - h_i}\exp\left(1 - \frac{z}{H_i - h_i}\right)\right] \quad i = 1,2,\ldots, N_v \tag{4}$$

where u_i and H_i = velocity and water depth along the i^{th} vertical, respectively; z = vertical distance measured from the channel bed; N_v = number of verticals sampled in the cross-sectional flow area; and M_i, h_i, and u_{max}, are parameters.

RESULTS AND ANALYSIS

Figure 2 shows the relationship of the maximum velocity (u_{max}) versus the cross-sectional mean velocity ($\bar{u}$) obtained experimentally in both open channel sets of measurements (Table 1). It includes two straight lines, each relating u_{max} to $\bar{u}$ in one of the two channels. The M value for both LDA and propeller measurements were determined to be 7.6 and 6.0, respectively, as shown in Figure 2.

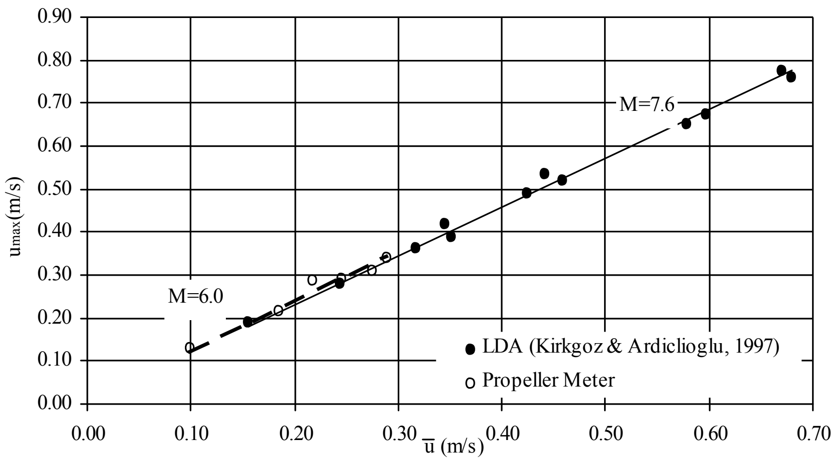

Figure 2 Relation between u_{max} and $\bar{u}$ based on experimental data

The 12 LDA experiments (KA1 to KA12) were compared with the entropy Eq. 4 (using constant M) especially near the surface, so as to assess the equations applicability concerning dip effect. When $B/H \geq 5.0$, the measured distributions along the cross section were in very good agreement near the water surface with the distributions as calculated by the entropy Eq. 4. In the case of narrow open channel flows ($B/H < 5.0$), the maximum velocity occurs below the free surface in some verticals. The creation of the dip in the velocity profiles across the channel is attributed to the secondary currents generated by the combined action of the bed and sidewall. The same trend, i.e., that dip effect can be observed only in verticals near the sidewalls, could be noticed for the six experiments with $2.0 \leq B/H < 5.0$ (KA3, 6, 9, 10, 11, 12). When $B/H < 2.0$, the calculated distributions diverge from the measured distributions for all verticals. The dip effect is observed to increase (i.e., maximum velocity position gets lower) with the proximity to the sidewall.

The applicability of the entropy Eq. 4 was assessed for the LDA experimental set (Table1) as a function of the aspect ratio (B/H) and the relative location of the vertical (2y/B), as shown in Figure 3. The relative error between measured (u_{meas}) and calculated (u_{cal}) velocity was defined as $\varepsilon = |u_{meas} - u_{cal}| / u_{meas}$. It was used to assess the prediction capability of Eq. 4 for the near-surface region ($z/H \geq 0.75$). Three different situations were observed: first, when aspect ratio (B/H) ≥ 5.0, there is no dip effect and Eq. 4 is found to be applicable throughout the whole section. The mean relative error of velocity distribution for $B/H \geq 5.0$ was calculated as 0.5%.

When $2.0 \leq B/H < 5.0$, no dip effect occurred for $2y/B < 0.80$, mean relative error was calculated as 0.3%, and Eq. 4 is applicable. When $2y/B \geq 0.80$, mean error increased 12 times (3.6%) for $z/H \geq 0.75$ region due to the dip effect. So, above the line in part II, entropy Eq. 4 was found not to be applicable, because it does not consider dip effect. Araujo and Chaudhry [7] and Moramarco et al. [8] determined that Eq. 1 performed better in the middle portion of the flow area, whereas in the regions close to the sidewall and near the surface, the velocity was poorly estimated due to the dip effect.

When the aspect ratio $B/H < 2.0$, the calculated distributions by Eq. 4 diverge from the measured distributions for all the vertical lines and mean error was found to be 4.4% for $z/H \geq 0.75$ region. Eq. 4 must be rearranged to consider dip effect otherwise its applicability near the surface can be questioned.

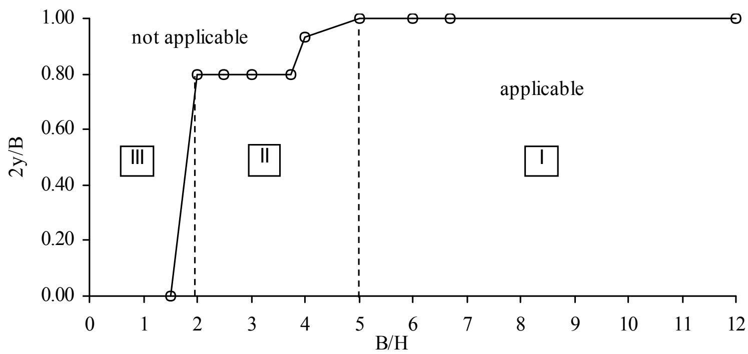

Figure 3 Applicability of entropy Eq. 4 as a function of aspect ratio and location of the vertical according to 12 LDA experiments (Table 1)

It is known that some velocity profiles, such as log law, are not applicable in the outer region, particularly in narrow open channels. As defined above, Chiu's modified Eq. 4 for velocity distribution also does not agree with measured data especially near the surface for aspect ratio (B/H) < 5.0. Wherever the maximum velocity u_{max} in a vertical profile occurs at a vertical distance h below the water, dip effects must be taken into consideration. For this purpose the term (H-h) in the right hand side of Eq. 4 should not be taken as a constant, as depicted by Moramarco et al. [8]. A variable fictitious depth (d) should be computed for every point where $z \geq H_{max}$. Eq. 4 (using M constant) can be rewritten to consider the dip effect as:

$$u_i = \frac{u_{max_i}}{M} \ln\left[1 + \left(e^M - 1\right)\frac{z}{d_i}\exp\left(1 - \frac{z}{d_i}\right)\right] \quad i = 1,2,....,N_v \qquad (5)$$

If $h_i = 0$ (no dip effect), such as in Eq. 4, Eq. (6a) should be used.

$$d_i = H_i - h_i \qquad (6a)$$

Otherwise ($h_i > 0$, dip effect observed), Eq. (6b) must be considered for determining d_i .

Using the LDA measurements where dip effect occurs, the relationship between d_i/H_{maxi} and z/H_{maxi} exhibited a linear relationship, as shown in Figure 4. Eq. 6 has a determination coefficient of $R^2 = 0.87$.

$$d_i = H_{max\,i}\left[2.14 - 1.18\left(\frac{z}{H_{max\,i}}\right)\right] \qquad (6b)$$

Velocity distributions along verticals beyond H_{max} can be calculated by Eq. 5 with the help of Eq. 6 for 'd_i' computation. Using Eq. 5, velocity distribution was calculated and results are given in Figs. 4 and 5. Whenever the dip effect is considerable, Eqs. 5 exhibit much better agreement with the observed velocities than Eq. 4. Mean relative error for Eq. 4 (ε_4) was calculated as 4.0% for $z/H \geq 0.75$ region. Using Eq. 5, mean relative error (ε_5) between measured and calculated velocities, was found to be 1.5%.

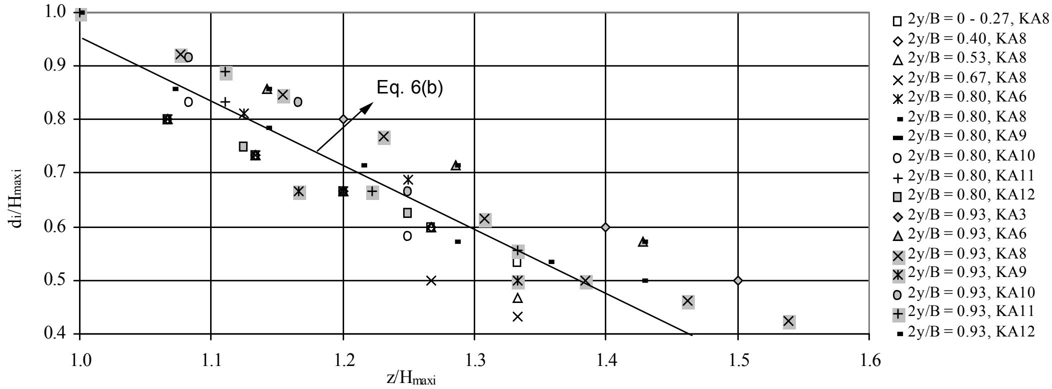

Figure 4 Relationship between d_i/H_{maxi} versus z/H_{maxi} for LDA measurements

For validation of Eq. 5 other measurements taken by propeller (Table 2) were used. The relative error (ε_4) between measured and calculated velocity by Eq. 4 was calculated for $z/H \geq 0.75$ region and aspect ratio $(B/H) \geq 5.0$. It was found to be 1%, higher than with LDA measurements, but accurate enough for acceptance. So, in this region, entropy Eq. 4 was also found applicable, as observed with LDA measurements. When $2.0 \leq B/H < 5.0$, dip effect was observed especially in last two verticals and mean relative error (ε_4) near the surface was 10% for Eq. 4. In Figure 5, P6 measurements were given as a sample. Velocity distributions along verticals beyond H_{max} were calculated by Eq. 5 and results are shown in Figure 5. Eq. 5 exhibits a much better agreement with the observed velocities for last two verticals, than Eq. 4 (Figure 5). Mean relative error between measured and calculated velocities with Eqs. 4 and 5 respectively were found 10% and 2% for $z/H \geq 0.75$ regions. This means Eq. 5 gives better results than Eq. 4 for flow regions directly affected by secondary flows.

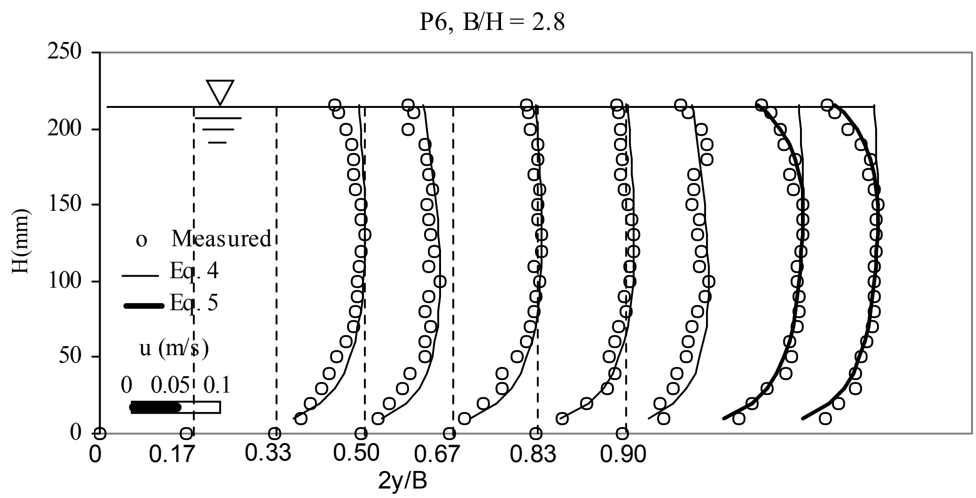

Figure 5 Velocity distributions across sidewall for measurements P6 (B/H = 2.8)

As an example of Eq. 5 applicability, two vertical profiles close to the sidewall ($2y/B = 0.90$) are given in Figure 6. For experiment P1, where dip effect did not occur, (aspect ratio $B/H = 6.2$) Eq. 4 yielded accurate velocity distribution, matching the experimental data. When $B/H < 5.0$, the dip effect strongly influences velocity profiles near sidewall as shown for the experiment P5 ($B/H = 3.6$) where the measured distribution diverge from those computed by

Eq. 4. Use of Eq. 5, on the other hand, reveal accurate agreement with the experiments. In this measurement (P5, 2y/B = 0.90) mean relative error between measured and calculated velocities with Eqs. 4 and 5 respectively were found to be 17% and 3% for $z/H \geq 0.75$ regions.

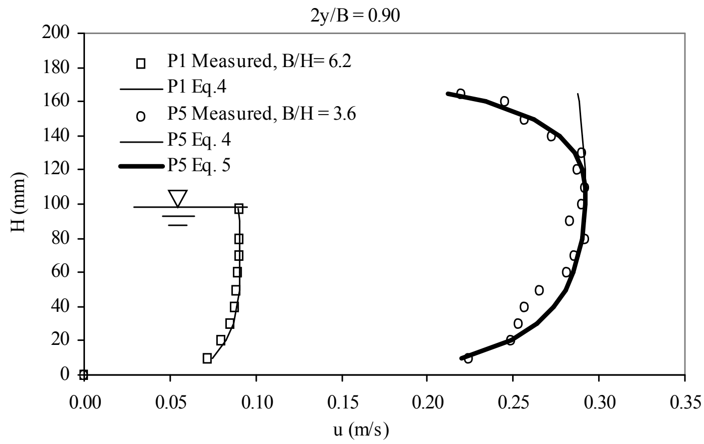

Figure 6 Velocity distributions measured and calculated for test P1 and P5

When propeller measurements are considered for aspect ratio $B/H < 5.0$ (P2, 4, 5, 6), dip effect occurred near the sidewall and mean relative error (ε_4) was calculated as 11.0% for $z/H \geq 0.75$ region. Velocity distributions along verticals beyond H_{max} were calculated by Eqs. 5 exhibit a much better agreement with the observed velocities near the surface for last two verticals, with mean relative error (ε_5) 3.5%.

CONCLUSIONS

The secondary currents generated by the combined action of the bed and sidewall influences flow velocities near the surfaces. This phenomenon, known as the 'velocity-dip', was investigated using velocity measurements in smooth open channel flows for aspect ratios $1.5 \leq B/H \leq 12$. When $B/H \geq 5$, the velocity distributions along the cross section increase upward in rectangular channels, and u_{max} occurs on the surface. In the case of narrow open channel flows ($B/H < 5$), the maximum velocity occurs below the free surface in some verticals. When $B/H < 2$, u_{max} was observed to occur below the free surface. The dip effect is observed to increase with the proximity to the sidewall. The applicability of the entropy equation (Eq. 4) for the near-surface region ($z/H \geq 0.75$) was assessed. For this purpose, the LDA experimental set was used. When aspect ratio (B/H) ≥ 5, entropy equation is found to be applicable throughout the whole section. When $2 \leq B/H < 5$, no dip effect occurred for $2y/B < 0.80$, and entropy equation is also applicable, but near the side wall ($2y/B \geq 0.80$), original entropy equation was not applicable due to dip effect. When the aspect ratio $B/H < 2$, the calculated distributions diverge from the measured distributions for all the vertical lines for the near-surface region. Herein, empirical adjustment for the entropy velocity distribution (Eq. 5) is proposed to account for dip effect, using the LDA data set for parameters calibration. Velocity distributions for the near-surface region were calculated by both Eqs. 4 and 5 and then compared with propeller measurements data set for validation purpose. The results showed a much better agreement of Eq. 5 with the observed velocities near the surface with mean relative error (ε_5) 3.5%, against Eq. 4 mean relative error of 11%.

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