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The Analysis and the Design of the Reinforced Concrete Elements and the Algorithm in the Calculation of these Elements under the Shear Forces

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ABSTRACT

During the process of the design for the reinforced concrete elements besides the calculation of the bending moment, another important calculation is the one made for these elements when they are under the shear forces. This calculation is made in the ultimate limit state, SLU. This paper will present all theoretical bases and algorithm calculations of reinforced concrete elements under the shear forces, based on the European rules (Eurocodes) and mainly EC2 and EC8. The study of reinforced concrete elements under the shear forces is complex process. It cannot be sufficient the analysis of one section in the study of reinforced concrete elements, in the contrary of what happens usually. The analysis of the reinforced concrete structures under the shear forces should be made by analyzing all its elements entirely. The behaviour of the reinforced concrete elements in its plastic phase is complicated and it directly affects in the size of the bearing capacity of element in the shear forces. In the size of the bearing capacity of element in the shear forces also the quantity of the longitudinal reinforcement, the quantity of transversal reinforcement, the type and the position of forces, the form of the transversal section, etc. The effect of the transversal (shear) forces is important in studying the behaviour of the element. The analysis is made by approximating the element with a truss, where the tension elements of this truss are represented by the reinforcement and the pressed elements are represented by the press concrete. The factors that mostly affect the capacity of the reinforced concrete element shear process are the longitudinal quantity of reinforcement, the size of gravel and the transversal size of the element. In this paper will be presented several numerical examples and the algorithm for calculating the concrete elements from shear forces.

INTRODUCTION

This study is based on European rules for the design of structures (Eurocodes). Especially are used the recommendations and formulas presented in Eurocode 2 (EC2) and in Eurocode 8 (EC8).

In addition of calculation of the bending moment, one of the most important calculations performed during the design of reinforced concrete elements is the calculation from shear forces. This calculation is included in the ultimate limit state of the element, SLU. Based in the European rules (Eurocodes), mainly in EC2 and EC8, we tried to present the theoretical bases and the algorithm of calculation of the reinforced concrete elements under the shear forces.

The study of reinforced concrete elements under the shear forces is complex process. It cannot be sufficient the analysis of one section. The analysis of the reinforced concrete structures under the shear forces should be made by analyzing all its elements entirely. The behaviour of the reinforced concrete elements in its plastic phase is complicated and it directly affects in the size of the bearing capacity of element in the shear forces. In the size of the bearing capacity of element in the shear forces also the quantity of the longitudinal concrete, transversal concrete the type and the position of forces, the form of the transversal section and other elements. The effect of the transversal (shear) forces is important in studying the behaviour of the element. The analysis is made by approximating the element with a truss. The pressed horizontal elements are represented from the pressed concrete while the pressed diagonals are represented from the inclined pressed concrete. The tension verticals are represented from the vertical transversal reinforcement. The tension horizontal elements are represented from the reinforcement. Approximation of the concrete element with a truss leads to the underestimation of the shear capacity of the element. Therefore, in order to avoid the inaccuracies, EC2 provides the forecasting of the bearing capacity of concrete for the element, that depends on the transverse section dimensions of the element and from the class of the concrete. EC2 also takes in the consideration the amount of the steel that the element contains.

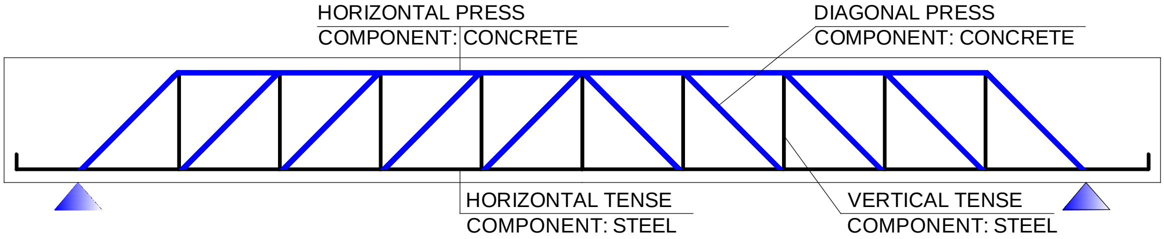

Fig. 1

Due to the simplifying treatment and also the uncertainties that accompany the European rules (Eurocodes), not taken into consideration the bearing capacity of the concrete in an element that equipped with a transversal shear reinforcement. The main factors that affect the bearing capacity of the element from the shear forces are: longitudinal reinforcement, the size of the gravels, the dimensions of the transverse section of the element, the direction of action of the main forces. When shear forces are small the value of the restrains does not exceed the tension resistant of the concrete. As a consequence the element does not crack. The cracking depends from the presence of the reinforcements in the element. The failure of the element from shear forces may happen in two ways:

- The collapse of the pressed elements. The crush is fragile. This kind of demolishment should be avoided with any cost.
- The collapse of the tense elements that are represented from the steel reinforcement. The demolishment is ductile.

We emphasize that the first type of failure is very dangerous and should be avoided with any cost. A good designed element from the action of shear forces must meet this requirement: the shear bearing capacity of the reinforcement must be smaller than the respective capacity of the concrete. In general, in the element should be putted a minimal reinforcement quantity, which in some cases can be avoided for example in plate or in the slabs.

Under the action of different static charges, might they be temporary or permanent, the cracks that are created in element are approximately as shown in Fig. 2.

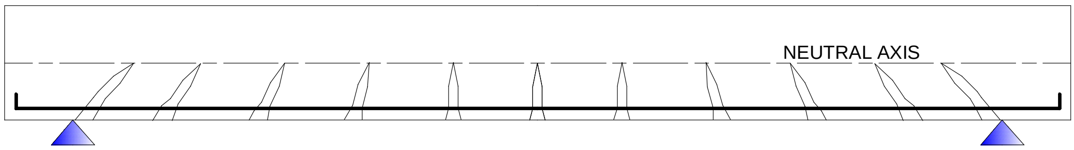

Fig. 2

In the middle of the spaced of the element the cracks are vertical. Moving from the space between in the element forward the supporting axes the cracks inclines and near the supporting axes their incline is nearly 45°. In the frame of the concrete element that works in flexibility is formed a symmetrical arc as shown in Fig. 3.

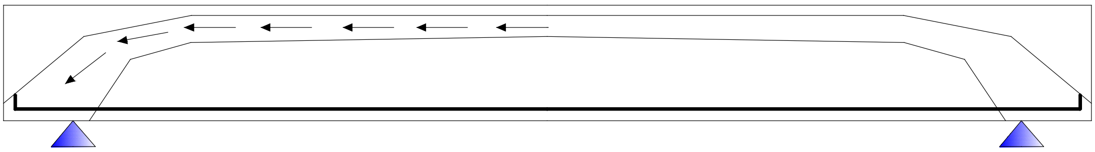

Fig. 3

As mentioned above the bending reinforced concrete element is approximated with a truss that contains diagonals in pressure (are formed from the pressed concrete), diagonals in the tension (are formed from the reinforcement), verticals in the tension (stirrups), the horizontal element in pressure (are formed from the pressed concrete), the horizontal elements in tension (are formed from the longitudinal reinforcement). In order to simplify the solution of the problem is accepted the fact that the pressed diagonals form in the horizontal axe the angle of 45°. This simplification gives reasonable and acceptable results. The pressing force of the pressed diagonal, dissolved in two components (vertical and horizontal) causes a vertical and horizontal sliding action as shown in the Fig. 4.

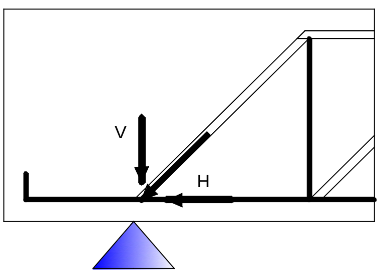

Fig. 4

The shear force causes a slide in an area with a length 0.9d (d is the working height of the transversal section of the reinforced concrete element). This slide is prevented (objected) from the interaction of the upper and lower part of the element.

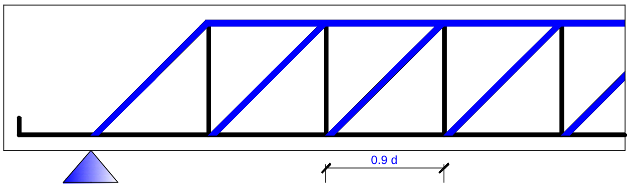

Fig. 5

ANALYSIS

Let's analyses the behavior of the reinforced concrete element that works in flexibility (beam), which does not contain shear transversal reinforcements. The element is under action of two charges that are placed symmetrically as shown in the Fig. 6.

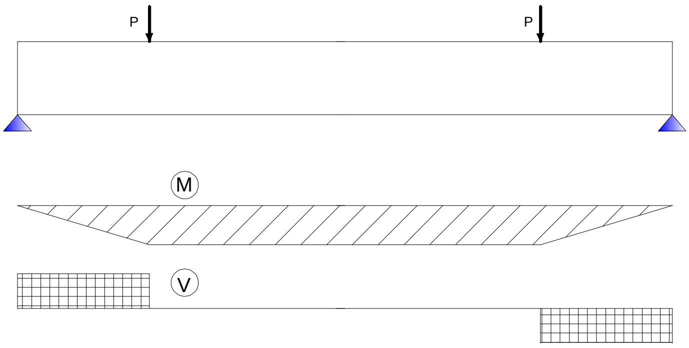

Fig. 6

In the middle part, the element (beam) is under action of a constant bending moment, without the presence of the shear forces. In the two supporting sides the element is under action of a constant shear force and of a bending moment which changes based of a low of first scale. The element (beam) under the action of a load will crack not only in the middle part but also in the areas near the supporting axes. In these parts the transmitting of the shear forces is carried out through the support of three mechanisms that act simultaneously:

- The bearing capacity in shear of the pressed area over the cracked
- The rubbing effect of the gravels of the concrete during the cracks
- The carrying effect that is offered from the longitudinal reinforcement in the cracks

The area of the element that is included between two successive cracks be having as a vertical fixed cantilever in the pressed cracked area. This cantilever is under the action of a force $\Delta \mathbf{F_t}$, where $\Delta \mathbf{F_t}$ is equal with the difference of the tension forces in the reinforcement in the two cracks, as shown in Fig. 7.

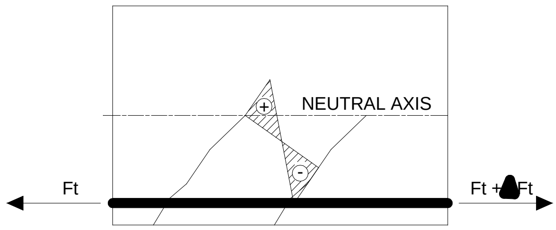

Fig. 7

With the increasing of the active loads in the element, comes a moment when the bending moment in the fixing point of the cantilever mentioned above, cancels the effect of the friction concrete granules. Thereafter remains the other two mechanisms which are not able to carry large loads. The failure comes with the forming of an inclining crack that forms an angle approximately 45°, which always enter deeper in the tension area of the cantilever that correspond with the pressed areas of the element over the neutral axis. The destruction can be presented in two ways:

- The incline crack enters in the upper pressed part of the element
- The press force is parallel transmitted with the cracking, acting from the top dawn and exercising tension force over the longitudinal reinforcement. If the longitudinal reinforcement is not very well join with the concrete and is not in the right quantity than the collapse happens rapidly.

The shear bearing capacity of a reinforced concrete element that works in flexibility (beam) is given by a formula that is taken from Eurocode number (EC2), that takes into consideration all the above.

$$V_{Rd,c} = \left[C_{Rd,c}k\left(100\rho_l f_{ck}\right)^{1/3} + k_1\sigma_{cp}\right]b_w d \qquad (1)$$

Where $V_{Rd,c}$ – shear force that is able to carry the reinforced concrete element without shear reinforcement.

In the formula (1) the term $C_{Rd,c} = 0.18/\gamma_c$ ($\gamma_c = 1.5$) represent holding base tangential strengthening. The tangential strengthening is modified from the coefficient k.

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2 \qquad (2)$$

Where d is the useful working height of the transverse section of the element in mm.

The k coefficient takes into consideration the reduction of the efficiency of the friction of the concrete granules when the useful d height increases.

$\rho_l = A_s / b_w d \leq 0.02$ Is the ratio of the longitudinal A_s with the useful working surface of the transverse section of the element.

b_w The width of the transverse section of the element.

f_{ck} The press cylindrical characteristic resistance of the concrete in N/mm^2.

When the reinforced concrete element is prestressed the $k_1\sigma_{cp}$ is different zero. Usually $k_1 = 0.15$.

$$\sigma_{cp} = N_{Ed}/A_c \leq 0.2\ f_{cd} \qquad (3)$$

Where N_{Ed} – is the prestressed force .

Term $k_1\sigma_{cp}$ takes into consideration the positive effect of the prestressed in the crack delay. Is needed that $V_{Rd,c} \geq V_{Rd,c,min}$

$$V_{Rd,c,min} = [\nu_{min} + k_1\sigma_{cp}]\, b_w\, d \quad (4)$$

$$\nu_{min} = 0.035\, k^{3/2}\, f_{ck}^{1/2} \quad (5)$$

CONCRETE CASE

Let's analyze the behavior of the ultimate limit state of a reinforced concrete element. In the Fig 8 is shown the schemes and the forces.

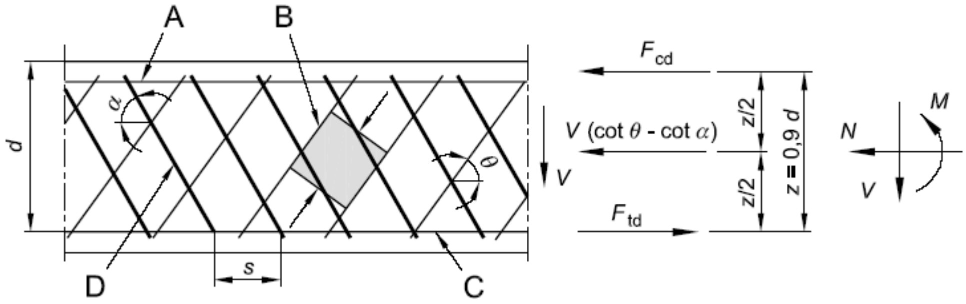

Fig. 8

The transversal reinforcement has an incline with α angle related to the longitudinal axe of the beam. This reinforcement represents the tension diagonals or the verticals of the truss (in this case $\alpha = 90°$), the angle $45° \le \alpha \le 90°$. The pressed diagonals are represented from the concrete element. They have an inclement with the angle, θ:

$1 \le \text{cotg}\,\theta \le 2.5$ or $21.8° \le \theta \le 45°$

The upper F_{cd}, as the result of the of the interaction of the bending with shear equals:

$$F_{cd} = (M_{Ed}/z) + 0.5\, V_{Ed}\, (\text{cotg}\,\theta - \text{cotg}\,\alpha) \quad (6)$$

Given the limits of the angle α is noticed $0 < \text{cotg}\,\alpha \le 1$. Thereafter:

$$(\text{cotg}\,\theta - \text{cotg}\,\alpha) \ge 0 \quad (7)$$

The lower force F_{td} as the result of the of the interaction of the bending with the shear equals :

$$F_{td} = (M_{Ed}/z) + 0.5\, V_{Ed}\, (\text{cotg}\,\theta - \text{cotg}\,\alpha) \quad (8)$$

As is mentioned above, results that the shear force always causes traction in the horizontal longitudinal reinforcement located above and below the element. The central part of the element involved between two reinforcements works under pressure of the force:

$$N = V_{Ed}\, (\text{cotg}\,\theta - \text{cotg}\,\alpha) \quad (9)$$

This is an advantage for the element. The shear force that is able to bear the concrete element that is equipped with a transversal reinforcement is determined from the bearing capacity of this reinforcement is given by the formula:

$$V_{Rds} = A_{sw}\, z\, f_{ywd}\, (\text{cotg}\,\theta + \text{cotg}\,\alpha) \sin\alpha / s \quad (10)$$

The shear force that is able to bear the concrete element equipped with a transversal reinforcement is determined from the bearing capacity of the concrete is given the formula:

$$V_{Rdmax} = \alpha_{cw}\, b_w\, z\, \nu_1\, f_{cd}\, (\text{cotg}\,\theta + \text{cotg}\,\alpha) / (1+\text{cotg}^2\,\theta) \quad (11)$$

$\nu_1 = 0.6\ (1 - f_{ck}/250)$ (12)

In the formula (12) the value of f_{ck} is expressed in MPa. To determine the maximal quantity of the transversal reinforcement that works in cutting is needed to equal V_{Rds} with V_{Rdmax}. Thereafter:

$A_{sw}\ z\ f_{ywd}\ (\cot\theta + \cot\alpha)\ \sin\alpha\ /\ s = \alpha_{cw}\ b_w\ z\ \nu_1\ f_{cd}\ (\cot\theta + \cot\alpha)\ /\ (1+\cot^2\theta)$ (13)

$A_{sw,max} = \alpha_{cw}\ \nu_1\ f_{cd}\ b_w\ s\ /\ (1+\cot^2\theta)\ \sin\alpha\ f_{ywd}$ (14)

For the minimum value of cotg θ = 1 derives:

$A_{sw,max} = \alpha_{cw}\ \nu_1\ f_{cd}\ b_w\ s\ /\ 2\ \sin\alpha\ f_{ywd}$ (15)

NUMERICAL EXAMPLES

Example 1. The concrete elements without shear transversal reinforcement.

Known: b_w =40cm; h = 60cm; d = 56.5cm; the longitudinal surface of the reinforcement A_s = 15.7cm2 and concrete class C40/50;

To be found: $V_{Rd,c}$ the shear force that can carry the reinforced concrete element without shear reinforcement.

$$V_{Rd,c} = \left[C_{Rd,c}k\left(100\rho_l f_{ck}\right)^{1/3} + k_1\sigma_{cp}\right]b_w d \quad (16)$$

$C_{Rd,c} = 0.18 / \gamma_c = 0.12$ where the $\gamma_c = 1.5$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$k = 1.59 \leq 2$

$\rho_l = A_s / b_w\ d = 0.00695$

f_{ck} = 40 N/mm2 = 40 MPa

$k_1 = 0.15$

σ_{cp} = 0 N/mm2

$V_{Rd,c}$ = 130 871 N (17)

$V_{Rd,c,min} = [\nu_{min} + k_1\sigma_{cp}]\ b_w\ d$ = 100 770 N (18)

$\nu_{min} = 0.035\ k^{3/2}\ f_{ck}^{1/2} = 0.446$

Example 2. The concrete element with vertical shear transversal reinforcement, α = 90°

Known: b_w = 40cm; h = 60cm; d = 56.5cm; the longitudinal area of the reinforcement A_s = 15.7cm^2; concrete class C40/50; the area of the transversal reinforcement in cutting A_{sw} = 1cm2; s = 10cm;

To be found: V_{Rds}, the shear force that can bear the reinforced concrete element equipped with shear transversal reinforcement and that is defined from the bearing capacity of this reinforcement

V_{Rdmax} - the cutting force that can bear the reinforced concrete element equipped with shear transversal reinforcement and that is defined from the bearing capacity of the concrete

$V_{Rds} = A_{sw}\ z\ f_{ywd}\ (\cot\theta + \cot\alpha)\ \sin\alpha\ /\ s$ (19)

z = 0.9d = 50.85cm

$f_{ywd} = f_{yd}$ = 4348 daN/cm2

cotg θ = 1

cotg α = 0

sin α = 1

V_{Rds} = 221 100 N = 22 110 daN (20)

$V_{Rdmax} = \alpha_{cw}\, b_w\, z\, \nu_1\, f_{cd}\, (\text{cotg}\, \theta + \text{cotg}\, \alpha) / (1+\text{cotg}2\, \theta)$ (21)

$\nu_1 = 0.6\, (1 - f_{ck}/250) = 0.504$

$\alpha_{cw} = 1$

$f_{cd} = \alpha_{cc}\, f_{ck} / \gamma_c = 0.85*40/1.5 = 22.67$ N/mm2 = 22.67 MPa

V_{Rdmax} = 1 161 820 N = 116 182 daN (22)

Example 3. The reinforced concrete element with inclined shear transversal reinforcement, $\alpha = 45°$

Known: b_w = 40cm; h = 60cm; d = 56.5cm; the longitudinal area of the reinforcement A_s = 15.7cm2; concrete class C40/50; the area of the shear transversal reinforcement A_{sw} = 9.42cm2; s = 20cm;

To be found: V_{Rds}, the shear force that can carry the reinforced concrete element equipped with shear transversal reinforcement and that is defined from the bearing capacity of this reinforcement

V_{Rdmax} - the shear force that can carry the reinforced concrete element equipped with shear transversal reinforcement and that is defined from the bearing capacity of the concrete

For $\alpha = 45°$, cotg α = 1, sin α = 0.707

Based on the formula (19) V_{Rds} = 1 472 710 N = 147 271 daN

Based on the formula (21) V_{Rdmax} = 2 323 640 N = 232 364 daN

Example 4. The reinforced concrete element with inclined shear transversal reinforcement, $\alpha = 45°$

Known: bw = 40cm; h = 60cm; d = 56.5cm; the longitudinal surface of the reinforcement As = 15.7cm2; concrete class C40/50; s = 20cm; the cutting active force V_{Ed} = 200 000 daN

To be found: the shear transversal reinforcement A_{sw}

Based on formula (19) :

$A_{sw} = V_{Ed}\, s / z\, fywd\, (\text{cotg}\, \theta + \text{cotg}\, \alpha) \sin \alpha$ (23)

A_{sw} = 12.79 cm2 (24)

Formula (15) shows

$A_{sw,max}$ = 14.86 cm2 (25)

If the angle of obliquity of the transversal reinforcement will be $\alpha = 90°$ then

A_{sw} = 18.09 cm2 (26)

$A_{sw,max}$ = 10.51 cm2 (27)

In this case we have to reduce the distance s between the traversal reinforcement reinforcement from 20cm to 10cm. After that:

$A_{sw} = 9.045\ cm^2$ (26)

$A_{sw,max} = 21.02\ cm^2$ (27)

CONCLUSION

Through this study we tried to give the reader teorical bases and numerical ones of reinforced concrete elements under the shear forces. We also presented the algorithm calculations of these elements.

The analysis was made by approximating the element with a truss, where the elements of this truss are represented by the reinforcement and the pressed elements are represented by the press concrete.

We also tried to present in this short paper several numerical examples and algorithms that would help the comprehension of the calculation of concrete elements from the shear forces.

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