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A Procedure for Application of Critical Path Method with Fuzzy Sets and Fuzzy Operations

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ABSTRACT

Critical Path Method (CPM), which is used to schedule construction activities depending to each other with network relationships, is deterministic in regards of the certain durations assigned to activities for its execution and the results it produces in certain values. Unfortunately, construction activities are performed under uncertain conditions. Project risks cause variation in activity durations and in turn, the entire network is affected from uncertainty. In this context, activity durations are represented by fuzzy sets and CPM network calculations are performed by fuzzy operations through a new procedure developed in this study. In construction projects, the duration of an activity can not be proposed certainly in advance. The predictions like "this activity can be completed most probably between 7 and 10 days but perhaps it takes 15 days maximum and 5 days minimum depending on the conditions" are frequently made. Fuzzy numbers enable modeling such kind of uncertain predictions mathematically. Since the activity durations are represented by fuzzy numbers and network calculations are performed by fuzzy operations, the activity early/late start/finish times and the project completion time are calculated as fuzzy numbers by the proposed procedure. An example CPM application with fuzzy sets is also presented in the paper. The findings show that CPM is applicable with fuzzy sets and the developed procedure operates well for modeling the uncertainty in CPM calculations.

INTRODUCTION

Construction projects are realized by carrying out various activities which are dependent to each other by finish-to-start, start-to-start, finish-to-finish or start-to-finish relations, and by lag or lead times. Therefore, the construction activities constitute networks. In order to provide managerial information such as the project completion time, the activity early/late start/finish times, the total/free/independent/shared float times and the activity/path criticalness, the dependency relations between the activities must be analyzed. Bar charts, Line of Balance Method (LOB) and Critical Path Method (CPM) have been the most popular methods of construction activity scheduling since 1950s [1-3]. Among these methods, CPM, which was first developed in 1956 by the DuPont Company with Remington Rand as consultants in USA, is accepted as the most suitable mean of scheduling and analyzing the activity networks [3]. This is due to its capabilities in showing the dependency relations between activities, detecting the critical activities, revealing the activity float times, and making the resource allocation feasible in a proper fashion [4].

In spite of its wide usage and popularity, CPM has some limitations and criticized features. The limitations of CPM are related to its deterministic calculation procedure, which is insufficient for modeling uncertainty. CPM is deterministic because of the single duration values assigned to activities in network calculations, as if these duration values are known certainly and are not changed by various risk factors. This deficiency may lead to inaccurate critical path identification and completion time measurement [5]. Unfortunately, the schedules of construction activity networks are under the influence of uncertainties in the factors such as weather, productivity, design, scope, site conditions and soil properties [6-8], and all of the risk factors in a construction project might be schedule risks because they are related to time schedules directly or indirectly. Moreover, all activities might become critical in practice due to uncertainties, even those that are not critical according to CPM.

In this context, this study aims at proposing a new procedure for performing the CPM network calculations (forward and backward pass calculations) with fuzzy sets. The activity durations are represented by a special kind of fuzzy sets called fuzzy numbers in this procedure and accordingly, the CPM forward and backward pass calculations are executed by fuzzy operations. The representation of activity durations by fuzzy numbers enables modeling the uncertainty effect. In construction management, the duration of an activity can not be proposed certainly in advance. The predictions like "this activity can be completed most probably between 7 and 10 days but perhaps it takes 15 days maximum and 5 days minimum depending on the conditions" are frequently made. Fuzzy numbers become suitable for modeling such kind of uncertain predictions mathematically. Since the activity durations are represented by fuzzy numbers and the network calculations are performed by fuzzy operations, the activity early/late start/finish times and the project completion time are calculated as fuzzy numbers by this new procedure. Therefore, the effect of uncertainty on the results of CPM is also modeled and the evaluation of activity/path criticalness is performed by using the geometric centers of the activity early/late times.

Fuzzy sets have been successfully used in the previous studies by researchers for modeling the uncertainty in activity durations and the risk factor effect on the project activity networks [9-11]. Some of the researchers have tried to implement the CPM network calculations through fuzzy sets and operations [12-14]. Lorterapong and Moselhi [14] developed a complete project network analysis technique by using fuzzy set theory named as FNET. This technique includes a new procedure for performing the forward and backward pass calculations of CPM with fuzzy sets in case the activities are dependent on each other with only finish-to-start relation and no lag or lead times are used between the activities. However, if other types of network dependencies like finish-to-finish, start-to-start or start-to-finish and lag/lead times are used, this technique stays insufficient. In this study, it is aimed at proposing a new procedure to be used for the implementation of CPM with fuzzy sets, in case all types of network dependencies and lag/lead times are used between activities.

The details of the procedure proposed for implementing the CPM network calculations with fuzzy sets is described after introducing the basic information about fuzzy set theory and fuzzy numbers in the following parts of the paper. An example application is also carried out. The paper ends with the conclusions and some recommendations for future work.

CPM WITH FUZZY SETS

Fuzzy Set Theory and Fuzzy Numbers

In classical set theory, the membership of an element to a specified set is described by two definite and opposite situations: belonging to the set (membership degree = 1.0) or not belonging to the set (membership degree = 0.0). However, in fuzzy set theory, the membership of an element to a specified set is described by the membership degrees between 0.0 and 1.0 [15, 16]. This enables us to model the uncertain expressions of real life mathematically, to perform fuzzy set operations between these uncertainties and finally to reach fuzzy results that cannot be achieved analytically otherwise.

Consider a fuzzy set A of the universe U.

$$A = \{(x, \mu_A(x)) \mid x \in A, \mu_A(x) \in [0, 1]\}$$

where $\mu_A(x)$ is a function called membership function; $\mu_A(x)$ exactly states the grade or degree to which any element x in A is a member of the fuzzy set A. The definition given above combines each element x in A with $\mu_A(x)$ in the interval [0, 1] which is assigned to x. Larger values of $\mu_A(x)$ indicate higher degrees of membership [17].

A fuzzy number is a continuous fuzzy set that possesses two properties: convexity and normality. The convexity indicates that the membership function has only one distinct peak, while the normality ensures that at least one element in the set has a degree of membership equal to 1.0. These two properties make the concept of fuzzy numbers attractive and naturally appropriate for modeling imprecise fuzzy quantities such as “approximately one week,” or “more or less than seven days”. Theoretically, fuzzy numbers can take various shapes. In modeling real-life problems, however, linear approximations such as trapezoidal and triangular fuzzy numbers are frequently used [12, 13]. Mathematical definitions and general shapes of triangular and trapezoidal fuzzy numbers are given below:

- Triangular Fuzzy Numbers

A triangular fuzzy number with membership function $\mu_A(x)$ is defined by:

$$\mu_A(x) = \begin{cases} (x-a)/(b-a) & \text{for } a \le x \le b \\ (x-c)/(b-c) & \text{for } b \le x \le c \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

This set is graphically shown below in Figure 1.

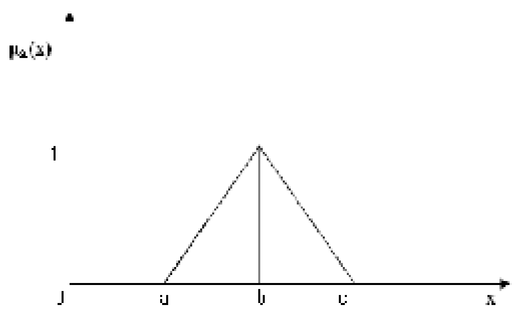

Figure 1 Triangular fuzzy number

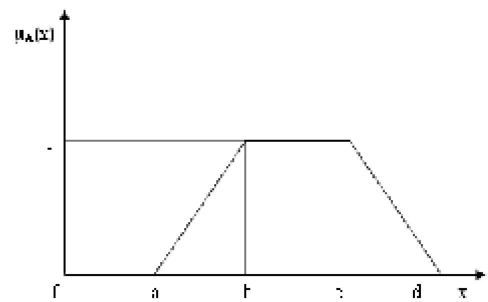

Figure 2 Trapezoidal fuzzy number

- Trapezoidal Fuzzy Numbers

A trapezoidal fuzzy number A with membership function $\mu_A(x)$ is defined by:

$$\mu_A(x) = \begin{cases} (x-a)/(b-a) & \text{for } a \le x \le b \\ 1 & \text{for } b \le x \le c \\ (x-d)/(c-d) & \text{for } c \le x \le d \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

This set is graphically shown in Figure 2.

Network Calculations of CPM with Fuzzy Sets

The early/late start/finish times, total float times, criticalness of the activities and the project completion time of a network are found by applying forward and backward pass calculations on the network. In other words, forward and backward pass calculations constitute the network calculations of CPM. In order to carry out the CPM network calculations, activity durations, activity interdependencies in the form of finish-to-start (FS), finish-to-finish (FF), start-to-start (SS) or start-to-finish (SF), and lag or lead times between the activities are required. The activity durations should be predicted as single values (most likely durations) for the CPM execution. However, if the activity durations and lag/lead times are represented by fuzzy sets, traditional forward/backward pass calculation of CPM becomes inapplicable. In this regard, a new procedure has been developed for the purpose of making the CPM network calculations applicable with fuzzy sets.

Forward Pass Calculations of CPM with Fuzzy Sets

Forward pass calculations should be performed through fuzzy operations in an activity network of which the activity durations and lag/lead times are represented by fuzzy sets. For this reason, fuzzy addition, fuzzy subtraction, fuzzy maximization and fuzzy minimization have been utilized in order to develop the procedure of the CPM forward pass calculation with fuzzy sets. The procedure is described below:

Let X and Y are the two trapezoidal fuzzy numbers such that:

$X = (a_1, b_1, c_1, d_1)$

$Y = (a_2, b_2, c_2, d_2)$

Then,

$$X \{+\} Y = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) \quad (3)$$

$$X \{-\} Y = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2) \quad (4)$$

$$\tilde{max} (X,Y) = (\max (a_1, a_2), \max (b_1, b_2), \max (c_1, c_2), \max (d_1, d_2)) \quad (5)$$

$$\tilde{min} (X,Y) = (\min (a_1, a_2), \min (b_1, b_2), \min (c_1,c_2), \min (d_1, d_2)) \quad (6)$$

where {+}, {–}, $\tilde{max}$, $\tilde{min}$ are fuzzy addition, fuzzy subtraction, fuzzy maximization and fuzzy minimization, respectively.

These fuzzy operations are only applied between the fuzzy values possessing the same membership degrees, which is a rule based on the logic of fuzzy operations [14].

If all of the activity dependencies are FS and no lag/lead time between activities are used in an activity network, fuzzy forward pass calculation is performed as follows [14]:

$$FES_x = \tilde{max} (FEF_P) \quad (7)$$

$$FEF_x = FES_x \{+\} FD_x \quad (8)$$

$$T_{proj} = FEF_e \quad (9)$$

where $p \in P$ (the set of predecessor activities); FES_x, FEF_x, FD_x are the fuzzy early start time, fuzzy early finish time and fuzzy duration of activity x, respectively; and T_{proj} and FEF_e are the fuzzy project duration and fuzzy early finish time of the last activity, respectively.

However, the construction activity networks may include lag or lead times between activities and dependencies other than FS. This problem is resolved by the following algorithm:

- i – Subtract lead time from lag time with fuzzy subtraction for each activity pair having predecessor/successor relation.

  $$FN_{pi} = [\text{fuzzy lag}_{pi} \{-\} \text{fuzzy lead}_{pi}] \quad (10)$$

  where pi denotes the predecessor activity so that i takes values depending on the number of predecessors.
- ii – Add the fuzzy number calculated in step i with fuzzy addition to the corresponding early time of the predecessor activity. For instance, if the relation is FF between an

activity and one of its predecessors, then early finish time of this activity is calculated through adding the fuzzy number calculated in step i to the early finish time of the predecessor activity.

$FEF_{xi} = FEF_{Pi}$ {+} FN_{pi} (11)

where xi denotes the successor activity. Once more, i takes values depending on the number of predecessors.

- iii – Fuzzy early start times of an activity are calculated with employing the fuzzy duration of this activity to the fuzzy early start times found in step ii. However, this step is executed if the dependency is SF or FF. If the dependency is SS or FS, the fuzzy early time found in step ii is already the fuzzy early start time.

  $FES_{xi} = FEF_{pi}$ {+} FN_{pi} If relation is FS (12)
  FES_{pi} {+} FN_{pi} If relation is SS
  $FEF_{xi} = FEF_{pi}$ {+} FN_{pi} If relation is FF (13a)
  FES_{pi} {+} FN_{pi} If relation is SF
  Then, $FEF_{xi} = FES_{xi}$ {+} FD_s (13b)
  where FD_s shows the fuzzy duration of the successor activity in question.

- iv – In step iii, different fuzzy early start times are calculated as many as the number of predecessors (pi) of the successor activity (xi) in question. Therefore, the final fuzzy early start time of an activity is found through fuzzy maximization of the fuzzy early start times calculated in step iii.

  $FES_x = \tilde{max} (FES_{xi})$ (14)

The fuzzy forward pass calculation procedure described above is clarified by an application on a short example network portion (one activity with four predecessors), which is shown in Figure 3. All of the fuzzy numbers in this example are accepted as trapezoidal; however the mode values, b and c, are accepted as equal to each other for the purpose of modifying the trapezoidal fuzzy numbers to triangular fuzzy numbers in order to simplify the calculations. The network consists of a single activity whose fuzzy early start and fuzzy early finish times are being searched and four predecessor activities whose dependency and lag/lead times differ as shown in Figure 3. FES and FEF designate the fuzzy early start and fuzzy early finish times, respectively. Fuzzy forward pass calculations of this example network portion are performed as follows:

- Predecessor 1 (p1) :
$FES_{x1} = FEF_{p1}$ {+} [fuzzy lag_{p1} {-} fuzzy $lead_{p1}$]
FES_{x1} = (5,6,6,8) {+} [(0,0,0,0) {-} (0,1,1,2)]
FES_{x1} = (5,6,6,8) {+} (-2,-1,-1,0)
FES_{x1} = (3,5,5,8)

- Predecessor 2 (p2) :
$FES_{x2} = FES_{p2}$ {+} [fuzzy lag_{p2} {-} fuzzy $lead_{p2}$]
FES_{x2} = (4,5,5,7) {+}[(0,1,1,2) {-} (0,0,0,0)]
FES_{x2} = (4,5,5,7) {+} (0,1,1,2)
FES_{x2} = (4,6,6,9)

- Predecessor 3 (p3) :
$FEF_{x3} = FEF_{p3}$ {+} [fuzzy lag_{p3} {-} fuzzy $lead_{p3}$]
FEF_{x3} = (8,10,10,12) {+}[(0,1,1,2) {-} (0,0,0,0)]
FEF_{x3} = (8,11,11,14)
$FEF_{x3} = FES_{x3}$ {+} Fuzzy Act. $Dur._s$ (FD_s)
(8,11,11,14) = FES_{x3} {+} (1,2,2,3)
FES_{x3} = (7,9,9,11)

- Predecessor 4 (p4) :
$FEF_{x4} = FES_{p4}$ {+}[fuzzy lag_{p4} {-} fuzzy $lead_{p4}$]
FEF_{x4} = (6,9,9,13) {+}[(0,2,2,3) {-} (0,0,0,0)]
FEF_{x4} = (6,11,11,16)
$FEF_{x4} = FES_{x4}$ {+} Fuzzy Act. Dur. $_s$ (FD_s)
(6,11,11,16) = FES_{x4} {+} (1,2,2,3)
FES_{x4} = (5,9,9,13)

- FES_s :
$FES_s = \tilde{max} (FES_{x1}, FES_{x2}, FES_{x3}, FES_{x4})$
FES_x = (7,9,9,13)

- FEF_x :
$FEF_x = FES_x$ {+} Fuzzy Act. Dur. $_s$ (FD_s)
FEF_x = (7,9,9,13) {+} (1,2,2,3)
FEF_x = (8,11,11,16)

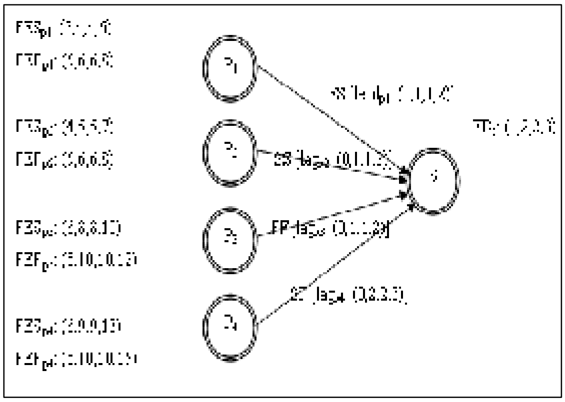

Figure 3 Four predecessors – one successor network portion

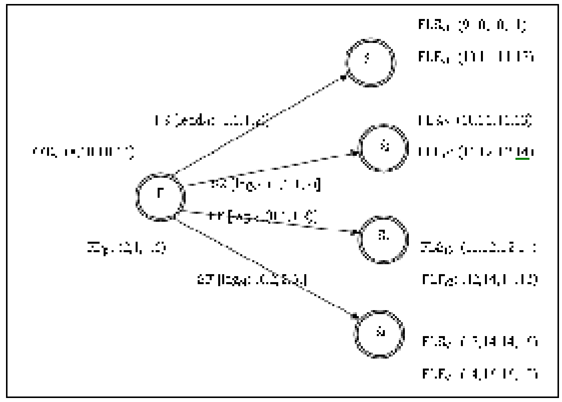

Figure 4 Four successors – one predecessor network portion

This example application shows that the fuzzy early start time of the successor activity S in Figure 3 is (7,9,9,13), i.e. the early start time of the activity S is certainly between 7th and 13th unit-times (day, month, etc.) and it is most plausibly at 9th unit-time from the start date of the network.

Backward Pass Calculations of CPM with Fuzzy Sets

If the activity durations and lag/lead times are represented by fuzzy sets in an activity network, fuzzy backward pass calculations should be performed through fuzzy operations just like the fuzzy forward pass calculations. For this reason, fuzzy subtraction has been utilized between the activities and the successor activities in order to develop the backward pass calculation procedure of CPM with fuzzy sets. However, a problem occurs due to the usage of fuzzy subtraction. Fuzzy subtraction produces unrealistically large uncertainties associated with fuzzy late start and fuzzy late finish times of activities. These uncertainties accumulate quickly as the backward pass calculation progresses. Moreover, earlier activities may be assigned with negative early finish and late finish times at the end of the calculation which has no meaning from the scheduling point of view. Lorterapong and Moselhi [14] tried to overcome this problem by developing a procedure while developing their so-called model, FNET. However, only FS relation was considered and lag/lead times were ignored in FNET. For this reason, their method has been carried one step further in this study to circumvent these limitations. The used assumptions and the developed backward pass calculation procedure are described below.

Assumptions:

- All the values in fuzzy numbers (lower, upper and mode values – a,b,c,d) should have a positive value.
- Each value should not exceed its successor (a ≤ b ≤ c ≤ d).
- The values of the fuzzy early start time or fuzzy early finish time of an activity found by fuzzy forward pass calculation should not exceed the values of the fuzzy late start or fuzzy late finish times found by the fuzzy backward pass calculation.
- The right spread of fuzzy late times (the difference between d and c) should be at least as uncertain as their respective fuzzy early times.

Procedure:

- i – First, lag/lead times between the activities are processed. Since, the operation is now the backward pass, lag times are considered just like the lead times of forward pass and lead times are considered just like the lag times of forward pass. In other words, lag time is subtracted from lead time with fuzzy subtraction for each activity pair having predecessor-successor relation.

FN_{si} = [fuzzy $lead_{si}$ {-} fuzzy lag_{si}] (15)

where si denotes the successor activity so that i takes values depending on the number of successors.

- ii - The fuzzy number calculated in step i is added with fuzzy addition to the corresponding late time of the successor activity. For instance, if the relation is FF between an activity and one of its successors, then late finish time of this activity is calculated through fuzzy adding of the fuzzy number calculated in step i to the late finish time of the successor activity.

$FLF_{xi} = FLF_{si}$ {+} FN_{si} (16)

where xi denotes the predecessor activity. Once more, i takes values depending on the number of successors.

- iii – Fuzzy late finish times of an activity x are calculated with employing fuzzy duration of this activity to the fuzzy late times found in step ii. However, this step is executed if the dependency is SF or SS. If the dependency is FS or FF, the fuzzy late time found in step ii is already the fuzzy late finish time.

$FLF_{pi} = FLS_{si}$ {+} FN_{si} If relation is FS, FF

= (FLS_{si} {+} FN_{si}) {+} FD_p If relation is SS, SF (17)

where FD_p denotes the fuzzy duration of the predecessor activity in question.

- iv – Final fuzzy late finish time of an activity is found with fuzzy minimization of the fuzzy late finish times calculated in step iii.

$FLF_x = \min (FLF_{xi})$ (18)

- v – The fuzzy number found in step iv is accepted as the preliminary fuzzy late finish time ($PFLF_x$).
- vi - FEF and PFLF are compared to find which of the two fuzzy numbers has a greater right spread. Suppose that FEF_p is represented by (a, b, c, d) and the $PFLF_x$ is represented by (p, q, e, f). In this case, the comparison is made between (f – e) and (d – c) [14].
- vii - If $(d – c) \geq (f – e)$, which means that the right spread of FEF_p is more uncertain, the right spread of the final fuzzy late finish time (FLF_x) is set equal to the right spread of FEF_p. In this case, FLF_x is calculated by Eq.19 [14].

$FLF_x = FEF_p$ {+} (f – d, f – d, f – d, f – d) (19)

FLF_x = (a, b, c, d) {+} (f – d, f – d, f – d, f – d)

FLF_x = (a + f – d, b + f – d, c + f – d, d + f – d)

FLF_x = (a + f – d, b + f – d, c + f – d, f)

- viii - If $(d – c) < (f – e)$, which means that the right spread of FEF_p is less uncertain, the right spread of FLF_x is set equal to the right spread of $PFLF_x$. In this case, FLF_x is calculated by Eq.17 [14].

$FLF_x = FEF_p$ {+} (e – c, e – c, e – c, f – d) (20)

FLF_x = (a, b, c, d) {+} (e – c, e – c, e – c, f – d)

FLF_x = (a + e – c, b + e – c, c + e – c, d + f – d)

FLF_x = (a + e – c, b + e – c, e, f)

- ix - Fuzzy late start time (FLS_x) is computed by substituting FLF_x and fuzzy duration (FD_p) into Equation 21[14].

FLS_x {+} $FD_p = FLF_x$ (21)

- x - The procedure described up to now is applied to all activities starting from the last activity towards the start activity through following the paths in backward direction.

The fuzzy backward pass calculation procedure described above is clarified by an application on a short example network portion (one activity with four successors) shown in Figure 4.

All of the fuzzy numbers are taken as trapezoidal; however mode values b and c are taken equal for the purpose of modifying the trapezoidal fuzzy numbers to triangular fuzzy numbers in order to provide simplicity in this example. The network consists of a single activity whose fuzzy late start and fuzzy late finish times are being searched and four successor activities whose dependency and lag/lead times differ as shown in Figure 4. Fuzzy backward pass calculations of this network are given below:

- Successor 1 (s1) :

$FLF_{x1} = FLS_{s1}$ {+} [fuzzy lead$_{s1}$ {-} fuzzy lag$_{s1}$]

FLF_{x1} = (9,10,10,11) {+} [(0,1,1,2) {-} (0,0,0,0)]

FLF_{x1} = (9,11,11,13)

- Successor 2 (s2) :

$FLS_{x2} = FLS_{s2}$ {+} [fuzzy lead$_{s2}$ {-} fuzzy lag$_{s2}$]

FLS_{x2} = (10,11,11,13) {+}[(0,0,0,0) {-} (0,1,1,2)]

FLS_{x2} = (10,11,11,13) {+}(-2,-1,-1,0)

FLS_{x2} = (8,10,10,13)

$FLF_{x2} = FLS_{x2}$ {+} Fuzzy Act. Dur. $_P$ (FD_p)

FLF_{x2} = (8,10,10,13) {+} (2,4,4,6)

FLF_{x2} = (10,14,14,19)

- Successor 3 (s3) :

$FLF_{x3} = FLF_{s3}$ {+} [fuzzy lead$_{s3}$ {-} fuzzy lag$_{s3}$]

FLF_{x3} = (12,14,14,16) {+}[(0,0,0,0) {-} (0,1,1,2)]

FLF_{x3} = (12,14,14,16) {+}(-2,-1,-1,0)

FLF_{x3} = (10,13,13,16)

- Successor 4 (s4) :

$FLS_{x4} = FLF_{s4}$ {+}[fuzzy lead$_{s4}$ {-} fuzzy lag$_{s4}$]

FLS_{x4} = (12,14,14,16) {+}[(0,0,0,0) {-} (0,2,2,3)]

FLS_{x4} = (12,14,14,16) {+} (-3,-2,-2,0)

FLS_{x4} = (9,12,12,16)

$FLF_{x4} = FLS_{P4}$ {+} Fuzzy Act. Dur. $_P$ (FD_p)

FLF_{x4} = (9,12,12,16) {+} (2,4,4,6)

FLF_{x4} = (11,16,16,22)

- $PFLF_x$:

$PFLF_x$ = $\tilde{\min}$ (FLF_{x1}, FLF_{x2}, FLF_{x3}, FLF_{x4})

$PFLF_x$ = $\tilde{\min}$ [(9,11,11,13), (10,14,14,19), (10,13,13,16), (11,16,16,22)]

$PFLF_x$ = (9,11,11,13)

- FLF_x :

$PFLF_x$ = (9,11,11,13) and FEF_p =(6,8,8,11)

(11-8) > (13-11) then,

$FLF_x = FEF_p$ {+} (f – d, f – d, f – d, f – d)

FLF_x = (6,8,8,11) {+} (13 – 11, 13 – 11, 13 – 11, 13 – 11)

FLF_x = (8,10,10,13)

- FLS_x :

FLS_x {+} Fuzzy Act. Dur. $_P$ (FD_p) = FLF_x

FLS_x {+} (2,4,4,6) = (8,10,10,13)

FLS_x = (6,6,6,7)

Fuzzy backward pass calculation may sometimes produce negative values, especially for the lower and mode fuzzy values (a,b,c) or it may produce zero for the mode fuzzy values (b,c) of the activities at the beginning of the network. In the former case, negative values are converted to zero and in the latter case all the fuzzy values (a,b,c,d) are accepted as zero.

EXAMPLE APPLICATION

The new procedure of CPM with fuzzy sets was applied on a hypothetic activity network. Network information and the results of the application are given in Tables 1 and 2, respectively. The network is a short and simple one but it contains all the types of network dependencies, i.e. FS, FF, SS, SF with lag and lead times. Therefore, it stands as a good example for showing the application of all of the features of CPM with fuzzy sets.

The results given in Table 2 reveal that the total float times calculated by using geometric centers of fuzzy early and late times of the activities are sufficient for detecting the critical and uncritical activities. The calculation procedure of total float times (TF) by using the geometric centers of the fuzzy numbers can be found by using Equations 22 and 23 as follows [14]:

$$TF\ x \in X = CLF_x - CEF_x \tag{22}$$

where the C designation denotes the geometric center of the early and late times, $x \in X$ (the set of activities), CEF and CLF shows geometric centers of fuzzy early and late finish times respectively.

$$C = \frac{c^2 + d^2 - a^2 - b^2 + cd - ab}{3(d+c-a-b)} \tag{23}$$

Table 1 Network information of example network

Activity	Predecessors	Fuzzy Duration (day)	Dependency	Fuzzy Lag Time (day)	Fuzzy Lead Time (day)
Start	-	(0,0,0,0)	FS		
A	Start	(2,3,3,4)	FS		
B	Start	(5,7,7,10)	FS		
C	A	(6,8,8,10)	FS	(0,1,1,2)	
	B		SS	(1,2,2,3)	
D	C	(3,4,4,5)	FS	(1,3,3,4)	
E	C	(7,8,8,10)	FS		(0,1,1,2)
Finish	D	(0,0,0,0)	FF		(2,4,4,5)
	E		SF	(1,2,2,4)	

It should be mentioned that the activities whose total float times are close to zero and whose early and late times are almost the same have been considered as critical in this study for the sake of detecting the critical path. For example, total float, fuzzy early finish and fuzzy late finish times of the activity C were found as 1.67, (8,12,12,16) and (8,12,12,21), respectively (shown in Table 2). Therefore, the activity C can be considered as a critical activity.

Table 2 Results of example application

Activity	Fuzzy Duration (day)	Fuzzy Early Start Time (day)	Fuzzy Early Finish Time (day)	Preliminary Fuzzy Late Finish Time (day)	Fuzzy Late Start Time (day)	Fuzzy Late Finish Time (day)	Geometric Center of Fuzzy Late Finish Time (x)	Geometric Center of Fuzzy Early Finish Time (y)	Total Float Time (day) (x-y)	Criticalness
Start	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	0,00	0,00	0,00	Critical
A	(2,3,3,4)	(0,0,0,0)	(2,3,3,4)	(0,3,3,10)	(0,0,0,0)	(2,3,3,4)	3,00	3,00	0,00	Critical
B	(5,7,7,10)	(0,0,0,0)	(5,7,7,10)	(7,9,9,20)	(2,2,2,10)	(7,9,9,20)	12,00	7,33	4,67	Uncritical
C	(6,8,8,10)	(2,4,4,6)	(8,12,12,16)	(8,12,12,21)	(2,4,4,11)	(8,12,12,21)	13,67	12,00	1,67	Critical
D	(3,4,4,5)	(6,11,11,15)	(9,15,15,20)	(9,17,17,26)	(8,13,13,21)	(11,17,17,26)	18,00	14,67	3,33	Uncritical
E	(7,8,8,10)	(6,11,11,15)	(13,19,19,26)	(10,19,19,25)	(6,11,11,19)	(13,19,19,25)	20,33	19,33	1,00	Critical
Finish	(0,0,0,0)	(0,0,0,0)	(7,13,13,20)	(7,13,13,20)	(7,13,13,20)	(7,13,13,20)	13,33	13,33	0,00	Critical

CONCLUSION

Construction activities are performed under uncertain conditions. Various risks cause variation in activity durations and in turn, the values found by CPM like the activity early/late times become uncertain. In this context, activity durations are represented by fuzzy sets and the CPM network calculations are performed by fuzzy operations through a new procedure developed in this study. In this procedure, fuzzy sets are utilized to model the uncertainty in activity durations, activity early/late times and project completion time. An example CPM application with fuzzy sets is also presented. The findings show that CPM is applicable with fuzzy sets and the developed procedure operates well for modeling the uncertainty in CPM network calculations.

The new procedure for the CPM network calculations with fuzzy sets proposed in this study can be compared with the other uncertainty analysis methods like the Monte Carlo simulation based models. It can be used for developing a schedule risk analysis model operating with simulation and fuzzy sets. Furthermore, it can be computerized easily by utilizing table processor software or computer programming languages. These issues can be proposed as future work.

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