Math Applications on the Displacement of Beam Calculations

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ABSTRACT

Beams under load are subject to displacement. In the calculations of these, mathematical formulations are used. In this paper, displacement of different beams under different loads has been analysed by using mathematical formulations. Thus, coherent formulations of loading on beams and displacement of beams are obtained. Furthermore to obtain these formulas, boundary conditions have been taken into consideration.

INTRODUCTION

Math has been used in many engeneering deciplins [1]. Among these civil engineering plays a crucial role. Civil Engineering is a field of wide scope that ranges from the design and construction of structures, roadways and pollution control processes to the management of our natural and engineered resources. Width of the field has led to division into a number of sub disciplines including structural engineering, geotechnical engineering, transportation engineering, environmental engineering, materials, construction management, and water resources engineering. Civil engineering included a number of science disciplines including such as chemistry, physics, ecology, geology, microbiology, material science, economics and mathematics, and statistics [2]. Matematical definacitions have a big range of applications in strength. Only one of these is math applications on the displacement of beam calculations.

Some part of these areas for instance displacement of beam calculations can be formulized. As part of these calculations it should not assess the maximum deflections that will occur in the beams of the structure and make sure that they are not excessive. The application of math on all branches of engineering will require numerical integration techniques to solve some problems such as beam deflection. Using techniques form the basis of the calculations that would be undertaken in real life albeit often carried out using sophisticated and powerful computer analysis software [3].

BACKGROUND THEORY

To calculate beam deflections a standard fundamental formula is used to determine deflections base on beam curvature. This is given by the expression:

Where:
M = The bending moment at the section, distance *x* from the origin
y = The vertical deflection at the section distance *x* from the origin.
The radius of the shape of the curved beam at a distance x from the origin, normally taken at the left or right hand end of the beam
E = The elastic or Young's modulus of the material from which the beam is fabricated.
I = The second moment of area of the beam's cross-section. This value depends on the shape of the cross section and is normally obtained from tables. Its units are m^4 or mm^4 or cm^4.

$$\frac{M}{EI} = -\frac{d^2y}{dx^2} \quad (1)$$

In the above formula *E* and *I* are normally constant values whilst *y*, *x* and *M* are variables. *M* can be expressed in terms of distance *x* and hence double integration techniques can be used to solve the above expression to calculate the deflection *y*.

In other words**,**

$$y = -\iint \frac{M}{EI} dxdx \qquad (2)$$

The bending moment is a means of describing mathematically the amount of bending and deflection that will occur in a beam under a given loading system and is defined as the sum of the moments of all forces to the left or right of the section under consideration. It doesn't matter whether the left or right is taken as the answer will be the same in both cases. For example in Figure 1 below the simply supported beam shown carries a uniformly distributed load of P KN/m. (note the units). When a load is described as uniformly distributed it means that the load intensity is the same throughout.

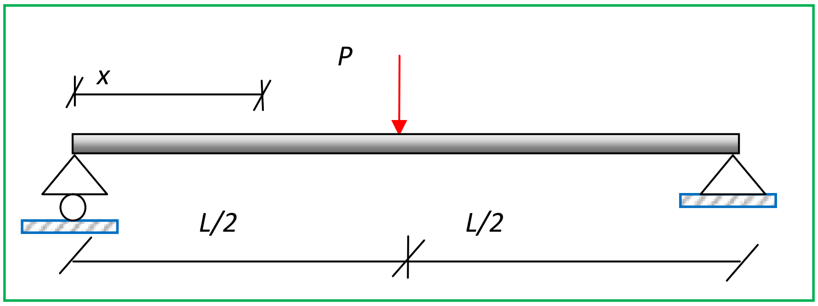

Figure 1.

Solution 1.

At x, a distance *x* from the left hand support the bending moment, M_x, will be given as:

$M_x = \frac{P_x}{2}$. If this value is put in equations (1),

$$\frac{d^2y}{dx^2} = -\frac{P.x}{2.EI}$$

And if the first entegral is calculated,

$$\int \frac{EI}{P} \frac{d^2y}{dx^2} = \int \left(\frac{-x}{2} \right) dx = \frac{EI}{P} \frac{dy}{dx} = -\left(\frac{x^2}{4} \right) + c_1$$

First boundary condition is **:** $\frac{dy}{dx}(\frac{L}{2}) = 0$

If this value is put in the equation above, constant C_1 is obtained.

$$C_1 - (\frac{L}{2})^2 \frac{1}{2} = 0 \qquad \Rightarrow \qquad C_1 = \frac{1}{8} L^2 .$$

If this value (C_1) is put in the equations above,

$$\frac{EI}{P}\frac{dy}{dx} = -\frac{x^2}{4} + \frac{1}{8} L^2$$

If the second entegral of both sides is calculated,

$$\int \frac{EI}{P}\frac{dy}{dx} = \int (\frac{-x^2}{4} + \frac{1}{8} L^2) dx = \frac{EI}{P}\frac{dy}{dx} = -\frac{x^3}{12} + \frac{L^2.x}{8} + C_2$$

Second boundary condition is: $y(0)=0$,
In other words because the left and right hand ends are both supports then they can not deflect downwards.

If this values are put in the equations above $\mathbf{C_2=0}$ is obtained.

If this value is put in the equation again, $\frac{EI}{P}.y = \frac{-x^3}{12} + \frac{1}{8} L^2 .x$

From here; $y = \frac{P}{EI}(\frac{-x^3}{12} + \frac{1}{8} L^2 .x)$ can be written and it is regulated, Equation of elastic curve in (3) is obtained.

$$y = \frac{PL^3}{48EI}\left[6.\left(\frac{x}{L}\right) - 4\left(\frac{x}{L}\right)^3\right] \tag{3}$$

Note: The above expression can now be used to calculate the deflection at any point on the beam. In practice it is the maximum deflection that is of interest and common sense would say that for this example this occurs at mid-span and can be calculated by substituting $x=L/2$ into equation 3 above. If it is not obvious where the maximum deflection occurs this will be where there is a change in slope of the beam where $dy/dx=0$. Hence equation (3), or its equivalent in a similar but different problem, could be differentiated and equated to find the distance *xmax* where the rotation, *dy/dx,* is zero. Substituting this value for x_{max} into equation (3) will give the maximum deflection, y_{max}.

From the boundary condition,maksimum deflection can be obtained in the centre of beam. If the deflections x=L/2 is written in (3), equations (4) is obtained.

$$y_{\max} = \frac{PL^3}{48EI} \tag{4}$$

Example 2.

Let`s find the elastic curve equation of Uniform loaded simple beam and the maximum displacement value(figure 2)

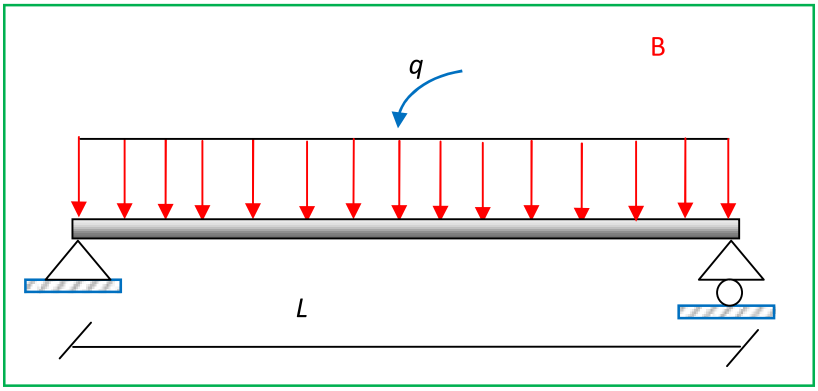

Figure 2

Solution 2.

$M_x = \frac{q.x}{2}.(L-x)$ is obtained. If this definition is put in the equation (1) ,

$\frac{d^2y}{dx^2} = \frac{-qx}{2EI}(L-x)$ the definition is regulated according to constants ,

$$\frac{2EI}{q}\frac{d^2y}{dx^2} = -x(L-x)$$

1. Integral is calculated, then it will be obtained;

$$\int \frac{2EI}{q}\frac{d^2y}{dx^2} = \int -x(L-x)dx \Rightarrow \frac{2EI}{q}\frac{dy}{dx} = -(\frac{x^2}{2}).L + \frac{x^3}{3} + C_1$$

First boundary condition is **:** $\frac{dy}{dx}(\frac{L}{2}) = 0$ and if this definition is written above;

$C_1 = \frac{1}{12}L^2$ is obtained.

If the obtained integral is put in constant place then 2. integral can be calculated;

$$\int(\frac{2EI}{q}\frac{dy}{dx})dx = \int(\frac{-x^2}{2}.L + \frac{x^3}{3} + \frac{L^2}{12})dx \Rightarrow y.\frac{2EI}{q} = \frac{x^3}{6}.L + \frac{x^4}{12} + \frac{L^2x}{12} + C_2$$

Second boundary condition is **:** y(0)=0
and if this boundry condition is written above **C_2=0** is obtained and if this value is put in the equation (5) then the equation of the elastic curve is obtained.

$$y = \frac{q.L^4}{24EI}\left[(\frac{x}{L})^4 - 2(\frac{x}{L})^3 + (\frac{x}{L})\right] \quad (5)$$

From boundary conditions, for $x = \frac{L}{2}$ the displacement is maximum and maximum displacement is given in equation (6).

$$y = \frac{5.q.L^4}{384EI} \tag{6}$$

Example 3.

In the figure below, the condition of loading in simple beam is given Let`s find the displacements in x=2m, x=4m . $I=4.10^5 cm^4$, $E=6.10^6 kg/cm^4$

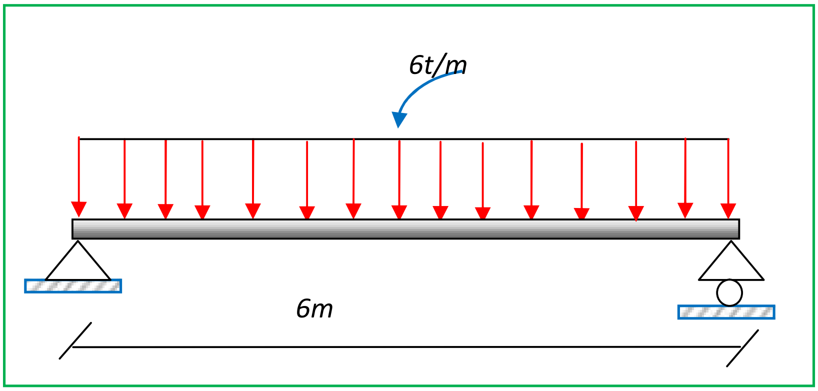

Figure 3.

Solution 3:

The equation of Elastic curve was found with definition (5) in previous problem.

$$y = \frac{q.l^4}{24EI}\left[(\frac{x}{L})^4 - 2(\frac{x}{L})^3 + (\frac{x}{L})\right]$$

$$y(2) = \frac{60.(600)^4}{6.10^6 4.10^5}\left(\frac{2}{6}\right)^4 - 2.\left(\frac{2}{6}\right)^3 + \left(\frac{2}{6}\right) = y(2) = 0{,}036\,cm$$

$$y(3) = \frac{60.(600)^4}{6.10^6 4.10^5}(3)^4 - 2.\left(\frac{3}{6}\right)^3 + \left(\frac{3}{6}\right) \Rightarrow y(3) = 0{,}042\,cm$$

$$y(4) = \frac{60.(600)^4}{6.10^6 4.10^5}\left(\frac{4}{6}\right)^4 - 2.\left(\frac{4}{6}\right)^3 + \left(\frac{4}{6}\right) \Rightarrow y(4) = 0{,}036\,cm$$

these results are given in table 1.

Table.1 Deflactions of the beam

I(cm^4)	**E(kg/cm^4)**	**x(m)**	**y(cm)**
4.10^5	6.10^6	2	0,036
4.10^5	6.10^6	3	0,042
4.10^5	6.10^6	0,042 4	0,036

Example 4.

(In the figure 4) If the beam which loading condition is given of is concrete and $I=1,6.10^5 cm^4$, $E=2.10^6 kg/cm^2$ then let`s find the displacement values of x=1m,x=2m and x=3m .

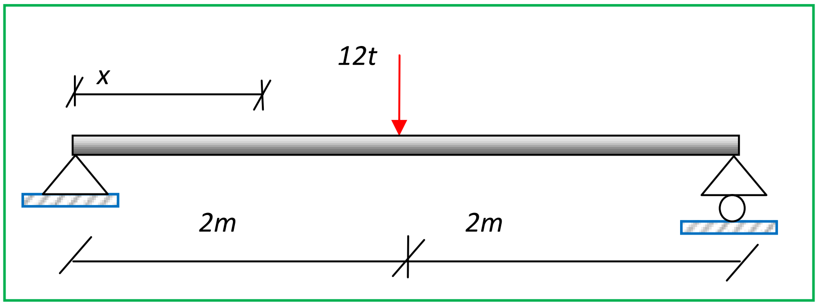

Figure 4.

Solution 4.

The general elastic curve equation of simple beam under the condition of having conservative load was founded by equation (3).

$$y = \frac{P.L^3}{48.EI}.\left[3.\left(\frac{x}{L}\right) - 4\left(\frac{x}{L}\right)^3\right]$$

$$y(1) = y(3) = \frac{12000.400^3}{48.1{,}6.10^5.2.10^6}.\left[3.\left(\frac{1}{4}\right) - 4\left(\frac{1}{4}\right)^3\right] = 0{,}036 \text{ cm}$$

The displacement of 2m can be found with definition (4).

$$y = \frac{P.L^3}{48.EI}. = \frac{12000.400^3}{48.1{,}6.10^5.2.10^6} = 0{,}042 \text{ cm}$$

CONCLUSION

Note that five of the problems are based on *cantilever* beams where the beam is held rigidly at one end and is unsupported at the other end. The boundary conditions in this case are that at the built-in end both rotation and deflection will be zero.

The major difference between the needs of current and past civil engineers relates to the techniques available for solving complex problems. For example, we believe that it is important for students to understand that integration can be used to determine the beam deflection, and that differentiation can be used to determine the slope of a curve at a particular location. Displacement on conservative or uniform loaded simple beam is obtained by simple mathematical equations.

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