A Comparison Study of Multi-Objective Metaheuristic Techniques for Continuous Review Stochastic Inventory System

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ABSTRACT

Supply chain management which involves managing the flow of material and information from sources to customers has been one of the most challenging issues facing both the academicians and the practitioners for years. Inventory control is a crucial part of tactical decision level affecting the performance of supply chain in distribution and production. The main focus of this study is to compare the performance of different multi-objective metaheuristic techniques to optimize inventory parameters for single-product continuous review stochastic inventory system with transportation costs. The simulation-based optimization method is used to solve the problem by combining the simulation model and metaheuristic algorithms in order to determine the inventory policy taking into account two conflicting objectives: customer service level and total inventory cost. We build a discrete event simulation model to evaluate the objective function of the problem. The Metaheuristic techniques such as the genetic algorithm and particle swarm optimization are applied to search the solution space. The results obtained by all these proposed techniques are compared and the effectiveness of each technique has been illustrated.

INTRODUCTION

One of the most important aspects affecting the performance of a supply chain is the management of inventories. Managing inventory in complex supply chains is typically difficult, and may have a significant impact on the customer service level and system-wide cost. For this reason, it has been studied in considerable depth. Many researchers have dedicated themselves to deal with inventory control involving more than one objective over the past several decades. The role of the inventory management is that try to find a way to minimize the level of their stocks without negatively impacting availability or customer service level. Due to the complication of real inventory system, single mathematic model cannot describe stochastic inventory problems. Now research on optimization algorithm has been used to resolve the multi-objective optimization problems.

Simulation-based optimization provides a structured approach to the system design and configuration when analytical expressions are very complex and mathematically intractable. In recent years researchers have attempted to combine simulation and optimization procedures to provide a complete solution [1]. Simulation-based optimization is an attractive combined strategy which integrates optimization techniques into simulation analysis where the simulation model is regarded as a function.

This paper presents a performance comparison of metaheuristics to solve the inventory problem under continuous review policy by using simulation-based optimization approach. The remainder of this paper is organized as follows. In the next section, we review the related literature on inventory optimization by using metaheuristics techniques. Section 3 introduces the multi-objective inventory control. In section 4 we explain how simulation optimization methods have been applied in inventory control. Section 5 shows the numerical results along with discussions. Finally, section 6 summarizes the conclusions.

LITERATURE REVIEW

One of the most important aims of multi-objective optimization is to obtain an efficient solution taking into accounts the balance among several conflicting objectives. One of them is the use of evolutionary algorithms to solve multi-objective problems. Evolutionary algorithm is a particularly important subset of random-based solution space searching meta-heuristic methods. A literature review survey of evolutionary multi-objective optimization techniques in all the fields of science, engineering and technology, and also a brief general concept the analysis of past and current research performed on evolutionary multi-objective optimization is presented in [2].

Although simulation is not an optimization tool by itself, simulation-based optimization is an active area of research in stochastic, uncertain environments. There are different ways to optimize supply chain inventory parameters represented by means of a simulation model. A computer aided simulation model and the genetic algorithm for optimization of the control parameters in stochastic inventory process is presented [3]. A new solution procedure based on GAs to find the set of Pareto optimal solutions for multi-echelon supply chain cyclic planning and optimization is proposed in [4]. They built a simulation model to estimate system performance while GAs uses the responses generated by the simulation model to control the optimization process. [5] considered a multi-echelon inventory system composed of five serial stages and applied simulation-based optimization to set the optimal policies. The performance of a single product serial supply chain operating with a base-stock policy is studied and optimized the base stock levels in the supply chain in [6]. They proposed a simulation-based genetic algorithm to optimize the base stock levels with the objective of minimizing the supply chain cost.

An evolutionary computation to find the non-dominated control policies for stochastic multi-objective (R, Q) inventory control systems is presented by [7]. He proposed two evolutionary optimizers, multi-objective electromagnetism-like optimization (MOEMO) and multi-objective particle swarm optimization (MOPSO). Computational results show that the evolutionary Pareto optimizers generate well and fast approximate the non-dominated policies in term of lot size and safety stock. Enhanced particle swarm optimization (EPSO) [8] is applied to solve a three stage inventory problem to address the outsourcing issues with different shipment policies. Particle Swarm Optimization algorithm is proposed to overcome in maintaining the optimal stock levels in the supply chain in [9]. Their proposed methodology reduced the total supply chain cost as it undoubtedly established the most probable excess stock level and shortage level along with the consideration of lead time in supplying the stocks as well as raw materials that are required for inventory optimization.

Problem formulation

We are interested in solving problem of the type:

$$\min\{z_1 = f_1(x), z_2 = f_2(x), \ldots, z_m = f_m(x)\}$$

$$g_i(x) \le 0, i = 1, \ldots, q$$

$$h_i(x) = 0, \; j = q+1, \ldots, k$$

where x is the decision variable, f_m is objective m, g_i is inequality constraint and h_j is equality constraint j. For the minimization multi-objective inventory control problem under continuous review system, a decision variable is $x_i = (R_i, Q_i)$. Two objective functions are introduced in the optimization problem. The average total inventory cost which includes holding cost, shortage cost and ordering cost. The second objective function is to maximize the service level. When the (R, Q) inventory system is simulated for a period of n days, the total cost can be written as:

$$TC(R,Q) = \sum_{t=1}^{T} (I(t) \times h + B(t) \times p + d \times A)$$

$$\min C(R,Q) = \frac{TC(R,Q)}{n}$$

$$\max \text{¥} = 1 - \frac{\sum Backorder}{\sum Demand}$$

where,

t:	time index
I(t):	on-hand inventory at the end of time period t
B(t):	backordered demand at the end of time period t
h:	unit holding cost per item per time
p:	unit shortage cost per item per time
A:	ordering cost

$$d_i = \begin{cases} 1 & \text{if an item ordered by stock point in specified time} \\ 0 & \text{else} \end{cases}$$

Performance measures are the average total system cost per unit time which includes holding cost, shortage cost, ordering cost and the service level of the γ-type which is defined as one minus the amount of cumulative backorders divided by the cumulative demand during the simulation periods [10].

One of the most challenging problems in multi objective optimization is related to the identification of the Pareto optimal set. A Solution X in objective space is said a Pareto-optimal (non-dominated solution), if and only if there is no other solution Y in the search space which could dominate X. In other words, X dominates Y, if X is better than Y in at least one objective function and not worse with respect to all other objective functions [11]. The set including all Pareto-optimal solutions is termed the Pareto set as shown in Figure 1.

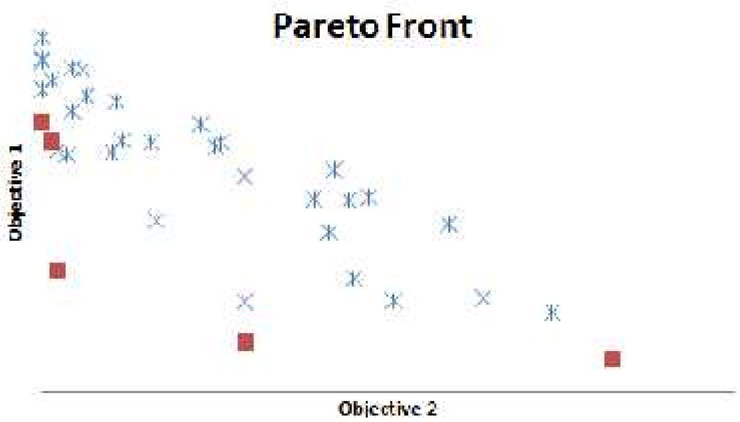

Figure 9 The Pareto front of a set of solutions in a two objective space

SIMULATION OPTIMIZATION approach based on metaheuristics

Although simulation technology has been widely used as one of the most powerful techniques for analyzing and improving enterprises supply chain management and logistics operations, simulation provides no concrete solutions to optimization problems. Users need to evaluate many feasible solutions in order to find a good solution to the problem. The aim of simulation based optimization is to integrate optimization techniques into simulation analysis. In mathematical programming the decision variables are assigned in an analytical function of decision variables. This function is known as the objective function. In simulation-based optimization, the performance measure becomes one (or a function of several) of the responses generated by a simulation model [1][12]. Figure 2 illustrates the general scheme of the simulation-based optimization procedure and interaction between the simulation model and the optimization model.

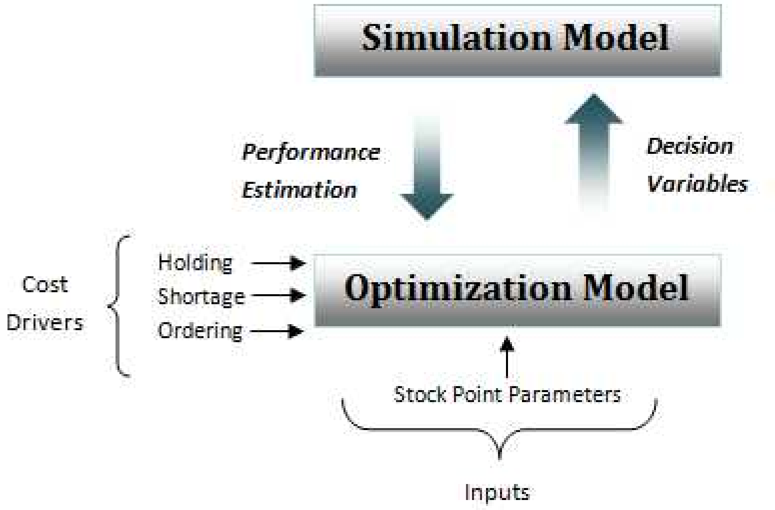

Figure 10 Simulation-Based Optimization Scheme

In the simulation optimization procedure, the control parameters which are obtained from optimization tool are assigned in simulation tool to update the objective functions for a pre-specified period of time. At the end of the simulation run the service level values and related inventory costs for the stock point are sent to optimization tool from simulation. Optimization tool then uses this information to check if all the constraints are satisfied and the process iterates until there is no further improvement in the solution or an iteration limit is reached.

NUMERICAL EXAMPLE

Discrete-even simulation tool of inventory and all multi-objective optimization algorithms such as MOPSO (multi-objective particle swarm optimization) and NSGA-II (Non-Dominated Sorting Genetic Algorithm) have been coded by using Microsoft Visual C-Sharp programming language. To compare the meta-heuristic algorithms we presented inventory problem with different coefficient of variance (CV) for daily demand. The run length of the simulation experiment is 300 days and 5 replications. We first use the normal distribution with different standard deviations by following numerical example:

« = 50 units per day, ± = 10 and 20 A = 1000 per order
h = 1 per unit per day, p = 5 per unit per day, and L(lead - time) = 2 days.

Figures 3 and 4 show non-dominated solutions received from optimization tool for high and low coefficient of variance. As is illustrated from Figures 3 and 4, the Pareto front of efficient solutions is well populated for both optimization techniques and the daily inventory cost seems to vary between the values of 280 and 320 as service level vary between 0.9 and 1. For high CV, the daily inventory cost varies between of 280 and 350.

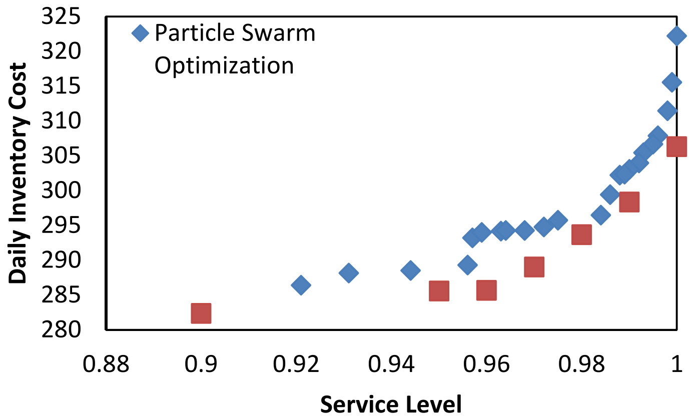

Figure 11 Non-dominated solutions for CV = 0.2

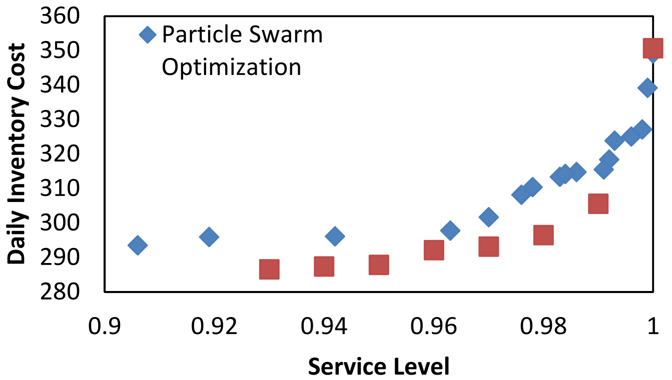

Figure 12 Non-dominated solutions for CV = 0.4

The result of different coefficient of variance is reported in Table 1. Clearly, and supported by the results in Table 1, NSGA-II's output approximates better to lower cost than MOPSO.

Table 3 Average results of non-dominated solutions for the different standard deviation

		R	Q	Cost	Service Level
σ=10	NSGA-II	109	348	291	0,96

	MOPSO	125	326	300	0,97
σ=20	NSGA-II	118	335	298	0,96
	MOPSO	137	337	314	0,98

In many inventory models, the demand process is described by the normal distribution, but many scholars have criticized the use of this normal approach [13]. If the coefficient of variation of demand is too large (greater than 0.5), the normal approximation will lead to be a negative value becomes too large to be ignored. In this study, gamma and lognormal distributions are chosen to represent the demand process of high coefficient of variance (CV=1).

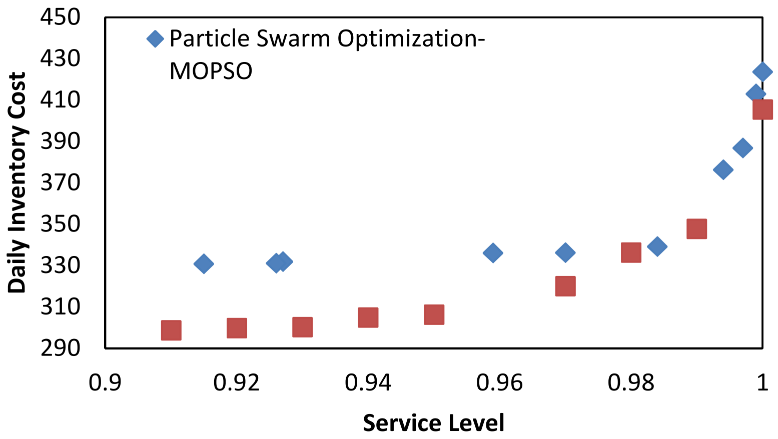

Figure 13 Non-dominated solutions for Lognormal distribution (CV=1)

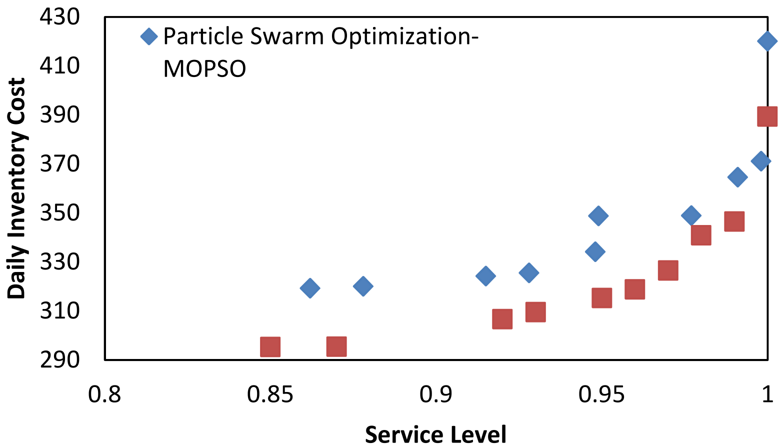

Figure 14 Non-dominated solutions for Gamma distribution (CV=1)

To study the effects of the variability of daily demand and the types of probability density function used on the cost performance of the inventory system, we used same mean value with previous experiment such as mean and standard deviation equals 50. Figure 5 and 6 sets of non-dominated solutions for lognormal and gamma distribution with same coefficient of variance. The result of different demand distribution is reported in Table 2. From results, we conclude that the form of daily demand distribution has an influence on the control parameters for high coefficient of variations.

Table 4 Average results of non-dominated solutions for the different demand distribution (CV=1)

		R	Q	Cost	Service Level
Lognormal	NSGA-II	122	424	324	0,95
	MOPSO	174	386	360	0,97
Gamma	NSGA-II	119	408	324	0,94
	MOPSO	156	344	348	0,94

Conclusion

This work presented a comparison study of optimization approaches based on metaheuristics to solve the stochastic inventory problem under continuous review policy. One of the objectives was to utilize simulation and optimization methods. The concept of multi-objective particle swarm optimization (MOPSO) and Non-Dominated Sorting Genetic Algorithm (NSGA-II) approaches were tested to find a set of near optimal solutions for multiple objectives by using simulation-based optimization. We also studied the effects of the variability of daily demand and the types of probability density function used on the cost performance of the inventory system.

This paper illustrated that multi-objective metaheuristics techniques can be used to successfully model a stochastic inventory problem. The proposed models can be extended in other inventory policies and different inventory input configurations.

REFERENCES

[1] Ammeri, A. Chabchoub, H. Hachicha, W. and Masmoudi, F. (2010) A Comprehensive Literature Classification of Simulation optimization Methods. *The 9th International Conference on Multiple Objective Programming and Goal Programming*, Tunisia.

[2] Kadadevaramath, R.S. and Mohanasudaram, K.M. (2007). Multi-Objective Trade-off Analysis: State of art: Methods, Applications, and future Research Directions in Production and Operations Management. *Manufacturing Engineering*, Issue: 2, pp: 70-78.

[3] Bona, K. (2004). Simulation Supported Optimization of Inventory Control Processes by Application of Genetic Algorithm. *Proceedings of 16th European Simulation Symposium.*

[4] Merkuryeva, G. and Napalkova, L. (2009). Supply chain cyclic planning and optimization. *Simulation-based studies in logistics: Education and applied research.* Berlin: Springer-Verlag, London, UK.

[5] Köchel, P. and Nieländer, U. (2005). Simulation-based optimization of multi-echelon inventory systems. *International Journal of Production Economics,* Vol.93.94, 505-513.

[6] Daniel, J. R.S. and Rajendran, C. (2005). A simulation-based genetic algorithm for inventory optimization in a serial supply chain. *International Transactions In Operational Research*, 101–127.

[7] Tsou, C (2009). Evolutionary Pareto optimizers for continuous review stochastic inventory systems. *European Journal of Operational Research,* 195, 364–371.

[8] Hiremath, N. C. Sahu, S. and Tiwari, M. K. (2010). Optimization for Efficient Supply Chain Using Swarm Intelligence: an Outsourcing Perspective. *Proceedings of the World Congress on Engineering*, Vol III.

[9] Narmadha, S. Selladurai, Dr.V. and Sathish, G. (2010). Efficient Inventory Optimization of Multi Product, Multiple Suppliers with Lead Time using PSO. *International Journal of Computer Science and Information Security,* Vol. 7, No. 1.

[10] Diks, E.B. De Kok, A.G. and Lagodimos, A.G. (1996). Multi-echelon systems: A service measure perspective. *European Journal of Operational Research* 95, 241-263.

[11] Yu, X., & Gen, M. (2010). Introduction to Evolutionary Algorithms. *Springer-Verlag,* London.

[12] Mele, F.D. Guillen, G. Espuna, A. and Puigjaner, L. (2006). A Simulation-Based Optimization Framework for Parameter Optimization of Supply-Chain Networks. *Industrial & Engineering Chemistry Research*, 45, 3133-3148.

[13] Tyworth, J.E. and O'Neill, L. (1997). Robustness of the Normal Approximation of Lead-Time Demand in a Distribution Setting. ***Naval Logistics Research***, 44, 165-186.