

RICE UNIVERSITY

PERMUTATION

by

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MASTER OF ARCHITECTURE

APPROVED, THESIS COMMITTEE



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May, 1997

# ABSTRACT

## Permutation

by

Christopher Nichols

This is a story of two spatial designers: The Mathematician and the Architect. In the four dimensional space of a Mathematician, most physical realities must be ignored in order for the space to remain pure. The visual, one of the most powerful tools of the Architect, can coexist within the Mathematician's four dimensional world without affecting its purity. However, the Architect must ignore his/her physical realities, such as gravity, mass, and human scale. The Mathematician establishes the rules within which the Architect must operate. The Architect manipulates form through a structure that is defined by the Mathematician. The Architect explores spatial qualities through the tools that the Mathematician gives him/her. The Architect tries to understand the meaning of the form that both of them have created. He/she pushes the form in order for it to achieve spatial qualities. Together, the Mathematician and the Architect try to understand the meaning of such spatial qualifiers as: big/small, wide/narrow, inside/outside, and light/dark. The Architect starts to understand why spaces achieve these qualities in his/her physical world.

## **Acknowledgments**

I would like to take the time to thank the people that helped me through the semester. The project would not have been possible if it were not for the guidance and support of my committee, which includes Lars Lerup, Michael Bell, and Farés el-Dahdah. I would like to thank Carlos Fighetti and Louis Delaura for helping me increase my processing power at a time of need. I would also like to thank Robert Sheh, Richard Odom, and Shisha van Horn for their assistance, informal advise, and friendship that they gave me throughout the project.

## Preface

The following thesis is discussed in terms of a conflict that exists within my education. Having a background in Mathematics, I entered the field of Architecture. In doing so, I was forced to reevaluate how I make decisions in terms of problem solving.

The preliminary research involved how each of the fields address problem solving. In terms of Architecture, I research the issue of program. This involved how one defines program as well as how program is addressed. In terms of Mathematics, I was interested in issues of abstraction and language. These were important in terms of understanding the essence of how one establishes a mathematical proof, which is the key to problem solving.

This document establishes the existence of two fictional characters: a Mathematician and a Designer. It discusses the process by which the project evolved. The characters represent a train of thought that helped guide the project to the next level.

# Introduction

Two spatial explorers, a Mathematician and a Designer, set a project in motion. Their goal was to understand the nature of a space in an abstract setting.

To the Mathematician an abstract setting was a world that was free of assumptions. Any assumption that would be established on the setting would influence the research and have an impact on the process that was involved. To the Designer, the abstract setting allowed him to explore certain spatial characteristics.

The Designer was interested in finding a threshold within which one can read the essence of a space.

Rather than deal with a problem, they would find a system that could help them establish the root of their research. Computer graphics have aided both Mathematicians and Designers in terms of visualization. To the mathematician, the computer offered the benefits of being free of any assumptions until they are defined by the user. To the Designer, computers allowed one to quickly edit parameters in order to explore different iterations of a design.

They begin their journey in search of an arbitrary starting point. The first goal was to find a system of “building” within the abstract setting. The only requirement for this system was that it not make any assumptions on the space. With this in mind, they begin their journey in search of a mode of operating.

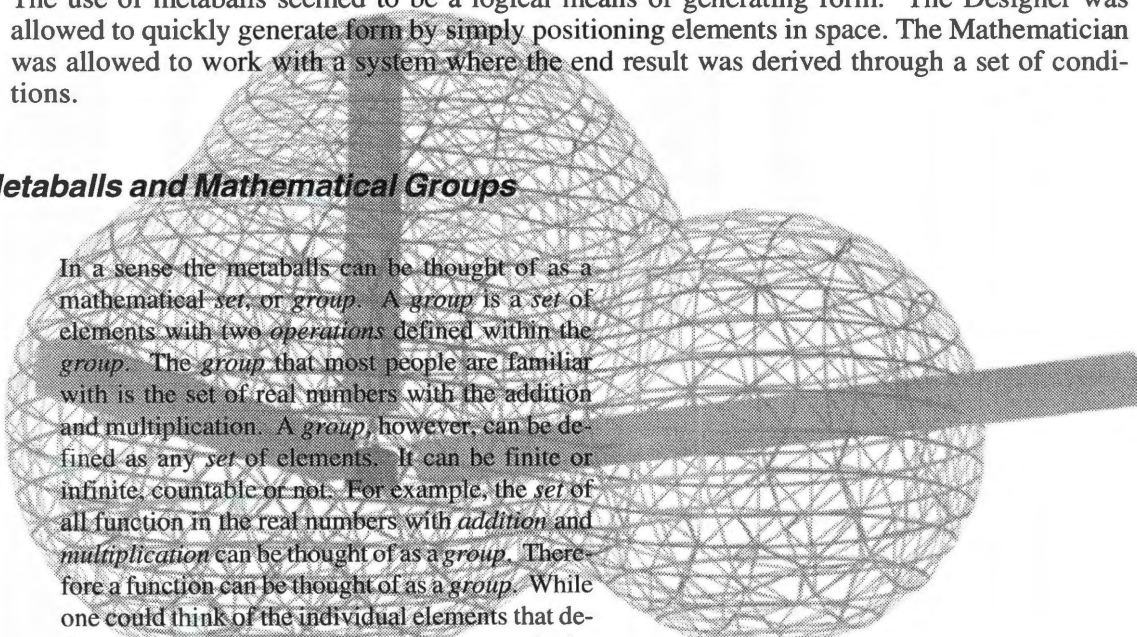


## Metaballs

In order to set up a system with which a design method can be achieved, a set of members were placed. Each of these members were then defined through metaballs. A metaball is a mesh which builds itself as a type of "field" through the use of other objects. In order to define a metaball, one needs to have a number of objects which are defined as being part of the metaball. Each object has an influence on the mesh according to its location, size and proximity to other objects. To computer animators, the tool serves as a means of quickly generating meshes of smooth forms. They are generally used as many small balls positioned close to one another. The other advantage of using metaballs for character animations is that the balls can be moved during an animation. The mesh is then regenerated for every frame. This allows one to easily animate a smooth form.

The use of metaballs seemed to be a logical means of generating form. The Designer was allowed to quickly generate form by simply positioning elements in space. The Mathematician was allowed to work with a system where the end result was derived through a set of conditions.

### **Metaballs and Mathematical Groups**



In a sense the metaballs can be thought of as a mathematical *set*, or *group*. A *group* is a *set* of elements with two *operations* defined within the *group*. The *group* that most people are familiar with is the set of real numbers with the addition and multiplication. A *group*, however, can be defined as any *set* of elements. It can be finite or infinite, countable or not. For example, the *set* of all function in the real numbers with *addition* and *multiplication* can be thought of as a *group*. Therefore a function can be thought of as a *group*. While one could think of the individual elements that define the metaball mesh as being the elements in the *group*, a better and possibly more interesting *set* of elements would be the *set* of distances between the elements. Therefore, the resulting mesh is simply an expression of the relationship of the overall distances.

It is important to note that distance is not a three-dimensional measurement, but a four-dimensional one, since the elements move over time. That is to say that they have a length in terms of time that can be measured like any of the other dimensions. We will refer to this *set* as the *distance function set*. Therefore, time dimension is not another variable, it is a part of overall distance measurement which is the *distance function set* defined in the *group*.

Normally, the elements that are used to generate

the field are not visible when rendered since they only serve to generate the mesh. By duplicating the elements, one can add them back in visually so that the position of the elements is understood when studying how the mesh reacts to the position of the individual elements.

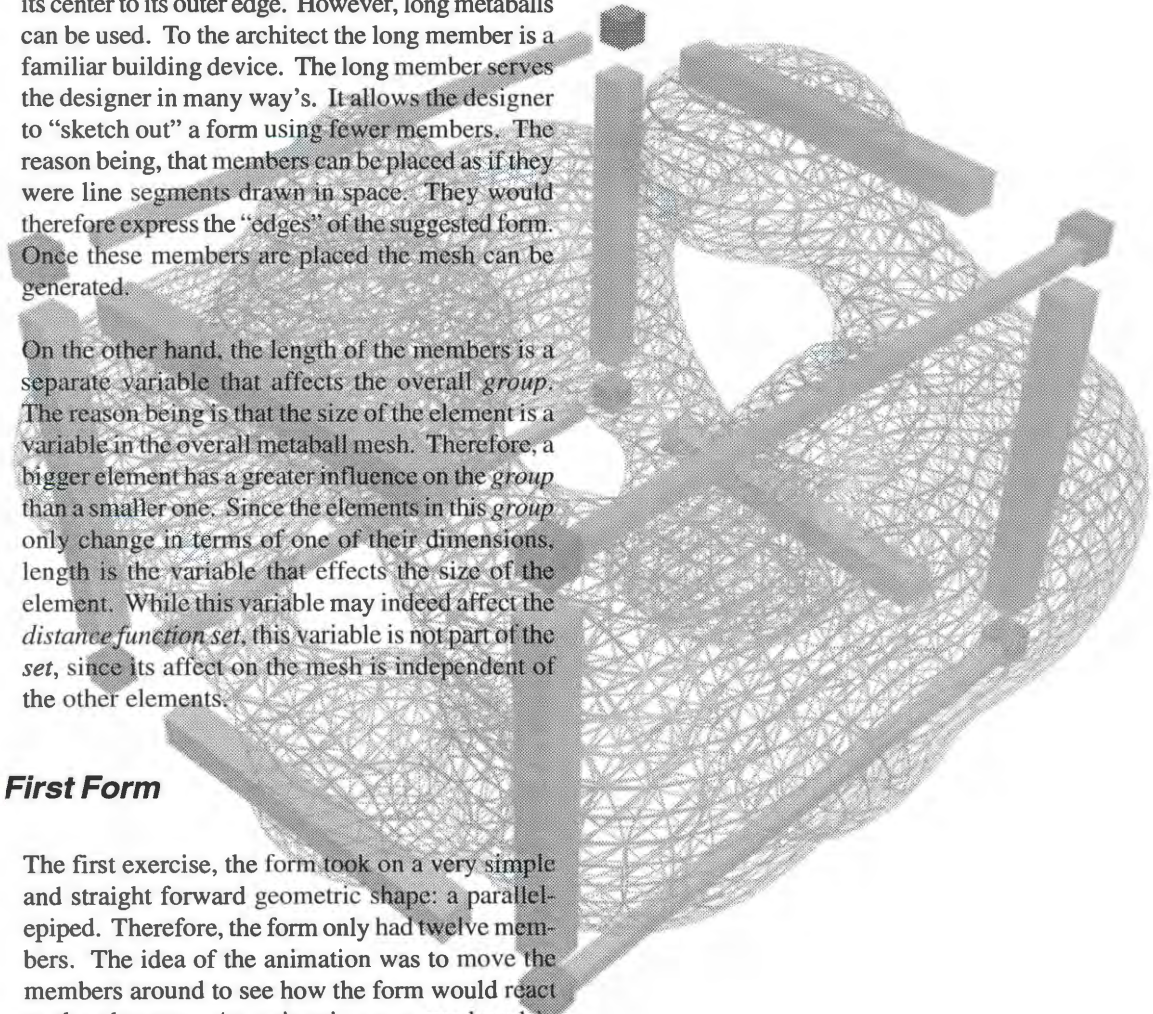
The elements used are not limited to small balls. Due to its simplicity, one generally does not use anything but small balls. However, one can also use other shapes to generate the field. The ball has the characteristic of being the same distance from its center to its outer edge. However, long metaballs can be used. To the architect the long member is a familiar building device. The long member serves the designer in many ways. It allows the designer to "sketch out" a form using fewer members. The reason being, that members can be placed as if they were line segments drawn in space. They would therefore express the "edges" of the suggested form. Once these members are placed the mesh can be generated.

On the other hand, the length of the members is a separate variable that affects the overall *group*. The reason being is that the size of the element is a variable in the overall metaball mesh. Therefore, a bigger element has a greater influence on the *group* than a smaller one. Since the elements in this *group* only change in terms of one of their dimensions, length is the variable that effects the size of the element. While this variable may indeed affect the *distance function set*, this variable is not part of the *set*, since its affect on the mesh is independent of the other elements.

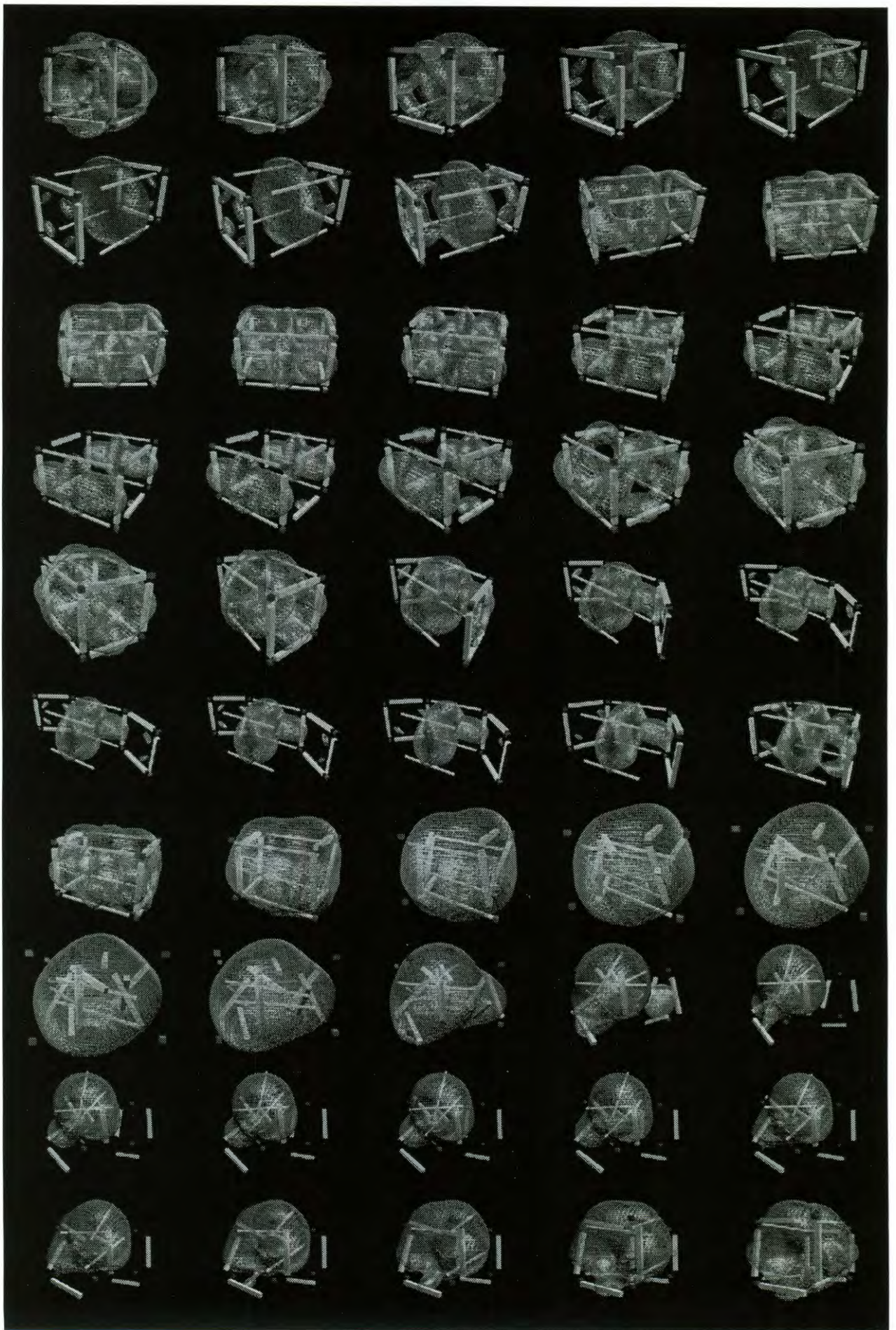
### **The First Form**

The first exercise, the form took on a very simple and straight forward geometric shape: a parallelepiped. Therefore, the form only had twelve members. The idea of the animation was to move the members around to see how the form would react to the changes. An animation was produced in order to observe these changes take place while the members moved from their initial location. Small cubes were added to the corners of the parallelepiped in order to have a reference for the edge of the form.

The first two moves involved transposition by moving the ends and the sides of the parallelepi-



Page 3: Animation stills of the first form. 250 frames  
320x240 pixels at 15 fps.





ped out. The third movement involved rotation. Here the caps were rotated out from the one edge of the object. Last, the members were scattered in a "random" fashion. Finally the members return to their initial location, and the animation looped.

It was clear that the proximity of the members had the greatest influence on form. So much so, that if the members moved to far from the form, the mesh around the object would virtually disappear. The individual members would also have a great influence on the shape of the mesh once it was moved. Even the section of the mesh that was furthest from the member would be visibly affected by the change. This may have been exaggerate by the fact that there were only eight members used to express the entire form.

The member were also far enough from each other that they lied at a sort of "threshold" of change. This can be seen at the last stage of the animation, when the members come together in a random fashion. Here the mesh's form seemed to "stabilize," and the change was not as radical as it was when they were positioned at the edged.

### ***The Second Form***

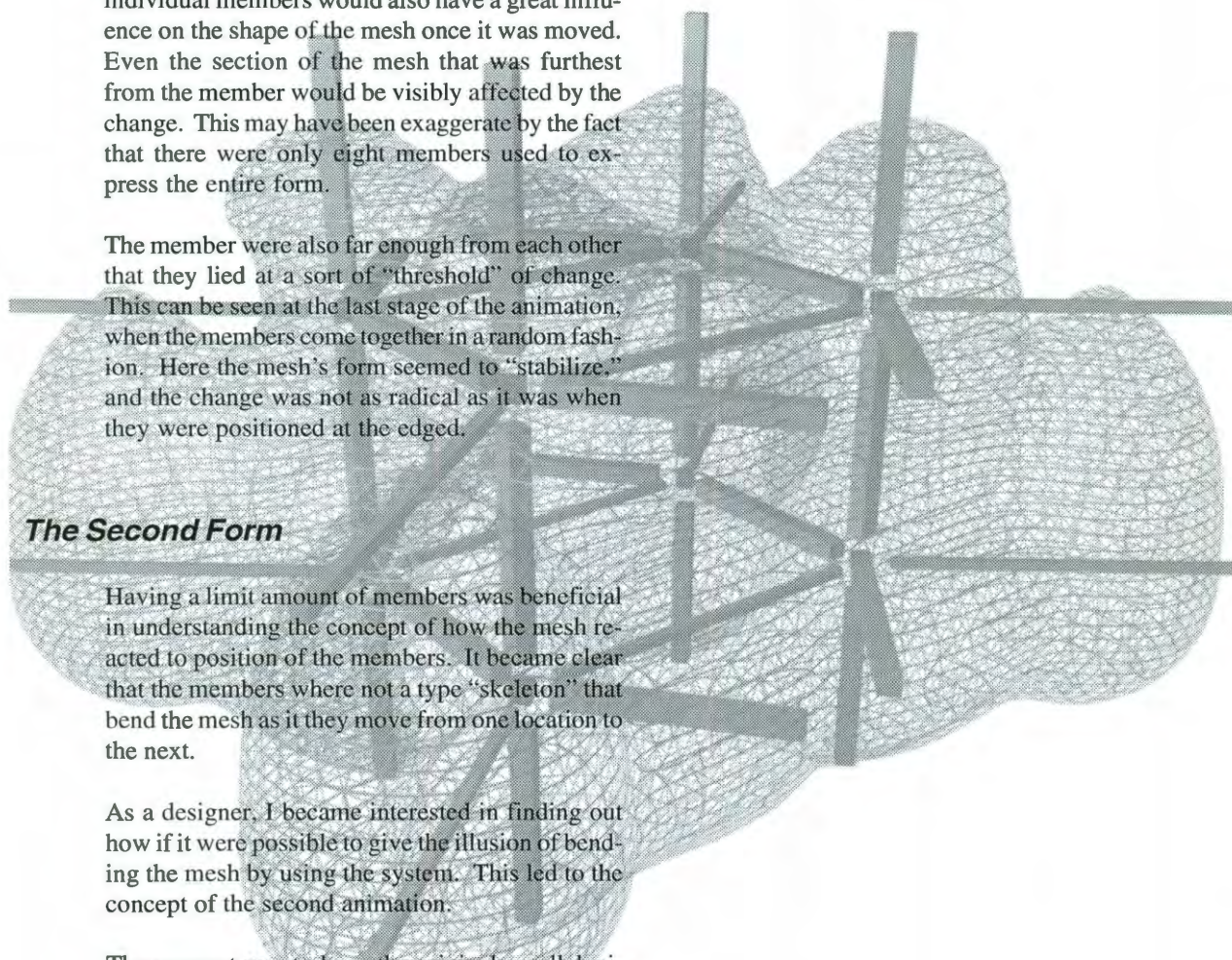
Having a limit amount of members was beneficial in understanding the concept of how the mesh reacted to position of the members. It became clear that the members where not a type "skeleton" that bend the mesh as it they move from one location to the next.

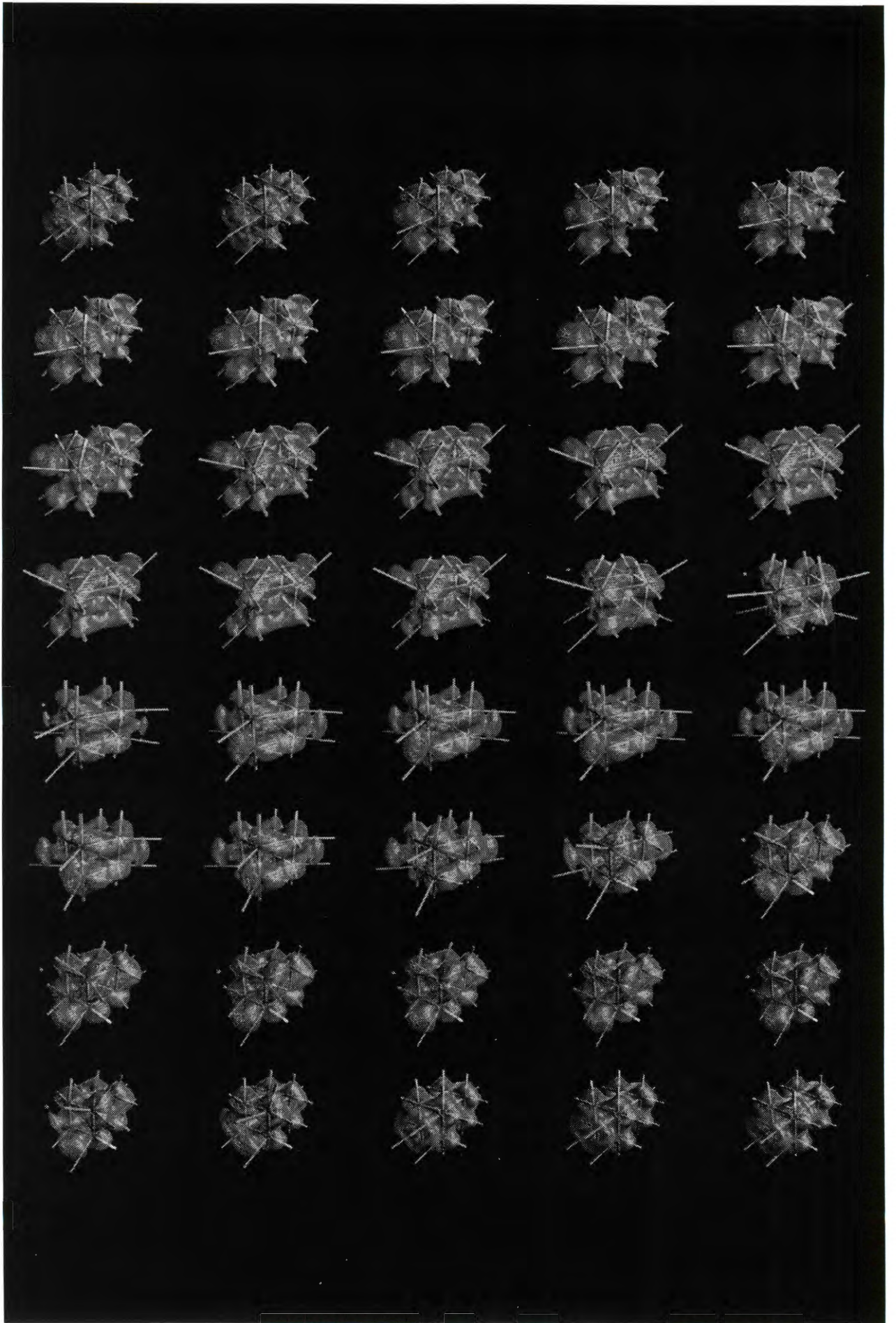
As a designer, I became interested in finding out how if it were possible to give the illusion of bending the mesh by using the system. This led to the concept of the second animation.

The concept was to keep the original parallelepiped, and add three member to every corner. These members would be the only members that moved. The idea was that by rotating the corners through these extra members, the mesh might bend in that direction.

While the mesh was not as radically affected by the movement as in the first form, it did not have give the illusion of bending. The major reason for this is that the mesh changes it configuration rather then simply move it vertices.

*Page 5: Animation stills of the second form. 200 frames  
320x240 pixels at 15 fs.*





## Enter the Designer/Explorer

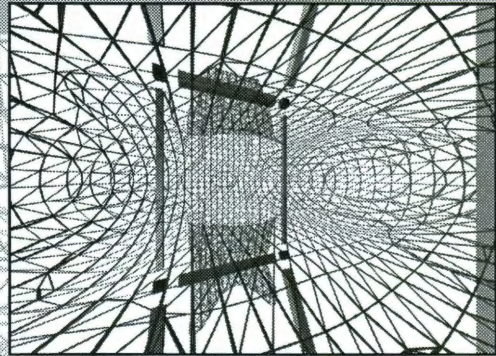
While the two first exercises/animations gave the designer an idea of how the forms functioned, there seemed to be a need to explore these changes from a closer perspective. The camera now needed to enter the mesh and to observe the changes from the inside. The act of entering the mesh made it possible for the observer to see the mesh as space rather than object or form.

The first "interior" animation done was of the second form. The designer wanted to observe the more subtle, or less radical, changes that took place inside the form as the corner members were moved. These changes were much more dramatic as they took place inside.

One could see the interior mesh restructuring itself. It was clear that the form was not "bending." From frame to frame the structures would find a new way to form itself. This was a shape that seemed to find its form in a new fashion. Its order was not of anything that would be held together by the constraints of atomic bonds. Its order came from something else. There was a need to explore this new ordering system in depth.

In order to enter the first form, one needed to deal with a constantly shifting and constraining space. There was a need to use a very wide angle lens. In this way, fast and sudden movements through the space would be greatly reduced. It would also allow the viewer to observe all aspects on the space, and allow him/her to gain a point of reference.

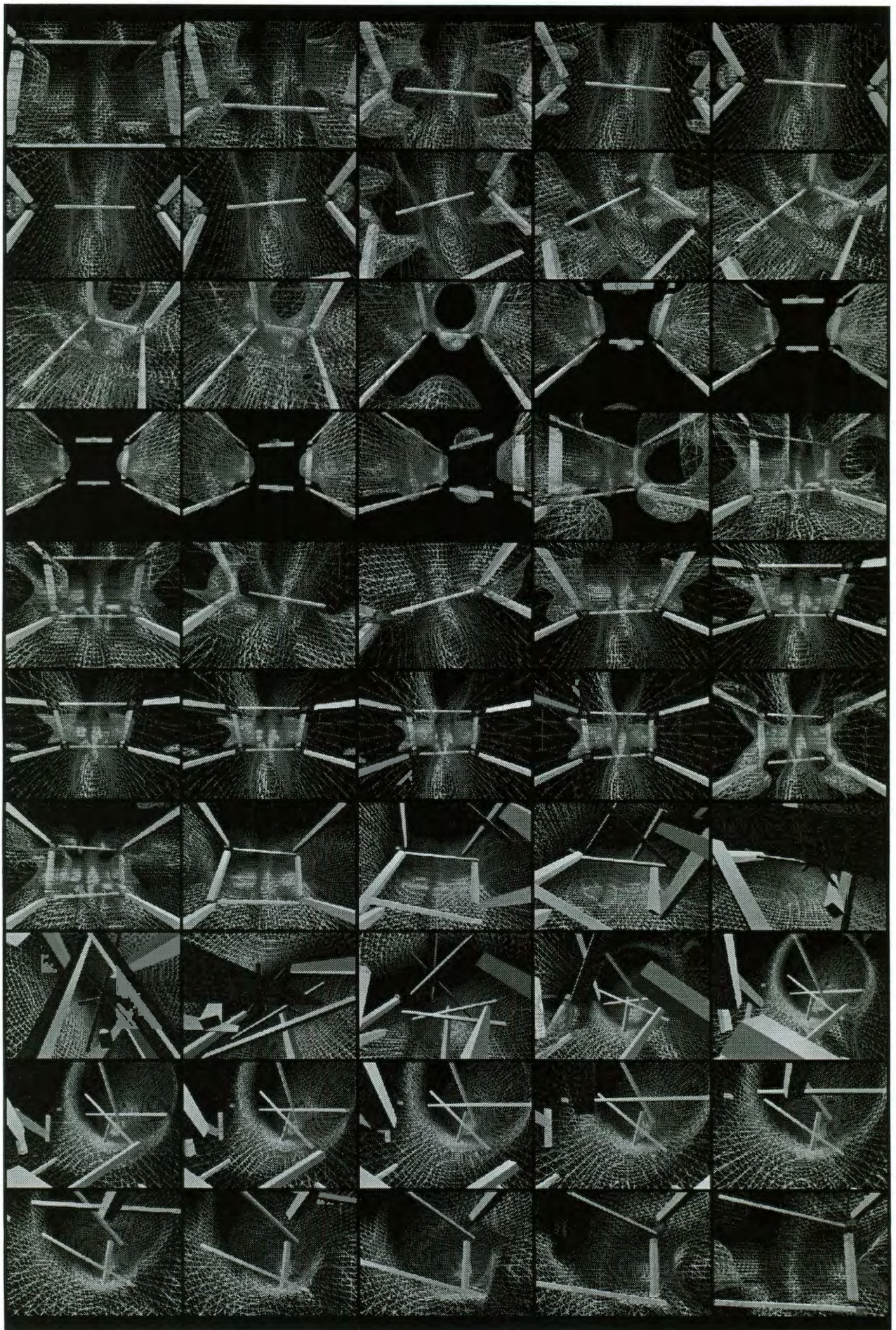
Again the mesh seemed to be of a new system. This feeling was explained when the mesh would join itself. In doing so, the mesh formed a new topology. It became clear therefore that the structure was not homeomorphic over time.



*Page 7: Animation stills of the interior of the second form. 250 frames 320x240 pixels at 15 fps.*

*Page 8: Animation stills of the interior of the first form. 250 frames 320x240 pixels at 15 fps.*





## Aesthetics

While largely undeveloped, this new system seemed to offer both the Mathematician and the Designer the ability to work and design together. The first issue that was addressed by the Designer was the issue of aesthetics. While the mesh's development was sophisticated, it lacked the aesthetics that the Designer thought would be fitting.

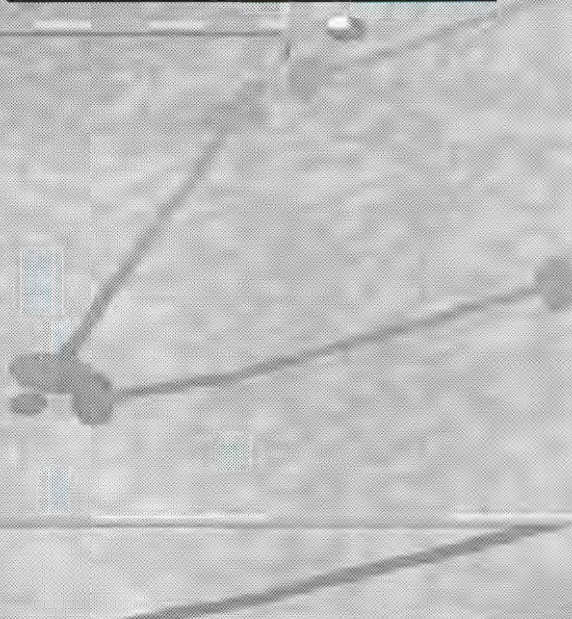
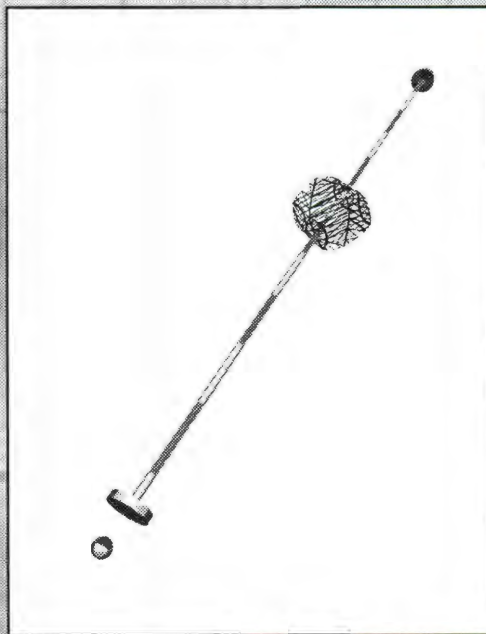
### The Members

To the Designer, the elements represented a type of three-dimensional line drawn in space. Also each member had specific length that had an impact on the formation of the mesh. A new representation of the member was therefore designed. The new design was inspired by the beauty of the perfectly hand-drawn line on a piece of paper. The hand-drawn line would be of even thickness and tone throughout its length. The ends of the line would be slightly thicker and bolder by the simple action of applying more pressure to the pencil and reversing the direction of the pencil. The result is a line that is immediately understood as a segment of a particular length. The heavier ends of the line capture the essence of the line and give the line visual brackets.

The Designer came up with a thin cylinder that would be capped at both ends with two small, thin disks. The idea was that one could imagine "holding" the member between two fingers, giving the member a similar visual bracket that the hand-drawn line would have.

The issue of length was addressed through texture. In a similar manner that a yard stick gives scale to an archeological photograph, a grid would be used to give the viewer an idea of a member's relative length compared to other members. Here the grid used orange and white.

Additionally the corner pieces were redesigned. The large squares were replaced by small spheres painted in a blue and white grid pattern similar to the members. They were reduced in size since they do not have any impact on the formation of the mesh. They were made as spheres since they do have an impact on the members as far as their kinetics. When a member rotates, its pivot point



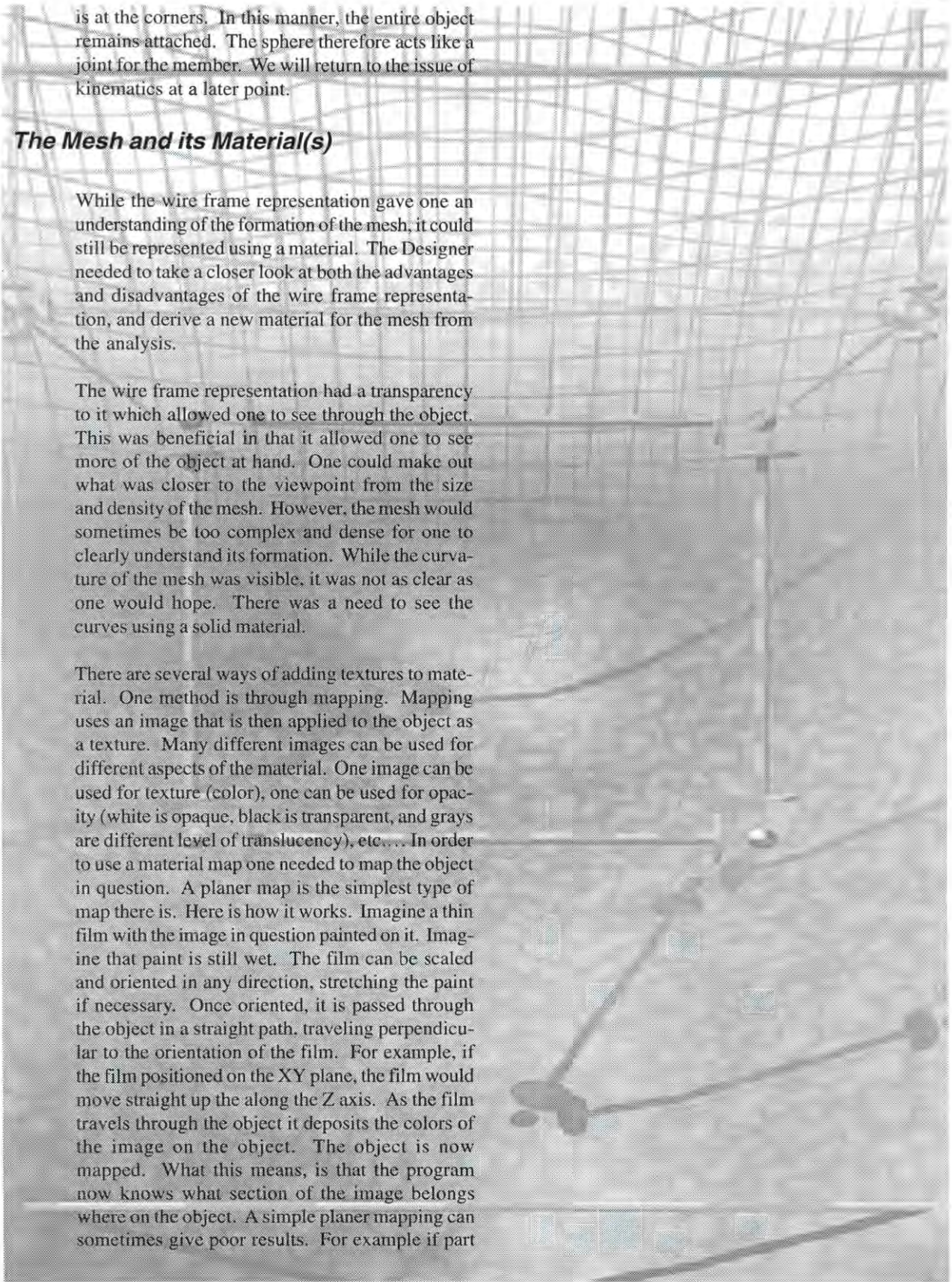
is at the corners. In this manner, the entire object remains attached. The sphere therefore acts like a joint for the member. We will return to the issue of kinematics at a later point.


### ***The Mesh and its Material(s)***

While the wire frame representation gave one an understanding of the formation of the mesh, it could still be represented using a material. The Designer needed to take a closer look at both the advantages and disadvantages of the wire frame representation, and derive a new material for the mesh from the analysis.

The wire frame representation had a transparency to it which allowed one to see through the object. This was beneficial in that it allowed one to see more of the object at hand. One could make out what was closer to the viewpoint from the size and density of the mesh. However, the mesh would sometimes be too complex and dense for one to clearly understand its formation. While the curvature of the mesh was visible, it was not as clear as one would hope. There was a need to see the curves using a solid material.

There are several ways of adding textures to material. One method is through mapping. Mapping uses an image that is then applied to the object as a texture. Many different images can be used for different aspects of the material. One image can be used for texture (color), one can be used for opacity (white is opaque, black is transparent, and grays are different level of translucency), etc. . . . In order to use a material map one needed to map the object in question. A planer map is the simplest type of map there is. Here is how it works. Imagine a thin film with the image in question painted on it. Imagine that paint is still wet. The film can be scaled and oriented in any direction, stretching the paint if necessary. Once oriented, it is passed through the object in a straight path, traveling perpendicular to the orientation of the film. For example, if the film positioned on the XY plane, the film would move straight up the along the Z axis. As the film travels through the object it deposits the colors of the image on the object. The object is now mapped. What this means, is that the program now knows what section of the image belongs where on the object. A simple planer mapping can sometimes give poor results. For example if part






of the object runs parallel to the path of the mapping, one will get "streaking." What this means is that same colors will run along the side of the object. This tends to make the object look poor. However, through a clear understanding of both the geometry and the image(s) used by the material, one can choose/design an appropriate image, and can carefully orient and scale the map on the object, in order to produce a good material.

The problem with the mesh is that it is regenerated for every frame. On a regular (permanent) object, when the object is scaled or distorted, the mapping information is altered along with it too. However, the mesh changes its shape all the time. It is essentially a new object for every frame. Therefore, the map has to be reapplied for every frame. The best means of mapping the object was to derive a system that would work for every frame. What was done was to pick a regular plane (the XZ plane), and apply the mapping along the Y axis. The map was positioned and scaled in every frame to the exact width and height of the mesh. The advantage that the mesh offered was that it usually had a great number of curves. Therefore, few or no faces in the object would be parallel since they were all parts of curves. Streaking would be minimal or even eliminated. This mapping was interesting to the Mathematician since it dealt with a system that is based on the formation of the object, and based on its position this respect to the XZ plane. It was interesting to the Designer since it left him with the challenge of only being able to manipulate the images used for the different maps.

Once the mapping system was set to the object, the material had to be composed. Essentially two types of image maps were used: the texture map (color), and the opacity map. The most important aspect of the material existed in the to transparency. The analysis of the benefits and disadvantages of the mesh revealed that one needed to use a combination of transparent and opaque surfaces. Due to the nature of the mapping of the object, the transparencies were placed as bands along the surface at different heights. This gave the illusion of the object being "sliced," and revealing openings that would exist within the object at different elevations. One form of transparency that proved to be effective in terms of reading the mesh was the





use of a translucent surface with a grid pattern that would be opaque. The grid would be small and thin enough to read at different depths of field. The translucent areas would be almost completely transparent. Therefore, as the curves of the object would overlap in the visual field, the surface would become slightly more opaque. This would give one the ability to see through the object and be able to understand the different layers of surfaces as they overlapped.

The opaque surfaces were filled with combinations of texture mapped surfaces, and procedural surfaces. The same grid that would be used in the translucent areas could be used on the opaque areas. The grid allowed one to understand the relative distance and scale of the object from different perspectives. The rest of the opaque surface was filled with a procedural surface. These surfaces do not use any information that is given by the mapping of the object. They allow for many different types of "random" patterns to be placed on the object. The color is based on the relative position of the object in space. These type of surfaces are generally based on fractals. Generally a speckled pattern would be used. Similar to the grid pattern, the speckles would have a scale that would allow one to visualize the depth of field in terms of the scale of the specks.

While the system for the material of the mesh was set up in terms of mapping, transparency, and texture, the actual material was never the same throughout the project. By constantly altering the images that would be used by the different material maps, new methods of reading the surface would be explored and tested. Every new transparency map would "slice" and open the surface in a new way, which would change the reading of the surface when view from the different perspectives.

The Designer, now satisfied with the new materials that were used by the mesh and the members, was ready to take the project to the next step.

## The Mistake

While the designer was intrigued by the new form, he was disappointed in the fact that the form read more as an object than a space. Since the goal was to explore the form for its spatial characteristics, the designer was eager to give the form a scale so that it could be explored spatially.

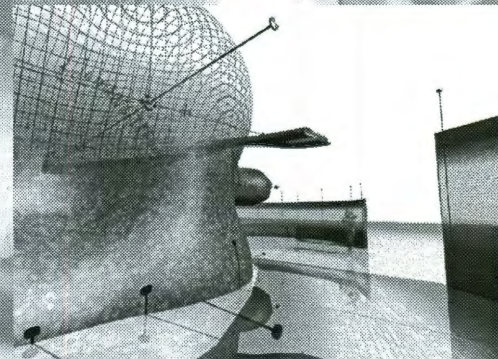
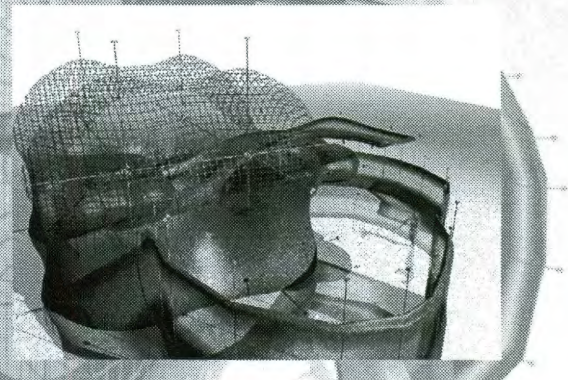
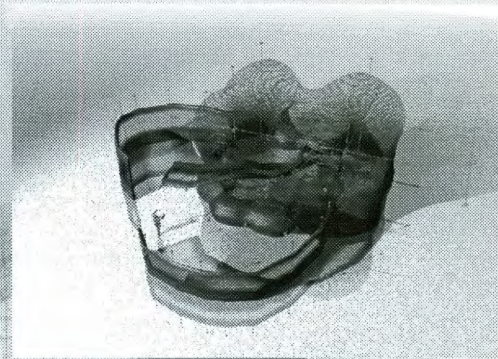
### Scale and Gravity

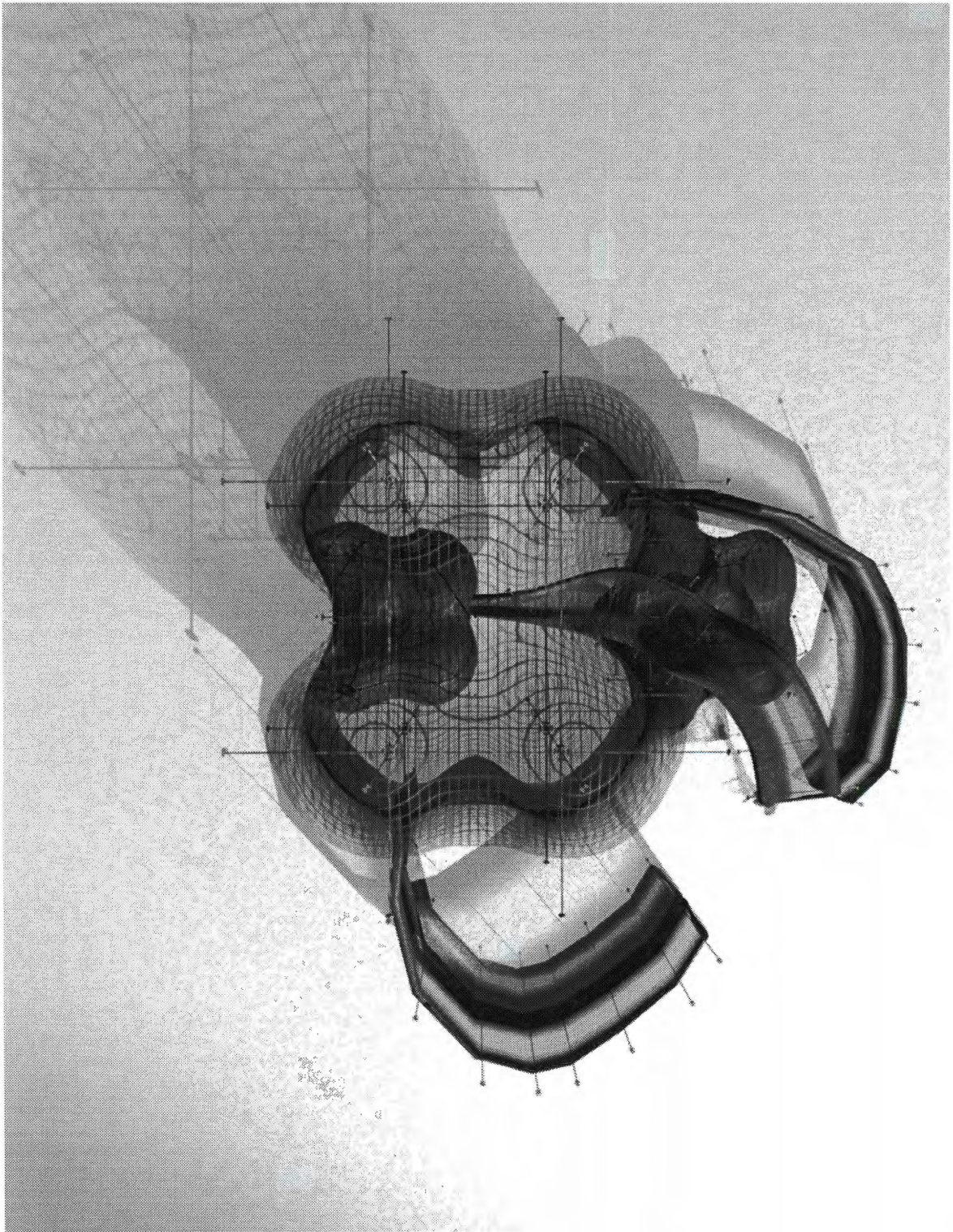
Certain problem can occur with certain architectural drawings. A design will generally not read in terms of a space until certain key elements are added to the drawing. The first key element is the issue of scale. The simplest and most logical method of giving reading scale in a drawing is through the use of scale figures. One can immediately read the scale of and space by proportioning it to the scale of the human body. Another key element that is added to an architectural drawing is the ground plane. This gives the drawing orientation, and allows one to understand how the structure is oriented in terms of gravity.

The Designer was eager to test the new form through the use of these elements. The goal being that once the form was placed in an environment with a reference to scale and gravity, the spatial characteristics of the form would reveal themselves.

The mesh, however, was not a two-dimensional drawing, but a three-dimensional form. Therefore, the scale figure had to be a three-dimensional figure, and the ground plane had to be positioned in space. An abstract representation of a human figure was scaled and positioned near the mesh. A ground plan was placed such that it "cut" the mesh. This would allow one to think of the mesh as resting on the ground plane.

At this point the Designer still believed that the object was reading as a large object rather than a space. There seemed to be a need for more detail within the design. The Designer then arbitrarily added a new unrelated form.

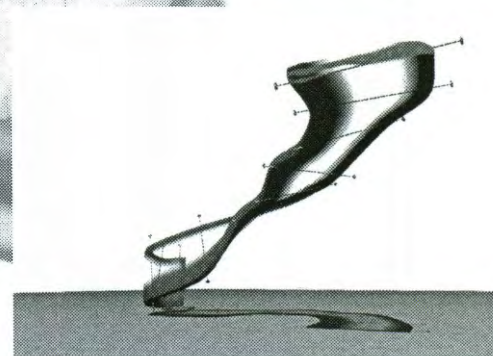
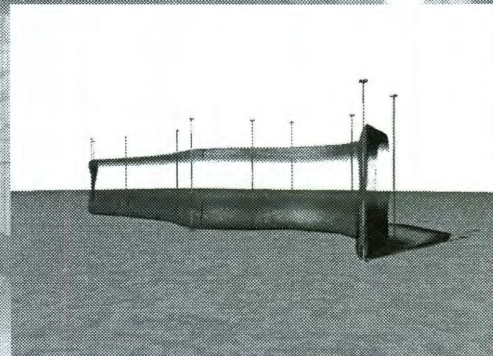
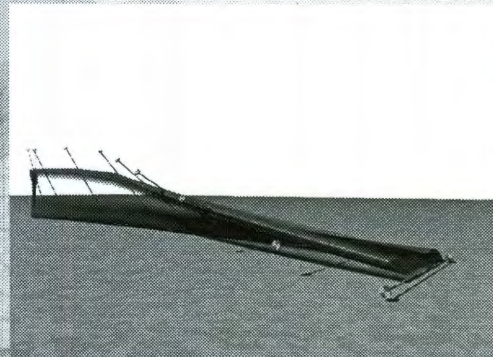
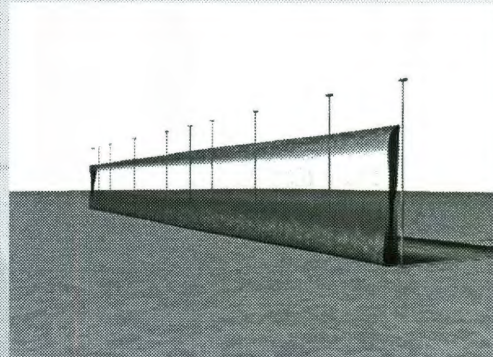




## The Strip

The strip was a new system that was introduced to the mesh. It was unrelated to the metaball mesh. Its form can be thought of as a ribbed system. Using the same representation of members that existed in the metaball mesh, long thin elements would be placed along a path. Each of the elements would be labeled in sequential order. Each of the elements would then serve as a node for a three-dimensional spline curve. The cross-section of the strip would then be extruded along the spline. As the nodes of the strip are moved and rotated, the spline curve is changed and the strip is reconfigured to follow the new direction of the path.

The Designer added these strips to the environment of the metaball mesh. They would serve to enclose the form and give a direction to travel around it. They acted both as walls as well as ramps around the mesh. In doing so, the Designer introduced the concept of a promenade to the design.



## The Mistake

It was obvious to the Mathematician that adding the ground plan and the scale figure had nothing to do with the system. The reason being that they made assumptions on the nature of the space which were not valid. The space that the form originally lived in was a "pure" space, where all that was assumed on the space was that it was four-dimensional. In order to explore the spatial characteristics of the system, one needed it to exist within this pure space. Therefore, the form had to exist without gravity or any reference to physical scale.

However, the issues addressed through the mistake were valid and needed to be dealt with. When would one stop reading the form as an object and more as a space? How could one give the idea of scale without a reference to the physical world? The only scale reference that could be used was within the form itself. Was there a need for the observer to be oriented within the space?



## Building the Extended Form

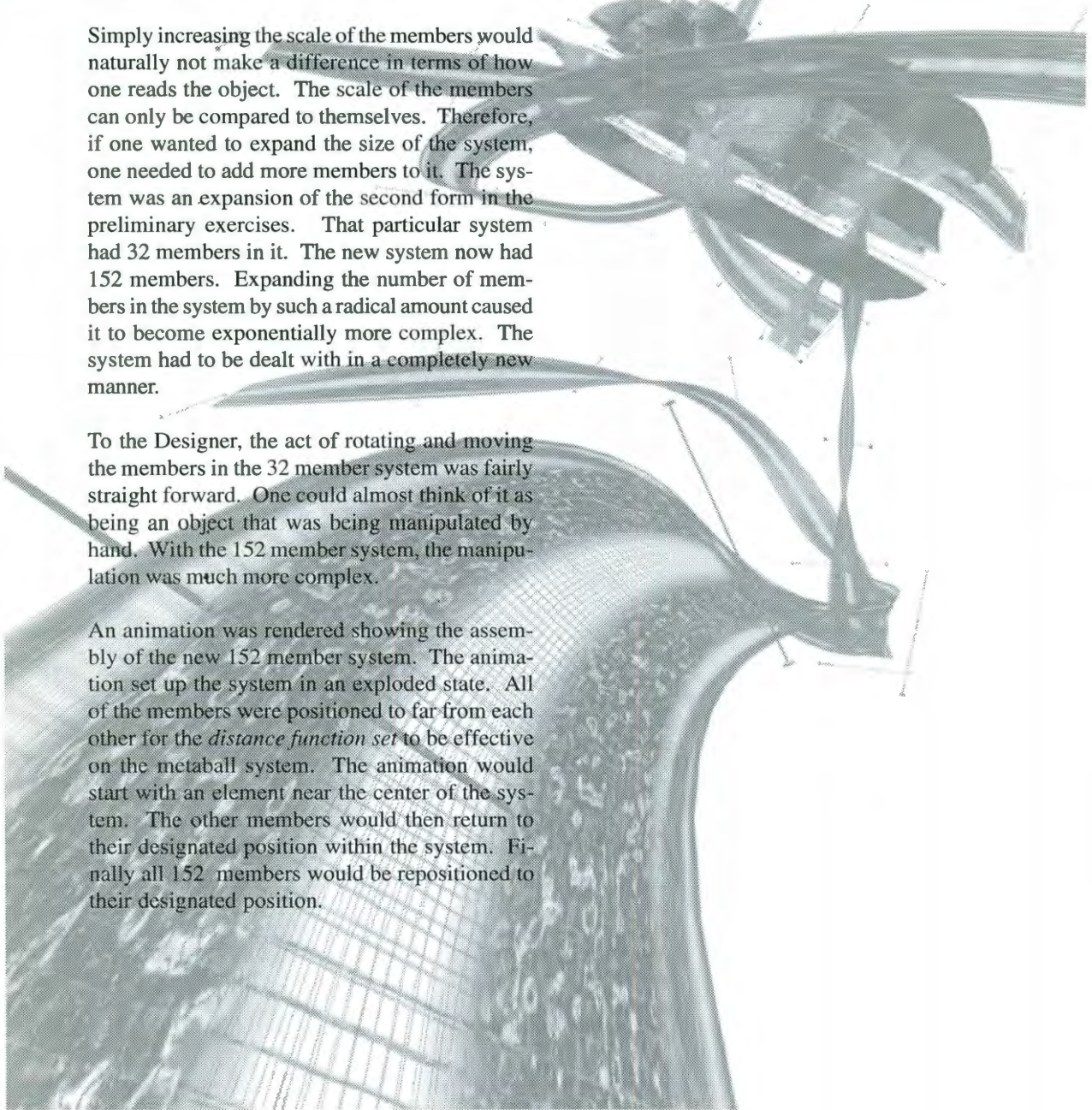
The Mathematician and the Designer had to now build a new form from the research that was address by the Designer. The first issue that needed attention was the issue of the form being read as a space. The primary aspect that one need to look at was scale. If one could give the illusion of the mesh being “large,” the overall reading of it may have greater spatial characteristics. Therefore the form was expanded.

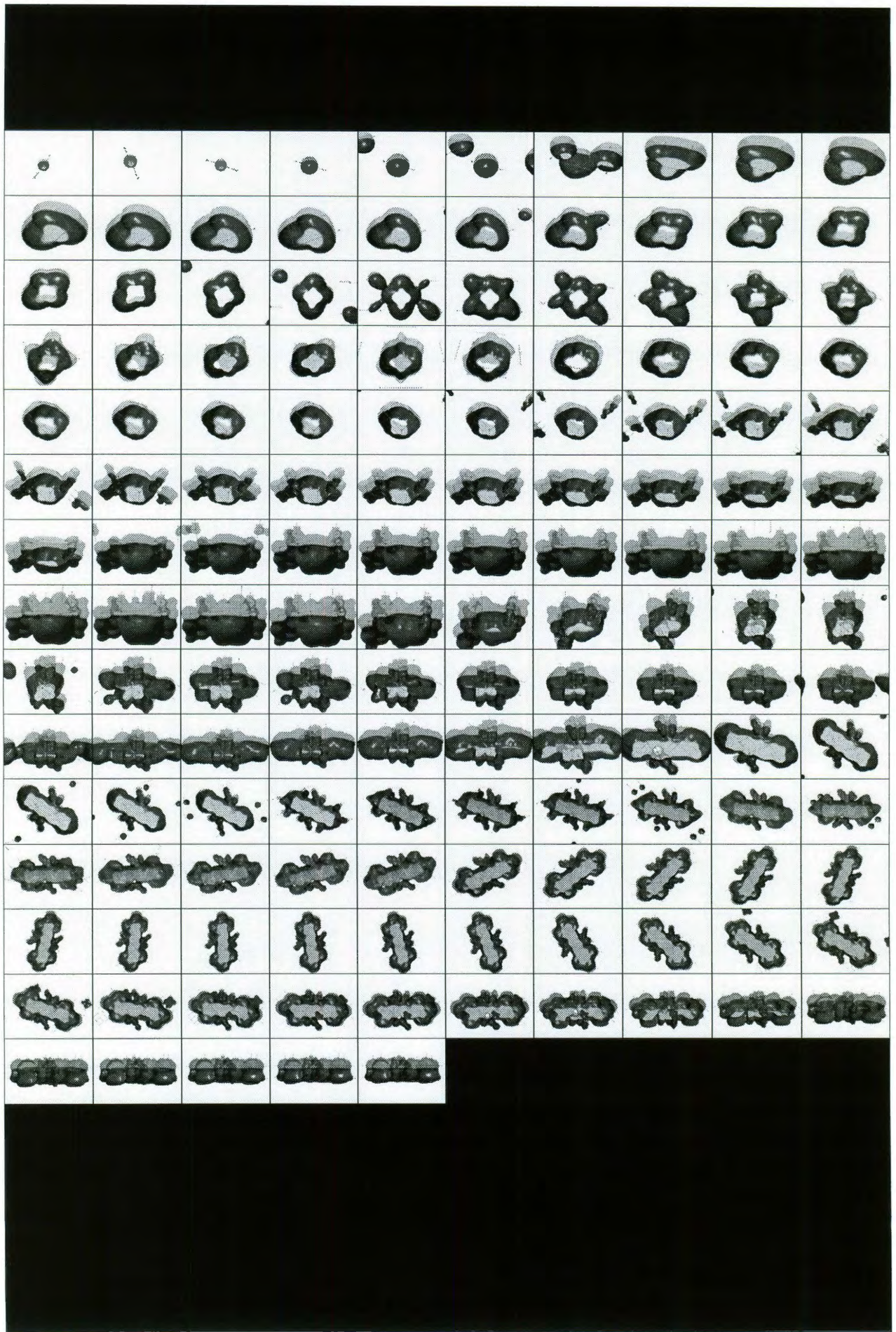
### Expanding the System

Simply increasing the scale of the members would naturally not make a difference in terms of how one reads the object. The scale of the members can only be compared to themselves. Therefore, if one wanted to expand the size of the system, one needed to add more members to it. The system was an expansion of the second form in the preliminary exercises. That particular system had 32 members in it. The new system now had 152 members. Expanding the number of members in the system by such a radical amount caused it to become exponentially more complex. The system had to be dealt with in a completely new manner.

To the Designer, the act of rotating and moving the members in the 32 member system was fairly straight forward. One could almost think of it as being an object that was being manipulated by hand. With the 152 member system, the manipulation was much more complex.

An animation was rendered showing the assembly of the new 152 member system. The animation set up the system in an exploded state. All of the members were positioned to far from each other for the *distance function set* to be effective on the metaball system. The animation would start with an element near the center of the system. The other members would then return to their designated position within the system. Finally all 152 members would be repositioned to their designated position.



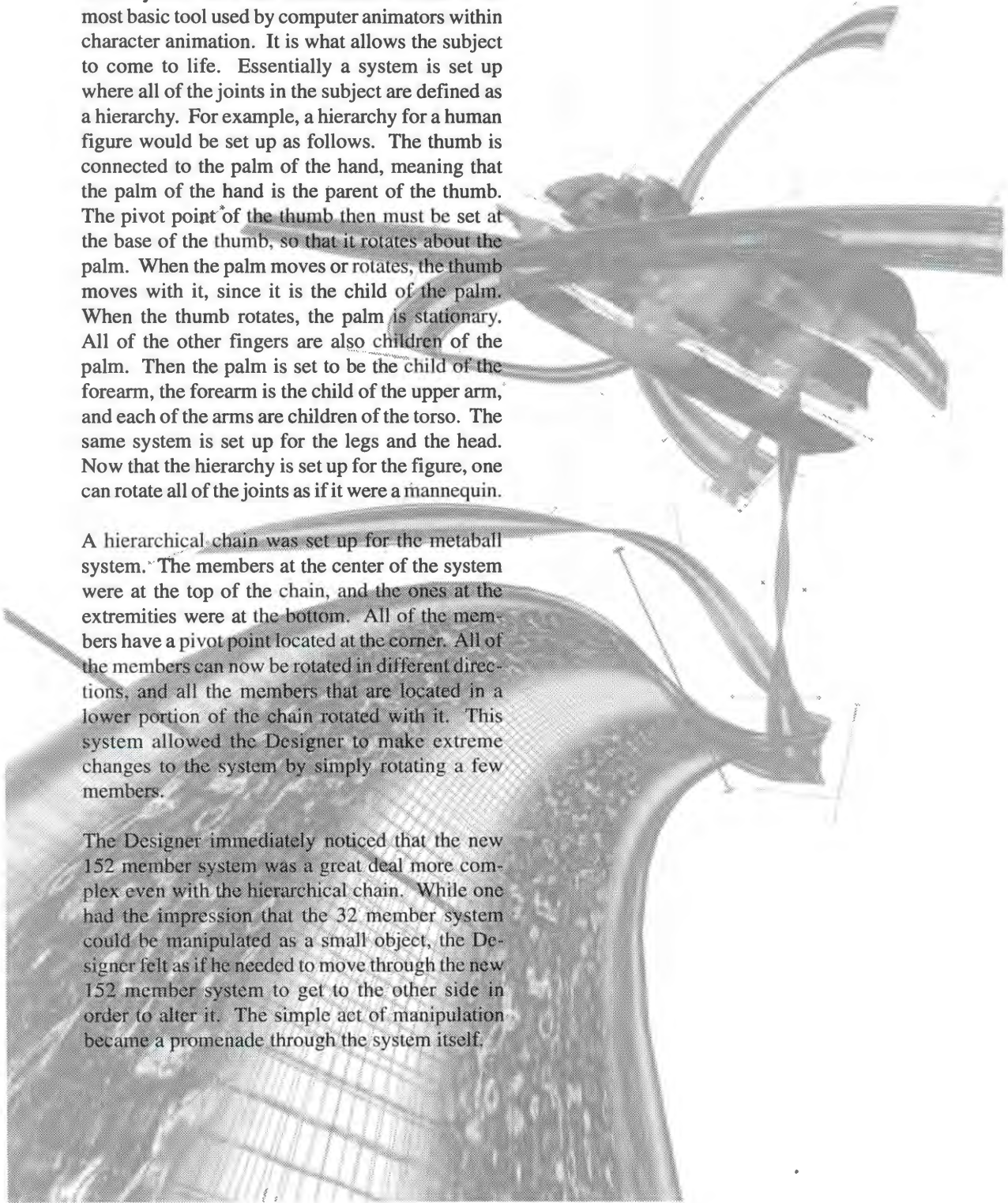


## ***Kinematics - Moving and Altering the System***

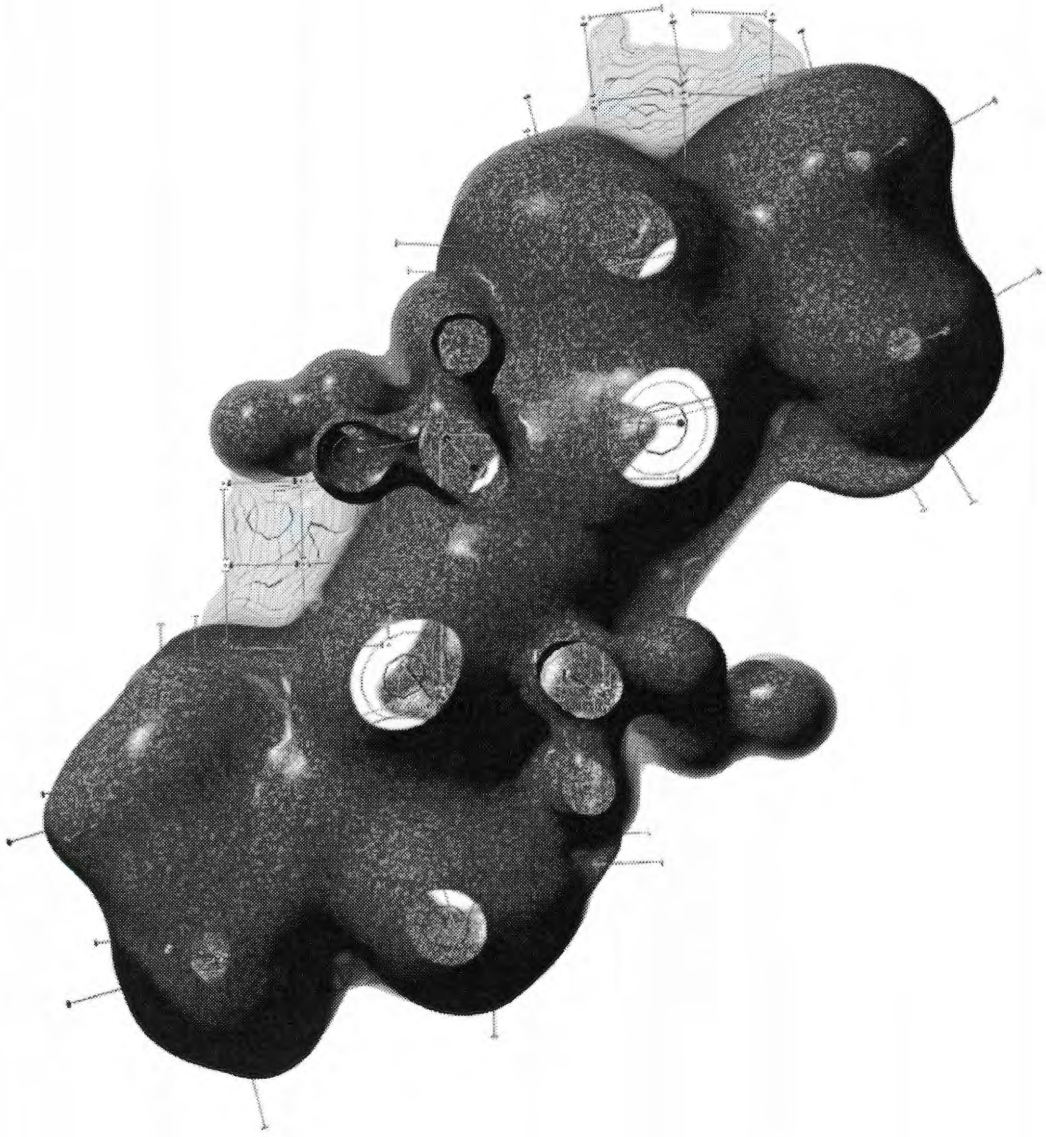
The use of more complex kinematics were added to the system. The use of kinematics is one of the most basic tool used by computer animators within character animation. It is what allows the subject to come to life. Essentially a system is set up where all of the joints in the subject are defined as a hierarchy. For example, a hierarchy for a human figure would be set up as follows. The thumb is connected to the palm of the hand, meaning that the palm of the hand is the parent of the thumb. The pivot point of the thumb then must be set at the base of the thumb, so that it rotates about the palm. When the palm moves or rotates, the thumb moves with it, since it is the child of the palm. When the thumb rotates, the palm is stationary. All of the other fingers are also children of the palm. Then the palm is set to be the child of the forearm, the forearm is the child of the upper arm, and each of the arms are children of the torso. The same system is set up for the legs and the head. Now that the hierarchy is set up for the figure, one can rotate all of the joints as if it were a mannequin.

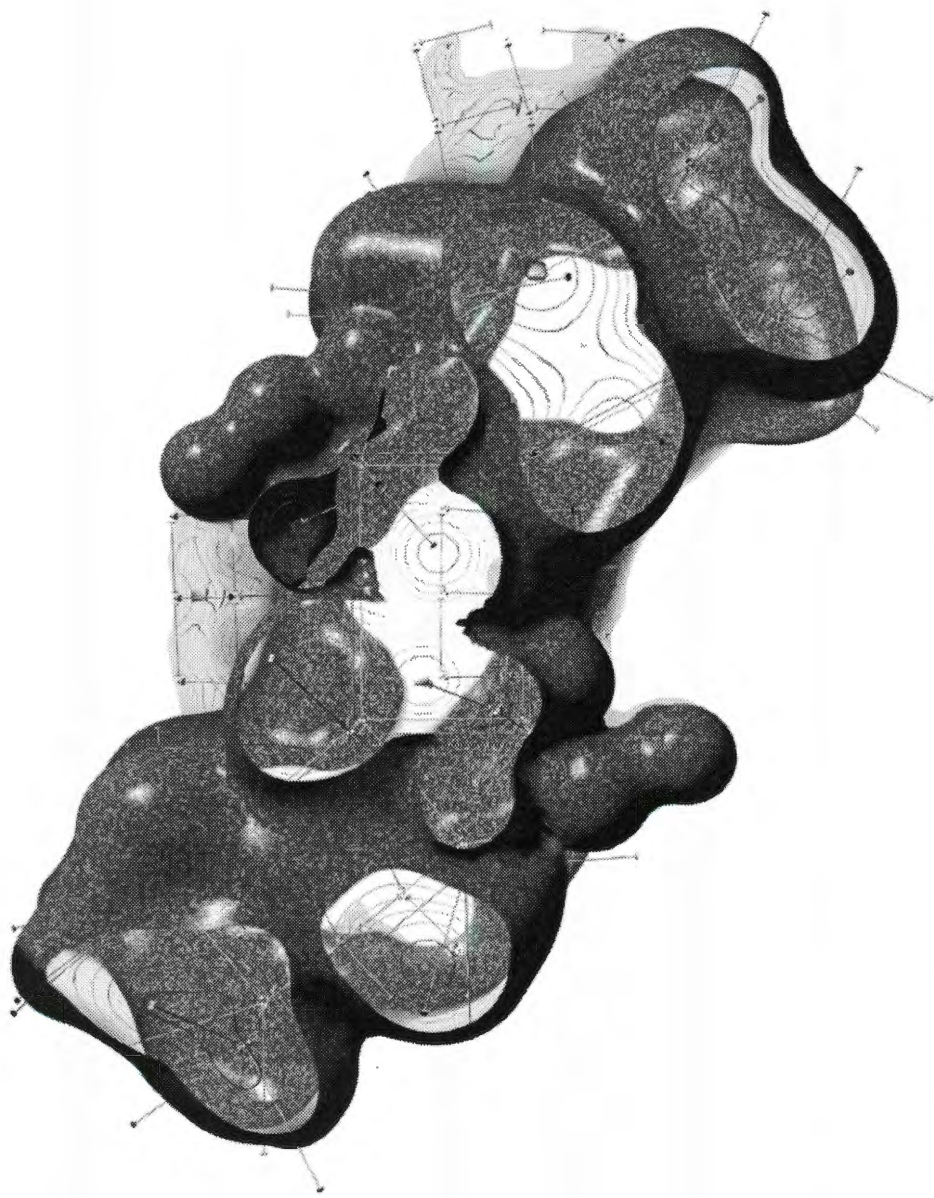
A hierarchical chain was set up for the metaball system. The members at the center of the system were at the top of the chain, and the ones at the extremities were at the bottom. All of the members have a pivot point located at the corner. All of the members can now be rotated in different directions, and all the members that are located in a lower portion of the chain rotated with it. This system allowed the Designer to make extreme changes to the system by simply rotating a few members.

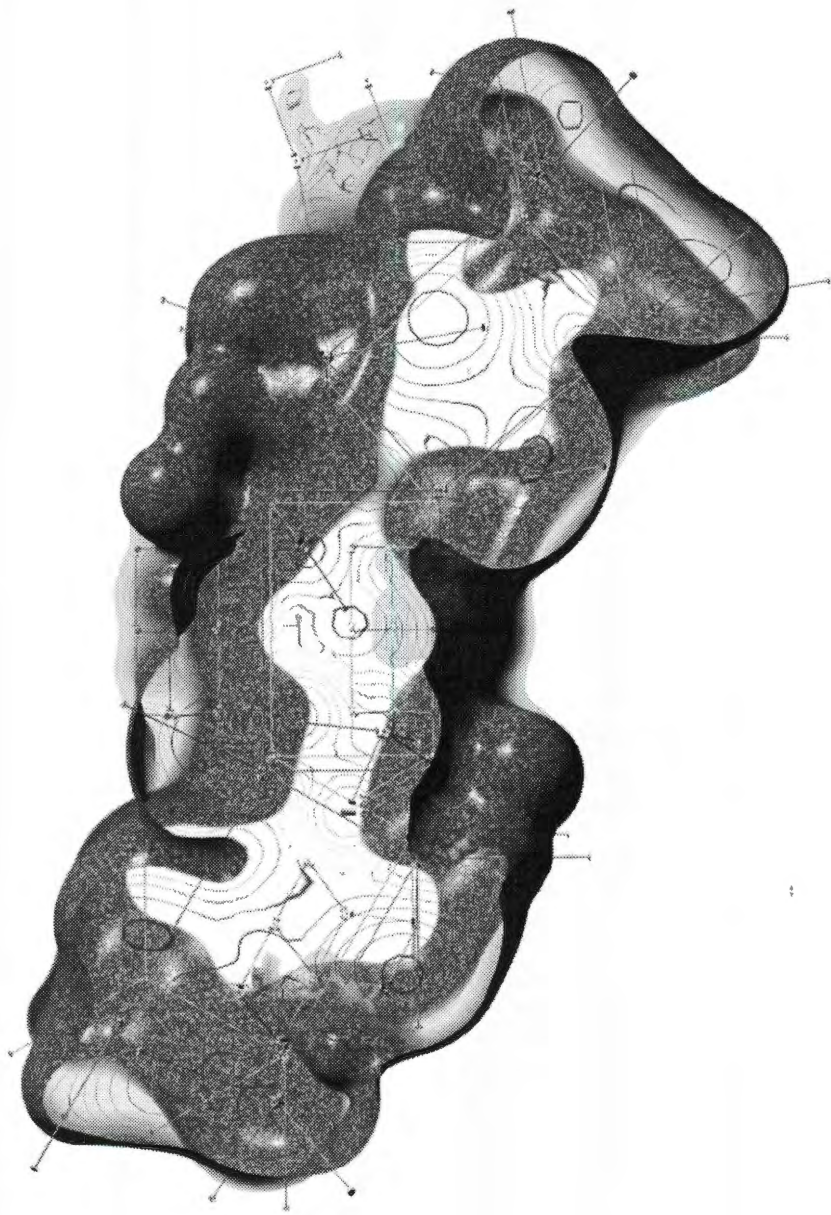
The Designer immediately noticed that the new 152 member system was a great deal more complex even with the hierarchical chain. While one had the impression that the 32 member system could be manipulated as a small object, the Designer felt as if he needed to move through the new 152 member system to get to the other side in order to alter it. The simple act of manipulation became a promenade through the system itself.

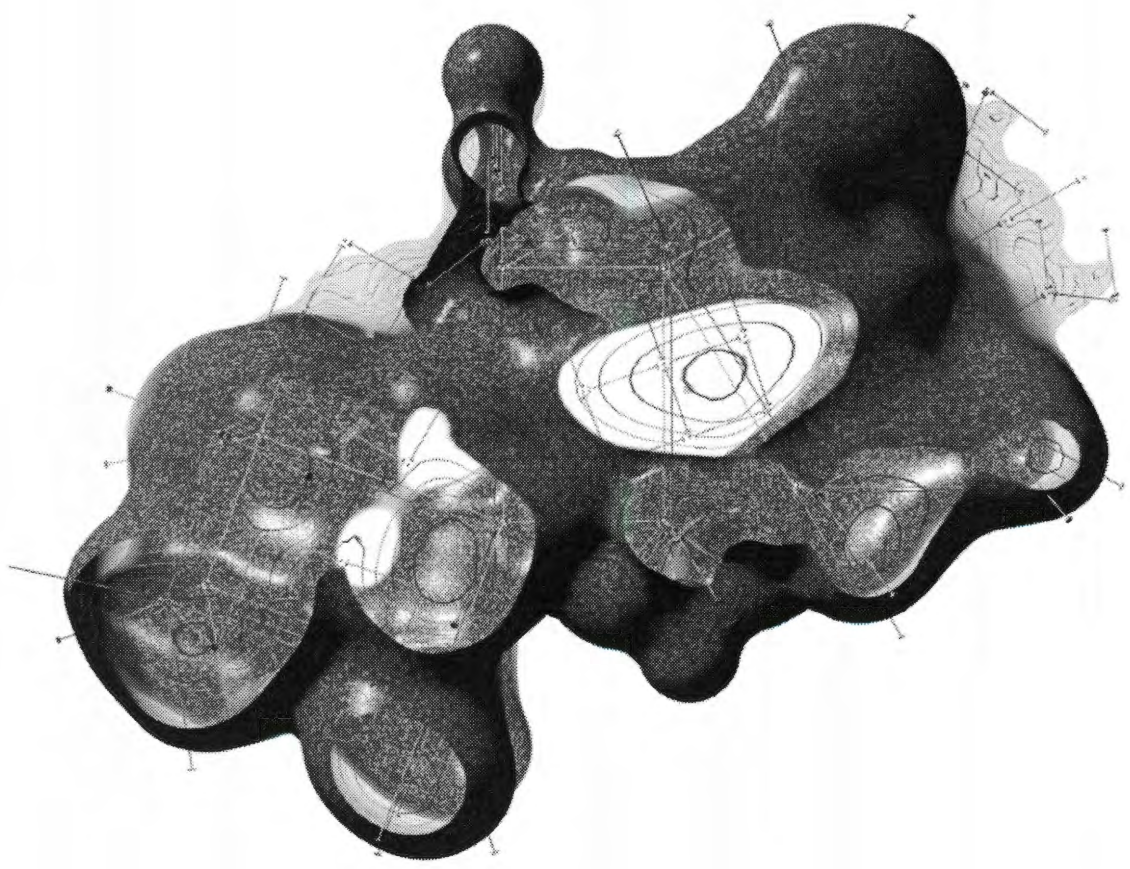








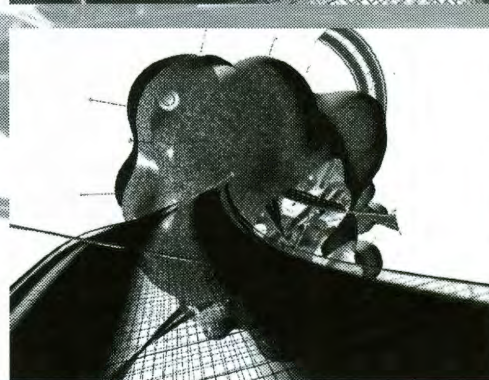
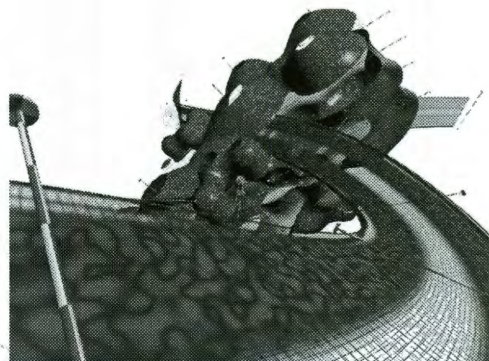




## Reintroducing the Strip

The issue of the promenade was then visually introduced into, the now gravity-less system. While the strip was effective as an issue of introducing a promenade, it now had to be effective in an environment without gravity.

The strip was once again an unrelated component of the metaball system. Its movement may loosely follow the movement of the metaball system, but its logic is not the same as the formation and transformation of the metaball mesh. It would intersect the mesh sometimes traveling through a transparent area which would give one the illusion of entering the mesh through and opening. In a sense, the strip served as a replacement for the ground plane. The ground plane gave one the illusion of being able to "walk up to" the mesh. Now, the camera could use the strip as a guide for its movement. Even if the camera was placed in a location away from a strip, one could visualize a path through the use of the strip.



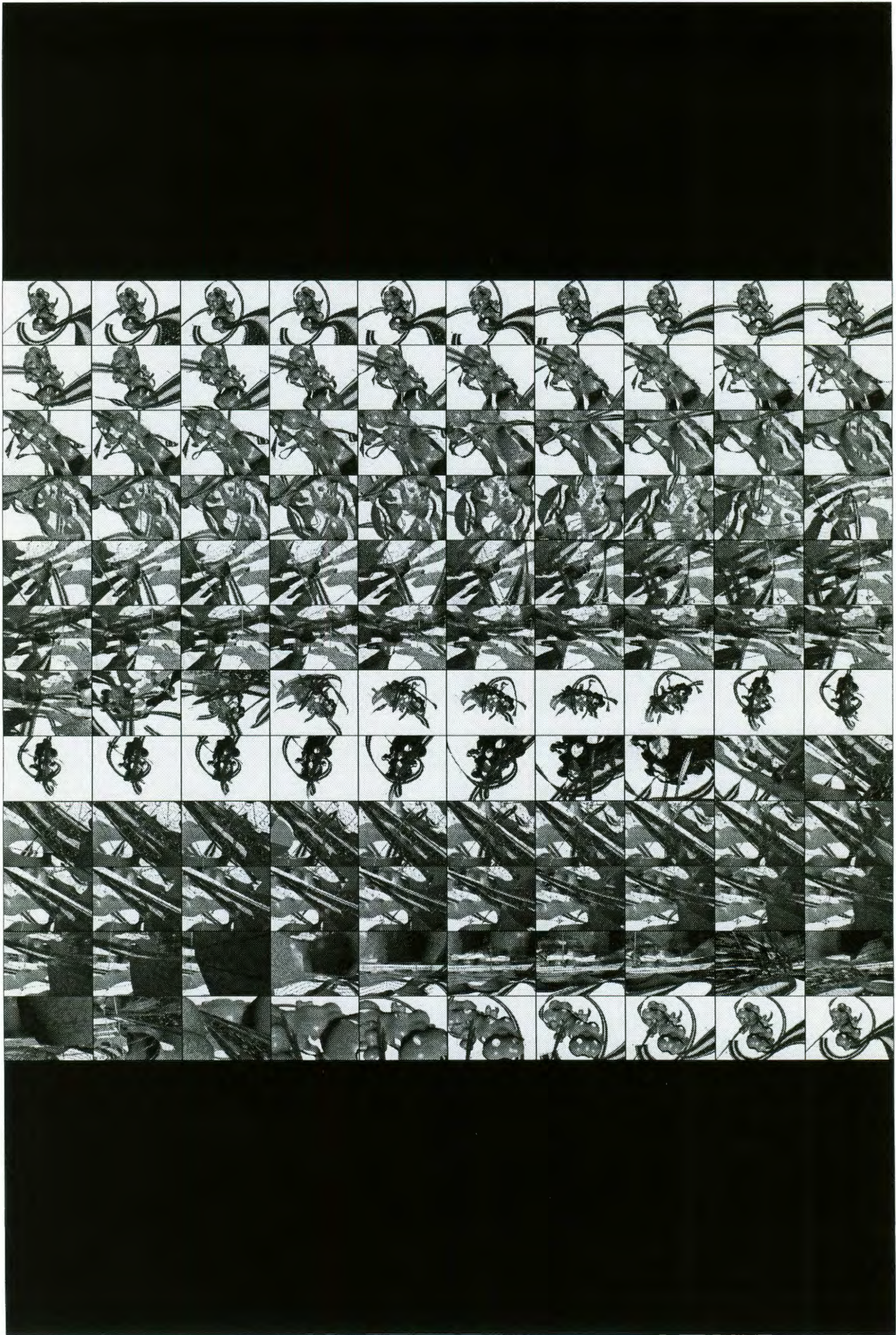
## Exploring the New System

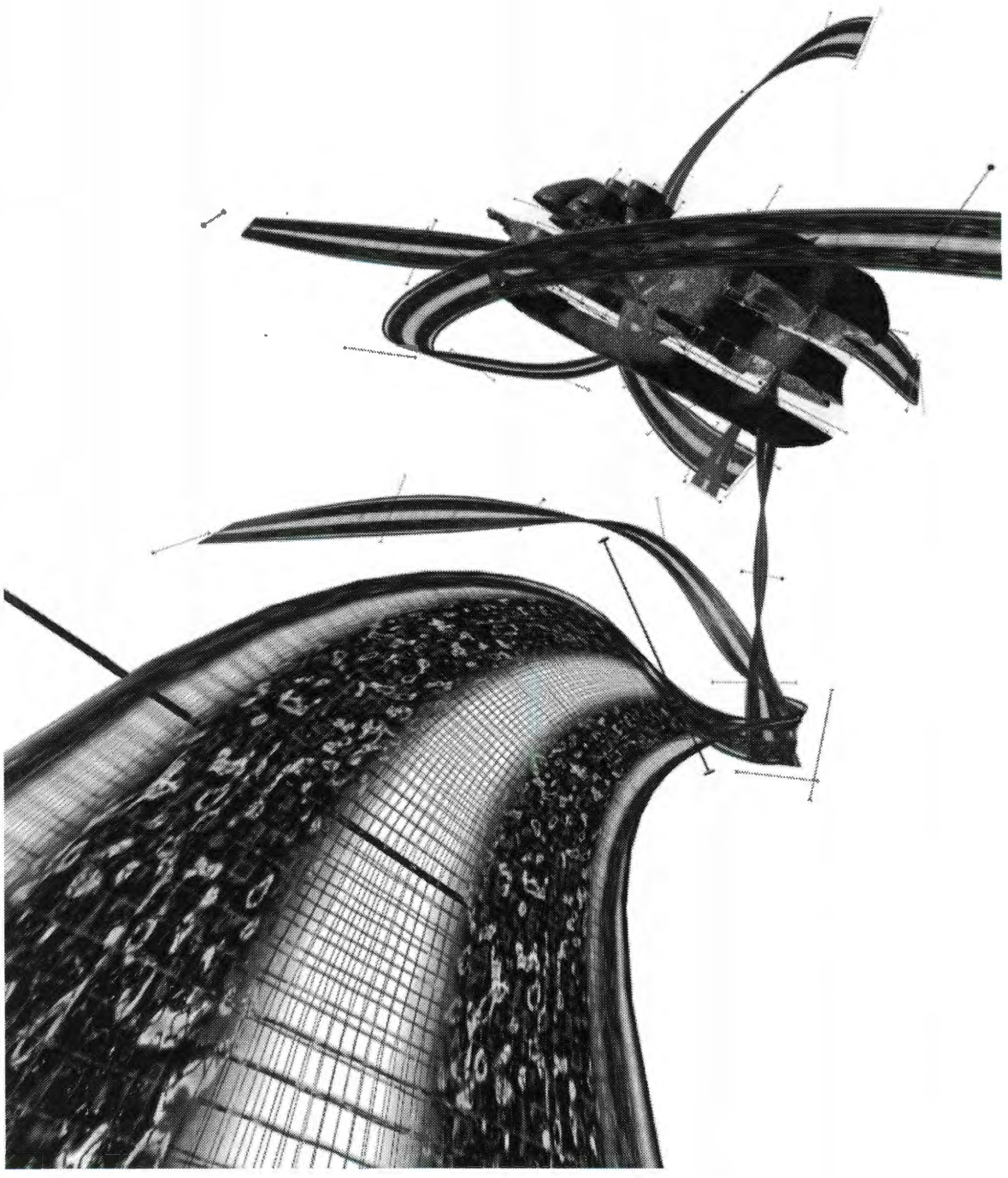
The Designer was eager to explore the new 152 member system. He could use the paths as guides to position the camera. The exploration that took place was both of the interior and exterior of the mesh. Many different material maps were used to explore the potential of the mesh's structure. Rarely did the color or transparency of either the mesh or the strip stabilize during the remainder of the project.

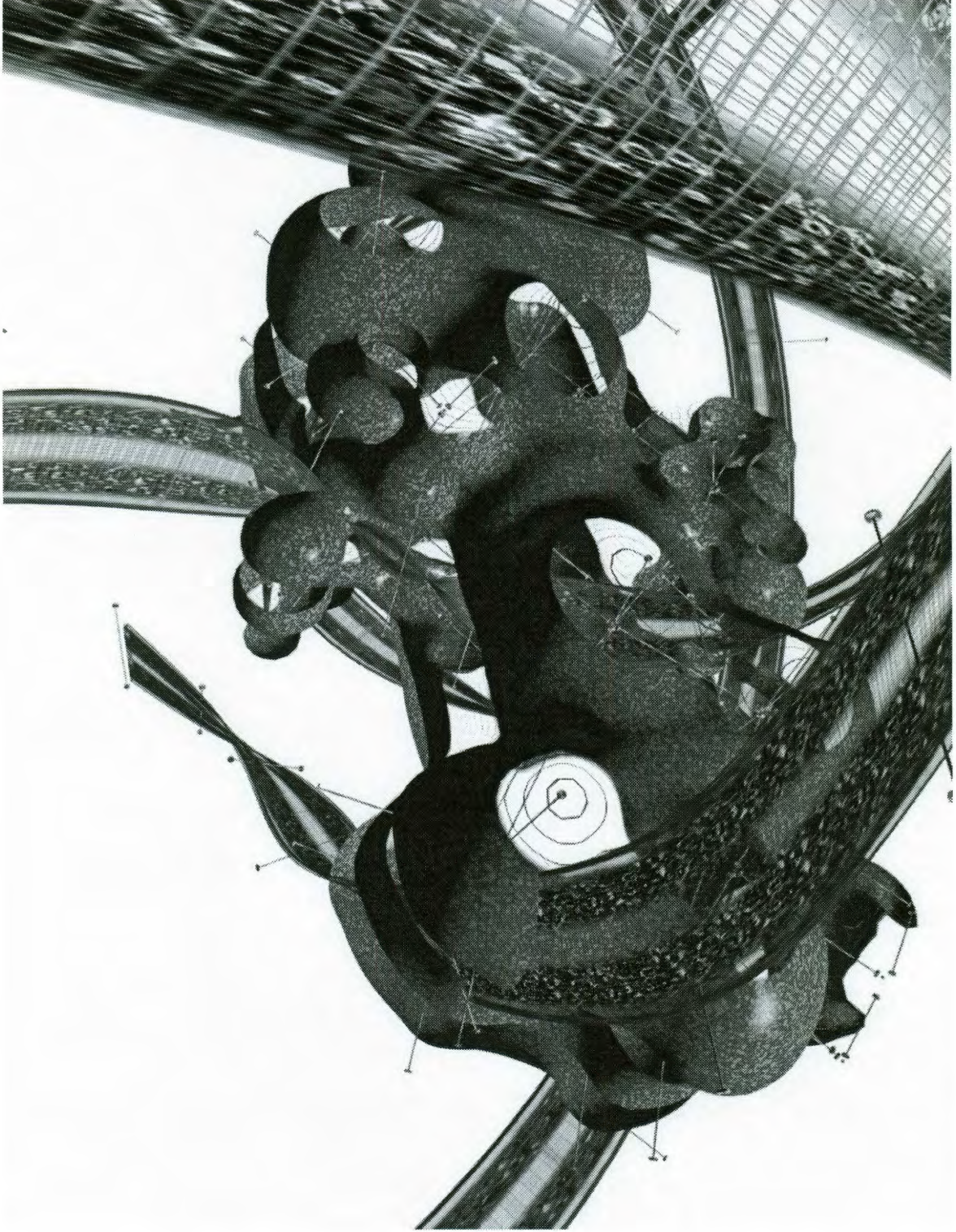
Many still shots were rendered. An animation was also rendered as a quick exercise in order to explore the effects of the kinematics of the system. The animation would attempt to explore the system both from an interior and exterior point of view.

The Designer was then interested in giving one the illusion of movement in a series of still shots. Therefore, a series of images were rendered with motion blur. Motion blur rendered several images from one frame to the next and composes them into a single image. This gives one the idea that the scene is in motion.

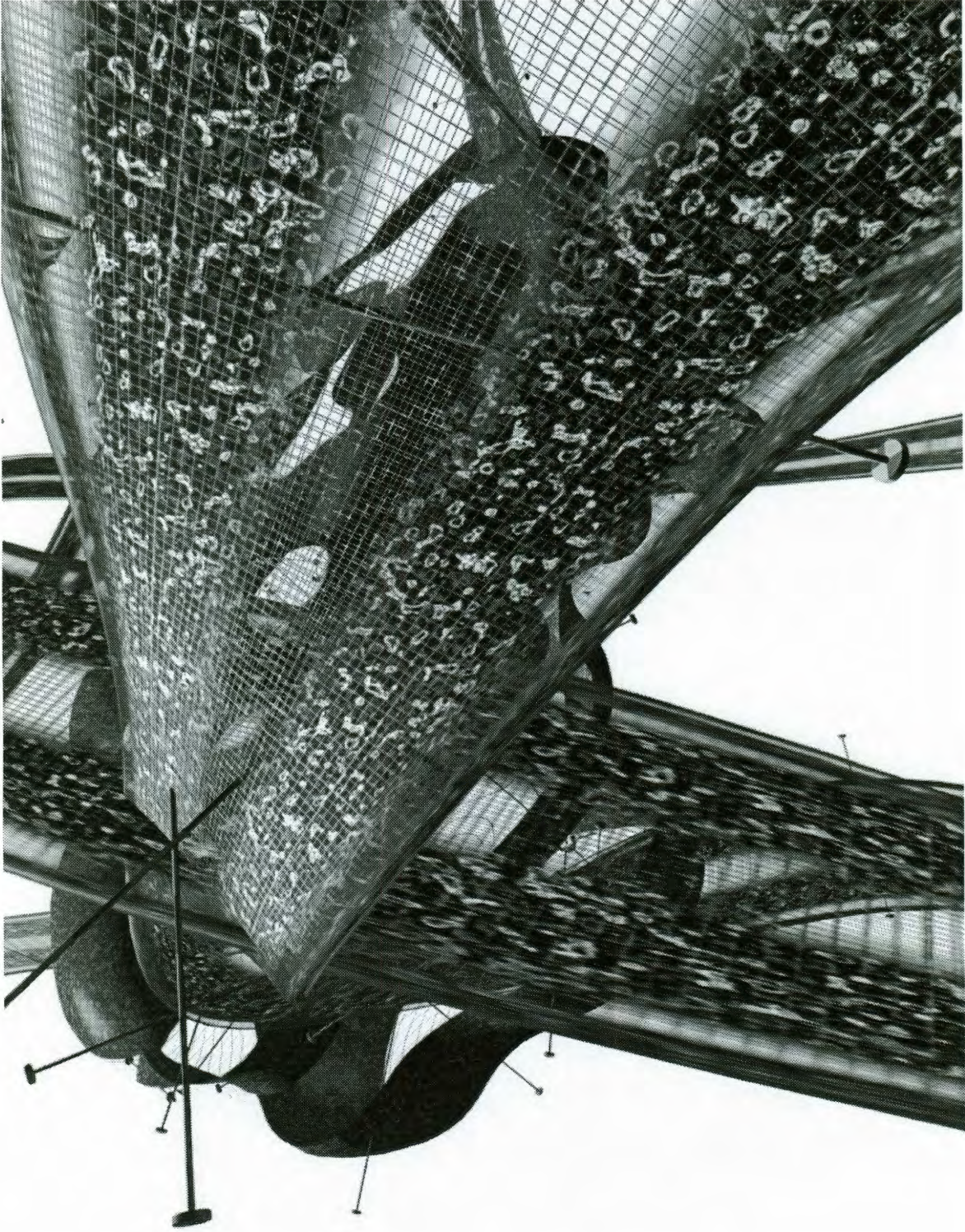


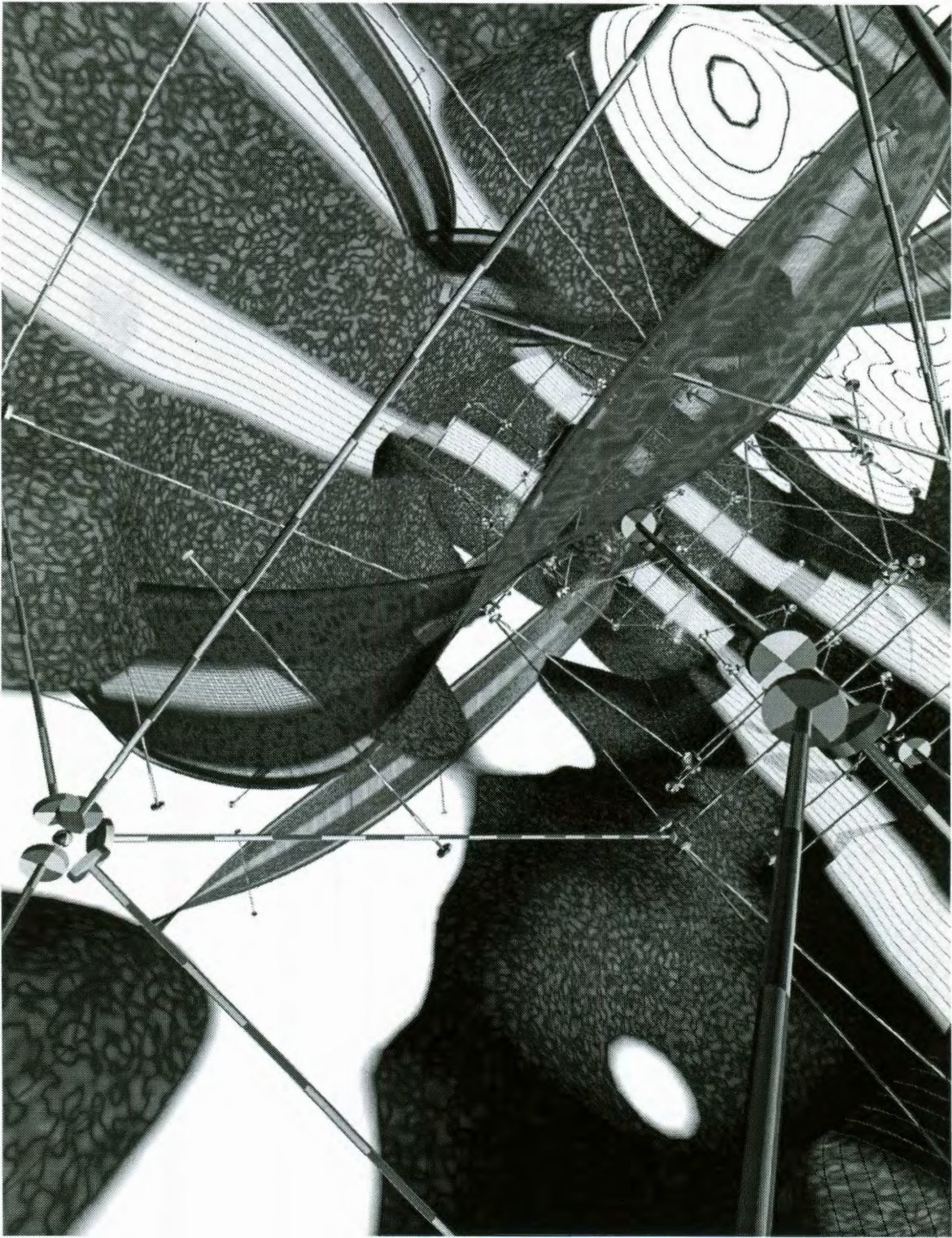


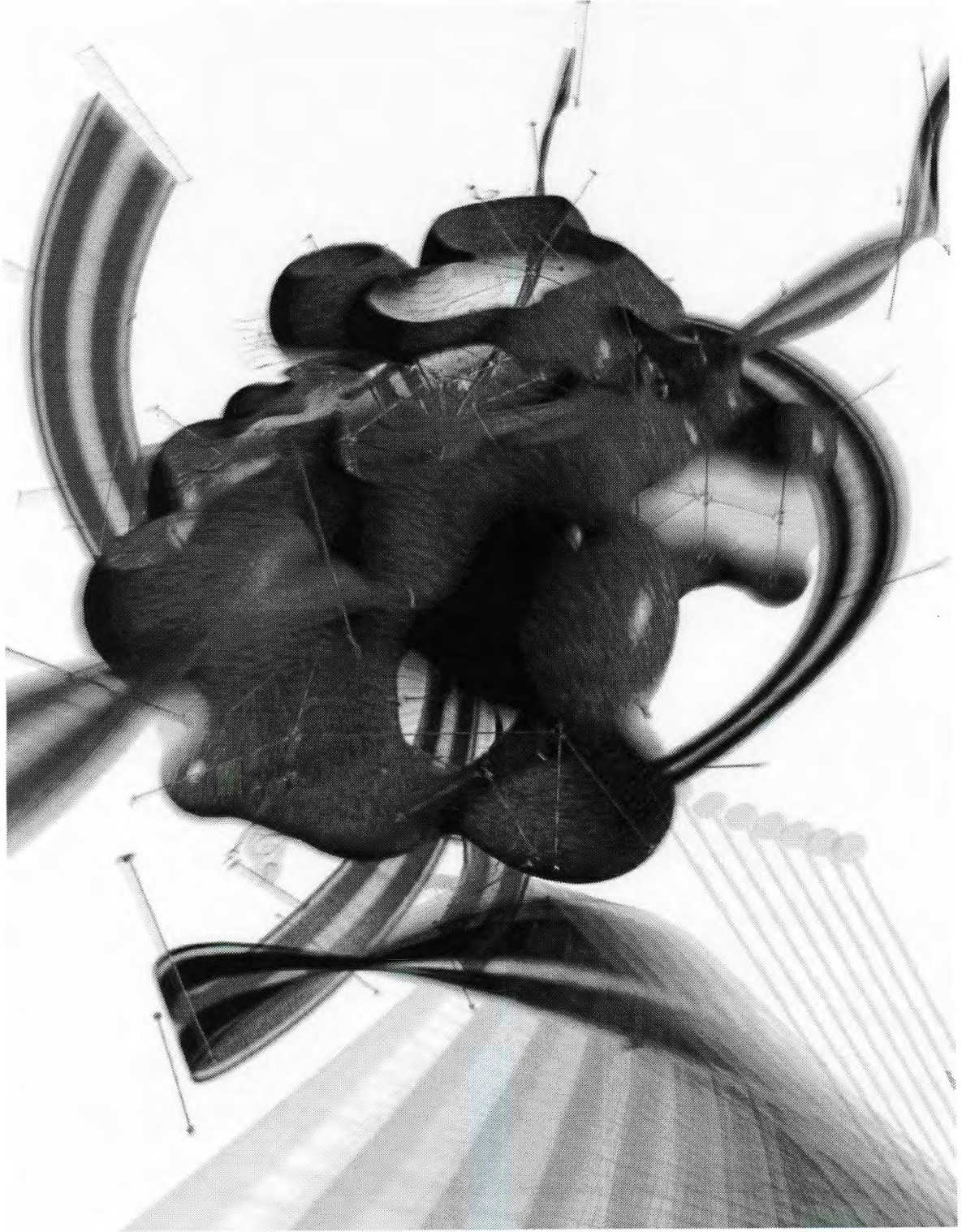


























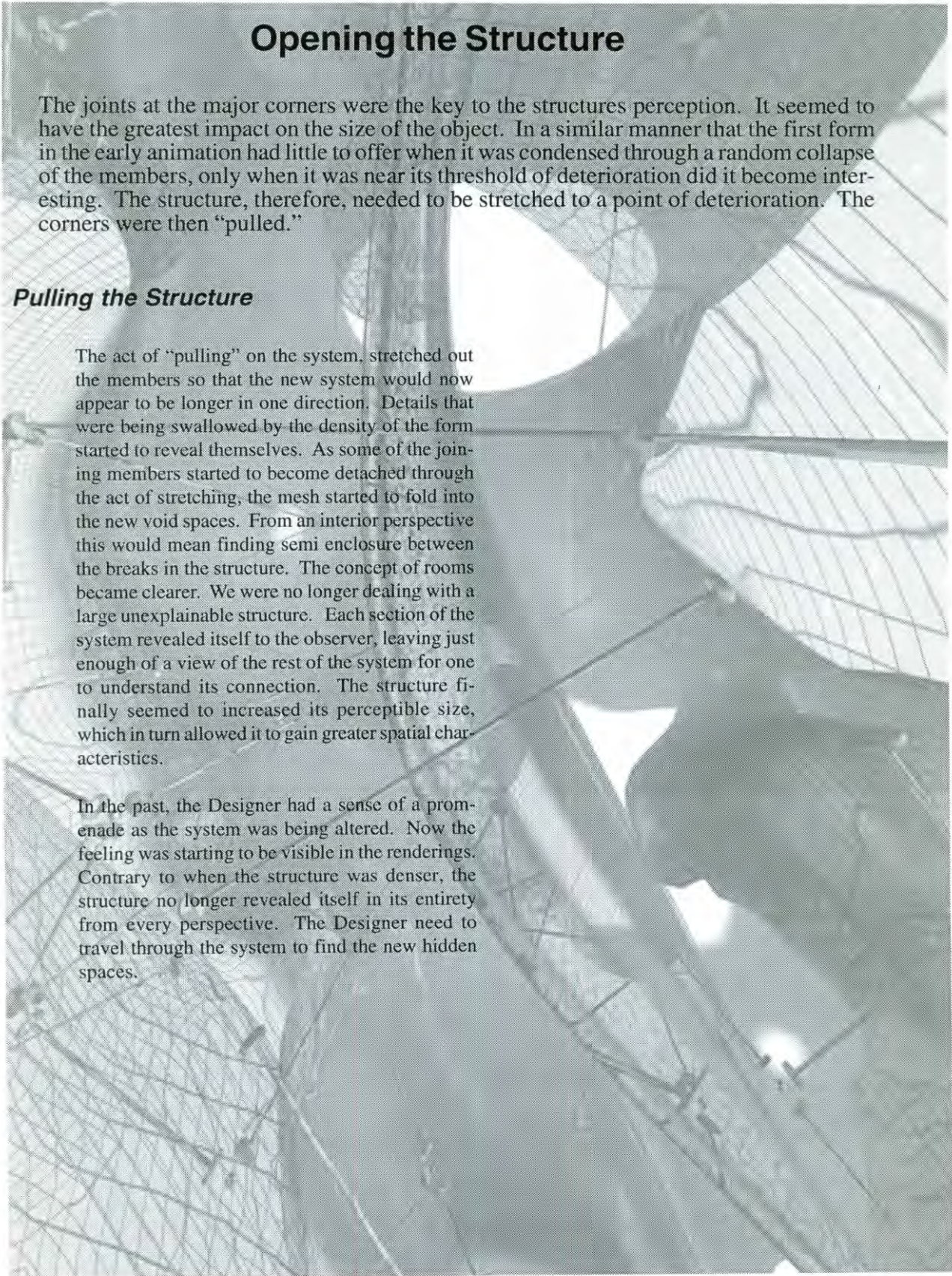
## Opening the Structure

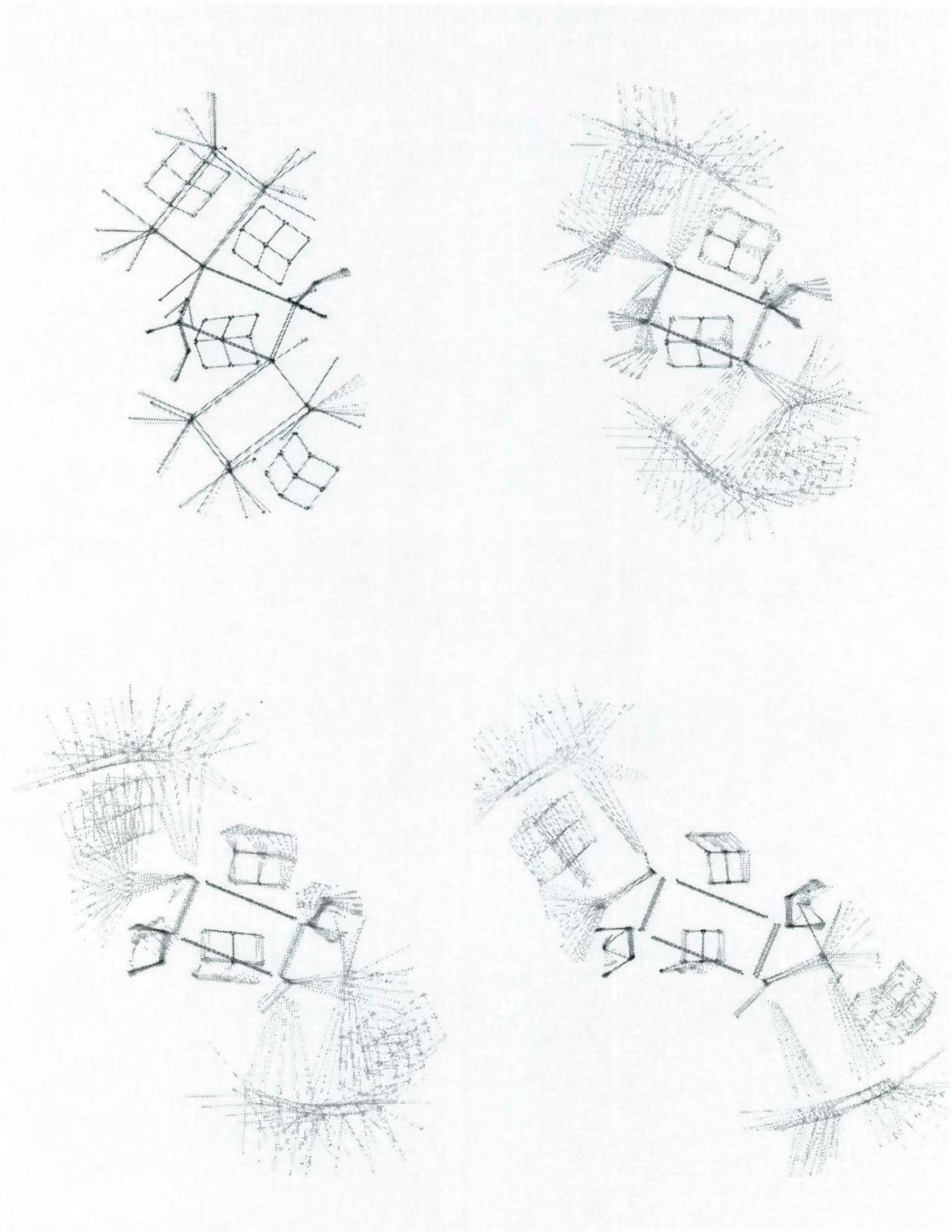
The joints at the major corners were the key to the structures perception. It seemed to have the greatest impact on the size of the object. In a similar manner that the first form in the early animation had little to offer when it was condensed through a random collapse of the members, only when it was near its threshold of deterioration did it become interesting. The structure, therefore, needed to be stretched to a point of deterioration. The corners were then "pulled."

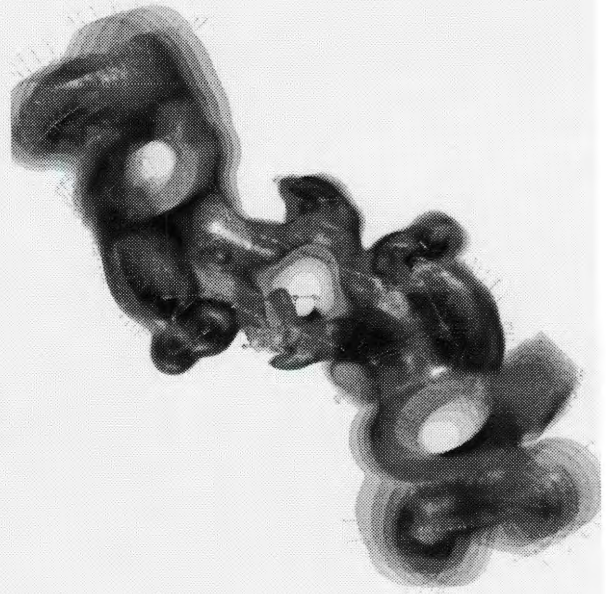
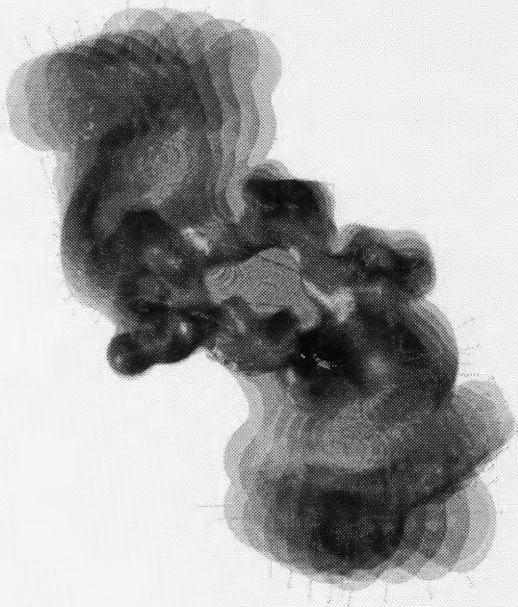
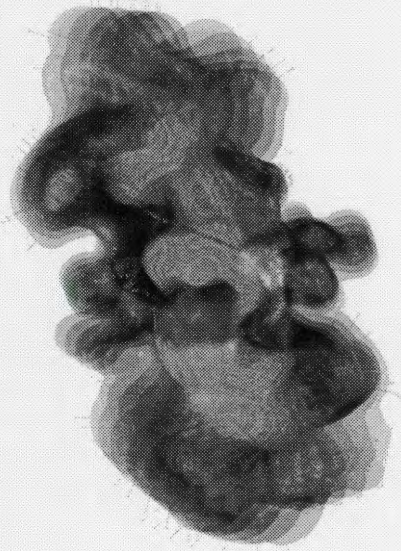
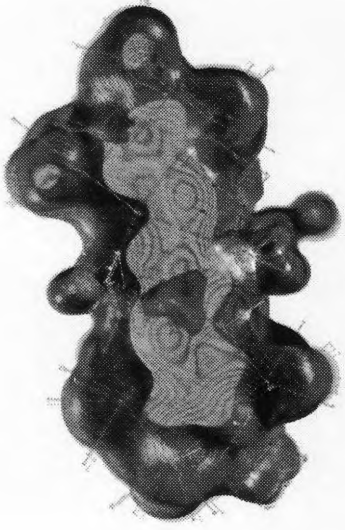
### *Pulling the Structure*

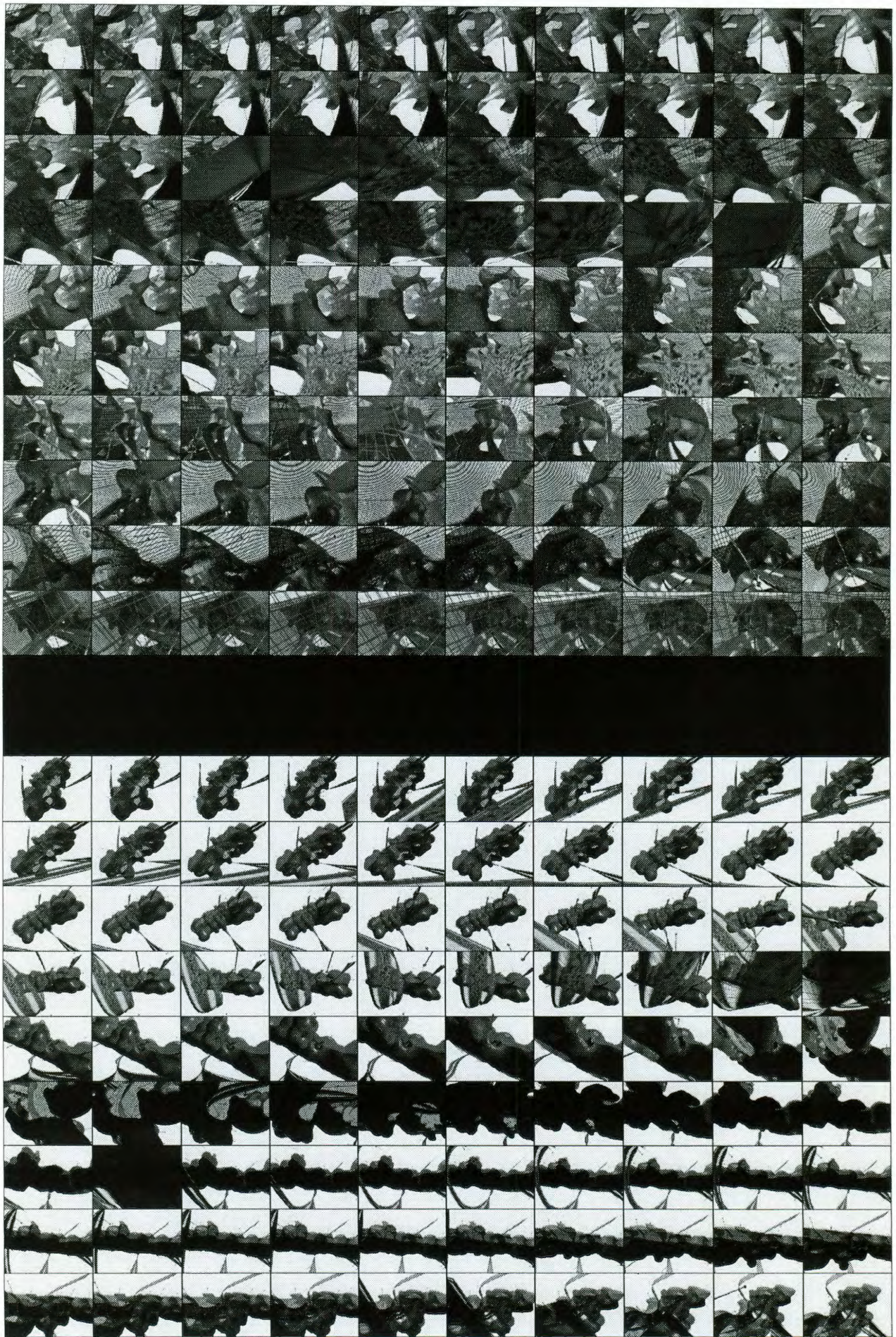
The act of "pulling" on the system, stretched out the members so that the new system would now appear to be longer in one direction. Details that were being swallowed by the density of the form started to reveal themselves. As some of the joining members started to become detached through the act of stretching, the mesh started to fold into the new void spaces. From an interior perspective this would mean finding semi enclosure between the breaks in the structure. The concept of rooms became clearer. We were no longer dealing with a large unexplainable structure. Each section of the system revealed itself to the observer, leaving just enough of a view of the rest of the system for one to understand its connection. The structure finally seemed to increase its perceptible size, which in turn allowed it to gain greater spatial characteristics.

In the past, the Designer had a sense of a promenade as the system was being altered. Now the feeling was starting to be visible in the renderings. Contrary to when the structure was denser, the structure no longer revealed itself in its entirety from every perspective. The Designer need to travel through the system to find the new hidden spaces.





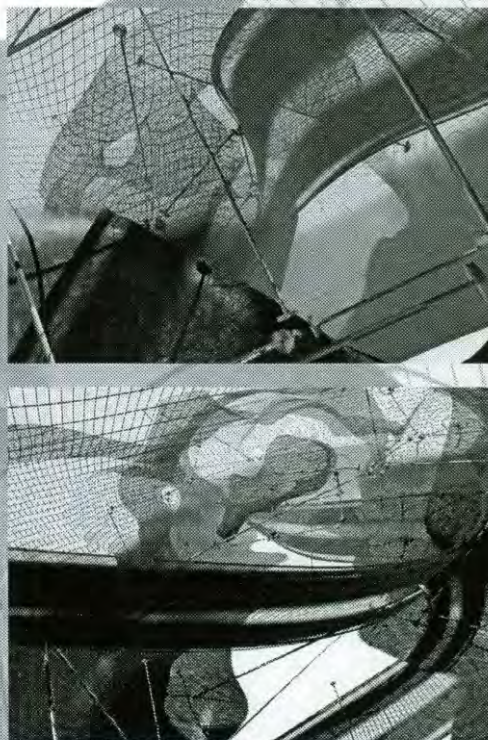




### *Exploring the Stretched System*

A single a strip was added that served as the path through the object. This strip extended itself along the now long structure. An animation was then composed where the camera would travel through the object. The subject would then travel from one side of the system to the next while loosely following the strip.

A second animation was then rendered from an exterior point of view. Another strip was added to the system that would wrap itself around the mesh. Once again, the camera would loosely follow the strip.



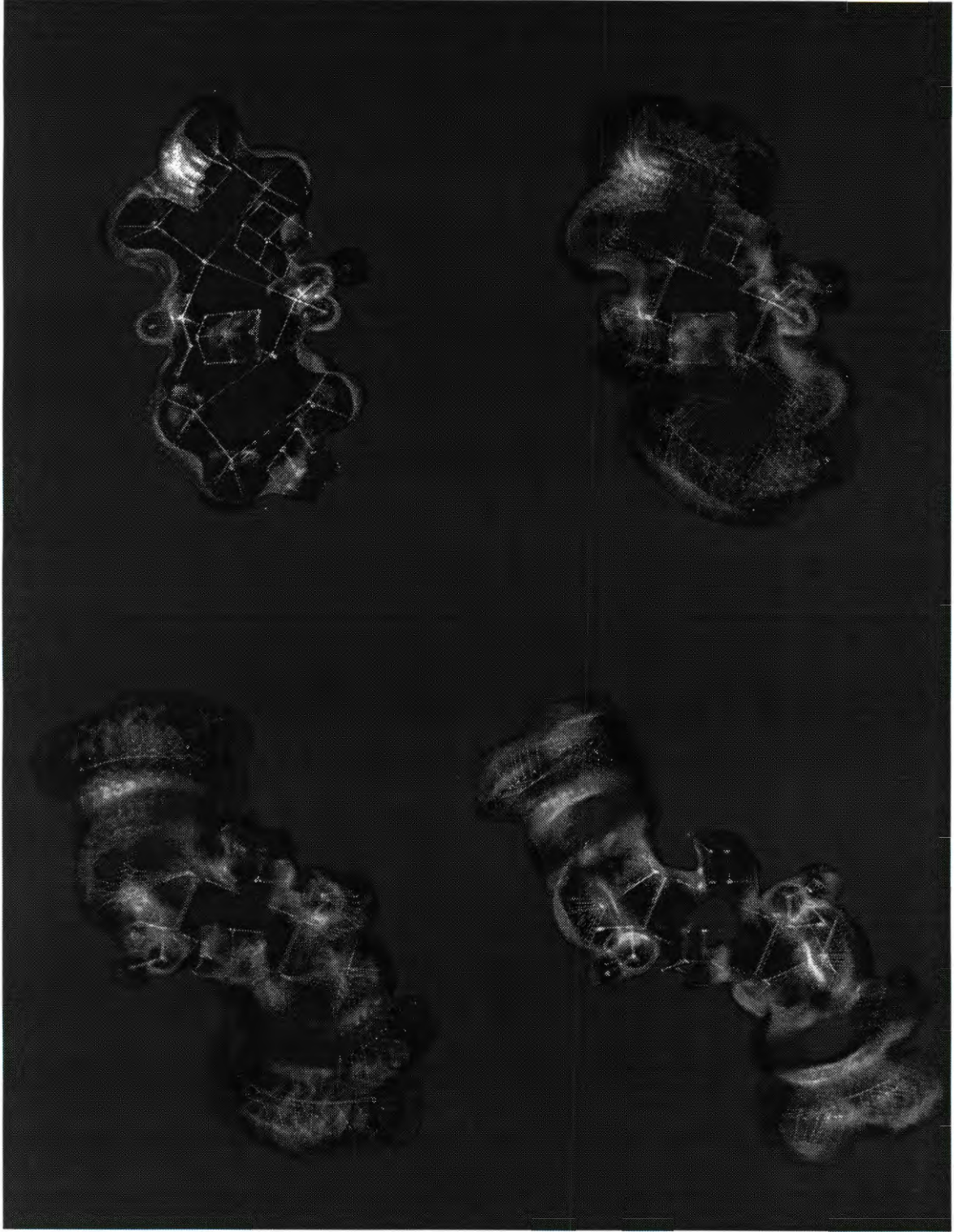
## Lighting

It was clear that the system was no longer an object but had clearly gain spatial characteristics without the use of gravity or scale. The Designer then had decided to explore the concept of lighting. Up until this point all of the images and animations had used exterior lighting. There was a single light source that was positioned far away from the system. One could think of it as a daytime observation. The Designer then thought that the interior of the object may read better if it was composed as a nighttime observation.

The Designer and the Mathematician added a series of lights that were linked to the object itself. Twelve light sources were added and linked to the system's hierarchy. They were positioned at the key joints of the system. Therefore the lights were not positioned as a new arbitrary system. They would follow the transformation of the system.

What was then done was to re-render the exact same animations of traveling inside and around the system with the new lighting system. One could then observe the two animations for a direct comparison of how lighting would alter one's perception of the space.

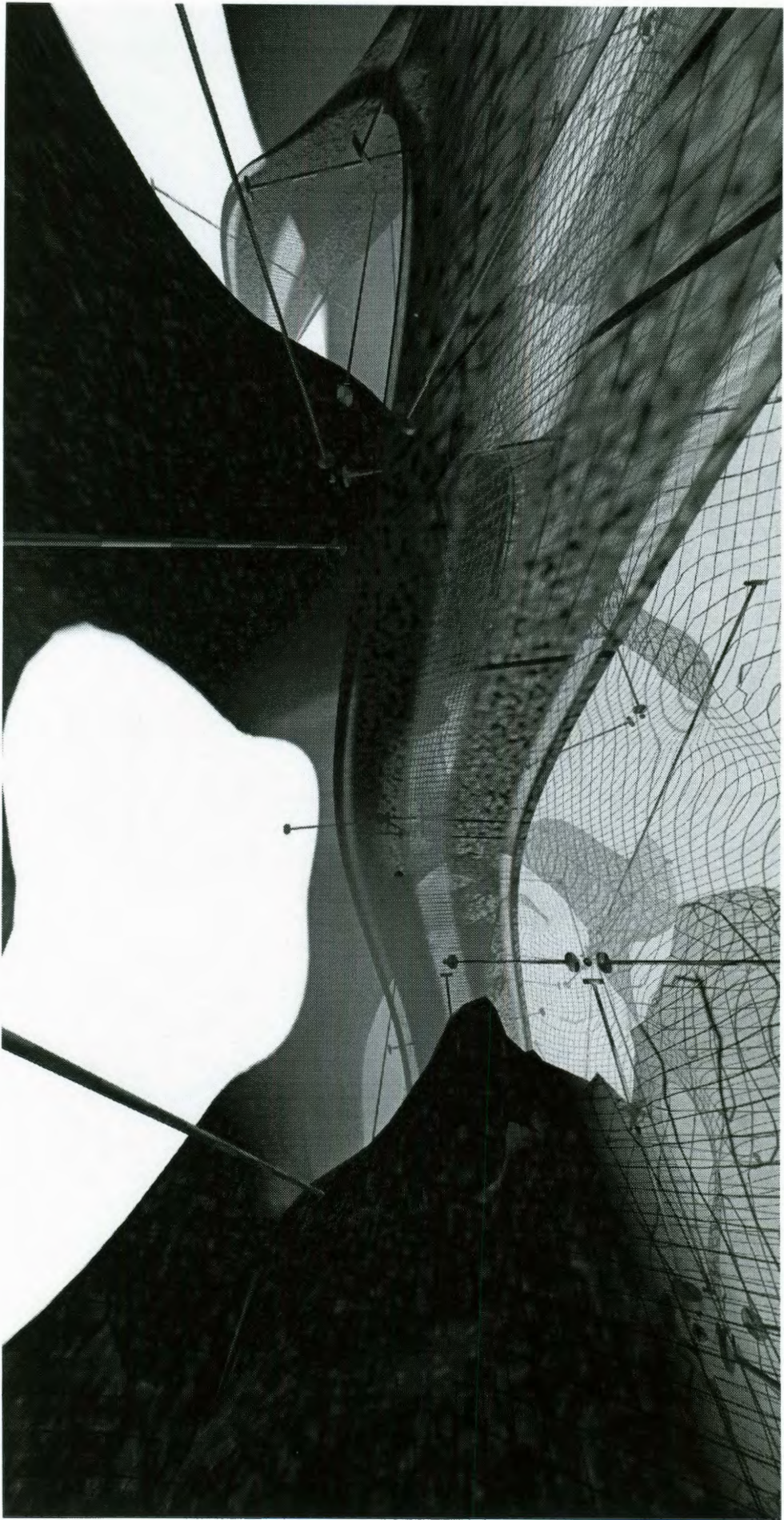


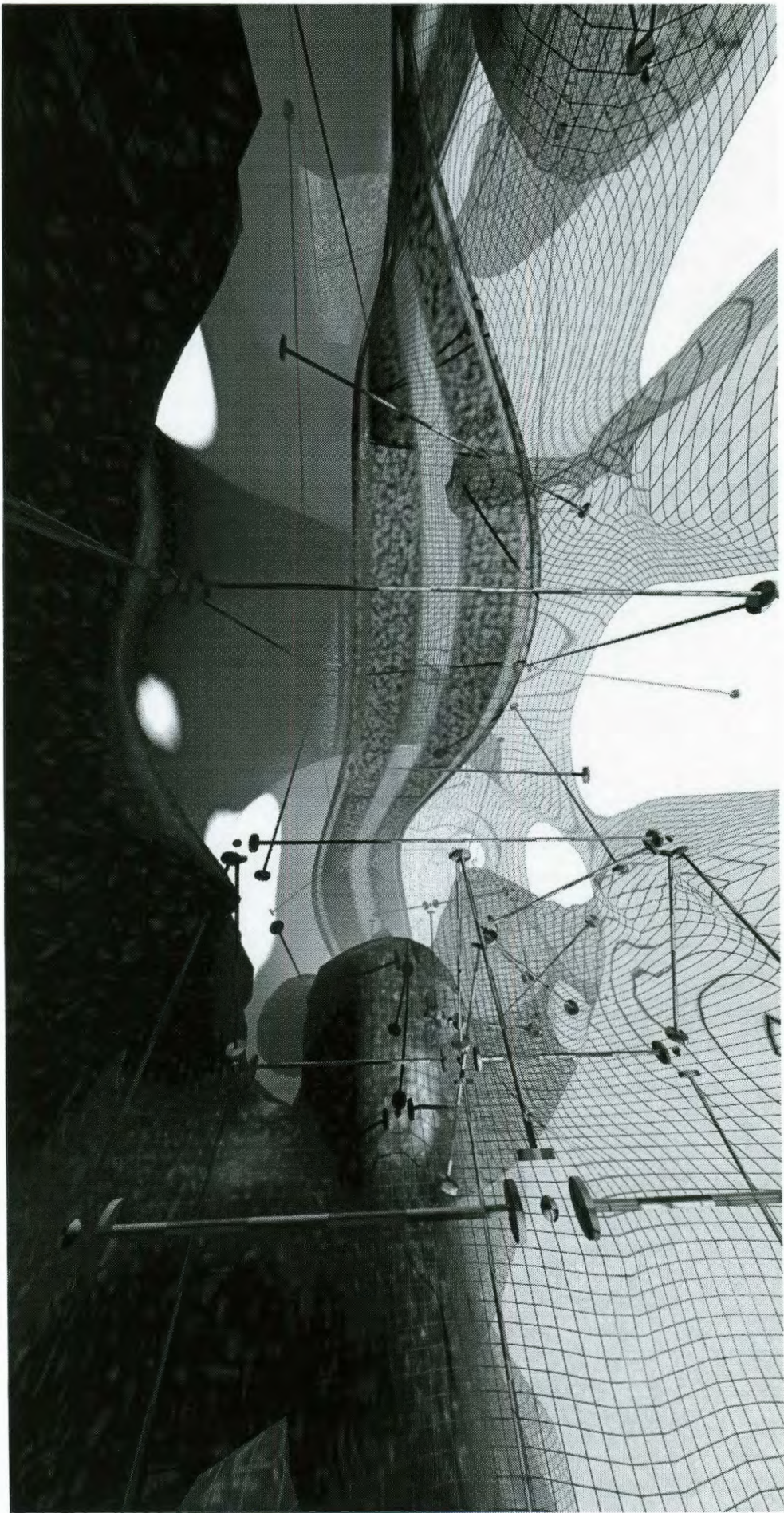


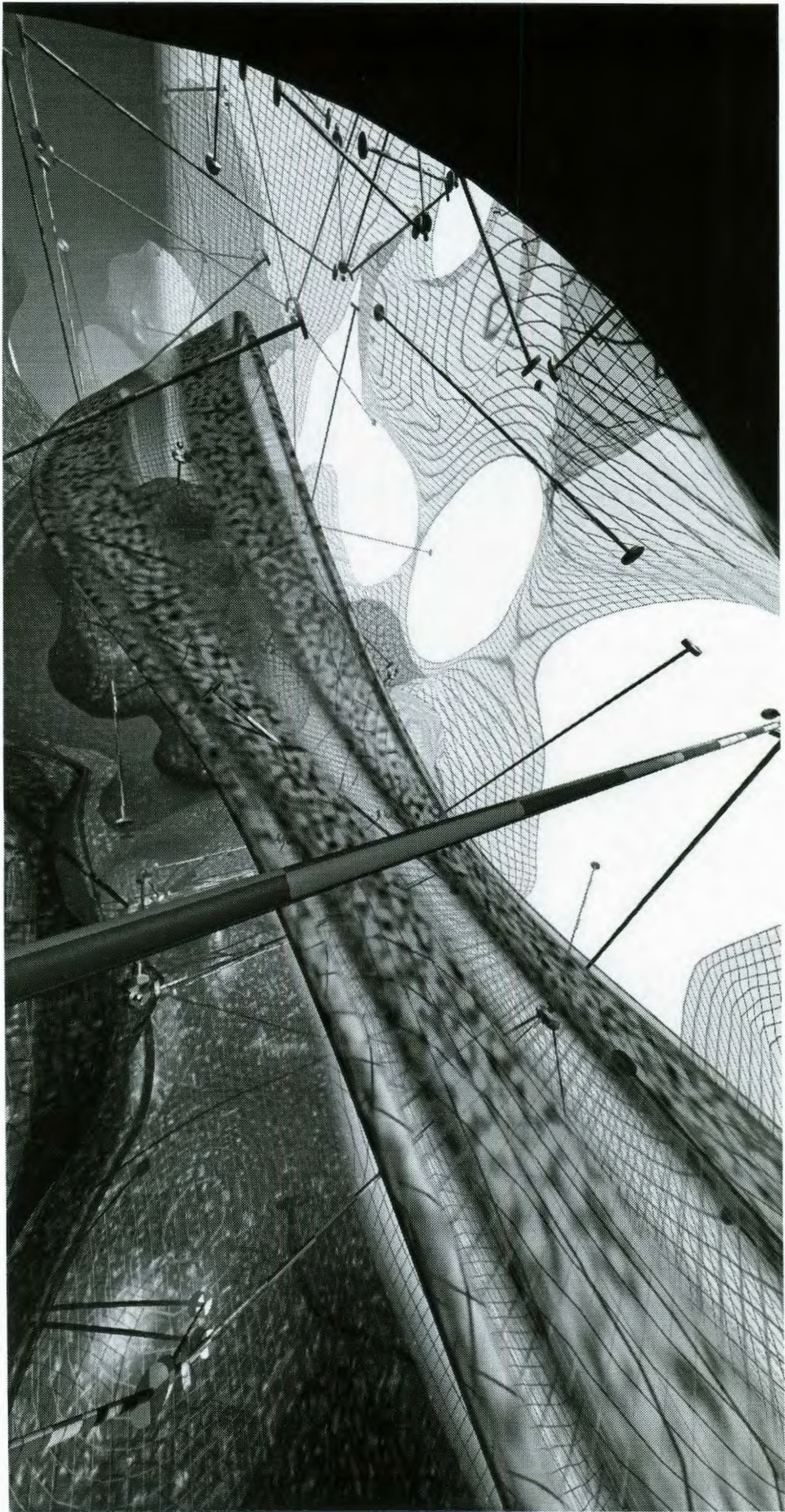


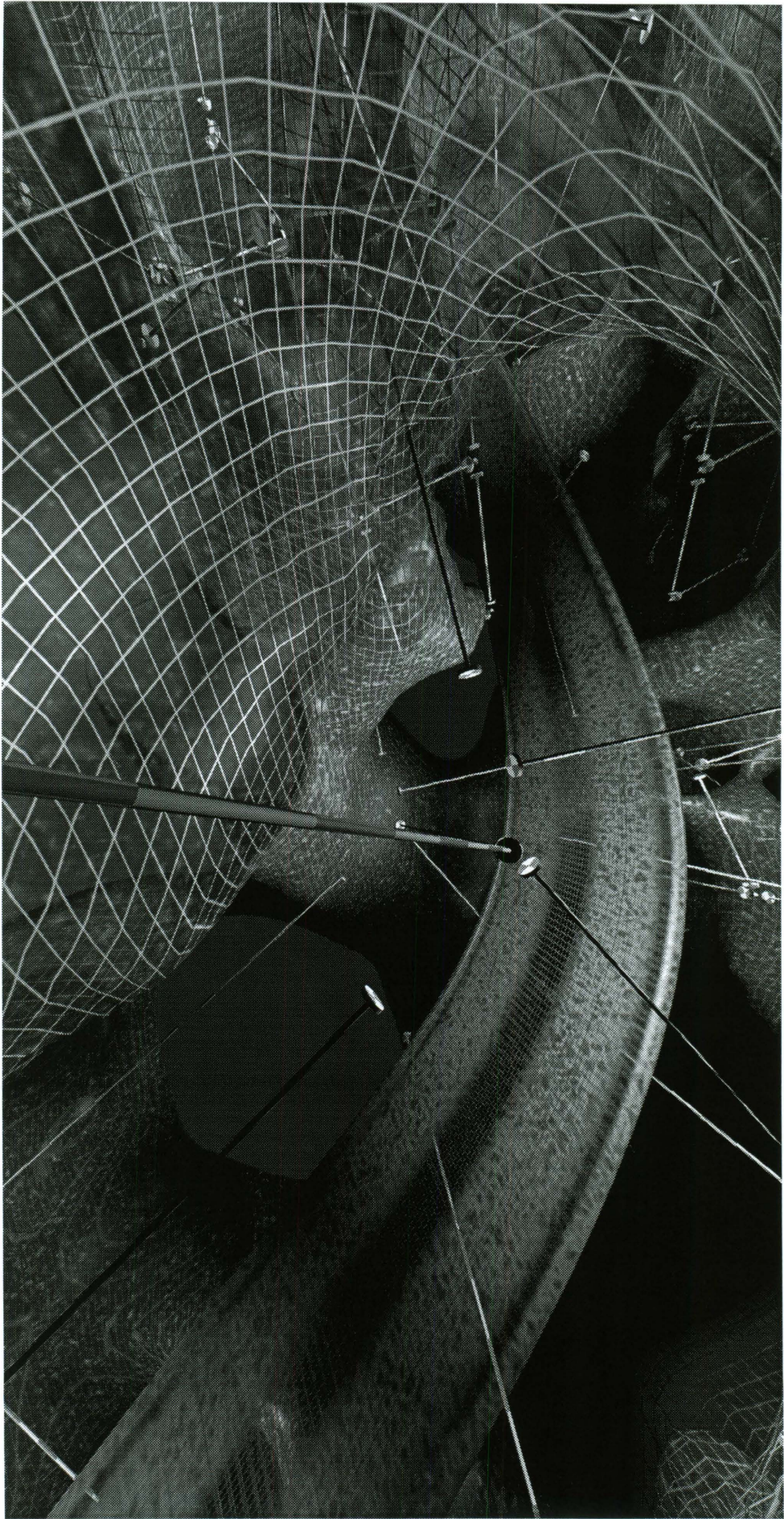




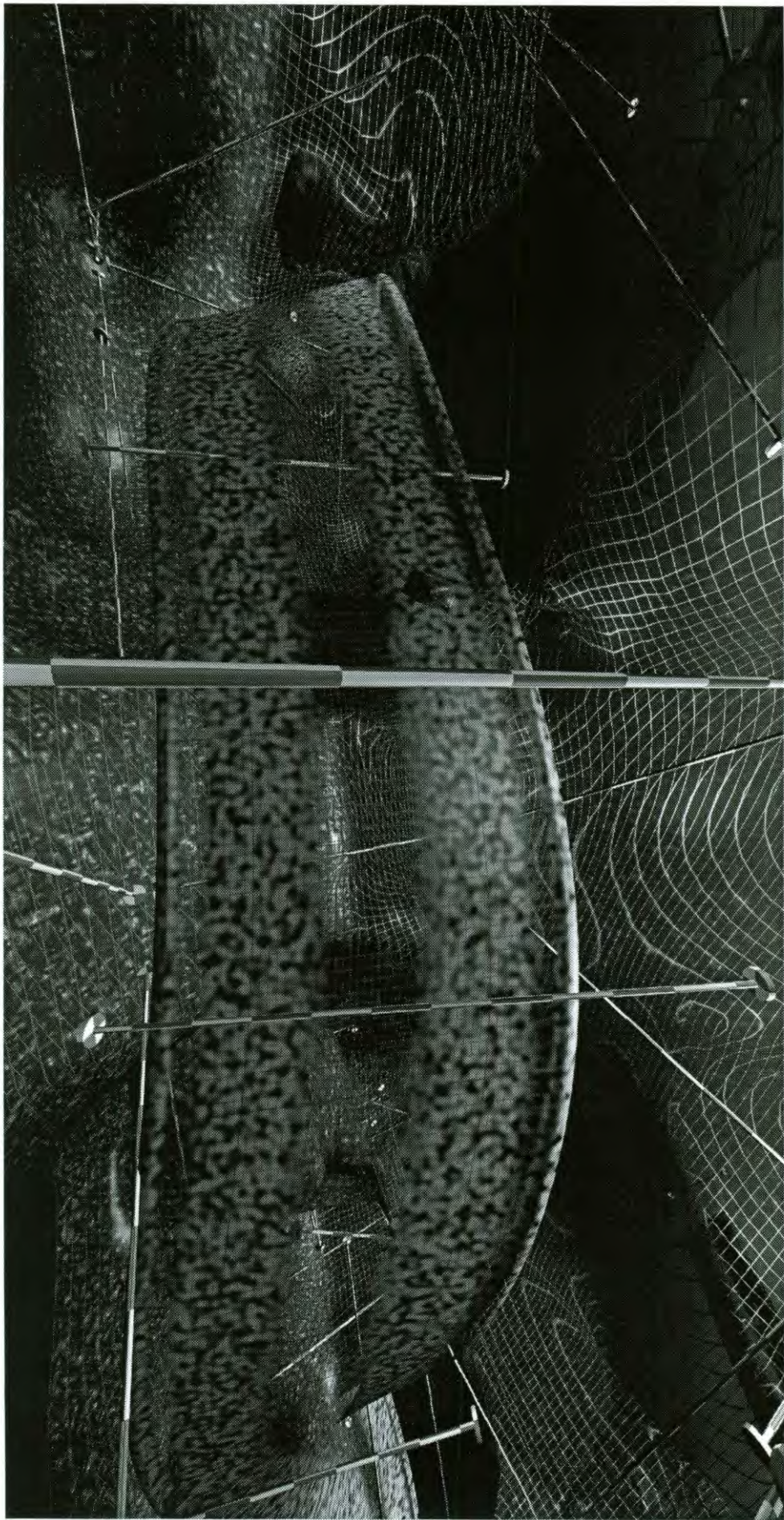












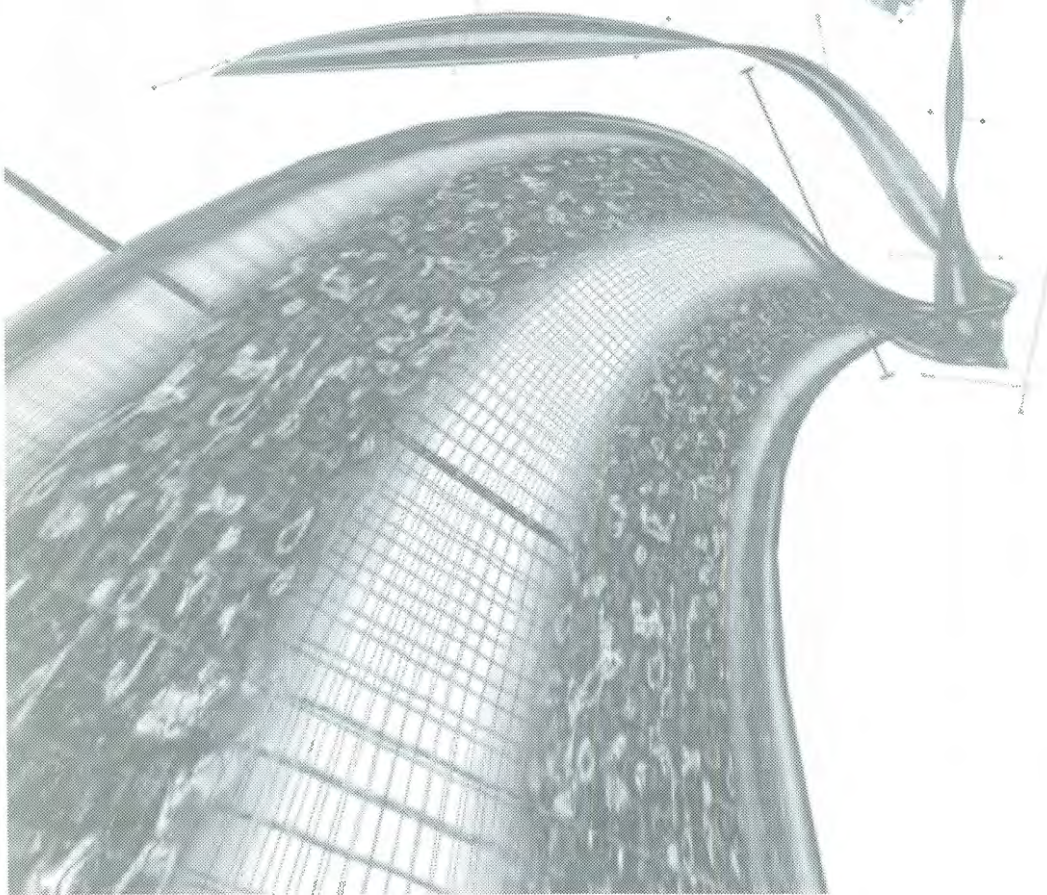




## Conclusion

The project concluded with a series of large images. The images were high resolution, interior renderings. The camera used a very large field of view to try to capture as much of the space as possible. The renderings. Several renderings were achieved of the space using both interior lighting and exterior lighting. By outputting these images in a large format, The Designer could hope to communicate how the space had expanded.

The goal that the Mathematician and the Designer had hoped to achieve was not simply to try to expand the space. The goal was to try to find the threshold where a space can read as a space. What they needed in the exploration was to achieve the use of a “system of design” rather than a “design.” The Mathematician needed to set up a means that the Designer could operate within and be forced to abandon most traditional design methods and tools. The Mathematician was aware that the only means with which one can specifically notice at which point the system reads as a space, is to build the system in a “pure” space. The Designer could not be influenced by physical attributes such as gravity. Only in the pure uninfluenced four-dimensional space would one be able to examine the nature of the system and its spatial characteristics.



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