

VALUATION OF CLOSELY-HELD STOCK FOR FEDERAL TAX PURPOSES: APPROACH TO AN OBJECTIVE METHOD

By LYLE R. JOHNSON, * ELI SHAPIRO ** AND
JOSEPH O'MEARA, JR. ***

PROBLEM AND PROPOSAL

The present double-standard approach to the valuation problem discriminates against owners of closely-held stock, that is, corporate shares which have no public market.¹ The disparity of treatment is traceable to the unfortunate fact that, whereas actual sales or bona fide bid and asked prices govern the valuation for federal tax purposes of listed stocks and of unlisted stocks which "are dealt in through brokers or have a market,"² the criteria employed in valuing closely-held stock are predominantly subjective. This makes a paradise for "experts" but they seldom agree and few, if any, of their assumptions and conclusions are anything more than unverified, and not always wholly

* Research Assistant in Statistics, The School of Business, The University of Chicago.

** Associate Professor of Finance, The School of Business, The University of Chicago.

*** Member of the Ohio Bar.

1. This excludes unlisted stock which is traded over-the-counter.

2. U.S. Treas. Reg. 105, § 81.10(c) (1944); U.S. Treas. Reg. 108, § 86.19(c) (1943).

There is no assurance the market would have absorbed additional shares on a given date at then prevailing prices; the market might have been broken under the weight of further offerings. The judge-made blockage rule is a response to this difficulty. This rule, applicable only in respect of large blocks of stock, *Mott v. Commissioner*, 139 F.2d 317 (6th Cir. 1943), permits the stock to be valued at the lower-than-market figure which could have been realized, according to the testimony of experts, by means of a special offering or secondary distribution, that is, off-the-exchange merchandising by specialists over a period of time. *Havemeyer v. United States*, 59 F. Supp. 537 (Ct. Cl. 1949), *cert. denied*, 326 U.S. 759 (1945); *Groff v. Munford*, 150 F.2d 825 (2d Cir. 1945); *Thomas A. Standish*, 8 T.C. 1204 (1947), *Acq.* 1947-2 CUM. BULL. 4. Thus blockage substitutes unverifiable estimates for the objective criterion of actual sales or bona fide bid and asked prices. But, in contrast with the procedure followed in valuing closely-held stocks, the issue is sharply defined (What could have been realized by means of a special offering or secondary distribution?) and relevant considerations are correspondingly delimited.

The regulations themselves leave the door open for escape in hard cases from the objective market-price-equals-value rule:

"In cases in which it is established that the value per bond or share of any security determined on the basis of selling or bid and asked prices as herein provided does not reflect the fair market value thereof, then some reasonable modification of such basis or other relevant facts and elements of value shall be considered in determining fair market value." U.S. Treas. Reg. 105, § 81.10(c) (1944); U.S. Treas. Reg. 108, § 86.19(c) (1943).

MONTGOMERY, FEDERAL TAXES—ESTATES, TRUSTS AND GIFTS 626 (1949-1950), lists 16 cases in which the value of stock was determined to be less than the quoted market price. Such cases, however, may fairly be considered exceptions and, aside from cases involving very large blocks of stock, the courts are not easily persuaded to depart from market quotations. *E.g.*, *Estate of Caroline McCulloch Spencer*, 5 T.C. 904, *Acq.* 1946-1 CUM. BULL. 4.

disinterested, hunches. As a consequence, disagreement between taxpayer and revenue agent is frequent and the process of resolving disagreement is long and costly and the event uncertain.³ In short, the

3. See Rice, *The Valuation of Close Held Stocks: A Lottery in Federal Taxation*, 98 U. OF PA. L. REV. 367 (1950).

The predicament of the taxpayer is illustrated by the following reply from an "expert" to a businessman seeking enlightenment:

"This memorandum is limited to an outline of factors involved 'to determine a fair value for a security which has no market quotations.'

"The factors involved in any such valuation, in the experience of the writer, are so numerous and each of such importance in individual cases that any attempt at establishing a set formula inevitably raises controversial problems and produces valuation results, even in securities of comparable companies, which are in direct conflict.

"It follows, therefore, that any valuation of a security which has no market quotation becomes an individual problem. (Emphasis supplied) Some type of formula may be applied, such as mentioned hereafter in this memorandum, but the final result must reflect the consideration given to the factors of greatest importance not represented by cold figures but exercising the greatest influence on any stock valuation. Some of these factors follow:

1—*Purpose of Valuation*

Unfortunately even experts differ and such differences may be based on honest opinions and extensive experience. To list a few purposes where valuations may differ, we have

- For sale purposes
- For purchase purposes
- For tax purposes
- Accounting purposes
- Litigation purposes
- etc., etc.

2—*Tax problems*

Increasing complexity of these problems as applied to stock valuation for any of the foregoing purposes becomes the major issue which definitely affects the final value to the owner of any security and this is not reflected in any formula but must be decided on its individual implications. A security sold at public sale at nominal price may conceivably produce more real value to an estate than by establishing an unlisted security value by formula where no real market exists or where it is severely limited.

3—*Continuity of Business*

This has an important bearing on value. It involves

- Ownership control
- Management control
- Securities distribution

So-called closely-held corporations or other business entities may depend almost entirely on continuity of management for success and this remains an important factor in valuation problems.

4—*Competitive Conditions*

Any value arrived at, after consideration of all individual aspects, must still reflect some reasonable comparative value to any existing comparable and competitive company, securities of which may have an established market.

5—*Market Possibilities*

The amount of outstanding stock involved in any valuation may have some controlling influence on value, such as

- 100% interest
- Majority interest
- Large minority
- Small block

6—*Goodwill*

Theoretically the value of goodwill is reflected in earnings. Formulae do take care of any stock value due to earnings, but it has been a part of the

whole merry-go-round of valuing closely-held stock is wasteful and unfair.

This paper and the study it reports are addressed to the possibility of improving the situation by making a fresh approach to the valuation of closely-held stock.

Mr. Justice Holmes once said that "value . . . as the word is used by the law . . . depends largely on more or less certain prophecies of the future" ⁴ In federal tax law that isn't necessarily so; for the most part, indeed, value depends not so much on peering into the future as on doing over the past. What was the value of such-and-such "expressed in the money that it would bring in the market" on a certain date in the past? ⁵ Specifically, what was its value on March 1, 1913 or when a gift was made or when a man died? That is, what could you have got for it in money if you had sold it at that time? ⁶

writer's experience to find a corporation with fifteen prosperous years followed by fifteen very lean years, and in spite of this the current value of the name could not be questioned although by no method of direct figuring could a goodwill value be established. This again becomes an individual and distinct problem.

7—General

The difficulty in applying a formula lies in finding two identical corporation twins, so to speak, to apply it on. One company may operate through mortgage loans and another by bank loans, and the result would differ.

"In the opinion of the writer, the foregoing are the important factors to be considered in determining a value and although he has seen formulae both simple and of great complexity, *it still remains a problem of good judgment applied to individual cases each on its own merits.* (Emphasis supplied)

"However, in order to avoid too abstract an approach to this problem of stock valuation, the basic formula generally used by the writer as a preliminary approach follows, subject, of course, to variations imposed by conditions.

Invested capital, 5-year average
 Net income, 5-year average
 Allow, say 6% on average invested capital as normal earnings
 Excess of 5-year average earnings over 6% on 5-year average
 capital a credit to good-will
 5-year or 3-year expectancy, five or three times such excess equiva-
 lent to goodwill value
 Add to adjusted book values of net assets
 Result—an approximate value

"The writer believes there is a practical method of determining the fair value of a stock in the absence of market quotations, and over a long period of years has been able to do so without undue difficulties arising, but such valuation can only be based on a full and complete knowledge of all factors surrounding the ownership, management, tangible assets and general conditions surrounding the subject and is based only in part on any fixed formula. Good judgment based on experience is a most important element in any computation of this nature."

4. *Ithaca Trust Co. v. United States*, 279 U.S. 151, 155 (1929).

5. *Ibid.*

6. *Andrews v. Commissioner*, 135 F.2d 314 (2d Cir. 1943), *cert. denied*, 320 U.S. 748 (1943). This is in accord with *Ithaca Trust Co. v. United States*, 279 U.S. 151, 155 (1929) (" . . . the value of property at a given time depends upon the relative intensity of the social desire for it at that time, expressed in the money that it would bring in the market.") and with *United States v. Petty Motor Co.*, 327 U.S. 372, 377

This calls for prophesy in reverse, prediction backward, a forecast to the rear. Orthodox prophesy is subject to verification: sooner or later it either turns out or it doesn't. But this business of predicting what might have been in circumstances which never occurred must remain forever in the realm of pure speculation. How many children would Maude Muller have had if she had married the Judge?

In federal tax law value is not a fact to be ascertained; value is an expedient, a means to an end, a tool, and its function (along with other devices such as exemptions and rates) is to determine tax liability in terms of dollars—to measure the tax-take. It cannot perform this office fairly so long as predominantly subjective criteria are employed in respect of closely-held stocks, that is, those which have no public market, while objective criteria (sales, bid and asked prices) are recognized as controlling in respect of stocks which do have a public market.⁷ Hence the problem is to evolve objective criteria which will function with substantial uniformity, as well in the case of stocks that are not traded publicly as in the case of those that are, and thus to relieve the disproportionate pressure presently exerted on the former.

The study reported in part II of this paper is a step in that direction. The study set out to ascertain whether and to what extent quantitative relationships exist which account for and thus can be used to estimate or predict the price of listed stocks in terms of reported earnings, dividends or book value. Back of the objective was this hypothesis: if in the case of listed stocks a statistical pattern can be

(1946) (“ . . . ‘market value’ does not fluctuate with the needs of [particular individuals] but with the general demand for the property . . .”). Decisions applying the blockage rule, note 2, *supra*, and cases involving the valuation of foreign currencies, *e.g.*, *Morris Marks Landau*, 7 T.C. 12 (1946), *Acq.* 1946-2 CUM. BULL. 3; *Ceska Cooper*, 15 T.C. 757 (1950), *Acq.* 1951 INT. REV. BULL. No. 8 at 1 (1950); *Estate of Ambrose Fry*, 9 T.C. 503, *Acq.* 1948-2 CUM. BULL. 2 (1947) exemplify the third-party approach enunciated in the *Ithaca Trust Co.*, *Petty Motor Co.* and *Andrews* cases. The most recent instance of this approach is *Estate of Blanchard v. Comm’r.* 184 F.2d 438 (3d Cir. 1950), *affirming* 8 T.C.M. 1088.

In contrast with these cases (which equate market value with the price buyers are willing to pay) the regulations go on repeating the old mumbo-jumbo about a willing buyer and a willing seller:

“The fair market value is the price at which the property would change hands between a willing buyer and a willing seller, neither being under any compulsion to buy or to sell.” U.S. Treas. Reg. 105, § 81.10(a) (1944); U.S. Treas. Reg. 108, § 86.19(a) (1943).

The trouble with this is that it is indeterminate—it tells nothing, leads nowhere. In real life people generally try to buy as low as possible and sell as high as they can. By what objective test can we determine the point at which the mythical buyer and the mythical seller will reach common ground—the point at which what one is willing to give will meet what the other is willing to take? The regulations are silent; they contain no clue to the answer. The willing-buyer-willing-seller jargon should be relegated to the Happy Hunting Ground of words without meaning.

7. As pointed out in note 2 *supra*, the regulations contain an escape clause which permits deviation from selling or bid and asked prices if they do not reflect “fair market value.” But this is the exception, not the rule.

traced between price on the one hand and reported earnings, dividends or book value on the other, application of the pattern to closely-held stocks will yield comparable and thus equitable valuations.⁸

That is the background from which the study emerged. Having emerged, it proceeded, quite independently of the problem of closely-held stock, simply as a project to discover, in the case of listed stocks, what if any statistical relationship exists between earnings and price, between dividends and price, between book value and price. Hence the study may have wider implications than any suggested here.

The findings may be summarized as follows:⁹

Of the three factors examined (earnings, dividends, book value) book value is the least dependable, and annual dividends the most reliable, index to the price of industrial common stocks listed on the New York Stock Exchange.¹⁰

There is no such thing, however, as a stable relationship between earnings and price, between dividends and price, between book value and price. On the contrary, the study highlights the shifting character of stock-market relationships. Thus there is no price-earnings ratio or dividend yield or price-book value ratio which can be used from year to year as a convenient rule of thumb.

At any given time, moreover, the ratio of price to each of the factors examined (earnings, dividends, book value) tends to decrease as earnings go up. Specifically, if two corporations earn, respectively \$1 and \$5 per share, the price of the latter will be something less, on the average, than five times as much as the former. And the pattern is the same for dividends and book value. That is to say, the yield of a stock paying, say, \$5 per share tends to be higher than that of a stock paying 50¢ per share. Similarly the percentage of book value represented by price is lower, on the average, in the case of a stock with a book value of \$100 per share, for example, than in the case of a stock with a book value of \$5 per share.

The fact that market relationships are not stable and that, at any given time, the statistical relationship between price and each of the

8. This assumes that the market-price-equals-value formula, applied in the case of stocks which have a public market (see notes 2 and 6 *supra*), is generally satisfactory in practice, as it is believed to be.

An interesting example of establishing the value of an unlisted security through comparison with the mean of the price-earnings ratios and of the dividend yields of several listed securities is found in *Estate of Wilson v. Comm'r.* C.C.H. Dec. 18487 (M) August 7, 1951. The Tax Court was convinced by this approach.

9. Presumably the findings can be applied only to minority interests since ordinarily that is what is traded on the New York Stock Exchange.

10. This is in sharp contrast with the Commissioner's customary emphasis on book value and disregard of dividends in valuing closely-held stock.

factors examined tends to be dependent upon the size of the latter—these facts emphasize the desirability of annual studies of prices on the principal exchanges. The pertinent data should be gathered systematically and published yearly by the Securities and Exchange Commission.

Two courses would then be open as a matter of administrative or legislative policy. The first is to accept the formula (to be determined annually by the proposed studies) which reflects most accurately the price of listed stocks in each occupational group (call it the optimum price formula) ¹¹ as the exclusive index of value of closely-held stocks

11. Cf. the provision in the Excess Profits Tax Act of 1950, INT. REV. CODE, § 447 for publication of industry rates of return:

“(b) Base Period Rate of Return.—The Secretary shall determine and proclaim for each industry classification in subsection (c) a rate of return (computed to the nearest thousandth) for the four year period 1946 through 1949. Such base period rate of return for each industry classification shall be obtained by aggregating the net income and interest deduction (such amounts being determined as provided under subsection (a)) for such four years and dividing the aggregate by the sum of the total assets (determined as provided under subsection (a)) for such four years.

“(c) Industry Classification.—For purposes of this subchapter the classification of taxpayers by industry shall be as provided in the table below. Each such industry classification is defined in accordance with the specifications shown in the Standard Industrial Classification Manual (prepared by the Division of Statistical Standards, Bureau of the Budget) for the major industry group or groups the numbers of which appear opposite such classification. . . .”

Preference for an objective approach was expressed by the Senate Committee on Finance in Senate Report No. 2679 on the Excess Profits Tax Act of 1950, at pp. 17 and 18:

“Section 722, the general relief provision of the World War II law, was designed to aid hardship cases by providing such corporations with a substitute, or constructive average, base period net income. Section 722 dealt with three principal classes of cases—(1) corporations which had suffered some adversity during their base period, (2) corporations which had made changes during the base period resulting in an increase in their profit potentials, and (3) corporations which were not in existence during the base period and, therefore, had no base period net income at all.

“In each instance the section provided that a hypothetical base period earnings credit be ‘tailor made’ for the particular taxpayer and that certain assumptions be made in connection with the case. Each case was a problem in research, and the legal or tax result generally was intertwined with complicated accounting and economic problems. Almost every factor which had any influence on the particular business was pertinent to the case and the time and expense involved in reconstructing the average base period earnings credit were tremendous.

“These complex relief provisions of the World War II law have resulted in extended delay in the settlement of relief claims which discriminated against taxpayers who had neither the time nor the financial resources necessary for the establishment of their cases. Moreover, the determination of what the taxpayer’s base period income would have been in the absence of the claimed abnormality was largely a matter of subjective judgment, and a great deal of complaint has arisen on this account. Hence this bill reduces to a minimum the amount of administrative discretion involved in the adjustment of the hardship cases which may be expected to arise under an excess profits tax. . . .”

“The bill provides automatic formulas for each of the most important types of cases which arose under section 722 of the World War II law. These formulas permit an objective computation of the amount of relief granted in each case, thus

of the same occupational classification.¹² Statistical averages, drawn from the action of prices on the principal exchanges, would thus become the sole criterion in the valuation of closely-held stock. Weighty considerations can be marshalled in support of a purely statistical approach, such as ease and economy of administration and consequent removal of the delays and uncertainties which characterize the present system. But a purely statistical approach would have serious disadvantages. For example, it might result in precisely the same value for the stock of two close corporations even though one was enjoying a healthy and continuing growth while the other was over the hill.

The alternative course is to adopt a modified statistical approach somewhat as follows. Application of the optimum price formula will, of course, yield the same estimated price for all listed stocks in a given occupational group which correspond statistically in those particulars which are significant for purposes of the formula. In contrast, the prices actually paid for these stocks (notwithstanding the identity of dividends or earnings or book value, as the case may be) will vary considerably. However, a substantial majority of the actual prices will be confined within a certain range.¹³ This price-spread could be accepted as establishing a floor under and ceiling over permissible valuations.¹⁴ This would tend to reduce the inordinate discrepancies frequently encountered between the valuation contended for by the Commissioner and that urged by the taxpayer,¹⁵ narrow the issue and focus attention on those considerations tending to make the stock in question more or less attractive to investors by comparison with others in the same occupational category. To be sure, there would still be the ele-

avoiding the practice of making the extent of the relief dependent upon an attempted analysis of all the varying factors in the individual case with the resulting uncertainty, delays, and disparity of treatment among taxpayers which characterized the application of the general relief provisions of the World War II law."

12. The study tends to support the practice of taking account of the market performance of those stocks only which belong to the same occupational group. See INT. REV. CODE § 811(k). Difficulties presently encountered in making comparative valuations are elaborated in Rice, *supra* note 3, at 391-92.

13. See Fig. 5 *infra*.

14. No doubt it will be objected that to tie the valuation of closely-held stock to dividends (as would be done if the optimum price formula should be based on dividends at the critical date) would be to turn over to close corporations a large measure of control over the value of their shares. It would seem a workable hypothesis, however, that no manipulative withholding of dividends had occurred in any case unless Sec. 102 of the Code (Surtax on Corporations Improperly Accumulating Surplus) had been invoked in the period reflected in the formula. If Sec. 102 had been applied in that period it would evidently be inappropriate to utilize any price formula based on dividends. In this situation recourse could be had to whatever formula not based on dividends most accurately reflected price at the critical date in the appropriate occupational classification.

15. See Rice, *supra* note 3, at 396 n.92.

ment of star-gazing but it would be circumscribed and held in check by being tied to a specific inquiry, namely, whether investors would have paid more or less within the limits fixed by the formula.¹⁶

This latter course would recognize that stock prices manifest human decisions conditioned by an intricate and changing set of circumstances, many of which defy quantitative expression. It would, accordingly, result in considerably less certainty than is attainable by a purely statistical approach; but it would avoid the inequities to which a purely statistical approach might lead, while at the same time confining the area of discretion (at present all but unlimited) within what seem to be reasonable bounds.

Something of this sort seems worth a try. The present study is published in the hope that it will lead to other and continuing factual inquiries which will refine and extend the usefulness of the experimental techniques, and amplify the data, which are respectfully presented in the balance of this article.

THE STATISTICAL INQUIRY

Rules for Estimating Stock Prices. In this article any algebraic equation proposed to describe or account for price in terms of one or more known quantities—such as annual earnings, annual dividends, book value, five-year average earnings, etc.,—is called a “rule” for estimating stock prices. The relationship “price equals 10 times annual earnings” is a rule as are “price equals 18 times annual dividends” and “price equals book value.”

Examples of rules evolved from the study are portrayed by the accompanying diagrams. The sloping line in Figure 1 is the graphic expression of the rule for estimating square root of price¹⁷ (and hence price itself) from 1945 earnings provided they do not exceed \$4 per share. For each value of earnings on the horizontal axis the line yields an estimated price.

In algebraic form the rule graphically expressed by the sloping line in Figure 1 is represented by

$$Y_e = a + bX$$

Y being estimated square root of price, X annual earnings and a and b

16. Some allowance would have to be made for the disadvantages of not having a public market, as was done in Drayton Cochran, 7 T.C.M. 325 (1948). Whether there is any way of expressing these disadvantages quantitatively must be left to future inquiries.

17. The statistical necessity for the use of square root of price is explained *infra*.

numerical coefficients computed from the data assembled in the study.¹⁸ In the case of McCrory Stores, for illustration, 1945 earnings, denoted by X, were \$2 per share. The coefficients a and b, set out in Appendix A *infra*, are 3.54 and 0.796 respectively. Hence Y is equal to

$$a + bX = 3.54 + 0.796 (2.00) = 5.13$$

and the estimated price of McCrory Stores is $(5.13)^2$ or \$26.30. The actual price was \$23.05.

Figure 2 is similar to Figure 1 except that the square root of price is plotted against 1945 dividends. Figure 3 is similar to Figures 1 and 2¹⁹ except that the horizontal axis represents square root of 1945 book value.²⁰ The sloping lines²¹ in these figures graphically represent rules for estimating price from 1945 dividends and 1945 book value, respectively.

The algebraic form of the rules is

$$Y_e = a + bX$$

18. The linear regression model

$$Y = \alpha_i + \beta_i X_i + E_i$$

(i being equal to 1, 2 or 3) was assumed because it appeared to describe the data satisfactorily. Y denotes square root of price; X_i denotes earnings (when $i=1$), annual dividends (when $i=2$) and square root of book value (when $i=3$). α_i and β_i are numerical coefficients describing the relationship between Y and X_i in the class of all stocks from which the sample used in the study was taken. E_i —a random variable with zero mean and variance σ_i^2 —denotes the amount by which an individual stock may depart from the true relationship.

In dealing with samples the exact values of α_i and β_i cannot be obtained. Neither can the behavior of E_i be exactly described; this would entail knowing the precise value of σ_i^2 . Instead unbiased sample estimates S^2 of σ^2 , a of α and b of β were obtained for each of the three cases ($i=1, 2, 3$). S^2 is the square of S, the standard error of estimate. The methods of forming these estimates and of evaluating their accuracy are described in SNEDECOR, *STATISTICAL METHODS* c. 6 (Iowa State College Press, 1946).

The variance ratio test was used to check the statistical significance of differences in S_i between samples for different years. *Id.*, §10.13.

To test the significance of pairwise differences in S_1 , S_2 and S_3 for a given sample, Hotelling's method was used. This is explained in JOHNSON, *STATISTICAL METHODS IN RESEARCH* 54 (1949). The 0.05 level of significance was used in all tests; if the chances were greater than 1 in 20 that the observed difference could have resulted from sampling variability alone, the difference was not considered conclusive.

19. The rules graphically expressed in Figs. 1, 2 and 3 can be approximated in terms of the familiar price-earnings, price-dividend and price-book value ratios. This is done in Appendix B, *infra*.

20. Square root of book value is used instead of book value so that a straight line will fit the data satisfactorily.

21. The lines are fitted to the data by the statistical method of "least squares." For a discussion of this method see SNEDECOR, *op. cit. supra* note 18, §6.5.

X being dividends or book value, as the case may be, and the other symbols having the same meaning as in the case of the price-earnings rule discussed above. To illustrate, the 1945 dividend of McCrory Stores was \$1. In the case of 1945 dividends the coefficients a and b, taken from Appendix A, are 3.42 and 1.48. Hence Y_0 is equal to

$$a + bX = 3.42 + 1.48 (1.00) = 3.42 + 1.48 = 4.90$$

and the estimated price is $(4.90)^2$ or \$24. As previously noted, the actual price of McCrory Stores was \$23.05.

The selection and processing of the data, from which the rules herein presented were derived, are described in the next section. In the following section a comparative evaluation of the rules is undertaken. The utility of such rules is examined in the final section.

Derivation of the Rules. Four separate investigations were made, one each for the years 1933, 1937, 1941 and 1945. The choice of years was made prior to the collection of any data and was guided by a desire to include the maximum number of different market situations: 1933 was a depression year, 1937 marked a recession, 1941 was stable and 1945 included a sharp upturn in prices.

In each year the inquiry concerned only industrial common stocks listed on the New York Stock Exchange. An "industrial" was taken to be an enterprise reported in Moody's *Manual of Investments: Industrials*; a "common" stock one which has a claim of lowest priority to earnings and to assets on dissolution.

The class of stocks regarded as eligible for the study was slightly narrowed by three other qualifications. For each of the years only those industrial common stocks were considered which were (a) not affected by a major reorganization or by bankruptcy proceedings within the years, (b) listed on the New York Stock Exchange throughout the year and (c) not the subject of a split-up within the year.²² Stock dividends of over 10% were treated as split-ups. There were seven instances of smaller stock dividends, for which no adjustments or omissions were deemed necessary.

Data on annual earnings per share, annual dividends per share and number of shares outstanding were obtained from Moody's *Manual of Investments*. Book value per share was computed from the balance sheets presented in Moody's. The high and low market price for each month of the year were obtained from W. B. Dana's *Bank and Quotation Record*.

22. These three exclusions do not in any way prejudice the conclusions of the study as long as it is remembered that the conclusions apply only to listed stocks not undergoing a reorganization or a split-up.

Book value per share was obtained by dividing net assets by the number of common shares outstanding. Net assets were obtained by subtracting from total assets the following items: current liabilities; long-term liabilities; tax reserves; depreciation and depletion reserves (where these reserves were included in total assets); preferred dividends in arrears; involuntary liquidation value of preferred or higher ranking stock; and reported value of treasury stock (where this item is carried in assets).

To arrive at a price for a given share in a given year 12 monthly prices were found by taking the average (arithmetic mean) of the high and the low price for each month. The average (arithmetic mean) of these 12 monthly prices was then defined as the "price" to be used in the initial inquiry. This method was employed in the thought that the business conditions reflected in an end-of-the-year earnings statement are sensed by investors during the year and that an annual measure of price would be most closely related to annual earnings.

Because fiscal years do not always coincide with calendar years it was necessary to adjust some of the data on earnings and book value. Suppose that M_1 months of a first fiscal year and M_2 months of a second fiscal year fall within the calendar year under consideration. If E_1 and E_2 are the annual earnings per share reported by a company for the first and second fiscal years and B_1 and B_2 the respective book values, then weighted averages E and B were used as the earnings per share and book value per share of the calendar year.

$$E = \frac{M_1}{12} E_1 + \frac{M_2}{12} E_2$$

$$B = \frac{M_1}{12} B_1 + \frac{M_2}{12} B_2$$

In each of the years selected for investigation many more stocks were eligible than it was practicable to study. As is commonly done in such circumstances, a sample was obtained by accepted statistical methods. All of the industrial common stocks listed on the New York Stock Exchange at the end of the year (in each case less than 1000) were numbered consecutively 001, 002, 003, etc. Numbers were then read from a three-digit column in the Interstate Commerce Commission's *Table of Random Decimal Digits* and stocks with corresponding numbers were selected for the sample. Once a stock was selected its number was thenceforth disregarded. Although 190 stocks were selected in each of the four years, some were subsequently disqualified as data were collected. The sample sizes finally obtained were as follows: 1933, 175; 1937, 172; 1941, 185; 1945, 176.

Analysis of the sample data dealing with prices and earnings revealed that the price-earnings relationship differed markedly between certain earnings classes. It was obvious, for example, that the 1933 and 1937 samples should be divided into (a) stocks reporting earnings equal to or greater than zero and (b) stocks with reported losses (negative earnings). A definite relationship between price and earnings was discerned in the first class of stocks; none could be discovered in the second. In other words, information on the amount of loss per share was not appreciably helpful in estimating price.

A more perplexing situation was encountered in the 1941 and 1945 samples. Very few of these stocks reported losses and none reported substantial losses. But it was apparent that the relationship between price and earnings broke down as regards the stocks with earnings above about \$4 per share. This tendency may be observed in Figure 4, a scatter diagram of the 1945 sample. The vertical axis represents price per share, the horizontal represents earnings per share and each dot is located by the price and earnings of an individual stock in the sample.

In the case of the stocks with earnings of \$4 or less per share there was a noticeable tendency for prices to rise as earnings increase. In the case of stocks with earnings above this point prices were very erratic—some high, others low. One might suspect that chance had selected an unusual or peculiar sample. Such doubts are removed by inspection of the 1941 sample. The latter, obtained independently of the 1945 sample exhibits the same peculiarities.

To make the analysis more meaningful, therefore, the 1941 and 1945 samples were each divided into two classes: (a) stocks with earnings of \$4 or less per share and (b) stocks with earnings in excess of \$4 per share. By far the larger proportion of stocks fell in the first group.

Even within these classifications, however, the variations in, or dispersion of, the prices of a group of stocks with, say, identical earnings depends upon the earnings group chosen. This is evident from Figure 4 which contains all of the stocks in the 1945 sample. Price dispersion (dispersion in the vertical direction) among stocks with \$2 earnings is greater than among stocks with \$1 earnings, and in general dispersion increases from lower to higher values of earnings. The practical implication of this is that there is no relationship between price and earnings which can be summed up in a single descriptive statistical measure: dispersion will differ with different values of earnings.

This difficulty was overcome for purposes of statistical analysis by substituting for price the square root of price. That is to say, in-

stead of studying directly the dispersion of prices, the alternative was adopted of studying variations in the square root of price. When square root of price is plotted against earnings, as in Figure 1, it is evident that vertical dispersion is very much the same for all values of earnings up to \$4 per share. Figure 1 includes all of the 1945 stocks with earnings of \$4 or less per share. As pointed out earlier, the remaining stocks were studied as a separate group.

Comparative Evaluation of the Rules. It can be assumed that any rule for estimating stock prices will be more or less wide of the mark. A rule for estimating price from earnings, for example, will yield one and only one estimated price for each value of earnings notwithstanding many stocks, with many different market prices, will report the same or very nearly the same earnings. Since the rule gives the same estimated price for all of these stocks the degree of error to be expected of the rule depends upon the variations in, or dispersion of, the market prices of this particular group of stocks. Similarly the error to be expected of rules for estimating price from dividends or from book value depends upon the price dispersion of stocks paying the same dividend or having the same book value.

Assuming that all rules for estimating stock prices are imperfect, which is the least imperfect? Specifically, which is the most accurate index of price—earnings, dividends or book value? To ascertain which of these factors leads to an estimate with the smallest expected error, measure statistically, and compare, the dispersion of prices of²³ (a) stocks with the same earnings, (b) stocks with the same dividend and (c) stocks with the same book value.

This dispersion of prices is illustrated by the plotted points in Figures 1, 2 and 3. In each case the vertical dispersion of the points about the sloping line corresponds with the dispersion of errors which would have been made if the rule represented by the line had been used to estimate prices of the stocks included in the figures. The most dependable rule, obviously, is the one accompanied by the smallest dispersion of prices, that is, the one which produces an estimated price with the least variance from actual prices. What is the rule which is accompanied by the smallest dispersion of prices? To answer this question an analysis of the data was undertaken, utilizing for the purpose the statistical measure known as the standard error of estimate, denoted by S .²⁴ The larger S , the more error may be expected of a rule, and vice versa.

23. Utilizing for this purpose the square root of price, for reasons explained in the text *supra*.

24. For a discussion of the standard error of estimate see SNEDECOR, *op. cit. supra* note 18, § 6.8.

The samples were divided into two parts for each of the four years, making essentially eight different sample groups. As to each of these three separate questions were asked: (a) Is information on earnings useful in estimating price, and if so what is the value of S? (b) Is information on dividends useful in estimating price, and if so what is the value of S? (c) Is information on book value useful in estimating price, and if so what is the value of S? These questions are answered in Table 1. Remember that the smaller S the more helpful is the corresponding variable in estimating prices.

Table 1

SAMPLE VALUES OF THE STANDARD ERROR OF ESTIMATE

Sample	Sample Size	Earnings	Dividends	Book Value
1933:				
Negative Earnings	81	1.12
Earnings zero or positive	94	1.03	1.02	1.43
1937:				
Negative Earnings	28	1.21
Earnings zero or positive	144	1.15	1.18	1.53
1941:				
Earnings \$4 or less	137	0.923	0.673	1.30
Earnings greater than \$4	48	2.05	1.12	1.99
1945:				
Earnings \$4 or less	144	1.14	0.869	1.14
Earnings greater than \$4	32	...	1.43	1.85

S is omitted where the evidence fails to show that a variable is useful in estimating price. Thus in 1933 and 1937 earnings and dividends do not appreciably assist in estimating prices of the stocks which reported losses. Since price bears little relationship to amount of loss and such stocks rarely pay dividends, book value is the only information found to be helpful in estimating prices in this situation. Again, in 1945, for stocks reporting earnings per share above \$4 the price-earnings relationship is so weak that earnings cannot be considered helpful in estimating price.

Analysis of the sample data leads, as Table 1 shows, to the following conclusions:

Both in 1933 and 1937, in the case of stocks suffering a loss, book value is the only variable helpful in estimating price.

The pattern changes in 1941 and 1945. In both of these years dividends are superior to either earnings or book value as an index to price. This is a valid inference for all stocks regardless of the earnings level.

In 1941 and 1945 earnings are not uniformly superior to book values for estimating price. In the case of stocks earning \$4 per share or less, earnings excel book value in 1941 but no difference can be asserted in 1945. In the case of stocks with higher earnings there is little difference in 1941 but book value excels earnings in 1945.

To summarize, except where earnings are negative, dividends are consistently as useful as any other single variable for estimation purposes. It is true that in 1933 and 1937 dividends are not appreciably better or worse than earnings, but in 1941 and 1945 dividends emerge as definitely superior to both earnings and book value. It is also noteworthy that dividends yield greater accuracy in estimating price in 1941 than in 1945.²⁵

The conclusions just summarized concern the relationship between price²⁶ on the one hand and, on the other, earnings, dividends and book value—each considered separately. In fact, however, earnings, dividends and book value are not independent of one another; these variables do not exist in isolation; to a greater or less extent, presumably, they interact upon each other. Hence the question arose as to the relationship between price on the one hand and, on the other, earnings, dividends and book value collectively. This relationship can be expressed satisfactorily as follows:

$$Y_0 = C_0 + C_1 X_1 + C_2 X_2 + C_3 X_3$$

where Y_0 is the estimated square root of price, X_1 is annual earnings, X_2 is annual dividends, X_3 is square root of book value and C_0 , C_1 , C_2 and C_3 are numerical constants determined by the sample data.²⁷ Again

25. It will be recalled that 1941 falls within a comparatively quiet and uneventful period for the stock market. The same cannot be said of 1945, a year marked by sharp and irregular upturns in stock prices.

26. See note 24 *supra*.

27. The regression model $Y = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2 + \gamma_3 X_3 + E$ was assumed for each year. Y denotes square root of price and E is assumed to be normally distributed with expected value zero and variance σ^2 . C_0, C_1, C_2 and C_3 are sample estimates of $\gamma_0, \gamma_1, \gamma_2$ and γ_3 . As before S^2 is the sample estimate of σ^2 . The method of obtaining these estimates and of testing their significance is discussed in SNEDECOR, *op. cit. supra* note 18, c. 13.

the standard error of estimate *S* is a satisfactory measure of the dispersion of errors that will be made by this rule. As before, the smaller *S*, the smaller the error to be expected of the rule when it is used to estimate a price.

Rules of this type were calculated for six of the eight sample groups included in Table 1.²⁸ The attendant values of *S* are given in column 3 of Table 2. Column 4 gives the value of *S* obtained when earnings are left out of the rule, column 5 the value of *S* when both earnings and book value are omitted—this being the same as the column for dividends in Table 1. Omitting dividends gives the results in column 6, omitting book value and dividends the figures in column 7.

Table 2

VALUES OF THE STANDARD ERROR OF ESTIMATES
FOR SEVERAL COMBINATIONS OF VARIABLES

(1) Sample	(2) Number in Sample	(3) Earnings, Dividends and Book Value	(4) Dividends and Book Value	(5) Dividends Alone	(6) Earnings and Book Value	(7) Earnings Alone
Earnings zero or over						
1933	94	0.834	0.924	1.02 ^a	0.971	1.03 ^a
1937	144	0.999	1.04	1.18 ^b	1.07	1.15 ^b
Earnings \$4 or less						
1941	137	0.577	0.629	0.673	0.903	0.923
1945	144	0.813 ^c	0.818 ^c	0.869	1.01	1.14
Earnings over \$4						
1941	48	1.14 ^d	1.13 ^d	1.12 ^d	2.01	2.05
1945	32	1.48 ^e	1.45 ^e	1.43 ^e	1.82	...

^{a, b, c, d, e} The difference in the values marked is statistically insignificant.

Analysis of the data in Table 2 leads to the following conclusions as regards stocks with earnings of \$4 or less:

28. The cases of negative earnings in 1933 and 1937 were ignored; it has already been demonstrated that earnings and dividends are of negligible value for these two cases.

In 1933 and 1937 all three variables contribute some independent information regarding price. There is no definite indication that earnings and dividends differ much in the influence they exert.

In 1941 all three variables likewise contain some independent information for, when earnings are omitted S rises from 0.577 to 0.629, a rise that cannot reasonably be considered negligible although it has minor significance so far as actual price estimation goes. On the other hand, when dividends are disregarded S jumps from 0.577 to 0.903, a rise that is definitely important for purposes of estimating stock prices. It is such a large rise, in fact, as to indicate that in 1941 dividends alone ($S = 0.673$) had more influence on price than earnings and book value jointly ($S = 0.903$). A similar statement for 1933 or 1937 would be wrong: it is clear that dividends were more important in 1941 than in 1933 or 1937.

In 1945 there is no evidence that earnings had any independent influence on price. For all three variables taken jointly, S equals 0.813; when earnings are omitted S shows a negligible increase to 0.818; but when dividends are omitted S shows an important rise from 0.813 to 1.01. Apparently, therefore, suppression of information about earnings would have had no effect, assuming that other factors remained unchanged.

As regards stocks earning more than \$4 per share, the following conclusions may be drawn:

The 1941 and 1945 samples furnish no evidence that either earnings or book value is helpful in judging price. Although it is possible that larger samples might show book value to be of some slight use, it seems fair to conclude that the practical importance of book value is very small.

Dividends are subject to less year-to-year fluctuation than earnings. The fact that dividends excelled annual earnings as an indicator of price in 1941 and 1945 suggested that earnings might be more useful if smoothed out by some averaging process. To test this, enough additional data were collected so that five-year average earnings and five-year average dividends could be computed for the 1945 sample. Because book value varies little from year to year the labor of gathering data for five-year average book values did not seem justified. The

expedient of dividing the sample into two parts was found to be as desirable as before, and again \$4 appeared to be a satisfactory dividing line.²⁹

Table 3 shows the results of a procedure similar to the one described for annual earnings and dividends. To facilitate comparison the results of the analysis for annual earnings and dividends are included in the table. Evidence from the 32 stocks with five-year average earnings of \$4 or more does not justify the conclusion that information on earnings is helpful in estimating the price of such stocks. It seems clear, therefore, that the 1945 price behavior of stocks with high earnings cannot be explained to any useful degree by information about either annual or five-year average earnings.

Table 3

1945 SAMPLE VALUES OF THE STANDARD ERROR OF ESTIMATE

Sample	Five-year Average Data			Annual Data		
	Sample Size	Earnings	Dividends	Sample Size	Earnings	Dividends
Earnings \$4 or less	143	0.944	0.842	144	1.14	0.869
Earnings greater than \$4	32	...	1.41	32	...	1.43

The 143 stocks in this table with average earnings of \$4 or less are not strictly comparable to the 144 in Table 2 having annual earnings of \$4 or less because the two groups are not composed of exactly the same stocks. A rigorous comparison is further prevented by the fact that average earnings/dividends are not independent of annual earnings/dividends. However, in view of the fact that the values of S for annual dividends and average dividends are almost equal, the improvement from 1.14 to 0.944 for earnings is suggestive that the averaging of data is more important as regards earnings than dividends. Nevertheless, five-year average dividends clearly excel five-year average earnings for purposes of estimating price.

An annual measure of price was used because, as pointed out earlier, it was felt that such a measure would be more closely related to

29. By coincidence, 32 stocks fell in the group with five-year average earnings of more than \$4 per share, although the composition of the group differed slightly from the group of 32 stocks with annual earnings over \$4 per share. One stock was omitted because data were not available for five years; therefore 143 stocks comprise the group with average earnings of \$4 or less.

annual earnings and annual dividends than a daily or monthly measure of price. On the other hand, in assigning value to a common stock one is ordinarily concerned with a short time period, *e.g.*, a month or a specific day of a month. If stock prices are fairly even throughout a year conclusions applicable to the average yearly price will likely apply quite well to prices computed for a shorter period; but if the market experiences sharp changes, as it did in the latter half of 1945, it may be dangerous to assume that the relative positions of earnings, dividends and book value with respect to price have not changed during a year. To examine the relationship of earnings, dividends and book value to price in respect of a shorter interval of time, additional computations were carried out for the 1945 sample. A single day would have been chosen but for the fact that not all shares have a market transaction each day. A month seemed to be the convenient short period for investigation.

Price was redefined as the arithmetic mean of the December (1945) high and low. Other variables have their previous meanings and variations in price were measured exactly as before. Results are given in Table 4.

Table 4
VALUES OF THE STANDARD ERROR OF ESTIMATE
FOR DECEMBER 1945, PRICES

Sample	Number in			
	Sample	Earnings	Dividends	Book Value
Earnings \$4 or less	144	1.18	0.977	1.22
Earnings greater than \$4	32	...	1.53	1.88

The values for S are all larger than before, indicating that slightly less accuracy should be expected in estimating the December average price than in estimating the average yearly price. The relative standings of the three variables in respect of price did not change, however. So far as concerns stocks with annual earnings of less than \$4 per share, analysis leads to the conclusion that dividends are superior to earnings and to book value, but fails to show any significant difference between earnings and book value. As regards stocks with earnings over \$4 per share dividends excel book value and book value excels earnings. The relationship between price and earnings is so weak that it was not detected from the sample of 32 stocks.

Utility of the Rules for Estimating Stock Prices. The success of a rule for estimating stock prices depends upon two things: (a) the extent to which the rule faithfully represents the market relationship it purports to express (for example, the relationship between price and earnings), and (b) the extent to which individual stock prices diverge from the statistical expression of this relationship. As to the first point, the rules resulting from the study are based on enough observations so that they may be expected to describe average market relationships rather closely. Therefore, to get an adequate idea of the success to be expected of the different rules attention may be confined to the second point. The standard error of estimate does not measure this deviation; it measures the deviation of the square roots of individual prices. But variations in square root of price can easily be expressed graphically as variations in price and this is done in Figure 5. This figure shows graphically what a given value of the standard error of estimate S means in terms of price estimation. Given an estimated price and the value of S which accompanies the rule used in the estimate, Figure 5 yields an interval or spread which may be expected to include the market prices of about two-thirds of all stocks with the same estimated price.³⁰

For example, suppose that a stock has an estimated price of \$40. Even though this is the best estimate of the stock's price, only by chance would it coincide with actual market price. To what extent is it apt to be wrong? This is answered by Figure 5. In this figure the horizontal axis represents the estimated price; the sloping lines represent different values of S . From these two quantities it is possible to obtain an interval or spread on the vertical axis which reflects actual market prices. With an estimated price of \$40, suppose that S is 1.0. Enter Figure 5 at 40 on the horizontal axis and follow the vertical lines. This will intersect two S lines labeled 1.0 at heights equivalent to about 28 and 54. Although \$40 may be the most reasonable estimate of price, actual prices vary so greatly that the interval \$28 to \$54 can be expected to include only two-thirds of the stocks whose estimated price is \$40 per share. As was seen in Table 1, S was seldom less than 1.0 in any of the rules of this investigation. On the other hand this does not mean that rules are useless. In the absence of better techniques for estimating prices, it is rational to use the rule with the smallest S , *i.e.*, the smallest probability of error.

30. Three assumptions underlie this statement. They are (1) that Y , the square root of price, is normally distributed about the true market relationship $\alpha + \beta X$ with variance σ^2 ; (2) that the sample rule $a + bX$ is a perfect estimate of $\alpha + \beta X$; and (3) that the standard error of estimate S , when squared is a perfect estimate of σ^2 . It is unlikely that any of these assumptions are perfectly true, but they should approximate the truth closely enough to make Figure 5 very useful.

In Figure 5 the area enclosed by the two S lines labeled 0.5 has been shaded. This permits one to visualize the intervals that would attend a rule for which S is equal to 0.5. None of the rules treated in this study yields an interval so narrow, although the special one mentioned in the following paragraph closely approaches it.

The wide variations portrayed by Figure 5 indicate that prices are influenced to a great extent by factors other than reported earnings, dividends and book value. As this inquiry is concerned with the effect of these three variables, no special effort has been made to isolate other forces, some of which, no doubt, cannot be measured statistically. The evidence does suggest, however, that price is associated with industrial classifications. Figure 6 is a scatter diagram of the 143 stocks in the 1945 sample that have five-year average earnings of \$4 per share or less. The vertical axis represents square root of price, the horizontal axis five-year average dividends. Roman numerals designate the four airline stocks in the sample and Arabic numbers the seven copper and iron mining companies.³¹ The airlines are priced high relative to their dividends; the mining companies, on the other hand, tend to be priced low relative to dividends. When these 11 stocks are omitted from the sample the value of S for the five-year average-dividend rule drops from 0.842 to 0.605. The conclusion may fairly be drawn that further improvements in accuracy could be obtained by dividing the sample according to occupational categories and studying each group of stocks separately.

31. (1) Calumet and Hecla Consolidated Copper, (2) Miami Copper, (3) Great Northern Iron Ore, (4) Phelps Dodge, (5) International Nickel, (6) Chile Copper, (7) Cerro de Paseo Copper; (I) National Airline, (II) Penn Central Airlines (Capital Airlines), (III) Northwest Airlines, (IV) American Airlines.

APPENDIX A

Let Y_0 ² be an estimate of a security price, averaged over some given time period. Let the estimating equation be

$$Y_0 = a + bX$$

Where Y_0 denotes the estimated square root of the average price, the average taken over twelve monthly prices as in our main study, X may denote (1) annual earnings, (2) annual dividends, or (3) square root of book value. Sample values of a and b for these three cases, in our four selected years, are given in Table I.

TABLE I
REGRESSION COEFFICIENT FOR ANNUAL DATA

Sample	Annual Earnings		Annual Dividends		Book Value	
	a	b	a	b	a	b
Annual earnings less than zero:						
1933	1.23	0.347
1937	2.14	0.310
Annual earnings zero or over:						
1933	2.77	0.998	3.57	1.18	0.798	0.829
1937	3.41	0.804	3.59	1.07	2.52	0.672
Annual earnings \$4 or less:						
1941	1.65	0.898	1.99	1.34	1.89	0.353
1945	3.54	0.796	3.42	1.48	2.69	0.499
Annual earnings over \$4:						
1941	3.77	0.345	2.87	1.07	3.41	0.385
1945	4.44	1.17	3.46	0.504

Now distinguish five other possibilities for 1945. With Y unchanged, let X represent (a) five-year average earnings and then (b) five-year average dividends. Finally let Y denote the square root of December 1945 price while X denotes (c) 1945 earnings, (d) 1945 dividends, and (e) square root of 1945 book value. Table II contains sample values of a and b for these cases.

TABLE II
REGRESSION COEFFICIENTS IN SUPPLEMENTARY ANALYSIS

Variable used in estimation	Earnings \$4 or less		Earnings above \$4	
	a	b	a	b
	December 1945 Price			
Annual Earnings	3.97	0.851
Annual Dividends	3.95	1.47	5.20	1.14
Book Value	3.23	0.50	3.96	0.521
	1945 Annual Price			
5-year Average Earnings	2.94	1.06
5-year Average Dividends	3.43	1.49	4.79	1.06

APPENDIX B

We can approximate the formula used by this simpler one: price equals twelve times annual earnings. This latter rule is easy to apply and, because it employs the familiar "price-earnings" ratio, year-to-year comparisons can be easily interpreted. Using this approach, the price of McCrory Stores would be estimated at $12 (\$2) = \24 per share. This is reasonably close to the previous estimate of \$26.30, but we cannot be sure that the two rules would agree as well for other values of earnings. Table a provides comparisons over a whole range of possible earnings.

Table a

1945 ESTIMATED PRICES: ORIGINAL EARNINGS RULE AND PRICE-EARNINGS RATIO OF 12 TO 1

Actual Earnings	Estimated Price	
	Original Rule	Price-earnings Ratio
0.00	\$12.50	\$ 0.00
1.00	18.80	12.00
2.00	26.30	24.00
3.00	35.20	36.00
4.00	45.20	48.00

The price-earnings ratio of twelve to one gives results that compare favorably with those of the original rule when earnings are \$2, \$3 or \$4 per share. For earnings of \$1 per share the price-earnings ratio yields an estimated price considerably smaller than the latter; for earnings equal to zero it gives an estimated price of zero as compared to \$12.50 for the more exact rule.

A similar argument can be made for a price-dividend ratio. Using information on dividends we estimated the 1945 price of McCrory Stores to be \$24. A price-dividend ratio of 21 to 1 will yield an

estimated price of 21 (\$1) or \$21. Here there is fairly close agreement between the prices given by the rule and by the price-dividend ratio, but as shown by Table b, this is not true for all values of dividends. For dividends smaller than \$1 the price-dividend ratio strongly tends to under-estimate price.

Table b
ESTIMATED PRICES: ORIGINAL DIVIDEND RULE *
AND PRICE-DIVIDEND RATIO
(1945 DATA)

Actual Dividends	Estimated Price	
	Original Rule	Price-dividend Ratio
0.00	\$11.70	\$ 0.00
0.50	17.30	10.50
1.00	24.00	21.00
1.50	31.80	31.50
2.00	40.70	42.00
2.50	50.70	52.50
3.00	61.80	63.00
3.50	74.00	73.50

* For stocks with earnings of \$4 or less.

To generalize from the comparisons in Tables a and b, it can be said that a price-earnings (price-dividend) ratio gives a highly unrealistic estimate when earnings (dividends) are zero. The price-earnings ratio improves as one considers larger earnings.

For earnings of \$2 or more per share it leads to satisfactory estimates. A similar tendency is observed with the price-dividend ratio, which does not come to close agreement with the carefully-fitted rule until dividends are about \$1 or larger.

These caveats should be kept in mind when reading Table c. Here the price-earnings and price-dividend ratios are submitted that best approximate several of the rules actually fitted to the data.

Table c
PRICE-EARNINGS AND PRICE-DIVIDEND RATIOS

Year and Classification	Ratio of Price to annual	
	Earnings	Dividends
1933: Earnings zero or over	12 to 1	20 to 1
1937: Earnings zero or over	12 to 1	21 to 1
1941: Earnings \$4 or less	7 to 1	12 to 1
Earnings over \$4	6 to 1	13 to 1
1945: Earnings \$4 or less	12 to 1	21 to 1
Earnings over \$4	21 to 1

No price-earnings ratio is exhibited for the 1945 stocks with earnings over \$4 per share. As explained for Table 1, the sample fails to demonstrate a relationship between the prices and earnings of such stocks. For a similar reason, no ratios are given for the 1933 and 1937 stocks suffering losses.

Table c clearly demonstrates that price-earnings and price-dividend ratios can undergo enormous year to year variations. A price-earning ratio of 12 to 1 in 1933, 1937, and 1945 drops to the neighborhood of 6 or 7 to 1 in 1941, while a price-dividend ratio of 21 to 1 is depressed to about 12 or 13 to 1.

No attempt has been made to express the price-book value rules derived from the samples by approximate price-book value ratios. In general, such ratios seriously misrepresent the empirically-fitted rules provided by the actual samples. This is apparent from Table d, which compares prices given by the price-equals-book-value rule with prices given by the statistical rules. In the case of 1941 and 1945, table d applies only to shares with annual earnings of \$4 or less, but nevertheless it provides enough evidence to seriously damage the price-equals-book-value rule.

Table d
ESTIMATES OF PRICE FROM BOOK VALUE

Book Value	A priori Rule	Statistical Rules			
	Price equals book value	1933	1937	1941	1945
\$ 0	\$ 0	\$ 0.50	\$ 6.32	\$ 3.66	\$ 7.21
4	4	6.05	14.90	6.70	13.60
9	9	10.80	20.50	8.70	17.50
16	16	16.90	28.10	10.90	23.00
25	25	24.50	34.50	13.20	27.00
36	36	33.20	43.00	16.00	32.30
49	49	43.50	52.00	19.00	38.20

It is apparent from Table d that the relationship between price and book value can vary tremendously. In 1941 price tended to be very low compared to book value, a tendency that concurs with results obtained from earnings and dividends. In all years there is a noticeable tendency for price to exceed book value for book values of \$9 per share or less. The evidence certainly does not point to close agreement between market price and book value in any consistent or general way.

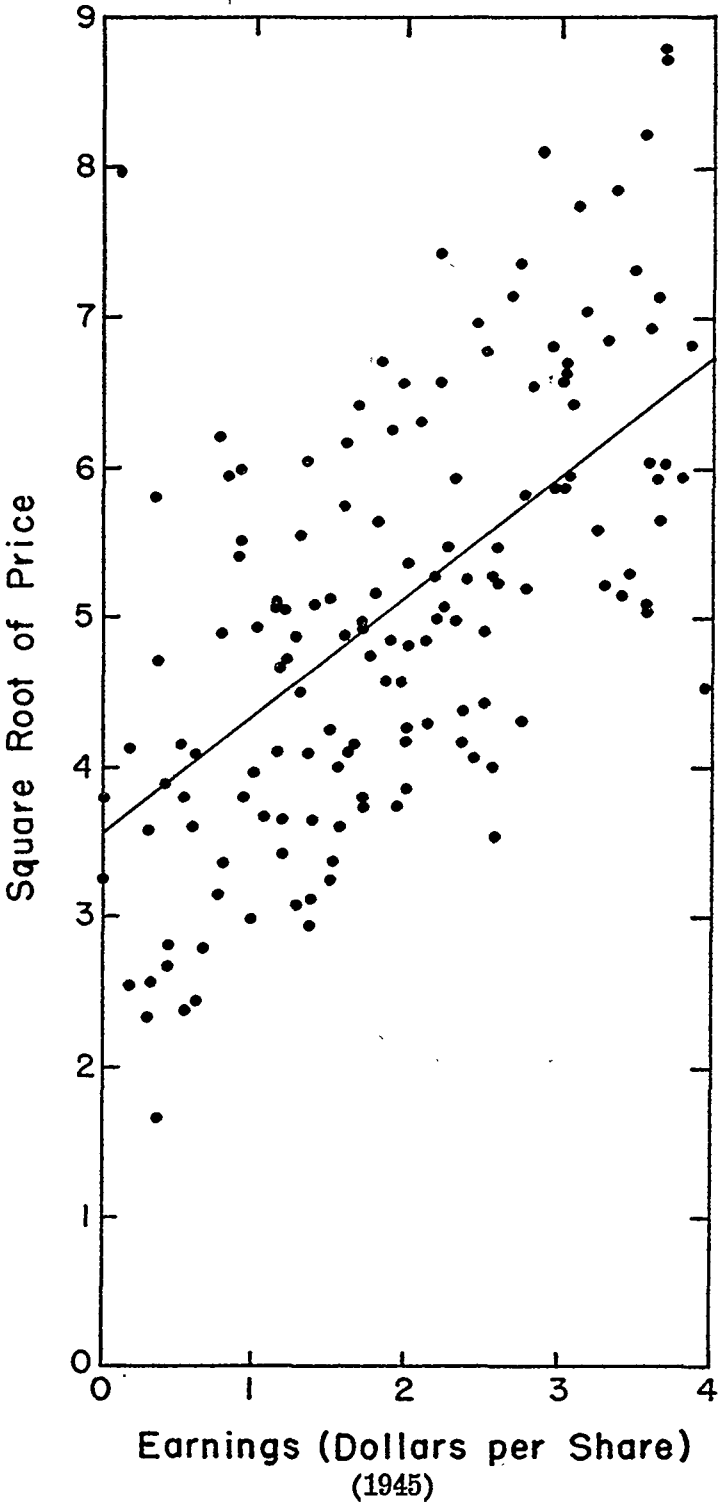


FIGURE 1

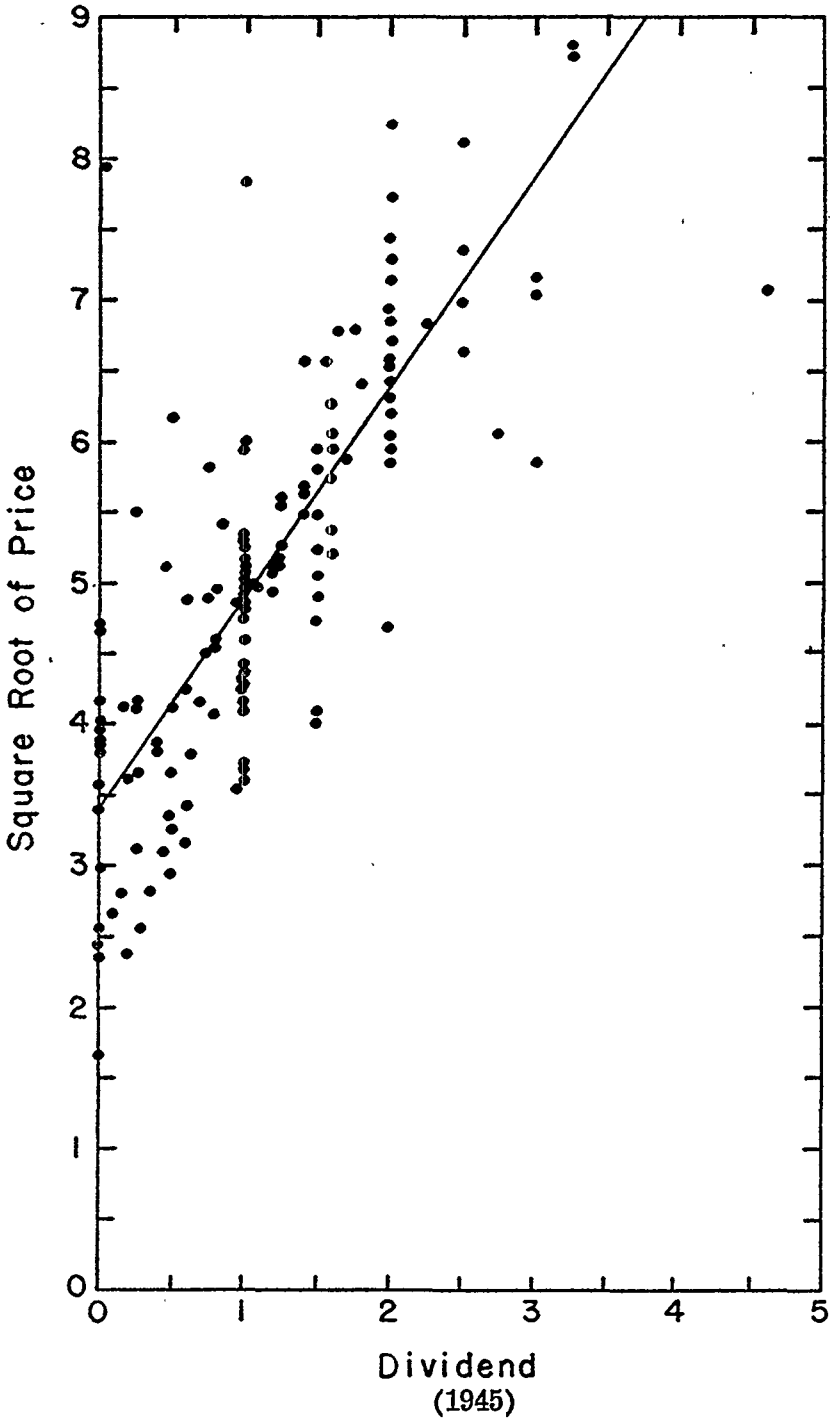


FIGURE 2

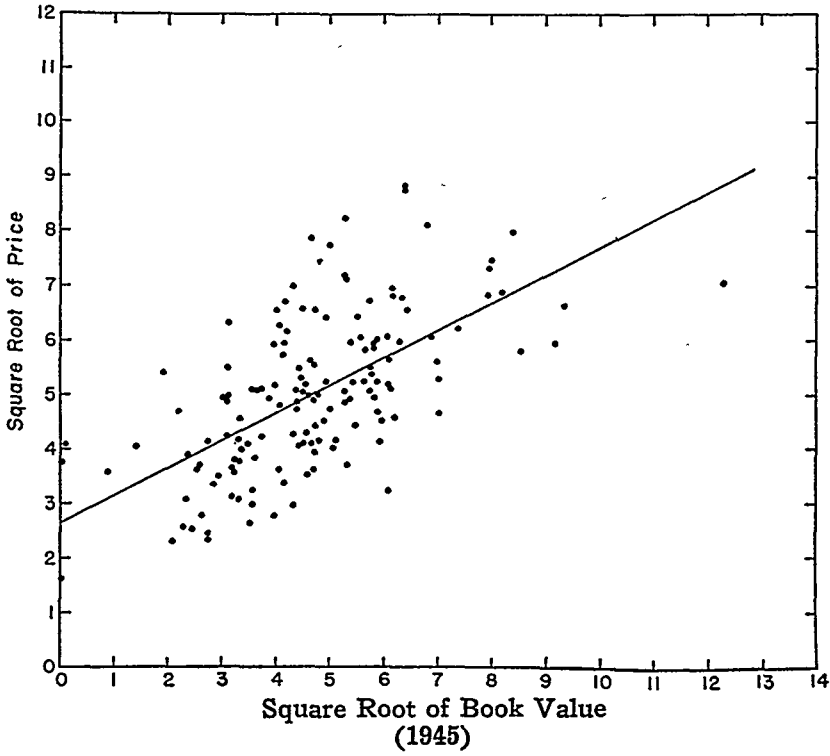
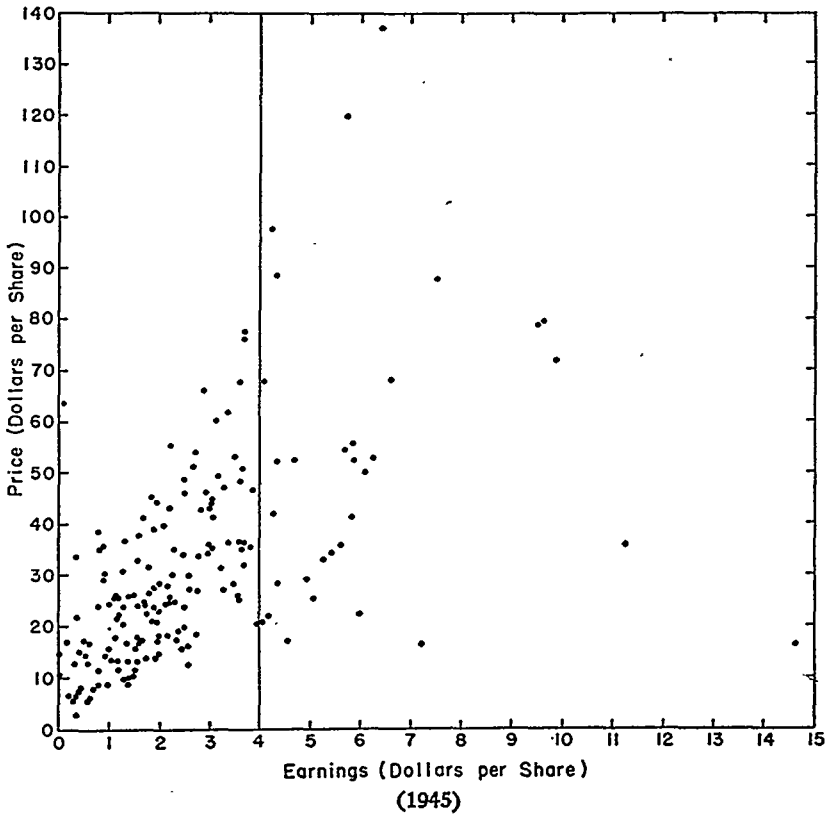


FIGURE 3



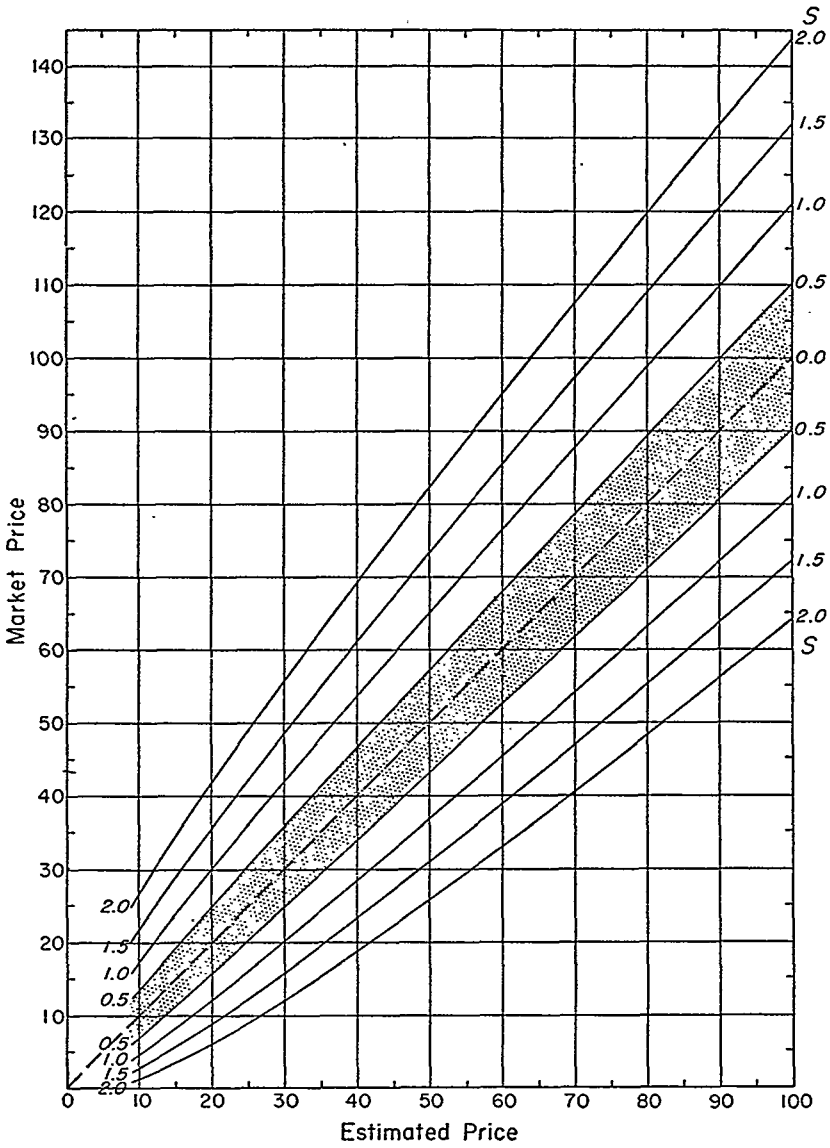


FIGURE 5

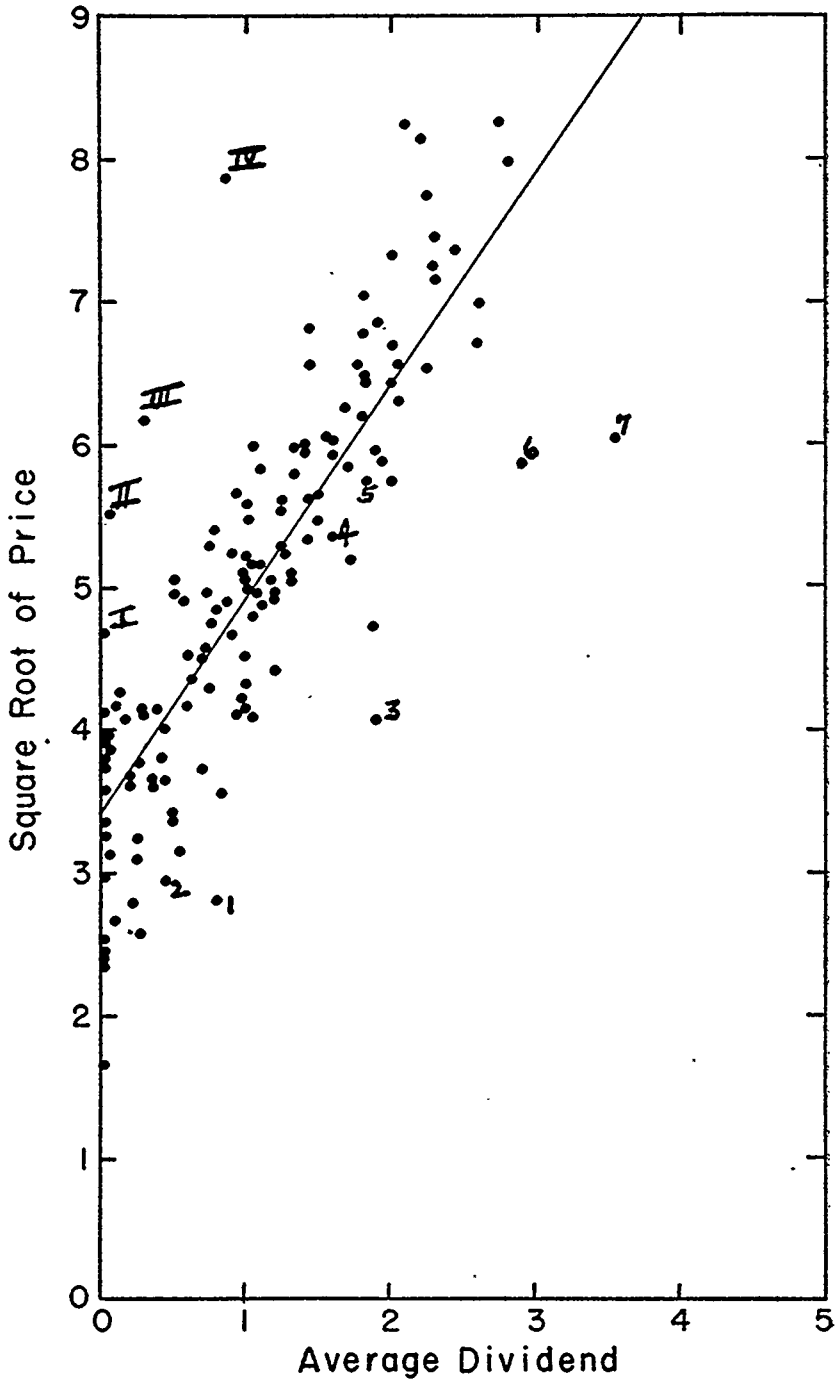


FIGURE 6