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Evaluation and visualisation of risk to water resources

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A traditional difficulty encountered in water resource planning is the inherent uncertainty in demand and supply capability. Over recent years this uncertainty has increased for the UK water industry, owing for example to forecasts of climate change and impending directives from Europe, and in the face of such uncertainty it is no longer justifiable to design water resource systems in a deterministic fashion. Design objectives should be risk-based, and it is likely that both investors and regulators will soon request formal risk evaluation prior to major investment or strategic decisions. Consequently, there is a need for probabilistic, or risk-based, approaches to water resource assessment. A methodology for evaluation and visualisation of risk to security of water resources is presented, based on hydrological frequency of occurrence and uncertainty-based analysis of headroom (that is, the difference between supply capability and demand). The methodology makes it possible to calculate and visualise the probability of system failure for particular hydrological conditions or the probability of system failure in an arbitrary year, and to make preventive planning or operational decisions. The proposed approach is illustrated with a case study.

NOTATION

A, B	defined events
a	day number representing start of R
ABS	sum of the primary abstractions
ABS′	sum of the secondary abstractions
c_i and d_i	regression coefficients
CAP	water supply capability
DEM	water demand
DEM'	demand data
k	shape parameter equal to $1/\alpha(u-w)$
m	number of years in sample of Q
N	length of resource critical period
P	annual drought frequency
$P(Q)$	cumulative probability distribution of Q
$p(Q)$	probability density function of Q
Q	annual drought severity
R	length of running average window for calculation of Q
t	time
u	location parameter
w	lower bound of type 3 extreme value distribution
x	fitted headroom curve minus the observed headroom
Z	reduced variate of $P(Q)$
α	scale parameter
θ	value of the $N(0,1)$ normal distribution for a given probability
μ_{DEM}	mean of the demand data
ξ	total risk of supply failure
σ	standard deviation of error in $P(Q)$
σ_S	sample standard deviation of Q
$\phi(Q)$	probability of supply failure given Q

1. INTRODUCTION

The UK water industry is currently in a dynamic and challenging period of water resource management and planning. Many of the water companies were faced with an extraordinary run of dry years from 1990 to 1996, and were forced to implement demand restrictions and call on emergency resources,[1] highlighting the inadequacies of water resources in numerous resource zones. Poor demand management, including leakage control and persuasion of the public to use less water, was held partly responsible.[2–4] The prolonged drought increased the concern that the climate in much of the UK is changing, resulting in generally drier summers.[5] Environmental concern is a more important issue than ever, and water resources are under increasing influence from Europe through directives such as the Habitats Directive[6] and the Water Framework Directive.[7]

It is clear that the assessment of the future adequacy of water resources is a complex, multi-criteria problem. Furthermore, each criterion is beset by uncertainty. Demand predictions, for example, are notoriously uncertain, with water company domestic use predictions for the year 2025 ranging from 125 l/day to 200 l/day,[8] and the UK Environment Agency's own 2025 demand scenarios ranging from a 70% increase to a 30% decrease.[9] The Kielder reservoir in north-east England epitomises the scale of the demand prediction problem. Calculation of supply capability is also inherently uncertain owing to the high variability of rainfall processes, and this uncertainty is traditionally accommodated using statistical analysis of hydrological records.[11] Planning for climate change requires speculative projection of hydrological data into the future, resulting in high uncertainty about resource reliability[12,13] and about the security of abstraction licences.

The potential impact of such uncertainties on the reliability of water resource systems and water company performance has led to the development of resource network simulation models that allow risk to be used as a planning and operational criterion. For example, Moore *et al.* describe software that allows integration of rainfall-runoff modelling with a detailed model of the Thames system of pumped storage reservoirs, allowing operation and planning to take account of system reliability under different rainfall scenarios.[14] Stahl and Elliot describe a decision support system that uses network flow programming to facilitate risk-based optimisation of planning and operation.[15,16] In both these cases, the authors emphasise the need for user-friendly interfaces so that the significance of results is accessible to resource managers.

In recently submitted water resource plans[8] the water companies in England and Wales have handled the problem of uncertainty using the concept of *target headroom*.[17,18] This concept evolved within the UK water industry during the 1990s in pursuit of a standard approach to the assessment of the supply–demand balance, and the identification of a sufficient margin of safety to allow for inherent supply–demand uncertainties. It is therefore a pragmatic approach to the assessment of whether or not water resources are adequate, and a basis for reviewing options to safeguard resources into the future. An important strength of the headroom concept is that it allows the idea of risk to be simply and graphically communicated, in a manner that is uniform across England and Wales.[8] However, in its present form, the target headroom approach cannot be considered as a formal risk evaluation because the risk level is not explicit in the results. Because risk cannot be considered as a numerical design variable, this approach does not permit objective risk management. For example, risk to security of water resources cannot be optimised against risk of environmental impact, or against cost.

In summary, the target headroom approach diminishes the potential role of risk evaluation in water resource management. Clearly, it is useful to develop the concept of headroom using formal risk evaluation methods whereby the risk of supply failure can be expressed as a probability, while maintaining the visualisation of risk made possible by the headroom concept. This paper describes an approach that aims to do so by building upon contemporary methods of water resource risk modelling in the context of current UK water industry needs: transparency of method, communicability of results to consumers and regulators, and robustness to the nature of the resource zone. An idealised case study is presented for illustration, although the approach should be viewed as a generic methodology rather than as a strict set of rules and formulations applicable to other resource zones.

2. DATA REQUIREMENTS

The risk evaluation method depends on the availability of resource system data to make it possible to derive a time series of annual hydrological drought frequency (P), supply capability (CAP), which is otherwise known as 'water available for use', and demand (DEM). The reliability of the results depends on the accuracy and pertinence of these three key variables. Therefore, as a preliminary step of the risk evaluation, the data must be reviewed. It is essential in any data-based risk evaluation that the data represent the pertinent conditions, which, in the present case, means that historical water resource data can be used only if they are applicable to future conditions. Consider the important parameters controlling the three key variables P, CAP and DEM:

$P = function$ (weather, catchment hydrology)
$CAP = function$ (weather, catchment hydrology, abstraction licence, infrastructure, operational efficiency)
$DEM = function$ (weather, demand management, demography, socio-economic change, industry, agriculture).

If climate is assumed to be stationary then, apart from major changes in land management, catchment hydrology is also likely to be stationary (or dependent only on managed abstractions and returns), and drought frequency can be calculated from historical hydrological data. However, it is unlikely that the other historically based control parameters will still be pertinent to current or future conditions. Therefore supply capability and demand must be calculated on the basis of various updated control parameters, in combination with historical hydrological data. Apart from the simplest of cases, this is unlikely to be straightforward, and some kind of supply network simulation will be required.[15] If climate is assumed to be changing, then drought frequency, as well as capability and demand, must be based on data generated from a stochastic hydrological model.[19]

3. INTRODUCTION TO THE CASE STUDY

This study is based on an urban resource zone that is supplied mainly by a large river and an associated large, on-line, raw water storage reservoir. The resource system characteristics are summarised in Table 1. A proactive demand management strategy is used whereby the public are persuaded to restrict their use of water to various degrees (for example through discouragement of the use of garden sprinklers) if reservoir storage falls below various threshold levels. Such measures lead to a restricted demand that is less than normal unrestricted demand, and which is a function of reservoir levels and therefore the river flow. The abstraction from the river is also a function of reservoir levels, owing to a licence agreement that is responsive to impending supply shortages. Both of these functions are represented by reservoir control curves. For planning purposes, supply is deemed to have failed if the reservoir falls below a prescribed emergency storage level.

Data for the case study consist of a 60-year daily time series of river flow. Daily reservoir storage levels, river abstractions and water supplied are simulated by a supply network model on the basis of the historical flow data and current unrestricted demand. The supply network model balances supply and demand on a daily basis by reducing or increasing reservoir storage, taking into account the reservoir control curves, demand restrictions and various operational rules (the details of

Reservoir gross capacity	110 000 Ml
Reservoir emergency capacity	25 000 Ml
Unrestricted demand including losses (1998)	1360 Ml/day
Supply network flow capacity	1650 Ml/day
Secondary sources (groundwater)	77 Ml/day
Average flow in primary source river	4200 Ml/day

Table 1. Summary of case study water resource system characteristics

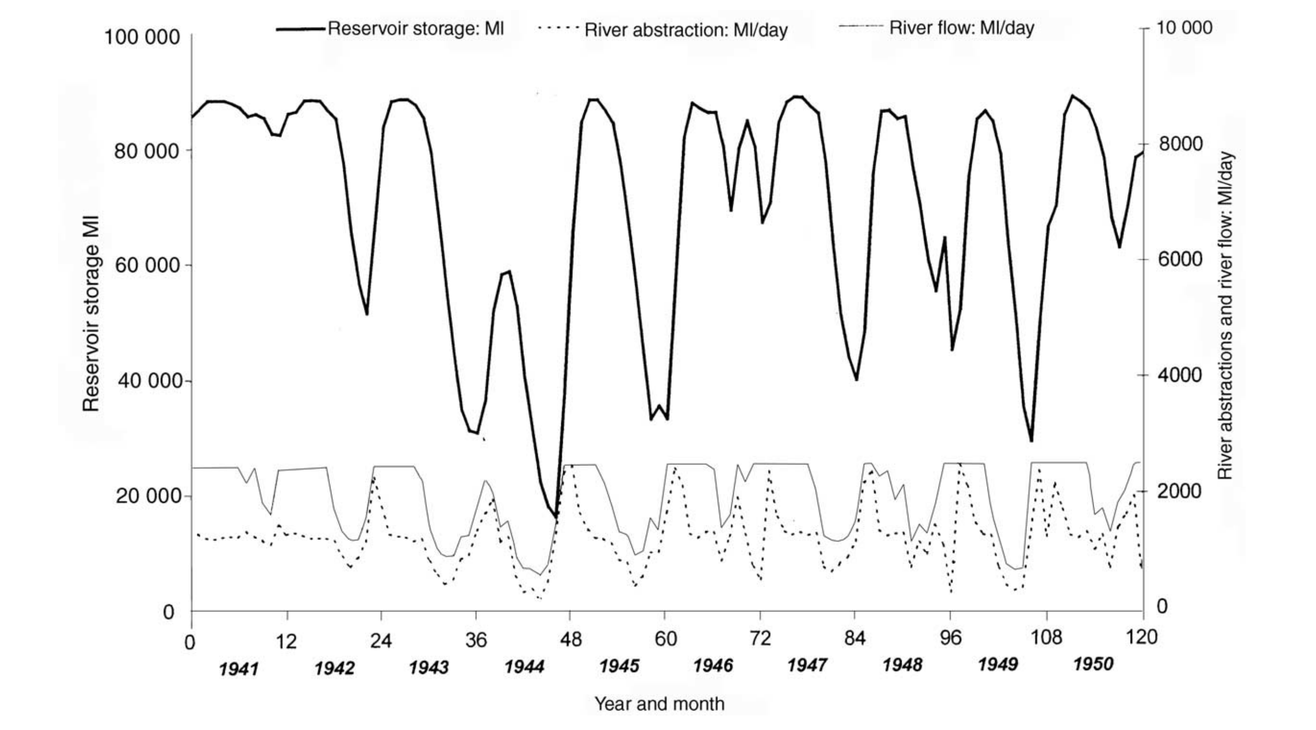

Fig. 1. Data for the case study from 1941 to 1950

which are not important here). The model outputs (daily reservoir storage levels, river abstractions and water supplied) are considered to be a reasonable re-creation of the resource scenario if current infrastructure, operational philosophy and unrestricted demand are imposed on historical hydrology. For the purpose of the demonstration, it is initially assumed that the historical time series of river flows and current unrestricted demand are relevant to future scenarios. The incorporation of non-stationary data of climate and demand is examined later.

A portion of the daily time series of historic river flow, together with the corresponding modelled reservoir storage and river abstractions, is shown in Fig. 1 to illustrate a number of important features of this resource network. For clarity the river flows are curtailed at a maximum abstraction rate of 2500 Ml/day. Data from 1943–1944 show that the reservoirs do not always completely refill over the winter, and data from 1944 show that the abstraction licence allows a high proportion of the flow to be abstracted if low reservoir storage occurs. Demand restrictions have a considerable influence on reservoir draw-down rates: the 1944 abstractions are visibly less than those in 1942, although the net reservoir draw-down over the drought period is similar.

4. METHOD OF RISK EVALUATION

The risk evaluation uses traditional probability theory, and is based on the following six steps:

(1) identification of a suitable measure, Q, of annual drought severity upon which to base a drought frequency analysis (this measure should be independent of the resource system: for example, independent of demand, network operation and infrastructure)
(2) drought frequency analysis to calculate the probability of occurrence of a drought event of any severity $P(Q)$
(3) calculation of supply capability and demand for each year of the drought frequency analysis, where capability and demand are defined, respectively, as the water available for supply above a prescribed emergency resource quota, and the water actually supplied, including losses, averaged over a critical period of each year
(4) identification of a mathematical relationship between headroom and $P(Q)$ using regression techniques
(5) analysis of the uncertainty in this relationship
(6) calculation of the probability of demand exceeding capability (that is, of emergency resources being drawn upon) as a continuous function of $P(Q)$, and calculation of the total risk of supply failure in any future year.

The novel aspect of this method is the identification of the relationship between the drought index and actual shortages of water, and the subsequent analysis of the uncertainty in this relationship. This allows the risk to security of supply to be associated with known drought events in an insightful manner. Also, the method gives an indication of the unmanageable component of risk (that is, risk associated with rainfall patterns), in parallel and/or in combination with the manageable component (that is, risk associated with leakage and other losses, demand, network operation and infrastructure).

5. IDENTIFICATION OF A SUITABLE MEASURE OF ANNUAL DROUGHT SEVERITY

The method begins with identification of a suitable drought severity index, Q. The chosen index should provide an annual measure of stress on the water resource system due to hydrological factors, and the variation in its values should be independent of the adequacy of the infrastructure or operational management of the water resources. Therefore Q should represent the dominant hydrological variable, or variables, affecting headroom during the drought period. In general, the most appropriate definition of Q will depend on the resource zone storage facilities. For example, in a zone with large reservoirs the previous winter's streamflow may be an important hydrological factor affecting security to supply during the summer. On the other hand, for a zone with a small reservoir capacity, security of supply will be dictated more by streamflow in recent days or weeks. Examples of Q are:

(*a*) lowest running average 7-day flow in the source river or rivers (suitable for cases with a nominal 7–14-day raw water storage reservoir)
(*b*) lowest running average 100-day low flow in the source river or rivers (suitable for larger raw water storage reservoirs, as in this case study)
(*c*) as above but using rainfall instead of river flow (most suitable when rainfall records (or forecasts) are considered to be more reliable)
(*d*) any of the above but with an addition for groundwater.

In the present case study, 60 years of daily flow data are available for the dominant source river. This river flow will therefore be used as a basis for the drought severity index. The capacity of the reservoirs is approximately 80 days of unrestricted demand. Therefore, as a starting point, it is hypothesised that the best measure of Q is the lowest 100-day running average river flow each year, defined as

$$Q = \min_{a=1,365-R} \left[\frac{1}{R} \sum_{i=a,a+R-1} q_i \right] \tag{1}$$

where Q is the drought index calculated for each year of the record, R is the length of the running average period (here $R = 100$), and a is the day number of the start of the R-day period. The sensitivity of the results to R will be tested later.

6. DROUGHT FREQUENCY ANALYSIS

Type 1, 2 and 3 extreme value distributions are commonly used to improve the statistical analysis of extreme events.[20,21] The general extreme value distribution is a method of encompassing types 1, 2 and 3 in one formula, and therefore gives greater flexibility to a frequency analysis where the most appropriate distribution is unknown. It is a three-parameter distribution that is defined for analysis of minima such as drought events by

$$p(Q) = \alpha \left(\frac{Q-w}{u-w} \right)^{\frac{1}{k}-1} \exp\left[-\left(\frac{Q-w}{u-w} \right)^{\frac{1}{k}} \right] \tag{2}$$

$$P(Q) = 1 - \exp\left[-\left(\frac{Q-w}{u-w} \right)^{\frac{1}{k}} \right] \tag{3}$$

where α is a scale parameter, u is a location parameter, w is the lower bound of the distribution (for example the fitted low extreme of river flow), k is a shape parameter equal to $1/\alpha(u-w)$, $P(Q)$ is the cumulative distribution function (that is, the probability of non-exceedence of Q in an arbitrary year), and $p(Q)$ is the probability density function equal to the derivative $\mathrm{d}P(Q)/\mathrm{d}Q$.

Extreme value distributions converge slowly with increasing number of samples, and therefore small sample sizes result in wide confidence intervals. Consequently it is expected that the drought frequency analysis will introduce significant uncertainty into the results of the risk evaluation. From ref. 22, the uncertainty in $P(Q)$ is given by the interval $[Q-\theta\sigma, Q+\theta\sigma]$, where

$$\sigma = \frac{\sigma_S}{\sqrt{m}} \left[1 + 1{\cdot}14\left(-\sqrt{6} \times \frac{0{\cdot}5772 - Z}{\pi} \right) + 1{\cdot}1\left(\sqrt{6} \times \frac{0{\cdot}5772 - Z}{\pi} \right)^2 \right]^{0.5} \tag{4}$$

$$Z = \frac{1}{k} - \frac{1}{k}\exp\left(\ln\left\{ \ln\left[\frac{1}{P(Q)} \right] - \ln\left[\frac{1}{P(Q)} - 1 \right] \right\} \right) \tag{5}$$

and σ is the standard error of $P(Q)$, σ_S is the sample standard deviation of Q, m is the number of years in the sample, and θ is the value of the $N(0,1)$ normal distribution for a given probability.

An assumption concerning the accuracy of the extreme value frequency analysis is independence of the Qs. This deserves particular attention if river flows are used as a measure of Q because it is well known that extreme drought events can affect subsequent flows through groundwater depletion. Methods of accounting for autocorrelation in extreme value analyses are discussed by Thas *et al.* and Tawn.[21,23] Another important assumption is that there are no factors affecting flows in a particular range, such as larger gauging errors at lower flows.

A GEV type 3 distribution was fitted to the Q data using the method of sextiles.[20] The quality of the fit can be assessed by visual comparison with the Gringorten plotting position,[24] illustrated in Fig. 2. The fitted probability density function and cumulative density function are given by $p(Q)$ and $P(Q)$ respectively:

$$p(Q) = 0{\cdot}14\left(\frac{Q-6{\cdot}9}{17{\cdot}6} \right)^{1{\cdot}5} \exp\left[-\left(\frac{Q-6{\cdot}9}{17{\cdot}6} \right)^{2{\cdot}5} \right] \tag{6}$$

$$P(Q) = 1 - \exp\left[-\left(\frac{Q-6{\cdot}9}{17{\cdot}6} \right)^{2{\cdot}5} \right] \tag{7}$$

These distributions are shown in Fig. 3, with the 90% confidence limits for $P(Q)$.

In general, type 3 is found to be the most appropriate distribution for drought frequency analysis[25,26] because it has a lower bound when applied to minima—in the case study the river's extreme low flow. That is, the type 3 distribution suggests that the 100-day running average flow will never be lower than 6·9 m^3/s. However, the probabilities of the most extreme events should be interpreted cautiously because their confidence limits are very large.

7. CALCULATION OF ANNUAL DEMAND, SUPPLY CAPABILITY AND HEADROOM

Supply capability (*CAP*) is defined as the average water available for supply over an identified control period (of N days), including all abstractions and storage above a predefined emergency level. Demand (*DEM*) is defined as the average water supplied over the same period, so that it includes

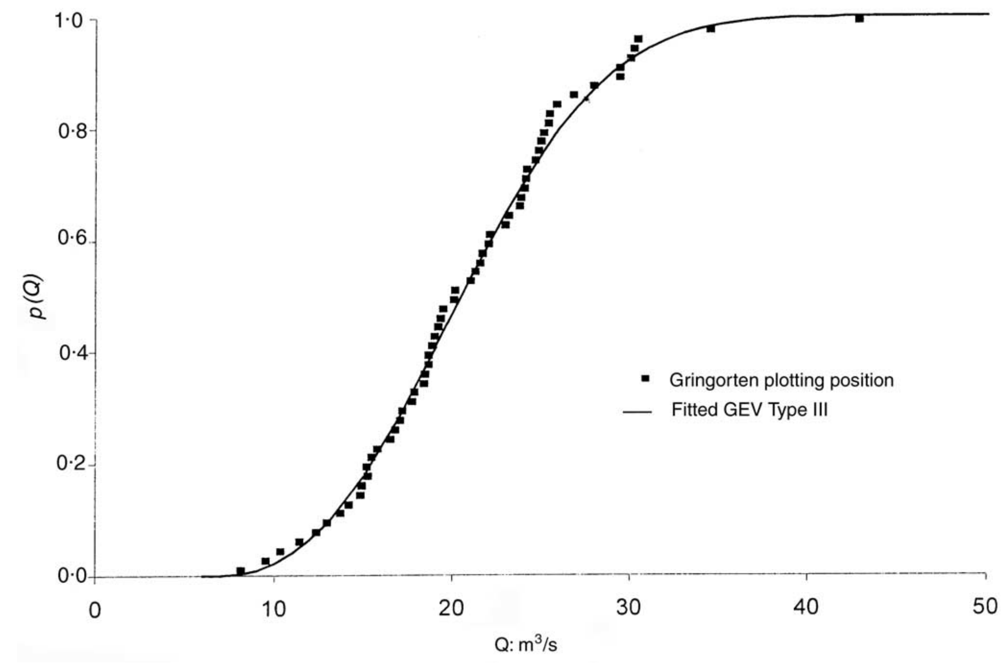

Fig. 2. Fitted GEV Type III distribution compared with Gringorten plotting position

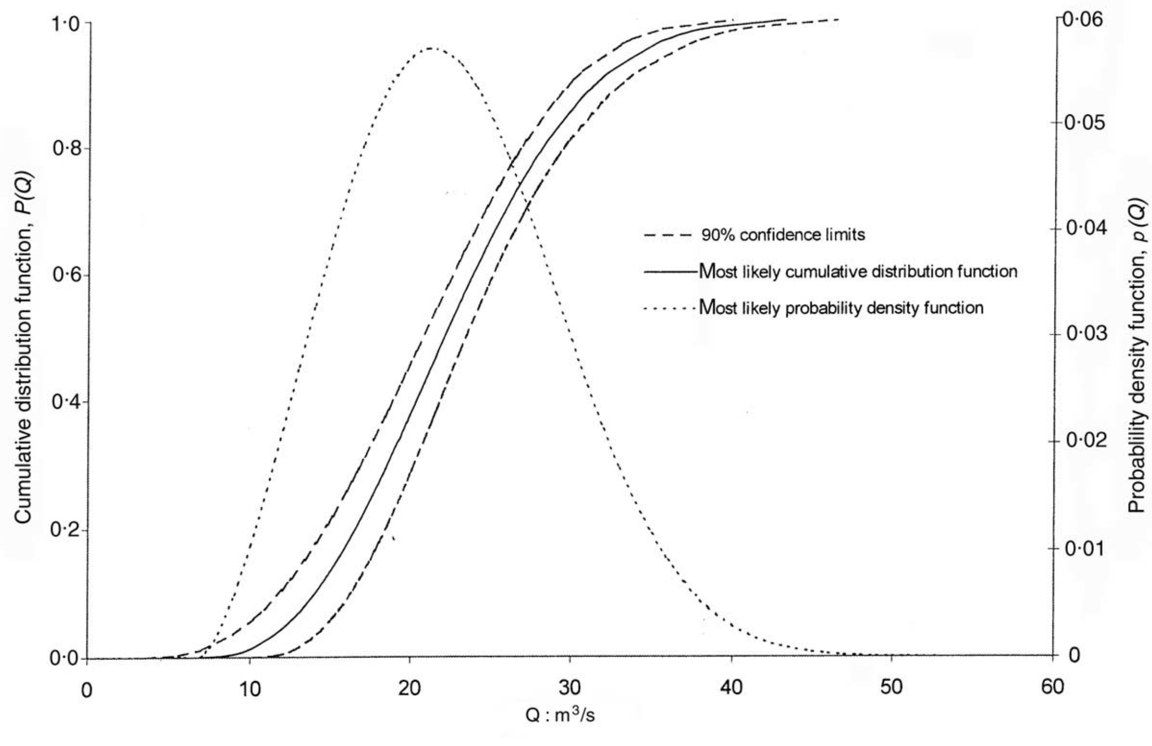

Fig. 3. Cumulative distribution function and probability density function for the case study with $Q = 100$-day running average river flow

reductions in demand caused by demand management, and also includes losses. Headroom, which is the difference between capability and restricted demand ($CAP - DEM$), is used as the basis for the risk evaluation. For the case study, the control period is identified as the period during which the reservoir storage is in decline within each year, neglecting partial recoveries such as those in 1946 and 1948 (Fig. 1). It can be seen from Fig. 1 that minimum reservoir storage in this system sometimes occurs as late as January. This is due to a combination of dry winters and the local operating policy, which deems that demand restrictions are unnecessary during the winter period. For this reason the water resource year—that is, the constraint on the control period, is defined as 1 April to 31 March the next year.

The control period represents the critical period for water resources within a chosen time constraint (one year for the case study). The time constraint needs to be defined according to the resource system in question, and according to the arguments that need to be put forward by the results. The application of critical periods in this way is discussed later.

For the case study, daily time series of 60 years of reservoir storage and river abstractions have been simulated using the available database of river flows and the water supply network model. From these time series, flow-balance calculations are used to find annual capability, demand and headroom, as defined below. Supply capability is the sum of the primary source abstractions (ABS) and the secondary source abstractions (ABS') plus the net storage at the start of the period (gross storage S_1 minus emergency storage E), all divided by the period length N:

$$CAP = \frac{1}{N}\left[(S_1 - E) + \sum (ABS + ABS')\right] \tag{8}$$

Demand is the water taken from the river and secondary source abstractions plus the water depleted from the reservoir during the period (S_1 minus gross storage at end of the period, S_2), all divided by the period length:

$$DEM = \frac{1}{N}\left[(S_1 - S_2) + \sum (ABS + ABS')\right] \tag{9}$$

The difference between equations (8) and (9) is the daily average surplus of water over the control period—that is, the headroom:

$$CAP - DEM = \frac{1}{N}(S_2 - E) \tag{10}$$

Equation (10) is solved for each year, and this set of solutions forms the basis of the risk evaluation.

To show the trend and uncertainty with which supply capability and demand (following modelled demand restrictions) decrease with increasingly severe hydrological drought, they can be plotted against $P(Q)$ (Fig. 4). This allows any drought severity to be associated with an expected resource scenario. The corresponding trend and uncertainty in headroom define the risk associated with any drought severity (Fig. 5).

8. REGRESSION AND UNCERTAINTY ANALYSES

The deterministic (that is, best-fit) relationships of capability and demand with drought frequency are described by nth order polynomials:

$$CAP = c_1 P(Q)^n + c_2 P(Q)^{n-1} + c_3 P(Q)^{n-2} + \ldots + c_n P(Q) + c_{n+1} \tag{11}$$

$$DEM = d_1 P(Q)^n + d_2 P(Q)^{n-1} + d_3 P(Q)^{n-2} + \ldots + d_n P(Q) + d_{n+1} \tag{12}$$

The coefficients c_i and d_i ($i = 1$ to $n + 1$) are computed using standard regression techniques. The resulting curves are shown together with the regression coefficients in Fig. 4. Headroom, $CAP - DEM$, is shown in Fig. 5.

This method of risk evaluation is based on the fact that there are important influences on water resources that have no correlation with $P(Q)$. Such influences cause the noise that is evident around the fitted curves in Figs 4 and 5 and contribute to risk supply failure at any given $P(Q)$. For example, there are likely to be influences that cause the reservoir not to be full at the start of the N-day critical period, but which are not dependent on that year's drought severity. Other sources of risk that may be independent of Q are outage events (intermittent losses of water due to operational needs and inefficiencies) and

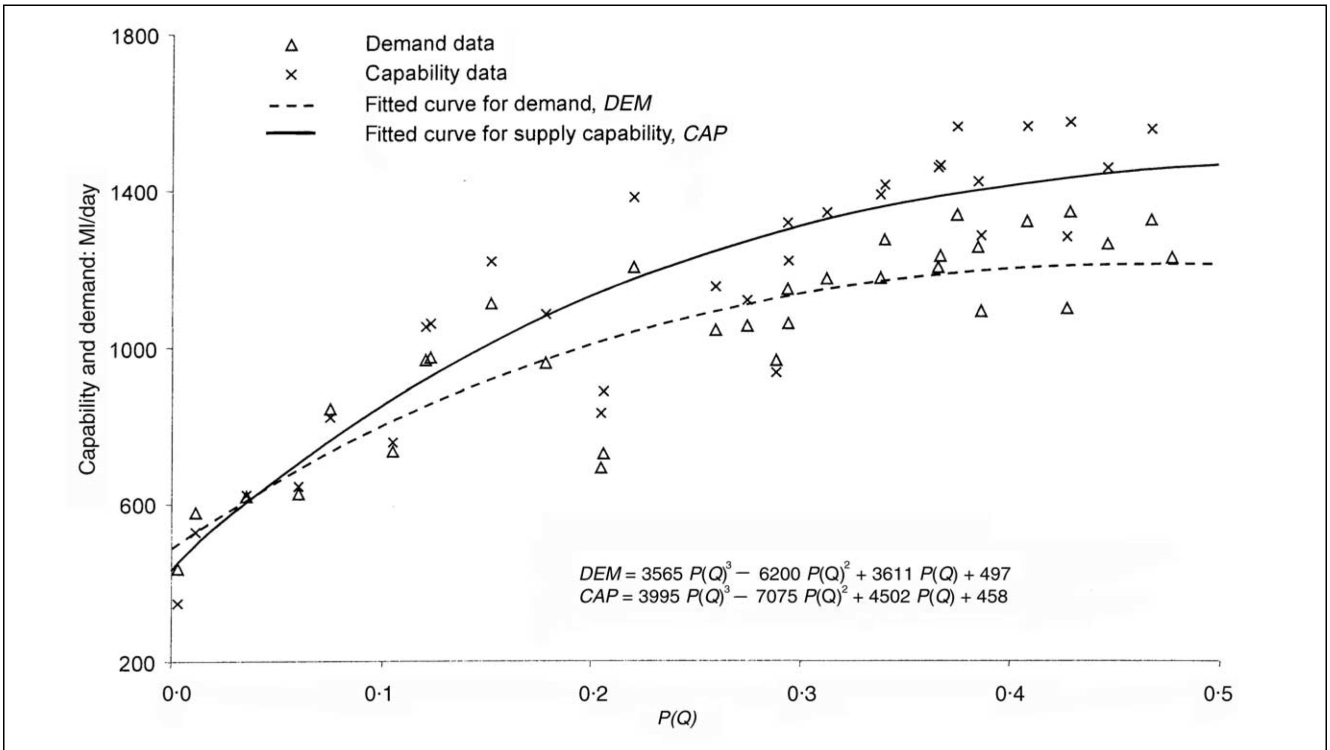

Fig. 4. Relationship of supply capability, CAP, and demand, DEM, to drought severity, $P(Q)$

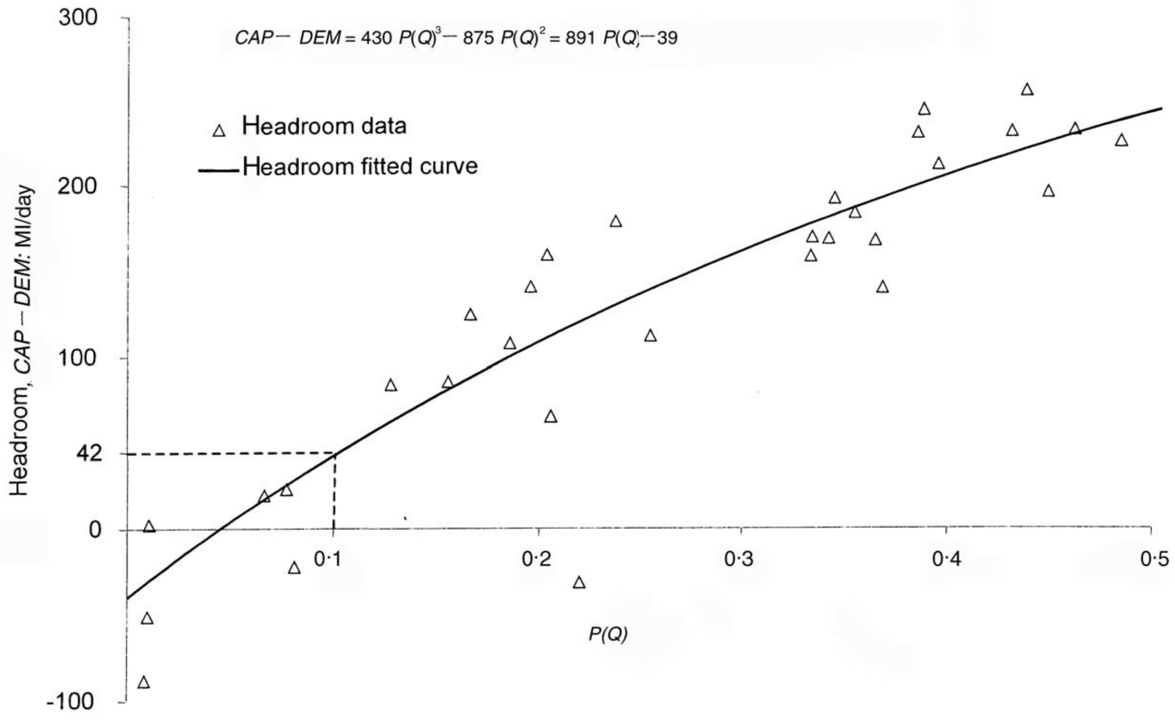

Fig. 5. Relationship of headroom, $CAP - DEM$, to drought severity, $P(Q)$

variability in unrestricted demand, although neither of these was incorporated into the case study. Any influences that *are* correlated with $P(Q)$ are implicit in the regression, and the risk that they pose to security of supply is therefore incorporated into $P(Q)$.

The next step in the risk evaluation is to calculate the probability of supply failure for each drought severity, $P(Q)$. This is achieved by identifying a conditional probability distribution for headroom, called the *sampling distribution*, the location of which for any drought severity is defined by the fitted curve in Fig. 5. Identification of a suitable sampling distribution may be difficult. The central limit theorem may be tentatively applied in assuming a normal distribution, and, if sufficient data are available, it is useful to test feasible distributions (such as triangular, normal and log-normal) using the chi-squared or other statistical tests.

For the case study it is assumed that the sampling distribution shape is constant over the range of $P(Q)$ on the basis of a test for homoscedasticity.[26] Normal, log-normal and triangular distributions are fitted to the deviations of the headroom around the fitted curve by optimising the chi-squared test statistic.[26] The results of the optimal distributions of each type are given in Table 2. It can be seen that a three-parameter log-normal distribution is reasonably descriptive of the data. This distribution, shown in

Range of deviations, x, around fitted curve*	Observed number in range	Expected number in range		
		Normal	Log-normal	Triangular†
Less than −100	0	1·4	0	1·1
−100 to −80	0	2·0	0	3·3
−80 to −60	4	3·6	1·9	5·6
−60 to −40	6	5·8	6·3	7·8
−40 to −20	15	7·9	11·4	9·1
−20 to 0	11	9·3	13·1	8·7
0 to 20	6	9·3	11·0	7·3
20 to 40	9	7·9	7·5	6·0
40 to 60	4	5·8	4·4	4·7
60 to 80	0	3·6	2·3	3·3
80 to 100	3	2·0	1·1	2·0
Over 100	2	1·4	0·9	0·7
Chi–squared statistic‡		0·93	0·84	0·96

*Fitted curve minus data points, so more headroom than expected appears negative.
†Using a mode of −30 Ml/day, maximum of 120 Ml/day, and minimum of −120 Ml/day.
‡This is the probability that a random observation from a Chi-square distribution with 9 degrees of freedom would lie below the Chi-square test statistic. Therefore the hypothesis that 'the deviations around the fitted curve do not come from this distribution' is rejected if this probability is lower than a preferred confidence level, say 0·9.

Table 2. Chi-squared test to identify optimum headroom distribution

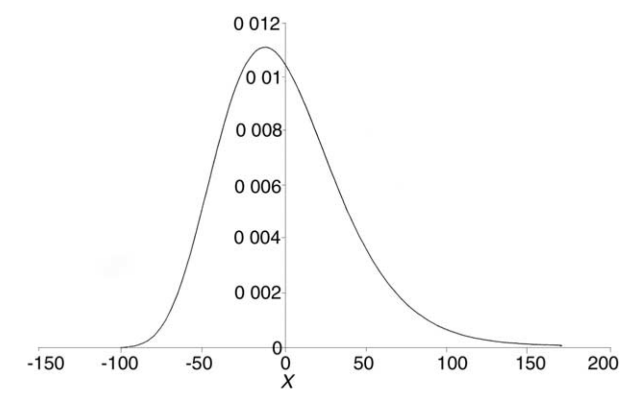

Fig. 6. Sampling distribution of x

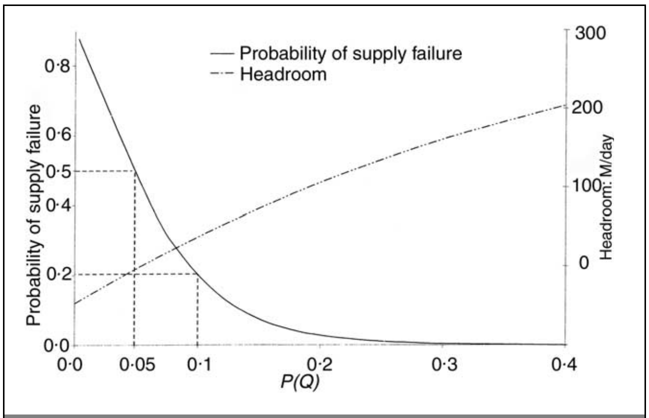

Fig. 8. Probability of supply failure and mean headroom

Fig. 6, is described by the following probability density function:

$$p(163+x) = \frac{1}{0.23(163+x)\sqrt{2\pi}}\exp\left\{\frac{-[\ln(163+x)-5{\cdot}07]^2}{2\times 0{\cdot}23^2}\right\} \quad (13)$$

where x is the fitted headroom curve minus the observed headroom. The mean x is 0, the median is $-4{\cdot}0$ Ml/day, and the mode is $-14{\cdot}0$ Ml/day.

9. RISK EVALUATION

The final stages of the risk evaluation method can be described fully using the case study. The probability that supply will fail at any drought severity can be calculated as the cumulative probability of negative headroom. As an example, suppose an estimate is required of the probability of supply failure assuming that a 1 in 10 year drought event occurs. From Fig. 5, at $P(Q) = 0{\cdot}1$, the curve is 42 Ml/day above the condition of zero headroom. Therefore negative headroom occurs for all $x > 42$, and the probability of failure at this drought severity is the cumulative sampling distribution for $x > 42$, equal to 0·21, and shown as the hatched area in Fig. 7. Thereby it is possible to define the probability of supply failure, $\phi(Q)$, as a function of $P(Q)$. This is plotted for the case study in Fig. 8. This signifies that there is almost no probability of failure at droughts with $P(Q) = 0{\cdot}3$, that there is a 21% chance of failure for droughts wih $P(Q) = 0{\cdot}1$, and a 50% chance of failure for droughts with $P(Q) = 0{\cdot}05$.

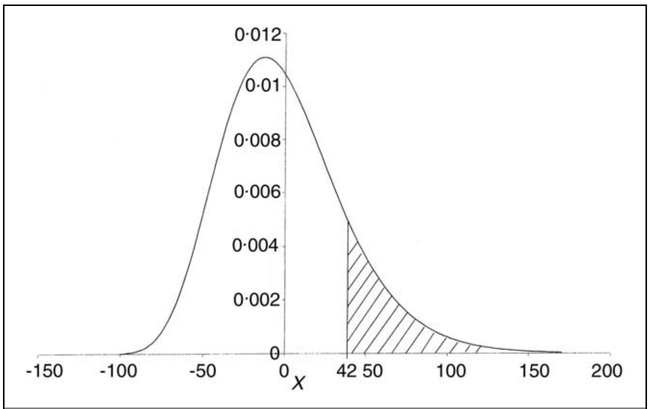

Fig. 7. Using the sampling distribution to calculate probability of failure for $P(Q) = 0{\cdot}1$

Multiplying $\phi(Q)$ by the drought probability density function $p(Q)$, and integrating over the whole range of Q, gives the *total risk* of failure of supply, ξ, in any one year. This calculation is a simple application of the theorem of total probability:[26]

$$P(A) = \sum [P(A|B) \times P(B)] \quad (14)$$

where $P(A)$ is the probability of occurrence of event A, $P(A|B)$ is the probability of occurrence of event A assuming that event B has occurred, and $P(B)$ is the probability of occurrence of event B.

In this application the total probability of supply failure, ξ, is equal to the sum of the probabilities of supply failure given that the drought has occurred multiplied by the probability that the drought will occur. For the case study $\xi = 0{\cdot}06$, equivalent to a return period of 17 years. The standard deviation for ξ is 0·072—that is, a standard error of 120%—based on the uncertainty in $p(Q)$ from equations (4) and (5).

10. DISCUSSION AND SENSITIVITY ANALYSIS

This method of risk evaluation quantifies the risk of failure of supply. Failure of supply can be defined as any event, or breach of level of service, so long as occurrences of that event or breach can be identified from the data. So far, in the case study, failure is defined as the event that the storage level in the reservoir falls below a predefined emergency storage. Consider a more consumer-oriented definition of failure: that the specified level of service to the consumer is not achieved, where the specified level of service is as follows

A Escalated water efficiency campaigns should not be used more than once every 5 years.

B Hosepipe bans should not be used more than once every 10 years.

Simulated occurrences of these two demand management actions are identified from the data by reference to the reservoir control curves, and the risk of the action in any year is calculated as previously described. For example, if action A is initiated when the reservoir level falls below 45 000 Ml then the risk of this occurring in any one year is 0·23. Then the risk of action A being used more than once in every 5 years, P(A), is calculated from binomial theory:[26]

$$P(\mathrm{A}) = 1 - \left[(1 - 0{\cdot}23)^5 + 5 \times 0{\cdot}23 \times (1 - 0{\cdot}23)^4\right] = 0{\cdot}32 \quad (15)$$

Similarly, if action B is used when the reservoir level falls below 30 000 Ml, then the risk of this occurring in any one year is 0·12, and the risk of it being used more than once every 10 years, P(B), is

$$P(\mathrm{B}) = 1 - \left[(1 - 0{\cdot}12)^{10} + 10 \times 0{\cdot}12 \times (1 - 0{\cdot}12)^9\right] = 0{\cdot}34 \quad (16)$$

It is seen that this proactive demand management strategy—that is, the reservoir control curve—is reasonably well balanced because there is a similar probability of failure due to each level of service criterion. Where the strategy is not well balanced, there is a unnecessarily high risk of failure to meet target level of service. Thus this method of risk evaluation can be used to identify this imbalance and optimise proactive demand management strategy.

The risk evaluation method is potentially valuable in summarising the effect on water resources of changing infrastructure, abstraction licences or operational strategy. For example, the benefit of added reservoir capacity over the whole range of drought severity can be illustrated using total risk. This is shown for the case study in Fig. 9, which has been derived by repeating the risk evaluation at a number of trial reservoir capacities, and plotting ξ against capacity.

Since the running average period for the drought frequency analysis (R in equation (1)) was chosen arbitrarily, a sensitivity analysis is carried out to investigate the effect of varying R from 25 days to 200 days. It is hypothesised that an optimised period will produce a more certain relationship between headroom and drought severity, and therefore reduce the modelled risk, ξ. An alternative hypothesis is that ξ is reasonably robust to R because the same uncertainties are merely being manipulated differently. Fig. 10, which shows how the uncertainty in headroom and ξ varies with R, reveals that the results are robust to the period length within the range 25–175 days. However, the result for 200 days implies that it is important to establish a measure of drought severity that is reasonably consistent with the system storage capacity. This is because, otherwise, conditions leading to low headroom are predicted to occur over-frequently.

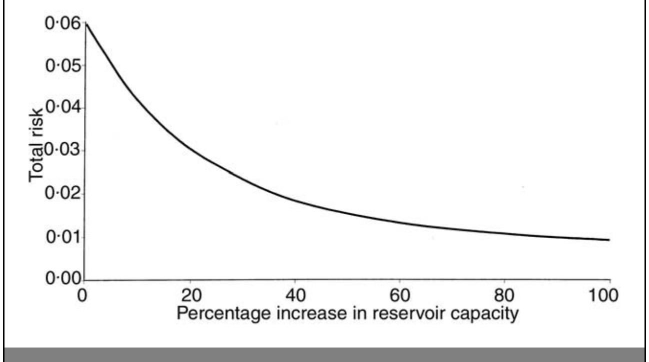

Fig. 9. Reduction in risk in response to increased reservoir capacity

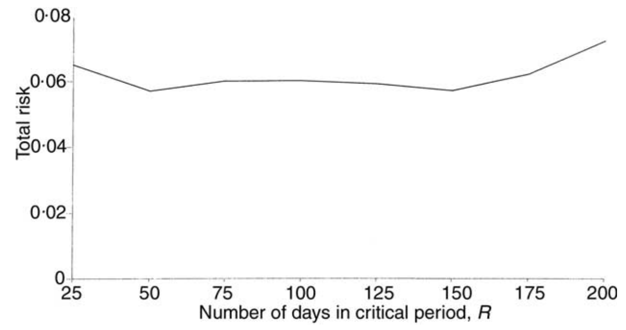

Fig. 10. Sensitivity of headroom uncertainty and total risk to period used for drought frequency analysis

It is worth comparing the aims and methodology of the proposed approach with those of traditional approaches to critical period analysis of reservoirs. Critical period analysis aims to identify the period over which decline of reservoir storage will lead to failure and the probability with which this will happen, in order to optimise resource system operation and identify the need for new resources.[27] This is usually achieved through analysis of one or more simulated time series of reservoir storages, and, if sufficient data are available, does not require prior constraints upon the critical period. The approach proposed in this paper is comparable, in that it identifies a critical period (N) and evaluates probability of failure (ξ) using simulated reservoir storages. However, at the heart of the proposed approach is the desire to maintain the insight provided by the headroom concept, and the ability to visually associate headroom and degree of risk with historical and forecast hydrological events, as well as providing an overall measure of risk. For this reason, prior constraints of the timescale of the hydrological event (R, over which the hydrological event is measured) and the reservoir's critical period (N, over which headroom is measured) are introduced. The chosen timescale depends upon expert knowledge of the system and also the arguments that need to be presented visually (Figs 4, 5, 8 and 9). While one year has been chosen for the case study, this is case-dependent. The disaggregation of the data time series into convenient units causes some loss of statistical rigour as complex serial dependences are treated simplistically: hence the view that the proposed approach is not an alternative to traditional design methods, but a complement.

It is evident that climate change and the associated changes in rainfall patterns are primary sources of risk to future security of water resources.[13,28] Therefore some attention should be given to the incorporation of impacts of climate change in the proposed risk evaluation methodology. The introduction of non-stationary climate would not alter the fundamental approach, as hydrological forecasts could replace the historical time series.[14] However, there would be three main complications. First, the large number of valid climate scenarios to be investigated would add significantly to the computational burden of the procedure, especially if a complex network model was being employed. Second, the non-stationarity would mean that the risk, ξ, would become a function of time, and this extra dimension would also add significantly to the computational burden. Third, climate change scenarios would need to be translated to water resources using rainfall–groundwater–

streamflow models, which itself raises modelling difficulties.[15,19] A related source of risk, which might be integrated into climate scenarios, is the insecurity of abstraction licences, especially in light of the requirements of the Water Framework Directive[7] and the new abstraction management strategy.[29]

The uncertainty in future demand is another priority for integration into the risk evaluation. Again, the large number of valid demand scenarios would impose additional computational burden in addition to the burden of the scenario development itself. A small number of representative scenarios could be proposed, as in the UK Environment Agency's strategy for water resource planning.[9] The four scenarios for 2010 and 2025 put forward in that strategy could easily be integrated into a single measure of risk using a Bayesian approach (equation (14)). However, such a nominal representation of possible future scenarios compromises statistical rigour, and there is a need for investigations into the implications of this. Although computational expense will always be an issue, the growing applicability of Monte Carlo methods and parallel processing in water resource management[30] provides a valuable opportunity for more rigorous scenario representation. The compatibility of this with water company and regulator requirements needs further investigation.

11. CONCLUSIONS

An approach to water resource risk evaluation and visualisation has been developed using integration of extreme value statistics with analysis of uncertainty in headroom, and this has been demonstrated using a hypothetical case study. The approach has built upon contemporary methods of water resource risk evaluation in the context of recent developments in the UK water industry. In particular, the method strives to illustrate clearly the concepts of headroom and risk, and their relationship with drought severity, and yet maintains a transparent, data-based foundation. This foundation allows formal evaluation of risk to security of level of service, and risk-based optimisation of demand management, system operation and strategic planning of resources. A limitation of the approach lies in the need to introduce assumptions (e.g. constraints on critical period, N, and distributional properties) to which the results are potentially sensitive. However, the assumptions involved in less objective evaluations of headroom, such as the existing scoring system,[17] are difficult to evaluate and defend, whereas the assumptions used in this proposal can be subject to rigorous sensitivity analysis. Another limitation (or arguably a strength) of the proposal is the absence of a distinct set of rules; rather, it is a methodology around which case-specific rules (network models, drought indices, etc.) must be formulated. Although the regulatory benefits of nationally uniform rules for the calculation of headroom are clear, it is very difficult (from an engineering point of view) to associate such a policy with useful, objective and transparent risk evaluation. Some generic challenges facing water resource planners have been discussed: the integration of non-stationarity of demand and climate, and the uncertain implications of the new era of integrated catchment management policy.[7,29]

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