Reliability analysis of moment redistribution in reinforced concrete beams

Hassan Baji
PhD candidate, School of Civil Engineering, University of Queensland, St Lucia, Brisbane, Queensland, Australia

Hamid Reza Ronagh
Senior lecturer, School of Civil Engineering, University of Queensland, St Lucia, Brisbane, Queensland, Australia

Design codes allow a limited amount of moment redistribution in continuous reinforced concrete beams and often make use of lower bound values in the procedure for estimating the moment redistribution factors. Here, based on the concept of demand and capacity rotation, and by means of Monte Carlo simulation, a probabilistic model is derived for the evaluation of moment redistribution factors. Results show that in all considered cases, the evaluated mean and nominal values of moment redistribution factor are greater than the values provided by the ACI code. On the other hand, the 5th percentile value of moment redistribution factor could be lower than those specified by the code. Although the reduction of strength limit state reliability index attributable to uncertainty in moment redistribution factors is not large, it is comparable to the reduction in reliability index resulting from increasing the ratio of live to dead load.

Notation

A_s	tensile rebar area
b	width of rectangular section
c	neutral axis depth
DL	dead load
d	effective depth of the rectangular section
d_b	rebar diameter
E_c	secant modulus of concrete
E_s	modulus of steel
EI	flexural stiffness
f'_c	characteristic concrete compressive strength
f'_{cm}	mean cylinder compressive concrete strength
f_y	reinforcement yield strength
g	strength limit state function of moment resistance
h	depth of rectangular section
k	neutral axis parameter
k_1, k_2, k_3	concrete stress block parameters
LL	live load
l	beam span length
l_p	equivalent plastic hinge length
M_D	dead load bending moment
M_e	elastic bending moment
M_L	live load bending moment
M_{Rn}	nominal bending capacity
M_u	ultimate bending moment
n	ratio of modulus of elasticity of steel to concrete
z	distance from point of contraflexure
α_1, β_1	code-specified concrete stress block parameters
β	moment redistribution factor
γ	load factor
ε_c	strain at peak stress of concrete
ε_{cu}	concrete ultimate strain
ε_t	extreme tensile rebar strain
ε_y	rebar steel yield strain
θ_{demand}	demand rotation of reinforced concrete beam
$\theta_{capacity}$	rotational capacity of reinforced concrete beam
μ_ϕ	curvature ductility
ρ	tensile rebar percentage
ρ_b	rebar percentage at balance state
ϕ_u	ultimate curvature
ϕ_y	yield curvature
ω_u	ultimate uniformly distributed load

Introduction

Normally, beams are loaded with different patterns of live load, after which an elastic analysis is performed for each of the chosen live load patterns and design is carried out for the envelope of these. Therefore, for any combination of live load patterns, certain critical stations along the beam might reach the ultimate strength while other stations hold extra capacity. In elastic analysis, this reserve capacity is not utilised; however, a full inelastic analysis based on hinge formation could take advantage of this reserve capacity. The very common way of dealing with this is to perform the analysis elastically, but to make use of moment redistribution factors to account for the redistribution. The amount of moment redistribution depends on the ductility of inelastic regions, the geometry of beams and the loading pattern. The moment redistribution in continuous reinforced concrete (RC) beams is one of the simplest applications of member ductility in the procedure of design. This prevents the congestion of rebars at critical sections along the beams and allows a more even distribution of rebars along the length. Design

codes worldwide permit a limited amount of moment redistribution and each presents a different formula for it. Usually provisions in codes do not comprehensively consider the effects of all important parameters in the redistribution.

Mattock (1959) and Cohn (1964) conducted the first experimental programmes on the subject of moment redistribution. They concluded that cracking and deflection of beams designed for limited moment redistribution are not significantly greater at service loads than for beams designed by the distribution of moments according to the elastic theory. Shakir and Rogowsky (2000) presented a model for calculating the plastic rotation capacity and permissible moment redistribution factor in RC beams. Their results showed a good agreement with experimental results and their conclusion was that, although the CSA A23.3-94 (CSA, 1994) provides a reasonable estimate of moment redistribution factor for unfavourable combinations of important parameters, the code can be very conservative when conditions are favourable for the moment redistribution to occur. Mostofinejad and Farahbod (2007) implemented a parametric study on moment redistribution in continuous RC beams using a ductility demand and capacity concept. Their results showed that the permissible moment redistribution in continuous RC beams based on the current codes is conservative for the majority of cases but is not safe for some.

Strength and ductility capacities of RC members depend on various geometric and material properties, most of which are of a random nature. Therefore, a level of uncertainty exists in the strength and ductility of RC members. There have been numerous studies on the strength of RC members, the results of which are already implemented in the design codes (Bartlett *et al.*, 2003; Szerszen and Nowak, 2003). In contrast, limited research can be found on the probabilistic inelastic deformation and ductility. Trezos (1997) calculated the probabilistic parameters of the curvature ductility of confined RC columns using Monte Carlo simulation (MCS). Parametric and sensitivity analyses were carried out and the results were compared with the proposed values of Eurocode 8 for structures in seismic zones. Kappos *et al.* (1999) investigated the uncertainty of strength and ductility of confined RC members using MCS and the response surface method (RSM) and evaluated the concrete model and curvature ductility provisions of Eurocode 8. Lu and Gu (2004) conducted a probabilistic analysis of RC member deformation limits for different performance levels. They reported that curvature and drift limits generally follow a normal distribution.

In the current study, first a closed-form expression of curvature ductility and moment redistribution is derived using ductility demand and the capacity method developed by Silva and Ibell (2008). Then, a probabilistic analysis is performed in order to find the reliability of the nominal and the code-specified moment redistribution factors. Finally, the effect of considering uncertainty in evaluating the moment redistribution factor associated with the strength limit state is investigated.

Provisions of codes on moment redistribution

By means of plastic hinge length, the plastic hinge rotation could be related to curvature ductility. Curvature ductility in RC beams is directly related to the percentage of tensile rebar area, which in turn is correlated to the strain in extreme tensile steel, ε_t, and neutral axis parameter, $k_u = c/d$. Current design codes worldwide have different moment redistribution forms. Some codes like the Canadian CSA A23.3-04 (CSA, 2004) and the Australian AS 3600 (AS, 2009) use the neutral axis parameter as the indicator of ductility in high-moment sections (i.e. the greater is the value of c/d, the lower are the ductility and the permissible moment redistribution factor). Other design codes such as ACI 318-08 (ACI, 2008) use ε_t as an indicator of ductility and permissible moment redistribution factors. In the previous code ACI 318-99 (ACI, 1999), the rebar percentage was used to calculate the moment redistribution factors. Figure 1 shows the curves for evaluating the moment redistribution factor based on different design codes ACI 318-08 (ACI, 2008), AS 3600 (AS, 2009), CSA A23.3-04 (CSA, 2004) and the CEB–FIP model code (CEB–FIP, 1990).

One of the concerns in moment redistribution of RC beams is the serviceability limit state. Allowing large amounts of moment redistribution, which happens in highly ductile sections with low rebar percentages, can create excessive deflection along the beam span. In order to limit these deflections, redistribution should be prevented under service loads. Shakir (2006) has proposed some equations to evaluate the maximum allowable moment redistribution factor considering the serviceability.

Moment redistribution requirements in RC beams

The basic idea for moment redistribution in continuous RC beams is that the demand rotation required for the development of plastic hinges at the ends and middle of the spans should be lower than the rotational capacity of the plastic hinge or hinges that yield first. The rotational capacity in members could easily
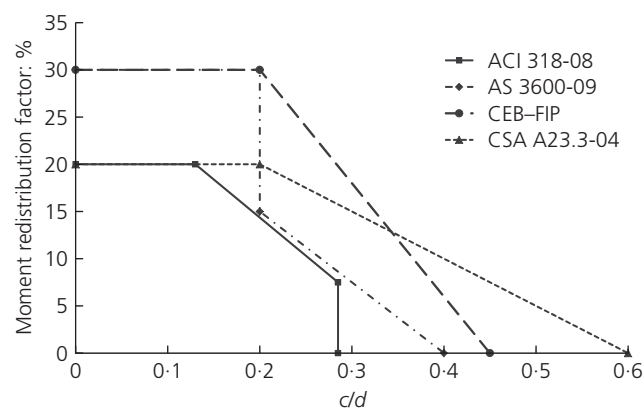

Figure 1. Permissible moment redistribution factor plotted against c/d based on different design codes

Magazine of Concrete Research
Volume 65 Issue 13

Reliability analysis of moment
redistribution in reinforced concrete
beams
Baji and Ronagh

be transferred to section curvature capacity using the concept of plastic hinge length.

Figure 2 shows a typical continuous RC beam with equal span length under a uniformly distributed load. In this study, for the sake of simplicity, a beam fixed at both ends is considered; this can approximately represent an interior span of a multi-span beam. Although the representation is not exact, the results can be trusted to be adequately accurate. It is assumed that the RC beam has a constant flexural stiffness, EI, along its length and plastic hinges are first formed at the ends of the beam. The end hinges should show adequate ductility and deform adequately to allow the formation of another hinge at the middle span of the beam.

The idealised stress–strain curve for steel and the moment–curvature curve at the critical sections are shown in Figure 3. In this study, the post-yield rigidity has been neglected. Usually, for conventional RC beams with normal reinforcement, the slope of the post-yield part of the moment–curvature curve can be neglected. This always brings about more conservative results. As can be seen in Figure 3, the width of the equivalent stress block is given by the product $k_1 k_3 f'_c$. The term f'_c here represents the real concrete compressive strength rather than the characteristic value. The k_3 factor takes into account the difference between the in situ compressive strength of concrete and the strength determined from standard cylindrical tests, as well as the load duration effect. The k_2 factor represents the stress block depth factor.

The ultimate deformation capacity is expressed through ultimate curvature of the section. The ultimate curvature is a state at which either the specific ultimate compressive strain in the concrete or the specific ultimate strength of an extreme tensile rebar are reached. Usually, for unconfined concrete, which is

assumed for RC beams, reaching the ultimate compressive strain in concrete governs the ultimate curvature; because the ultimate concrete compressive strain of unconfined concrete is relatively low and the rebar steel even for high-strength concrete has adequate ductility prior to rupture. In this study, it is assumed that the ultimate curvature is governed by crushing of the extreme fibre of the RC beam section. In addition the effect of the compressive rebar is neglected.

Here, first, the capacity of curvature ductility for the critical section is derived using the basic mechanics of RC. The curvature ductility is defined as the ratio of ultimate to yield curvature (Park and Paulay, 1975)

1.
$$\mu_\phi = \frac{\phi_u}{\phi_y}$$

According to Figure 3, the ultimate curvature is defined as the gradient of strain over the section height. Using geometry, compatibility and equilibrium, the yield and ultimate curvature can be easily obtained

2.
$$\phi_u = \frac{\varepsilon_{cu}}{c} = k_1 k_2 k_3 \frac{f'_c}{f_y} \frac{1}{\rho d} \varepsilon_{cu}$$

3.
$$\phi_y = \frac{f_y/E_s}{d(1-k)} = \frac{f_y/E_s}{\left[1 - \rho n \left(\sqrt{1 + \frac{2}{\rho n}} - 1 \right) \right] d}$$
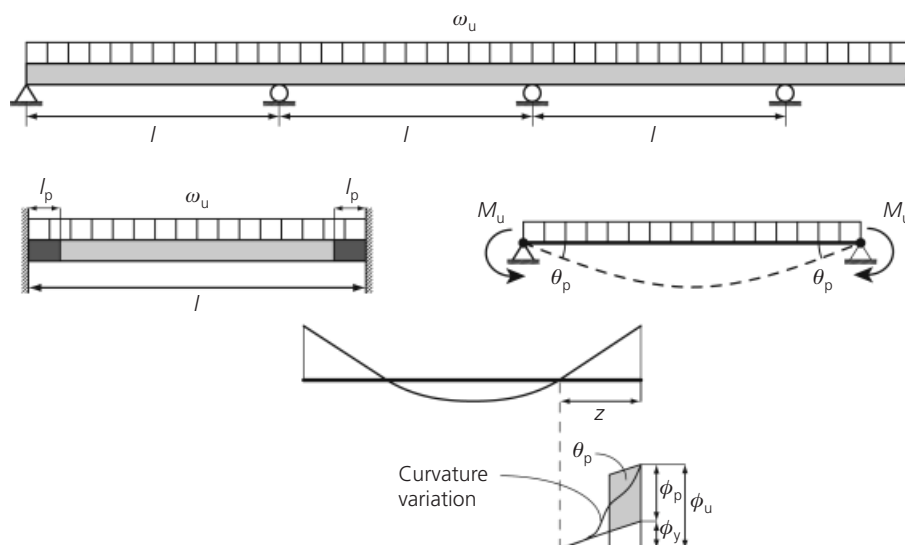

Figure 2. Reinforced concrete beam geometry

Magazine of Concrete Research
Volume 65 Issue 13

Reliability analysis of moment
redistribution in reinforced concrete
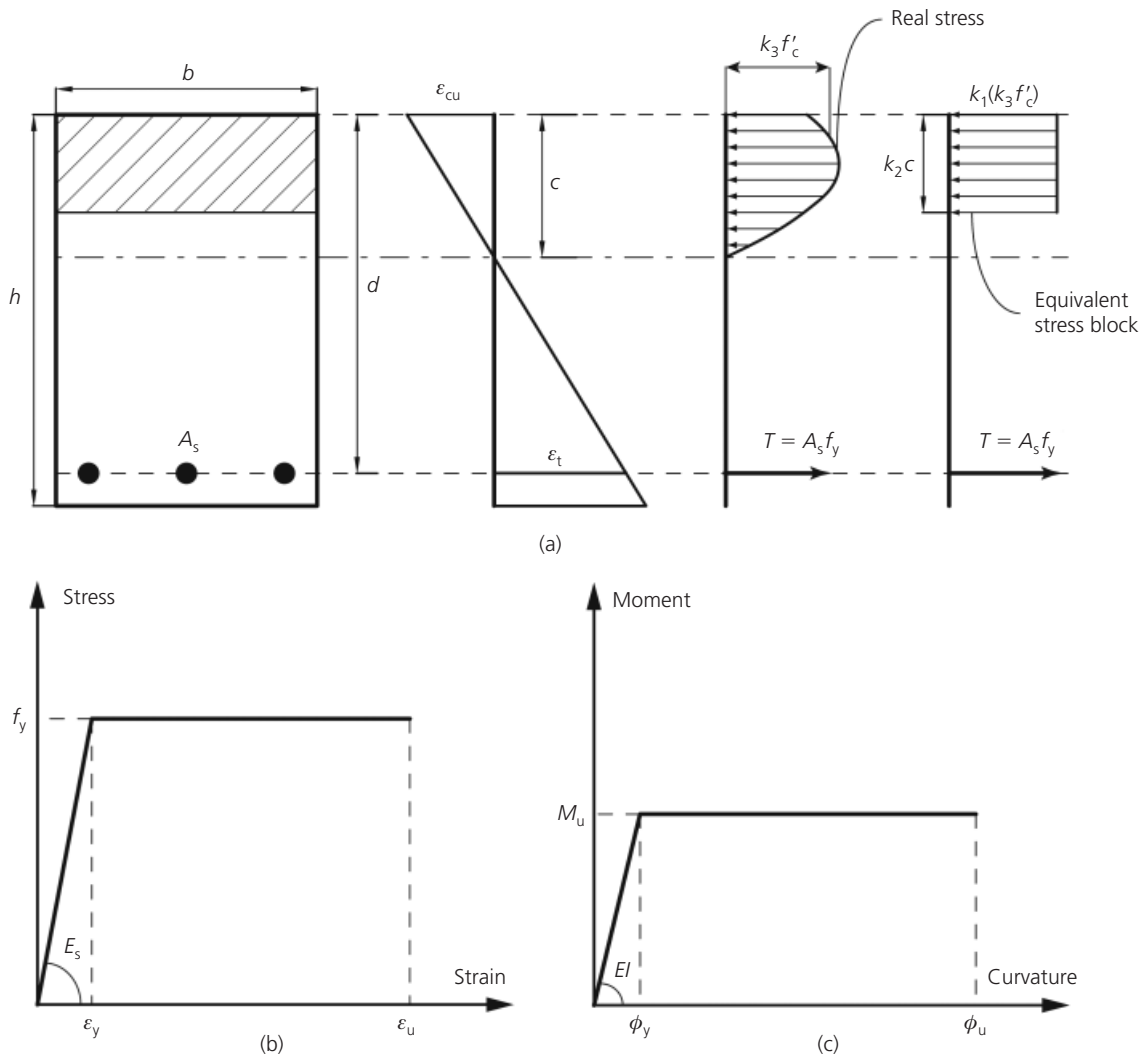beams
Baji and Ronagh

Figure 3. Basic assumptions of mechanical model for RC

In the case of yielding steel, the stress in the extreme compressive fibre of concrete could be much lower than the cylinder strength f'_c. The stress–strain curve for concrete is approximately linear up to $0.70 f'_c$. Therefore, by using the elastic theory and assuming that concrete stress does not exceed this value when the extreme steel yields, the neutral axis parameter at yield is calculated as shown in Equation 3.

In Equation 3, ρ is the tensile rebar percentage and n is the ratio of modulus of elasticity of steel to concrete. Substituting Equations 2 and 3 into Equation 1, the final expression for curvature ductility in singly reinforced rectangular beam section can then be derived as

4. $$\mu_\phi = k_1 k_2 k_3 \frac{f'_c}{f^2_y} E_s \varepsilon_{cu} \frac{1 - \rho n \left(\sqrt{1 + \dfrac{2}{\rho n}} - 1 \right)}{\rho}$$

In Equation 4, the $k_1 k_2 k_3$ factor represents the equivalent stress block parameters, the $(f'_c/f^2_y)E_s\varepsilon_{cu}$ factor is related to material properties and the last multiplier represents the cross-sectional dimensions.

The demand ductility (rotational or curvature) depends on the geometry of the RC beam, type of loading and plastic hinge length in the critical regions. Referring to Figure 2 and using the moment–area method, the demand rotation for the formation of plastic hinges at the ends and middle of the beam can be calculated as shown in Equation 5.

5. $$\theta_{demand} = \frac{l}{2EI} \left(\frac{\omega_u l^2}{12} - M_u \right) = \frac{l}{2EI} (M_e - M_u)$$

Using the concept of equivalent plastic hinge length, the rotational capacity of end hinges can also be calculated as shown in Equation 6 (Park and Paulay, 1975).

Magazine of Concrete Research
Volume 65 Issue 13

Reliability analysis of moment
redistribution in reinforced concrete
beams
Baji and Ronagh

6. $\qquad \theta_{\text{capacity}} = (\phi_{\text{u}} - \phi_{\text{y}})l_{\text{p}}$

In Equation 6, ϕ_{y} and ϕ_{u} are the yield and ultimate curvature at the end sections of the beam while l_{p} is the equivalent length of plastic hinge. The relationship between the parameters of ultimate uniform load ω_{u}, moment redistribution factor β, ultimate moment at beam ends M_{u} and elastic moment M_{e} can simply be written as Equation 7.

7. $\qquad M_{\text{u}} = \dfrac{\omega_{\text{u}}l^2}{12}(1 - \beta) = M_{\text{e}}(1 - \beta)$

Equating Equations 5 and 6 and substituting Equation 7, results in Equation 8 for demand curvature ductility at the end critical sections.

8. $\qquad \mu_{\phi} = 1 + \dfrac{1}{2}\left(\dfrac{\beta}{1 - \beta}\right)\left(\dfrac{l}{l_{\text{p}}}\right)$

Finally, rearranging parameters in Equation 8 results in Equation 9, in which the permissible moment redistribution factor can be found.

9. $\qquad \beta = 1 - \dfrac{1}{1 + 2\left(l_{\text{p}}/l\right)\left(\mu_{\phi} - 1\right)}$

In Equation 9, μ_{ϕ} is the curvature ductility of the section which is calculated using Equation 4. Applying different end conditions for the RC beam would result in different values for moment redistribution factors. In the considered case of this study, which is a fixed-end beam, the bending moment is $0\cdot083\omega_{\text{u}}l^2$. For instance, in three-bay continuous RC beams the maximum bending moment is $0\cdot10\omega_{\text{u}}l^2$, which is less than that of a fixed-end beam. The smaller values of maximum bending moments at supports result in lower plastic rotation and consequently lower curvature ductility demands, so they are more conservative when it comes to the investigation of moment redistribution factors (Silva and Ibell, 2008).

Basic random variables
Important parameters for the estimation of moment redistribution factors are presented in Equations 4 and 9. Equation 9 shows that the plastic hinge length and the curvature ductility at the critical section are the main random variables in the probabilistic analysis of the moment redistribution factor. In this section, all random variables are reviewed and the nominal, mean and standard deviation values as well as best-fit probability density function for each variable are selected based on the available literature.

Dimensions
The uncertainties in dimensions are relatively small and do not have a significant effect on the results of reliability analysis. Therefore, for the sake of simplicity, in the present study the statistical model used by Szerszen et al. (2005) is applied. According to this study, bias factor and coefficient of variation values of 1·0 and 0·015 are used for the rebar area, and 1·0 and 0·04 for the effective depth and width of RC sections.

Concrete properties
In this study, both strength and ductility are of concern. Therefore, a statistical model that includes concrete compressive strength and all parameters of the equivalent stress block is required. Attard and Stewart (1998) in their probabilistic analysis of a concrete stress block used the log-normal distribution as the probability density function for the concrete compressive strength, based on other studies (Diniz and Frangopol, 1997; Pham, 1985; Tabsh and Aswad, 1995). They used the mean value of $(f'_{\text{c}} + 7\cdot5\ \text{MPa})$ and the standard deviation of 6 MPa in their analysis. As in the current study, the statistical model of the equivalent stress block is derived from the research of Attard and Stewart (1998); their model for concrete compressive strength is also being used.

Attard and Stewart, based on the above-mentioned probabilistic model for the concrete compressive strength and the results of Setunge (1993), proposed models for the modulus of elasticity, E_{c}, and strain of concrete at peak stress, ε_{c}, for a wide range of concrete compressive strength. They used the stress–strain model proposed by Attard and Setunge (1996) to find the ultimate strain and equivalent stress block parameter variation. Their results for the equivalent stress block parameters included both the dog bone and the sustained load value for k_3. In this research, the main focus in the probabilistic analysis is directed towards dog bone results. For all of these parameters, Attard and Stewart proposed the use of normal probability functions.

Rebar steel
Based on Mirza and MacGregor (1979), the yield strength of rebar steel can be modelled by a beta distribution. In their study a four-parameter beta function was used for the probability density function fitting. According to their study, a mean value and coefficient of variation of 337 MPa and 0·107 and 490 MPa and 0·093 were obtained for G40 (280 MPa yield stress) and G60 (420 MPa yield stress), respectively.

Bournonville et al. (2004) gathered extensive data for various steel types for the USA and Canada. Their study included different types of steel and a wide range of rebar sizes. They used different types of steel produced according to the ASTM standard. In the current study, grade 615 ASTM steel is considered in the three different types of G40 (280 MPa), G60 (420 MPa) and G75 (500 MPa), and the statistical data are taken from Bournonville et al. (2004). According to the Bournonville et al. study, G40 and G60 steels follow beta distribution, whereas

Magazine of Concrete Research
Volume 65 Issue 13

Reliability analysis of moment
redistribution in reinforced concrete
beams
Baji and Ronagh

for G75, either normal or beta distributions could be used as the best-fit probability functions. It is assumed that the modulus of elasticity follows log-normal distribution with a mean equal to 201 GPA and a coefficient of variation of 0·033.

Plastic hinge length

Various formulations have been suggested for the determination of equivalent plastic hinge length. There is high uncertainty involved in the plastic hinge length because it is affected by many uncertain factors. Lu and Gu (2004) combined the experimental data from previous studies (Bayrak, 1999; Priestley and Park, 1987; Sheikh *et al.*, 1994) and plotted the plastic hinge length against z (the distance from the point of contraflexure) and d_b (the longitudinal rebar diameter). Using linear regression analysis, they proposed the following equation for the estimation of plastic hinge length

$$10. \quad l_p = 0·077z + 8·16d_b$$

In order to include uncertainty in Equation 10, a model uncertainty factor has to be associated with it. Based on the experimental results and a linear regression, Lu and Gu (2004) found that the model uncertainty factor can be presented with normal distribution with a mean equal to unity and coefficient of variation of 0·198. In this study, the same model is used for the plastic hinge random variable.

A summary of the probabilistic model for plastic hinge length is presented in Table 1, in which f_c' represents the specified or nominal concrete compressive strength and f_{cm}' shows the mean concrete compressive strength of concrete.

Probabilistic analysis of moment redistribution factor

In order to consider the effect of variation of concrete compressive strength, three different strengths of 20, 40 and 60 MPa are selected for this study. The nominal values of all random variables that are summarised in Table 1 are taken from ACI 318-08 (ACI, 2008), which signifies the assumption that the code-specified values represent the nominal values. In this study, two different span-to-depth ratios of 10 and 20 are used and the effective depth of RC beams is assumed to be 500 mm. In an ideal fixed-end beam, the distance from the point of contraflexure to the section at which the maximum moment occurs, z, is about one fifth of the whole span length. Consequently, using Equation 10, assuming 25 mm rebar diameter and having 500 mm effective depth, plastic hinge lengths of 281 mm and 358 mm are calculated, respectively. In these cases, the ratios of plastic hinge to span length are about 0·0562 and 0·0358, respectively. The assumptions for span length, effective depth and bar diameter are presented as an example here. Any other values that provide ratios close to the code considered lower bound or other limits could be used.

Design codes basically use the 5th percentile value (95% chance of being exceeded) as the nominal value of resistance-related parameters. The moment redistribution factor could be treated in a similar manner. In evaluating the 5th percentile values, simulated data are used to find the probability density function of the moment redistribution factor. Figure 4 shows the theoretical

Variable		Bias[a]/Mean	COV[b]/Standard deviation	PDF[c]	Reference
Dimensions	b	1·00	0·04	Normal	Szerszen *et al.* (2005)
	d	1·00	0·04	Normal	Szerszen *et al.* (2005)
	A_s	1·00	0·015	Normal	Szerszen *et al.* (2005)
Concrete	f_c'	Nominal + 7·5 MPa	6·0 MPa	Log-normal	Attard and Stewart (1998)
	E_c	$1·01(4370·3 f_{cm}'^{0·5164})$	0·15	Normal	Attard and Stewart (1998)
	ε_c	$\dfrac{4·11 f_{cm}'}{E_c \sqrt[4]{f_{cm}'}}$	$0·18 - 0·001 f_c'$	Normal	Attard and Stewart (1998)
	ε_{cu}	$\varepsilon_c \times 2·8133(f_{cm}')^{-0·2093}$	0·19	Normal	Attard and Stewart (1998)
	$(k_1 k_3)$	$1·2932(f_c')^{-0·0998}$	0·09	Normal	Attard and Stewart (1998)
	k_2	$1·0948(f_c')^{-0·091}$	0·03	Normal	Attard and Stewart (1998)
Steel	E_s	1·00	0·033	Log-normal	Lu and Gu (2004)
	f_y (G60)	1·22	0·068	Beta	Bournonville *et al.* (2004)
	f_y (G75)	1·16	0·048	Beta	Bournonville *et al.* (2004)
Plastic hinge	l_p	$0·077z + 8·16d_b$	0·198	Normal	Lu and Gu (2004)

[a] Mean to nominal value
[b] Coefficient of variation
[c] Probability density function

Table 1. Summary of statistical models of random variables

Magazine of Concrete Research
Volume 65 Issue 13

Reliability analysis of moment
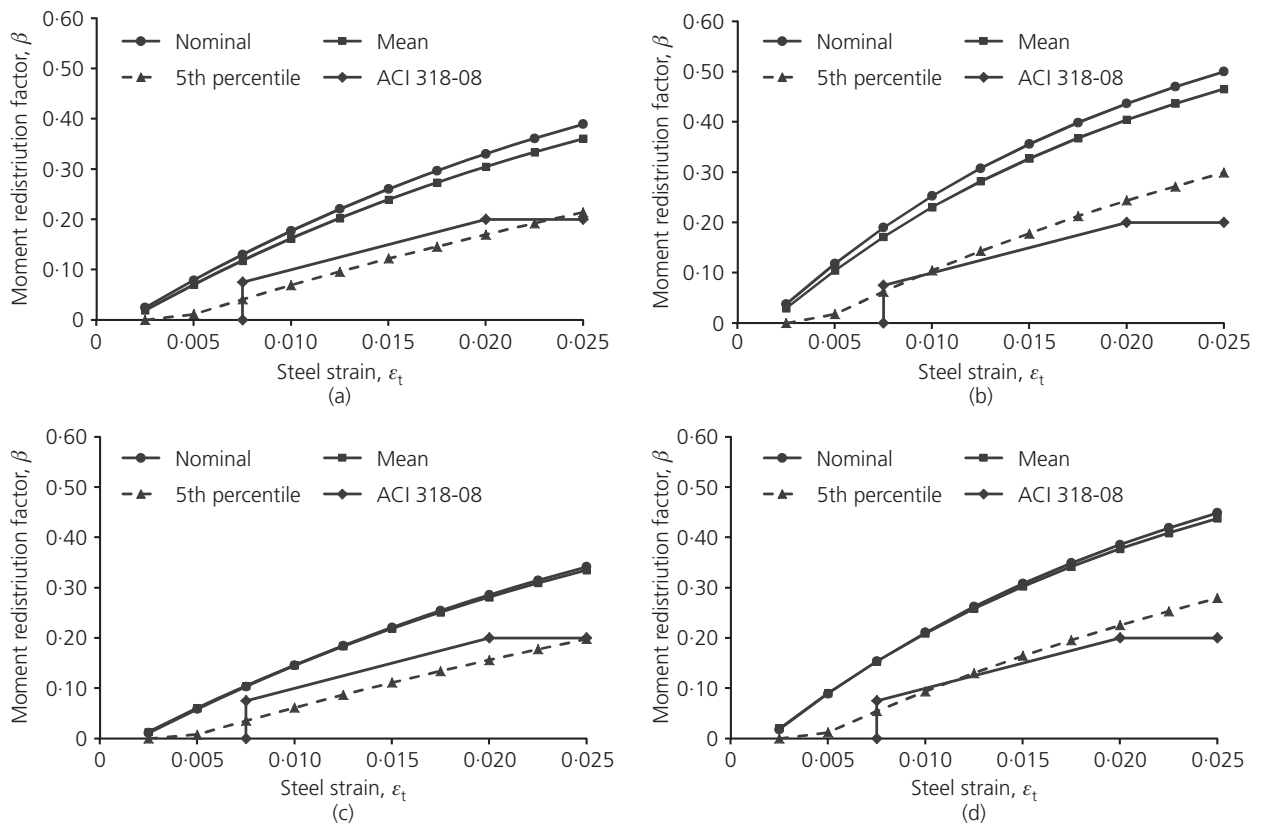redistribution in reinforced concrete
beams
Baji and Ronagh

Figure 4. Mean, nominal and 5th percentile values of moment redistribution factor ($f'_c = 40$ MPa): (a) G60 steel rebar and $l/d = 20$; (b) G60 steel rebar and $l/d = 10$; (c) G75 steel rebar and $l/d = 20$; (d) G75 steel rebar and $l/d = 10$

5th percentile moment redistribution value and ACI 318-08 specified value of moment redistribution factor as a function of steel strain for different types of steel. In all considered cases, the curve proposed by ACI 318-08 falls below the evaluated mean value. Furthermore, as the steel rebar yield strength increases, the mean and nominal curves merge closer. In almost all cases, mean moment redistribution curves lie between the nominal and the code curves. For the large span-to-depth ratio of 20, the code-specified moment redistribution factors are smaller than the 5th percentile values; for the smaller span-to-depth ratio of 10, however, the code-specified moment redistribution factors are slightly larger. The 5th percentile values are much smaller than the nominal and the mean values and this indicates the high uncertainties in the evaluation of moment redistribution factors.

Figure 5 shows the probability of exceedance as a function of steel strain for different types of steel. The results in Figure 5 are based on the span-to-depth ratio of 20. The code-specified nominal and mean values are treated as deterministic values and the probability of exceeding these values is calculated using the simulated data. The probability of exceedance gives estimation for safety of the calculated moment redistribution factors and

those specified by the code. Results in Figure 5 show that the code-specified values for moment redistribution provide higher safety margins of above 75%, compared to mean and nominal values. For the mean and the nominal values, the chance of exceedance is about 50%. The chance of exceedance is not uniform for the code-specified moment redistribution factors, whereas the nominal and the mean values show almost uniform probability against steel strain. The 50% chance of being exceeded indicates that the probability density function of the moment redistribution factor is close to a symmetric distribution such as the normal distribution.

In order to investigate the effect of different types of concrete and rebar steel on the reliability of code-specified values, Figure 6 presents the chance of exceedance for ACI 318-08 (ACI, 2008) specified values of moment redistribution factor considering different concrete and steel rebar strengths. For the range of 0·0075–0·020 steel strain, the chance of exceeding code-specified moment redistribution factors is above 75% for all of the considered cases. The results for span-to-depth ratio of 10 are much higher than those for span-to-depth ratio of 20 and close to 95%.
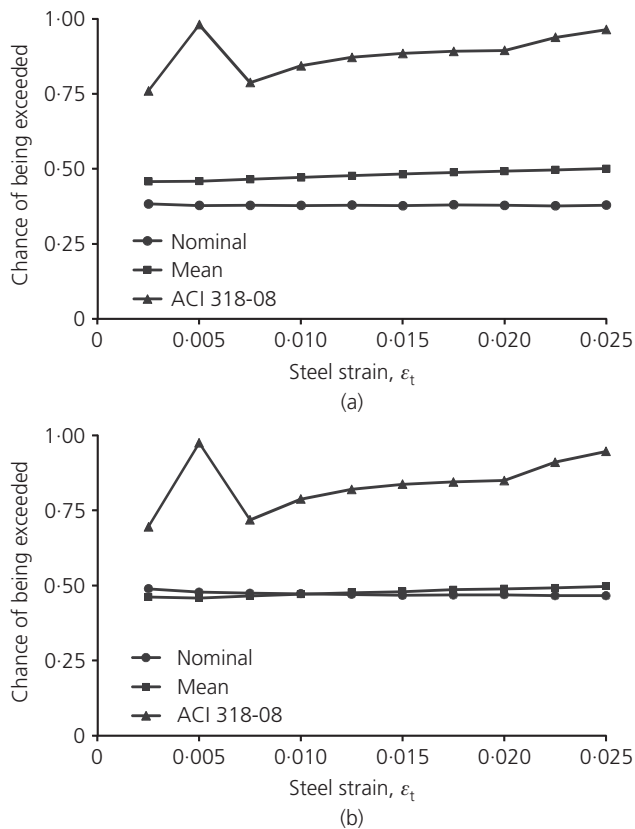
Magazine of Concrete Research
Volume 65 Issue 13

**Reliability analysis of moment
redistribution in reinforced concrete
beams**
Baji and Ronagh

Figure 5. Reliability of code-specified, mean and nominal values of moment redistribution factor ($f'_c = 40$ MPa): (a) G60 steel rebar; (b) G75 steel rebar
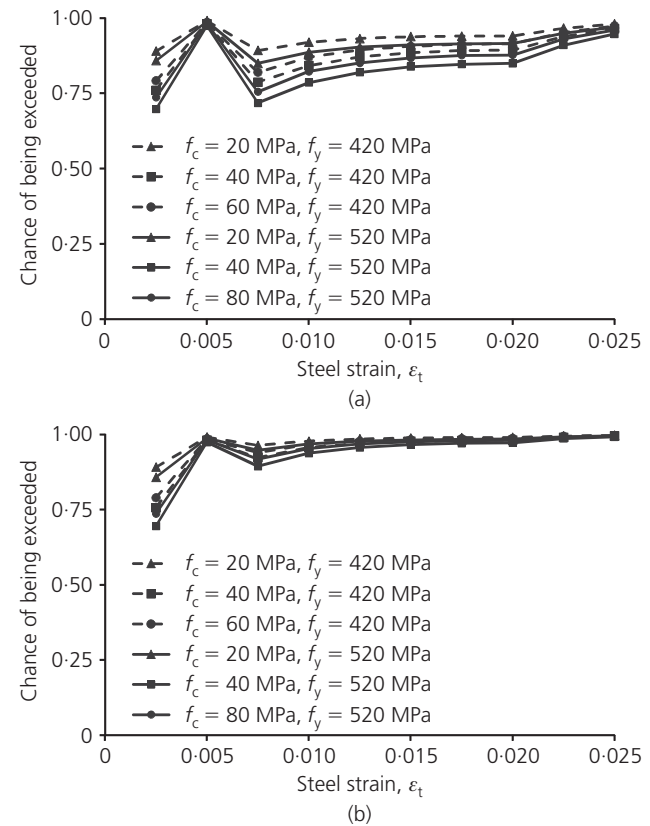

Figure 6. Reliability of code-specified values of moment redistribution factors for different types of concrete and steel rebar: (a) $l/d = 20$; (b) $l/d = 10$

The reason for the values shown in Figures 5 and 6 for strains below 0·0075 being so different from the values above this level of strain is that the ACI code does not allow any redistribution of moment in this region (i.e. moment redistribution factor = 0). Furthermore, as the moment redistribution factor is kept at 0·2 for strains above 0·02, the rate of increase in the chance of exceedance increases more rapidly for strains beyond 0·02.

Strength reliability by considering uncertainty in moment redistribution factor

The reliability of RC beams under dead and live loads is a classic problem in structural reliability and has been investigated by many researchers. Here, the reliability of RC beams is evaluated taking into consideration the uncertainty of the moment redistribution factor.

The strength limit state function of moment resistance of any RC beam can be stated as shown in Equation 11. In this limit state, both the dead load and the live load are considered.

11.
$$g = \frac{M_R}{1 - \beta} - M_D - M_L$$

In Equation 11, M_R, M_D and M_L represent the moment resistance of RC section, dead load effect and live load effect, respectively. The β factor stands for the moment redistribution factor. The moment redistribution factor can be calculated using Equation 9. The resistance of a RC section depends on its dimensions, and the material properties of concrete and steel. According to Figure 3, the moment resistance of a singly reinforced rectangular section is calculated as shown in Equation 12.

12.
$$M_R = \rho b d^2 f_y \left[1 - \frac{\rho f_y}{2 k_1 k_3 f'_c} \right]$$

All variables of Equation 12 were defined in the previous sections. These variables are random in nature.

Dead load is treated as a normal variable with a mean of 1·05 times its nominal value and coefficient of variation of 0·10 according to Ellingwood *et al.* (1980). Live load in Equation 14 is the maximum life-time live load and is modelled by extreme type I distribution with bias factor (mean to nominal) and coefficient of variation of 1·0 and 0·23, respectively (Ellingwood *et al.*, 1980). For the design to suit, the demand must be less than

Magazine of Concrete Research
Volume 65 Issue 13

Reliability analysis of moment
redistribution in reinforced concrete
beams
Baji and Ronagh

the capacity. Equation 13 shows the governing equation relating the load effects (demand) to the capacity (moment resistance).

$$13. \quad \frac{\phi M_{Rn}}{1 - \beta_n} \geq \gamma_D M_{Dn} + \gamma_L M_{Ln}$$

In Equation 13, subscript n denotes the nominal value of the variable. The factor γ represents the load factor. In this study, the nominal value of moment redistribution factor, β, is either calculated using Equation 9 by inserting the nominal values of all variables or is based on the ACI 318-08 design code. MCS is utilised to find the probability of failure and the reliability index of the strength limit state shown in Equation 11. Three cases are considered in the reliability analysis; in the first case, the effect of moment redistribution is not considered, but elastic analysis and design are considered; whereas in the other two cases, the moment redistribution factor is calculated using Equation 9, that is based on the section capacity and the ACI 318-08 design code.

Figure 7 shows the results of reliability analysis for two different concrete compressive strengths and the live-to-dead-load ratio of 1·0. The reliability indices for the elastic analysis are almost constant and do not depend on the rebar percentage and the concrete properties. As expected, considering the uncertainty in

the moment redistribution factor causes a reduction in the reliability index (in cases where the section capacity is used to derive the moment redistribution factor). The amount of reduction for high steel strain (low rebar percentage) is more than that for low steel strain. The reason behind this result is that the moment redistribution factors for high steel strain are higher and the moment redistribution factor has a proportionally larger contribution in the design. On the other hand, when low steel strain is used in the design, lower ductility is produced and consequently the role of moment redistribution uncertainty is decreased. In case of $\beta = 0$, this case converges to the elastic case.

Figure 7 shows that if the code-specified moment redistribution factors are used in the design, the reliability indices will be even higher than those of an elastic analysis. As was shown in the previous sections, the code-specified values are more conservative than the calculated ones based on section capacity. Therefore, it is obvious that if the code-specified moment redistribution factors are used in a design, higher reliability indices will result for the strength limit states.

In order to investigate the effect of different live-to-dead-load ratios, several reliability analyses with various live-to-dead-load ratios were conducted. The results in Figure 8 are based on the moment redistribution factor resulting from a section capacity
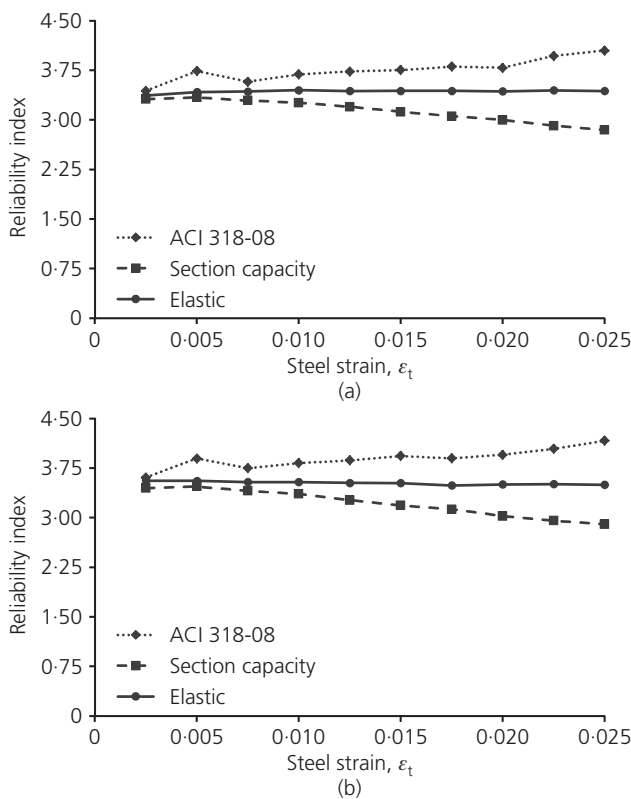

Figure 7. Reliability indices of strength limit state for different cases (G60 steel, $l/d = 20$ and live load/dead load, LL/DL = 1·0): (a) $f'_c = 20$ MPa; (b) $f'_c = 60$ MPa
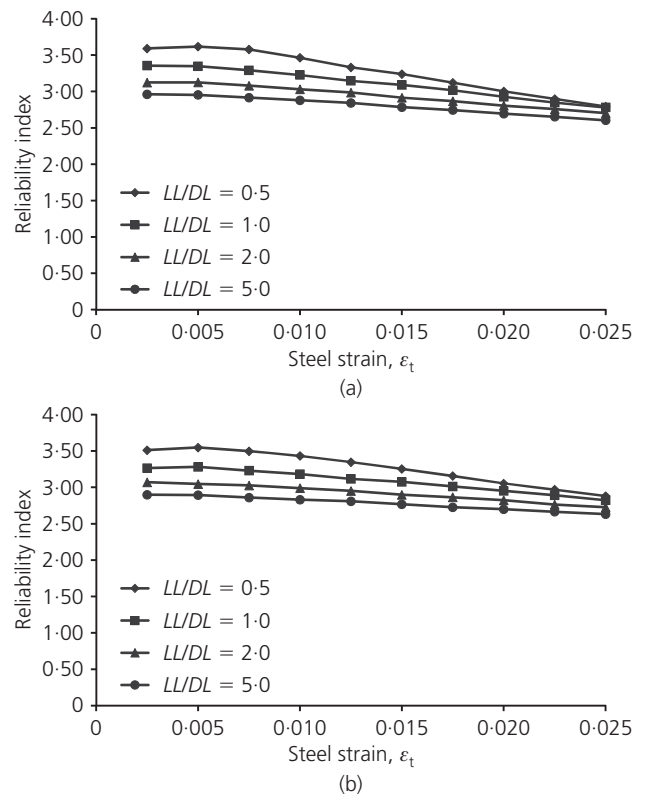

Figure 8. Effect of live-to-dead-load ratios on strength limit state reliability indices ($f'_c = 40$ MPa and $l/d = 20$): (a) G60 steel rebar; (b) G75 steel rebar

Magazine of Concrete Research
Volume 65 Issue 13

**Reliability analysis of moment
redistribution in reinforced concrete
beams**
Baji and Ronagh

analysis (nominal moment redistribution factor is used in design). As can be seen, although the reduction of reliability index due to uncertainty in moment redistribution factors is not large, it is comparable to the reduction in reliability index resulting from increasing the ratio of live load to dead load.

Conclusions

A reliability-based analysis of moment redistribution was performed using the MCS method. The statistical properties and distributions of all random variables were derived from the currently available literature. According to the results of reliability analysis, the evaluated mean and nominal values of the moment redistribution factor are greater than the values provided by the ACI 318-08 code in all of the considered cases. The results show that, in some cases, the 5th percentile values of moment redistribution factor could be smaller than those specified by the code. However, owing to the lack of adequate statistical models for parameters related to the moment redistribution, no specific judgment can be made on the code-specified values. Generally, the probability of exceedance of moment redistribution factors specified by ACI 318-08 is above 75%, and increasing with increase in the steel strain. On the other hand, the probability of exceedance for the mean values of moment redistribution is about 50%.

As expected, considering the uncertainty in the moment redistribution factor causes a reduction in the reliability index of the strength limit state. The effect of considering the uncertainty of moment redistribution factor in the moment resistance reliability index is in the order of about 0·5 in the reliability index scale. Although the reduction of reliability index due to uncertainty in moment redistribution factors is not large, it is comparable to the reduction in reliability index resulting from increasing the ratio of live load to dead load.

Lack of probabilistic models for important parameters such as plastic hinge, ultimate strain of concrete, concrete stress block parameters and other important variables complicates the judgement regarding reliable values for moment redistribution factors. The results presented here are highly dependent on the statistical models chosen for the main random variables. Nevertheless, they show the importance of probabilistic evaluation of moment redistribution factors.

REFERENCES

ACI (American Concrete Institute) (1999) ACI 318-99: Building code requirements for structural concrete. ACI, Farmington Hills, MI, USA.

ACI (2008) ACI 318-08: Building code requirements for structural concrete and commentary. ACI, Farmington Hills, MI, USA.

AS (Australian Standards) (2009) AS 3600: Australian concrete structures standard. Standards Australia, Sydney, Australia.

Attard M and Setunge S (1996) Stress–strain relationship of confined and unconfined concrete. *ACI Materials Journal* **93(5)**: 432–442.

Attard MM and Stewart MG (1998) Two parameter stress block for high-strength concrete. *ACI Structural Journal* **95(3)**: 305–317.

Bartlett FM, Hong HP and Zhou W (2003) Load factor calibration for the proposed 2005 edition of the National Building Code of Canada: Companion–action load combinations. *Canadian Journal of Civil Engineering* **30(2)**: 440–448.

Bayrak O (1999) *Seismic Performance of Rectilinearly Confined High Strength Concrete Columns*. PhD thesis, University of Toronto, Canada.

Bournonville M, Dahnke J and Darwin D (2004) *Statistical Analysis of the Mechanical Properties and Weight of Reinforcing Bars*. The University of Kansas, Structural Engineering and Materials Laboratory, Kansas, USA.

CEB–FIP (1990) *CEB–FIP Model Code*. CEB–FIP, Lausanne, Switzerland.

Cohn M (1964) Rotation compatibility in the limit design of reinforced concrete continuous beams. In *Proceedings of the International Symposium on the Flexural Mechanics of Reinforced Concrete, Miami*. ASCE–ACI, USA, pp. 359–381.

CSA (Canadian Standards Association) (1994) CSA A23.3-94: Design of concrete structures. Canadian Standards Association, Toronto, Ontario, Canada.

CSA (2004) CSA A23.3-04: Design of concrete structures. Canadian Standards Association, Toronto, Ontario, Canada.

Diniz SMC and Frangopol DM (1997) Reliability assessment of high-strength concrete columns. *Proceedings of the International Conference on Structural Safety and Reliability, Kyoto, Japan*.

Ellingwood B, Galambos TV, MacGregor JG and Cornell CA (1980) *Development of a Probability-based Load Criterion for American National Standard A58*. National Bureau of Standards, Washington, D.C., USA, NBS Special Publication 577.

Kappos AJ, Chryssanthopoulos MK and Dymiotis C (1999) Uncertainty analysis of strength and ductility of confined reinforced concrete members. *Engineering Structures* **21(3)**: 195–208.

Lu Y and Gu X (2004) Probability analysis of RC member deformation limits for different performance levels and reliability of their deterministic calculations. *Structural Safety* **26(4)**: 367–389.

Mattock AH (1959) Redistribution of design bending moments in reinforced concrete continuous beams. *Proceedings of the Institution of Civil Engineers* **13(1)**: 35–46.

Mirza SA and MacGregor JG (1979) Variability of mechanical properties of reinforcing bars. *Journal of the Structural Division* **105(5)**: 921–937.

Mostofinejad D and Farahbod F (2007) Parametric study on moment redistribution in continuous RC beams using ductility demand and ductility capacity concept. *Iranian Journal of Science and Technology, Transaction B: Engineering* **31(B5)**: 459–471.

Park R and Paulay T (1975) *Reinforced Concrete Structures*. John Wiley & Sons, New York, USA.

Magazine of Concrete Research
Volume 65 Issue 13

Reliability analysis of moment
redistribution in reinforced concrete
beams
Baji and Ronagh

Pham L (1985) Reliability analyses of reinforced concrete and
composite column sections under concentric loads. *Institution
of Engineers, Australia, Civil Engineering Transactions*
CE27: 68–72.

Priestley M and Park R (1987) Strength and ductility of concrete
bridge columns under seismic loading. *ACI Structural Journal*
84(1): 61–76.

Setunge S (1993) *Structural Properties of Very High Strength
Concrete*. PhD thesis, Monash University, Melbourne,
Australia.

Shakir A (2006) *Moment Redistribution in Reinforced
Concrete Structures*. PhD thesis, University of Alberta,
Canada.

Shakir A and Rogowsky DM (2000) Evaluation of ductility and
allowable moment redistribution in reinforced concrete
structures. *Canadian Journal of Civil Engineering* **27(6)**:
1286–1299.

Sheikh SA, Shah DV and Khoury SS (1994) Confinement of

high-strength concrete columns. *ACI Structural Journal*
91(1): 100–101.

Silva PF and Ibell TJ (2008) Evaluation of moment distribution in
continuous fiber-reinforced polymer-strengthened concrete
beams. *ACI Structural Journal* **105(6)**: 729–739.

Szerszen MM and Nowak AS (2003) Calibration of design code
for buildings (ACI 318): Part 2 – Reliability analysis and
resistance factors. *ACI Structural Journal* **100(3)**:
383–391.

Szerszen MM, Szwed A and Nowak AS (2005) Reliability
analysis for eccentrically loaded columns. *ACI Structural
Journal* **102(5)**: 676–688.

Tabsh S and Aswad A (1995) Statistical properties of plant-
produced high strength concrete in compression. *PCI Journal*
40(4): 72–76.

Trezos CG (1997) Reliability considerations on the confinement of
RC columns for ductility. *Soil Dynamics and Earthquake
Engineering* **16(1)**: 1–8.

WHAT DO YOU THINK?

To discuss this paper, please submit up to 500 words to
the editor at www.editorialmanager.com/macr by 1
January 2014. Your contribution will be forwarded to the
author(s) for a reply and, if considered appropriate by
the editorial panel, will be published as a discussion in a
future issue of the journal.