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# Experiments on the Life of Self-Quenching Geiger Mueller Counters

Raymond C. Opperman

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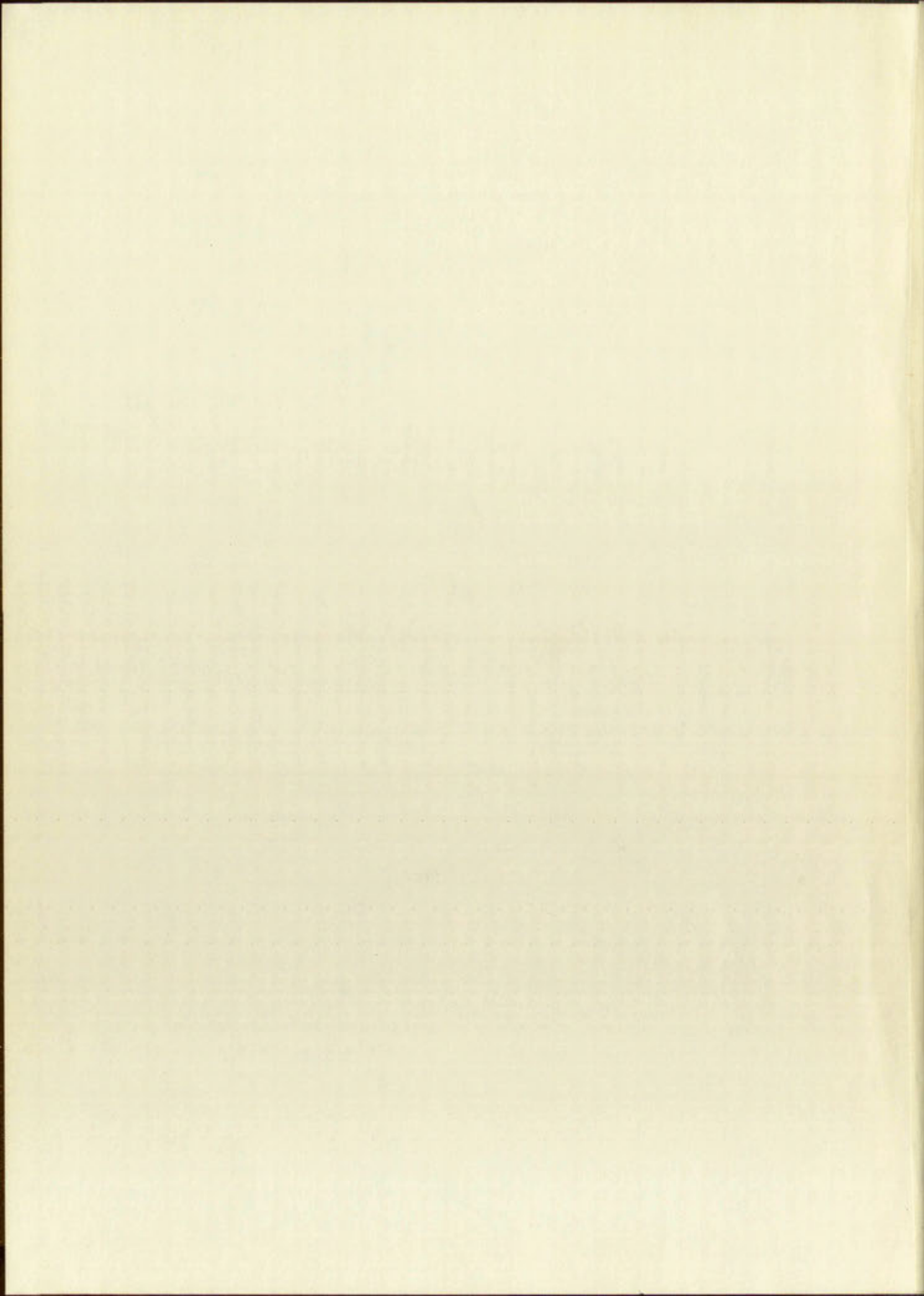
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EXPERIMENTS ON THE LIFE OF SELF-QUENCHING  
GEIGER NUELLER COUNTERS

By

Raymond H. Opperman

A Thesis

In partial fulfillment of the  
Requirements for the Degree of  
Master of Science in Physics

The University of New Mexico  
1953

THE UNIVERSITY OF CHICAGO

DEPARTMENT OF CHEMISTRY

LABORATORY OF ORGANIC CHEMISTRY



RESEARCH REPORT

1952

THE UNIVERSITY OF CHICAGO

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LABORATORY OF ORGANIC CHEMISTRY

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1952

This thesis, directed and approved by the candidate's committee, has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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This is to certify that the above named person  
has been duly elected to the office of  
Member of the Board of Directors of the  
Company for the term ending on the 31st day of  
December 1912.

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CHAPTER I  
INTRODUCTION

A. Self-quenching Counters

Whenever a Geiger counter is operated in the plateau region,<sup>1</sup> pulses are initially quenched by the formation of a positive ion sheath around the center wire which reduces the electric intensity in the vicinity of the wire. If the filling gas consists entirely of monatomic or diatomic molecules, extra counts will in general be initiated by the liberation of electrons from the cathode by the positive ions.

A quenching mechanism must be introduced to prevent these extra pulses in a counter. Self-quenching counters employ an internal quenching process. This is usually accomplished by including a small percentage of polyatomic molecules with the filling mixture. Tubes filled partially or entirely with an organic vapor have been used extensively as self-quenching counters. These fillings generally result in counters which have comparatively large pulses with a low percentage of spurious counts and a relatively long plateau. Unfortunately, the useful life of organic vapor counters is usually limited to about  $10^8$  or  $10^9$  counts, compared to lives of more than  $10^{10}$  counts for counters using an inorganic

---

<sup>1</sup> A list of definitions is given in the following section.

A. Self-diffusion coefficient

Whenever a foreign ion is introduced into the lattice region, it must be considered as a positive ion and the electric field in the vicinity of the ion is such that the electric field is directed towards the ion. This field will in turn cause a drift of the ions towards the ion, and this drift will be in the opposite direction to the direction of the electric field.

A decreasing number of ions will be present in the lattice region and in a crystal. Self-diffusion coefficient is an internal property of a crystal. This is usually explained by the fact that a small number of ions are replaced by foreign ions with the same lattice energy. The ions which are replaced are actually with an energy which is not too far from the energy of the self-diffusing atoms. These ions are usually in a number which have approximately the same energy as the percentage of atoms which are replaced. The self-diffusion coefficient is usually limited to about 10% of the total number of ions of one type in a crystal.

---

A list of self-diffusion coefficients is given in the following section.

quenching gas. Some experiments to determine the relative importance of the factors limiting the life of alcohol-argon filled counters will be described in this thesis.

Counters in which an inorganic gas is used as the quenching gas generally have a shorter plateau than those employing an organic vapor. However, the lives of counters using an inorganic quenching gas may be considerably longer for some mixtures than those found for organic vapor counters. Experiments on the life of counters filled with ammonia and argon are described.

#### B. Notation and Terminology

The following is a list of definitions of terms used in this thesis:

V The potential difference applied across a counter.

V<sub>s</sub> The lowest potential at which counts are observed with the equipment employed. In these experiments the starting potential was determined by visual observation on an oscillograph with constant amplification.

V<sub>g</sub> The lowest potential at which all pulses are of equal size regardless of the number of ions produced by the primary ionizing particle.

V<sub>c</sub> The potential at which the counter gives continuous or self-sustaining counts.

V<sub>u</sub> The overvoltage which is equal to V minus V<sub>g</sub>.

quenching gas. The experiments described in this paper  
importance of the fact that the rate of reaction between  
filled containers will be described in this paper.  
Containers in which an inert gas is used as the  
quenching gas generally have a lower rate of reaction than  
employing an organic vapor. However, the lower rate of reaction  
using an inert gas is due to the fact that the rate of reaction  
for some mixtures may be faster than for others. The results of  
experiments on the rate of reaction between the gas and the  
organ are described.

#### Reaction and Kinetics

The following is a list of the reactions observed between  
in this reaction:  
V The potential difference applied across a capacitor  
V The lowest potential of which is used as a reference  
with the equipment employed. It is also possible that the  
the potential was determined by a standard potential  
oscillation with constant amplitude.  
V The lowest potential of which is used as a reference  
equal also to the potential of the standard of the reaction  
the primary ionizing particle.  
V The potential at which the reaction between the gas and  
or self-sustaining current.  
V The overvoltage which is used as a reference.

**Plateau** The Potential region in which the counting rate is nearly independent of the applied voltage.

**Plateau Slope** The change in counting rate for one hundred volts change in supply voltage while the counter is operated in the plateau region.

**Geiger Region** The Potential region of Geiger counting action, i.e.,  $V_0$  minus  $V_g$ .

### C. Definition of Counter Tube Life

The life of a Geiger-Mueller counter is the number of events which can be counted without changing certain characteristics of the counter in such a way as to make it useless. These characteristics are the length and slope of the plateau, the normal operating voltage and the background counting rate.

Some causes of counter failure are common both to externally and to internally quenched counters, for example, pitting of the center wire. However, the changes in counter characteristics discussed in this thesis, will in general apply only to the self-quenching type since most of these changes are connected with the dissociation of the quenching gas used.

The amount and rate of change which can be tolerated in any of the listed characteristics depends on the application of the counter. If the length of the plateau decreases with use of the counter it eventually becomes impractical to





regulate the voltage applied across the tube so as to remain in the plateau region. This constitutes the end of the useful life for such a counter. Increase in plateau slope or fluctuation in background counting rate will also limit the useful life.

#### D. Purpose of Experiment

Because of the several superior properties of counters containing organic vapor as a quenching agent, it is desirable to extend the life of such counters. It is, therefore, of interest to determine the relative importance of factors which limit their useful life. It is also worthwhile to further the investigation of counters containing an inorganic gas as the quenching agent.

In these experiments life tests were conducted with counters filled with different percentages of alcohol vapor. One counter was given a life test while supplied with a constant alcohol vapor pressure. In some cases, counters were refilled and in other cases new center wires were installed to determine the major factors limiting counter life. Tests were also conducted to determine the useful life of counters filled with ammonia and argon.

regulate the voltage applied across the tubes in the  
main in the plate region. This arrangement has the effect of  
the useful life for each counter. It is found that the  
slope of the characteristic is dependent on the voltage applied  
limit the useful life.

#### D. Factors of the circuit

Because of the several applications of the  
containing organic vapors a vacuum system, it is  
able to extend the life of the counter. It is  
of interest to determine the relative influence of factors  
which limit their useful life. The following  
further the investigation of counter operation and  
Gauls gas as the counting gas.  
In these experiments the tubes were enclosed in  
counters filled with different gases. A list of values  
One counter was given a life test which included a  
against alcohol vapor pressure. In all cases, counter  
refilled and in other cases the counter was  
to determine the effect of the filling gas. The  
were also conducted to determine the effect of  
filled with argon and nitrogen.

CHAPTER II

THEORY AND PRESENT STATE  
OF EXPERIMENTAL EXPERIENCE

A. General

A. Ehmert and A. Trost<sup>1</sup> in 1936 and A. Trost<sup>2</sup> in 1937 described experiments concerning vapor-quenched counters and proposed a theory of the counting mechanism. C. G. Montgomery and D. D. Montgomery<sup>3</sup> in 1940 and H. G. Stever<sup>4</sup> in 1942 extended this theory for nonself-quenching and self-quenching counters respectively. Korff and Present<sup>5</sup> (1944) described the role played by the polyatomic molecules in a self-quenching counter.

As explained by Korff and Present, the process of self-quenching must involve the dissociation of polyatomic molecules. The initial avalanche, after spreading along the entire sensitive portion of the counter in the Geiger counting region, is stopped by the action of the positive ion sheath which is formed during the avalanche. Ions of a monatomic or diatomic gas upon reaching the cathode liberate electrons and thus initiate extra counts. Polyatomic molecules, on the other hand, have a large probability of predissociation, i.e., dissociation before radiation, and thus have a small probability of liberating electrons from the cathode after being neutralized. In a filling

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mixture composed of about ten percent polyatomic molecules, the monatomic or diatomic ions produced in a discharge are neutralized by collision before reaching the cathode provided the ionization potential of the polyatomic gas is lower than that of the other gases present. The polyatomic ions are neutralized at the cathode and dissociate so that the counting action ceases.

A property of the quenching gas which is essential for proportional counters is the absorption of ultraviolet photons emitted by the gas molecules during an avalanche. Such an absorption prevents the initial avalanche from spreading throughout the entire volume of the counter for an appreciable range of applied voltages. The absorption of these photons results in additional dissociation of the polyatomic molecules.

#### B. Life of Self-quenching Counters

Dissociation of the polyatomic molecules sets a limit on the life of self-quenching counters, assuming no recombination of those molecules. Korff and Present<sup>5</sup> calculated a life of about  $10^{10}$  counts for a counter containing ten percent alcohol vapor as limited by dissociation.

When an organic vapor is used as the quenching agent the dissociation products form a deposit on the electrodes. When ammonia is used as the quenching agent the dissociation

mixtures composed of both low and high molecular weight  
the monomers or oligomers that are present in the  
neutralized polymer solution. The polymer  
lower than that of the monomers. The polymer  
ions are neutralized at the same rate and the  
the resulting polymer solution.

A study of the effect of the rate of neutralization  
for fractional conversion is a function of the  
fraction called by the rate of neutralization.  
Such an effect may be due to the fact that  
spreading resistance of the polymer is the constant  
or appreciable range of neutralization. The effect  
of these factors results in a change in the  
polymeric solution.

... life of the polymer solution  
Disappearance of the polymer solution is a result  
on the life of self-polymerization, which is the  
tion of these solutions. The rate of neutralization  
life of about 10<sup>10</sup> hours for a neutralization rate  
some electric energy is used in neutralization.  
When anionic polymerization is used, the  
the dissociation of the polymer is a result of the  
When anionic is used, the neutralization rate is

products are gases only. In either type counter, the continuous potential will be lowered if the center wire becomes pitted with use.

Korff and Arumbein<sup>6</sup> report recombination of the molecules in a counter filled entirely with ammonia. However, these authors report an increase in the background counting rate with use of this counter.

prophylaxis are given only in the most severe cases, and  
treatment is essential in the early stages of the disease.  
It is essential to use the most effective drugs available  
and to continue treatment until the patient is completely  
free of infection. In the case of severe infection, the  
use of antibiotics is essential. The use of antibiotics  
is essential in the treatment of severe infection. The use  
of antibiotics is essential in the treatment of severe  
infection. The use of antibiotics is essential in the  
treatment of severe infection. The use of antibiotics is  
essential in the treatment of severe infection.



## CHAPTER III

### DESCRIPTION OF EXPERIMENTAL APPARATUS

The Geiger tubes used for these experiments were designed for cosmic-ray research. The cathodes were of two-inch diameter brass tubing with  $1/32$  inch walls. The anodes were of 0.005 inch Kovar wire. The tubes had an overall length of twenty-four inches and a sensitive volume of about nineteen inches in length. Complete details of the construction of these counters and of the filling procedure are given in an article by Regener.<sup>7</sup>

The equipment used for measuring counter characteristics during the life tests is indicated in Figure 1. A source of Gamma radiation supplies a constant high rate of counts. The oscillograph was used to determine the values of  $V_s$  and  $V_c$  and to make visual observations of multiple counts. The scaling circuits were used to measure the slope of the plateau during the life tests of some of the counters.

The potential measurements are limited to an accuracy of about two percent by the measuring equipment. However, the value of  $V_c$  is somewhat arbitrary since multiple counts, high background and the characteristic of breaking in and out of a self-sustaining discharge make it difficult to repeat readings.

The Gamma source was calibrated at a large distance



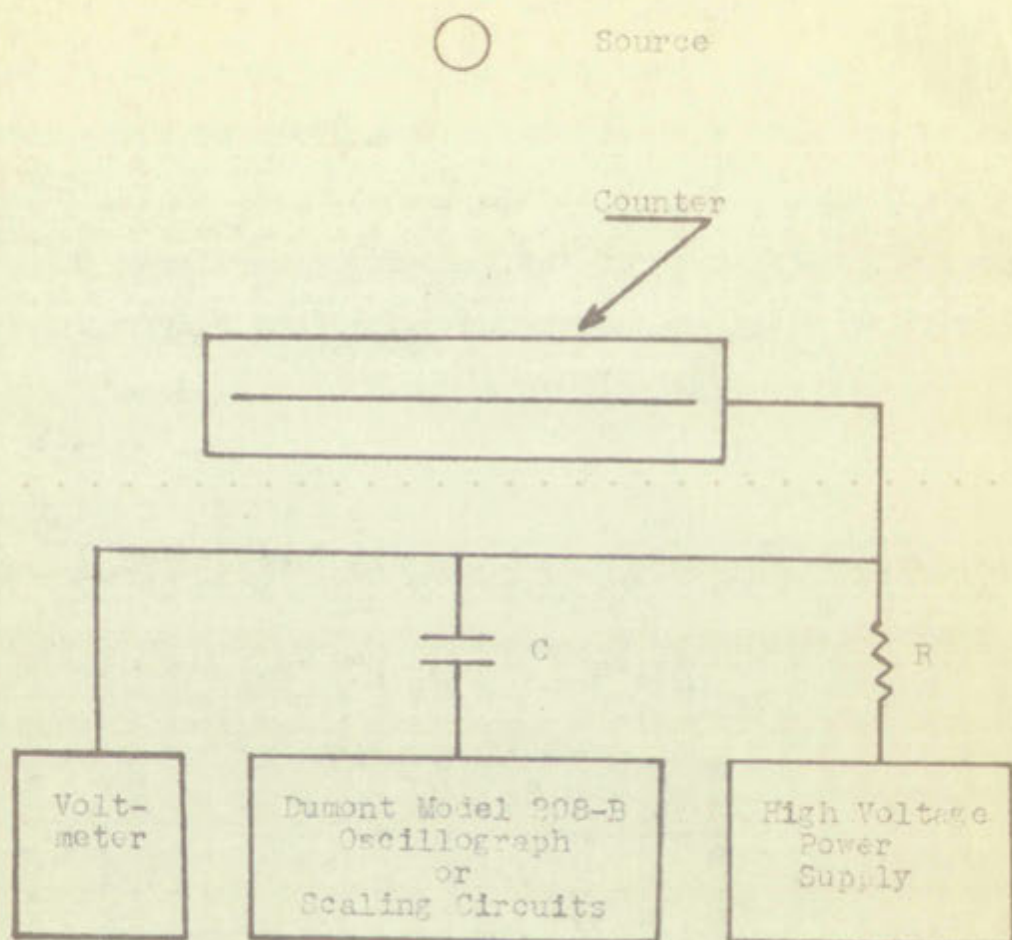


Fig. 1. Diagram of Experimental Apparatus. R is a one megohm resistor, C a 0.0005 Mfd. capacitor.

CHECK

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from the counter with the aid of scaling circuits and a mechanical recorder. The solid angle subtended by a Geiger counter at the Gamma source was calculated for the smallest distance actually used<sup>1</sup> and an estimate was made of the dead time of the counter. The calculated counting rate for the separation used throughout the life tests was 500 per second. These experiments were intended to compare lives of different fillings rather than to measure absolute life times accurately.

---

<sup>1</sup> A calculation of the solid angle subtended by a Geiger counter at a point source is given in the appendix.

From the counter with the aid of a certain amount of  
mechanical resources. The first was a quantity of a  
counter of the same kind, the second was a quantity of  
distance actually used, and an estimate of the time  
time of the counter. The third was counting rate for the  
operation used. These were the three main points of  
these experiments. The first was to determine the  
ellipses rather than to measure directly the length.

Reply.

---

I A calculation of the relative error in the  
value of the counter is given in the appendix.

## CHAPTER IV

### EXPERIMENTAL PROCEDURE AND RESULTS

#### A. General Procedure

Three sets of experiments will be described in this chapter. (1) Starting and continuous potentials were measured for different mixtures of gases. (2) Starting and continuous potentials were measured during the life of several counters. Measurements were made of the counting rate versus supply voltage during the life test of some of these counters. (3) A variation in starting potential with supply voltage was measured for "used" counters.

All tubes used were first cleaned, and a new center assembly was installed. The cathode surface was ground, polished, and cleaned with ethyl alcohol. After having been filled the tubes were given about a day for their characteristics to reach a constant value before beginning a life test. The initial values of starting and continuous potential were recorded and a visual check was made for multiple counts. In some cases the counting rate with respect to the supply voltage was recorded using the scaling circuits. The counter was then exposed to a counting rate of about five hundred per second by placing the Gamma source at a distance of 15 cm from the center line of the tube. The testing procedure described above was repeated at intervals

EXPERIMENTAL PROCEDURE

A. General Procedure

Three sets of experiments were conducted:

(1) Standard and experimental conditions were

assumed for all tests. The standard test

conditions were assumed to be the same as those

obtained in the standard test. The standard test

was assumed to be the same as those obtained in

(2) A variation in test conditions was assumed

was assumed for each test.

All tests were first-class, and a good

assembly was installed. The standard test

was assumed to be the same as those obtained in

the standard test. The standard test

was assumed to be the same as those obtained in

the standard test. The standard test

was assumed to be the same as those obtained in

the standard test. The standard test

was assumed to be the same as those obtained in

the standard test. The standard test

was assumed to be the same as those obtained in

the standard test. The standard test

was assumed to be the same as those obtained in

the standard test. The standard test



of 20 hours or  $0.36 \times 10^8$  counts during the life test. The average pulse height observed on the oscillograph was kept about the same for all the tests by adjusting the supply voltage. The average pulse had an amplitude of eight volts with the equipment used.

The starting and continuous potentials have been defined in Chapter I. After a counter has been "used" however, these potentials become somewhat arbitrary. The continuous potential will be further specified for the data recorded in this thesis as the potential at which pulses are seen along the entire oscillograph trace. In some cases this condition persists only momentarily at intervals of about a second. It may be necessary to increase the potential by as much as one hundred volts before the condition of continuous counting becomes permanent. The starting potential was observed in most cases after a counter had been counting at its background rate for a few minutes at the same potential at which the test was being conducted.

Some measurements were made of starting and continuous potentials for different percentages of quenching gas. Final readings were taken about forty-five minutes after filling since further changes in characteristics with time were about three percent as observed for several fillings. Measurements of starting and of continuous potentials were made for mixtures of alcohol with argon, ammonia with argon,



and ammonia with nitrogen. The observed values of these potentials together with the gas pressures are listed in Tables I, II and III.

#### B. Tests With Alcohol-Argon Fillings

Two counters, each filled with pressures of 10 mm Hg of alcohol vapor and 90 mm Hg of argon were given life tests as described in the preceding section. Values of  $V_s$  and  $V_c$  versus total number of counts for one of these counters are plotted in Figure 2. The other counter gave essentially the same results with some values differing by a few volts from those plotted. Figure 3 shows the corresponding curves for a counter filled with pressures of 15 mm Hg of alcohol and 85 mm Hg of argon.

The values circled in Figures 2 and 3 were taken at the end of the tests after the counters had been disconnected from the supply voltage for a few days. In subsequent life tests the counters were given "rest" periods of one day during which the values of  $V_s$  and  $V_c$  were checked several times. The value of  $V_s$  generally increased and that of  $V_c$  generally decreased during these "rest" periods becoming nearly constant with time after one day.

Immediately after counting for some time at a normal background rate with an overvoltage of about one hundred volts, the measured starting potential was higher than after



TABLE I

STARTING AND CONTINUOUS POTENTIALS  
FOR COUNTERS FILLED WITH MIXTURES OF ALCOHOL AND ARGON

Alcohol Vapor Pressure (mm Hg)	Argon Pressure (mm Hg)	Starting Potential Vs (Volts)	Continuous Potential Vc (Volts)	Vc Minus Vs (Volts)
5	95	800	1125	325
10	90	1025	1500	475
15	85	1190	1800	610
20	80	1310	2000	690
5	145	850	1150	300
15	135	1275	1975	700

STATISTICAL DATA FOR COURTESY KILLED WITH A RIFLE AT 1000 YARDS

Altitude (meters)	Barometric Pressure (mm Hg)	Alveolar Pressure (mm Hg)	Partial Pressure (mm Hg)	Alveolar Oxygen Pressure (mm Hg)
0	760	760	100	660
10	750	750	100	650
20	740	740	100	640
30	730	730	100	630
40	720	720	100	620
50	710	710	100	610
60	700	700	100	600
70	690	690	100	590
80	680	680	100	580
90	670	670	100	570
100	660	660	100	560

TABLE II

INITIAL STARTING AND CONTINUOUS POTENTIALS  
FOR COUNTERS FILLED WITH MIXTURES OF AMMONIA AND ARGON

Ammonia Pressure (mm Hg)	Argon Pressure (mm Hg)	Starting Potential (Volts)	Continuous Potential (Volts)	Vc Minus Vs (Volts)
100	0	2100	2240	140
60	40	1525	1715	190
50	50	1445	1635	190
40	60	1330	1525	195
30	70	1245	1440	195
20	80	1105	1325	220
10	90	875	960	85
100	50	1715	1960	245
30	120	1310	1545	235
20	130	1150	1430	280
20	180	1175	1485	310

INITIAL AND FINAL TEMPERATURES OF THE

FOR COUNTRY FIELD NO. 10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-29-30-31-32-33-34-35-36-37-38-39-40-41-42-43-44-45-46-47-48-49-50-51-52-53-54-55-56-57-58-59-60-61-62-63-64-65-66-67-68-69-70-71-72-73-74-75-76-77-78-79-80-81-82-83-84-85-86-87-88-89-90-91-92-93-94-95-96-97-98-99-100

Initial Temperature (°C)	Final Temperature (°C)	Initial Pressure (mm Hg)	Final Pressure (mm Hg)
100	100	100	100
90	90	90	90
80	80	80	80
70	70	70	70
60	60	60	60
50	50	50	50
40	40	40	40
30	30	30	30
20	20	20	20
10	10	10	10
100	100	100	100
90	90	90	90
80	80	80	80
70	70	70	70
60	60	60	60
50	50	50	50
40	40	40	40
30	30	30	30
20	20	20	20
10	10	10	10



TABLE III

STARTING AND CONTINUOUS POTENTIALS FOR  
COUNTERS FILLED WITH MIXTURES OF AMMONIA AND NITROGEN

Ammonia Pressure (mm Hg)	Nitrogen Pressure (mm Hg)	Starting Potential (Volts)	Continuous Potential (Volts)	Vo Minus Vs (Volts)
90	10	1400	1435	35
80	20	1430	1500	70
60	40	1485	1555	70
50	50	1510	1575	65
20	80	1715	1790	75
130	20	1715	1780	65

STATISTICAL DATA FOR THE YEAR 1950

COUNTY OF ...

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Year	Population	Area (sq. mi.)	Population Density (per sq. mi.)	Percentage of Total Population
1940	1000	100	10	10
1945	1500	100	15	15
1950	2000	100	20	20
1955	2500	100	25	25
1960	3000	100	30	30
1965	3500	100	35	35
1970	4000	100	40	40
1975	4500	100	45	45
1980	5000	100	50	50
1985	5500	100	55	55
1990	6000	100	60	60
1995	6500	100	65	65
2000	7000	100	70	70

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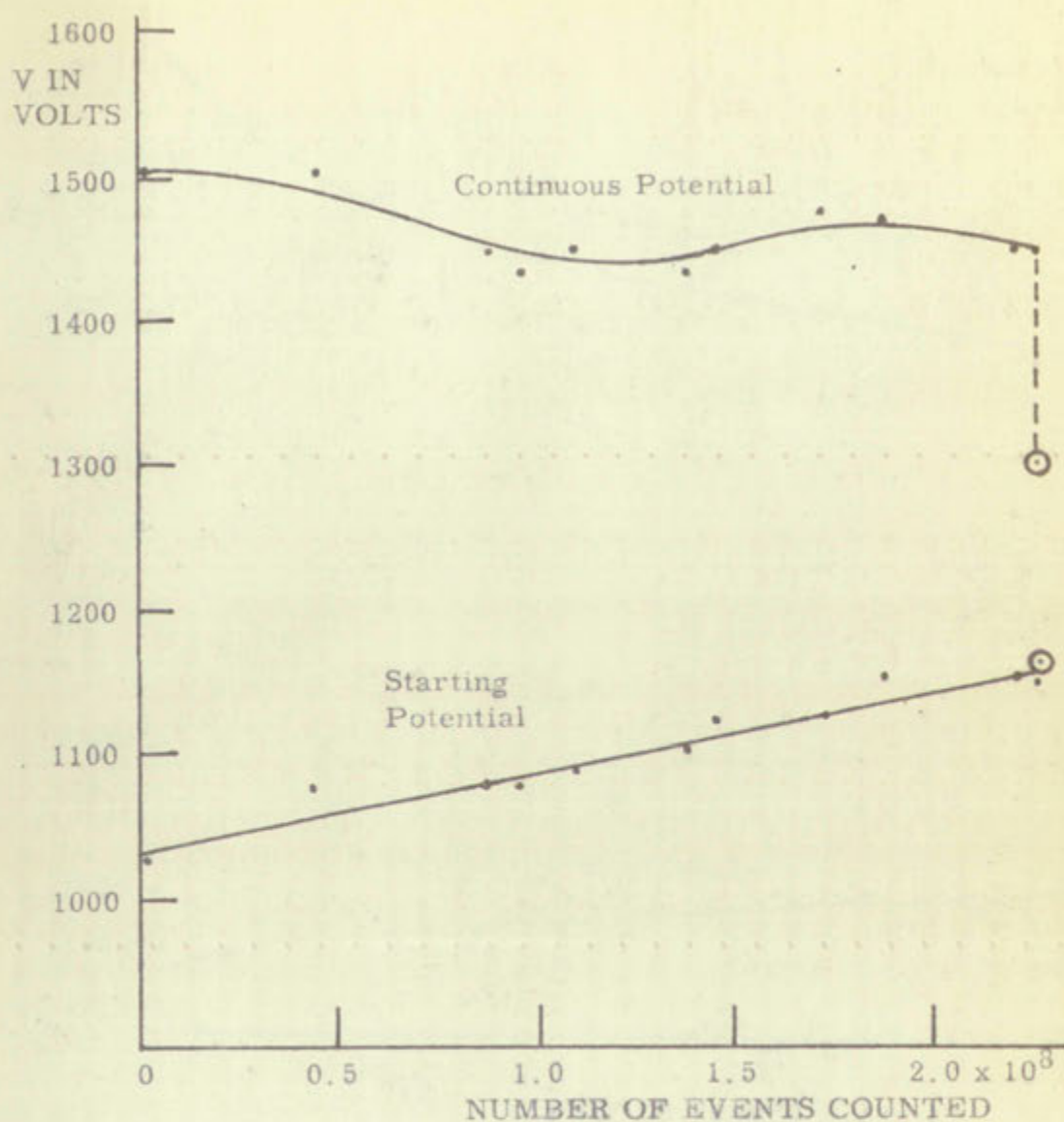


Fig. 2. Starting and continuous potentials during the life test of a counter filled with 10 mm Hg of alcohol and 90 mm Hg of argon. Circled values were taken after a few days rest.

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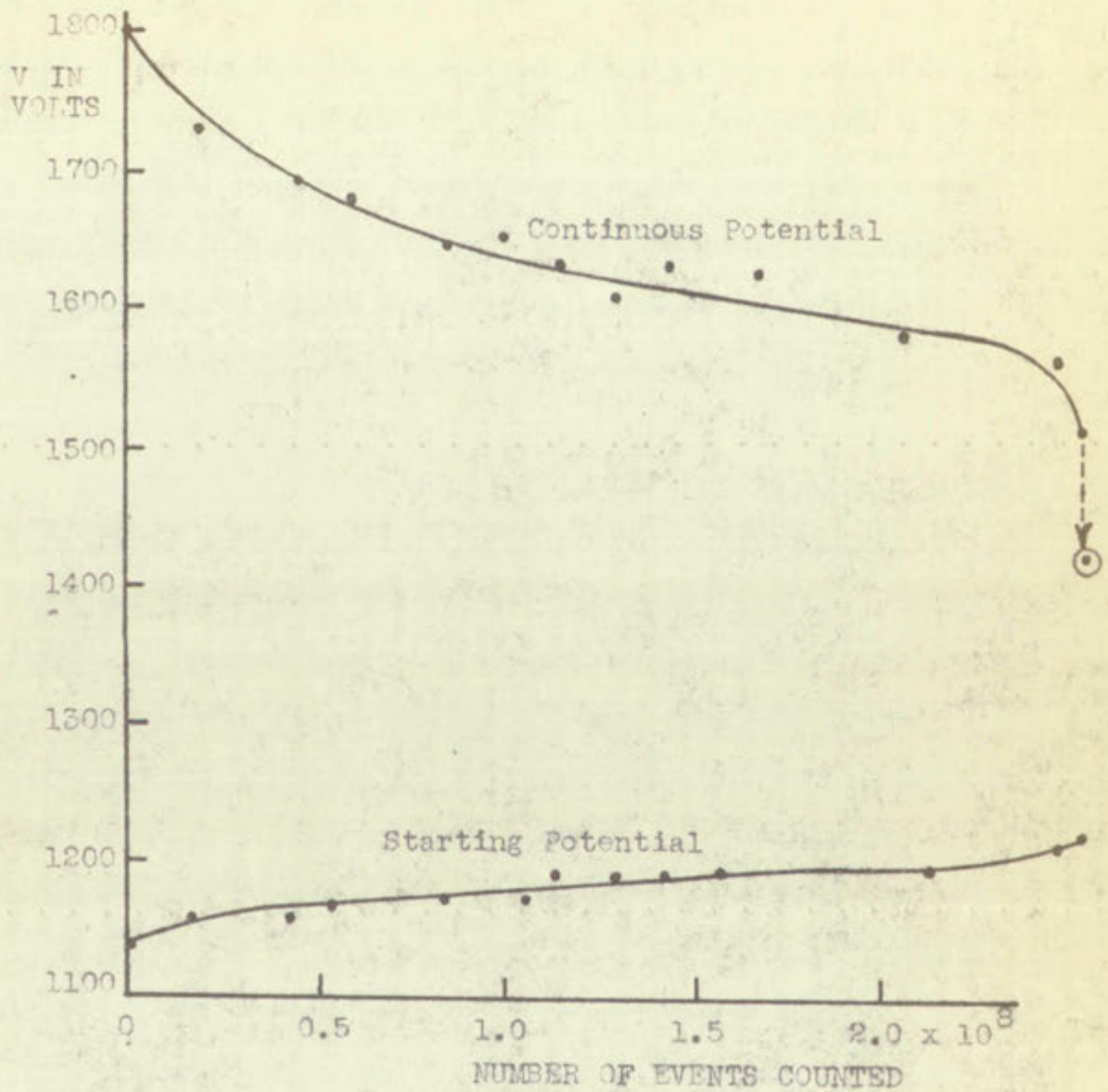


Fig. 3. Starting and continuous potentials during the life test of a counter filled with 15 mm Hg of alcohol and 85 mm Hg of argon.

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a subsequent rest of only a few minutes. This rapid variation in  $V_s$  was not observed for "new" counters. It increased with the age of the alcohol counters to about one hundred volts in some cases. Several counters which had been "worn out" while counting cosmic rays gave the same result. When the supply voltage was increased from zero to about one hundred volts overvoltage, the amplitude of the pulses decreased to about fifty percent of their initial value within a few minutes.

Figure 4 is a plot of starting potential versus supply voltage for a counter filled with 10 mm Hg of alcohol and 90 mm Hg of argon which had counted  $2 \times 10^8$  counts. After a "rest" of several hours, the supply voltage was increased until counts were first observed. This gave the lowest value of starting potential in Figure 4. The supply voltage (V) was then increased to 1150 volts and left constant for two minutes. V was then quickly lowered and again increased until counts were first observed. The new value of starting potential thus obtained was plotted above the supply voltage  $V = 1150$  volts. The same procedure was repeated for  $V = 1200$  to  $V = 1300$  volts.

The values plotted for  $V_s$  in subsequent tests were taken after a counter had counted at its normal background rate for a few minutes with the same overvoltage as that being used in the life test. This procedure greatly reduced

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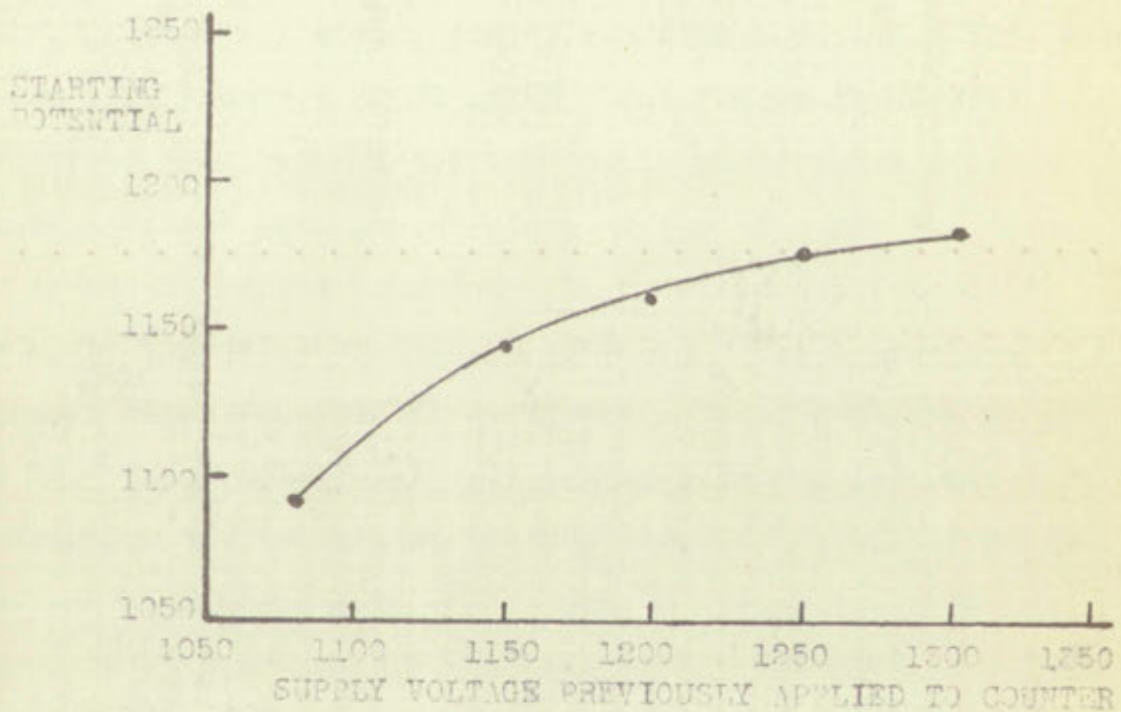


Fig. 4. Starting potential versus supply voltage for a counter filled with 10 mm Hg of alcohol and 90 mm Hg of argon after a total of  $2 \times 10^6$  counts.

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the fluctuation in successive readings of  $V_s$  for a counter.

A counter was supplied with a constant alcohol vapor pressure equal to 12.2 mm Hg by connecting the counter to a reservoir of liquid alcohol as indicated in Figure 5. The reservoir of alcohol was submerged in a bath of dry ice and ethyl alcohol while the counter was evacuated and filled with argon to a pressure of 88 mm Hg. The reservoir was then submerged in a bath of cracked ice in water while alcohol vapor diffused into the counter. After 48 hours the values of  $V_s$  and  $V_c$  became constant with time and approximately equal to the values indicated in Table I for a pressure of alcohol vapor equal to 12 mm Hg and a total pressure of 100 mm Hg. The alcohol reservoir was kept in the ice bath throughout the subsequent testing period. Values of starting and continuous potential observed during the life test for this counter are given in Figure 6. The circled values indicate readings taken after one day's "rest" following the life test. No further change in these potentials was observed after an additional day.

The changes in physical properties of a counter with use listed in Chapter II, Section B can be classified as changes in the filling mixture and changes in the condition of the electrode surfaces. In the experiment just described, the pressures of the major filling components remained constant throughout the life test. Further tests were performed



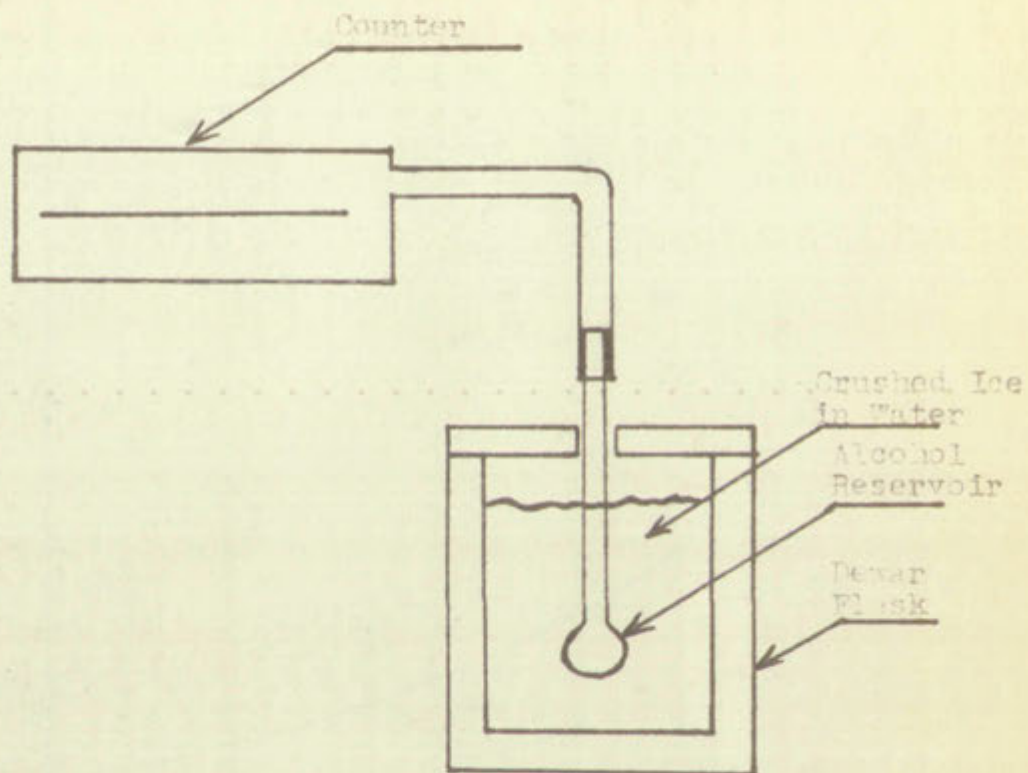


Fig. 5. Diagram of counter supplied with a constant alcohol vapor pressure.

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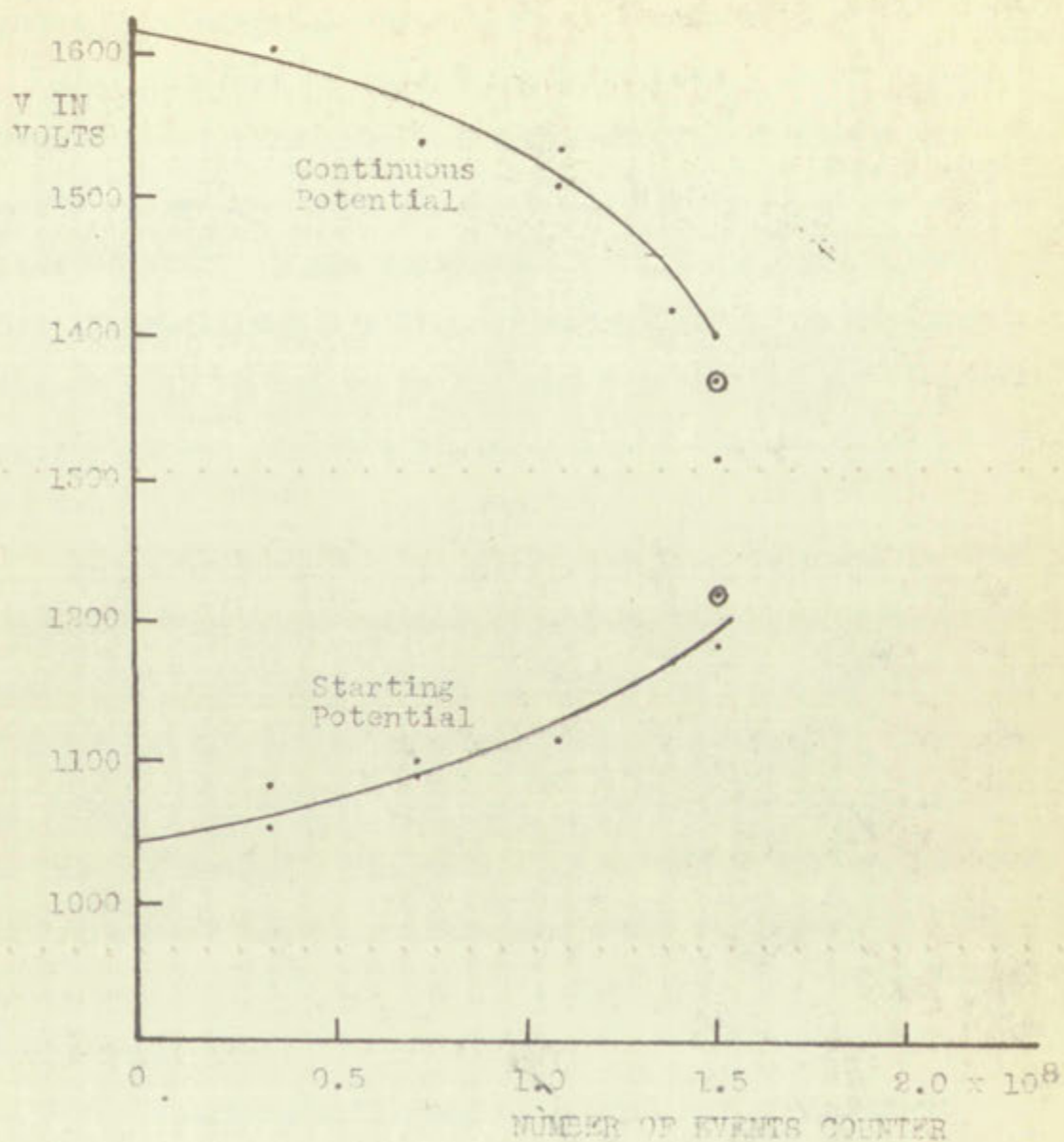


Fig. 6. Starting and continuous potentials during the life test of a counter supplied with a constant vapor pressure of alcohol equal to 12.2 mm Hg. Total pressure of argon and alcohol equal to 100 mm Hg.

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to determine the relation between physical changes and the changes in electrical characteristics observed.

Two counters which had counted about  $2 \times 10^8$  counts, Figure 2, were evacuated and refilled with 10 mm Hg of alcohol vapor and 90 mm Hg of argon. Table IV gives the values of starting and continuous potentials before and after the refilling. A drop of approximately thirty-five volts in both  $V_s$  and  $V_c$  is indicated as due to the change in composition and pressure of the filling mixture during the life test.

New center assemblies were installed in these two counters. They were again filled with the same mixture of gases. The resulting values of  $V_s$  and  $V_c$  were nearly equal to the original values when the tubes were "new." The number of spurious counts was also comparable to that found for "new" counters. The values of  $V_s$  and  $V_c$  during the life test of one of these counters are shown in Figure 7. The values observed for the other counter were the same within experimental error.

Two counters which had been worn out at a normal cosmic-ray and background counting rate gave comparable results when new center wires were installed. Figure 8 gives values of  $V_s$  and  $V_c$  during the life test of one of these counters after refilling. The history of these two counters was not known except that they had been filled

to determine the relative error in the  
changes in electrical conductivity  
Two counters with two channels each

Figure 2. Schematic diagram of the  
absorber vapor, which is at a pressure of 10 mm Hg  
vices of branching of the vapor, which is at a  
after the reformation. A series of experiments  
voles in order to determine the relative error in  
concentration and pressure of the vapor. There are  
the best.

New counter apparatus with two channels  
counters. They were used for the  
genes. The results of the experiments are given  
to the original values when the vapor is at a  
but at different times was also measured by means  
for new counters. The results of the experiments  
best of one of the channels of the counter  
values observed for the other channel are given  
experimental error.

The counters with two channels each  
accuracy and low error in the  
results when new counters were used  
Given values of the relative error in  
these counters. The results of the experiments  
counters was not more than 1%.

TABLE IV

VALUES OF STARTING AND CONTINUOUS  
POTENTIAL FOR TWO COUNTERS BEFORE AND AFTER  
REFILLING WITH 10 MM HG OF ALCOHOL AND 90 MM HG OF ARGON

Counter	Starting Potential (Volts)		Continuous Potential (Volts)	
	Before Refilling	After Refilling	Before Refilling	After Refilling
1	1120	1070	1260	1225
2	1155	1120	1295	1260

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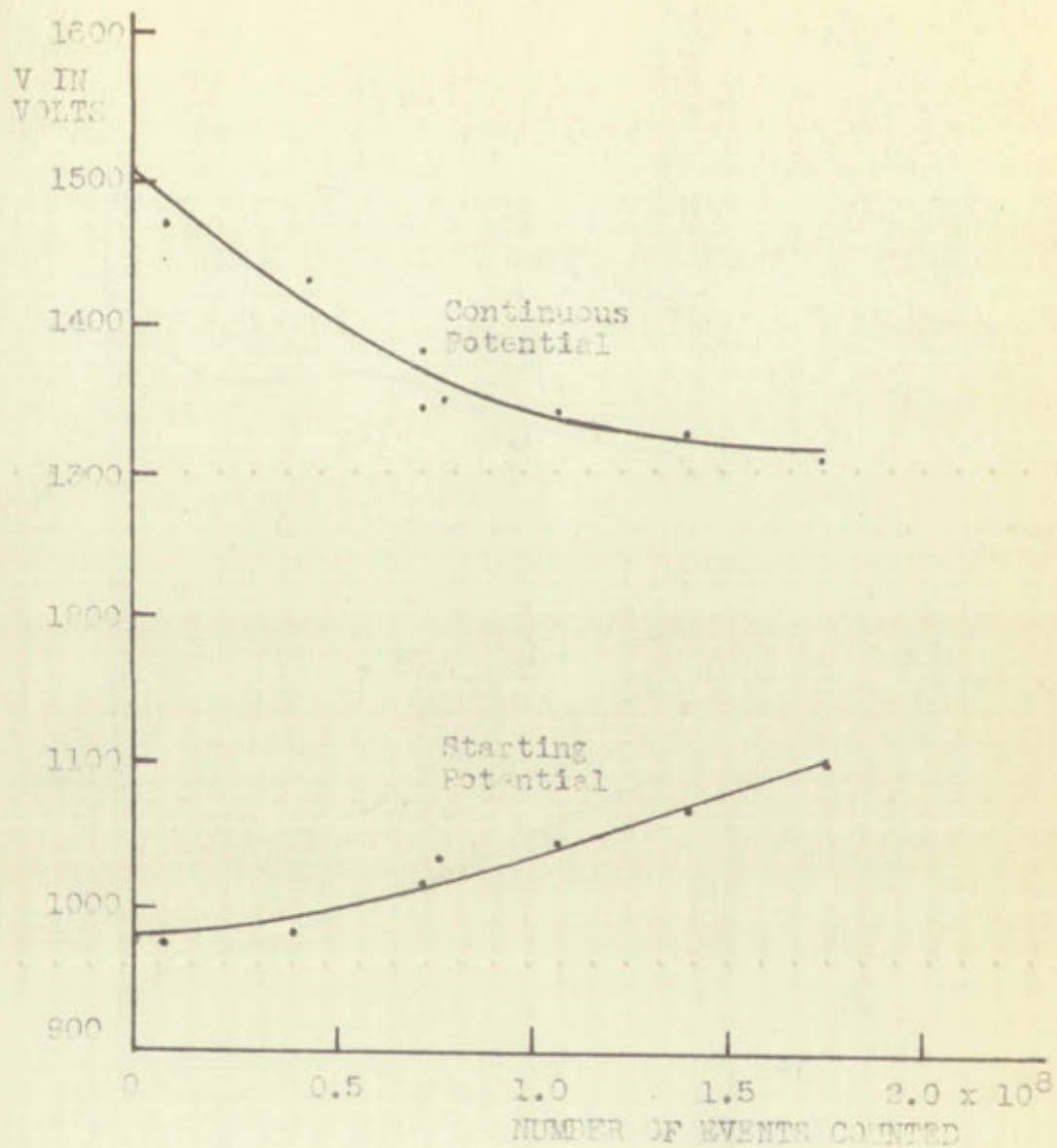


Fig. 7. Starting and continuous potentials during the life test of a used counter with a new center wire. Counter refilled with 10 mm Hg of alcohol and 80 mm Hg of argon.

over a year before the present...  
The change in...  
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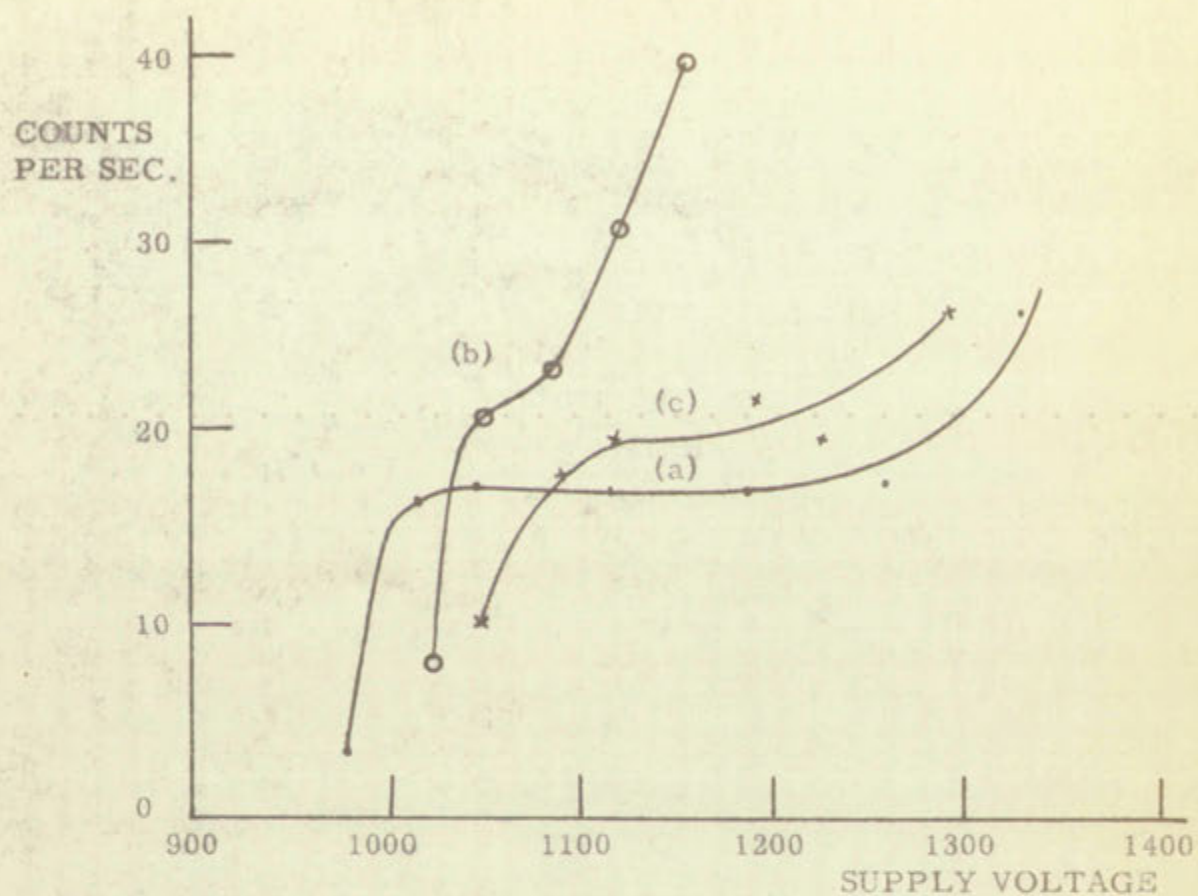


Fig. 9. Plateau curves taken during the life test of a counter filled with 10 mm Hg of alcohol and 90 mm Hg of argon. Curve (a) was taken after a life of  $0.4 \times 10^8$  counts. Curves (b) and (c) were taken after a life of  $1.4 \times 10^8$  counts. Curve (b) was taken immediately after exposing the counter to a high counting rate. Curve (c) was taken after a "rest" of one day.

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$2.6 \times 10^8$  counts the plateau had disappeared due to the high background count. No significant recovery was observed after several days "rest."

The third ammonia-argon counter tested showed an increase in both starting and continuous potentials of about forty volts per  $10^8$  counts. This increase was apparently due to an air leak since the potentials continued to increase at the same time rate with no counts being recorded. Although this counter counted a total of  $3.2 \times 10^8$  counts, there was no noticeable increase in its random background counting rate. There was an increase in the number of multiple counts toward the end of the test. An increase in multiple counts was observed whenever a few mm Hg pressure of air or oxygen was added to the filling mixture of a counter.

#### D. Discussion of Results

##### (1) Alcohol-Argon Fillings

All the alcohol-argon fillings tested had a useful life of approximately  $2 \times 10^8$  counts. The slightly longer lives indicated by Figures 2 and 3 were primarily due to the method of measuring the starting potential for those counters and because no rest periods were given during the life tests. Since no substantial difference in life was found for the counter supplied with constant alcohol vapor

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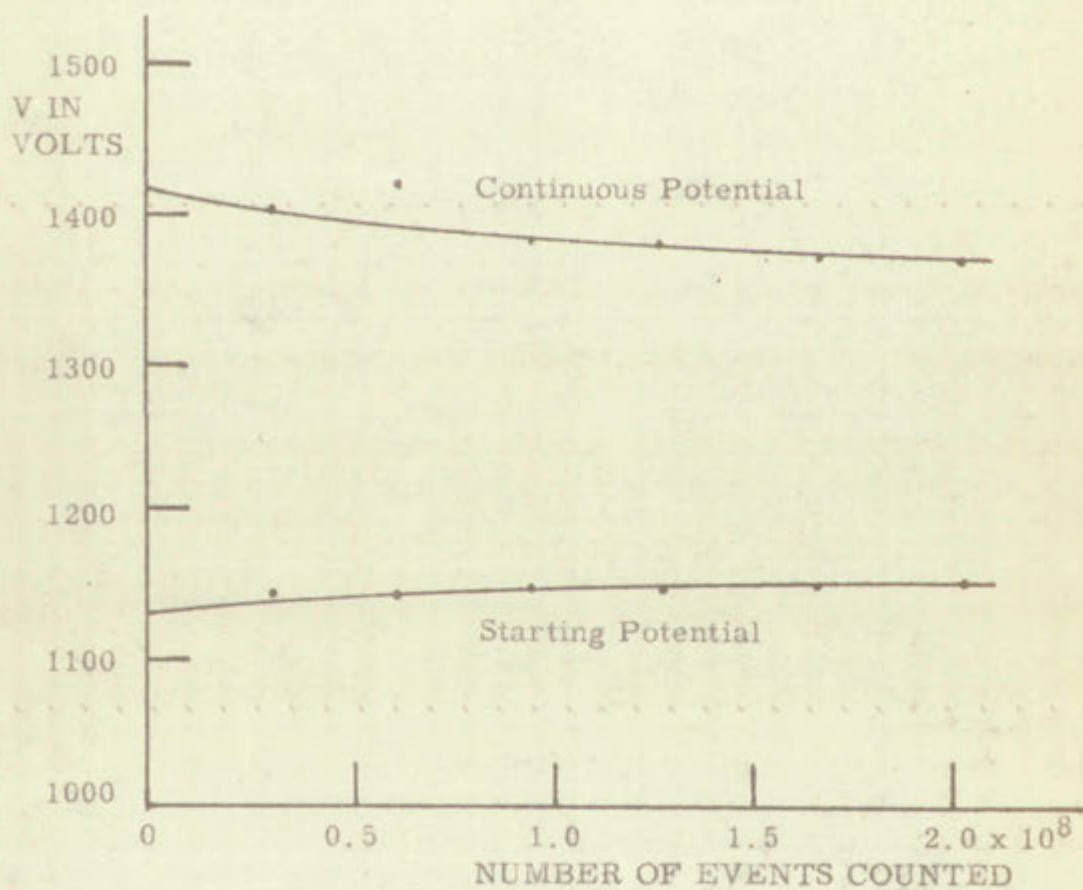


Fig. 10. Starting and continuous potentials during the life test of a counter filled with 20 mm Hg of ammonia and 130 mm Hg of argon.

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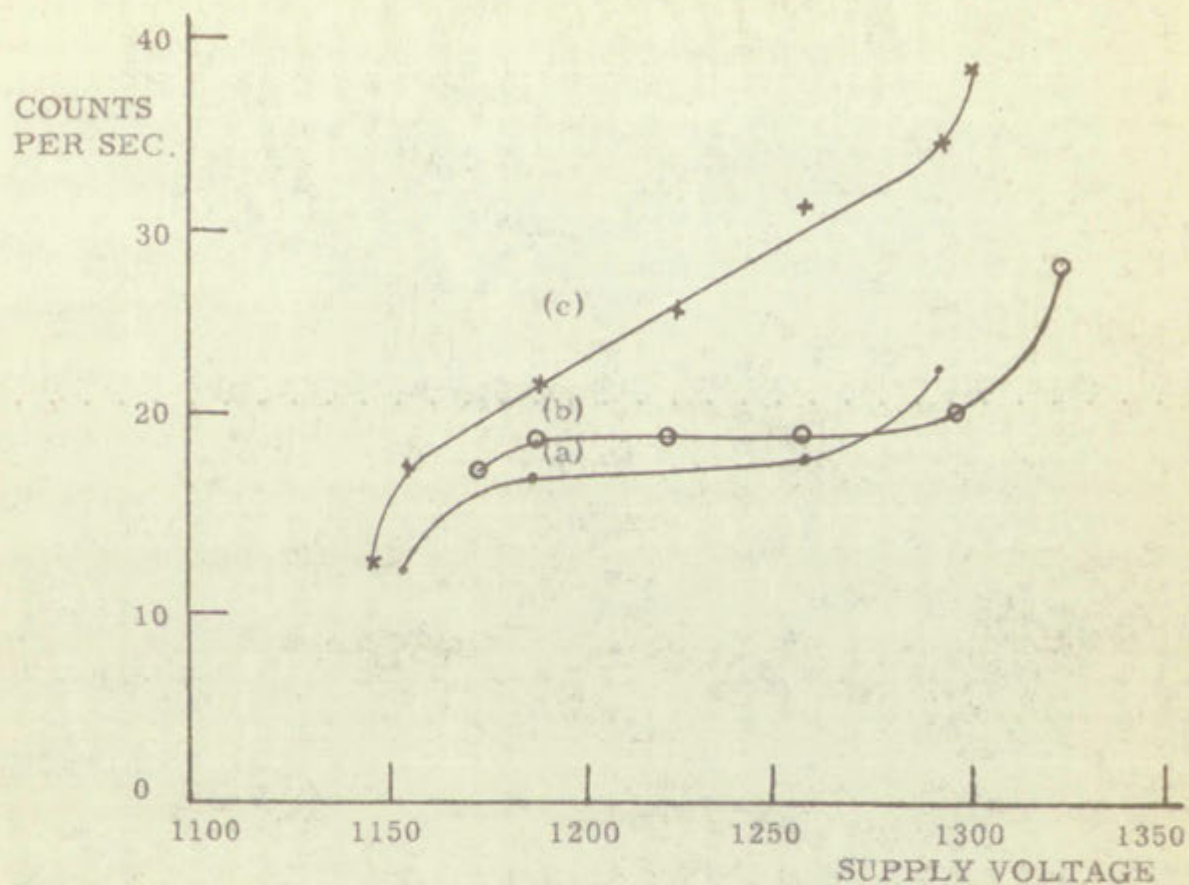


Fig. 11. Plateau curves taken during the life test of a counter filled with 20 mm Hg of ammonia and 130 mm Hg of argon. Curve (a) taken after  $0.94 \times 10^8$  counts, curve (b) after  $1.6 \times 10^8$  counts and curve (c) after  $2 \times 10^8$  counts.

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$2.6 \times 10^8$  counts the plateau had disappeared due to the high background count. No significant recovery was observed after several days "rest."

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pressure, the decrease in plateau for these alcohol-argon counters was not primarily due to the normal decrease in number of alcohol molecules.

The result of refilling two counters with the same mixture of gases as they contained originally was a decrease both in starting and in continuous potentials so that the length of the plateau remained nearly the same. This result indicates an increase in total pressure with use as observed by Spatz.<sup>8</sup>

The results found upon replacing the center wire show its condition to be a major life factor for the tubes tested in these experiments. These last results agree with those reported by Hagen and Loughridge<sup>9</sup> who used the same filling mixtures and tube construction. Shepard<sup>10</sup> reported nearly original characteristics for some "used" counters after electrically glowing the center wire for ten seconds. These counters contained a tungsten center wire in an atmosphere of alcohol and argon.

The increase of the starting potential after an increase in the supply voltage could be due to an insulating deposit on the electrodes as mentioned briefly by Loeb.<sup>11</sup> Korff and Present<sup>5</sup> list a hydrocarbon deposit on the electrodes as a cause for failure of organic vapor counters. It is difficult, however, to trace the decrease in continuous potential to the same cause. A more plausible explanation

pressure, the decrease in pressure for these alcohol-  
 containers was not primarily due to the normal decrease in  
 number of alcohol molecules.

The result of retilling two containers with the same  
 mixture of gases as they contained originally was a decrease  
 both in starting and in continuous potentials as that the  
 length of the plates remained nearly the same. This re-  
 sult indicates an increase in total pressure with no  
 observed by Spatz.<sup>6</sup>

The results found upon replacing the carbon wire with  
 its condition to be a major life factor for the cases tested  
 in these experiments. These last results agree with those  
 reported by Hagen and Langmuir,<sup>7</sup> who used the same efflu-  
 ing mixture and tube construction. Spatz<sup>10</sup> reported  
 nearly original characteristics for some "used" cathodes  
 after electrically glowing the carbon wire for ten seconds.  
 These cathodes contained a tungsten center wire in an  
 atmosphere of alcohol and nitrogen.

The increase of the starting potential after an in-  
 crease in the supply voltage could be due to an increasing  
 deposit on the electrodes or a reduction in the area of the  
 cathode and present list a hydronium deposit on the  
 electrodes as a cause for failure of organic vapor cathodes.  
 It is difficult, however, to trace the decrease in continuous  
 potential to the same cause. A more plausible explanation

for the decrease in the continuous potential would be pitting of the center wire due to overvoltage.

Spatz<sup>8</sup> reports lives of  $10^9$  to  $10^{10}$  counts in a series of life tests on alcohol-argon counters. He used an overvoltage of sixty volts. Thus the lives found in these experiments are low by a factor of ten or more from the values predicted by Spatz. This discrepancy could be due to pitting of the center wire at the overvoltage (150 volts) used in these experiments. Also, the counters were operated at their continuous potentials for a few seconds while  $V_c$  was measured.

#### (2) Ammonia-Argon Fillings

The ammonia-argon filled counters showed an essentially constant starting potential for at least  $2 \times 10^8$  counts. The continuous potential was also nearly constant but showed more fluctuation between readings. The overvoltage used during the tests was about one hundred volts. Values of  $V_c$  were initially lower and the total pressure higher than for the alcohol counters, so that one would expect less damage to the center wire during the life test.

These counters showed an increase in background counting rate beginning after about  $2 \times 10^8$  counts. No substantial decrease in this background rate was observed after a rest as observed by Korff and Krumbein<sup>6</sup> for ammonia filled counters.

for the device in the various stages of its development  
and the manner in which it is used.  
The device is a simple one, consisting of a  
series of thin plates, each of which is  
an oval shape, and is held together by  
these experiments and has a length of  
the width of the plates. The plates are  
due to the fact that the plates are  
voltage used in a series of experiments  
operated at their own rate, and the  
while it was reported.

(2) *Amplitude Modulation*

The amplitude modulation is a simple  
series of plates, each of which is  
counts. The amplitude modulation is a  
but showed that the amplitude modulation  
voltage used during the time of the  
values of the amplitude modulation  
higher than for the amplitude modulation  
expect that the amplitude modulation  
These counts are as follows:  
counting rate for the amplitude modulation  
essential device in the amplitude modulation  
after a test of the amplitude modulation  
filled counter.

A counter which had a small leak did not show a noticeable increase in random background counting rate after  $3.2 \times 10^8$  counts. It did show an increase in multiple counts, apparently due to the contamination with air.

A counter which had a well back did not show a  
noticeable increase in counts per minute during the  
after 3.2 x 10<sup>8</sup> counts. It did show an increase in count-  
ing rate, especially due to the contribution of the air.

CHAPTER V

SUMMARY OF THESIS AND CONCLUSIONS

Measured values of starting and continuous potentials for several mixtures of argon with alcohol, argon with ammonia, and nitrogen with ammonia are listed.

Graphs of starting and continuous potentials recorded during the life tests of counters filled with argon and alcohol, and with argon and ammonia are given. Graphs are also plotted which show changes in the plateau with use of the counters.

The similar life characteristics obtained with the counter supplied with a constant pressure of alcohol vapor compared with those filled in the normal way shows that the major factor limiting the life of these counters is not a lack of quenching vapor. Tests in which "used" tubes were refilled and tests in which new center assemblies were installed indicate that the condition of the center wire is a major factor resulting in the failure of alcohol-argon counters.

Replacement of the center wire after  $2 \times 10^8$  counts in four counters resulted in characteristics comparable to new tubes. It seems probable that the worn out condition was due largely to a deposit of hydrocarbons on the electrodes and to pitting of the center electrode. It

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...Re ...  
...in ...  
...new ...  
...was ...  
...electrodes ...



cannot be concluded from these limited data that replacing the center wire would result in counters with uniform characteristics. However, the length of the plateau can be substantially increased by replacing the center wire and the gas mixture. The condition of the cathode had a minor effect on counter characteristics after a few times  $10^8$  counts. These conclusions assume no contamination of the cathode surface while replacing the center wire.

A longer life can probably be obtained by using as low an overvoltage as possible and avoiding potentials as high as  $V_c$ .

The ammonia-argon fillings tested became useless because of an increase in background counting rate.

In conclusion I wish to acknowledge the advice and assistance received from Dr. V. H. Regener during the course of these experiments. Valuable aid was also given by Mr. Munson Thorpe in the preparation and filling of the counters.

cannot be concluded from these findings that the

the center wire was to be in contact with the

anode. However, the fact that the center wire

substantially increases the rate of reaction

has been noted. The reaction is of the order

of a counter electrochemical reaction. The

these conditions are similar to those of the

surface while varying the center wire.

A low rate of reaction is observed at

low an overpotential at potentials of the order

of 100 mV.

The mechanism of the reaction is not yet

known. It is probable that the reaction is

of the order of a counter electrochemical

reaction. The reaction is of the order

of a counter electrochemical reaction. The

reaction is of the order of a counter

BIBLIOGRAPHY

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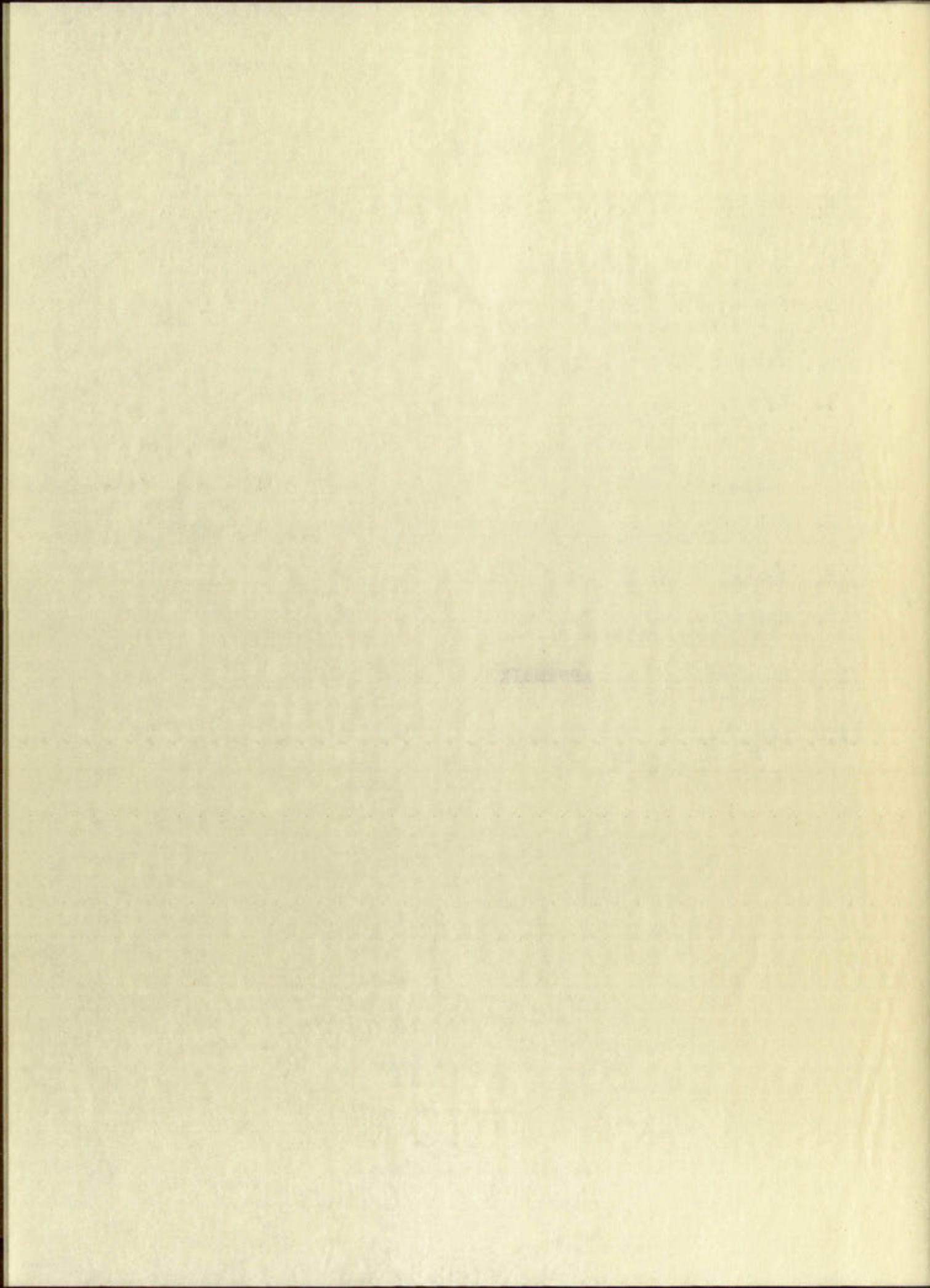
## BIBLIOGRAPHY

1. Ehmert, Alfred and Trost, Adolf, Zeitschrift fur Physik, 100, 553 (1936).
2. Trost, Adolf, Zeitschrift fur Physik, 105, 399 (1937).
3. Montgomery, G. G. and Montgomery, D. D. Physical Review, 57, 1030-40 (1940).
4. Stever, H. G. Physical Review, 61, 38-52 (1942).
5. Korff, S. A. and Present, R. D. Physical Review, 65, 274-282 (1944).
6. Korff, S. A. and Krumbein, A. D. Physical Review, 76, 1412 (1949).
7. Regener, V. H. Review of Scientific Instruments, 18, 267-270 (1947).
8. Spatz, W. D. B. Physical Review, 64, 236 (1943).
9. Hagen, G. E. and Loughridge, D. E. Physical Review, 73, 1131 (1948).
10. Shepard, L. Review of Scientific Instruments, 20, 217-218 (1949).
11. Loeb, L. B. "Fundamental Processes of Electrical Discharges in Gases," New York: John Wiley and Sons (1939).

REFERENCES

1. Kramet, J. H. and J. H. ... 1951 (1952)
2. ... 1951 (1952)
3. ... 1951 (1952)
4. ... 1951 (1952)
5. ... 1951 (1952)
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## APPENDIX



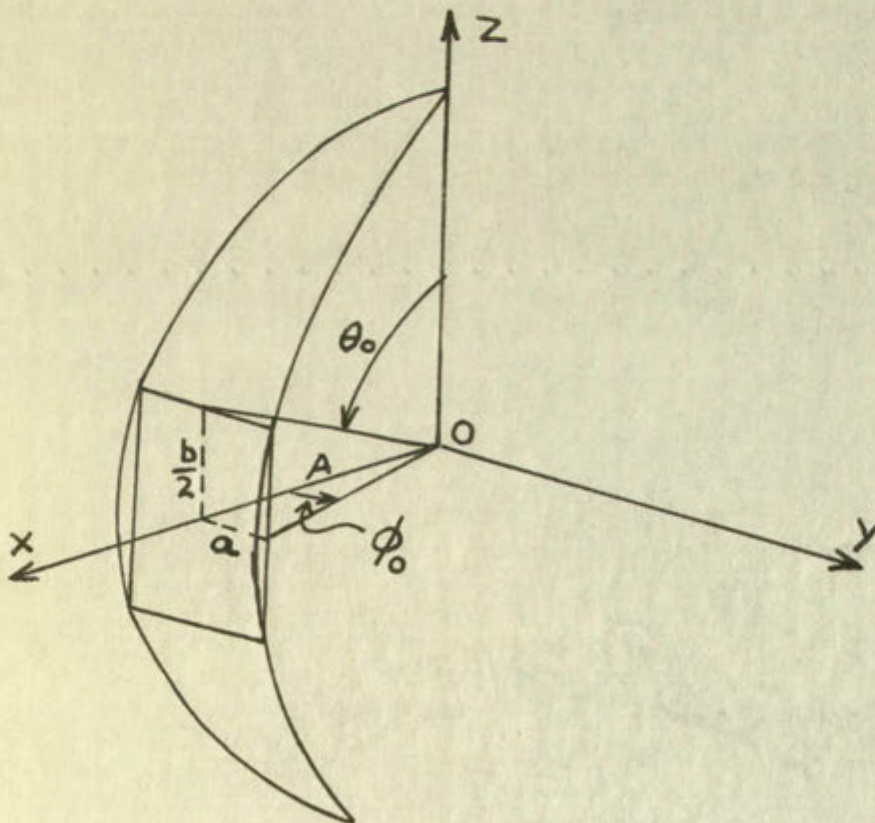


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### APPENDIX

The solid angle subtended by a Geiger counter at a point source was calculated using spherical polar coordinates. The solid angle subtended by the cylindrical counter is approximately equal to that subtended by a rectangle of area  $2ab$  where  $a$  is the radius and  $b$  the length of the sensitive volume.

As indicated in the following diagram, the rectangle is oriented perpendicular to the  $x$  axis. The source is located at the origin. The distance from the origin to the center of the rectangle is  $A$ .





The expression

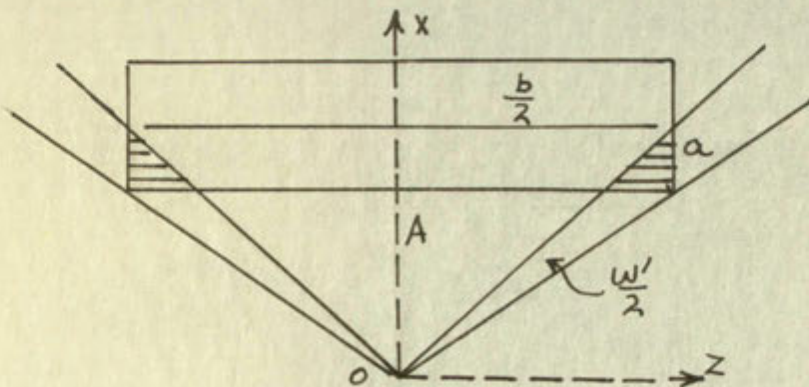
$$\omega = 4 \int_0^{\phi_0} d\phi \int_{\theta_0}^{\frac{\pi}{2}} d\theta \sin \theta \quad (1)$$

where  $\cos \theta_0 = \frac{b}{2R} = \frac{b}{2\sqrt{\frac{b^2}{4} + A^2}}$  and  $\tan \phi_0 = \frac{a}{A}$

gives the solid angle subtended by the rectangle within 0.5 percent for the values of  $a$ ,  $b$  and  $A$  in the present experiments. Using the above values for  $\theta_0$  and  $\phi_0$  in equation (1) gives for the solid angle subtended by the rectangle at the origin:

$$\omega = \frac{2b}{\sqrt{A^2 + \frac{b^2}{4}}} \tan^{-1} \frac{a}{A} \quad (2)$$

The sensitive volume of a counter extends beyond the ends of the rectangle as indicated in the following diagram where the counter is drawn as a cylinder.



The expression

$$W = \frac{1}{2} \rho g \left( \frac{b^2}{\tan \alpha} - \frac{b^2}{\tan \beta} \right)$$

$$\cos \theta = \frac{b}{R} = \frac{b}{\frac{b}{\tan \alpha} + A}$$

Given the soil angle  $\alpha$  and the weight  $W$ , it is possible to find the value of  $A$  which gives the required result. Being the horizontal distance  $A$  and  $N$  is known. Given for the soil angle  $\alpha$  and the weight  $W$ , it is possible to find the value of  $A$ .

$$W = \frac{1}{2} \rho g \left( \frac{b^2}{\tan \alpha} - \frac{b^2}{\tan \beta} \right)$$

The sensitive value of  $A$  is constant and is the same for all values of  $W$ . Where the distance is constant.



The solid angle subtended by the shaded volume is very nearly:

$$\omega' = \frac{\pi a^2 b}{2 \left( A^2 + \frac{b^2}{4} \right)^{\frac{3}{2}}}$$

This correction is approximately 5 percent of the total solid angle.

The final expression for  $\omega$  is

$$\omega_f = \frac{2b}{\sqrt{A^2 + \frac{b^2}{4}}} \tan^{-1} \frac{a}{A} + \frac{\pi a^2 b}{2 \left( A^2 + \frac{b^2}{4} \right)^{\frac{3}{2}}}$$

This is within one percent of the correct value for  $a =$   
one inch,  $b = 19$  inches and  $A = 15$  cm.

The solid angle subtended by the small sphere is very small:

$$\omega' = \frac{\pi a^2 \sin \theta}{r^2} = \frac{\pi a^2}{r^2} \left( \frac{b}{A} + \frac{b^2}{4A^2} \right)$$

This correction is equivalent to a change of the total solid angle.

The final expression for  $W$  is

$$W_f = \frac{\pi a^2}{4} \sqrt{\frac{b}{A^2 + \frac{b^2}{4}}} + \frac{\pi a^2 \sin \theta}{2} \left( \frac{b}{A} + \frac{b^2}{4A^2} \right)$$

This is valid for any value of the angle  $\theta$ .

one inch,  $b = 1$  inch and  $a = 1/2$  inch.

10/1







