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# **Parents' Decision-Making on the Consumption of Private Tutoring of School Going Children: A Game Theoretic Approach**

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## Abstract:

While private tutoring leads to a low social rate of return with substantive opportunity as well as transaction costs, research reveals that investments in private tutoring genders a high private rate of return for parents who are willing to allocate resources in terms of time and money to improve their wards chances of getting higher scores in exams. Considering such stakes, this paper introduces game-theoretic models of parents' decision making on the consumption of private tutoring given the interactions among the public and private school going children as well as possible interventions by the government. By applying a three-stage behavioral game theoretical set up, we develop private tutoring game models using the key stakeholders comprising the higher authority of both public and private schools, teachers involved with private tutoring, and the parents. Our preliminary results reveal that teachers' dutifulness increases with salary but with professional development, the result is ambiguous. A teacher's expected income is decreasing in school infrastructure through fall in private tuition demand but increasing in teacher's salary. However, from higher authorities' point of view, if they care only about the total education of the students and is unmindful of the sources of student education, they would not mind lowering the teachers' wage, even if the policy reduces teacher dutifulness. Given such developments, parents are expected to support policies that encourage more private tutoring from the teachers since it leads to higher parental utility and overall education level for the students.

JEL Classifications: I21, C72

## **Introduction**

Private tutoring (PT) or supplementary education is a growing industry in many developing as well as developed countries (Bray and Kobakhidze, 2014a; Bray, 2009; Dang and Rogers, 2008). According a cross-sectional survey conducted by Dang and Rogers (2008), around twenty-five percent (25%) to ninety percent (90%) of students in twenty-two (22) developed and developing countries received private tutoring at various levels of their education. Bray (2003, 2009) also found private tutoring to be prevalent and increasing at significant rates among countries all over the world irrespective of their income status (Table 1). Moreover, spending by households on private tutoring even rivals public sector education expenditures in some countries such as the Republic of Korea and Turkey (Dang and Rogers, 2008). In some countries, governments contract with private companies to provide academic support for disadvantaged students (Patrinos et al., 2009). For example, the number of private companies providing supplemental academic services (academic tutoring) in the United States increased by 90 percent between 2003 and 2004. This is partly attributed to the increase in federal funds allocated to support supplemental education as well as the federal law that require all school districts in USA to provide supplemental education services both to schools that have not made adequately yearly progress for three consecutive years, and to schools with high percentages of poor children (more than 40 percent) or students with special needs (Burch, Steinberg, and Donovan, 2007).<sup>1</sup>

Despite private tutoring's capacity to increase parental choice and improve student achievement, concerns abound whether private tutoring aggravates social inequalities as well as impose significant costs on households if student outcomes fail to meet their required goals. To address such concerns, most studies explored the private tutoring influence on a public school environment and identify policy options for the government by assessing the efficiency and equity trade-offs of private tutoring. However, few studies looked into the influence of private tutoring on parental, school teachers, and government choices from a game theoretical model perspective. In our paper,

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<sup>1</sup> According to the United States Department of Education (2007), the supplemental services include afterschool tutoring, remediation, and other academic support activities that take place outside regular school hours. Between 2000 and 2003, the supplemental services industry grew by an average of 14 percent annually (Hentschke 2005). Main requirements of the program are the private providers offering high-quality and research-based services as well as having few barriers to entry in the supplemental education services (Burch, Steinberg, and Donovan, 2007).

we look into the possible interactions among parents, teachers, and the government to identify optimal choices of the parties in a private tutoring market using a game theoretical setup. We focus on the improvement of the overall level of school education based on the optimal choices pursued by the affected parties with private tutoring. Our paper does not address the issue of increasing social inequalities that might arise due to lack of access to private tutoring for households arising from income and wealth inequalities and, other contingent factors.

## **Literature Review**

Private tutoring, which is an international phenomenon, is defined as the after-school or outside-school supplementary lessons provided by teachers informally to students needing academic assistance (de Castro and de Guzman, 2014; Dang and Rogers, 2008). Private supplementary tutoring is also termed as *shadow education* activity that mimics the academic activities offered by formal schools (Bray and Kobakhidze, 2014a; de Castro and de Guzman, 2013). According to Bray (2003), private tutoring covers coaching and instructing in academic subjects by tutors for financial gains and in addition to the provision of mainstream schooling. Marimuthu al. (1991) metaphorically treats private tutoring as a shadow education system in the sense that the latter term signifies the hidden nature of private tutoring and the way in which it mirrors the formal education system in scope, intensity and size. Based on the conclusions of Marimuthu et al. (1991), the application of a shadow metaphor for private tutoring fits in three ways: (1) private tutoring exists because of a perceived or real deficiency in mainstream education systems; (2) the shape and size of mainstream education systems change so do the sizes, function and shape of private tutoring; and, (3) public attention is more apt to focus on the mainstream issue than on its shadow.

Conversely, Hartmann (2013, 2008) treats private tutoring to be not just a passive entity but the one that poised to negatively affect the body it imitates. Hartmann (2008) uses the phrase “informal market of education” in contrast to the shadow education system, where she puts more emphasis on the aspect of “commodification.” Critics like her are worried about the perception of education being turned into a marketable good or into the object of a commercial transaction as a result of significant presence of private tutoring. Moreover, there exist shared concerns about whether widespread presence and application of private tutoring exacerbates social and income inequality in developed and developing countries around the world (Dang and Rogers, 2013). Although

evidence indicate that private tutoring to have positive impact on different measures of student aptitude such as student test scores and academic performance in Africa (Paviot et al., 2008), India (Banerjee et al., 2010), Ireland (Smyth, 2009), Hong Kong (Bray, 2013), and the United States (Zimmer et al., 2010), such apprehensions associated with overall welfare impacts of private tutoring cannot be refuted or denied.

Following the interactions between the demand (parents and students) and supply (teachers) in the private tutoring market and how it shaped the industry in Philippines, de Castro and de Guzman (2014) categorized the stakeholders of private tutoring into three types: (1) Lean on; (2) Ride on; and, (3) Pass on. Their categorization is based on four criteria of for whom, for what, with whom and by whom (de Castro and de Guzman, 2014). Table 2 summarizes their classifications of type of interdependence behavior among the parents and students representing the demand side, the teachers, learning centers, and the multinational institutions representing the supply side, and the government representing the regulator side.

However, studies on private tutoring reveals the ambiguity associated in measuring the actual impacts of private tutoring since the industry can assume multifarious forms given the socio-economic and the regulatory institutions of a given country and the lack of systematic data due to informal nature of the industry (de Castro and de Guzman, 2014; Bray and Kobakhidze, 2014b; Dang and Rogers, 2013; Mori and Baker, 2010). Despite such obstacles, there are considerable number of studies that looked into the possible impacts and policy implications of private tutoring. Regarding government's role, findings reveal mixed policy responses in dealing with the private tutoring market. Although private tutoring is unregulated in majority of the countries, there are some countries where it is either controlled or actively regulated (Dang and Rogers, 2008). Governments of countries such as Cambodia, South Korea, Mauritius, and Myanmar banned private tutoring at various times because of their concern that that long-term negative impacts of private tutoring far outweighs the positive benefits in terms of accentuating social and income inequalities, unsettling the public education system, and failing to increase academic performance or building human capital led economic development (Dang and Rogers, 2008; Bray, 1999).

Taking into account the interplays among the student and parent group, the teachers and the learning centers group, and the government and other regulators group, we focus in developing a game theoretic model with a three-stage temporal framework that captures the essence of the interactions among these key stakeholders. Although there are few studies that discussed the demand side of the private tutoring from the perspectives of the students and parents decision making using a non-cooperative game framework (Choi, 2010; Yu and Ding, 2011), the possible interplays among the key stakeholders inclusive the government under a temporal setting is not extensively explored in the literature of private tutoring. In fact, most of the work on private tutoring are empirical that mainly examined possible influences of household socio-economic and other characteristics, and the level of schooling on the demand for private tutoring. Only a handful of studies attempted to capture the interactions among the stakeholders but the focus of their work are different. For example, Biswal (1999) attempted to capture the corruption aspect of private tutoring, Sylvain et al. (1998) showed both public school and private tutorial market can coexist under a particular labor market, and Jayachandran (2014) focused on the educational inequality arising from income inequality. However, there are no role of government decision making to influence the demand and supply of private tutoring in these models. In addition, school quality, an important influential factor determining the level of private tutoring, is not taken into consideration in most of the game theoretic models.

The paper is organized as follows. In Section 2, we introduce our three-stage game theoretic model involving three stakeholders: parents determining the amount of private tutoring for their wards (the demand side), teachers and learning centers determining the amount of private tutoring to offer (the supply side), and the government and regulators determining their possible market interventions to influence socially optimal level of private tutoring (influencing the demand-and-supply private tutoring market). Section 3 analyzes the optimal private tutoring offered by the teachers and the learning centers given the school quality and income earning opportunities. In Section 4, we discuss the optimal decision by the government and the regulators considering the developments in the demand-and-the supply sides of the private tutoring market. We conclude in Section 5.

## **2. The Inter-temporal Game Theoretic Model of Private Tutoring**

Following the interdependence among the three key stakeholders in determining the supply-and-demand model of private tutoring: the government regulators, the teachers, and the parents, our inter-temporal game theoretic model have all three agents involve in the decision making process. However, the process starts with the government deciding whether to ignore, discourage, or, encourage private tutoring. In our simple game theoretic setup, we assume that the government sets up a low-incentive regime to discourage private tutoring by imposing fines given its limited budget in allocating the public fund between teacher's salary and the school infrastructure for public schools. Conversely, a higher private authority can perform the role of a government to allocate resources between teacher's salary and the school infrastructure for private schools. Since quality of school education might be difficult to measure, we assumed that the quality of school infrastructure could be used as a proxy to capture the quality of education provided at school. Once the government or the higher private authority finalized their choices of public funds allocation, the school teachers decide how much school work they should shirk to optimize their expected earnings through teacher salary and private tutoring. Parents of the pupils, one of the three agents in the decision making process, can influence the private tutoring supply of the teachers by revealing tutorial demand on behalf of their wards. Given the tutorial demand and regulator choices, the teachers determine their private tutorial supply at the last stage of the decision making process. All in all, we have three (3) stages of decision making process involving three (3) agents in period 1.

To keep the exposition simple, we allow the households to live for only two periods. Hence, period 2 will reveal the outcomes of the all three agents' decision making processes taken during period 1. That is, the government-regulator of public schools or higher authority of a set of private schools will maximize social returns from their respective education policy. The teachers will maximize their expected income by allocating their resources between hours put in school duty and hours put in for private tutoring. Lastly, the parents of the students, will maximize expected benefits by experiencing the children to realize human capital potential by earning the market driven income based on their educational achievements. In some countries, this parental expectation of benefits could also be realized through the children's achievement of getting access to better colleges and universities based on their higher grades (Choi, 2010; Dang and Rogers, 2008).

Starting with the households, we assume a household's spending on private tuition is equivalent to its savings. After allocating for subsistence consumption,  $\bar{C}$ , and paying for taxes,  $T$ , out of present income,  $Y$ , a household's lifetime utility function can be expressed as,

$$U = \left[ C_1 - \bar{C} \right] + \delta \left[ C_2 - \bar{C} \right] \quad \dots \quad \dots \quad (1)$$

Where  $C_1$  and  $C_2$  refer to period 1 and period 2 consumption,  $\delta$  is the discount factor.

If household's or parents demand  $q_{PT}$  amount of private tuition in terms of total private tuition hours for their wards by paying  $p_{PT}$  per hour tuition fee, then, the period 1 household budget is,

$$C_1 + (p_{PT} \times q_{PT}) \leq Y - T - \bar{C} = Y_{DI_{public}} \quad \dots \quad \dots \quad (2a)$$

For private school going children, there is an additional private school fee,  $t_{private}$ , which could be accommodated to the household budget on top of the private tuition fees.

$$C_1 + (p_{PT} \times q_{PT}) + t_{private} \leq Y - T - \bar{C} = Y_{DI_{private}} \quad \dots \quad \dots \quad (2b)$$

With  $Y_{DI_{private}} < Y_{DI_{public}} \quad \dots \quad \dots$

Since a child's education is received through two sources: (a) education received at school; and, (b) education received through private tutorials, we assume that education received at school is positively affected by teacher's dutifulness at school,  $e$  ( $e \leq 1$ ), and the quality of the school education is captured through students exposure to the school infrastructure quality,  $s$ . Hence, the child's education function is,

$$E = s \cdot e + q_{PT} \quad \dots \quad \dots \quad (3)$$

Under two-period model, we assume the child becomes an adult in period 2. If the child is successful in receiving the highest level of education,  $h_{edu}$ , in period 1, then, as an adult in period 2, would realize an income according to the following earnings function,

$$Y(E) = h_{edu} \cdot E - \frac{E^2}{2}, \text{ for } E \leq h_{edu} \quad \dots \quad \dots \quad (4)$$

$$= \frac{h_{edu}^2}{2} \text{ for } E > h_{edu}$$

Combining equations (1) through (4), the parent's utility maximization problem is,

$$Max U = C_1 + \delta C_2 - (1 + \delta) \bar{C}$$

Subject to

$$C_1 = Y_{DI_{public}} - p_{PT}q_{PT}$$

$$C_2 = Y(E) = h_{edu} \cdot E - \frac{E^2}{2}$$

The first-order condition in terms of private tutoring leads to,

$$\frac{\partial U}{\partial q_{PT}} = \delta \cdot (h_{edu} - se) - \delta q_{PT} - p_{PT} = 0, \quad \text{if } se < h_{edu} \quad \dots \quad \dots \quad (5)$$

Parent's will not demand any private tutorials for their wards if  $se \geq h_{edu}$ , i.e., quality of education received from schools exceeds or equivalent to the highest level of education received by a student. Assuming,  $N$  number of parent couples (or households), parents' inverse market demand for private tutoring is,

$$p_{PT} = \delta \cdot (h_{edu} - se) - \delta \cdot \frac{Q_{PT}}{N} \quad \text{where, } Q_{PT} = N \cdot q_{PT} \quad \dots \quad \dots \quad (6)$$

Reformulating (6), we get,

$$p_{PT} = \delta \cdot [(h_{edu} - se) + s \cdot (1 - e)] - \delta \cdot \frac{Q_{PT}}{N} \quad \dots \quad \dots \quad (7)$$

From equation (7), three things can be concluded: (a) if  $C_1 = 0$  and  $p_{PT}q_{PT} > Y_{DI_{public}}$ , the inverse demand function is not valid; (b) if  $s < h_{edu}$ , i.e. quality of school infrastructure could not provide the highest level of education, tutorial demand will not go away. This will hold even teachers are providing their maximum effort in school ( $e = 1$ ) without any shirking of their school responsibilities and duties; and, (c) if  $s = h_{edu}$ , i.e. quality of school infrastructure meeting the expectations of highest level of education received by students, but teachers decide to shirk on their school responsibilities and duties ( $e < 1$ ), tutorial demand from parents will not go away.

Now, if government decides to maximize social rather than private benefits, then, we expect that the government's goal would be to improve the overall level of school education in terms of quality education. This could be achieved through policy incentives of allocating sufficient funds for improving the school infrastructure quality and increasing the teacher salary. Regarding private tutorials, government can encourage private tutoring practice by pursuing either doing nothing (no penalty for shirking) or providing incentives for teachers through creation of public-private



partnerships that can gradually improve the mechanisms of the private tutoring supply. On the other hand, government can discourage private tutoring by either prohibiting with rules and restrictions or by discouraging the private tutorial practice through introduction of fines and penalties. For this paper, we are focusing on the aspect of government discouraging the private tutorial practice with fines.

### 3. Optimal Private Tutoring based on Private Tutorials Market Structure

By modeling for the government policy to discourage private tutoring among the teachers, we can assume a penalty structure,  $R$ , with the degree of penalty,  $\lambda$ , which is influenced by teacher's dutifulness,  $(1-e)$ , and teacher's salary,  $w$ . This could take the following form,

$$R = \lambda \cdot w(1-e) \quad \dots \quad \dots \quad (8)$$

Following the penalty structure, the teacher's profit function becomes,

$$\begin{aligned} \Pi &= w + p_{PT} \cdot Q_{PT} - R \\ &= w + p_{PT} \cdot Q_{PT} - \lambda \cdot w(1-e) \end{aligned} \quad \dots \quad \dots \quad (9)$$

Here, the teacher decides how much private tutoring to offer ( $Q_{PT}$ ) based on his decision to transfer the time and effort saved through shirking responsibilities and duties at school. We assume that the demand for private tutoring ( $Q_{PT}$ ) should be sufficient enough to generate enough interest in the teacher to supply  $A(1-e)$  amount of private tuition. Moreover, even with complete shirking ( $e=0$ ), we assume that the teacher falls short of providing highest desirable level of education,  $h_{edu}$ , to each student. With total  $N$  number of students being representative of all households, this relationship could be defined as,  $a_{edu} < h_{edu}$  with  $a_{edu} = \frac{A}{N}$ , amount of education each student receives through private tutorials.

Regarding the total supply of tutoring, we consider the teacher to be the leader with strong reputation and goodwill in the private tutorial market in the subject area of interest. However, there is also a bunch of followers providing private tutorials to students on the same subject but with no well-established reputation. They could fall under a perfectly competitive market structure and act as price-takers. For simplicity, we assume all  $n$  number of fringe firms face symmetric marginal

cost of private tutoring,  $MC = c_{PT} \cdot q_{PT}^f$  and their supply decision is given by,  $p_{PT} = c_{PT} \cdot q_{PT}^f$ . Based on this setup, the total fringe tutorial supply curve is  $p_{PT} = c_{PT} \cdot \frac{Q_{PT}^f}{n}$ . So, the total supply of private tutoring could be represented as,

$$Q = Q_{PT} + Q_{PT}^f \quad \dots \quad \dots \quad (10)$$

Given the school teacher's supply of  $Q_{PT}$  amount of private tuition, the residual fringe demand could be determined with the assumption that the fringe teachers pick up the unmet private tutorial demand that could not be provided by the school teacher due to his or her time constraints. This relationship could be expressed as,

$$\begin{aligned} p_{PT} &= \left[ \delta [(h_{edu} - s) + s(1 - e)] - \delta \cdot \frac{Q_{PT}}{N} \right] - \delta \cdot \frac{Q_{PT}^f}{N} \quad \dots \quad \dots \quad (11) \\ &= H - \delta \cdot \frac{Q_{PT}^f}{N} \end{aligned}$$

Where,  $H = \delta \cdot [(h_{edu} - s) + s(1 - e)] - \delta \cdot \frac{Q_{PT}}{N}$ .

Equating the total fringe demand, equation (11), with total fringe supply, equation (10), would lead to the following relationship,

$$Q_{PT}^f = H \cdot \frac{nN}{cN + \delta N} \quad \dots \quad \dots \quad (12)$$

Equation (12) confirms the private tutorial market trends of observing the competitive fringe tutorial,  $Q_{PT}^f$ , to be increasing with the addition of new fringe teachers ( $n$ ) but decreasing with more market share of the leader,  $Q_{PT}$ , i.e. the school teacher with established reputation and goodwill.

Considering the leader-follower sequential game setup, one can argue that the school teacher decides on her tutorial supply prior to the fringe tutors' choice which allows the teacher to maintain her private tutorials market share. Based on this premise, we can substitute equation (12) into equation (7), to get the tutorial demand that is relevant to the school teacher – the 'leader' of the private tutorial market supply.

$$\begin{aligned}
 p_{PT} &= H - \frac{\delta}{N} \cdot \frac{nN}{(cN + \delta n)} \cdot H \\
 &= \frac{cN}{(cN + \delta n)} \cdot H \quad \dots \quad \dots \quad (13) \\
 &= v \cdot H
 \end{aligned}$$

Here,  $v = \frac{cN}{cN + n\delta} < 1$  captures the effect of the fringe tutors on the leader-follower sequential set of actions of private tutorial market supply. If the school teacher, the leader, could exercise complete market power in the private tutorial market supply, then, we will have  $v = 1$ .

Taking into account the above developments and substitution for  $H$ , the school teacher's maximization problem becomes,

$$\begin{aligned}
 \Pi &= w + p_{PT} \cdot Q_{PT} - \lambda \cdot w(1-e) \\
 &= w + \delta v \cdot [(h_{edu} - s) + s(1-e)] \cdot Q_{PT} - \delta v \frac{Q_{PT}^2}{N} - \lambda \cdot w(1-e)
 \end{aligned}$$

Subject to

$$Q_{PT} \leq A(1-e) \quad \dots \quad \dots \quad (14)$$

Performing the first-order condition of the above problem with respect to  $(1-e)$  leads to,

$$\frac{\partial \Pi}{\partial (1-e)} = \delta v A [(h_{edu} - s) - 2(a_{edu} - s) \cdot (1-e) - \lambda w] = 0 \quad \dots \quad \dots \quad (15)$$

By re-arranging the terms,

$$\underbrace{[(h_{edu} - s) + 2(a_{edu} - s)(1-e)]}_{\text{Marginal return to shirking}} = \underbrace{\frac{\lambda}{\delta v A} \cdot w}_{\text{Marginal cost to shirking}} \quad \dots \quad \dots \quad (16)$$

Given the assumptions, equation (16) shows that the teacher will shirk to the extent where marginal return to shirking is equal to the marginal cost of shirking. If marginal return is greater than the marginal cost of shirking, the teacher will increase the level of shirking at school and allocate more time for private tutoring unless she realizes  $MR = MC$  condition. On the other hand, once the teacher finds out that her marginal return to shirking is lower than her marginal cost of shirking, then, she will lower the level of her shirking unless  $MR = MC$  condition is achieved.

By further differentiating equation (15) with respect to  $(1-e)$ , we get the second-order condition,

$$\frac{\partial^2 \Pi}{\partial (1-e)^2} = -2\delta vA(a_{edu} - s) < 0, \text{ iff } a_{edu} > s \quad \dots \quad \dots \quad (17)$$

Equation (17) reveals that the second order condition holds when the amount of private tutorials provided by the teacher for each of her student  $\left(a_{edu} = \frac{A}{N}\right)$  exceeds the amount of quality school infrastructure ( $s$ ) that is sufficient to motivate the same teacher to impart quality education services at school.

Conversely, if  $a_{edu} < s$ , the teacher will either shirk completely ( $e = 0$ ) or not shirk at all ( $e = 1$ ). From the teacher's profit function setup at equation (14), complete dutifulness at school ( $e = 1$ ) will lead to teacher's profit to be equal to her wage earned from school service,

$$\Pi = w \quad \dots \quad \dots \quad (18a)$$

On the other hand, complete shirking at school ( $e = 0$ ) will cause teacher's profit to become,

$$\Pi = \delta vA(h_{edu} - a_{edu}) + w(1 - \lambda) \quad \dots \quad \dots$$

(18b)

Comparing equations (18a) and (18b),

The teacher chooses  $e = 1$  when  $w > \frac{\delta vA \cdot (h_{edu} - a_{edu})}{\lambda}$ ; otherwise, the teacher completely shirks ( $e = 0$ ).

With partial shirking ( $0 < e < 1$ ) under  $a_{edu} > s$ , the school teacher's optimal shirking decision making will be based on the following set of conditions,

Choice for complete dutifulness ( $e^* = 1$ ):

$$(1 - e^*) = 0 \quad \text{if } w \geq w_H; \text{ where, } w_H = \delta vA \cdot \frac{(h_{edu} - s)}{\lambda}$$

Choice for partial shirking ( $0 < e^* < 1$ ):

$$(1 - e^*) = \frac{(h_{edu} - s) - \frac{\lambda}{\delta v A} \cdot w}{2(a_{edu} - s)} \quad \text{if } w_L < w < w_H;$$

$$\text{where, } w_L = \delta v A \cdot \frac{(h_{edu} + s - 2a_{edu})}{\lambda} \quad \text{and, } w_H = \delta v A \cdot \frac{(h_{edu} - s)}{\lambda}$$

Choice for complete shirking ( $e^* = 0$ ):

$$(1 - e^*) = 1 \quad \text{if } w \leq w_L; \text{ where, } w_L = \delta v A \cdot \frac{(h_{edu} + s - 2a_{edu})}{\lambda} \quad \dots \quad \dots \quad (19)$$

From equation (19), we can see that the highest wage ( $w_H$ ) that could be earned from school is decreasing with better quality school infrastructure ( $s$ ); whereas, the lowest wage ( $w_L$ ) that could be earned from school is increasing with better quality school infrastructure ( $s$ ).

Assuming partial shirking ( $0 < e^* < 1$  and  $w_L < w < w_H$ ) for the school teacher, the teacher's optimal dutifulness becomes,

$$e^* = \frac{\frac{\delta v A}{\lambda} w - (s_{edu} + b - 2a_{edu})}{2(a_{edu} - s_{edu})} \quad \dots \quad \dots \quad (20)$$

Equation (20) reveals that the school teacher's dutifulness ( $e^*$ ) increases with teacher's salary ( $w$ ) but with better school infrastructure ( $s$ ), such as the targeted professional development programs for the teachers, the relationship is ambiguous or cannot be determined.

Resulting school education from equation (20) shows,

$$s e^* = s \cdot \frac{\frac{\delta v A}{\lambda} w - (s + h_{edu} - 2a_{edu})}{2(a_{edu} - s)} \quad \dots \quad \dots \quad (21)$$

Equation (21) shows that education received at school is increasing with teacher's salary ( $w$ ) but possible direction of better school infrastructure ( $s$ ) influence cannot be determined.

Taking into account the above equations, total education for a student is,

$$\begin{aligned}
 E^* &= se^* + Q_{PT}^* + \frac{Q_{PT}^{f*}}{N} \\
 &= se^* + a_{edu}(1-e^*) + \frac{Q_{PT}^{f*}}{N} \quad \dots \quad \dots \quad (22) \\
 &= \left\{ s + \frac{(h_{edu} - s) - \frac{\delta v A}{\lambda} \cdot w}{2} \right\} \cdot v + h_{edu}(1-v)
 \end{aligned}$$

Equation (22) shows that in case the school teacher is opting for complete dutifulness ( $e^* = 1$ ), the student receives from school and the entire private tutoring hours coming from the fringe tutors.

Now, teacher's expected income is,

$$\Pi^* = \delta v A \cdot \frac{\left( h_{edu} - s - \frac{\delta v A}{\lambda} \cdot w \right)^2}{4(a_{edu} - s)} + w \quad \dots \quad \dots \quad (23)$$

Equation (23) shows that the teacher's expected income is decreasing in school infrastructure quality ( $s$ ) but in wages ( $w$ ).

#### 4. Optimal Policy Choice of the Government or the Higher Authority

Going back to stage 1 of the game, government or the higher authority decides how to allocate public funds,  $G_{school}$ , between the teacher's salary and the school infrastructure. The government clearly knows the implications of the allocation decisions on the school teacher's choice of amount of dutifulness at school and effort commitment for private tutoring.

Based on the sequence of events, the government official solves the following maximization problem,

$$\text{Max } E(s, w)$$

Subject to,

$$\begin{aligned}
 w + rS &\leq G_{school} \quad (\text{budget constraint}) \\
 \Pi(s, w) &\geq w^0 \quad (\text{Teacher's participation constraint}) \quad \dots \quad \dots \quad (24)
 \end{aligned}$$

Solving for equation (24) using a graphical approach, we find that the government official or the higher authority will prefer to offer low-incentive for the school teacher by offering them lower salary ( $w^*$ ) compared to the reservation salary ( $w^0$ ). Since teacher's dutifulness is negatively affected as a result of such policy initiative, the ultimate goal of the government or the higher authority becomes substituting school learning with more private tutoring services if the latter is considered to be more efficient. With  $a_{edu} > s$ , this policy initiative allows the government official or the higher authority to allocate more for improving the quality of school infrastructure ( $s^*$ ).

However, if evidence reveals that the school infrastructure is not reaching the level of quality as expected with higher budget allocation, then, we expect more shirking from the school teachers with lower salaries. As a result, greater education will be offered through private tutoring and the government might find it optimal to offer a salary ( $w^*$ ) below the reservation salary ( $w^0$ ). The school teacher will have to make up the short fall through private tutoring fees. Hence, the optimal policy choice of the higher authority and its overall impact on student education lend support to reality regarding why private tutoring is becoming popular choice around the world especially from the perspectives of the middle-and-low income countries facing tight budgets. These combined results emanating from the government official or the higher authority's policy choice are illustrated in Figure 1.

## **5. Conclusion**

Considering the growing trends in private tutoring or supplementary education around the world, the policymakers are finding the importance of it in shaping the human capital and productivity of nations (Dang and Rogers, 2008; de Castro and de Guzman, 2013). Evidence reveals mixed responses from the government, where few countries went for outright ban on private tutoring and while majority of the countries allowed private tutoring to flourish without any market interventions (Bray, 2009; Dang and Rogers, 2008). In between, there are countries that are actively controlling and regulating the private tutoring market (Dang and Rogers, 2008). Only a handful are providing high-incentives to encourage participation of teachers in after school programs, such as in USA (Burch et al., 2007). Interestingly, there is not much evidence of a government imposing penalty or fine on school teachers to discourage after school private tutoring

at their homes. Since the options of fines and subsidies to discourage as well as encourage private tutoring cannot be overlooked, we consider four (4) policy options that are available to the government. They are: (1) prohibition; (2) regulation; (3) encouragement; and, (4) doing nothing. One can argue that policies (1) and (2) fall under low-incentives regime; whereas, policies (3) and (4) under high-incentive regime to influence the private tutoring supply.

Since the interactions and relationships among the student and parents representing the demand side, the school teachers and fringe tutors representing the supply side, and the government and higher authorities representing the market interventionist side, play their respective roles in the private tutoring market, we introduced a three-stage game theoretic model in this paper. In our model, we focused on the low-incentive regime with government regulations (Policy 2) of fines and penalties to discourage private tutoring. To keep the exposition simple, we do not address increasing social inequalities that might arise through private tutoring. We achieved this objective by ignoring income inequalities of the households by assuming that education contributes only to future consumption as well as ensuring upward social mobility of the students.

Under government regulations with fines and penalties, our preliminary results reveal that a school teacher's dutifulness and education provided at school increases with salary. However, with improvement in school infrastructures, such as, professional development aimed for teachers, the directional relationship between school teacher's dutifulness and school infrastructures cannot be established. Findings also reveal that teacher's expected income is decreasing in school infrastructure through fall in private tuition demand and increasing in teacher's salary. If the government official or the higher authority cares only about the total education of the students and therefore, unmindful of its sources, the optimal policy turns out to be offering lower wages for the school teacher given a penalty or fine imposed on shirking responsibilities at school. Although such policy would discourage a teacher's school dutifulness, the social planner would achieve the goal of providing maximum education to students by allocating more resources to improve the quality of the school infrastructure but at the expense of substituting learning away from schools towards more efficient private tutoring sector. To make up for the short fall of income, the school teacher would increase more time and effort on private tutoring. Such outcome is more likely to impact the demand side of the market positively since the parents (or, the households) would prefer



to receive more education for their children given the existing institutions and school infrastructure under such policy choice.

Findings from our study relate to the empirical evidence of increasing demand and supply of private tutoring around the world given the resource constraints. The results might be more relevant to middle-and-low income nations where teacher absence and shirking cannot be fully eradicated due to higher monitoring costs, and the school infrastructure that cannot be fully upgraded due to tight government and school budgets. For future directions of research, we want to see whether the results hold under a policy regime with no penalty and fine. How the findings play out if private tutoring is encouraged through subsidies? If school accessibility is not an issue for government subsidized primary (elementary) and secondary schooling programs, can private-public partnerships be forged to improve teacher professional development and private tutoring mix to enhance the overall quality of education? How the results be different under publicly-funded but privately-managed schools such as the one that is offered through the Charter school programs in USA? We want to explore these questions in our next subsequent steps.

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## Appendix

**Table 1: Cross national indicators of private supplementary tutoring**

Location	Year of study	Rural / Urban area	Primary / Secondary school
<b>Bangladesh</b>	2005	28% of rural and 52% of urban primary school students consumed private tutoring	31% of primary school students consumed private tutoring
<b>Cambodia</b>	1997-1998		31% of 77 primary schools surveyed consumed private tutoring
<b>Canada</b>	1990s	Tutoring businesses in major cities grew 200 to 500 %	
<b>China</b>	2004		74% of primary 66% of lower secondary and 54% of upper secondary students consumed private tutoring
<b>Cyprus</b>	2003		87% of secondary school students consume private tutoring
<b>Egypt</b>	2004	64 % in urban and 52% in rural	All levels
<b>Hong Kong</b>	2004 - 2005		36% of primary 28% of lower secondary, 34 48% in upper secondary school students consumed private tutoring
<b>India</b>	1997	70 % of children in urban areas consume private tutoring	40% of primary students consume private tutoring
<b>Japan</b>	2007	90% children in urban areas consume private tutoring	65% of junior secondary students consume private tutoring
<b>Kenya</b>	1997		69 % of sixth graders consumed private tutoring
<b>Malta</b>	1997-19998		50.5 % of primary and secondary school students consume private tutoring
<b>Republic of Korea</b>	1997		72.9 % of primary school students, 56% of middle school and 32 % of high school students consume private tutoring
<b>Romania</b>	1994	32% of secondary school students consume private tutoring	58 % of secondary school students consume private tutoring
<b>Taiwan</b>	1998		81.2% of secondary schools students consume private tutoring
<b>Vietnam</b>	2001-2002		38% of primary school students paid 29% of household expenditure to consume private tutoring

Source: Bray (2003 and 2009)

**Table 2: Characteristics of private tutoring** (adapted from de Castro and de Guzman, 2014).

	<b>Lean on</b>	<b>Pass on</b>	<b>Ride on</b>
For whom	Low achieving students, slow learners	Students with busy parents, lacking assistance in their school work	Both low and high achieving students, students whose parents can afford tutorial fee
For what	Hidden remedial activities	Supplementary activities	Structured, remedial and enrichment activities
With whom	School teachers	School teachers, small scale institutions	Multinational institutions, learning centers, experts in the field, university students
By whom	Unregulated	Regulated as a business entity	Regulated as a Business or academic entity

**Table 3: The Inter-temporal Game Theoretic Setup**

<b>Period 1</b>			
<b>3-stage setup</b>	<b>Actors</b>	<b>Objective/ Main Goals</b>	<b>Actions</b>
<i>Stage 1</i>	Higher Authority at Government funded public School (or, privately funded private school)	Maximize <i>social return</i> from education through policies geared through private tutoring  Decides on four basic policy responses to private tutoring: (1) Ignore – most countries; (2) Prohibit – South Korea; (3) Regulate – Mauritius, Hong Kong (4) Encourage – Singapore, Taiwan	(1) <i>Ignore</i> – do nothing (2) <i>Prohibit and Regulate</i> – Low-incentive policies; (3) <i>Encourage</i> – High-incentive policies
<i>Stage 2</i>	Teachers involved with private tutoring	Maximize <i>expected income</i> through teaching salary and private tutoring	How much time to allocate for private tutoring (optimal effort for private tutoring)
<i>Stage 3</i>	Parents	Maximize <i>net benefits of private tutoring investments</i> allocated (private tuition) for children	Amount of money to set aside for private tuition
<b>Period 2</b>			
<b>Actors</b>		<b>Objective / Main Goals</b>	
<i>Higher Authority at Public or Private school</i>		Maximize social return from education policy	
<i>Teachers involved with private tutoring</i>		Maximize expected income	
<i>Parents</i>		Children grow up and realize an income through an earnings function  Or, Children grow up and get access to better colleges and universities through higher grades	

Figure 1: Optimal Policy Choice of Higher Authority and Its Impacts on the School Education

