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The Leakage Neutron Spectrum of a Spherical Critical Assembly of U233

Larry E. Bobisud

LEAKAGE NEUTRON SPECTRUM OF A SPHERICAL CRITICAL ASSEMBLY ▪ BOBISUD

THE LEAKAGE NEUTRON SPECTRUM OF A SPHERICAL CRITICAL ASSEMBLY OF U^{233}

By

Larry E. Bobisud

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MASTER OF SCIENCE

Roy Parish
DEAN

Dec 27, 1962
DATE

Thesis committee

Howard C. Bryant
CHAIRMAN

Arnold Skabelund

Roy Thomas

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I. INTRODUCTION

In this paper the spectrum of the neutrons escaping from a critical sphere of U^{233} is investigated experimentally and theoretically. The experimental investigation involved exposing a nuclear emulsion to the neutron flux and, after development, examining the details of the proton-recoil tracks visible in the emulsion. From the information thus derived the initial energies of the colliding neutrons were determined; combined with a correction for protons leaving the emulsion, the cross section for neutron-proton scattering, the number of protons per cubic centimeter of emulsion, and the solid angle considered, the number of neutrons per energy interval yields the energy spectrum of the incoming neutrons. The spectrum so obtained is briefly compared with the similarly obtained spectra of spherical critical assemblies of U^{235} and Pu^{239}.

For the theoretical determination of the leakage neutron spectrum, the neutron transport equation is derived under suitable assumptions. Obtained by numerical integration, the solution of this equation is presented in the form of the relative neutron flux in each of several energy intervals. Finally, the theoretical spectra of the three critical assemblies mentioned above are compared with those observed.

I. INTRODUCTION

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from a critical sphere of [illegible]
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For the theoretical [illegible]
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relative neutron flux [illegible]
Finally, the [illegible]
assemblies [illegible]

II. EXPERIMENTAL ARRANGEMENT AND ANALYSIS

To determine experimentally the leakage neutron spectrum of the bare U^{233} critical assembly in existence at Los Alamos, a number of approximately 200 μ thick Ilford K2 nuclear emulsions were exposed for several minutes to the neutron flux present at a distance of 125 centimeters from the surface of the critical assembly. The plates were stacked two together, emulsions facing each other but separated by a thin platinum foil to prevent the possibility that a proton from one emulsion might recoil into the other; the whole plate assembly was wrapped in black paper. The accompanying illustration shows the exposure configuration.

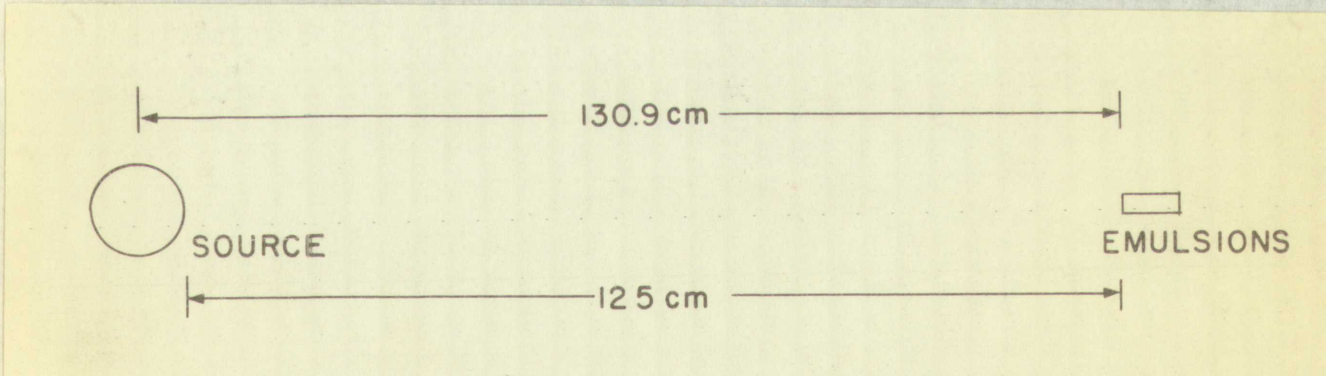

Fig. 1.--Exposure configuration

By making several exposures at differing integrated neutron flux levels (approximately 5×10^7 to 2×10^8 neutrons/cm^2) at the plates, the density of tracks was optimized for ease in subsequent analysis.

II. EXPERIMENTAL

To determine experimentally [illegible]
of the bare U^{233} critical [illegible]
a number of exposures only [illegible]
emulsions were exposed for [illegible]
flux present at a distance [illegible]
surface of the critical assembly [illegible]
two together, emulsions facing [illegible]
a thin platinum foil to prevent [illegible]
proton from one emulsion might [illegible]
whole plate assembly was wrapped [illegible]
accompanying illustration shows [illegible]

Fig. 1.--Exposure configuration.

By making several exposures at [illegible]
flux levels (approximately 3 x [illegible]
at the plates, the density of tracks [illegible]
in subsequent analysis.

A background measurement was made with the identical basic geometry to allow correction for events caused by normal background radiations (cosmic rays, etc.) and neutrons scattered into the emulsion; for the background determination, however, a polyethylene block in the form of the right frustum of a pyramid was inserted between the source and the detector in the illustrated manner. This shield was

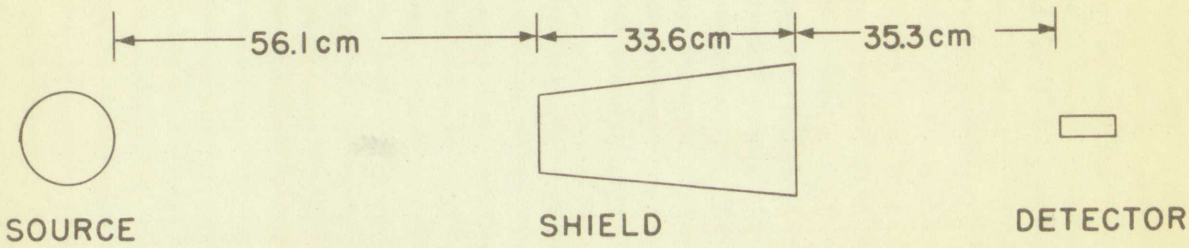

Fig. 2.--Background exposure configuration

sufficiently large (greater than three mean free paths in length) to prevent arrival at the detector of a neutron coming directly from the source.

After development[1] of the emulsions, the tracks of protons which had apparently recoiled from collisions with incident neutrons were examined using a high-power (100X) oil-immersion objective on a microscope equipped with a calibrated fine adjustment of focus and with a calibrated

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[1]For development techniques see J. C. Allred and A. H. Armstrong: <u>Laboratory Handbook of Nuclear Microscopy</u> (Los Alamos: Los Alamos Scientific Laboratory, 1951) and L. Rosen, <u>Nuclear Emulsion Techniques for the Measurement of Neutron Energy Spectra</u>, Nucleonics <u>11</u>, No. 7, 32 (1953).

A background measurement [illegible]

basic geometry [illegible]

normal background [illegible]

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however, a polyethylene [illegible] the first

frustum of a pyramid [illegible]

the detector [illegible]

Fig. 2.--Background [illegible]

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After development[1] [illegible] protons which had [illegible] incident neutrons [illegible] oil-immersion [illegible] calibrated [illegible]

[1]For [illegible] H. H. [illegible] (Los Alamos [illegible] L. Rosen [illegible] of [illegible]

mechanical stage. Since the index of refraction of the cedar oil is very nearly equal to that of the emulsion, use of an immersion objective obviates correction for the difference between the index of refraction of air and that of the emulsion. In addition to the projected range R_p of the track, the dimensions A and B and the angle θ, as illustrated, were determined. The angle θ was read from

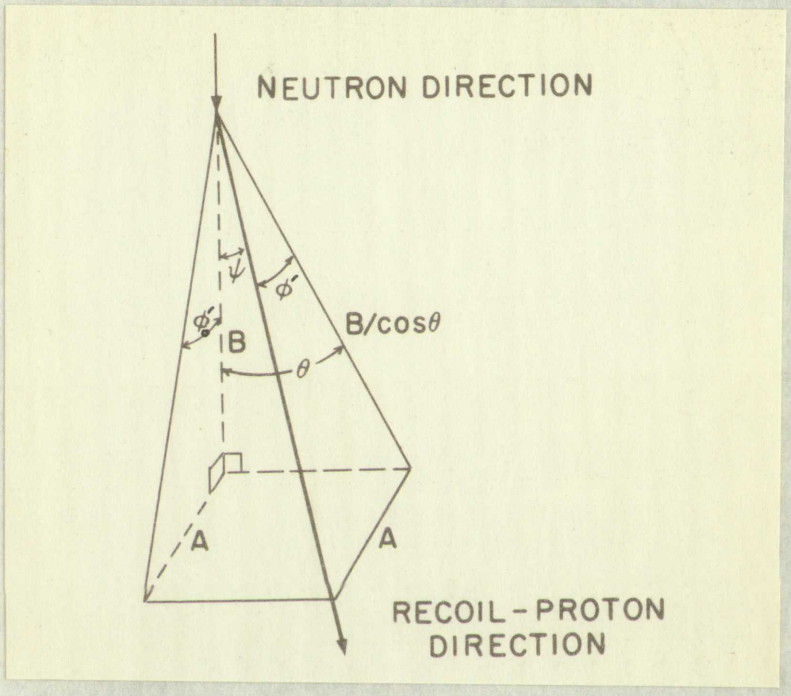

Fig. 3.--Illustration of measured quantities

a scale attached to the microscope eyepiece, which was rotated until a reference line inscribed on the reticule contained in the eyepiece coincided with the initial direction of the track. Both B and R_p are the projections of the track along a line parallel to the surface of the emulsion and parallel to the edge of the plate (and hence parallel to the direction of the incident particles). R_p is the

thus-projected range of the particle; B is some convenient portion of the track length. In general, B was taken equal to R_p for short tracks ($\leq$ 80 μ, the width of the eyepiece reticule used) and was less than R_p for long tracks. R_p for short tracks was measured in terms of subdivisions of the eyepiece reticule (8 μ per subdivision) and for long tracks (over 80 μ) with the stage micrometer.

The angle ϕ' shown in Fig. 3 was not measured directly; instead, the dip distance A was measured using the calibrated fine adjustment. Since the thickness of the emulsion shrinks during development and during storage,[2] A is not the true dip of the proton. Let ϕ be the true dip angle; then

$$\tan\phi = \frac{KA}{B/\cos\theta},$$

where K is the shrinkage factor, defined as

$$K = \frac{\text{True emulsion thickness before development}}{\text{Measured emulsion thickness after development}}.$$

For the plates studied, which were maintained in an atmosphere of 40% relative humidity, K was of the order of 1.55, with daily variations of the order of ± .02. The emulsion thickness before development was measured using a micrometer; the thickness after development was measured using the fine adjustment of the microscope. Since $\tan\phi$ thus contains the ratio of two quantities measured with the fine adjustment, any linear error in the fine adjustment cancels itself in

[2]The emulsion cannot shrink appreciably in its plane since it is secured to a glass backing.

thus-projected [illegible] portion of the [illegible] to B_y for short [illegible] reticule used) [illegible]

B_y for short tracks and [illegible] of the eyepiece [illegible] μ [illegible] tracks (over 50 μ) [illegible]

The angle ϕ [illegible] Instead, the dip [illegible] fine adjustment. [illegible] shrinks during development [illegible] true dip of the proton. [illegible] ϕ [illegible]

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where K is the [illegible]

K = [illegible]

For the plates [illegible] of 40% relative humidity [illegible] daily variations [illegible] thickness before [illegible] the thickness of [illegible] adjustment of the [illegible] ϕ [illegible] the ratio of two [illegible] any linear error [illegible]

The emulsion [illegible] since it is assumed [illegible] the calculation of ϕ. In order to avoid complications arising from curvature of field, the depths of the two ends of a track were measured with the track placed symmetrically in the field. An alternative procedure, also used, consisted of placing each end of the track in succession at the same place in the field and there determining the depth of the end grain. In both cases the dip is given, obviously, by the difference in the two depth readings.

Only tracks lying within a pyramidal solid angle defined by the (essentially arbitrary) half angles $\theta \leq 16°$, $\phi_0 \leq 16°$ were used in the calculations. Only tracks running within the pyramid in the direction of the incident beam were counted; the direction of a track could in most cases be inferred from the grain density along the track since a proton ionizes more heavily near the end than at the beginning of its track.

The raw data, as described above, were processed on an IBM 704 computer, which automatically calculated the true range

$$R = R_p \sec \theta \sec \phi = R_p \sec \psi,$$

where ψ is the polar angle, for each proton whose track was measured. If a track had a sharp bend where the proton underwent a large-angle scattering from a collision with a massive nucleus, the range was measured in two (or more) portions and the computer added these partial ranges to obtain the total range. From a built-in table[3] of range

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[3]The table used is that of J. J. Wilkins, <u>Range-Energy</u>

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[3]The table used is that of J. H. [illegible]

versus energy for the type K2 emulsion the machine determined the energy of the proton; the energy of the neutron was then found from the relation

$$E_n = E_p/\cos^2\psi,$$

where all energies are in the laboratory system and the proton and neutron are taken to have the same mass m and non-relativistic velocities. This latter assumption is clearly acceptable in the region investigated, where $E_n \lessapprox 9$ Mev. The relation above follows readily from Hamilton's formula for kinetic energy and the law of conservation of momentum in the laboratory system:

$$E_n = \frac{|\vec{P_n}|^2}{2m}$$

$$|\vec{P_n}| = |\vec{P_p}| \cos\psi + |\vec{P_{n'}}| \sin\psi$$

$$|\vec{P_p}| \sin\psi = |\vec{P_{n'}}| \cos\psi$$

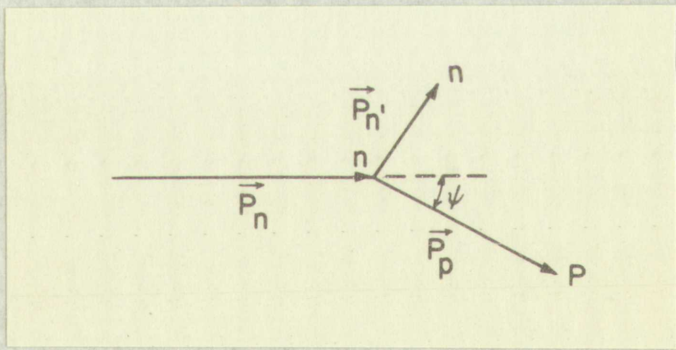

Fig. 4.--An n-p collision

Here E_n is the initial energy of the neutron, P_n the momentum of the incident neutron, E_p the energy of the recoil proton

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Relations for Ilford Nuclear Emulsions, A.E.R.E. G/R 664, Harwell, Berks, England (1951). Permission to reproduce this table is not forthcoming.

and $\vec{P}_p$ its momentum; $\vec{P}_n$, is the momentum of the neutron after the collision. is again the angle, in the laboratory system, between the direction of motion of the recoiling proton and that of the incident neutron. Combining the last two relations,

$$|\vec{P}_n| = |\vec{P}_p| \cos\psi + |\vec{P}_p| \tan\psi \sin\psi,$$

whence

$$E_n = \frac{|\vec{P}_n|^2}{2m} = \frac{|\vec{P}_p|^2}{2m} \frac{1}{\cos^2\psi} = E_p/\cos^2\psi.$$

Having performed these calculations, the machine then tabulated the data in order of increasing E_n.

In order to arrive at an expression for the incident flux of neutrons as a function of energy, it is necessary to consider a time-integrated flux $F(E_n \pm \Delta E_n/2)$ of neutrons of energy $E_n \pm \frac{1}{2}\Delta E_n$ incident on a nuclear emulsion of thickness t centimeters containing n hydrogen atoms per cubic centimeter. $F(E_n \pm \frac{1}{2}\Delta E_n)$ is the number of neutrons per square centimeter normal to the flux in the energy range $E_n \pm \frac{1}{2}\Delta E_n$. Let $\sigma_{n\text{-}p}(E_n)$ represent the n-p scattering cross section for neutrons of energy E_n, and let $N_p(E_p \pm \frac{1}{2}\Delta E_p)$ be the number of protons measured in the energy interval $E_p \pm \frac{1}{2}\Delta E_p$ corresponding to the energy interval $E_n \pm \frac{1}{2}\Delta E_n$, where $E_p = E_n \cos^2\psi$, ψ being the laboratory angle between the path of the incident neutron and that of the recoil proton. If Θ be the solid angle of acceptance in the laboratory corresponding to a maximum acceptable angle of α for dip angle ϕ or horizontal angle θ, then Ω, given by

$$\Omega = 16 \sin\alpha \tan^{-1}(\sin\alpha)$$

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$\bar{P}_1 = \bar{P}_3 \ldots$

where

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tabulated the ...

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energy E_n, and ...

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the energy ...

the laboratory ...

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acceptable ...

Θ, then Ω ...

$\Omega = \ldots$

is the corresponding solid angle in the center of mass system.[4] Assuming[5] that n-p scattering is isotropic in the center of mass system, the fraction of tracks which are counted is then

$$\Omega/4\pi .$$

Let $P(E_p)$ be the correction, to be specified in detail below, for protons of energy E_p leaving the emulsion, and

...the case in the immediate experiment.

...ce, if A be the area (cm^2) of emulsion analyzed,

$$N_p(E_p \pm \tfrac{1}{2}\Delta E_p)\, P(E_p) = \frac{F(E_n \pm \tfrac{1}{2}\Delta E_n)\, n\, A\, t\, \sigma_{n-p}(E_n)\,\Omega}{4\pi\, T(E_n)},$$

from which it is immediate that

$$F(E_n \pm \tfrac{1}{2}\Delta E_n) \quad \frac{4\pi}{\Omega\, \sigma_{n-p}(E_n)\, n\, A\, t}\, N_p(E_p \pm \tfrac{1}{2}\Delta E_p)\, P(E_p)\, T(E_n).$$

This is the required expression.

The correction $P(E_p)$ for the probability that a track made by a proton of energy E_p will leave the emulsion and hence not be counted may be calculated[6] on either of the

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[4] For the calculation of , see Appendix A.

[5] The n-p scattering process is known to be isotropic in the center of mass system for the energy range considered (0 to 8 Mev) to within 2%. Stephen R. White: "Photographic Plate Detection," in Fast Neutron Physics (New York: Interscience, 1960), I.

[6] The problem is not unrelated to Buffon's needle problem of probability theory.

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Let $P(E_p)$ be the correction, to be specified in detail below, for protons of energy E_p leaving the emulsion, and let $T(E_n)$ be the correction for attenuation of the incident neutron beam as it traverses the emulsion. The nuclear emulsion is considered to present an end-on view to the incident flux, as was the case in the immediate experiment. Then one can write, if A be the area (cm^2) of emulsion analyzed,

$$N_p(E_p \pm \tfrac{1}{2}\Delta E_p)\, P(E_p) = \frac{F(E_n \pm \tfrac{1}{2}\Delta E_n)\, n\, A\, t\, \sigma_{n-p}(E_n)\,\Omega}{4\pi\, T(E_n)},$$

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[6]The problem is not unrelated to Buffon's needle problem of probability theory.

is the corresponding solid angle in the center of mass system.[4] Accordingly, if the recoil proton is accepted in the center of mass system, the fraction of tracks which are counted is then

$$\Omega/4\pi .$$

Let $P(E_p)$ be the correction, to be described in detail below, for protons of energy E_p leaving the emulsion, and let $F(E_n)$ be the energy distribution of the incident neutron beam at the point of exposure. The nuclear emulsion is considered to present n hydrogen atoms per cm² to the incident flux, and the target is the irradiated area. Then one can write, for the number N_p of tracks [illegible]

$$N_p(E_p \pm \tfrac{1}{2}\Delta E_p)\, P(E_p) = \frac{F(E_n \pm \tfrac{1}{2}\Delta E_n)\, n A\, \sigma_{np}(E_n)\, \Omega}{4\pi\, T(E_p)}$$

from which it is immediate that

$$F(E_n \pm \tfrac{1}{2}\Delta E_n) = \frac{4\pi}{\Omega\, \sigma_{np}(E_n)\, n A} N_p(E_p \pm \tfrac{1}{2}\Delta E_p)\, P(E_p)\, T(E_p).$$

This is the standard expression.

The correction $P(E_p)$ for the probability that a track made by a proton of energy E_p will leave the emulsion and hence not be counted may be estimated[5] or estimated on the basis of the [illegible]

[4] For the calculation of Ω, see Appendix II.

[5] The most satisfying method is [illegible] in the center of mass system [illegible] (0 to 8 Mev) [illegible]. Stephen R. [illegible], "Photographic Plate Detection," in Fast Neutron Physics (New York: Interscience, 1960), I.

[6] The problem is not unrelated to [illegible] problem of probability theory.

following assumptions: (1) an insignificant number of protons undergo multiple scattering, or (2) as many protons are scattered back into the emulsion by a second scattering as are scattered out of the emulsion by a second scattering.[7] An empirical study based on the forms of 100 actual tracks for emulsions 200 μ thick has been carried out[8] for maximum dip angles of 10° and 15°, and the extrapolation of these results to a maximum dip angle of 16° was used in the present analysis. Clearly, since the emulsion may be considered infinitely wide in comparison with its depth, it is $R \cos \theta$ (see Fig. 3) and not the true range R which is directly concerned in the probability correction. Employing the range-energy relation and an average value for $\cos \theta$, one can obtain a graph of the fraction of protons which leave the emulsion versus the initial energy of the proton. Let $f(E_p)$ be the fraction of protons of energy E_p leaving the emulsion, and suppose n out of m tracks stay within the emulsion. Then it is clear that

$$m(1 - f(E_p)) = n,$$

or

$$m = \frac{n}{1 - f(E_p)},$$

whence

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[7] L. Rosen, op. cit.; H. T. Richards, A Photographic Plate Spectrum of the Neutrons from the Disintegration of Lithium by Deuterons, Phys. Rev. 59, 796-804 (1941).

[8] L. Rosen, op. cit.

$$P(E_p) = \frac{1}{1 - f(E_p)} \cdot$$

It is then a simple matter, making use of the relation

$$E_n = \overline{E_p}/\overline{\cos^2 \psi},$$

where $\overline{\cos^2 \psi}$ is the "average" value[9] of $\cos^2 \psi$ over the solid angle considered, to obtain the accompanying graph (Fig. 5) of $P(E_p)$ versus E_n, which is expected to be correct to within 3%.

The values of σ_{n-p} were taken from the compilation of Gammel[10] (partly reproduced in Table 1) and are considered correct to within 2%. The number of hydrogen atoms per cubic centimeter of the emulsion used is known to be 3.43 x 10^{22}, corresponding to a density of 0.057 gm/cm^3.[11] The correction $T(E_n)$ for attenuation of the neutron beam in traveling through the emulsion is discussed in some detail by Rosen.[12] By limiting the area of emulsion which is examined to a strip near the edge (0.2 to 0.5 centimeters in the present case), this corrective factor may be considered very close to unity.

Evaluation of the errors encountered in the use of photographic plates for detection of neutrons must rest to a great extent on the opinions of microscopists, as many of

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[9] See Appendix B.

[10] J. L. Gammel: "The n-p Total and Differential Cross Section in the Energy Range 0 to 40 Mev," in <u>Fast Neutron Physics</u> (New York: Interscience), II. Not yet published.

[11] J. J. Wilkins, <u>op</u>. <u>cit</u>., Fig. 1.

[12] L. Rosen, <u>op</u>. <u>cit</u>.

$$[illegible] = \frac{[illegible]}{1 - T(E_p)} \cdot$$

It is then a simple matter, [illegible]

$$E_p = E_n \overline{\cos^2 \psi},$$

where $\overline{\cos^2 \psi}$ is the [illegible] value of $\cos^2 \psi$ over the [illegible] angle considered, to obtain the [illegible] of $P(E_p)$ versus E_n which is expected to [illegible] within 3%.

The values of σ_{n-p} were taken from [illegible] of Gammel[10] (partly reproduced in Table I) and are [illegible] correct to within 2%. [illegible] cable contaminant of the emulsion [illegible] x 10^{22}, corresponding to a density [illegible] correction $T(E_p)$ for absorption of [illegible] traveling through the emulsion [illegible] by Rosen.[12] By limiting the area of [illegible] examined to a strip near the edge [illegible] in the present case), this correction [illegible] very close to unity.

Evaluation of the scanning [illegible] photographic plates for detection of [illegible] a great extent by the opinions of [illegible]

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[9]See Appendix B.

[10]J. L. Gammel "The n-p [illegible]" Section in the [illegible] Physics (New York: Interscience), [illegible]

[11]J. J. Wilkins, [illegible]

[12]L. Rosen, [illegible]

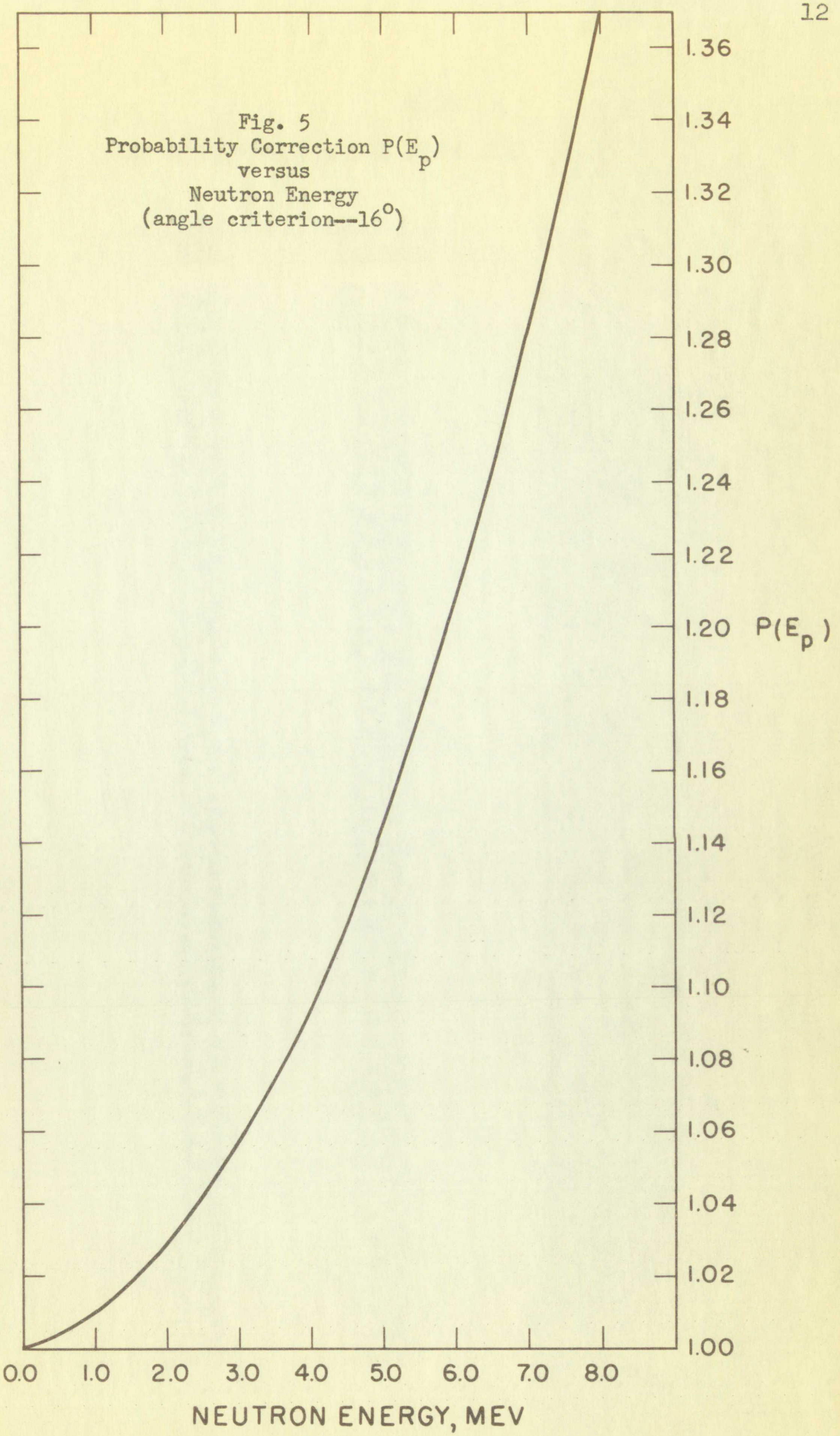

Fig. 5
Probability Correction $P(E_p)$
versus
Neutron Energy
(angle criterion--16°)

TABLE 1

N-P SCATTERING CROSS SECTIONS (GAMMEL'S FORMULATION)

E_n, Mev	$\sigma_{n-p}(E_n)$, Barns	E_n, Mev	$\sigma_{n-p}(E_n)$, Barns
0.0	20.34	4.5	1.75
0.1	12.79	4.6	1.72
0.2	9.70	4.7	1.70
0.3	8.00	4.8	1.67
0.4	6.92	4.9	1.65
0.5	6.16	5.0	1.62
0.6	5.60	5.1	1.60
0.7	5.16	5.2	1.58
0.8	4.80	5.3	1.56
0.9	4.51	5.4	1.54
1.0	4.26	5.5	1.52
1.1	4.05	5.6	1.50
1.2	3.86	5.7	1.48
1.3	3.69	5.8	1.46
1.4	3.55	5.9	1.44
1.5	3.41	6.0	1.42
1.6	3.29	6.1	1.40
1.7	3.18	6.2	1.39
1.8	3.08	6.3	1.37
1.9	2.99	6.4	1.35
2.0	2.90	6.5	1.34
2.1	2.82	6.6	1.32
2.2	2.75	6.7	1.31
2.3	2.68	6.8	1.29
2.4	2.61	6.9	1.28
2.5	2.55	7.0	1.26
2.6	2.49	7.1	1.25
2.7	2.43	7.2	1.23
2.8	2.38	7.3	1.22
2.9	2.33	7.4	1.21
3.0	2.28	7.5	1.20
3.1	2.23	7.6	1.18
3.2	2.19	7.7	1.17
3.3	2.15	7.8	1.16
3.4	2.11	7.9	1.15
3.5	2.07	8.0	1.13
3.6	2.03	8.1	1.12
3.7	1.99	8.2	1.11
3.8	1.96	8.3	1.10
3.9	1.93	8.4	1.09
4.0	1.89	8.5	1.08
4.1	1.86	8.6	1.07
4.2	1.83	8.7	1.06
4.3	1.80	8.8	1.05
4.4	1.77	8.9	1.04

the factors do not permit of more accurate determination. In particular, the accuracy with which the solid angle Θ in the laboratory frame (and hence Ω in the center of mass frame) can be delineated is subject to such uncertainty. Reasonable estimates of the non-statistical errors encountered are:[13]

σ_{n-p}	2%
$P(E_p)$	3%
$T(E_n)$	3%
n	5%
Ω	13%
t	3%

The root-mean-square error is hence 15%. Of these errors, the errors in n and t do not effect the relative distribution of neutrons, and the error in Ω has only a slight effect. (The error in Ω has some effect since Ω is better defined for long than for short tracks because of larger errors in determining horizontal and dip angles for short tracks.) A reasonable estimate of the preciseness with which the relative neutron distribution can be obtained is hence 10%.[14]

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[13] L. Stewart, Neutron Spectrum and Absolute Yield of a Plutonium-Beryllium Source, Phys. Rev. 98, No. 3, 740-743 (1955).

[14] See Rosen, op. cit.

the factors do not permit of more accurate determination. In particular, the accuracy with which the solid angle Θ in the laboratory frame (and hence Ω in the center of mass frame) can be delineated is subject to such uncertainty. Reasonable estimates of the non-statistical errors encountered are:[13]

σ_{n-p}	2%
$P(E_p)$	3%
$T(E_n)$	3%
n	5%
Ω	13%
t	3%

The root-mean-square error is hence 15%. Of these errors, the errors in n and t do not affect the relative distribution of neutrons, and the error in Ω has only a slight effect. (The error in Ω has some effect since Ω is better defined for long than for short tracks because of larger errors in determining horizontal and dip angles for short tracks.) A reasonable estimate of the preciseness with which the relative neutron distribution can be obtained is hence 10%.[14]

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[13] L. Stewart, Neutron Spectrum and Absolute Yield of a Plutonium-Beryllium Source, Phys. Rev. 98, No. 3, 740-743 (1955).

[14] See Rosen, op. cit.

III. EXPERIMENTAL RESULTS

The IBM tabulation of tracks according to energy of the incident neutron was divided into 0.1 Mev intervals, the number in each energy interval being the $N_p(E_p \pm \frac{1}{2} \Delta E_p)$ defined in the preceding chapter. From Fig. 5 the value of $P(E_p)$ corresponding to the central energy of the interval was found; from Table 1 was obtained the value of $\sigma_{n-p}(E_n)$ for this energy. The time-integrated neutron flux is then proportional to

$$N_p(E_p \pm \tfrac{1}{2} \Delta E_p)\ P(E_p)\ /\ \sigma_{n-p}(E_n),$$

the constant of proportionality being

$$4\pi/(\Omega\ n\ A\ t).$$

A small area was scanned analyzing all tracks greater than $3\ \mu$; to decrease the statistical uncertainty at high energies without undue expenditure of time, a larger area of the plate was examined with only tracks longer than a set minimum being recorded. Accordingly, the "constant" A (area scanned) was treated as a parameter having the form of a step function.

The flux obtained from the above expression clearly depends on the width of the energy interval considered. To remove this dependence, the flux indicated above was divided by the width ΔE_n of the interval (0.1 Mev). The result is

The [illegible]

the incident neutron [illegible]

number in each energy [illegible]

defined in the preceding [illegible]

$P(E_p)$ corresponding [illegible]

was found, from Table I [illegible]

for this energy. The [illegible]

proportional to

$$N_c(M_p \pm \Delta M_p) \ldots$$ [illegible]

the constant of proportionality being [illegible]

$N\pi/\Omega \ldots$ [illegible]

A small step was obtained [illegible]

3 μs to decrease the [illegible]

without undue [illegible]

was examined with only [illegible]

being recorded. [illegible]

assumed) was treated as [illegible]

step function.

The flux obtained [illegible]

depends on the width of the [illegible]

remove this dependency, [illegible]

by the width ΔE_p of [illegible] hence in units of neutrons cm^{-2}/Mev.

After the analysis described above, the data were grouped into larger intervals of energy (to improve statistics) and smoothed. This smoothing is justified since the errors involved in determining the range and angles of a track are large enough to prevent accurate assignment to a 0.1 Mev interval.

Although the background radiation was expected to be low since the critical assembly had been taken out-of-doors and hoisted some thirteen feet above the earth with the emulsions and shield even higher, a portion of a plate exposed to this radiation was analyzed. The neutron flux indicated by this plate was subtracted from the observed spectrum; the correction was of the order of 3% below 1 Mev, 1% from 1 to 2 Mev, 0.8% from 2 to 3 Mev, and less than 0.5% at higher energies.

The leakage neutron spectrum thus obtained for the U^{233} assembly is tabulated in Table 2 with statistical errors and plotted in Fig. 6. The leakage spectra of the structurally similar critical assemblies Godiva (U^{235}) and Jezebel (Pu^{239}) have also been measured[1] using a similar experimental arrangement; these spectra are tabulated in Tables 3 and 4, where the errors shown are the result of counting statistics only.

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[1]G. M. Frye, Jr., J. H. Gammel, and L. Rosen, Energy Spectrum of Neutrons from Thermal Neutron Fission of U^{235} and from an Untamped Multiplying Assembly of U^{235} (Los Alamos: Los Alamos Scientific Laboratory, 1954), TID-10073; L. Stewart, Leakage Neutron Spectrum from a Bare Pu^{239} Critical Assembly, Nuclear Sci. and Eng. 8, 596 (1960).

hence in units of neutrons per [illegible].

After the analysis [illegible] [illegible] [illegible] grouped into larger intervals [illegible] [illegible] and smoothed. This [illegible] [illegible] [illegible] involved in determining the [illegible] [illegible] large enough to [illegible] [illegible] [illegible] interval.

Although the background [illegible] [illegible] [illegible] low since the critical assembly [illegible] [illegible] and [illegible] some distance [illegible] [illegible] emulsions and shield [illegible] [illegible] to this radiation was [illegible]. [illegible] by this plate was subtracted from the [illegible] correction was of the order of [illegible] 2 Mev, 0.5% from 2 to [illegible] [illegible] energies.

The leakage neutron spectrum [illegible] [illegible] U^{235} assembly is tabulated in [illegible] and plotted in Fig. [illegible]. The [illegible] similar critical assemblies [illegible] have also been [illegible] [illegible] these spectra are [illegible] errors shown are the [illegible] [illegible]

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[1] [illegible]

TABLE 2

Leakage Neutron Spectrum of Jezebel U^{233}

Neutron Energy Interval, MeV	Neutrons/cm^2 per MeV	Statistical Uncertainty, %	Neutron Energy Interval, MeV	Neutrons/cm^2 per MeV	Statistical Uncertainty, %
--	--	--	3.2 - 3.6	94.3×10^6	9
0.3 - 0.4	646×10^6	11	3.6 - 4.0	83.6	10
0.4 - 0.6	520	10	4.0 - 4.4	57.9	12
0.6 - 0.8	391	7	4.4 - 4.8	41.6	14
0.8 - 1.0	306	7	4.8 - 5.2	42.6	15
1.0 - 1.2	324	8	5.2 - 5.6	32.3	15
1.2 - 1.4	299	8	5.6 - 6.0	23.5	22
1.4 - 1.6	230	10	6.0 - 6.4	14.3	30
1.6 - 1.8	209	11	6.4 - 6.8	13.6	30
1.8 - 2.0	182	10	6.8 - 7.2	6.0	50
2.0 - 2.2	166	10	7.2 - 7.6	8.4	40
2.2 - 2.4	157	10	7.6 - 8.0	4.4	60
2.4 - 2.6	148	10	8.0 - 8.4	1.7	100
2.6 - 2.8	138	10	8.4 - 8.8	5.2	60
2.8 - 3.0	119	11	8.8 - 9.2	2.5	90
3.0 - 3.2	108	12	9.2 - 9.6	3.1	80

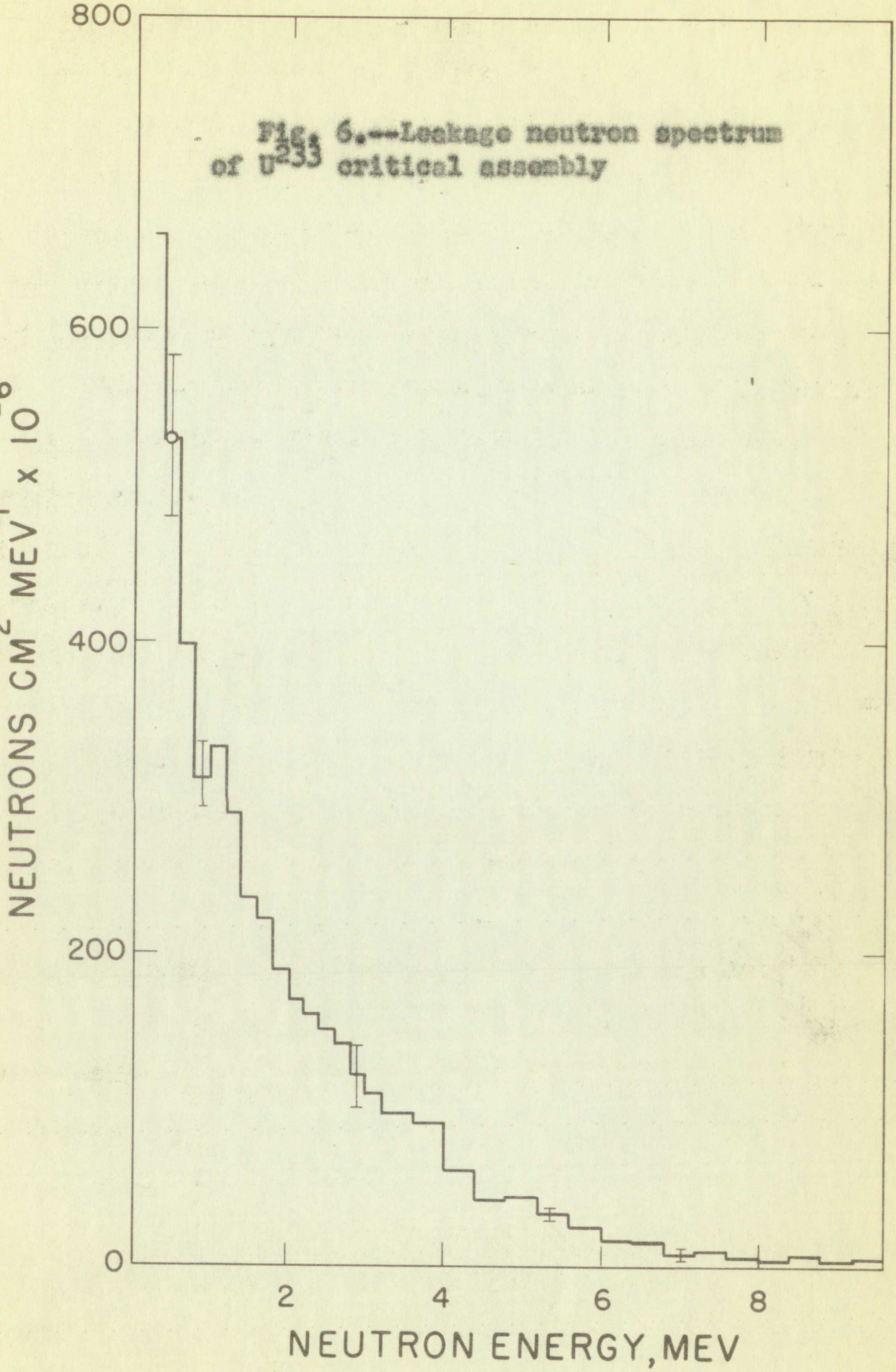

Fig. 6.--Leakage neutron spectrum of U^{233} critical assembly

TABLE 3

LEAKAGE NEUTRON SPECTRUM OF GODIVA U^{235}[a]

Neutron Energy Interval, Mev	Neutrons/cm^2 per Mev	Statistical Uncertainty, %	Neutron Energy Interval, Mev	Neutrons/cm^2 per Mev	Statistical Uncertainty, %
			3.0 - 3.4	84.6 x 10^5	9
0.2 - 0.4	721 x 10^5	4	3.4 - 3.8	70.5	8
0.4 - 0.6	718	3	3.8 - 4.2	62.1	9
0.6 - 0.8	612	4	4.2 - 4.6	40.7	11
0.8 - 1.0	499	4	4.6 - 5.0	42.9	11
1.0 - 1.2	371	5	5.0 - 5.4	33.6	14
1.2 - 1.4	320	6	5.4 - 5.8	21.7	16
1.4 - 1.6	257	7	5.8 - 6.2	12.9	24
1.6 - 1.8	218	8	6.2 - 6.6	10.8	27
1.8 - 2.0	224	8	6.6 - 7.0	11.4	27
2.0 - 2.2	193	7	7.0 - 7.4	13.9	25
2.2 - 2.4	159	8	7.4 - 7.8	1.9	70
2.4 - 2.6	141	9	7.8 - 8.2	2.2	70
2.6 - 2.8	122	9	8.2 - 8.6	6.6	40
2.8 - 3.0	136	9	8.6 - 9.0	2.4	70

[a]Table from Frye, et. al., op. cit.

TABLE 4

LEAKAGE NEUTRON SPECTRUM OF JEZEBEL Pu^{239}[a]

Neutron Energy Interval, Mev	Neutrons/cm^2 per Mev	Neutron Energy Interval, Mev	Neutrons/cm^2 per Mev
0.3 - 0.5	(289 ± 14) x 10^6	3.5 - 3.9	(50.3±5.2) x 10^6
0.5 - 0.7	250 ± 14	3.9 - 4.3	44.0 ± 5.0
0.7 - 0.9	224 ± 14	4.3 - 4.7	33.9 ± 4.5
0.9 - 1.1	177 ± 13	4.7 - 5.1	28.8 ± 4.3
1.1 - 1.3	176 ± 12	5.1 - 5.5	16.6 ± 3.3
1.3 - 1.5	164 ± 13	5.5 - 5.9	16.5 ± 3.4
1.5 - 1.7	152 ± 12	5.9 - 6.3	8.2 ± 2.5
1.7 - 1.9	125 ± 11	6.3 - 6.7	11.2 ± 3.0
1.9 - 2.1	122 ± 12	6.7 - 7.1	9.7 ± 2.9
2.1 - 2.3	96.1 ± 8.4	7.1 - 7.5	4.7 ± 2.1
2.3 - 2.5	87.0 ± 8.2	7.5 - 7.9	4.3 ± 2.1
2.5 - 2.7	85.7 ± 8.3	7.9 - 8.3	2.1 ± 1.5
2.7 - 3.1	70.1 ± 5.6	8.3 - 8.7	4.9 ± 2.5
3.1 - 3.5	61.5 ± 5.5	8.7 - 9.1	1.2 ± 1.2

[a]Table from Stewart, Leakage Neutron Spectrum.

For Fig. 7 the three spectra have been normalized so that the area under each curve from 0.3 to 9.5 Mev is the same. The figure shows[2] that the leakage spectra of the U^{233} and Pu^{239} assemblies are quite similar and that both are "harder" than that of the U^{235} assembly. By being "harder" is meant having relatively more high-energy neutrons. These observations are in keeping with the known cross sections and fission spectra of these elements.

The discontinuity at 1.1 Mev is of questionable significance since the statistical uncertainties are fairly large in this region. Note, however, that there is an indication of such a discontinuity at the same energy in the Pu^{239} spectrum. Possibly some resonance phenomenon is indicated.

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[2]See also Table 6 of the following chapter.

For Fig. 7 the three spectra have been normalized so that the area under each curve from 0.3 to 9.5 Mev is the same. The figure shows[2] that the leakage spectra of the U^{233} and Pu^{239} assemblies are quite similar and that both are "harder" than that of the U^{235} assembly. By being "harder" is meant having relatively more high-energy neutrons. These observations are in keeping with the known cross sections and fission spectra of these elements.

The discontinuity at 4.1 Mev is of questionable significance since the statistical uncertainties are fairly large in this region. Note, however, that there is an indication of such a discontinuity at the same energy in the Pu^{239} spectrum. Possibly some resonance phenomenon is indicated.

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[2]See also Table 6 of the following chapter.

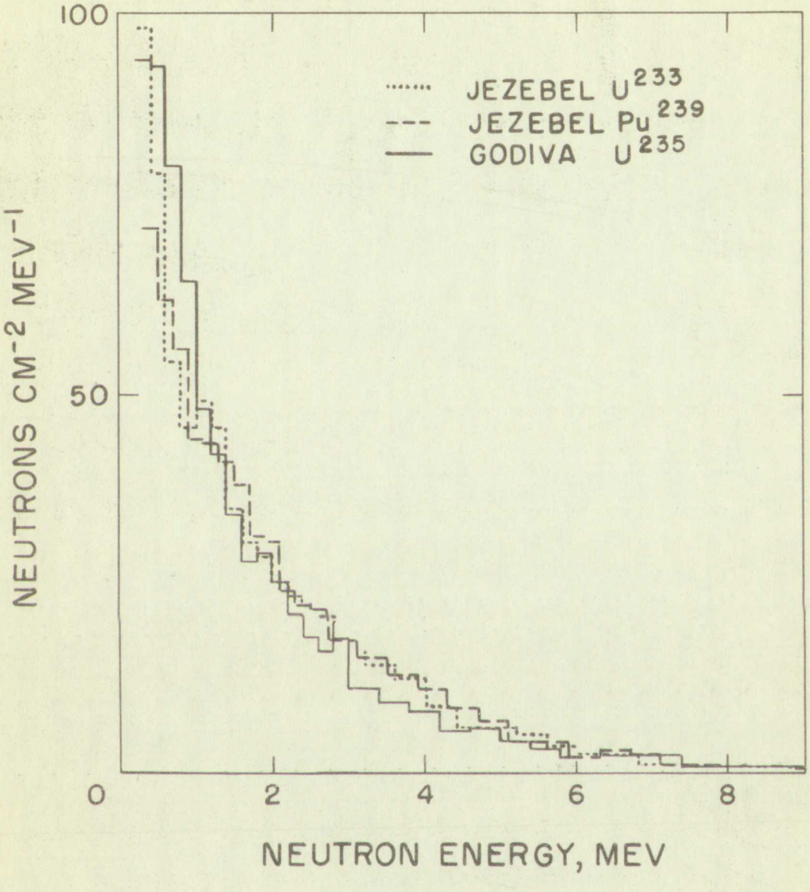

Fig. 7.--Leakage neutron spectra of three critical assemblies

IV. APPLICATION OF THE NEUTRON TRANSPORT EQUATION TO SPHERICAL CRITICAL ASSEMBLIES

The leakage spectrum of so structurally simple an assembly as the bare sphere of U^{233} here investigated may be determined by solving numerically the neutron transport equation. Assuming for the moment that all scattering is isotropic in the laboratory system, the mathematical form of the transport equation (or Boltzmann equation)[1] for neutrons may be readily derived. Let the following definitions be laid down:

t --- time

$\vec{r}$ --- position vector of the neutron

v --- speed of the neutron

$\vec{\Omega}$ --- unit vector in the direction of motion of the neutron

$n\, dV\, dv\, d\Omega \equiv n(\vec{r}, v\vec{\Omega}, t)\, dV\, dv\, d\Omega$ --- the number of neutrons at time t in the volume element dV around the point defined by $\vec{r}$ having a speed between v and $v + dv$ in a direction lying within a differential solid angle $d\Omega$ about the direction $\vec{\Omega}$

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[1]For the general form of this equation see B. Davison: Neutron Transport Theory (Oxford: At the Clarendon Press, 1957), pp. 15-16, on which the present development is based.

IV. [illegible]

[illegible]

[illegible] assembly as the [illegible] be determined by [illegible] equation. [illegible] isotropic in [illegible] of the transport [illegible] neutrons may be [illegible] be laid down:

t --- time

$\vec{r}$ --- position vector [illegible]

v --- speed [illegible]

$\vec{\Omega}$ --- unit vector [illegible]

[illegible]

n dV dv dΩ = [illegible]

[illegible]

[illegible]

$S\,dV\,dv\,d\Omega\,dt \equiv S(\vec{r}, v\vec{\Omega}, t)\,dV\,dv\,d\Omega\,dt$ --- the number of neutrons having speed between v and v + dv and moving in a direction within the differential solid angle $d\Omega$ about the direction $\vec{\Omega}$ arising in dV in the time dt due[2] to both fission and scattering.

Consider a packet of neutrons with velocity $v\vec{\Omega}$ and position $\vec{r} + tv\vec{\Omega}$ at times close to t = 0. Then the net increase in the number of neutrons in this packet is, by definition,

$$\frac{dn}{dt}\,dV\,dv\,d\Omega\,dt$$

as t increases by dt.

It is clear that neutrons may be removed from the packet by capture and by scattering out of the packet. Since a neutron which undergoes a scattering process must change either speed or direction of travel, for otherwise no scattering can be said to have taken place, and since the probability that a neutron emitted during fission will have the same speed and direction as the neutron causing the fission is of the second order in small quantities, decrease of the number of neutrons in the packet in time dt due to scattering and capture (with or without subsequent fission) is simply the number of neutrons undergoing collisions in the time dt. Using $\sigma(v)$ for the total probability of

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[2]It is assumed herein that no sources independent of the neutron flux are present. This assumption implies, among other considerations, that spontaneous fission is negligible.

[illegible]

Consider [illegible]

position $\vec{r} + h\vec{\Omega}$ [illegible]

increase in the [illegible]

definition,

[illegible]

as t increases by [illegible]

It is clear [illegible]

packet by neutron [illegible]

Since a neutron [illegible]

change either [illegible]

no scattering [illegible]

probability that [illegible]

the same speed and [illegible]

fission is of [illegible]

of the number of [illegible]

scattering and [illegible]

is simply the [illegible]

the time dt. [illegible]

[illegible] the neutron [illegible] many other [illegible] negligible.

collision for neutrons of velocity v, it is clear that the decrease in time dt due to these causes is given by

$$\sigma(v)\, n\, v\, dV\, dv\, d\Omega\, dt,$$

where nv is the number of neutrons passing through unit area perpendicular to $\vec{\Omega}$ per second and $n\, v\, dV\, dv\, d\Omega\, dt$ is hence the number passing through dV in time dt. Here, and throughout this section, σ will be assumed to be in units of cm^{-1}; i. e., obtained from the conventional cross section (cm^2) by multiplying this cross section by the number of nuclei per cm^3 of the element considered. Thus σ as used here is the reciprocal of the mean free path.

The increase in the number of neutrons in the packet due to scattering into the packet is due to collisions occuring in dV, and similarly the increase due to fission is due to fissioning atoms lying in dV. Hence this increase is

$$S(\vec{r}, v\vec{\Omega}, t)\, dV\, dv\, d\Omega\, dt.$$

Recalling from vector analysis that

$$\frac{dn}{dt} = D_t n + v\vec{\Omega} \cdot \nabla n,$$

collecting results, and cancelling differentials, there results the expression

$$D_t n + v\vec{\Omega} \cdot \nabla n + \sigma(v) nv = S(\vec{r}, v\vec{\Omega}, t)$$

for neutron balance: the net rate of increase of neutrons in the given packet plus the number of neutrons lost from the packet per second equals the rate at which neutrons are created in the packet. For a time-independent problem such as that considered,

$$v\vec{\Omega}\cdot\nabla n + \sigma(v)nv = S(\vec{r}, v\vec{\Omega}).$$

Assuming[3] that all processes involved in S are isotropic in the laboratory frame, and further restricting the discussion to a system possessing spherical symmetry ($\vec{r} \longrightarrow r$), one has

$$v\vec{\Omega}\cdot\nabla\, n(r, v\vec{\Omega}) + \sigma(v) n(r, v\vec{\Omega})v = S(r, v).$$

Since v is a constant scalar, one can write this as

$$\vec{\Omega}\cdot\nabla(vn(r, v\vec{\Omega})) + \sigma(v)vn(r, v\vec{\Omega}) = S(r, v).$$

Introducing the angular neutron flux

$$N(r, v\vec{\Omega})\, dV\, dv\, d\Omega \equiv v\, n(r, v\vec{\Omega})\, dV\, dv\, d\Omega,$$

i.e., the number of neutrons in the volume dV having speeds between v and v + dv and directions of travel within the differential solid angle $d\Omega$ about Ω passing through one square centimeter perpendicular to Ω per second, the transport equation becomes

$$\vec{\Omega}\cdot\nabla N(r, v\vec{\Omega}) + \sigma(v) N(r, v\vec{\Omega}) = S(r, v).$$

In order to put this equation into the multigroup form, one considers G (arbitrary) energy groups 1,2,...,g,... ...,G and integrates the equation over the values of v in each energy group. For group g the transport equation thus takes the form

$$\vec{\Omega}\cdot\nabla N_g(r, \vec{\Omega}) + \sigma_g N_g(r, \vec{\Omega}) = S_g(r),$$

where $N_g(r, \vec{\Omega})$ is the flux in direction $\vec{\Omega}$ of neutrons belonging to the g^{th} group and $S_g(r)$ is the source term for neutrons with energies lying in the g^{th} velocity group:

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[3]A discussion of the validity of this assumption may be found in Davison, op. cit., Chapter 1. See also below for the case of elastic scattering.

$$N_g(r,\vec{\Omega}) \equiv \int_{V_g} N(r,v\vec{\Omega})\, dv,$$

$$S_g(r) \equiv \int_{V_g} S(r,v)\, dv,$$

$$\sigma_g \equiv \frac{1}{\Delta V_g}\int_{V_g} \sigma(v)\, dv,$$

where $\int_{V_g}$ indicates integration over all velocities belonging in group g. Further simplification is possible when the angular neutron flux depends only on the radial coordinate r and the angle between r and the direction $\vec{\Omega}$ of the flux. Since the cosine of this angle is

$$\mu \equiv \frac{\vec{r}\cdot\vec{\Omega}}{r},$$

one can replace $N(r,\vec{\Omega})$ by

$$N(r,\mu) = N(r,\frac{\vec{r}\cdot\vec{\Omega}}{r}).$$

In terms of the variables r and μ one has for the first term in the transport equation

$$\vec{\Omega}\cdot\nabla N(r,\frac{\vec{r}\cdot\vec{\Omega}}{r}) = \mu D_r N + (D_\mu N)\vec{\Omega}\cdot\nabla(\frac{\vec{r}\cdot\vec{\Omega}}{r}).$$

Using well-known vector identities the gradient in the last term can be expanded as

$$\nabla(\frac{\vec{r}\cdot\vec{\Omega}}{r}) = \frac{1}{r}\nabla(\vec{r}\cdot\vec{\Omega}) - \frac{1}{r^2}(\vec{r}\cdot\vec{\Omega})\nabla r$$

$$= \frac{1}{r}\{\vec{r}\times(\nabla\times\vec{\Omega}) + \vec{\Omega}\times(\nabla\times\vec{r}) + (\vec{r}\cdot\nabla)\vec{\Omega}$$

$$+ (\vec{\Omega}\cdot\nabla)\vec{r}\} - \frac{1}{r^2}(\vec{r}\cdot\vec{\Omega})\frac{\vec{r}}{r}$$

$$= \frac{1}{r}(\vec{\Omega}\cdot\nabla)\vec{r} - \frac{1}{r^3}(\vec{r}\cdot\vec{\Omega})\vec{r},$$

since $\vec{\Omega}$ is a constant vector and

$$\nabla \times \vec{r} = 0,$$

$\vec{r}$ being a position vector. Thus

$$\vec{\Omega} \cdot \nabla N(r,\mu) = \mu D_r N + \left(\frac{1}{r}\vec{\Omega}\cdot\vec{\Omega} - \frac{(r\cdot\vec{\Omega})^2}{r^3}\right) D_\mu N$$

$$= \mu D_r N + \frac{1-\mu^2}{r} D_\mu N,$$

and the transport equation for the problem considered assumes the form

$$\left(\mu D_r + \frac{1-\mu^2}{r} D_\mu + \sigma_g\right) N_g(r,\mu) = S_g(r).$$

σ_g, the total cross section for the removal of a neutron from the gth group, is obviously given by

$$\sigma_g = \sigma_g^e + \sigma_g^i + \sigma_g^a + \sigma_g^f,$$

where σ_g^e is the cross section for elastic scattering, σ_g^i that for inelastic scattering, σ_g^f the cross section for capture followed by fission, and σ_g^a the cross section for capture followed by any other process, all for neutrons in the velocity group g. When corrected for anisotropic scattering in the manner noted below, σ_g is known as the transport cross section.

It is now necessary to examine in detail the source term $S_g(r)$, in which are included all sources of neutrons in velocity group g, whether from fission or from scattering from another or the same[4] group. $S_g(r)$ may clearly be written in the standard form

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[4]The present consideration must be carefully distinguished from that of the discussion on page 24.

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$$\nabla \times \bar{r} = 0,$$

$\bar{r}$ being a position vector. Thus

$$\bar{\Omega}\cdot\nabla N(r,\mu) = \mu D_r N + \left(\frac{1}{r}\bar{\Omega}\cdot\bar{\Omega} - \frac{(\bar{r}\cdot\bar{\Omega})^2}{r^3}\right) D_\mu N$$

$$= \mu D_r N + \frac{1-\mu^2}{r} D_\mu N,$$

and the transport equation for the problem considered assumes the form

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*The present consideration must be carefully distinguished from that of the discussion on page 24.

$$S_g(r) = \sum_{g'} \sigma_{gg'} N_{g'}(r),$$

where $N_{g'}(r)$ is the total flux in group g', defined as

$$N_{g'}(r) = \frac{1}{2}\int_{-1}^{1} N_{g'}(r,\mu)\, d\mu.$$

The $\sigma_{gg'}$ (cm^{-1}) are known as transfer coefficients. To determine the form of $\sigma_{gg'}$ from (hopefully) known cross sections, one makes the following definitions:

$\eta_{gg'}$ --- the fraction of neutrons scattered into velocity group g by neutrons of group g' undergoing inelastic (or velocity-degrading) scattering, such that

$$\sum_g \eta_{gg'} = 1,$$

where g and g' are such that $v_g \leq v_{g'}$

$\nu_{g'}$ --- the average number of neutrons per fission due to a neutron of group g'

$\nu_{gg'}$ --- the relative number of neutrons born into group g per fission due to a neutron of group g', such that

$$\sum_g \nu_{gg'} = \nu_{g'}.$$

It should be clear that the transfer coefficients are then given by

$$\sigma_{gg'} = \sigma_{g'}^{e}\delta_g^{g'} + \sigma_{g'}^{i}\eta_{gg'} + \sigma_{g'}^{f}\nu_{gg'},$$

where $\delta_g^{g'}$ is the Kronecker delta.

Of the interactions involved above, σ_g^i and σ_g^e may be anisotropic, i.e., may depend on μ. In this case the above equations may still retain approximate validity if σ_g^i and σ_g^e

are replaced by corresponding integrals over μ and there is subtracted from $\sigma_{g'}$ and $\sigma_{g'g'}$ a certain parameter correction $\epsilon_{g'}$ defined[5] as

$$\epsilon_{g'} \equiv \int_{-1}^{1} \mu \{\sigma_{g'}^{e}(\mu) + \sigma_{g'}^{i}(\mu)\} d\mu.$$

This <u>transport approximation</u> is apparently accurate, a fact which constitutes its major justification. The alternative to using this approximation is to solve the anisotropic problem.[6]

The transport equation is now completely specified for the case of spherical geometry. One generalization of the equation is possible and, indeed, necessary: a review of the preceding development will reveal that the various cross sections employed may be allowed to depend on the radial coordinate. In particular, it is necessary to let σ be a step function in r, such that σ is zero beyond the radius of the sphere of fissile material.

Preparation of the transport equation for numerical solution by the S_n method is discussed in Carlson's report.[7] Briefly, the system of differential equations for neutron transport is reduced to a set of coupled difference equations in intervals of r and μ, and these solved by iteration.

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[5] Bengt G. Carlson, <u>Solution of the Transport Equation by S_n Approximations</u> (Los Alamos: Los Alamos Scientific Laboratory, 1955), LA-1891, pp. 7-8.

[6] For which see Carlson, <u>ibid</u>., Sec. 9.

[7] Carlson, <u>ibid</u>.

are replaced [illegible] is subtracted from σ_t and [illegible]

$\sigma_{s'}$ defined[5] as

$$\sigma_{s'} \equiv \int [illegible]$$

This [illegible] which constitutes [illegible] to using this [illegible] problem.[6]

The transport [illegible] for the case of [illegible] of the equation [illegible] of the preceding [illegible] cross sections [illegible] coordinate. [illegible] step function [illegible] of the sphere of [illegible]

Preparation [illegible] solution by the [illegible] Briefly, the [illegible] transport is [illegible] in intervals of [illegible]

---

[5] [illegible] by [illegible] Laboratory, [illegible]

[6] For [illegible]

[7] Carlson, [illegible]

For the present investigation, 8 intervals of μ (the S_8 approximation) and 18 of r were taken, and the problem solved by means of the existing code[8] for the IBM 704. The cross sections, fission spectrum, etc., used are the six-group set presented in a Los Alamos report[9] and reproduced in Table 5. The following boundary conditions are necessary to complete the specification of the problem for the bare sphere:

a) Conservation of flux at the center:

$$N(0,\mu) = N(0,-\mu)$$

b) Derivation of all neutrons from the critical assembly:

$$N(a,-\mu) \equiv 0,$$

where a is the radius of the assembly.

Among the output of the computer is the relative neutron leakage flux in each energy interval used. For comparison with the experimentally determined spectrum, the theoretical leakage spectrum is normalized so that

$$\sum_{g=3}^{6} N_g = 1,$$

---

[8]Bengt G. Carlson, The S_n Method and the SNG Code (Los Alamos: Los Alamos Scientific Laboratory, 1959), LAMS-2201.

[9]Gordon E. Hansen and William H. Roach, Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies (Los Alamos: Los Alamos Scientific Laboratory, 1961), LAMS-2543. These cross sections must be multiplied by the number of nuclei per cm^3. Since Jezebel U233 contained some U234 (1.24%) and U238 (0.06%), the cross sections for U233 were corrected slightly by taking a weighted average. The stated composition corresponds to a density of 18.45 gm/cm^3.

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TABLE 5

SIX GROUP PARAMETERS FOR U^{233}

(CROSS SECTIONS IN BARNS)

Group No.	1	2	3	4	5	6
Energy Range, Mev	0-0.1	0.1-0.4	0.4-0.9	0.9-1.4	1.4-3.0	3.0-∞
Fission Spectrum	0.014	0.090	0.180	0.168	0.344	0.204
ν_g	2.51	2.53	2.57	2.61	2.70	3.02
σ_f	3.23	2.24	1.94	1.89	1.83	1.75
σ_a	0.40	0.15	0.11	0.08	0.06	0.04
$\sigma_{transport}$	11.8	8.1	5.3	4.6	4.5	4.25
$\sigma_{g\to g}$	8.17	5.66	2.91	1.82	1.53	1.19
$\sigma_{g\to g-1}$		0.05	0.29	0.45	0.18	0.20
$\sigma_{g\to g-2}$			0.05	0.30	0.50	0.27
$\sigma_{g\to g-3}$				0.06	0.35	0.45
$\sigma_{g\to g-4}$					0.05	0.31
$\sigma_{g\to g-5}$						0.04
$\nu_g\sigma_f$	8.107	5.667	4.986	4.933	4.941	5.285

and then N_g, the neutron flux in energy group g, is divided by E_g, the width of the gth group in Mev. (For group 6, E_g is taken as 6.5 Mev; i.e., it is assumed that there are no neutrons with energies greater than 9.5 Mev.) This normalization yields neutrons cm^{-2}/Mev. The experimental spectrum is lumped into the same four groups in a similar manner, where now

$$N_g = \int_{\Delta E_g} F(E)\, dE \Bigg/ \int_{0.4\ \mathrm{Mev}}^{9.5\ \mathrm{Mev}} F(E)\, dE .$$

These quantities are displayed, along with the ratio of the theoretically calculated and experimentally measured quantities, in Table 6. Included in this table are the same quantities for the bare-sphere assemblies Jezebel Pu^{239} and Godiva U^{235}.[10] It is seen that the theoretical result is not in violent disagreement with that observed; certainly, no condemnation of the cross sections, etc., employed in the theoretical treatment can be made on this basis.

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[10] L. Stewart, <u>Leakage Neutron Spectrum</u>.

TABLE 6

COMPARISON OF THEORETICAL AND EXPERIMENTAL LEAKAGE SPECTRA

	0.4-0.9 Mev	0.9-1.4 Mev	1.4-3.0 Mev	3.0-∞ Mev
Jezebel Pu^{239}				
Theoretical	0.481	0.390	0.216	0.034
Experimental	0.488	0.342	0.211	0.038
Ratio[a]	0.99	1.14	1.02	0.89
Jezebel U^{233}				
Theoretical	0.513	0.384	0.218	0.031
Experimental	0.510	0.372	0.203	0.036
Ratio[a]	1.01	1.03	1.07	0.86
Godiva U^{235}				
Theoretical	0.634	0.392	0.197	0.026
Experimental	0.658	0.392	0.189	0.027
Ratio[a]	0.96	1.00	1.04	0.96

[a]Theoretical flux/Experimental flux.

APPENDIX A

CALCULATION OF Ω

Elementary considerations of collisions between equally massive particles yield the results

$$d^2\Omega_{LAB} \equiv d^2\Theta = \sin\theta \, d\theta \, d\phi ,$$

$$d^2\Omega_{CM} \equiv d^2\Omega = 4 \cos\theta \, d^2\Theta = 4 \cos\theta \, \sin\theta \, d\theta \, d\phi ,$$

where all angles are measured in the laboratory reference frame. Along the edge AB (Fig. 8) one has

$$z = \rho \cos\theta = \text{const.},$$

$$x = \rho \sin\theta \cos\phi = \text{const.},$$

whence

$$\frac{x}{z} = \tan\theta \cos\phi = \tan\alpha ,$$

where α is the maximum permissible value of the horizontal or dip angle (assumed equal in this analysis); i.e., α is the half angle of the pyramid of acceptance. Thus

$$\theta = \tan^{-1} \left(\frac{\tan\alpha}{\cos\phi}\right)$$

along edge AB. It follows that

$$\Omega = 8 \int_0^{\pi/4} \int_0^{\tan^{-1}\left(\frac{\tan\alpha}{\cos\phi}\right)} 4 \cos\theta \, \sin\theta \, d\theta \, d\phi$$

$$= 32 \int_0^{\pi/4} \int_0^{\cos^{-1}\left(\frac{\cos\phi}{\sqrt{\cos^2\phi + \tan^2\alpha}}\right)} \cos\theta \, \sin\theta \, d\theta \, d\phi$$

$$= 16 \tan^2\alpha \int_0^{\pi/4} \frac{d\phi}{\tan^2\alpha + \cos^2\phi} .$$

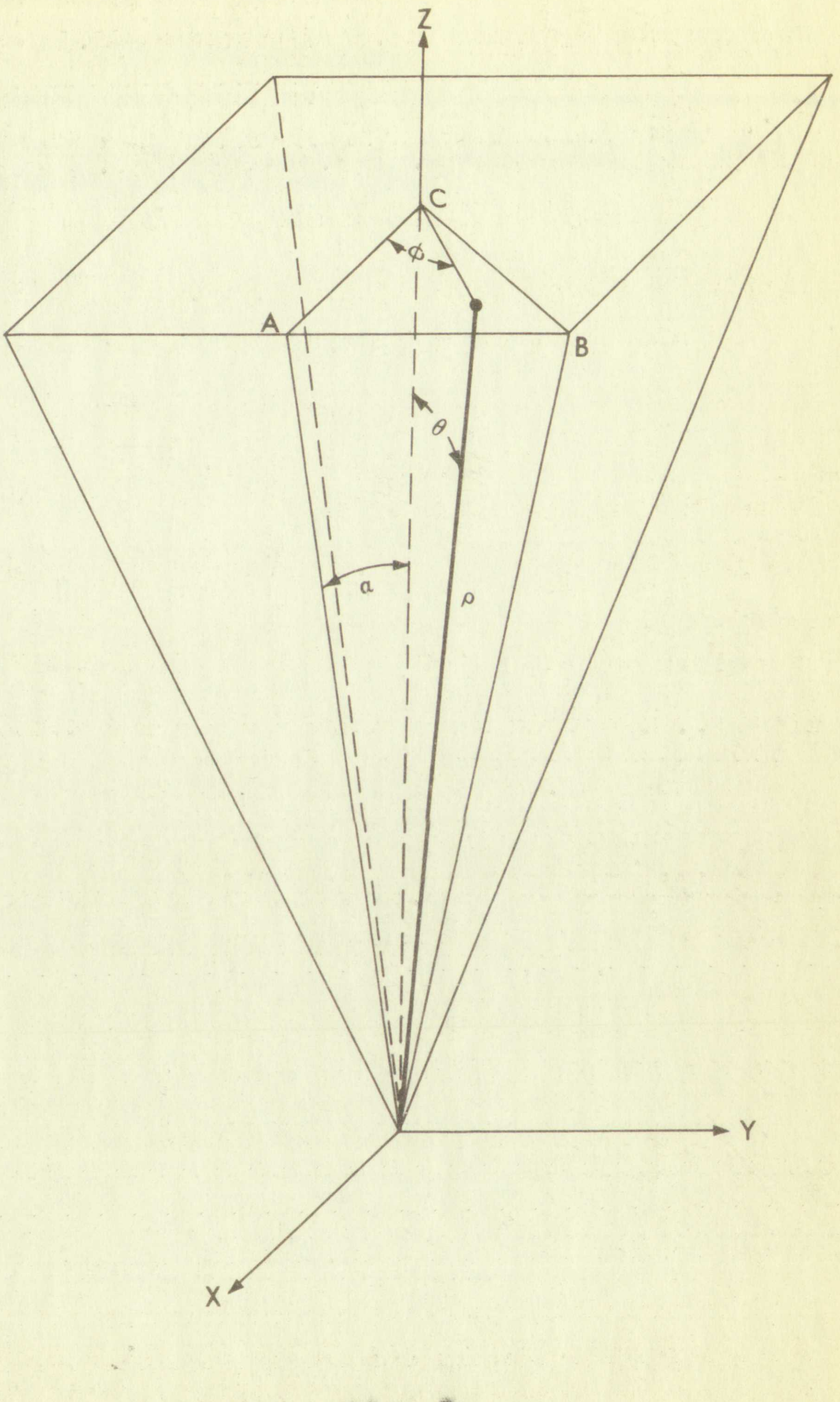

Fig. 8.

Evaluating the final integral,[1]

$$\Omega = 16 \sin\alpha \tan^{-1}(\sin\alpha).$$

For the case at hand $\alpha = 16^{\circ}$ and

$$\Omega = 1.186 \text{ sterradians.}$$

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[1] B. O. Pierce, A Short Table of Integrals (3d ed.; Boston: Ginn and Co., 1929), No. 315.

APPENDIX B

APPROXIMATE CALCULATION OF $\overline{\cos^2\psi}$

To sufficient accuracy (1%) the square-based pyramidal solid angle of observation may be considered replaced by a right circular cone such that both bases have the same area. Let a be the length of the edge of the pyramid and b the slant height of the cone. Then, if α be the half-angle for the pyramid and ψ_{max} that of the cone,

$$\pi a^2 \sin^2\psi_{max} = 4\, b^2 \sin^2\alpha ,$$

whence, to a good approximation for α and ψ_{max} small,

$$\pi \sin^2\psi_{max} = 4 \sin^2\alpha .$$

Thus, since in the laboratory system $\frac{d\sigma}{d\Theta} = 4\sigma\cos\Theta$,

$$\overline{\cos^2\psi} = \frac{\int_0^{\psi_{max}} \cos^2\psi \; 2\pi\sin\psi \; 4\sigma\cos\psi \; d\psi}{\int_0^{\psi_{max}} 2\pi\sin\psi \; 4\sigma\cos\psi\, d\psi}$$

$$= \tfrac{1}{2}(\cos^2\psi_{max} + 1).$$

For $\alpha = 16^\circ$, $\psi_{max} = 18^\circ 7'$, and $\overline{\cos^2\psi} = 0.952$.

APPENDIX C

RAW DATA

E_n (MeV)	N_p	Volume Scanned (cm^3)	E_n (MeV)	N_p	Volume Scanned (cm^3)
0.35	86	0.475×10^{-4}	4.35	12	5.740×10^{-4}
0.45	54	"	4.45	12	"
0.55	47	"	4.55	19	"
0.65	142	1.924×10^{-4}	4.65	11	"
0.75	122	"	4.75	7	"
0.85	98	"	4.85	9	"
0.95	70	"	4.95	10	"
1.05	89	"	5.05	17	"
1.15	82	"	5.15	12	"
1.25	98	"	5.25	10	"
1.35	48	"	5.35	8	"
1.45	57	"	5.45	7	"
1.55	51	"	5.55	7	"
1.65	40	"	5.65	9	"
1.75	45	"	5.75	9	"
1.85	35	"	5.85	2	"
1.95	69	3.834×10^{-4}	5.95	4	"
2.05	50	"	6.05	2	"
2.15	63	"	6.15	4	"
2.25	60	"	6.25	3	"
2.35	48	"	6.35	4	"
2.45	46	4.315×10^{-4}	6.45	1	"
2.55	66	4.800×10^{-4}	6.55	4	"
2.65	59	"	6.65	4	"
2.75	43	"	6.75	2	"
2.85	46	"	6.85		"
2.95	42	"	6.95	3	"
3.05	37	"	7.05		"
3.15	37	"	7.15	1	"
3.25	33	"	7.25	1	"
3.35	33	"	7.35	2	"
3.45	34	5.740×10^{-4}	7.45	1	"
3.55	29	"	7.55	2	"
3.65	32	"	7.65	2	"
3.75	20	"	7.75	1	"
3.85	34	"	8.25	1	"
3.95	36	"	8.65	3	"
4.05	23	"	9.05	1	"
4.15	20	"	9.25	1	"
4.25	16	"	9.35	1	"