NATURAL RESOURCES JOURNAL

Volume 21
Issue 2 *Spring 1981*

Spring 1981

The Economics of Outer Space

Todd Sandler

William Schulze

Recommended Citation

Todd Sandler & William Schulze, *The Economics of Outer Space*, 21 Nat. Resources J. 371 (1981).
Available at: https://digitalrepository.unm.edu/nrj/vol21/iss2/10

This Article is brought to you for free and open access by the Law Journals at UNM Digital Repository. It has been accepted for inclusion in Natural Resources Journal by an authorized editor of UNM Digital Repository. For more information, please contact amywinter@unm.edu, lsloane@salud.unm.edu, sarahrk@unm.edu.

THE ECONOMICS OF OUTER SPACE*

TODD SANDLER and WILLIAM SCHULZE**

INTRODUCTION

With the advent of the space shuttle, a new natural resource—outer space—will become increasingly available for exploitation. This article surveys economic issues in the development of outer space. We approach this task by identifying the principal uses of this resource. Thus, the article is organized to provide a brief economic look at earth based activities (telecommunication satellites, military applications, and scientific exploration) as well as possible space based exploitation (space manufacturing, solar power satellites, and space colonization).

Three areas of economic analysis have special relevance to the outer space resource. First, outer space is a common property resource, in many ways similar to the high seas. This characteristic implies that allocation problems may be severe because of externalities and public good problems. As an example we construct a formal economic model for optimal allocation of telecommunication satellites and examine the efficiency of the existing institutional structure. Second, in many cases the technology for outer space exploitation shows decreasing cost characteristics, consistent with natural monopoly resulting from scale economies. While exploitation to this point has been principally by government monopoly, this may not be the case in the future. As the private sector moves to develop further space resources, regulation may become important. Thus we attempt to identify those areas where natural monopolies might occur.

Third, proposals by the National Aeronautics and Space Administration (NASA) and futurists for space manufacturing, power production, and space colonization, if realized, could have a major impact on world economic growth and distribution of wealth. However, economic feasibility, no matter how attractive the arguments of proponents, remains a distant and open question for such applications.

---

*The authors appreciate helpful comments provided by Allen Kneese, Robert Ayres, David Brookshire, Shelby Gerking, John Tschirhart, Elizabeth Knoer, and Kate Ranta. Full responsibility rests with the authors.

**Professor and Associate Professor, respectively, of Economics, University of Wyoming, Laramie 82071.

To demonstrate the economic uncertainty surrounding such proposals we focus on an economic analysis of O'Neill's scheme for space colonization presented in his book, *The High Frontier* (1977). The development of a "space economy," as proposed by O'Neill, can be simply analyzed using the traditional Harrod-Domar model of economic growth. We then apply benefit-cost analysis to the scheme to test the sensitivity of economic feasibility to O'Neill's underlying assumptions.

SOME ECONOMIC ASPECTS OF TELECOMMUNICATION SATELLITES

In this section, the allocative efficiency of market and non-market arrangements for satellite telecommunication networks are examined. In particular, the analysis shows that a "club arrangement," charging members (or users) a toll per signal sent, can efficiently determine membership size, utilization rates for the network, and the number of satellites in the system. Such an arrangement can allocate two important natural resources: frequency band width and orbital space. The current structure of the International Telecommunications Satellite Organization's communication network (INTELSAT) is briefly compared with the club scheme presented here.

INTELSAT and Alternative Allocative Structures

The 1969 completion of INTELSAT made telecommunications the most important current application of satellite technology.[1] INTELSAT links some 80 nations in an external communication network carrying approximately two-thirds of all transoceanic messages. Currently, the system consists of eight geostationary satellites positioned some 22,300 miles above the equator. At this altitude, the satellites orbit the earth in the same time interval that the earth rotates about its axis, and hence the satellites remain stationary over a point on the earth's surface. This high altitude geosynchronous or geostationary orbit means that only three satellites are required to

---

[1] INTELSAT's technology, operation, and organization are discussed by Edelson, *Global Satellite Communications*, 236 SCIENTIFIC AM. 58 (Feb. 1977); Fawcett, *Outer Space: New Perspectives*, 49 INT'L AFF. 358 (1973); Fawcett, *Satellite Broadcasting*, 27 THE WORLD TODAY 76 (1971); J. GALLOWAY, THE POLITICS AND TECHNOLOGY OF SATELLITE COMMUNICATIONS (1972); Galloway, *Worldwide Corporations and International Integration: The Case of INTELSAT*, 24 INT'L ORGANIZATION 503 (1970); Levy, *INTELSAT: Technology, Politics, and the Transformation of a Regime*, 29 INT'L ORGANIZATION 655 (1975); Mickelson, *Communications by Satellite*, 48 FOREIGN AFF. 67 (1969); Miles, *Transnationalism in Space: Inner and Outer*, 25 INT'L ORGANIZATION 602 (1971); Miles, *International Administration of Space Exploration and Exploitation*, 8 MONOGRAPH SERIES IN WORLD AFFAIRS (1970); G. O'NEILL, THE HIGH FRONTIER: HUMAN COLONIES IN SPACE (1977).

provide point-to-multipoint service almost everywhere on the earth (except near the poles), because each satellite can communicate with the microwave transmitters and receivers (earth stations) on one-third of the earth. Four satellites are positioned over the Atlantic, while two each are positioned over the Pacific and Indian Oceans. Since the largest flow of messages transverses the Atlantic Ocean, this region requires more communication satellites than elsewhere. Of the eight satellites, four serve as spares and increase the system's reliability to better than a 99.9 percent effectiveness rate. If a low orbiting system were installed, 20 to 50 satellites (depending upon orbital altitude) would be needed to cover the globe. INTELSAT satellites receive weak radio signals in the mega-hertz frequency band from earth station transmitters. After receiving these signals, the satellite amplifies and retransmits them in the giga-hertz band to earth station receivers.[2] Between earth stations and other ground points, signals travel via microwave links and cables.

What type of allocative structure (*e.g.*, perfect competition, regulated monopoly) is most efficient for an external telecommunication network such as INTELSAT? Since significant scale economies and large initial investment outlays characterize these networks, perfect competition is not a likely or desirable allocative arrangement for these networks. Among other things, perfectly competitive markets require a large number of sellers, none of whom have an appreciable market share. Unfortunately, the existence of scale economies, which lower unit costs, gives a cost advantage to whichever firm enters the industry first. Moreover, this advantage can later be exploited to prohibit others from joining the industry. In addition, large scale investment requirements can block entry into the industry for all but the richest firms. Both scale economies and investment prerequisites create a situation of *natural monopoly* wherein one or a few firms set price and dominate the industry. If perfect competition and unrestricted entry are encouraged, losses will characterize the industry as marginal cost pricing falls short of per unit cost owing to these scale economies.

The scale economies of satellite communications are documented by Snow and Edelson.[3] For example, the investment cost per circuit

---

[2] The high frequency range of the radio spectrum is assigned to satellites because high frequency waves are able to penetrate the ionized layers (i.e., the Van Allen Belts) and clouds of the earth's atmosphere. Moreover, high frequency waves disperse less than lower frequency ones, and hence, less ground interference results.

[3] *See* Snow, *Investment Cost Minimization for Communication Satellite Capacity: Refinement and Application of the Chenery-Manne-Srinivasan Model,* 6 BELL J. ECON. 621 (1975) and Edelson, *supra* note 1.

year dropped from \$32,500 to \$800 as the size of INTELSAT satellites increased from series I to V.[4] Increased INTELSAT utilization therefore permitted larger satellites to reduce the cost per unit of utilization. Similar scale economies characterize launch costs.[5]

Since elements of natural monopoly are present, a conceivable allocative structure for satellite telecommunications is that of regulated monopoly; however, there are also problems with this structure. The extent of scale economies indicates that a global network would minimize per unit costs, and this, in turn, means that the regulatory agency must control an international monopoly. Thus, the agency would require international authority to regulate price and output. No regulatory agency has ever had these powers and, in practice, this structure can be dismissed as an alternative.

If a good's benefits are excludable, and if the good can be simultaneously utilized by more than one individual, then the good is a "club good."[6] INTELSAT qualifies as a club good, since access to the network can, for the most part, be restricted by coding or scrambling signals and the network can be simultaneously used. As utilization increases for club goods, the benefits per unit of utilization (*e.g.*, per signal sent) diminishes due to congestion (*e.g.*, interference or noise). A club arrangement can optimally allocate utilization rates based on the congestion phenomenon. This arrangement consists of voluntary participants agreeing to pay either a membership charge or a user fee (or toll) per unit of utilization. When properly formulated, a club model can determine club size, tolls, and the optimal amount of the shared good to provide.

A Club Model for Satellite Telecommunication Systems

When access to satellite telecommunication systems can be limited to paying members, a club arrangement can efficiently allocate radio frequencies and orbital space. Other forms of non-market structure, for example, a supranational structure, may be more suitable when access is nonexcludable. Throughout this section, efficiency refers to Pareto optimality, which corresponds to a position in which no participant can be made better off without harming at least one other participant. Pareto-optimal solutions are derived by maximizing an

---

[4] INTELSAT IV-A satellites currently contain 6,000 circuits, where each circuit consists of two one-way telephone channels; hence, 6,000 simultaneous telephone conversations can be conducted by each satellite.

[5] *See* T. HEPPENHEIMER, COLONIES IN SPACE (1978).

[6] A good's benefits are excludable if the owner can keep others from using the benefits. For more discussion on club goods, *see* Buchanan, *An Economic Theory of Clubs*, 32 ECONOMICA 1 (1965).

arbitrary participant's utility subject to the constancy of all other participants' utilities. Moreover, all relevant constraints must be satisfied. Focusing on efficiency does not imply that distributional consequences are ignored. Rather, we treat these separately below.

There are two aspects in allocating frequencies; frequencies must be allocated *between* satellite and nonsatellite communication uses, *and* frequency bands must be assigned to satellite users. The first allocative problem can be conceptualized by treating the radio wave portion of the electromagnetic spectrum as a *joint good* ($\bar{b}$) obeying the transformation $\bar{b} = \sum_{j=1}^{z} b^j$, where b^j is the width of the radio spectrum allocated to the jth use.[7] That is, a joint good is purely rival *between* uses; however, each use may display nonrival characteristics so that many users can benefit from it simultaneously. In the case of the radio spectrum, a frequency band allocated to ground communication (like radio transmission) eliminates these frequencies from being assigned to satellite transmission if interference between uses is to be avoided. An optimal allocation between spectral uses requires an equality between the marginal benefits of the frequency band associated with each use. When marginal benefits are unequal, frequencies should be reallocated until equality is reached, with wider bands being assigned to those uses with larger marginal benefits. As wider frequency bands are allocated to a particular use, marginal benefits consist of the value of the reduction in noise and interference experienced by the users of a particular radio spectrum allocation. These benefits can be evaluated based both on the commercial value per unit of time utilization associated with a radio spectrum assignment, and on the time savings from not having to repeat signals as clearer reception is achieved.

Once frequencies are distributed between uses, both orbital assignments and frequency allocations for satellite users can be determined with a club model.[8] There are two congestion phenomena involved with satellites: signal interference and satellite collision. Signal interference depends on the network's average utilization rate (k), which equals the number of signals sent ($\sum_i x^i$ summed over users) per unit

---

[7] This analysis is similar to that found in Oakland, *Joint Goods,* 36 ECONOMICA 253 (1969).

[8] Currently, the International Telecommunication Union (ITU) assigns orbits and frequencies (except for the U.S.S.R. and the U.S. Military). However, we can find *no* theoretical justification for how these assignments are made. *See* Brown & Fabian, *Toward Mutual Accountability in the Nonterrestrial Realms,* 29 INT'L ORGANIZATION 877 (1975); Fawcett (1973), *supra* note 1; Galloway (1970 & 1972), *supra* note 1; and Miles (1970 & 1971), *supra* note 1.

time divided by the capacity of the network for the relevant time interval. Network capacity (X) is, in turn, dependent on both the frequency band (b^j) allocated to satellites and the number of satellites (N) used, since satellites can use the same band provided the satellites are sufficiently spaced. With current technology and frequency allocations, each INTELSAT IV-A satellite can conduct up to 6,000 telephone conversations, but as this capacity is approached through use, background noise and interference increase. Hence, as utilization increases (*i.e.*, more signals are sent), interference congestion increases. In contrast, an increase in either the number of satellites or the frequency band lowers interference. The former increase allows each satellite to carry fewer messages, and the latter allows a greater message capacity per satellite. By making interference a function of k [*i.e.*, $c = c(k)$],[9] both opposing influences on interference are captured.

The second form of congestion concerns orbital spacing.[10] Congestion costs due to possible collision (s) increase as the number of satellites (N) in a given orbital altitude increases; *i.e.*, $s = s(N)$. By placing *both* the c and s functions in each user's utility function [$u^i(\cdot)$], the model can then be formulated to find the Pareto-efficient solutions for the number of satellites for a given altitude above the earth (say, 22,300 miles), the toll per signal sent, and the number of users or members of the network.[11]

To find these solutions, any user's utility function must be maximized subject to the following factors: the constancy of the other users' utilities; a private good consumption-distribution constraint; a production possibility constraint; the frequency distribution constraint; and the requirement that no user utilizes the entire network capacity. The model is briefly presented in the appendix. Equations (1)-(3) are simplified representations for the provision, toll, and membership conditions, respectively.

$$\text{(Provision)} \qquad \sum_i MBIR^i = MC_N + \sum_i MCC^i_N \qquad (1)$$

$$\text{(Toll)} \qquad \sum_i MIC^i = MB^p \text{ (for all p)} \qquad (2)$$

---

[9] If an increase in N creates interference due to inadequate satellite spacing, then $c = c(k,N)$, and another marginal cost of increasing N must be balanced with the marginal benefits.

[10] In geostationary orbit, satellites must be separated by at least 100 miles to avoid collision due to drift. *See* Fawcett (1971), *supra* note 1.

[11] The model sketched here only concerns frequency-division multiple access (FDMA) arrangements. Time-division multiple access (TDMA) require a more complex intergenerational club model. *See* Sandler, *A Theory of Intergenerational Clubs* (1979) (unpublished copy on file in NRJ office). *See also* Edelson, *supra* note 1, on the different frequency-division arrangements.

$$\text{(Membership)} \quad \Delta TB^p \geqq TIC^p \text{ (for all p)} \qquad (3)$$

As the number of satellites in the network is increased, the benefits consist of reduced interference due to increased network capacity, while the costs relate to greater collision probability as well as increased construction and launch expenses. The collision probability (and associated costs) may be zero until a sufficient number of satellites populate a given altitude. In order to determine the optimal number of satellites at a given altitude, N should be increased until the resulting marginal benefits of interference reduction ($\sum_i MBIR^i$) are equal to the sum of the marginal cost of launch and construction (MC_N) and the marginal collision costs ($\sum_i MCC^i_N$) associated with the increase in N [see equation (1)]. Both marginal collision costs and marginal benefits of interference reduction are summed over all users, since these benefits and costs affect all network participants. Surprisingly, collision is a real problem requiring spacing of 100 miles or more.

In equation (2), tolls are set equal to the sum of the marginal interference costs ($\sum_i MIC^i$) imposed upon the users as utilization increases, and the pth user sends signals[12] until his marginal benefits from utilization (MB^p) equal the marginal interference costs placed on the network members. Since the sending of an additional signal causes the same marginal interference costs, irrespective of user, the toll per signal sent is identical for all members. Nevertheless, the total tolls paid for satellite communications vary between users according to their revealed intensity of utilization. Finally, the membership condition requires that a potential user should be admitted whenever the total benefits from membership (ΔTB^p) are greater than or equal to the total interference costs imposed on the network by the potential user's membership.[13]

INTELSAT and Club Arrangements

The above discussion indicates that for a satellite telecommunications network, sharing arrangements can achieve allocative efficiency with respect to utilization, membership, and provision whenever appropriability is feasible.[14] If costs and benefits can be monitored accurately, an optimal sharing arrangement could, in practice, be

---

[12] Signals can be measured in terms of the number of words or letters.

[13] If the addition of a member leads to a cost or benefit in terms of private good production, then an additional term must be included in the membership condition. *See* Sandler, *supra* note 11.

[14] The ability to force payment for giving benefits of a good to others is called appropriability.

realized. The sole difficulty concerns financing; scale economies mean that subsidies are required to finance an optimal satellite provision.

The current structure of INTELSAT conforms closely to that of an economic club with firms and governments as members.[15] Members pay fees according to their utilization, and voting in the Board of Governors is weighted according to members' utilization rates and investment shares. Although the other bodies of INTELSAT, such as the Assembly of Parties, the Meeting of Signatories, and the Manager, make policy recommendations, the Board of Governors is the decision-making body of INTELSAT. A weighted voting scheme based upon utilization appears to promote optimality, since heavier users will be serving more individuals (whose marginal benefits and costs must be aggregated), and consequently, these users account for a greater share of costs and benefits resulting from policy changes. As orbital space and frequencies become scarcer, an allocative arrangement similar to that proposed above should be initiated. Such a scheme must develop proxies to measure interference costs and interference reduction benefits. Furthermore, if other types of satellites share geostationary orbits (*e.g.*, solar power satellites), then the determination of the optimal number of communication satellites must include other considerations.

Other Economic Consequences of INTELSAT

Allocation of orbital space and radio frequencies for INTELSAT does not necessarily achieve full allocative efficiency, since other kinds of externalities,[16] not accounted for by the model, may occur. For example, INTELSAT satellites pose a collision problem for other satellites, for example, solar power satellites, sharing geostationary orbit and vice versa. In determining the optimal number of powersats (solar power satellites) *or* communication satellites, collision costs must include collision externalities imposed upon all types of satellites in a given altitude band. If the number of communication satellites is, however, decided without including the collision externality imposed on powersats, too many communication satellites will populate the geostationary orbital band. A supranational structure may be

---

[15] 15. On INTELSAT organization, see Edelson, *supra* note 1; Galloway (1972), *supra* note 1; Levy, *supra* note 1; Miles (1970), *supra* note 1; and Riegel, *Communications by Satellite: The Political Barriers,* 11 Q. REV. ECON. & BUS. 23 (Winter 1971).

[16] 16. On transnational externalities, *see* D'Arge, *Observations on the Economics of Transnational Environmental Externalities* and Scott, *Economic Aspects of Transnational Pollution,* both in PROBLEMS IN TRANSFRONTIER POLLUTION (Organization for Economic Cooperation and Development, eds. 1974).

needed to allocate orbital space to include all the diverse interests using an orbital altitude.[17]

Analogously, interference externalities may involve other types of satellites, not part of INTELSAT. Unlike in-space collision externalities, interference can result from satellites *not* in the same altitude band, since radio waves transmitted at two different altitudes can still interfere provided their paths cross. Hence, geostationary satellites' transmissions may interfere with those of nongeostationary satellites. A comprehensive club model must include these other interference externalities. In so doing, network membership will be reduced and tolls will increase, because additional interference costs are present at the margin.

These other externalities raise a host of common property problems, owing to an absence of property rights assigned to space resources (*e.g.*, orbital space). Moreover, these resources have economic value for two or more agents, who may want to exploit available benefits. Essentially, outer space shares the same kinds of common property difficulties now being confronted by the international community with regard to the oceans, especially beyond the 200-mile limit. Without property rights assignments, the strongest and most technologically advanced nations will claim these resources and will exploit them as soon as the necessary technology is developed. Such action can widen the income gap between poor and rich nations.

MILITARY RELATED USES OF OUTER SPACE

At present, military activities in outer space consist of surveillance, navigation and communication.[18] Surveillance satellites are used to gather information on missile deployment, and play an important role in the verification of conditions such as those proposed in the SALT II Treaty.[19] Satellite based global positioning systems guide cruise missiles to targets, while other satellite guidance systems are used to navigate nuclear submarines. Furthermore, satellite communication networks are operated by NATO and the Warsaw Pact.[20]

Since all outer space military activities utilize satellites, the allocative issues previously discussed apply here as well. Although many

---

[17] *See* Sandler and Cauley, *The Design of Supranational Structures: An Economic Perspective,* 21 INT'L STUD. Q. 251 (1977).

[18] *See* Brown, *Reconnaissance from Space,* 27 THE WORLD TODAY 68 (1971); Galloway (1972), *supra* note 1; Riegel, *supra* note 15; and Miles (1971), *supra* note 1.

[19] Surveillance satellites can discern clearly an object one foot across from an altitude of 100 miles. *See* Aspin, *The Verification of the SALT II Agreement,* 240 SCIENTIFIC AM. 38 (Feb. 1979).

private corporations contract with the military for satellite related parts, market transactions tend to exhibit non-competitive elements with many government contracts going to large corporations. Additionally, military related externalities limit the efficiency of markets. For example, in the early 1960s project West Ford, a Department of Defense undertaking, placed 400 million small copper dipoles in a belt around the earth.[21] These dipoles posed an interference threat to radio astronomy and other forms of transmitters. Even though this project served the Department of Defense's communication needs, it produced international externalities that were uncompensated.

In 1967, members of the United Nations signed the outer space treaty prohibiting all military activity in outer space, including the moon and other celestial bodies.[22] Whether this treaty will restrain the superpowers from developing space deployed weapons is doubtful, especially since these nations are *already* financing research on particle beam satellites, which shoot high energy subatomic particles at targets.[23] Will the exploitation of outer space increase the stability of deterrence or will conflict result due to space exploitation? Improvements in military surveillance as provided by satellites are probably augmenting stability, owing to the ability to verify arms limitations agreements. Prior to these improvements, the stumbling block to SALT treaties concerned verification, because neither side wanted on-site inspections.[24] Monitoring improvements also increase stability by reducing the chance that war will result from error when an opposing side falsely perceives an attack.

Unfortunately, space exploitation also heightens the possibility of conflict because of common property problems, externalities, and the vulnerability of space objects. As nations vie for space resources, conflict can develop as two or more nations lay claim to the same resources. Another potential avenue of conflict concerns satellite related externalities, for example, collision and falling debris. Even with the "Treaty on the Liability for Damage Caused by Objects Launched into Outer Space," in effect since 1971, the recent refusal by the U.S.S.R. to compensate Canada for the cleanup of a Russian surveillance satellite demonstrates that liability assignments are not

---

20. NATO INFORMATION SERVICE, NATO: FACTS AND FIGURES (1976).

21. Galloway (1972), *supra* note 1.

22. Treaty on Principles Governing the Activities of States in the Exploration and Use of Outer Space, Including the Moon and other Celestial Bodies, *opened for signature* Jan. 27, 1967, 18 U.S.T. 2410, T.I.A.S. No. 6347, 610 U.N.T.S. 205.

23. Parmentola & Tsipis, *Particle-Beam Weapons*, 240 SCIENTIFIC AM. 54 (Apr. 1979).

24. Myrdal, *The International Control of Disarmament*, 231 SCIENTIFIC AM. 21 (Oct. 1974).

well-defined.[25] Finally, conflict may be enhanced because "spy" satellites drifting in the international domain of outer space make for tempting targets. With more military navigation being controlled by satellites having no self-defensive capabilities, any side that destroys the other's satellites may have a deciding first-strike advantage.

EXPLORATORY USES OF OUTER SPACE

Satellites are being used to discover new resource pools on earth,[26] to explore the formation of stars and galaxies,[27] and to predict long and short term weather patterns. As satellites generate geophysical information benefiting food and energy supplies, improvements in income and hence income distributions may be made possible. Much satellite produced data has strong publicness characteristics, since this information is nonrival and can be distributed widely.[28] Due to these publicness elements, free-riding behavior can lead to an undersupply of information. As an example, using an analogous situation, consider the case of an ally taking a "free ride" by not buying defense weapons, but rather relying on the arsenal of other allies. Consequently, in the case of space exploration, nonmarket structures may be required to cure inherent suboptimality problems. There have been some cooperative ventures between the U.S. and the U.S.S.R. concerning space lab experiments, and the Europeans have pooled efforts in the European Space Research Organization; however, all agreements have been very loose, and the efficacy of tighter structures should be explored.[29] Moreover, the discovery of space resources through probes will further compound the common property problem, and this may necessitate international structures to exploit these resources.

Appropriability problems also inhibit market operation for information producing satellites. Even though scrambling and coding devices can exclude potential users from pirating satellite signals, once the satellite owner sells satellite produced data, the buyer can sell (or give) the information to others without the owner's permission.

---

[25] *See Canadians End Search for Debris of Soviet Satellite,* N.Y. Times, April 2, 1978, at 10, col. 1.

[26] *See* Fawcett (1973), *supra* note 1, and Jastrow & Newell, *The Space Program and the National Interest,* 50 FOREIGN AFF. 532 (1972).

[27] Strom & Strom, *The Evolution of Disk Galaxies,* 240 SCIENTIFIC AM. 72 (Apr. 1979).

[28] A good's benefits are nonrival when one person's consumption of a unit of the goods does not detract from the consumption opportunities of other people, *e.g.*, a sunset.

[29] *See* Galloway (1972), *supra* note 1.

SPACE-BASED EXPLOITATION OF OUTER SPACE

In this section the focus is on the outward reaching, more speculative, aspects of man's possible future exploitation of the outer space resource. Since the objective is to define an "economics of outer space," discussion must be limited to concrete proposals put forward by NASA and others, eschewing speculation on man's possible colonization beyond the solar system. Such proposals include, first, a plan (originally conceived by Peter Glaser) to ring the earth with solar power space satellites (SPSS) placed in geosynchronous orbit.[30] These would beam microwave energy to earth for conversion into what is claimed to be an almost unlimited supply of environmentally benign electric power. A second proposal, also considered, is development of human colonies in space relatively near the earth and moon in stable positions termed Lagrange points. L-5 is one such point which has a number of favorable characteristics.[31] The second of these proposals is closely related to the first: it appears the principal hope for economic feasibility for space colonization depends on the ability to manufacture such power stations in space more cheaply than to manufacture them on earth and boost them to geosynchronous orbit. Thus, Gerald O'Neill has proposed space colonies to manufacture SPSS. A key feature is the notion that once a first earth colony is placed at L-5, raw materials from the moon can be used to build both new colonies and solar power satellites—where growth of a "space economy" is self perpetuating based on sale of electric power to earth. The plan of this section is to consider proposals for space manufacturing, solar power satellites, and space colonies, addressing the first issue of the economic feasibility of such proposals. However, questions relating to the future economic growth of the earth, the distribution of growth and income between developed and third world nations, and finally the types of economic and political structures which might evolve, become important if space colonization proves economically feasible.

Space Manufacturing

Goods manufactured on earth cost about $2 per pound.[32] This statement refers to ordinary commodities (such as turbogenerators,

---

[30] *See Hearings on Space Shuttle Payloads (Part 2) Before the Senate Committee on Aeronautical and Space Sciences,* 93rd Cong., 1st Sess. 10-62 (Oct. 31, 1973) (statement of Peter E. Glaser).

[31] There exist a number of gravitationally balanced positions in space relative to the earth, moon, and sun called Lagrange points. Some of these are dynamically unstable. One stable position near earth is termed L-5.

[32] For example, the Spring 1979 Sears Catalogue has electric stoves for $1.99/lb., table saws for $1.90/lb., refrigerators for $1.78/lb., automobile batteries at $1.20/lb., tires at $1.96/lb., and air conditioners for $3.63/lb.

refrigerators, and automobiles) made principally from steel, aluminum, plastics, wood, glass, and other nonexotic materials. If such goods are to be employed economically in space, they must first be brought there at reasonable cost. Costs for placing one pound of payload in low earth orbit, geosynchronous orbit or L-5, and on the moon are now about $1,100, $4,000, and $8,000 per pound respectively. It is argued that lift costs are about to fall an order of magnitude from the current levels, which are based on the Apollo project that put man on the moon at a total cost of $40 billion. These lower costs, about $110, $430, and $860 per pound respectively, could result from use of a proposed heavy lift vehicle (HLV) which is an adaptation of the space shuttle engines to a simple payload carrying "mule"–implying even lower costs than the more versatile shuttle. Given the difficulties the space shuttle has had in meeting deadlines and cost projections, it is important to note that the optimistic order of magnitude reduction to $430 per pound lift cost to geosynchronous orbit implies that ordinary manufactured goods would still cost about $432 per pound if placed in the ideal spot for a solar power space satellite. Clearly, only lift costs are relevant, and most ordinary economic activities in the near term, if they rely on manufactured goods produced on earth, are hopelessly infeasible.[33]

Can one avoid lift costs from earth for space manufacturing? In part, the answer may be yes. Certainly the asteroid belt, probably the broken remains or unformed material of a "missing" planet between Mars and Jupiter, contains every desirable raw material–mineral, liquid, or gas. But again, transport costs to near-earth space would probably be large. However, it turns out that the *average* soil of the earth's moon appears to be an excellent industrial raw material. The percentage breakdown of lunar soil by weight is about 42 percent oxygen, 19 percent silicon, 14 percent iron, 6 percent aluminum, 6 percent titanium, and 4 percent magnesium. Thus, oxygen necessary for life, useful both in industrial chemical reactions and as a propellant; silicon, necessary for manufacture of glass; and the basic metals constitute 91% of lunar soil. Why not then promote moon-based manufacturing? Two problems make such an enterprise infeasible. First, human work and life at low gravity (G) is very awkward. Second, lift costs from earth to moon are significantly higher than those from earth to L-5. Thus, current proposals focus on transporting

---

33. Of course, certain special activities including satellite communications and surveillance as outlined in the previous section are obviously feasible. Other industrial activities such as growing large perfect crystals in zero-G or those requiring a high vacuum may thrive under the economics of the space shuttle and HLV. However, from the perspective of, say, the U.S. economy, these are likely to remain unimportant even compared to satellite communications.

moon soils as raw materials to an industrial space colony located at L-5. It is argued that (i) the costs of maintaining one gravity in the living or working portions of a colony by rotation are very small; (ii) zero gravity is available, if advantageous, for industrial processes; and (iii) lift costs from earth are halved compared to those from the moon. Of course, the argument also depends on an inexpensive method of transporting lunar soil to L-5. O'Neill, for example, proposes a mass-driver–a system of electromagnetically driven and levitated buckets–to literally shoot lunar soil to L-5.[34] Thus, only a small mining and transport facility is proposed for the moon itself.

Solar Power Space Satellites

Solar power space satellites as proposed by Glaser, O'Neill, and others would consist of either orbiting concentrating mirrors heating a working fluid to drive turbogenerators or, alternatively, panels of solar cells for direct solar electric conversion.[35] In either case, electric power would be converted to microwaves and beamed to an antenna grid on earth for reconversion to electricity. Conversion to microwaves and reconversion to electric power is estimated to occur at 60 to 70 percent efficiency. Environmental problems are claimed to be minimal. The microwave energy density would be relatively low so occupants of aircraft passing through the power beam would supposedly go unharmed. A 5000 megawatt power satellite would require less than 50 square kilometers of remotely placed receiver antenna. The antenna array would allow most light to pass through, so the land below could be used for grazing. However, it has been pointed out that such facilities could make tempting military targets, and environmental questions are far from resolved.

If a turbogenerator type power satellite were launched from earth, even using exotic materials, the weight of components to be placed in geosynchronous orbit would likely exceed 20 pounds per installed kilowatt capacity. Thus, in the near term the cost would exceed $13,600 per installed kilowatt (given lift costs of $430 per pound and a transmission efficiency of .63) compared to about $1,000 per installed kilowatt for a coal fired power plant, complete with emission controls, built on earth.[36] Clearly, as noted above, the economics of power satellites manufactured on earth are now hopeless.

Two alternative proposals for achieving feasibility for SPSS have been put forward. First, O'Neill suggests that if space manufacturing

---

34. G. O'NEILL, *supra* note 1.

35. *Id.*

36. Costs for solar cells, needed for the photoelectric conversion alternative, as we point out below, are at least for now prohibitive even without considering lifts costs.

is possible, that is, manufactured goods can be produced in near-earth space at costs and using techniques somewhat comparable to those on earth, then the solar power satellite becomes an economic possibility.[37] Power satellites, in turn, may generate the revenue through sale of power to earth to make space colonization and manufacturing feasible.

Second, NASA has evaluated an earth-based photovoltaic SPSS system which depends on development, in the long term, of a space freighter capable of achieving low earth orbit at a cost of $15 per pound. At the turn of the century the proposed space freighter would use hydrogen and methane from coal gasification as fuel to lift SPSS components and be completely reusable. Solar power satellite modules could then be assembled in low earth orbit and be self propelled to geosynchronous orbit using 20 percent of the electrical output of each module to power ion-drive electric propulsion rocket engines.[38] Eighty percent of the solar cell array could then remain protected during passage through the Van Allen belt from radiation which damages (irreversibly reduces efficiency of) solar cells. This "free" boost to high orbit reduces the number of space freighter launches to low orbit, necessary for construction of a 5,000 megawatt SPSS, by about half. Unfortunately, near term costs of solar cells are optimistically projected to be about $2,000 per peak kilowatt in 1982. Given lift costs of $15 per pound, feasibility requires that solar cells cost about $200 per peak kilowatt to manufacture. NASA thus projects feasibility for earth-based SPSS development *if* lift costs are reduced almost one order of magnitude ($110 to $15 per pound) and *if* the costs of solar cells is similarly reduced by one full order of magnitude ($2,000 to $200 per peak kilowatt) from near term levels.[39] Economic feasibility is only a theoretical possibility at this point, but NASA's studies have at least defined the requirements for feasibility in a straightforward way—one order of magnitude reductions in both lift costs and the cost of manufacturing solar cells.

Colonization of Outer Space—A Space Economy

O'Neill's proposal for space colonization can be briefly summarized as follows:[40] the initial colony at L-5 would house 10,000 people in a revolving sphere about 460 meters in diameter having a structural

---

37. G. O'NEILL, *supra* note 1.

38. Kraft, The Solar Power Satellite Concept—The Past Decade and the Next Decade (paper prepared for the American Institute of Aeronautics and Astronautics, 1979).

39. *Id.*

40. G. O'NEILL, *supra* note 1.

weight of 150,000 tons.[41] Construction of the first colony depends, first, on construction of a moon base (about 10,000 tons transported from earth) to supply raw materials, and placement of a construction station located at L-5 weighing about 30,000 tons. O'Neill estimates that "Island One" could be constructed in this way for $96 billion. Output in manufactured goods from Island One when fully operational is estimated at 200,000 tons per year. O'Neill proposes that 60 percent of this output be used for construction of 5000 megawatt capacity power satellites at 80,000 tons per unit which would produce revenues from power sales sufficient to allow the remaining 40 percent of total output to be "saved," that is, used in the construction of new colonies.

This proposal, given the numbers outlined above, implies an incredibly rapid rate of growth, both for space colonies and for power satellites, approaching 35 percent per year. Furthermore, as is shown below, economic feasibility depends heavily on the assumption that this growth rate can be achieved and maintained for a period of about 20 years. Clearly, this rate of growth needs careful examination. Fortunately, the growth process, which O'Neill describes as the "bootstrap principle"—one colony constructed mostly from earth becomes the springboard for an almost entirely space based growth of new colonies—corresponds precisely to a basic model of economic growth which has come to be known as the "Harrod-Domar" model.[42] This model can be explained with the following notation:

Y = total output
K = total capital
$\alpha = K/Y$ = fixed capital-output ratio of the economy
s = fraction of output "saved" and invested for growth
t = time (year, starting at 0).

For consistency with O'Neill, in modeling the space economy he describes, output Y will be measured in total tons of manufactured goods, as will be capital K. Capital in the model consists of (1) lunar bases (3 per colony at 10,000 tons each); (2) colony construction stations (30,000 tons each); and (3) colonies themselves (170,000 tons including glass windows). Thus, to allow each colony to replicate itself in about two years, as O'Neill proposes, one must satisfy a requirement of about 230,000 tons of manufactured capital per colony. In turn, each colony is supposed to be able to produce an out-

---

[41] Additionally, 20,000 tons of glass for windows and 400,000 tons of "waste" slag for radiation shielding, soil, etc. would be utilized.

[42] For a description, *see* H. WAN, ECONOMIC GROWTH (1971).

put of 200,000 tons per year. Thus, in terms of the notation introduced above, our space economy has a capital-output ratio of:

$$\alpha = \frac{K}{Y} = \frac{230{,}000}{200{,}000} = 1.15.$$

To determine the rate of growth of the space economy, note that the following relations hold:

$$Y = \frac{1}{\alpha} K \text{ (the production relation)} \qquad (4)$$

and

$$\frac{d}{dt} K = \dot{K} = sY \text{ (the investment rate, } \dot{K}\text{, equals the savings ratio, s, times output).} \qquad (5)$$

Differentiating (4) with respect to time yields $\dot{Y} = \frac{1}{\alpha}\dot{K}$ which combined with (5) implies a percentage rate of growth of output for the space economy of

$$\frac{\dot{Y}}{Y} = \frac{s}{\alpha} \qquad (6)$$

Since O'Neill proposes a savings ratio of $s = .4$ and the capital output ratio is $\alpha = 1.15$, the growth rate is $\dot{Y}/Y = .4/1.15 = .35$ or 35 percent per year. No economic system has ever achieved such a spectacular rate of growth. To explain this result, note first that the savings rate proposed is very high—the U.S. economy has a savings ratio (including business savings) of less than 15 percent while 40 percent is proposed for the space economy. Second, the capital-output ratio is very low. Most industrialized nations have α's of about 4, nowhere near the 1.15 proposed for the space economy. To check the Harrod-Domar model, note that the rate of predicted growth for the U.S. economy would be

$$\frac{\dot{Y}}{Y} = \frac{s}{\alpha} = \frac{.15}{4} = .0375$$

or 3¾ percent per year. This is just slightly higher than the actual long run average rate of growth for the U.S. of about three percent per year for the last century. Thus, the Harrod-Domar model holds up as a rough predictive tool for economic performance in spite of the fact that much more sophisticated models are available.[43]

For purposes of argument, one might choose to agree with O'Neill's

---

[43] *Id.*

choice of savings ratio, since very high savings rates have been obtained historically through forced means—for example, the rapid rate of growth of the Soviet Union between the world wars resulted from a high forced savings rate. However, the choice of capital output ratio, implying an enormously higher than expected productivity for capital, seems untenable.[44]

To explore the impact of varying the capital output ratio, the model of the space economy can be completed as follows. Assuming the first colony, "Island One," is in place, the number of colonies, N, will grow from the base year, t = 0, as follows:

$$N(t) = 1 \cdot e^{\frac{s}{\alpha} t} . \qquad (7)$$

Since power satellites produced by colonies consume net output (that remaining after savings for growth are subtracted) the number of power satellites P(t) changes over time as follows:

$$\dot{P}(t) = \frac{(1 - s)Y(t)}{80{,}000 \text{ tons}} \qquad (8)$$

given that each 5000 megawatt power satellite requires 80,000 tons of manufactured goods. Note, that if one accepts O'Neill's assumption that each colony can produce 200,000 tons of output per year, then $Y = 200{,}000 \cdot N(t)$. This, in turn, implies, setting $s = .4$ and $\alpha = 1.15$ in equations (7) and (8) above, that

$$N(t) = 1 \cdot e^{.35t}$$

and

$$P(t) = 4.3(e^{.35t} - 1).$$

How many colonies and power satellites does O'Neill's scheme generate? Table I shows that if growth is allowed to proceed for only eight years, earth will have 16 colonies and about 65 power satellites (Case I). If growth is allowed to proceed for 20 years, earth would have 1,097 colonies and 4,711 power satellites (Case II)! Cases I and II approximate O'Neill's assumptions, and the resultant growth rate and numbers for colonies and "powersats" are roughly consistent with his projections.

---

[44] The capital-output ratio for heavy industry exceeds that predicted for the whole space economy. For example, in the U.S. primary metals industry, the capital-value added (a measure of final output) ratio exceeded 2 in 1971. Both for the stone, clay and glass sector and for the chemical industry, this ratio was about 1.3. Of those industries relevant for the proposed space economy, only metal fabricating fell below 1.15, with a ratio of .8. The entire U.S. economy has a much higher overall ratio of about 4 because capital stocks in the form of roads, hospitals, schools, private homes, etc. are added to those in basic industries such as those noted here.

TABLE 1

ALTERNATIVE GROWTH PATHS FOR A SPACE ECONOMY

	Years of Growth	*# of Colonies*	*# of Powersats*	*Total Generating Capacity* MW_e	*Assumed Capital Output Ratio*	*Growth Rate*	*Benefit-Cost Ratio to Earth*
Case I	8	16	65	$.33 \times 10^6$	1.15	.35	.4
Case II	20	1,097	4,711	23.5×10^6	1.15	.35	2.8
Case III	20	13	55	$.27 \times 10^6$	3.0	.13	.68

To test economic feasibility of Cases I and II—short and long run time frames, small and large scale colonization respectively—the assumption is made that all power can be sold at 1.5¢ per kilowatt hour and that this figure reflects benefits to earth for power production. Thus, with a discount rate r (a 10 percent rate will be used, comparable to rates of return for public utilities) present value of benefits are

$$B = \int_0^T e^{-rt} V \cdot P(t) dt \qquad (9)$$

where V, the value of annual electric power sales for each 5000 MW powersat is taken to be $\$.46 \times 10^9$ (assuming a load factor of 0.7). A 20-year lifetime for powersats is also used so that, for example, if we allow an eight-year growth period, T in the integral above is 28 years and the time profile P(t) has the number of powersats *declining* after 20 years to zero in 28 years. (In Case II the number of powersats would decline after 20 years to zero in 40 years).

Costs include the initial cost of setting up "Island One," $K_0 = \$96 \times 10^9$, and an additional cost for outfitting each new colony from earth of $M = \$5.5 \times 10^9$, again following O'Neill. Discounted costs are then

$$C = K_0 + \int_0^{t^*} e^{-rt} M \cdot \dot{N} dt \qquad (10)$$

where t* is the length of the growth period, eight or 20 years respectively in Cases I or II.

Using these formulas, benefit-cost ratios are .4 and 2.8 for Cases I and II respectively as shown in Table 1. Apparently, small scale colonization is infeasible. Thus, feasibility depends principally on two aspects of O'Neill's arguments:

1. that a rapid expansion to large scale colonization can occur; and

2. that enormous quantities of electricity can be sold to finance such growth.

In evaluating the second of these arguments, note from Table 1 that Case II electricity sales are equivalent to 23.5 x 10^6 megawatt installed capacity. To put this in perspective, power sales are about ten times the projected total installed capacity of the United States in the year 2000! Can this much power be sold? Obviously, power sales to the Third World must constitute the principle additional demand. Further, approximately half the world's population must begin to consume electricity at per capita rates approaching those for the United States—a noble objective indeed, and it is argued that enormous quantities of cheap power (note that a B/C ratio of 2.8 implies one could sell power at less than *half* current rates and still break even) may be just what the Third World needs to boost developing nations into the industrial age.[45] Also, a view taken by some is that this source of power may allow the earth to escape the limits to growth—pollution and depletion of energy resources.[46]

Returning to the first point, the presumed 35 percent rate of growth put forward for the space economy must be severely questioned. If a more plausible (at least to most economists) capital output ratio of 3 is assumed, the rate of growth drops to 13 percent per annum. This implies that each colony can only produce about 83,000 tons of manufactured output per year. Making this change in the model of the space economy results in Case III, shown in Table 1, where again, as in Case II, a 20-year growth period is assumed. The lowered growth rate results in a benefit-cost ratio of only .68, so feasibility is not achieved.

This is not to argue that space colonization is necessarily infeasible. Rather, the analysis suggests that the two key assumptions outlined above—an extraordinarily low capital-output ratio and the ability to sell vast quantities of electric power—need close scrutiny.

CONCLUSION

Both earth and space based exploitation of outer space have been examined. For satellite communication networks, a club arrangement can foster efficiency by determining the optimal number of satellites at each altitude band, the composition of network members, and the toll per signal sent. This arrangement will efficiently allocate the scarce resources of orbital space and the radiowave portion of the

---

[45] Vajk, *The Impact of Space Colonization on World Dynamics*, 9 TECHNOLOGICAL FORECASTING & SOC. CHANGE 361 (1976).

[46] *Id.*

electromagnetic spectrum. The resulting supranational structure is extremely loose and reflects the current INTELSAT arrangement, where members decide their utilization rates and pay accordingly; hence, little overall consultation of the members is required to manage the organization. In contrast, earth based military and exploratory pursuits and space based activities may generate significant problems due to externalities and nonappropriability, and these difficulties may require tighter structures.

The analysis of space based exploitation showed that existing projections for solar power space satellites lifted from earth show feasibility only if order of magnitude reductions in lift costs and costs of manufacturing solar cells occurs. O'Neill's scheme for space colonization and SPSS growth assumes a high saving rate, a low capital-output ratio, and an enormous projected production of electricity. When more realistic capital output ratios and production projections are used, the benefit-cost ratio becomes less than one, which raises questions about the feasibility of these endeavors. Other scenarios and ratios must, of course, be analyzed before pessimistic conclusions are justified; however, the results here raise important questions that call for further investigation.

MATHEMATICAL APPENDIX

To illustrate the model, two uses for radio wave frequencies are assumed. Superscripts on the c, k, X and b functions stand for the use, with j = 1 corresponding to satellite communications and j = 2 corresponding to a nonsatellite use. Satellite communications exhibit interference $[c^1(\cdot)]$ and collision $[s^1(\cdot)]$ congestion externalities, while the ground use experiences only interference $[c^2(\cdot)]$.

All values in the model are expressed in terms of a private good serving as the *numeraire.* Y is the amount of the private good produced, and y^i is the amount consumed by the ith individual. For the communication uses, x^{ij} is the ith individual's utilization of the jth use, and X^j is the provision (or capacity) of the jth use. The utility function of the ith individual is depicted in (1-A), where k^j is the average utilization rate for the jth use, and N is the number of satellites.

$$u^i = u^i[x^{i1}, x^{i2}, y^i, c^1(k^1), s(N), c^2(k^2)] \tag{1-A}$$

where

$$k^1 = \sum_i x^{i1}/X^1(b^1,N) \text{ and } k^2 = \sum_i x^{i2}/X^2(b^2).$$

A number of constraints must be satisfied when finding Pareto-optimality. These constraints are given by (2-A)-(6-A).

$$u^i(\cdot) \geq \bar{u}^i \qquad (\forall i \text{ except } i = 1) \qquad (2\text{-A})$$

$$X^j \geq x^{ij} \qquad (\forall i \text{ and } j = 1,2) \qquad (3\text{-A})$$

$$Y \geq \sum_i y^i \qquad (4\text{-A})$$

$$F[X^1(\cdot), Y, X^2(\cdot)] \leq 0 \qquad (5\text{-A})$$

$$\bar{b} \geq \sum_j b^j \qquad (6\text{-A})$$

The constraints of (2-A) are those of Pareto-optimality requiring each individual's utility level to remain no less than some beginning level ($\bar{u}^i$), except for one individual. In (3-A), the impurity constraint indicates that no user's utilization can exceed the capacity of the particular network. The private good production-consumption constraint is given by (4-A). This constraint requires private good consumption to be less than or equal to the good's production. The remaining two constraints depict the production transformation function and the frequency band distribution constraint, respectively.

The Lagrangian is represented in (7-A), where the greek letters stand for Lagrangian multipliers (or shadow prices).

$$\begin{aligned} L = {} & u^1(\cdot) + \sum_{i=2} \lambda^i [u^i(\cdot) - \bar{u}^i] + \sum_i \beta^{i1} [X^1(N, b^1) - x^{i1}] \qquad (7\text{-A}) \\ & + \sum_i \beta^{i2} [X^2(b^2) - x^{i2}] + \gamma(Y - \sum_i y^i) - \mu F(\cdot) \\ & + \sigma(\bar{b} - \sum_j b^j). \end{aligned}$$

The relevant first order conditions for this Kuhn-Tucker nonlinear programming problem are shown in (8-A)-(14-A). By assuming λ^i (for all i), γ, μ, and $\sigma > 0$, and X^j (for all j), N, x^{ij} (for all i and j), Y, y^i, and b^j (for all j) > 0, the conditions listed are the only relevant ones. These assumptions remove the possibility of "corner solutions" with respect to the utility constraints, the private good's constraint, the transformation constraint, and the frequency constraint.

$$\frac{\partial L}{\partial y^i} = \lambda^i \frac{\partial u^i}{\partial y^i} - \gamma = 0 \qquad (\forall i \text{ and } \lambda^1 = 1) \qquad (8\text{-A})$$

$$\frac{\partial L}{\partial Y} = \gamma - \mu \frac{\partial F}{\partial Y} = 0 \qquad (9\text{-A})$$

$$\frac{\partial L}{\partial x^{pj}} = \lambda^{p} \frac{\partial u^{p}}{\partial x^{pj}} - \beta^{pj} + \sum_{i} \lambda^{i} \frac{\partial u^{i}}{\partial c^{j}} \frac{\partial c^{j}}{\partial k^{j}} \frac{1}{X} = 0 \quad (j = 1,2 \text{ and } \forall p) \qquad (10\text{-A})$$

$$\frac{\partial L}{\partial N} = -\sum_{i} \lambda^{i} \frac{\partial u^{i}}{\partial c^{1}} \frac{\partial X^{1}}{\partial N} \frac{k}{X^{1}} \frac{\partial c^{1}}{\partial k} + \sum_{i} \lambda^{i} \frac{\partial u^{i}}{\partial s} \frac{\partial s}{\partial N} + \sum_{i} \beta^{i1} \frac{\partial X^{1}}{\partial N}$$

$$- \mu \frac{\partial F}{\partial X^{1}} \frac{\partial X^{1}}{\partial N} = 0 \qquad (11\text{-A})$$

$$\frac{\partial L}{\partial b^{j}} = -\sum_{i} \lambda^{i} \frac{\partial u^{i}}{\partial c^{j}} \frac{\partial X^{j}}{\partial b^{j}} \frac{k^{j}}{X^{j}} \frac{\partial c^{j}}{\partial k^{j}} + \sum_{i} \frac{\partial X^{j}}{\partial b^{j}} \beta^{ij} - \mu \frac{\partial F}{\partial X^{j}} \frac{\partial X^{j}}{\partial b^{j}}$$

$$= \sigma \quad (j = 1,2) \qquad (12\text{-A})$$

$$\beta^{ij} \frac{\partial L}{\partial \beta^{ij}} = \beta^{ij}(X^{j} - x^{ij}) = 0 \quad (\forall i,j); X^{j} - x^{ij} \geq 0 \qquad (13\text{-A})$$

$$\beta^{ij} \geq 0 \quad (\forall i,j) \qquad (14\text{-A})$$

By assuming that no one utilizes the entire capacity of a radio spectrum use [*i.e.*, $X^j > x^{ij} \rightarrow \beta^{ij} = 0$, see (13-A) and (14-A)], the conditions expressed in the paper can be derived. The elimination of the Lagrangian multipliers, via substitution of (8-A) into (10-A), yields the toll condition. Similarly, substitution of (8-A) and (9-A) into (11-A) produces the provision condition. Finally, a similar substitution into (12-A) gives the frequency band width allocation condition. By defining the resulting "weighted" marginal rates of substitution and marginal rate of transformation expressions as marginal benefit and cost terms, respectively, equations (1) and (2) of the text result.

The model must be reformulated to include both members and nonmembers of the radiowave networks if the optimal membership conditions are to be derived.[47]

---

[47] For this reformulation, *see* Sandler, *supra* note 11.