

12-11-2001

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## Recommended Citation

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DEPARTMENT OF ELECTRICAL AND  
COMPUTER ENGINEERING



SCHOOL OF ENGINEERING  
UNIVERSITY OF NEW MEXICO

**Non Stationary Signal Analysis, Energy Demodulation and the  
Multicomponent AM–FM Signal Model**

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UNM Technical Report: EECE-TR-01-001

Report Date: December 11, 2001

<sup>1</sup>This work was submitted towards the ONR Young Investigator Program for the fiscal year 2002.

## **Abstract**

This report addresses the problem of non-stationary signal analysis and multicomponent AM–FM signal separation and demodulation, which are important and significant problems in the area of non stationary signal modeling. This problem appears in various guises in the areas of communications, where it appears in the problem of the cochannel or adjacent channel problem, in biomedical signal processing, where it appears in the area of heart-beat sound modeling, radar signal processing, where it is used to model high frequency radar clutter, in speech processing where it appears in the form of AM–FM speech analysis and synthesis, and image processing, where it finds application in image demodulation and texture analysis. Although numerous multicomponent AM–FM signal analysis algorithms exist, these algorithms are unable to effect signal separation and demodulation in the general case and work only in narrow ranges of component spectral separation or relative power ratios. These algorithms specifically experience singularity problems when the components overlap spectrally, when one of the components is much stronger than the others or when the component instantaneous frequency tracks overlap. Numerous component enumeration methods have been studied for stationary multicomponent sinusoidal signals exist but such a framework does not exist for non-stationary signal component enumeration. The Teager–Kaiser energy operator, related energy separation algorithm and higher-order generalizations of the operator have recently gained recognition for monocomponent demodulation due to their simplicity, efficiency, and excellent time resolution. The goal of this proposal is to develop energy based multicomponent AM–FM demodulation algorithms, that incorporate component enumeration, with the above mentioned difficult scenarios in mind. The developed algorithms will then be applied to (a) cochannel and adjacent channel interference problem, (b) ocean clutter suppression and multiple target tracking problem, (c) second heart beat sound separation and the maternal-foetal ECG signal separation problem.

## **Keywords**

Multicomponent AM–FM signals, signal separation and demodulation, component enumeration, energy operator, higher–order energy operators, generating differential/difference equations, applications.

# 1 Introduction

## 1.1 Significance and Motivation:

The multicomponent AM–FM signal model and related signal separation algorithms find applications in numerous areas of the signal processing and the communications field. Specifically there is a need for such algorithms in the areas of wireless and digital communications, where the demand for bandwidth requires spectral reuse [2], in the area of biomedical signal processing where signal processing algorithms can augment existing diagnostic capabilities in detecting heart disease [38], vocal tract pathologies [44] etc., and in the radar signal processing applications such as ocean clutter suppression and multiple target tracking [36, 12]. In the monocomponent case, numerous algorithms including the Teager–Kaiser energy operator based *energy separation algorithm* (ESA), could be used for solving the monocomponent demodulation problem [24, 25]. However, new problems such as component interaction surface when considering more than one component. When the components are distinct entities in the time–frequency plane just bandpass filtering would be sufficient to separate the components. The case where the components overlap spectrally is however, a challenging case.

Existing multicomponent AM–FM signal separation and demodulation algorithms, however, are unable to effect signal separation in the general case and work only in narrow ranges of component spectral separation or relative component power ratios. Specifically these approaches run into singularity problems when there is significant spectral overlap, when one of the components is much stronger than the others, or the component frequency tracks cross-over, or when there is significant noise in the signal. These existing methods further assume that the number of components present in the signal is known apriori. The problem of enumerating the number of components present in a non-stationary environment is a more difficult task because the phenomenon of decomposing the signal into components is a local one and as pointed out in [7] signal components that are well separated at one instant could overlap spectrally at a later instant.

In this proposal, the need for more robust multicomponent AM–FM energy demodulation approaches that are more general and capable of handling the difficult scenarios discussed above is addressed. A component enumeration approach that incorporates the energy operator and its higher-order generalizations is introduced. Applications of the proposed algorithms to the problems of adjacent channel and cochannel interference mitigation, radar clutter suppression and multiple target tracking, second heart beat sound modeling and foetal-maternal ECG separation, speech processing applications are suggested and studied.

## 1.2 AM–FM Signal Model

Monocomponent AM–FM signals are sinusoidal signals of the form:

$$s(t) = a(t) \cos[\phi(t)],$$

whose instantaneous amplitude (IA),  $a(t)$  and instantaneous frequency (IF),  $\omega(t) = d\phi(t)/dt$  are time-varying quantities. Amplitude modulation (AM) and/or frequency modulation (FM) find extensive use in human-made communication systems [29] and are often present in signals created and processed by biological systems.

*Multicomponent* AM–FM signals<sup>1</sup> are superpositions of monocomponent AM–FM signals:

$$x[n] \equiv \sum_{i=1}^M A_i[n] \cos(\underbrace{\int_0^n \Omega_i[k] dk + \theta_i}_{\phi_i[n]}), \quad M \geq 2,$$

where  $\{A_i[n], \Omega_i[n]\}$  are the IF and IA information signals corresponding to the  $i^{\text{th}}$  component. Each component IF signal is of the general form  $\Omega_i[n] = \Omega_{ci} + \Omega_{mi}q_i[n]$ , where  $\Omega_{ci}$  is the carrier frequency of the  $i^{\text{th}}$  component,  $\Omega_{mi}$  is its maximum frequency deviation, and  $q_i[n]$  is its normalized information signal with  $|q_i[n]| \leq 1$ .

<sup>1</sup>The multicomponent AM–FM model used in this work is quite general in terms of being able to accommodate FSK, CPM, CPFSK, MSK, GMSK and other digital modulation schemes also [3].

For each AM–FM component  $x_i[n]$ , we will assume that its instantaneous amplitude  $A_i[n]$  and frequency  $\Omega_i[n]$  do not vary too fast or too greatly compared with its carrier frequency  $\Omega_{ci}$ . As further, explained in [6, 7], for the decomposition of the composite signal  $x[n]$  into its AM–FM components to be well defined, it is assumed that the instantaneous bandwidth, i.e., the instantaneous frequency spread of each component is narrow with respect to the instantaneous bandwidth of the composite signal.

The multicomponent AM–FM signal separation and demodulation problem appears in various forms in signal processing. In the speech processing area it appears in conjunction with AM–FM speech synthesis, formant frequency and bandwidth tracking and AM–FM vocoder design [19, 20, 21]. In the area of narrowband analog/digital communications it appears in the form of the cochannel and adjacent channel interference problem [4]. In radar signal processing, the multicomponent AM–FM model finds application in high frequency radar ocean clutter suppression and multiple target tracking [36, 12]. In the biomedical signal processing area a multicomponent transient chirp model has been used to model the first and second heart beat signal [9].

### 1.3 Energy Demodulation Primer

The Teager–Kaiser energy operator is a nonlinear, differential operator that computes the energy of a signal  $x(t)$  via:

$$\Psi_c(x) = [x'(t)]^2 - x(t)x''(t),$$

where the dot denotes the time derivative. The discrete–time energy operator applied to the signal  $x[n]$  is defined via:

$$\Psi_d(x) = x^2[n] - x[n+1]x[n-1].$$

The *energy separation algorithm* (ESA) developed in [24, 25] uses this operator to separate amplitude modulations from frequency modulations to accomplish monocomponent AM–FM signal demodulation:

$$\begin{aligned} \omega_i(t) &\approx \sqrt{\frac{\Psi(\dot{x})}{\Psi(x)}} \\ |a(t)| &\approx \frac{\Psi(x)}{\sqrt{\Psi(\dot{x})}}. \end{aligned}$$

Discrete versions of the ESA (DESA's) [24, 25] and applications of the multiband version of the ESA [26] to the problems of AM–FM speech analysis–synthesis, AM–FM vocoding, speech formant frequency and bandwidth tracking have been investigated in [19, 20, 21].

Higher order generalizations of the energy operator, i.e., *higher-order energy operators* (HOEO) for the continuous–time and the discrete–time case are defined via [28, 27, 29]:

$$\begin{aligned} \Upsilon_k(x) &= \dot{x}(t)x^{(k-1)}(t) - x(t)x^{(k)}(t) \\ \Upsilon_k(x) &= x[n]x[n+k-2] - x[n-1]x[n+k-1]. \end{aligned}$$

These operators for sinusoidal input signals measure the higher-order energies of a classical harmonic oscillator normalized to half unit mass [28, 27, 29]. For example, when  $k = 2$  they yield the energy operator, when  $k = 3$  we obtain the *energy velocity operator*, and when  $k = 4$  they yield the *energy acceleration operator*. The *energy demodulation of Mixtures* (EDM) algorithm developed in [28, 27] uses these HOEO's to accomplish separation and demodulation of two component AM–FM signals.

The underlying assumption in all of these energy demodulation approaches is that signal modulations contained in the IF and IA information signals do not vary too much or too fast in comparison to the corresponding carrier frequency. For AM–FM signals with reasonable modulation parameters, these approaches yield negligible demodulation errors. Specifically for the ESA the demodulation errors can be reduced by upto 50 % via simple binomial smoothing of the relevant energy signals [19].

## 1.4 Existing Approaches and Limitations

Existing approaches to multicomponent AM–FM demodulation fall into the conceptual categories of:

1. State space estimation: (a) cross coupled digital phase lock loops (CCDPLL) [16, 17], (b) extended Kalman filters (EKF) [18].
2. Hankel and Toeplitz matrix methods: (a) Hankel rank reduction (HRR) algorithm [12], (b) instantaneous Toeplitz determinant (ITD) algorithm [13].
3. Adaptive linear prediction: (a) LMS algorithm-based demodulation [8], (b) RLS algorithm-based demodulation [5].
4. Energy operator based methods: (a) multiband energy demodulation [26], (b) energy demodulation of mixtures (EDM) algorithm [28, 27].
5. FM-to-AM transduction based approaches [35].
6. Maximum likelihood estimation [15, 10].

For small frequency separation between the components, the covariance matrix in the linear predictive approaches becomes singular [11], the Fischer information matrix in the maximum likelihood scheme becomes singular [10]. In the same region, the matrix systems of the HRR and the ITD algorithms become singular [29], while the observability gramian of the state space approaches becomes singular [18]. The EDM algorithm in particular can separate and demodulate two-component voice-modulated FM signals for spectral separations as small as 25% of the RF bandwidth of the components. For smaller spectral separations, the energy equations of the EDM become singular [28, 27]. The filters of the multiband energy demodulation approach do not have the requisite resolution for component separation [29].

The performance of the EKF based approach, the LMS algorithm, and the CC–DPLL algorithm is also dependent on the relative power ratio between the components [29] (near-far problem). The CC–DPLL algorithm relies upon the *capture effect*, i.e., one component is stronger than the other. The states of the state-space model corresponding to the parameters of the weaker component becomes less observable.

Specifically in the cochannel region these algorithms fail to accomplish signal separation. These problems are a direct consequence of the assumption that the components of the cochannel signal are distinct entities in the time-frequency plane and ignoring the interaction between them. Specifically the problems in the LMS, the HRR, the ITD, and the EDM algorithms are due to the signal separation section that is based on polynomial rooting of some form [29]. Problems encountered in the component separation part eventually translate into problems in the subsequent demodulation sections. The specific situation where the IF tracks of the component signals cross-over is also important because at these instants the existing algorithms by themselves exhibit component flipping and are not able to allocate the cross-over frequency to either component and require additional processing for time-frequency grid tracking [35].

The *periodic algebraic separation energy demodulation* (PASED) algorithm developed recently [31, 32] is capable of accomplishing multicomponent AM–FM signal separation and demodulation even in these difficult situations. This is accomplished by separating the signal separation and demodulation tasks. Component separation is accomplished via algebraic separation techniques and component demodulation is accomplished via the DESA, while the component periodicities are estimated via the DDF algorithm [31, 32]. The assumptions made in the PASED procedure are that: (a) component periodicities used in the algebraic separation section are different and (b) components of the multicomponent signal can be modeled as monocomponent AM–FM signals, (c) the number of components in the multicomponent signal is known a priori. The demodulation errors arising out of the application of the ESA algorithm are practically negligible for AM–FM signals with realistic values of modulation parameters, but they can be reduced further by using simple binomial smoothing [19] of the energy signals before applying the ESA.

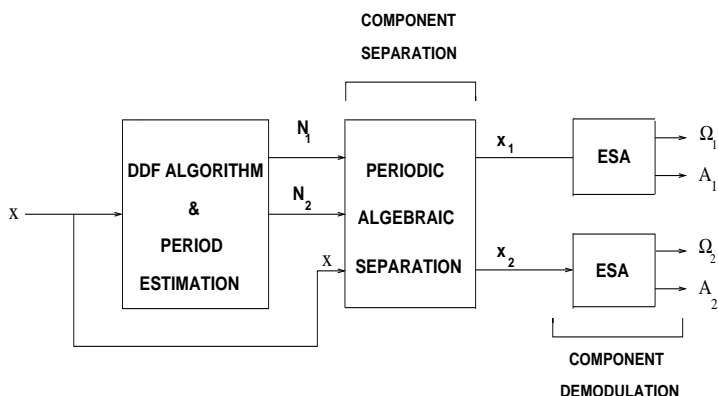


Figure 1: Block Diagram of the PASSED Algorithm

The separation matrix system of the algebraic separation procedure, however, is ill-conditioned and consequently sensitive to noise. The energy operators involved in the ESA demodulation section are also sensitive to the presence of noise. Furthermore the separation system of the PASSED approach requires least-squares inversion of a large system of equations and prior knowledge of the number of components present in the signal. Finally the PASSED algorithm assumes that the component signals are periodic and consequently works only for this particular case. Hence the need for: (a) energy demodulation approaches that are more general in terms of incorporating component quasiperiodicity into the framework, (b) the need for approaches that work for wideband modulation parameter scenarios, (c) the need for approaches that are more robust to noise, and (d) the need for a robust component enumeration procedure.

## 2 Work Proposed

### 2.1 Developments

1. Development of adaptive techniques using the sparse and binary structure of the separation system to implement the least-squares inversion in the PASSED algorithm to circumvent matrix inversion.
2. Development of general separability criteria for the energy based demodulation approaches such as carrier frequency and bandwidth constraints that were developed in [31] for periodic components.
3. Development of a robust component enumeration procedure to determine the appropriate model order using rank-based methods or HOEO's [27]. Presently the energy demodulation approaches assume that the number of components in the signal is known apriori.

### 2.2 Improvements

1. Improving the robustness of the energy demodulation approaches to noise using spline smoothing [37]. Presently the approach used is to use the multiband filtering philosophy where the filters need to be optimized by trading-off signal distortion for noise-suppression [26].
2. Application of the extended algebraic separation approach to the biomedical signal processing problem of maternal-foetal ECG signal separation [33].
3. Application of the energy demodulation algorithms developed to the problems of (a) cochannel and adjacent channel signal separation with emphasis on digital modulation schemes such as FSK, CPM, CPFSK and

(b) ocean clutter suppression and multiple target tracking, (c) heart beat component separation, (d) speech processing applications.

## 2.3 Envisioned Applications

### Application to Component Enumeration

The *generating difference equation* (GDE) for a monocomponent sinusoidal signal that is invariant to both the amplitude and frequency is given by [27, 39]:

$$D[n] = \Psi(x[n]) - \Psi(x[n-1]) = 0.$$

In the case of narrowband AM-FM signals, where the information signals are slowly time-varying this relation holds approximately. The test signal  $D[n]$  can therefore be used to detect the presence of a single component by thresholding the samples of  $D[n]$  that are close enough to zero within a threshold  $\eta_o$  that is dependent of the SNR of the signal environment, i.e.,

$$T[n] = \begin{cases} 1 & |D[n]| \leq \eta_o \\ 0 & \text{otherwise.} \end{cases}$$

The monocomponent detection problem can then be posed as a binary hypothesis testing problem of the form:

$$\begin{aligned} \mathbf{H}_1 &: \hat{p} \geq p_o(\eta_o) \\ \mathbf{H}_0 &: \hat{p} < p_o(\eta_o), \end{aligned}$$

where  $\hat{p}$  is the proportion of decision variable values that is 1 and  $p_o$  is a threshold that depends on the SNR of the signal environment. Modelling each of the  $N$  samples of the decision variable,  $T[n]$  as independent trials of a binomial random variable and treating a zero decision variable sample as a success, for a large  $N$  the variable

$$Z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{N}}}$$

is a standard Gaussian random variable via the central limit theorem. The hypothesis detection problem can then be posed in terms of a Neyman-Pearson test of the form:

$$\begin{aligned} &\mathbf{H}_1 \\ Z &> Q^{-1}(\alpha). \\ &\mathbf{H}_0 \end{aligned}$$

where  $\alpha$  is the probability of a false-alarm. Fig. (2) describes a component enumeration example where the composite signal described in Fig. (2)(a) has two components present in its first half and just one component present during the second half with a SNR of 27 dB. The additional benefit in energy operator based component enumeration is that the spikes in the energy operator output can be used to detect the presence or the onset of an event [43]. The presence of a energy discontinuity in  $D[n]$  indicates the presence of an event after 500 samples, which in this example is a change in the number of components. Fig. (2)(b) describes the test signal  $D[n]$  after 5-time binomial smoothing of the energy signal  $\Psi(x[n])$ . Fig. (2)(c) describes the decision variable  $T[n]$  using a threshold of  $\eta_o = 0.06$ . The proportion of ones in the decision variable for this example over the second half of the signal was 98.6 %, while in the first half this proportion was 5.0 %.

For signals that contain more than one component, we will use the fact that for stationary sinusoids a Hankel matrix of signal values is of rank  $2M$  when there are  $M$  components in the signal or when the component IF and IA signals are slowly time-varying [41]. These Toeplitz determinant operators are then computed for various orders and then the value of the determinant is then thresholded and the proportion of the zero determinant samples is



computed. The number of components in the signal can then be determined from the model order for which a significant proportion of determinant samples is close to zero within a threshold  $p_o$  that depends on the SNR [34]. Figure 3 describes a two-component example where two components are present during both halves but the component IF's overlap over the second half. Fig. (3)(c) describes the performance of the determinant proportion method for  $\eta_o = 0.1, p_o = 0.8$  for a SNR of 30 dB, MPR of 6 dB. Selection of the threshold for the proportion test, however, depends on the specific signal parameter environment. Figure. (3)(c,d) illustrates the methodology for threshold parameter selection in the two-component example.

The singularity of these instantaneous operators will however, depend on the spectral proximity of the components and the relative strengths of the components. The *normalized carrier separation* (NCS) parameter described in [27] is defined as the separation between the component carrier frequencies normalized by the average Carson bandwidth of the components. In a similar fashion, the relative power or amplitude ratio between the components plays a role in singularity. Specifically the interaction between the components of a two-component AM-FM signal is a maximum when the components are of equal magnitude [7]. Figure (5) describes the proportion of singular samples of a two-component sinusoidally modulated AM-FM signal in terms of the NCS and relative amplitude ratio parameters.

### Application to Interference Removal

With the advent of the wireless/cellular revolution in communications the demand for communication systems that support more users has risen. The increased demand for bandwidth inturn necessitates systems that have spectral efficiency and support spectral reuse [2]. In such systems where multiple users share the same carrier frequency *cochannel interference* (CCI) is a common occurrence [4]. Due to the close spectral proximity of the signals of users adjacent in frequency *adjacent channel interference* (ACI) is also a common problem that results in deterioration of performance [4].

The intent is to focus on the two-component AM-FM signal model for the interference removal application. Specifically the intention is to model the desired signal (SOI) as one of the components and to model the dominant interference (SNOI) as the second component. Although this is a simple model for the CCI/ACI problem it is accurate enough to reflect the problems that are specific to the multicomponent separation and demodulation problem. Although the AM-FM signal model adopted here involves amplitude and frequency modulation it is general enough to accomodate digital modulation approaches such as FSK, MSK, CPM, where user data bits are loaded onto the IF of the corresponding AM-FM signal component [2, 3].

Consider the two-component CPM-ACI example in Fig. (6), where a rectangular pulse shaping function with BPSK signaling was used. The *mean power ratio* (MPR) between the components is 0 dB and the SNR of the signal environment in Fig. (6)(a,b) is 40 dB. The symbol period for the example was  $T_b = 5$  ms at a sampling rate of 10 kHz. In this example the EDM algorithm was used to demodulate the two-component CPM signal. The estimated component IF's are first 11-pt median filtered and then smoothed using a 3-pt binomial filter. A conventional BPSK detector is then used for symbol detection for each user. The composite signal for the example is described in Fig. (6)(a), the IF estimates of the EDM algorithm after 11-pt median filtering and binomial smoothing are described in Fig. (6)(b), while Fig. 6(c) depicts the average detection error probability for the EDM/MEDM algorithms for various SNR scenarios over 50 experiments and 1500 symbol intervals.

Although the IF estimates are used to make decisions on the bits transmitted, it is ultimately the decision error probability that we are concerned about in this case. The detection error probability is negligible for SNR values larger than 30 dB. Further robustness to noise can be acheived by using the multiband-EDM (MEDM) approach proposed in [28, 27]. Fig. (6)(c) describes the detection error propbabilities of the MEDM approach which are considerably better than the corresponding EDM probabilities.

## Radar Signal Processing Applications

In high frequency radar applications, ocean clutter manifests in the return signal in the form of well defined frequency shifts from the transmitted radar frequency  $f_r$  via [12]

$$f_{oc} = \pm \sqrt{\frac{gf_r}{\pi c}},$$

where  $f_{oc}$  corresponds to the Doppler frequency of the ocean clutter signals,  $f_r$  is the radar carrier frequency and  $g$  is the acceleration due to gravity. This effect can be attributed to constructive interference of the scattering return signals. Recent work on modeling ocean clutter has shown that they can be modeled as two narrowband signals with time varying frequencies centered about two frequencies corresponding to both first and second order scattering [12]. This framework for clutter modeling fits perfectly with the multicomponent AM-FM signal model described in the previous section.

It has also been known in the radar community that a target moving with a constant velocity produces a constant Doppler frequency shift given by

$$f_d = \frac{2f_r v}{c},$$

where  $v$  is the radial velocity of the target and  $f_r$  is the radar carrier frequency. Similarly a target that has a constant radial acceleration will produce linearly changing frequency, i.e., linear-FM or chirp return signals [14]. Further work in the target tracking area has focussed on the problem of multiple target tracking, where the return signal for each target is modeled as a chirp signal [22].

This is the motivation behind adopting a multicomponent chirp signal model for the multiple target tracking application of the form [22]:

$$x(t) = \sum_{k=1}^M a_k \cos(\omega_{ck}t + \mu_k t^2),$$

where  $\omega_{ck}$  corresponds to the carrier frequency of the  $k^{\text{th}}$  target and the IF signal corresponding to the  $k^{\text{th}}$  target is a linear-FM signal of the form:

$$\omega_k(t) = \omega_{ck} + 2\mu_k t, \quad |t| \leq \frac{T}{2},$$

where  $T$  is the duration of the radar pulse waveform.

Consider the two-component chirp example in Fig. (7). The relevant parameters associated with the composite FM signal are FM = 8%, NCS = 0.06, SNR = 40 dB, MPR = 0 dB. With these parameters, there is a significant amount of spectral overlap between the components. In addition the IF laws of the two components cross-over. The composite FM signal is described in Fig. (7)(a), the DDF intensity image used for component periodicity estimation is shown in Fig. (7)(b). Fig. (7)(c) describes the spectrogram of the composite FM signal which shows that there is a significant amount of spectral overlap. Fig. (7)(d) describes the component IF estimates of the PASED algorithm. The chirp parameters estimated via the PASED algorithm after least-squares fitting and the actual chirp parameters are tabulated in Fig. (7)(e). The least-squares estimates of the component IF's are plotted in Fig. (7)(f).

Further robustness towards noise can be obtained by binomial smoothing of the energy operators used in the implementation of the *discrete energy separation algorithm* (DESA) [32] used in the PASED algorithm. The separated components can be further filtered using a bandpass filter to eliminate noise outside of the signal bandwidth<sup>2</sup>.

## Biomedical Signal Processing Applications

The multicomponent AM–FM signal model, specifically a two–component nonlinear transient chirp signal model, has recently been used to model the pulmonary and aortic components of the second heart beat sound [9]. Specifically it was shown that both the aortic and the pulmonary components of the second heart beat sound could be modeled as narrowband chirp signals of short duration, with decreasing IF's, and with energy distributions concentrated along their decreasing IF signals. The second heart beat signal can in particular be used as a diagnostic tool where it can augment existing methods for detecting paediatric heart disease [38]. The signal separation aspects of the proposal as it relates to the separation of quasiperiodic signals also find application in a related biomedical signal processing problem of maternal–foetal ECG signal separation and foetal heart–rate monitoring [33]. Presently used time–frequency tools for analyzing these multicomponent signals suffer from the existence of cross-terms in the time–frequency representations [9].

Consider the example in Fig. (8) where the algebraic separation (MAS) algorithm was applied to a mixture of quasi-periodic segments of the ECG data. The ECG signals were obtained from samples of the MIT-BIH polysomnographic, MGH/MF ECG databases available at <http://ecg.mit.edu/dbsamples.html>. The signal from the MIT-BIH database has a sampling frequency of  $f_s = 340$  Hz and the signal from the MGH/MF database has a sampling frequency of  $f_s = 250$ Hz. The ECG data used in this example is approximately periodic with periods  $N_1 = 12$  and  $N_2 = 21$  samples which were estimated using the DDF algorithm. The composite signal for the example is shown in Fig. (8)(a). The DDF intensity image used to estimate the periods is shown in Fig. (8)(b). The component estimates ( $\lambda = 10^8$ ) are shown in Fig. (8)(c,d) with RMS component separation errors are 10, 13%.

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<sup>2</sup>In this case signal bandwidth refers to twice the Carson bandwidth of the FM signal, i.e., the separation between spectral components whose strength is 1 % of the carrier strength [1].

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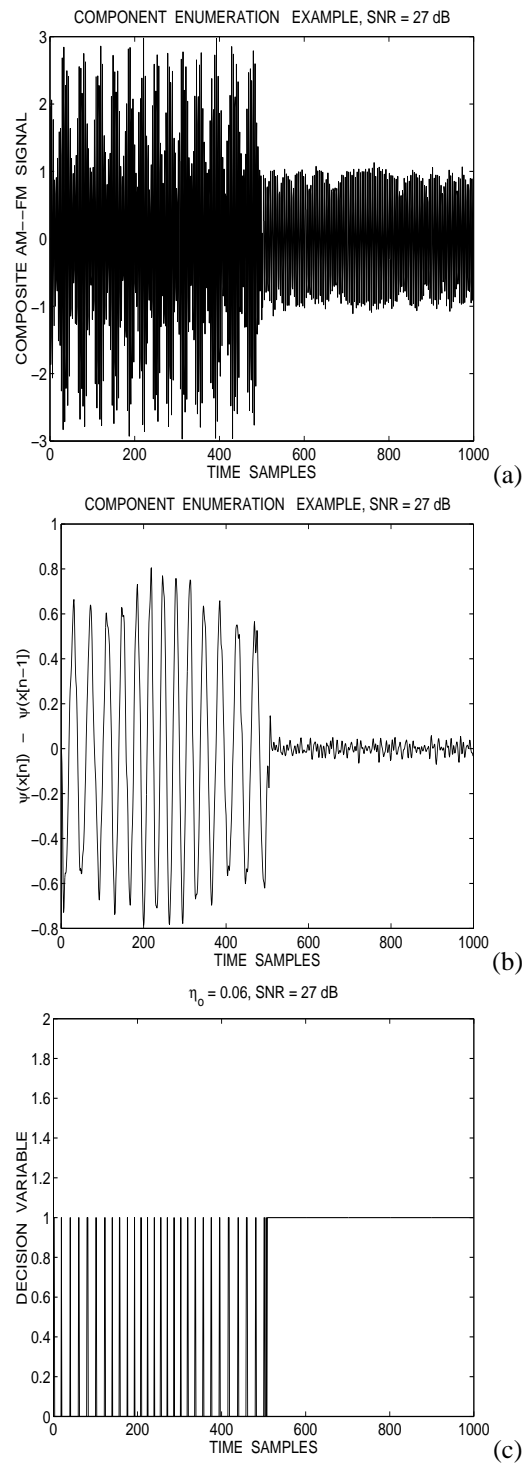


Figure 2: Component enumeration example: (a) composite AM-FM signal, (b) test signal  $D[n]$  after 5-time binomial smoothing, (c) decision variable for monocomponent detection over the two halves of the signal. The first half of the signal contains two components while the second contains just one component.

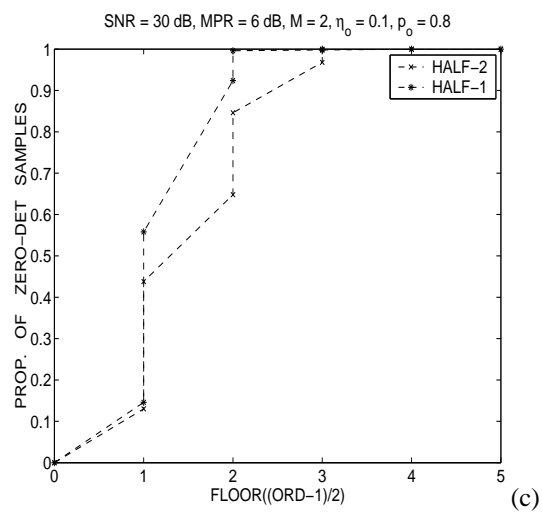
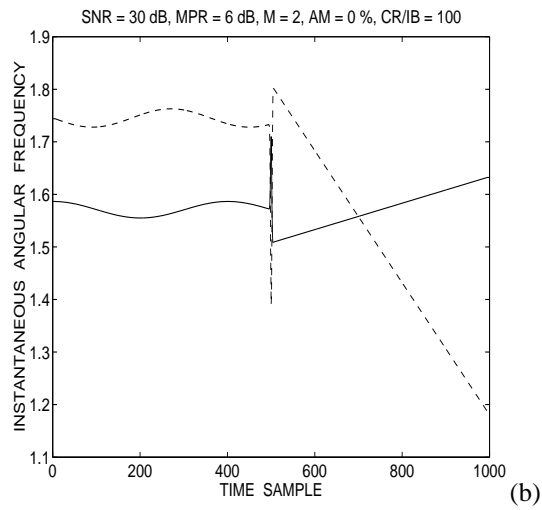
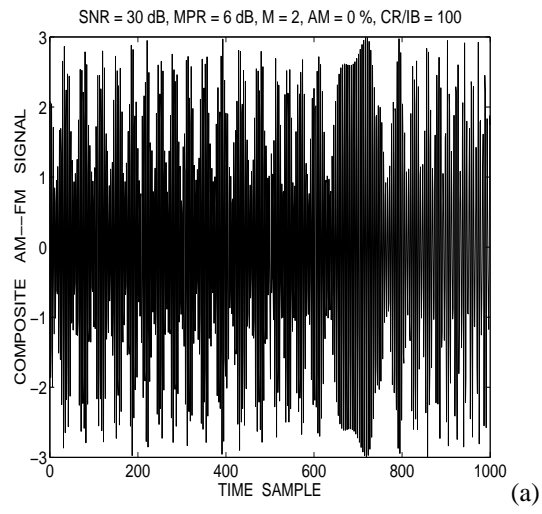


Figure 3: Two-component signal example where both halves of the signal contain 2 components. In the first half the component IF's are well-separated, while in the second half they cross-over.



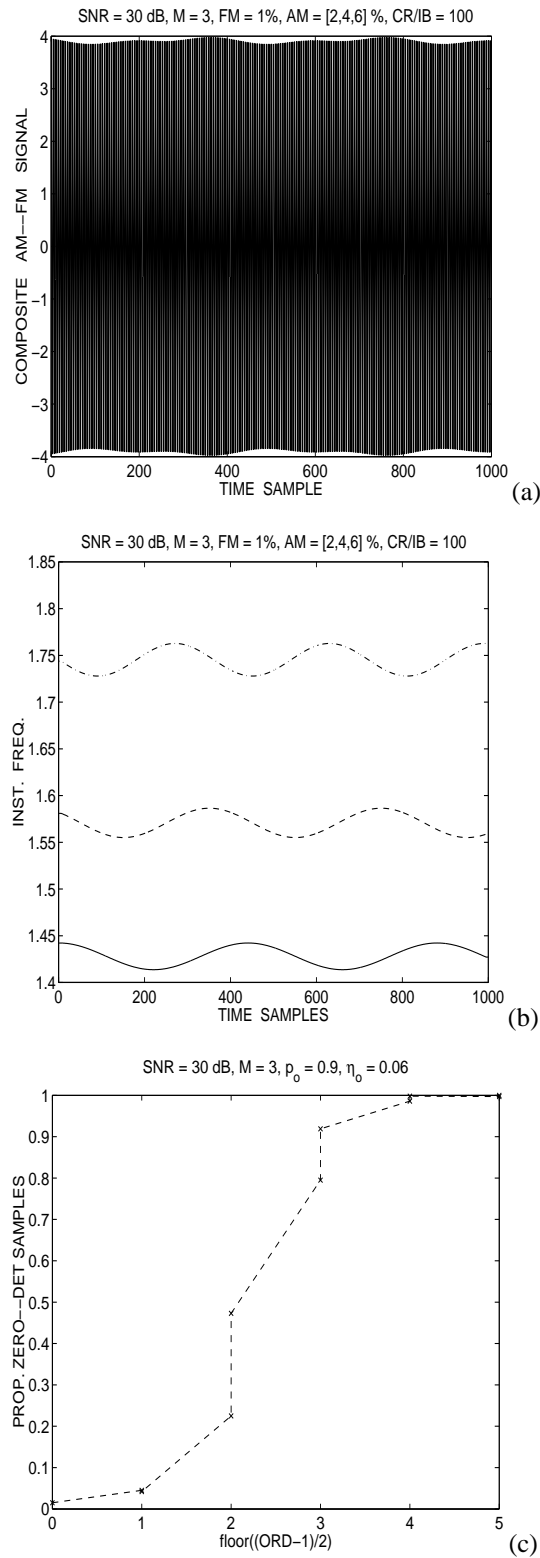


Figure 4: Three-component example: (a) composite AM-FM signal, (b) component IF's, (c) proportion of zero-determinant samples for various model orders using the Toeplitz-determinant proportion technique with a SNR of 30 dB,  $\eta_o = 0.06$  and  $p_o = 0.9$ .

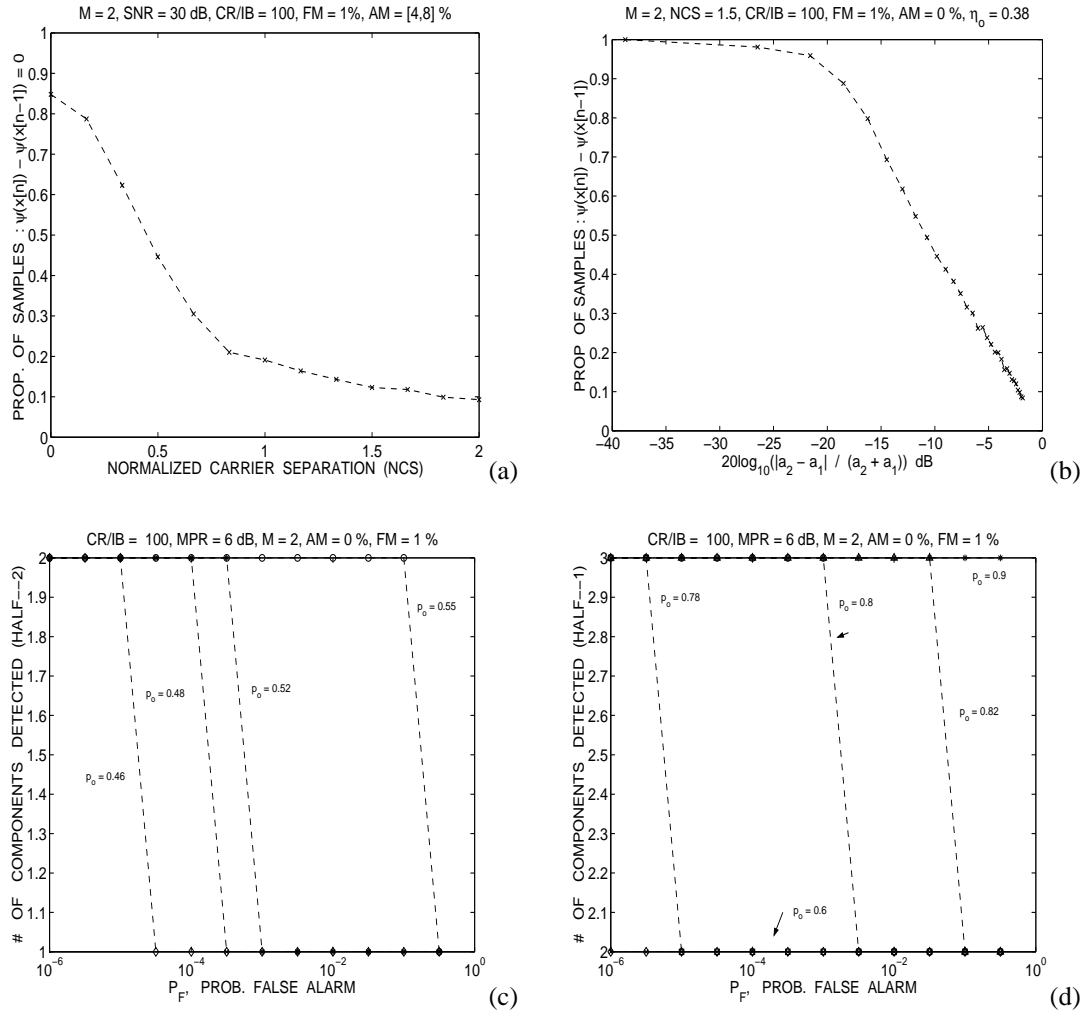


Figure 5: Effect of NCS and relative component power: (a) proportion of samples where  $D[n] = 0$  for different NCS parameters with  $\eta_0 = 0.05$ , (b) proportion of samples where  $D[n] = 0$  for different relative component amplitudes. (c,d) threshold selection: thresholds  $p_o$  for the two halves in the second two-component example and the corresponding number of components detected for different false alarm probabilities. The intersection of the set of valid thresholds for both halves is selected for use.

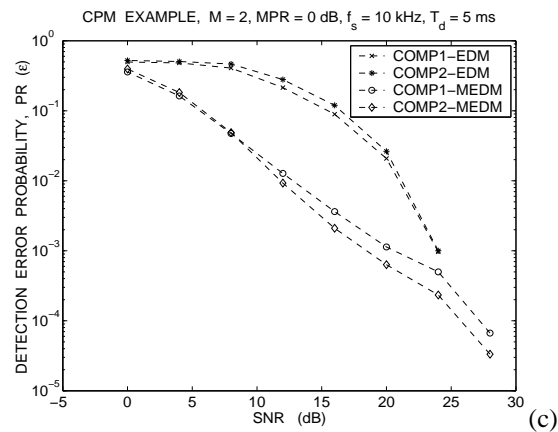
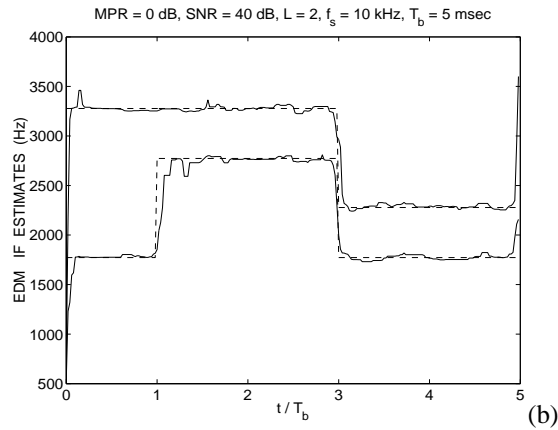
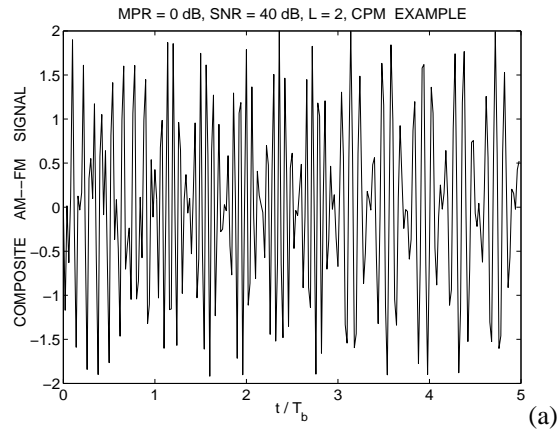


Figure 6: CPM-BPSK-ACI Example: (a) composite signal, (b) 11-pt median filtered IF estimates of the EDM algorithm over 5 symbol intervals, (c) averaged detection error probability of the EDM/MEDM algorithms over 50 experiments and 1500 symbol intervals. For higher SNR's, the detection error is negligible.

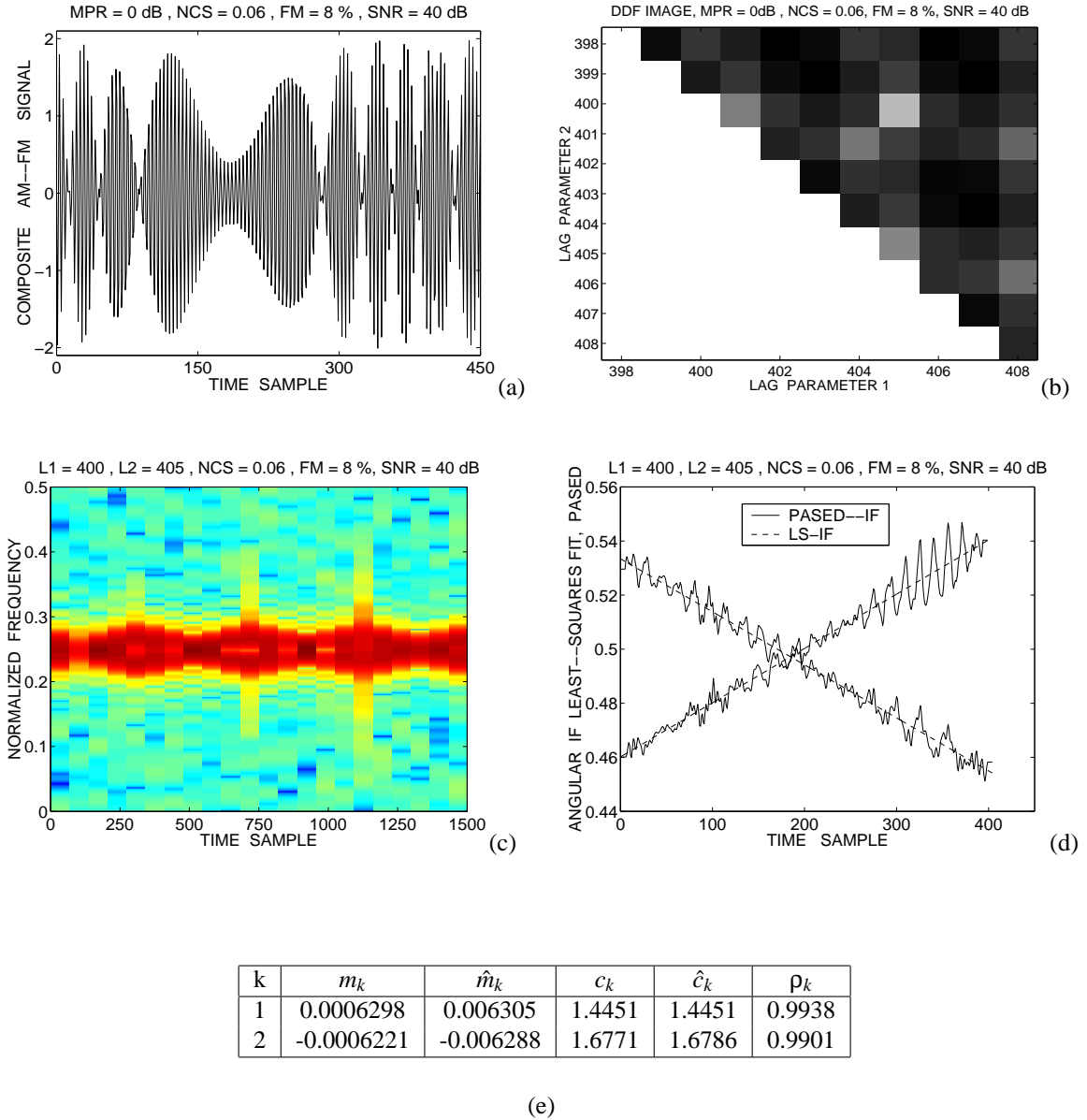


Figure 7: Chirp target tracking example: (a) composite FM signal, (b) DDF intensity image associated with component period estimation, (c) spectrogram of the composite FM signal with a FFT size of  $N = 1024$ , a Hanning window of length  $L = 75$ , with  $R = 2$  showing significant spectral overlap, (d) least-squares IF fit versus actual IF, (e) slope and intercept obtained from a least-squares fit of the PASSED IF estimates with the corresponding correlation coefficients.

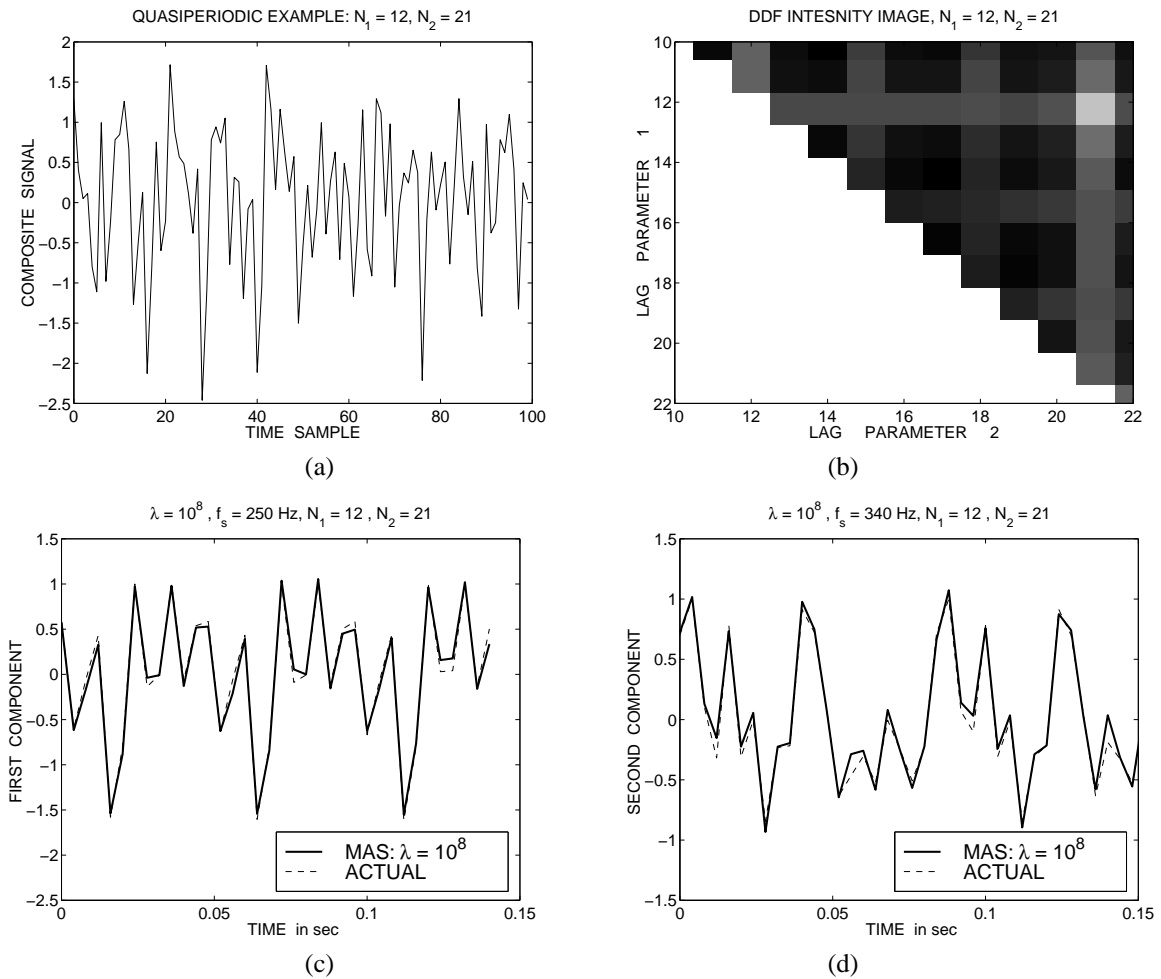


Figure 8: Separation of quasi-periodic segments of two ECG signals: (a) the composite signal, (b) periods of  $N_1 = 12$  and  $N_2 = 21$  samples estimated via the DDF algorithm, (c,d) MAS component estimates.