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Distributed Joint Rate and Power Control Game-Theoretic Algorithms for Wireless Data

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Abstract—In this letter, we consider two distributed game theoretic algorithms to jointly solve the problem of optimizing the transmission rates and transmit powers for future wireless data communication systems. We then establish the existence, uniqueness and Pareto optimality of Nash equilibria of both games.

Index Terms—Game theory, joint rate and power, Pareto efficiency.

I. INTRODUCTION

TRANSMITTERS in multimedia wireless networks may require different quality of services (QoS) in order to establish a communication link with a receiver. Providing flexible transmission rates for each transmitter/receiver pair and efficient use of the shared radio resources requires joint power and rate control optimization algorithms. Earlier work in this arena used centralized algorithms (c.f. [1]). Due to the difficulty of implementing centralized algorithms, and to avoid control signals that cause delays in the system operation, distributed algorithms were proposed. Game theory was shown to be an appropriate tool for finding power control algorithms in [7]–[10] and for rate flow control algorithms in [4] and [5]. In particular, the authors in [7] proposed a utility based joint power and rate optimization algorithm, but the resulting Nash equilibrium (NE) point was Pareto inefficient and to guarantee the uniqueness of NE the rates of all users were forced to be equal. In this letter, we use game theory framework for finding a pure distributed algorithms for the **joint** rate and power control optimization problem. To solve the problem, we propose two, layered, but noncooperative priced games as follows: The first game $G1$ allocates the optimal transmission rates for all users, then provides the second game with a vector of constants c. The second game $G2$ uses c to evaluate the optimal transmit power levels that support the resulting Nash equilibrium transmission rates of game $G1$.

II. SYSTEM SETUP AND OUR APPROACH

Consider N transmitter/receiver pairs (users) in a mobile cellular network. The ith transmitter, $i \in \mathcal{N} = \{1, 2, \ldots, N\}$ transmits at a power level p_i from its convex strategy space P_i to the ith receiver and sends data at a rate r_i from its convex strategy space R_i. The received power level at the kth receiver from the ith transmitter is given by $G_{k,i}p_i$, where $G_{k,i} > 0$ is

the path gain from the ith transmitter to the kth receiver. This gain may represent spreading gain and/or cross correlation between codes in CDMA systems or any gain that captures the effect of a fading channel.

The game $G1$ that optimally allocates the transmission rates for all users is given by

$$G1 : \max_{r_i \in R_i} \{L_i(r_i, \mathbf{r}_{-i}, \lambda)$$
$$= u_i \log(1 + K_i r_i) + \beta_i \log(r_i - r_{i-\min}) - \lambda r_i\} \quad (1)$$

where $K_i = \left(\sum_{k \neq i}^{N} r_k\right)^{-1}$, $\lambda > 0$ is the pricing factor broadcasted by the base station (BS) to all users, and u_i (which measures the willingness of user i to pay) is the utility factor of the ith user locally selected based on the desired transmission rate. Finally, β_i is a constant selected such that $\beta_i \ll u_i$, and $r_{i-\min}$ is the minimum required transmission rate. The first term of L_i is chosen to maximize the transmission rate of user i, while the second term works as a barrier to prevent the ith user's rate from going below $r_{i-\min}$, and to fairly allocate the transmission rates. The goal of λr_i is to prevent the greedy use of the available channel capacity.

Game $G2$ below, allocates the transmit power levels that support the resulting Nash equilibrium rates r_i^o, $\forall\ i \in \mathcal{N}$ of game $G1$.

$$G2 : \max_{p_i \in P_i} \{g_i(p_i, p_{-i}, c_i) = \log(\gamma_i) - c_i p_i\}$$

$$c_i = (I_i \exp(r_i^o))^{-1}; \quad I_i = \frac{\sum_{k \neq i}^{N} G_{i,k} p_k + \sigma^2}{G_{i,i}}. \quad (2)$$

I_i is the effective interference that user i needs to overcome. Vectors $\mathbf{r}_{-i}$ and $\mathbf{p}_{-i}$ are the vectors of transmission rates and transmit powers of all users except for the ith user, and γ_i is the signal-to-interference ratio (SIR) defined by

$$\gamma_i = \frac{G_{i,i} p_i}{\sum_{k \neq i}^{N} G_{i,k} p_k + \sigma^2}. \quad (3)$$

In applications where the spectrum and power are limited resources, it is recommended to use a spectrally and power efficient modulation technique such as M-QAM. An empirical link rate model for M-QAM of user i is given by [2]:

$$r_i = \mu \log(1 + \theta_i \gamma_i) \quad (4)$$

where $\theta_i = -1.5/ln(5BER_i)$ with BER_i is the target BER of user i and μ is a system constant. In this letter we use the following approximation of (4) at high SIR [1]:

$$r_i = \log(\gamma_i) \quad (5)$$

where r_i is normalized by the channel bandwidth with units, nats/s/Hz. A user can change the transmission rate by adapting

different modulation formats (e.g., 2-QAM, 4-QAM, …). Therefore, the transmission rate of each user belongs to a discrete set, but we assume in this letter that the transmission rates are continuous for simplicity. In Section III we establish the existence, uniqueness and optimality of the equilibrium point $(\mathbf{r}^o, \mathbf{p}^o)$ of both games.

III. EXISTENCE OF NASH EQUILIBRIUM

A. Non-Cooperative Rate Control Game With Pricing (NRGP) $G1$

The optimization problem of the ith user defined in game $G1$ is to find the transmission rate r_i^o from the strategy space R_i that maximizes the utility function defined in (1). To do so, we set

$$\frac{\partial L_i}{\partial r_i} = \frac{u_i K_i}{1 + K_i r_i} + \frac{\beta_i}{r_i - r_{i-\min}} - \lambda = 0. \tag{6}$$

The maximizing transmission rate of user i, r_i^o is thus given by

$$r_i^o = -\frac{1}{2} B_i + \sqrt{\frac{1}{4} B_i^2 + C_i} \tag{7}$$

where $B_i = (1/K_i) - (r_{i-\min} + (u_i/\lambda) + (\beta_i/\lambda))$ and $C_i = (r_{i-\min}/K_i) - (u_i r_{i-\min}/\lambda) + (\beta_i/K_i \lambda)$. Note that $\partial^2 L_i/\partial r_i^2 < 0, \forall i \in \mathcal{N}$, which means that L_i is a strictly concave function of r_i. Therefore, L_i is a quasiconcave function optimized on a convex set R_i, and game theory results guarantee the existence of a Nash equilibrium point [3]. In the remainder of this section we prove the uniqueness of this Nash equilibrium point. We first need the following result.

Proposition 1: For game $G1$ defined in (1), the best response of user i, given the transmission rates vector of the other users $\mathbf{r}_{-i}$ is given by: $\rho_i(\mathbf{r}_{-i}) = \min(r_i^o, r_{i-\max}), \forall i \in \mathcal{N}$ where $r_{i-\max}$ is the maximum allowed transmission rate in the ith user's strategy space R_i.

Proof: Define the best response function $\rho_i(\mathbf{r}_{-i})$ of the ith user as the best action that user i can take to attain the maximum pay off given the other users' actions $\mathbf{r}_{-i}$. That is, $\rho_i(\mathbf{r}_{-i}) = \{r_i : L_i(r_i, \mathbf{r}_{-i}) \geq L_i(r_i', \mathbf{r}_{-i}), \forall r_i' \in R_i\}$, where this set contains only one point [9]. From (7), r_i^o is the unconstrained maximizer of the target function L_i, i.e., $r_i^o = \operatorname{argmax}_{r_i \in \mathbb{R}^+} L_i$. Since $\partial^2 L_i/\partial p_i^2$ is negative $\forall r_i \in \mathbb{R}^+$, this maximizer is unique. Now, assume that r_i^o is not feasible, that is, $r_i^o \notin R_i$, then user i will get his/her maximum at $r_{i-\max}$ since the target function is increasing on the set $\{r_i : r_i < r_i^o\}$. This implies that $r_i = r_{i-\max}$ is the best response of user i given $\mathbf{r}_{-i}$. ∎

The following theorem, proven in [6], guarantees the uniqueness of a Nash equilibrium operating point of game $G1$.

Theorem 1: If a power control algorithm with a standard best response function has a Nash equilibrium point, then this Nash equilibrium point is unique.

See [6] for the definition of a standard function. Theorem 1 allows us to state the following lemma, whose proof is omitted.

Lemma 1: In game $G1$, the best response vector of all users given by

$$\rho(\mathbf{r}) = (\rho_1(\mathbf{r}), \rho_2(\mathbf{r}), \ldots, \rho_N(\mathbf{r}))$$

is a standard vector function. Therefore, by theorem 1, game $G1$ has a unique Nash equilibrium point $\mathbf{r}^o = (r_1^o, r_2^o, \ldots, r_N^o)$.

B. Non-Cooperative Power Control Game With Pricing $G2$

To find the maximizing p_i^o for game $G2$ we evaluate:

$$\frac{\partial g_i}{\partial p_i} = \frac{1}{I_i \gamma_i} - c_i = 0 \tag{8}$$

and by substituting for the value of c_i, the maximizing transmit power level is thus given by

$$p_i^o = I_i \exp(r_i^o). \tag{9}$$

The transmit power level p_i^o represents the minimal power (i.e., without waste) required to support the optimal transmission rate r_i^o. Note that $\partial^2 g_i/\partial p_i^2 = -1/p_i^2 < 0, \forall i \in \mathcal{N}$. Therefore, g_i is a strictly concave function, and using the same argument for L_i in $G1$, there exists a Nash equilibrium point $\mathbf{p}^o = (p_1^o, p_2^o, \ldots, p_N^o)$ in game $G2$. In what follows we prove the uniqueness of the Nash equilibrium point of game $G2$ by proposing the best response of user i in game $G2$ similarly to proposition 1.

Proposition 2: For game $G2$ defined in (2), the best response of user i, given the transmit power levels vector of the other users $\mathbf{p}_{-i}$ is given by: $\nu_i(\mathbf{r}_{-i}) = \min(p_i^o, p_{i-\max}), \forall i \in \mathcal{N}$ where $p_{i-\max}$ is the maximum transmit power level in the ith user's strategy space P_i.

Then, the uniqueness of the Nash equilibrium operating point can be proved similarly to game $G1$ since the best response vector of users in $G2$ given as $\nu(\mathbf{r}) = (\nu_1(\mathbf{r}), \nu_2(\mathbf{r}), \ldots, \nu_N(\mathbf{r}))$ is also a standard function. The following Lemma then guarantees Pareto optimality (efficiency) of the equilibrium point $(\mathbf{r}^o, \mathbf{p}^o)$ of both NRGP and NPGP games $G1$ and $G2$, respectively.

Lemma 2: The Nash equilibrium point $(\mathbf{r}^o, \mathbf{p}^o)$ of the NRGP game $G1$ and NPGP game $G2$ is Pareto optimal. Mathematically speaking, for $G1$, $\nexists \mathbf{r}^* = (r_1^*, r_2^*, \ldots, r_N^*)$: $L_j(\mathbf{r}^*) \geq L_j(\mathbf{r}^o), \forall j \in \mathcal{N}$ and $L_m(\mathbf{r}^*) > L_m(\mathbf{r}^o)$ for some $m \in \mathcal{N}$, with $\mathbf{r}^* > \mathbf{r}^o$ component wise. For $G2$, $\nexists \mathbf{p}^* = (p_1^*, p_2^*, \ldots, p_N^*)$: $g_j(\mathbf{p}^*) \geq g_j(\mathbf{p}^o), \forall j \in \mathcal{N}$ and $g_n(\mathbf{p}^*) > g_n(\mathbf{p}^o)$ for some $n \in \mathcal{N}$, with $\mathbf{p}^* < \mathbf{p}^o$ component wise.

Proof: We already know from (6) that

$$f_j(\mathbf{r}^o) \triangleq \frac{u_j K_j^o}{1 + K_j^o r_j^o} + \frac{\beta_j}{r_j^o - r_{j-\min}} - \lambda = 0 \tag{10}$$

where $K_j^o = \left(\sum_{k \neq j}^N r_k^o\right)^{-1}$, therefore (10) can be written as

$$f_j(\mathbf{r}^o) = \frac{u_j}{\sum_{k=1}^N r_k^o} + \frac{\beta_j}{r_j^o - r_{j-\min}} - \lambda = 0. \tag{11}$$

Without loss of generality, let $r_k^* = \rho_k r_k^o, \forall k \in \mathcal{N}$ where $\rho_k > 1, \forall k \in \mathcal{N}$. Then we have the following:

$$L_j(\mathbf{r}^*)$$

$$= u_j \log \left(1 + \frac{\rho_j r_j^o}{\sum_{k \neq j}^N \rho_k r_k^o}\right) + \beta_j \log \left(\rho_j r_j^o - r_{j-\min}\right) - \lambda \rho_j r_j^o. \tag{12}$$

In order to find out how $L_j(\mathbf{r}^*)$ behaves with ρ_j, we need to find the first-order derivative of $L_j(\mathbf{r}^*)$ with respect to ρ_j as follows:

$$\frac{\partial L_j(\mathbf{r}^*)}{\partial \rho_j} = r_j^o \left(\frac{u_j}{\sum\limits_{k=1}^{N} \rho_k r_k^o} + \frac{\beta_j}{\rho_j r_j^o - r_{j-\min}} - \lambda \right)$$

$$= r_j^o f_j(D^\rho \mathbf{r}^o), \quad \forall j \in \mathcal{N} \tag{13}$$

where $D^\rho = \text{diag}(\rho_1, \rho_2, \ldots, \rho_N)$. One can check easily that $f_j(D^\rho(\mathbf{r}^o) < f_j(\mathbf{r}^o) = 0, \forall j \in \mathcal{N}$. Henceforth, $\partial L_j(\mathbf{r}^*)/\partial \rho_j < 0, \forall j \in \mathcal{N}$, that is, $L_j(\mathbf{r}^*)$ is decreasing over $\rho_j > 1$ for all users, and by this we conclude that $\mathbf{r}^o$ is a Pareto optimal NE point of NRGP game $G1$. To prove that $\mathbf{p}^o$ is a Pareto optimal NE point of $G2$, it is enough to prove that $p_j^o \in P_j$ is the minimum required transmit power to support $r_j^o \in R_j$ for all $j \in \mathcal{N}$. By re-writing (9) as: $\gamma_j^o = p_j^o/I_j = \exp(r_j^o), \forall j \in \mathcal{N}$ And from (5), we conclude the proof. ∎

It was proven in [6] that both synchronous and asynchronous algorithms with standard best response functions converge to the same point. Therefore, we consider asynchronous power and rate control algorithms which converge to the unique Nash equilibrium point $(\mathbf{r}^o, \mathbf{p}^o)$ of games $G1$ and $G2$. In this algorithm, the users update their transmission rates and powers in the same manner as in [9]. Assume user j updates its transmission rate at time instances in the set $T_j = \{t_{j_1}, t_{j_2}, \ldots\}$, with $t_{j_k} < t_{j_{k+1}}$ and $t_{j_0} = 0$ for all $j \in \mathcal{N}$. Let $T = \{t_1, t_2, \ldots\}$ where $T = T_1 \bigcup T_2 \bigcup \ldots \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{\mathbf{r}}$ to be the transmission rates vector picked randomly from the total strategy space $R = R_1 \bigcup R_2 \bigcup \ldots \bigcup R_N$.

Algorithm 1: Consider the game $G1$ given in (1) and generate a sequence of transmission rates vectors as follows: **(a)** Set the transmission rate vector at time $t = 0$: $\mathbf{r}(0) = \underline{\mathbf{r}}$, let $k = 1$ **(b)** For all $j \in \mathcal{N}$, such that $t_k \in T_j$: Given $\mathbf{r}(t_{k-1})$, calculate $r_j^o(t_k) = \text{argmax}_{r_j \in R_j} L_j(r_j, \mathbf{r}_{-j}(t_{k-1}), \lambda)$, then let the transmission rate $r_j(t_k) = \rho_j(t_k) = \min(r_j^o(t_k), r_{j-\max})$ **(c)** If $\mathbf{r}(t_k) = \mathbf{r}(t_{k-1})$ stop and declare the Nash equilibrium transmission rates vector as $\mathbf{r}(t_k)$, else let $k := k + 1$ and go to **(b)**. **(d)** For all $j \in \mathcal{N}$, calculate c_j and provide it to algorithm 2.

When Algorithm 1 converges to $\mathbf{r}^o$, Algorithm 2 below finds the optimal levels $\mathbf{p}^o$ to support $\mathbf{r}^o$. Suppose user j updates its power level at time instances in $T_j^p = \{t_{j_1}, t_{j_2}, \ldots\}$, with $t_{j_k} < t_{j_{k+1}}$ and $t_{j_0} = 0$ for all $j \in \mathcal{N}$. Let $T^p = \{t_1, t_2, \ldots\}$ where $T^p = T_1^p \bigcup T_2^p \bigcup \ldots \bigcup T_N^p$ with $t_k < t_{k+1}$ and define $\underline{\mathbf{p}}$ a randomly chosen power vector in $P = P_1 \bigcup P_2 \bigcup \ldots \bigcup P_N$.

Algorithm 2: The game $G2$ as given in (2) generates a sequence of power vectors as follows: **(a)** Set the power vector at time $t = 0$: $\mathbf{p}(0) = \underline{\mathbf{p}}$, let $k = 1$. **(b)** For all $j \in \mathcal{N}$, such that $t_k \in T_j^p$: Given $\mathbf{p}(t_{k-1})$, then calculate $p_j^o(t_k) = \text{argmax}_{p_j \in P_j} g_j(p_j, \mathbf{p}_{-j}(t_{k-1}), c_j)$, then let the transmit power $p_j(t_k) = \nu_j(t_k) = \min(p_j^o(t_k), p_{j-max})$ **(c)** If $\mathbf{p}(t_k) = \mathbf{p}(t_{k-1})$ stop and declare the Nash equilibrium power vector as $\mathbf{p}(t_k)$, else let $k := k + 1$ and go to **(b)**.

IV. SIMULATION RESULTS

We consider a wireless data system with $N = 50$ receiver/transmitter pairs. The path gains $G_{i,k}$ were generated from a uniform distribution on $[0, 0.001]$ for all $i \neq k \in \mathcal{N}$
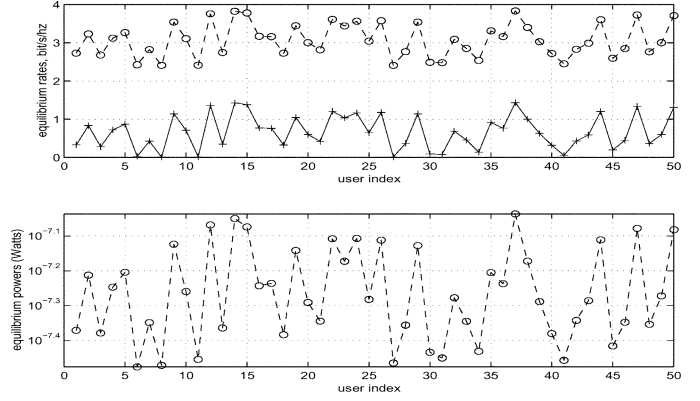

Fig. 1. Normalized equilibrium rates of the game $G1(\circ)$ and the normalized minimum required rates of the users $(+)$ in the upper graph and the equilibrium powers of the game $G2(\circ)$ in the lower graph versus the user index with pricing factor $\lambda = 10^3$, utility factors $u_i = 10^5$, and $\beta_i = 10^2, \forall i \in \mathcal{N}$.

and $G_{i,i} = 1 \forall i \in \mathcal{N}$ [1]. The additive-white-gaussian noise (AWGN) variance was set $\sigma^2 = 5 \times 10^{-10}$. Game $G1$ was run for different values of the minimum transmission rates for different users. Results show that all users were able to reach reasonable transmission rates with low transmit power levels resulting from game $G2$ as shown in the lower graph of Fig. 1.

V. CONCLUSIONS

In this letter two joint game-theoretic distributed rate and power control algorithms for wireless data systems were proposed. We presented target functions which are composed of the difference between a utility function and a pricing function to set the rules of the games among the users. We established the existence, uniqueness and Pareto optimality (efficiency) of the Nash equilibrium point of both games. All 50 users in the studied example were able to attain transmission rates that are higher than their minimum required transmission rates at very low transmit power levels.

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