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Statistical Learning Control of Delay Systems: Theory and Algorithms

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Abstract

Recently, probabilistic methods and statistical learning theory have been shown to provide approximate solutions to "difficult" control problems. Unfortunately, the number of samples required in order to guarantee stringent performance levels may be prohibitively large. In this paper, using recent results by the authors, a more efficient statistical algorithm is presented. Using this algorithm we design static output controllers for a nonlinear plant with uncertain delay.

1 Introduction

In systems such as flexible structures with non-collocated sensors/actuators, teleoperators, or communications networks, time delay is a serious problem to be dealt with. It is obvious that controllers designed with the assumption of instantaneous information and power transfer may fail to stabilize physical systems with time delay. Because of these motivations, a recent surge in the study of such systems has occurred [1]. Some of these papers try to stabilize the system independent of the delay amount [2], while others rely on Razumikin approach [3] and obtain upper bounds on the size of delay for stabilizability [4, 5]. Yet another approach uses delay as part of the stabilizing schemes and finds regions of delays where stabilizability is possible [6, 7].

This paper discusses the problem of stabilizing nonlinear continuous-time, delay systems with fixed-structure controllers, while also minimizing some performance objectives. Specifically, we find the gain K such that

$$\begin{aligned}
\dot{x} &= Ax(t) + f_d(x(t-\tau)) + Bu(t), \quad t \in [0,\infty), \\
\tau &> 0, \quad x(t) = \phi(t), \quad t \in [-\tau, 0], \quad x(0) = \phi(0), \\
y &= h(x)
\end{aligned} \quad (1)$$

is stabilized using the output feedback controller $u = -Ky(t)$, while also minimizing certain cost functionals. Moreover, we shall assume that the parameter τ is only known to lie in a certain region. In [8], we had developed techniques for solving such problems but were able to obtain sufficient conditions for the existence of static state feedback controllers only.

In the current paper, we use our recent statistical learning control (SLC) techniques in order to design static output feedback (or any fixed-order controllers) for delay-differential systems. The paper is organized as follows: Section 2 describes the statistical design approach and presents the system to be controlled. In section 3 we illustrate the design methodology and present our numerical results. Section 4 concludes the paper and presents our future research directions.

2 Statistical Design

Let us consider the following system with nonlinear state delay [9]

$$\begin{aligned}
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0.7 \begin{bmatrix} \frac{x_1(t-\tau_d)}{\sqrt{1+x_1^2(t-\tau_d)}} \\ x_2(t-\tau_d) \end{bmatrix} \\
&\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad t \in [0, +\infty) \\
y(t) &= \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t-\tau_d) \\ x_2(t-\tau_d) \end{bmatrix} \\
z(t) &= \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}
\end{aligned} \quad (2)$$

with

$$x(t) = \begin{bmatrix} 100t + 3 \\ -200t + 1 \end{bmatrix}, \qquad t \in [-\tau_d, 0], \quad (3)$$

where $y(t)$ is the measured output and $z(t)$ is the regulated output. For this system a full state feedback control was proposed in [9]. The case of the output feedback was not addressed. Hereafter, we shall design some static output feedback controllers $u(t) = Ky(t)$.

We shall assume the time delay τ_d to be a uniform random variable in the interval $[0, 1]$. Our target is to find a controller gain K such that

1. The plant with no time delay (i.e. $\tau_d = 0$) is stabilized. We shall call this plant the *nominal* plant.

2. The output transient response $z(t)$ from the initial state $x_0 = \begin{bmatrix} 3 & 1 \end{bmatrix}^T$ (see (3)) does not exceed 5, for the nominal plant, and

3. A certain cost function is minimized. This cost function accounts for the closed-loop stability and the performance (in terms of output transient response $z(t)$) in the presence of variations of the time delay.

We initially choose K to have uniform distribution in the symmetric interval $[-1, 1]$. In order to use the randomized algorithm methodology, this problem has been reformulated in the following way (see also [10], [11]). We shall denote by $X \in \mathcal{X} = [0, 1]$ the time delay τ_d and by $Y \in \mathcal{Y} = [-1, 1]$ the controller gain K. Let us define a cost function

$$\Psi(Y) = \max\{\psi_1(Y), \psi_2(Y)\} \qquad (4)$$

where

$$\psi_1(Y) = \begin{cases} 1 & \text{if the nominal plant is not} \\ & \text{stabilized or if the output} \\ & \text{transient response exceeds 5} \\ & \text{for the nominal plant} \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

and

$$\psi_2(Y) = E\left(\zeta(X, Y)\right) \qquad (6)$$

where E indicates the *expected value* with respect to X, and

$$\zeta(X, Y) = \begin{cases} 1 & \text{if the randomly generated} \\ & \text{plant is not stabilized} \\ 1/2 & \text{if the output transient} \\ & \text{response exceeds 5} \\ 0 & \text{otherwise} \end{cases}$$

Our aim is to minimize the cost function (4) over $\mathcal{Y}$. The optimal controller is then characterized by the parameter Y^* for which

$$\Psi^* := \Psi(Y^*) = \inf_{Y \in \mathcal{Y}} \Psi(Y) \qquad (7)$$

Finding the scalar Y^* which minimizes (7) would imply the evaluation of the expected value in (6) and then the minimization of (4) over the set $\mathcal{Y}$. What we shall find is a *suboptimal* solution, a probably approximate near minimum of $\Psi(Y)$ with confidence $1 - \delta$, level α and accuracy ϵ (see [12]).

Definition 1 *Suppose* $\Psi : \mathcal{Y} \rightarrow \mathbb{R}$, *that* P *is a given probability measure on* $\mathcal{Y}$, *and that* $\alpha \in (0, 1)$, $\delta \in (0, 1)$ *and* $\epsilon > 0$ *are given. A number* Ψ_0 *is a a probably approximate near minimum of* $\Psi(Y)$ *with confidence* $1 - \delta$, *level* α *and accuracy* ϵ, *if*

$$\text{Prob} \left\{ \inf_{Y \in \mathcal{Y}} \Psi(Y) - \epsilon \leq \Psi_0 \leq \inf_{Y \in \mathcal{Y} \setminus \mathcal{S}} \Psi(Y) + \epsilon \right\}$$
$$\geq 1 - \delta \qquad (8)$$

with some measurable set $\mathcal{S} \subseteq \mathcal{Y}$ *such that* $P(\mathcal{S}) \leq \alpha$. *In* (8), $\mathcal{Y} \setminus \mathcal{S}$ *indicates the complement of the set* $\mathcal{S}$ *in* $\mathcal{Y}$.

Based on the randomized algorithms discussed in [10], a probably approximate near minimum of $\Psi(Y)$ with confidence $1 - \delta$, level α and accuracy ϵ, can be found with the following Procedure:

Procedure

1. Let $k = 0$

2. Choose n controllers with random uniformly distributed coefficients $Y_1, \ldots, Y_n \in \mathcal{Y}$, where (we indicate by $\lfloor \cdot \rfloor$ the floor operator)

$$n = \left\lfloor \frac{\log(2/\delta)}{\log[1/(1 - \alpha)]} \right\rfloor$$

Evaluate for these controllers the function ψ_1 in (5) and discard those controllers for which $\psi_1 = 1$. Let $\hat{n}$ be the number of the remaining controllers.

3. Choose m plants generating random parameters $X_1, \ldots, X_m \in \mathcal{X}$ with uniform distribution, where

$$m = 2^k \left\{ \left\lfloor \frac{100}{\epsilon^2} \log\left(\frac{8}{\delta}\right) \right\rfloor + 1 \right\}$$

4. Evaluate the stopping variable

$$\gamma = \max_{1 \leq j \leq \hat{n}} \left| \frac{1}{m} \sum_{i=1}^{m} r_i \zeta(X_i, Y_j) \right|$$

where r_i are *Rademacher* random variables, i.e. independent identically distributed random variables taking values $+1$ and -1 with probability $1/2$ each. If $\gamma > \epsilon/5$, let $k = k + 1$ and go back to step 3

5. Choose the controller which minimizes the function

$$\frac{1}{m} \sum_{i=1}^{m} \zeta(X_i, \cdot)$$

This is the *suboptimal* controller in the sense defined above.

Remark 1 The proposed algorithm consists of two distinct parts: the estimate of the expected value in (6), which is given with an accuracy ϵ and a confidence $1 - \delta/2$, and the minimization procedure which is carried out with a confidence $1 - \delta/2$ and introduces the level α. As can be seen from the Procedure, the number m of samples in $\mathcal{X}$ which are needed to achieve the estimate of the expected value (6), known as the *sample complexity*, is not known *a priori* but is itself a random variable. The upper bounds for this random sample complexity however, are of the same order of those that can be found in [13].

3 Numerical results

In our case the procedure needed just one iteration to converge, i.e. $k = 1$. Therefore, for $\delta = 0.05$, $\alpha = 0.05$ and $\epsilon = 0.1$, n evaluated to 72 controllers and m evaluated to 50,753 plants. In Figure 1, the stopping variable γ is shown. The *suboptimal* controller is $K = -0.9830$, and the corresponding value of the cost function is $\Psi_0 = 0.3976$.
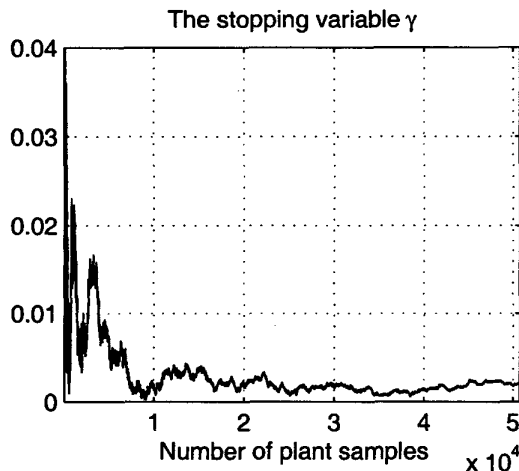
Figure 1: The stopping variable for the design of the controller $K = -0.9830$

In Figure 2 we show the output response $z(t)$ with the designed controller for two different values of the time delay from the initial state $x_0 = \begin{bmatrix} 3 & 1 \end{bmatrix}^T$. With the higher value of τ_d stability is retained, but the output response exceeds the limit of 5. In both cases, the behavior is low damped.

If we are not satisfied with the value of Ψ_0, we can try to search for a better controller either enlarging the interval for the gain K, or choosing a dynamic compensator. Indeed one of the characteristics of the algorithm is that the number of plants tested depends only on
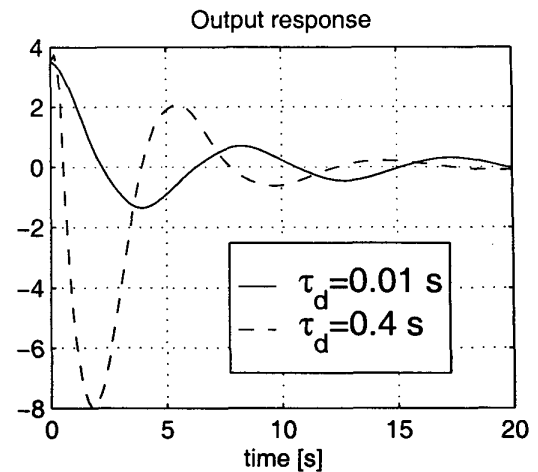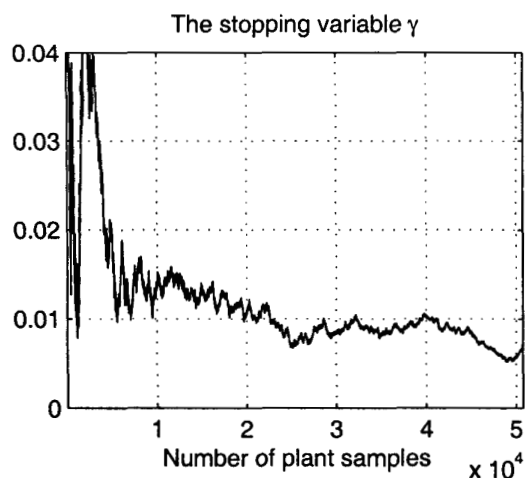

Figure 2: The output response $z(t)$ with the controller $K = -0.9830$ for two values of τ_d

the accuracy and the confidence chosen but not on the complexity of the plant nor of the controller. Therefore, once we have fixed a certain performance level that we want to achieve, the complexity of the controller can be increased until all requirements are met. Here, we tried to find a better controller choosing K to have uniform distribution in the larger interval $[-10, 10]$. Also in this case the procedure needed just one iteration to converge. We chose the same values as before for δ, α and ϵ. In Figure 3, we show the stopping variable γ we got in this case. The *suboptimal* controller is $K = -2.7679$ with a corresponding value for the cost function of $\Psi_0 = 0.2932$. Searching on a larger interval for the controller gain K, we got an improvement in the cost function.

In Figure 4 we show the output response $z(t)$ from the initial state $x_0 = \begin{bmatrix} 3 & 1 \end{bmatrix}^T$ for the two different values of the time delay chosen before with this new controller. A better transient behavior can be observed and the performance specification is now verified also with the larger time delay $\tau_d = 0.4$ s.

4 Conclusions

In this paper we have illustrate the application of our recent methodology on statistical learning control to design controllers for delay-differential systems where the time delay parameter is uncertain. Specifically, we have shown that SLC is a viable control methodology to design fixed-structure or even static output feedback controllers for such systems. The case of multiple uncertainties can be easily addressed with our approach [14].
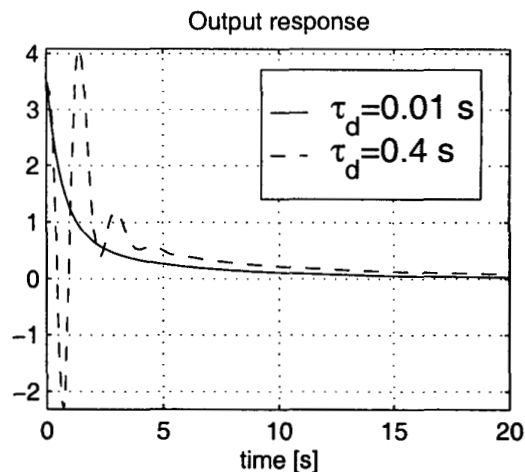
The stopping variable γ

Figure 3: The stopping variable for the design of the controller $K = -2.7679$

Output response

Figure 4: The output response $z(t)$ with the controller $K = -2.7679$ for two values of τ_d

We are currently investigating the application of SLC to nonlinear systems which are uncertain, and to communications networks.

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