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Time delay and power control in spread spectrum wireless networks

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Abstract

Delay in the computation of the signal-to-interference ratio in communication systems is unavoidable. In the the case of mobile communication, delay is a very critical problem due the fast variation of the communication channel and the need for effective, fast and accurate power control. In this paper we present our approach to dealing with the delay in mobile communication systems as well as our controller to achieve and maintain the desired signal quality. We will concentrate on the *code division multiple access* (CDMA) systems.

I. Introduction

The popularity of the CDMA in commercial wireless systems is growing rapidly, and has been recognized as a viable alternative to the more traditional methods such as *frequency division multiple access* (FDMA) and *time division multiple access* (TDMA). Although there are different types of spread spectrum, we will concentrate on direct sequence (DS) spread spectrum. DS-CDMA has many advantages: i) universal one-cell frequency reuse, ii) narrow band interference rejection, iii) inherent multipath diversity, iv) soft hand-off capability, and v) soft capacity limit. Unlike FDMA and TDMA, in CDMA the users are separated by pseudorandom codes rather than in frequency or time, and the transmitted signal is spread to occupy a much larger bandwidth. In this case the central mechanism for interference management is *power control.* The role of power control is to keep the received power of all users at an equal level so as to avoid problems such as the *near-far problem.* This problem occurs due to the lack of power control: If all mobiles were to transmit at a fixed power, the mobile closest to the base station will overpower all others. The other role of power control is to account for the fast varying changes in the mobile radio channel. One of the main issues in power control is thus to address the inherent delay in the system due to the averaging process used to estimate the *signal-to-interference ratio* which is an accepted measure for the quality-of-service for each user, and the fact that the control command is sent at fixed intervals. This delay becomes critical in the case of fast-fading channels.

As is well known, the mobile channel is best modeled statistically using a Rayleigh or Ricean magnitude models [1]. These models are called *small-scale* fading models and the parameters of the Rayleigh or Ricean distribution are themselves modeled as random variables that change at a much slower rate, according to the so called *large-scale* fading characteristics. Large-scale fading usually has a log-normal probability density function whose parameters depend on the location of the mobile relative to the base station. In particular, the average power is determined under the assumption that electromagnetic waves will experience a path loss inversely proportional to the distance traveled raised to some power (denoted α later). An accurate model of the wireless channel and its state is usually unobtainable. Any power control algorithms developed should be able to adjust the power levels of each mobile using local measurements only, so that in a reasonable amount of time, all users will maintain the desired signal-to-interference ratio. In this paper, only the uplink (mobile-to-base), also known as *reverse link*, control will be studied, but all results may be applied to the downlink (base-to-mobile) case, also known as *forward link*. The reverse link, is more challenging due to the fact that mobile stations have more computation and power limitations than the base station.

Some of the early work in power control was provided by [2]. In [3], [4], [5] centralized power control was studied, and due to the complexity of the system, centralized power control was suggested to be used only for providing theoretical limits. When all users could be accommodated with acceptable signal-to-interference ratio, [6] suggested a distributed power control algorithm that guarantees will convergence and computes the required transmission power of each mobile station. In [7] a second-order constrained power control (CSOPC) algorithm was presented. This approach uses the current and past power values to determine the necessary transmission power of each mobile. CSOPC was compared with the algorithm presented in [6] and was shown to converge at a faster rate. Convergence analysis of distributed power control algorithms is investigated in [8].

In [9] a framework for uplink power control in cellular radio systems was presented. Our approach to solving the power control problem will be within such framework. Many of the above mentioned approaches did not use knowledge already established in the area of control theory, and did not address the delay problem.

In this paper we present our approach for dealing with the delay in the estimation of the signal-to-interference ratio and our design of the power control algorithm. The paper is organized as follows. In Section II we discuss mobile radio propagation models, which will help understand the nature of the received signal power. In Section III our approach to predicting the changes in the channel due to flat fading will be presented. Section IV discusses power control, and presents our algorithms. Simulation results are given in Section V and our conclusion are given in section VI.

II. Mobile Radio Propagation

Since signal-to-interference ratio is the measure used to determine the quality of the signal, and consequently, the power adjustments that need to be made, accurately estimating the power of the received signal is a critical part of power control. In a typical mobile-radio situation, a moving mobile station is in communication with a base station at a fixed position. This movement is usually in such a way that the direct line between the mobile station and the base station is obstructed by buildings and other obstacles. Therefore, the mode of propagation of the electromagnetic energy from the transmitter to the receiver will be largely by way of scattering, either by reflection from flat sides of the obstacles or by diffraction around such obstacles.

A. Channel Model

For a wideband fading channel, the baseband impulse response of the channel having L resolvable taps is given by [10]

$$h(t) = \sum_{l=1}^{L} \xi(d,\alpha,\eta)\beta_l \delta(t-\tau_l)\exp(j\theta_l) \tag{1}$$

where d is the distance between the transmitter and the receiver, α is the path loss exponent and η is the shadowing variable which is a normally distributed variable with zero mean and variance σ_s^2. The factor $\xi(d,\alpha,\eta)$ is due to large-scale fading over the link between the ith base station and the jth mobile station, β_l is the magnitude of the lth path of the small-scale fading channel response at time τ_l, and θ_l is phase shift of the lth path relative to the 0th path and is uniformly distributed between 0 and 2π. $\xi(d,\alpha,\eta)$ is given by

$$\xi^2(d,\alpha,\eta) = d^{-\alpha}10^{\eta/10} \tag{2}$$

Assuming the communication channel is of the form given in Equation (1) the average received power is given by

$$\overline{p_r}(n) = Gp_t \tag{3}$$

where G is the communication link gain and is defined as $\overline{\xi}^2(d,\alpha,\eta)\sum_{l=1}^{L}\overline{\beta_l}^2$, and p_t is the transmission power. As seen in Equation (3), in order to compute the signal-to-interference ratio, we need to have an averaging window. Also, in CDMA systems, the power command is sent at fixed intervals based on previous estimation of the signal quality, e.g. in the IS-95 standard the power command is sent every 1.25msec. These factors cause an unavoidable delay in the system, and introduces power control errors due to the fact that the power command is based on previous estimates. Consequently, it is important to predict such changes in order to minimize the power control error. In the next section, we discuss in detail Clark's model for flat fading which shows the existence of spatial correlation in the received signal power due to scattering. Using this property, a prediction algorithm will be used to accurately track changes in the communications channel.

B. Clarke's Model for Flat Fading

In a typical radio-communication scenario, the mobile station is in motion and is trying to communicate with a base station that is at a fixed location. In this situation, Doppler shift is introduced. Clarke's model deduced the statistical characteristics of the fields and signals in the reception of radio frequencies by a moving vehicle from a scattering propagation model [11].

Assuming that the total field is vertically polarized and is composed of the superposition of N waves, the E-field component can be described as [11]:

$$E_z = E_0 \sum_{n=1}^{N} C_n \exp\{j\vartheta_n + 2\pi f_{md} t \cos \alpha_n\} \tag{4}$$

where $f_{md} = v f_0 / c$ is the maximum doppler shift, v is the velocity of the mobile station, f_0 is the carrier frequency, c is the speed of light, E_0 is the common amplitude of the N waves (assumed constant), η is the intrinsic impedance of free space, the time variations are of the form $\exp\{j2\pi f_0 t\}$, ϑ_n is the random phase (uniformally distributed throughout 0 to 2π) of the nth wave arriving at any angle α_n to the x-axis (Figure 1), and C_n is a random variable describing the amplitude of the nth wave. The ensemble average of the C_n terms is assumed to satisfy

$$\sum_{n=1}^{N} \overline{C_n^2} = 1 \tag{5}$$

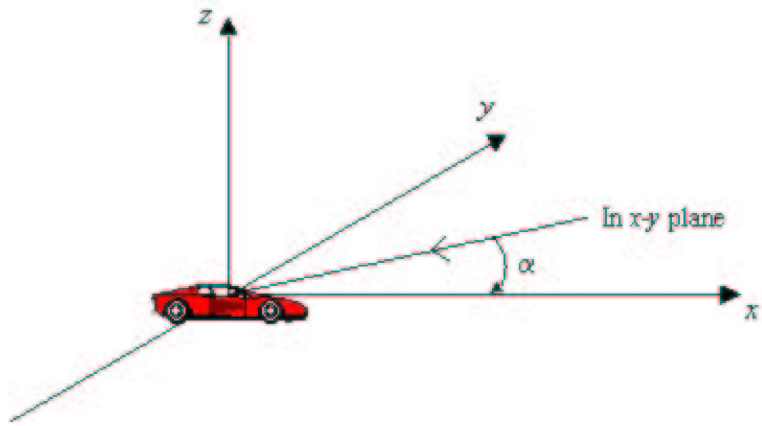

Fig. 1. Plane wave

The random received signal envelope is assumed to have a Rayleigh distribution given by

$$p(r) = \begin{cases} \frac{r}{\sigma_r} \exp\left(-\frac{r^2}{2\sigma_r^2}\right) & (0 \leq r \leq \infty), \\ 0 & (r < 0) \end{cases} \tag{6}$$

where σ_r is the rms value of the received voltage signal, and $r(t)$ is the complex envelope of the received signal. Figure 2 shows a Rayleigh distribution signal envelope at 845MHz with receiver speed of 100km/hr as a function of time.

III. Received Power Prediction

The fast changes in the received power are dominated by the small-scale fading. Using Equation (4), the spatial correlation function of the power is derived (see Appendix). Taking advantage of this case we can derive a linear predictor that uses past values to predict the future changes in the channel. Since the variation of $\bar{\beta}$ in Equation (3) is much faster compared to the other terms, we will concentrate on estimating it.

Assuming that the we have N observations of β^2, in the mean-square-sense, its estimate

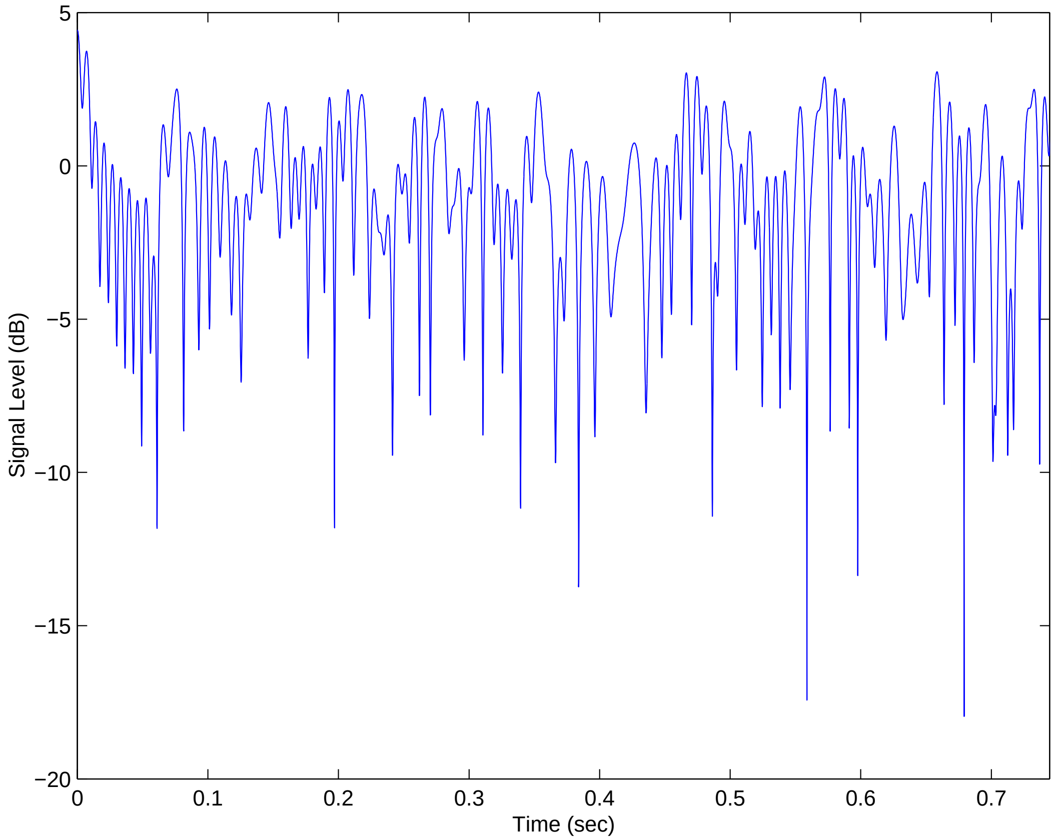

Fig. 2. A typical Rayleigh fading envelope at 845MHz and receiver speed of 100km/hr

at time $n+1$ given its previous N measured values is given by

$$\hat{\beta}^2(n+1) = \sum_{i=1}^{N} h_i \beta^2(n-i) \tag{7}$$

where $\hat{\beta}^2$ is the predicted value of β^2, h_i is computed as follows [12]

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_N \end{bmatrix} = \begin{bmatrix} R_{\beta^2}(0) & R_{\beta^2}(1) & R_{\beta^2}(2) & \cdots & R_{\beta^2}(N-1) \\ R_{\beta^2}(1) & R_{\beta^2}(0) & R_{\beta^2}(1) & \cdots & R_{\beta^2}(N-2) \\ R_{\beta^2}(2) & R_{\beta^2}(1) & R_{\beta^2}(0) & \cdots & R_{\beta^2}(N-3) \\ \vdots & \vdots & \vdots & & \vdots \\ R_{\beta^2}(N-1) & R_{\beta^2}(N-2) & R_{\beta^2}(N-3) & \cdots & R_{\beta^2}(0) \end{bmatrix}^{-1} \begin{bmatrix} R_{\beta^2}(1) \\ R_{\beta^2}(2) \\ R_{\beta^2}(3) \\ \vdots \\ R_{\beta^2}(N) \end{bmatrix} \tag{8}$$

and

$$R_{\beta^2}(\tau) = 1 + J_0^2(2\pi f_{md}\tau) \tag{9}$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind.

IV. Power Control

The goal of the power control algorithms is to find the transmission power of each mobile such that the following inequality is satisfied

$$\gamma_i = \frac{G_{ki} p_i}{\sum_{j \neq i}^{Q} p_j G_{kj} + n_i} \geq \gamma^* \tag{10}$$

where γ_i is the signal-to-interference ratio for mobile i, G_{ki} is the communication gain between mobile station i and its assigned base station k, γ_i^* is the desired signal-to-interference for the ith mobile station with receiver noise n_i and transmission power p_i which is constrained as follows

$$0 \leq p_i \leq p_{i\_\max} \tag{11}$$

where $p_{i\_\max}$ is the maximum transmission power of mobile i.

Since it is desirable that the mobile station transmits the signal at the minimum needed power to maintain the required quality-of-service, inequality (10) becomes an equality. Consequently, the transmission power is written as

$$p_i(n+1) = \min\{p_{i\_\max}, \frac{\gamma^*}{\gamma_i(n)} p_i(n)\} \tag{12}$$

where $\gamma_i(n)$ is the signal-to-interference ratio of mobile i at iteration n. It is important to note that unlike centralized power control, only the total interference is needed to compute the power levels. Convergence of the iterative algorithm given by Equation (12) is studied in [9]. Along the same lines, different power control algorithms can be designed to have faster convergence rate. In the next subsections we will present the CSOPC approach followed by our own.

A. Constraint Second-Order Power Control

The constraint second-order power control (CSOPC) presented in [7] uses current and past values in order to provide a faster convergence of the power command. In this section a brief overview of the CSOPC approach will be given. Equation (10) could be written as a set of linear equations as follows

$$\mathbf{XP} = \mathbf{\Xi} \tag{13}$$

where $\mathbf{P}$ is defined as

$$\mathbf{P} = \begin{pmatrix} p_1 \\ \vdots \\ p_Q \end{pmatrix} \tag{14}$$

and

$$\mathbf{X} = \mathbf{I} - \mathbf{A} \tag{15}$$

$$\mathbf{A} = \{a_{ij}\}, \qquad a_{ij} = \gamma^* w_{ij} \tag{16}$$

$$\mathbf{\Xi} = \{b_i\}, \qquad b_i = \gamma^* \frac{n_i}{G_{ik}} \tag{17}$$

and w_{ij} is defined as

$$w_{ij} = \begin{cases} \frac{G_{kj}}{G_{ki}} & i \neq j, \\ 0 & i = j. \end{cases} \tag{18}$$

Thus Equation (10) has been converted to a set of linear equations, that could be iteratively solved for $\mathbf{P}$ [7].

CSOPC is developed by applying the *successive overrelaxation method* (SOR) [13] to Equation (13). The CSPOC results in [7] were compared with the *distributed constraint power control* (DCPC) in [6]. CSOPC was proven to be more effective; consequently, later in this paper the CSOPC algorithm will be used as the comparison benchmark. Through some manipulations the following iterative algorithm was obtained [7].

$$p_i(n+1) = \min\left\{p_{i\_\max}, \max\left\{0, a(n)\frac{\gamma^*}{\gamma_i(n)}p_i(n) + (1-a(n))p_i(n-1)\right\}\right\} \tag{19}$$

where as described earlier, $p_{i\_\max}$ is the maximum allowable power for mobile i, $\gamma_i(n)$ is the signal-to-interference ratio of mobile i at iteration n, $p_i(0)$ is chosen randomly between 0 and $p_{i\_\max}$, and $a(n)$ is a decreasing sequence such that $\lim_{n\to\infty} a(n) = 1$. As an example, the following $a(n)$ sequence was used in [7]:

$$a(n) = 1 + \frac{1}{1.5^n}, \quad n = 1, 2, \ldots, l \tag{20}$$

where l is the total number of iterations. Equation (19) determines the necessary power using the current and the past power values, which accounts for the terminology of "second-order". Note that if $a(n) = 1$, Equation (19) reduces to Equation (12) and that the *min* and *max* operators are used to guarantee that the power will be within the allowable range based on Equation (11).

B. Non-Linear Control

In this section we present our power control algorithm. Our approach is to view each mobile-to-base station connection as a separate subsystem as follows

$$s_i(n+1) = \frac{\xi^2\beta^2(n+1)(p_i(n)+u_i(n))}{I_i} \tag{21}$$

where ξ^2 is defined by Equation (2), I_i is the interference experience by mobile i due to other users (assumed constant in short periods of time) and $u_i(n)$ is the increment by which the power of mobile i should be changed. Since $\beta^2(n+1)$ is unknown, we use the linear predictor described in Equation (7) to modify Equation (21) as follows

$$s_i(n+1) = \alpha(n)(p_i(n) + u_i(n)) = \alpha(n)p_i(n) + v_i(n) \tag{22}$$

where $\alpha(n) = (\xi^2/I_i)\sum_{i=1}^{N} h_i\beta^2(n-i)$. Now if we choose $v_i(n) = -\alpha(n)p(n) + z_i(n)$ then Equation (23) becomes

$$s_i(n+1) = z_i(n) \tag{23}$$

where $z_i(n)$ is our control law. The goal is to find the right control command that will make each s_i follow a desired signal to interference ratio γ^*. For simplicity we will assume that γ^* is the same for all mobile stations. To accomplish such a task, a new state is added to the system. This state represents an integrator of the error $e_i(n) = s_i(n) - \gamma^*$ [14]. A discrete time integrator is nothing more than a summation of the previous values, therefore

$$\zeta_i(n+1) = \zeta_i(n) + e_i(n) = \zeta_i(n) + s_i(n) - \gamma^* \tag{24}$$

Let us define $x_i(n)$ as

$$x_i(n) = \begin{pmatrix} \zeta_i(n) \\ s_i(n) \end{pmatrix} \tag{25}$$

Using the above notation the system can now be expressed as a second-order linear state-space system by

$$x_i(n+1) = \begin{pmatrix} \zeta_i(n+1) \\ s_i(n+1) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} x_i(n) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} z_i(n) + \begin{pmatrix} -\gamma^* \\ 0 \end{pmatrix} \tag{26}$$

$$y_i(n) = (0 \quad 1)\, x_i(n) \tag{27}$$

We then choose the feedback controller

$$z_i(n) = -\left[k_\zeta \quad k_s\right] x_i(n) + k_s \cdot \gamma^* \tag{28}$$

If we choose the appropriate feedback gains k_ζ and k_s, then Equation (26) will be stable. Since the closed-loop system is stable then $x_i(n+1) = x_i(n)$. Therefore, we have $\zeta_i(n+1) = \zeta_i(n) + s_i(n) - \gamma^*$, and in the steady state $s_i(n) = \gamma^*$. Finally the transmission power at iteration $n+1$ is

$$p_i(n+1) = p_i(n) + u_i(n) = \min\left\{p_{i\_\max}, \min\left\{0, \frac{-1}{\alpha(n)}\{(k_\zeta \quad k_s)\, x_i(n) - k_s \cdot \gamma^*\}\right\}\right\} \tag{29}$$

Equation (29) is a non-linear power control command. One of the states used to obtain this control command is the output of an integrator which is very useful in suppressing noise that occurs during the computation of the signal-to-interference ratio.

V. Simulation and Results

For comparison purposes, we have simulated our power control algorithms as well as the CSOPC. We have set the ratio of the desired energy per bit, E_b, to interference, I_0, to $E_b/I_0 = 7$dB. Based on the IS-95 standard, we use a bit rate of 9600kbps, a channel bandwidth of 1.2288MHz, and update the power control command every 1.25msec. Using these parameters, $\gamma^* = (E_b/I_0)(R_b/B_c)$ where R_b is the bit rate, and B_c is radio channel bandwidth. The system is then simulated at different mobile speeds. Figure 3 shows a comparison between the CSOPC algorithm and our nonlinear control (NLC) introduced in this paper for a mobile speed of 100km/hr. As seen in the figure, the NLC outperforms

CSOPC. Note in particular that CSOPC violates the 7 dB limitation. The fluctuations in the plot are due to the errors in the prediction algorithm. The two power control algorithms are also tested with a mobile speed of 20km/hr, see Figure 4, in this case the advantage of the new control is also obvious. By using CSOPC there are high fluctuations due to the delay in computing the signal-to-interference ratio.

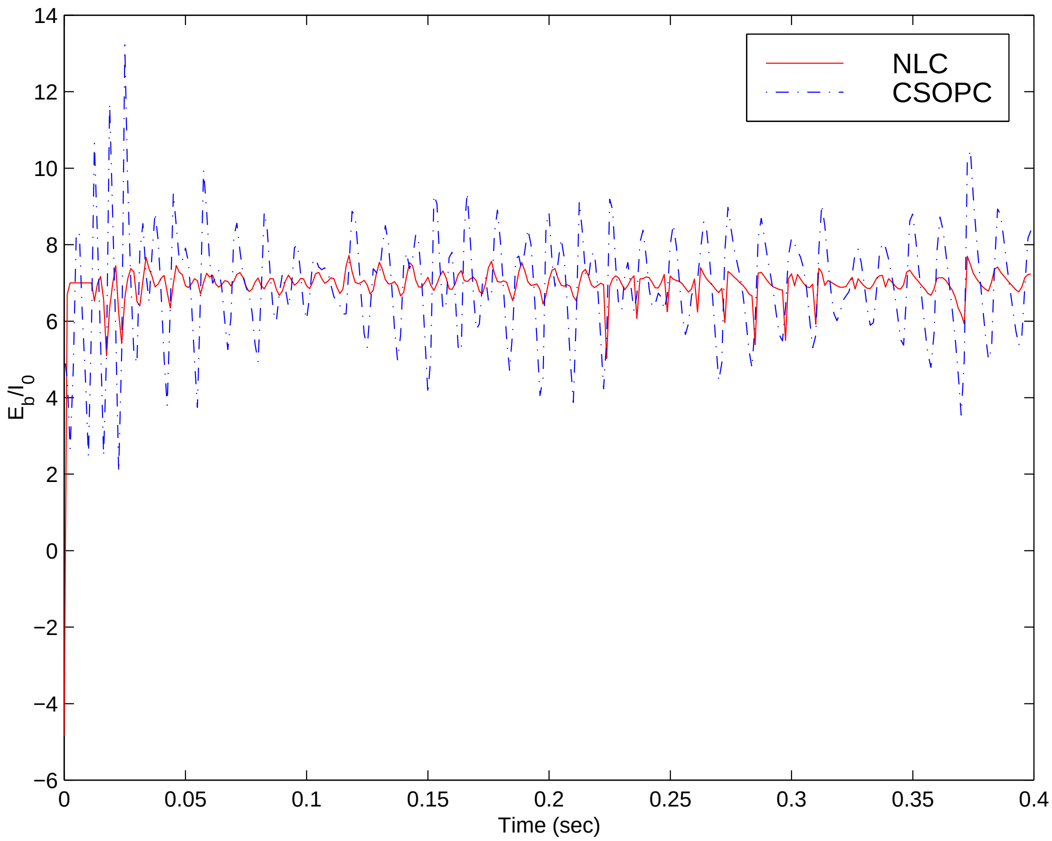

Fig. 3. A typical Rayleigh fading envelope at 845MHz and receiver speed of 100km/hr

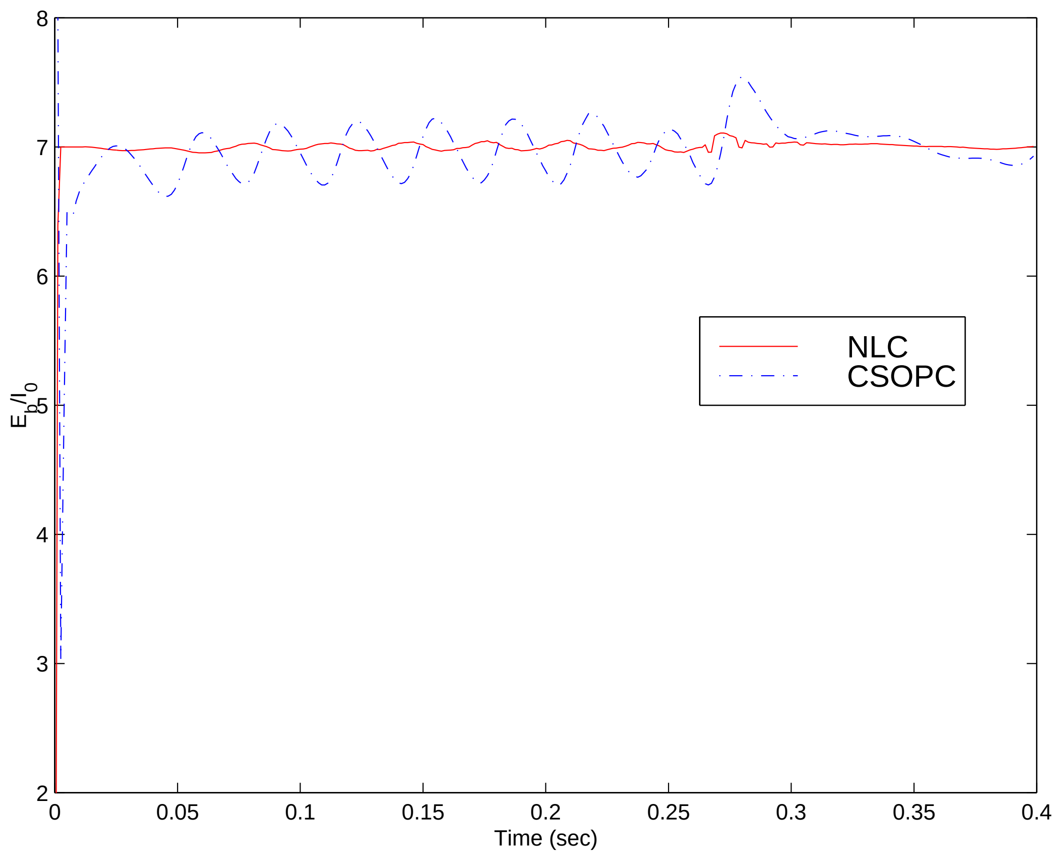

Fig. 4. A typical Rayleigh fading envelope at 845MHz and receiver speed of 100km/hr

VI. Conclusion

Delay in wireless communication systems is a serious problem. Due to the fast variations in the communication channel, the need to overcome the delay effects becomes evident. Controlling the transmission power in CDMA wireless networks is very critical, and as shown in the simulation section, it is greatly affected by the delay. In this paper we have presented our approach in dealing with the delay in spread spectrum wireless networks. As shown in the paper, by predicting the changes in the channel, and taking the delay into account while designing the power control algorithms we can improve the performance of our power control algorithm with a minimum increase in computational expense.

Appendix

I. Autocorrelation Function

Assuming that N is sufficiently large and that the phases ϑ_n are independent, and as a consequent of the Central Limit Theorem, the E component, Equation (4), is complex Gaussian random variable. The field component E_z, may be expressed by

$$E_z(t) = x(t) + jy(t) \tag{30}$$

$$\mathbb{E}\{x(t)\} = \mathbb{E}\{y(t)\} = 0 \tag{31}$$

$$\mathbb{E}\{x^2(t)\} = \mathbb{E}\{y^2(t)\} = \sigma_r^2 = E_0^2/2 \tag{32}$$

Since both x and y are Gaussian distributed and are independent

$$\mathbb{E}\{x(t)y(t)\} = 0 \tag{33}$$

Using the above properties, we start deriving the autocorrelation function of the received power.

$$\begin{aligned} R_{\|E_z\|^2}(\tau) =& \mathbb{E}\{(x^2(t) + y^2(t))(x^2(t+\tau) + y^2(t+\tau)\} = \\ & \mathbb{E}\{x^2(t)x^2(t+\tau) + \mathbb{E}\{y^2(t)y^2(t+\tau)\} + \\ & \mathbb{E}\{x^2(t)y^2(t+\tau)\} + \mathbb{E}\{y^2(t)x^2(t+\tau)\} \end{aligned} \tag{34}$$

Using the following property for Gaussian variables

$$\begin{aligned} \mathbb{E}\{x_1x_2x_3x_4\} =& \mathbb{E}\{x_1x_2\}\mathbb{E}\{x_3x_4\} + \mathbb{E}\{x_1x_3\}\mathbb{E}\{x_2x_4\} + \\ & \mathbb{E}\{x_1x_4\}\mathbb{E}\{x_2x_3\} \end{aligned} \tag{35}$$

the autocorrelation function $R_{\|E_z\|^2}$ becomes

$$R_{\|E_z\|^2} = 2R_{x^2x^2}(\tau) + 4\sigma_r \tag{36}$$

where

$$\begin{aligned}
R_{x^2x^2}(\tau) &= \mathbb{E}\{x^2(t)x^2(t+\tau)\} \\
&= \sigma_r^4 + 2\mathbb{E}^2\{x(t)x(t+\tau)\} \\
&= \sigma_r^4 + 2E_0^4\mathbb{E}^2\left\{\sum_{n=1}^{N} C_n \cos\vartheta_n \sum_{m=1}^{N} C_n \cos(\vartheta_m + 2\pi f_{md}\tau\cos\alpha_m)\right\} \\
&= \sigma_r^4 + \frac{1}{2}E_0^4\left\{\sum_{n=1}^{N} \mathbb{E}\{C_n^2\cos(2\pi f_{md}\tau\cos\alpha_n)\right\}^2 \\
&= \sigma_r^4 + \frac{1}{2}E_0^4\left\{\int_{-\pi}^{+\pi} f_\alpha(\alpha)\cos(2\pi f_{md}\tau\cos\alpha_n)d\alpha\right\}^2 \\
&= \sigma_r^4 + \frac{1}{2}E_0^4 J_0^2(2\pi f_{md}\tau)
\end{aligned} \tag{37}$$

Finally,

$$R_{\|E_z\|^2}(\tau) = 4\sigma_r^4 + E_0^4 J_0^2(2\pi f_{md}\tau) \tag{38}$$

Now the autocorrelation of the small-scale fading becomes

$$R_{\beta^2}(\tau) = 1 + J_0^2(2\pi f_{md}\tau) \tag{39}$$

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