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# Performance of Game Theoretic Power Control Algorithms forWireless Data in Fast Flat Fading Channels

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*Abstract*—**We consider a game-theoretic power control algorithm for wireless data realistic channels. We study the performance of the algorithm for wireless data in the fast, flat-fading channel mobile users encounter in accessing the cellular system. The game-theoretic power control algorithm depends on an average utility function that assigns a numerical value to the quality of service (QoS) of users. The fading coefficients under this channel model are studied for two appropriate channel models that are used in CDMA cellular systems: Rayleigh fast flat fading channel and Rician fast flat fading channel.**

## I. Introduction

The mathematical theory of games was introduced by John Von Neumann and Oskar Morgenstern in 1944 [8], and in the late 1970's game theory became an important tool in the analyst's hand whenever he or she faces a situation in which a player's decision depends on what the other players did or will do. A core idea of game theory is the way strategic interactions between rational agents (players), generates outcomes according to the players' utilities [4],[9]. A player in a non-cooperative game responds to the actions of other players by choosing a strategy (from his strategy space) in an attempt to maximize his/her utility function that quantifies the quality level, i.e. its level of satisfaction.

In a cellular system users desire to have a high SIR (signal-to-interference ratio) at the BS (base station) coupled with the lowest possible transmit power. It is very important in such systems to have a high SIR, because this will be reflected in very low error rate, a more reliable system, and high channel capacity, which means that users can be sent at higher bit rates [5]. It is also important to decrease the transmit power because low power levels lead to longer battery life and helps alleviate the ever present near-far problem in CDMA systems[7].

In power control algorithms exploiting game theory, the tendency of each user is to increase his/her transmit power in response to other users' actions, leading to a sequence of power vectors that converges to a point where no user has incentive to increase his/her individual power. This operating point is called a Nash equilibrium. In many cases, and due to the lack of cooperation between the users (players), this point is not efficient, in the sense that it is not the most desirable social point [3]. The most desirable social point is called a Pareto optimal point, and may be viewed as the equilibrium point where no user can improve his utility function without harming at least one other user in the network.

The power control problem for wireless data CDMA systems was first addressed in the game theoretic framework in [2], then in a more detailed manner in [1] and [3]. In this paper the work in [1], which only dealt with deterministic channels, is extended by considering two cases of fading channel models: A Rayleigh fast flat fading channel model and a Rician fast flat fading channel model.

The remaining of this paper is organized as follows: In section II we present the utility function used in this paper. The signal model and the performance of the system under the two channel models mentioned above are presented in section III. Non-cooperative power control game (NPG) is discussed in section IV. Simulation results are outlined in section V. Finally, our conclusions and future work are presented in section VI.

## II. Utility Function

We use the concept of a utility function to map the player's preferences onto the real line. A utility function is chosen in a way that puts all the elements of the game taking place between self-interested players in its most desired order. A formal definition of utility functions is available from [4].

**Definition 1:** *A function $u$ that assigns a numerical value to the elements of the action set $A$ :*
$u : A \to \mathbb{R}$ *is a utility function if for all* $a,\ b \in A$, $a$ *is at least as preferred compared to* $b$ *if and only if* $u(a) \geq u(b)$.
In a cellular CDMA system there are a number of users sharing a spectrum and the air interface as a common radio resource. Henceforth, each user's transmission adds to the interference of all users at the receiver (BS). Each user desires to achieve a high quality of reception at the BS, i.e., a high SIR, by using the minimum possible amount of power to extend the battery's life. The goal of each user to have a high SIR at the BS produce conflicting objectives that make the framework of game theory suitable for studying this problem and proposing solutions. Let us consider a single-cell system with $N$ users, where each user transmits frames (packets) of $M$ bits with $L$ information bits [1]. The rate of transmission is $R$ bits/sec for all users. Let $P_c$ represents the *average* probability of correct reception of all bits in the frame at the BS, in other words, $P_c$ refers the average frame (packet) correct reception rate. As we know, $P_c$ depends on the SIR, the channel characteristics, the modulation format, the channel coding, etc.

A suitable utility function for a CDMA system is given by (see [1] and references therein):

$$u = \frac{L\,R}{M\,p}\,P_c \tag{1}$$

where $u$ thus represents the number of information bits received successfully at the BS per joule of expanded energy. With the assumption of no error correction, the *random* packet correct reception rate $\tilde{P}_c$ is then given as $\prod_{l=1}^{M}(1-\tilde{P}_e(l))$, where $\tilde{P}_e(l)$ is the *random* bit error rate (BER) of the $l$th bit at a given SIR $\gamma_i$ (c. f. (13) and (25)).

## III. Signal Model and Performance

In this paper we are assuming that all users in a cell are using the same modulation scheme, non-coherent Binary Frequency shift Keying (BFSK), and that they are transmitting with the same rate $R$. The signal $r_i(t)$ received at the BS from the $i$th user is given as:

$$r_i(t) = \alpha_i \, s_i(t) + n(t), \quad i = 1, 2, ..., N \tag{2}$$

where $\alpha_i$ is the path fading coefficient between $i$th user and the BS and it is constant for each bit in a fast flat fading. And $s_i(t)$ is the sent message for each bit, $n(t)$ is the BS receiver's background noise modelled as zero-mean AWGN, and $N$ is the number of active users currently in the cell. The SIR $\gamma_i$ at the receiver for the $i$th user is given as [6]:

$$\gamma_i = \frac{W}{R} \frac{p_i \, h_i \, \alpha_i^2}{\sum_{k \neq i}^{N} p_k \, h_k \, \alpha_k^2 + \sigma^2} \tag{3}$$

where, $W$ is the spread spectrum bandwidth, $R$ is the data rate (bits/sec), $p_k$ is the transmitted power of the $k$th user, $h_k$ is the path gain between the BS and the $k$th user, and $\sigma^2$ is the variance of the AWGN. For simplicity let us express the interference from all other users as $x_i$, i.e.

$$x_i = \sum_{k \neq i}^{N} p_k \, h_k \, \alpha_k^2 \tag{4}$$

therefore (3) can be written as:

$$\gamma_i \quad = \quad \frac{W}{R} \frac{p_i \, h_i}{x_i + \sigma^2} \;\; \alpha_i^2 = \gamma_i^{'} \alpha_i^2 \tag{5}$$

For a given $\gamma_i$ and $x_i$, the BER, $\tilde{P}(e/\gamma_i, x_i)$, of the $i$th user using BFSK is given by [6]:

$$\tilde{P}(e/\gamma_i, x_i) = \frac{1}{2} \, e^{-\frac{\gamma_i}{2}} \tag{6}$$

The average BER for this modulation scheme is evaluated for two channel models: The Rayleigh fast flat fading channel and The Rician fast flat fading channel. In the next two subsections we shall evaluate the average utility function under the two channel models.

### A. Rayleigh Fast Flat Fading Channel

In this case $\alpha_i$ is modelled as a Rayleigh random variable with a probability distribution given by:

$$p(\alpha_i) = \frac{\alpha_i}{\sigma_r^2} \; e^{-(1/2\sigma_r^2)\alpha_i^2}, \quad i = 1, 2, ..., N \tag{7}$$

Where $\sigma_r^2 = E\{\alpha_i^2\}/2$ is the measure of the spread of the distribution. In all following calculations it is assumed that $\sigma_r^2 = 1/2$. Using (5) and (7) the distribution of $\gamma_i$ for a given $x_i$ is defined as:

$$f(\gamma_i/x_i) = \frac{1}{\gamma_i^{'}} \; e^{-(\frac{1}{\gamma_i^{'}})\gamma_i} \tag{8}$$

For the $l$th bit in the frame, we can rewrite the SIR (5) and the interference (4) for the $i$th user as follows:

$$\gamma_i(l) = \frac{W}{R} \frac{p_i \, h_i \, \alpha_i^2(l)}{x_i(l) + \sigma^2} \tag{9}$$

$$x_i(l) = \sum_{k \neq i}^{N} p_k \, h_k \, \alpha_k^2(l) \tag{10}$$

Assuming that both $\{\alpha_i(l)\}_{l=1}^{M}$ and $\{x_i(l)\}_{l=1}^{M}$ are iid (identical independent distributed) random variables, and of course $\alpha_i(l)$ and $x_i(l)$ are jointly independent random variables. Henceforth, the *averaged* correct reception $P_c$ is given as $(1-P_e)^M$, where $P_e$ is *averaged* BER for each bit in the frame, that is $P_e = E\{\tilde{P}_e\}$. We will calculate the averaged $P_e$ in the next few lines.

We can find the conditioned error probability $\tilde{P}(e/x_i)$ by taking the average of (6) with respect to $f(\gamma_i/x_i)$:

$$\begin{aligned} \tilde{P}(e/x_i) &= E\left\{\tilde{P}(e/\gamma_i, x_i)\right\} \\ &= \int_0^{\infty} \tilde{P}(e/\gamma_i, x_i) \, f(\gamma_i/x_i) d\gamma_i \\ &= \frac{1}{2 + \gamma_i^{'}} \end{aligned} \tag{11}$$

Notice that we dropped the bit index $l$ because the average BER does not depend on $l$. For large SIR, (11) behaves like:

$$\tilde{P}(e/x_i) \quad \approx \quad \frac{1}{\gamma_i^{'}} \quad = \quad \frac{x_i + \sigma^2}{\frac{W}{R} \, p_i \;\, h_i} \tag{12}$$

Now, we can find the *averaged* BER $P_e$ by taking the expectation of (12):

$$P_e \quad = \quad E\left\{\tilde{P}(e/x_i)\right\} \quad = \quad \frac{1}{\overline{\gamma_i}} \tag{13}$$

where $\overline{\gamma_i}$ is the average SIR given by:

$$\overline{\gamma_i} = \frac{W}{R} \frac{p_i \, h_i}{\sum_{k \neq i}^{N} p_k \, h_k + \sigma^2} \tag{14}$$

Therefore, the average utility function of the $i$th user is given by:

$$u_i = \frac{L \;\, R}{M \;\, p_i} \; (1 - \frac{1}{\overline{\gamma_i}})^M \tag{15}$$

*B. Rician Fast Flat Fading Channel*

In this case $\alpha_i$ is modelled as a Rician random variable with a probability distribution given by:

$$p(\alpha_i) = \frac{\alpha_i}{\sigma_r^2}\, e^{(-\frac{\alpha_i^2+s^2}{2\sigma_r^2})} I_0(\frac{\alpha_i\, s}{\sigma_r^2}) \tag{16}$$

where $s^2$ represents the power in the nonfading signal components, and is sometimes called a specular component of the received signal or the noncentrality parameter of the distribution [6]. $I_0(z)$ is the zero-order, first-kind Bessel function. Similarly to the Rayleigh case, we need to find the distribution of $\gamma_i$ (see (5)) for fixed $x_i$ (see (4)):

$$f(\gamma_i/x_i) = \frac{e^{-s^2}}{\gamma_i^{'}}\ \ e^{-(\frac{1}{\gamma_i^{'}})\gamma_i}\ I_0(2s\sqrt{\frac{\gamma_i}{\gamma_i^{'}}}) \tag{17}$$

where we assumed that $\sigma_r^2 = 1/2$ as we mentioned earlier. Similarly, as we did in the Rayleigh fast flat fading case, the averaged frame correct reception is given as $P_c = (1-P_e)^M$. Where $P_e$ can be found as follows:

$$\begin{aligned}\tilde{P}(e/x_i) &= \int_0^\infty \tilde{P}(e/\gamma_i, x_i) f(\gamma_i/x_i) d\gamma_i \\ &= \frac{e^{-s^2}}{2\gamma_i^{'}} \int_0^\infty\ e^{-\gamma_i(\frac{1}{2}+\frac{1}{\gamma_i^{'}})}\ I_0\left(2s\sqrt{\frac{\gamma_i}{\gamma_i^{'}}}\right) d\gamma_i \end{aligned} \tag{18}$$

using the fact that $I_0(\zeta)$ can be written as:

$$I_0(\zeta) = \sum_{n=0}^{\infty} \frac{(\frac{\zeta}{2})^{2n}}{(n!)2} \tag{19}$$

By substituting (19) in (18) and after few mathematical manipulations we obtain:

$$\tilde{P}(e/x_i) = \frac{1}{2+\gamma_i^{'}}\ \ e^{s^2(-1+\frac{2}{2+\gamma_i^{'}})} \tag{20}$$

At high SIR ($\gamma_i^{'} \gg 1$), (20) can be approximately written as:

$$\tilde{P}(e/x_i)\ \ \approx\ \ \frac{1}{\gamma_i^{'}}\, e^{-s^2}\ \ =\ \ \frac{x_i+\sigma^2}{\frac{W}{R}\, p_i\ h_i}\ \ e^{-s^2} \tag{21}$$

Now, to find the final average error rate $P_e$ we need to find $\mu_{x_i}$ the mean of $x_i$.

$$\begin{aligned}\mu_{x_i} &= E\{x_i\} = E\left\{\sum_{k\neq i}^{N} \alpha_k^2 p_k\, h_k\right\} \\ &= (1+s^2)\sum_{k\neq i}^{N} p_k\, h_k \end{aligned} \tag{22}$$

where we used the fact that [6]

$$\begin{aligned} E\{\alpha_k^n\} &= (2\sigma_r^2)^{n/2}\, e^{(-\frac{s^2}{2\sigma_r^2})} \frac{\Gamma((2+n)/2)}{\Gamma(n/2)} \\ &\times\ \ 1F1[(2+n)/2, n/2; s^2/2\sigma_r^2] \end{aligned} \tag{23}$$

where $\Gamma(.)$ is the Gamma function, and $1F1[a, b; y]$ is the confluent hypergeometric function [10]. By substituting for $\sigma_r^2 = 1/2$ and $n = 2$ in (23) we can get the result in (22). We used the following special case of the confluent hypergeometric function $1F1[a, b; y]$ in calculating (22):

$$1F1[2, 1; s^2] = (1+s^2)\, e^{s^2} \tag{24}$$

Finally to obtain $P_e$, we simply need to replace $x_i$ in (21) by $\mu_{x_i}$, that is

$$P_e\ \ \approx\ \ \frac{e^{-s^2}(\mu_{x_i}+\sigma^2)}{\frac{W}{R} h_i\, p_i}\ \ =\ \ \frac{1}{\gamma_i^s} \tag{25}$$

where

$$\gamma_i^s = \frac{\frac{W}{R} h_i\, p_i\ \ e^{s^2}}{(1+s^2)\sum_{k\neq i}^{N} h_k\, p_k + \sigma^2} \tag{26}$$

Then, the utility function of the $i$th user is given by

$$u_i = \frac{L\ R}{M\ p_i}\,(1-\frac{1}{\gamma_i^s})^M \tag{27}$$

## IV. NON-COOPERATIVE POWER CONTROL GAME

Suppose $\mathcal{N} = \{1, 2, ..., N\}$ represent the index set of the users currently served in the cell and $\{P_j\}_{j\in\mathcal{N}}$ represents the set of strategy spaces of all users in the cell. Let $G = [\mathcal{N}, \{P_j\}, \{u_j(.)\}]$ denote a noncooperative game, where each user chooses its power level from a convex set $P_j = [p_{j-min}, p_{j-max}]$ and where $p_{j-min}$ and $p_{j-max}$ are the minimum and the maximum power levels in the $j$th user strategy space, respectively. With the assumption that the power vector $\mathrm{p} = [p_1, p_2, ..., p_N]$ is the result of NPG, the utility of user $j$ is given as [1]:

$$u_j(\mathrm{p}) = u_j(p_j, \mathrm{p}_{-j}) \tag{28}$$

where $p_j$ is the power transmitted by user $j$, and $\mathrm{p}_{-j}$ is the vector of powers transmitted by all other users. The right side of (28) emphasizes the fact that user $j$ can just control his own power. We can rewrite (1) for user $j$ as:

$$u_j(p_j, \mathrm{p}_{-j}) = \frac{L\,R}{M\,p_j}\, P_c(\gamma_j) \tag{29}$$

The formal expression for the NPG is given in [1] as:

$$\text{NPG}:\quad \max_{p_j\in \mathrm{P}_j} u_j(p_j, \mathrm{p}_{-j}),\ \ \text{for all}\ \ j\in\mathcal{N} \tag{30}$$

This game will continue to produce power vectors until it converges to a point where all users are satisfied with the utility level they obtained. This operating point is called an equilibrium point of NPG.

In the next section, we define the Nash equilibrium point and describe its physical interpretation.

### A. Nash Equilibrium in NPG

The resulting power vector of NPG is called a Nash equilibrium power vector.

**Definition 2:** *[1] A power vector* $\mathrm{p} = [p_1, p_2, ..., p_N]$ *is a Nash equilibrium of the* NPG defined above *if for every* $j \in \mathcal{N}, u_j(p_j, \mathrm{p}_{-j}) \geq u_j(p_j^{'}, \mathrm{p}_{-j})$ *for all* $p_j^{'} \in \mathrm{P}_j$.
One interpretation of Nash equilibrium is that no user can increase its utility by changing its power level unilaterally. Sometimes, a user may find different values of transmit power levels from its strategy space that give the user higher values of the utility function for given power levels of the other users. For this reason, the best response correspondence $r_j(\mathrm{p}_{-j})$ was introduced [1]. It assigns to each $\mathrm{p}_{-j} \in \mathrm{P}_{-j}$ the set

$$r_j(\mathrm{p}_{-j}) = \Big\{p_j \in \mathrm{P}_j : u_j(p_j, \mathrm{p}_{-j}) \geq u_j(p_j^{'}, \mathrm{p}_{-j}) \text{ for all } p_j^{'} \in \mathrm{P}_j\Big\} \quad (31)$$

In light of this correspondence one can announce the power vector $\mathrm{p} = [p_1, p_2, ..., p_N]$ as a Nash equilibrium power vector if and only if $p_j \in r_j(\mathrm{p}_{-j})$ for all $j \in \mathcal{N}$.

If we multiply the power vector p by a constant $0 < \beta < 1$ we may get higher utilities for all users. This means that the Nash equilibrium is not efficient, that is, the resulting p is not the most desired social operating point. And this results from the lack of cooperation between the users currently using the system. To impose a kind of cooperation between users in order to reach a Pareto dominant Nash point, a pricing technique was introduced in [1]. We then use the following algorithm to find Nash equilibrium point of NPG. Assume user $j$ updates its power level at time instances that belong to a set $T_j$, where $T_j = \{t_{j_1}, t_{j_2}, ...\}$, with $t_{j_k} < t_{j_{k+1}}$ and $t_{j_0} = 0$ for all $j \in \mathcal{N}$. Let $T = \{t_1, t_2, ...\}$ where $T = T_1 \bigcup T_2 \bigcup ... \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{\mathrm{p}}$ to be the smallest power vector in the total strategy space $\mathrm{P} = P_1 \bigcup P_2 \bigcup ... \bigcup P_N$.

**Algorithm 1:** *Consider NPG as given in (30) and generate a sequence of power vectors as follows [1]:*

*1. Set the power vector at time* $t = 0$: $p(0) = \underline{p}$, *let* $k = 1$

*2. For all* $j \in \mathcal{N}$, *such that* $t_k \in T_j$:

*(a) Given* $p(t_{k-1})$, *calculate* $r_j(t_k) = arg \max_{p_j \in P_j} u_j(p_j, p_{-j}(t_{k-1}))$

*(b) Let the transmit power* $p_j(t_k) = min(r_j(t_k))$

*3. If* $p(t_k) = p(t_{k-1})$ *stop and declare the Nash equilibrium power vector as* $p(t_k)$, *else let* $k := k + 1$ *and go to 2.*

## V. Simulation Results

We present the effect of a fast fading channels on the equilibrium utilities and powers which are the outcomes of NPG algorithm 1 studied in [1]. We use the same definition of utility function (see (1)) with $P_c$ modified to fit the channel model in two cases: Rician fast flat fading channel model and Rayleigh fast flat fading channel model. The system studied is a single-cell with 9 stationary users using the same data rate $R$ and the same modulation scheme, non-coherent BFSK. The system parameters used in this study are given in Table I. The distances between the 9 users and the BS are $\mathrm{d} = [310, 460, 570, 660, 740, 810, 880, 940, 1000]$ in meters. The path attenuation between user $j$ and the BS using the simple path loss model [7] is $h_j = 0.097/d_j^4$. Fig.1 shows that under Rayleigh and Rician fast flat fading channels with the strategy space $\mathrm{P}_j = [\tilde{p_j}, 2]$ , where $\tilde{p_j} > 0$ and spreading gain $W/R = 100$, users do not reach the Nash equilibrium point since all users except the nearest user to the BS are using the highest power level in the strategy space. More clearly, in Fig.2 one can see that users obtain very low utilities as a result of NPG compared to deterministic path gains. Fig.3 and Fig.4 show that with the same parameters as in Fig.1 and Fig.2 but with spreading gain $W/R = 1000$ the results are more encouraging where a Nash equilibrium was possible and it is comparable to that of deterministic channel gains. This tells us that a fast channel variation can decrease the performance of the game-theoretic approach dramatically for low values of $W/R$.

## VI. Conclusions

We studied a noncooperative power control game (NPG) introduced in [1] for more realistic channels, where we studied the impact of power statistical variation in Rayleigh and Rician fast flat fading channels on the powers and utilities vectors at equilibrium. The results showed that an equilibrium can be obtained in both games only at higher processing gains ($W/R > 100$). Also, Results showed that users at different distances from the base station (BS) can have the same transmit power at equilibrium as a result of fading. Utilities for Rayleigh and Rician fast flat fading channel gains at equilibrium are lower (at higher equilibrium power vector ) than the utilities for deterministic channel gains. But, the SIRs obtained at equilibrium are higher for all users at equilibrium in the Rician and Rayleigh flat fading cases than SIR under deterministic channel gains.

We are currently investigating the choice of different utility functions which may allow us to solve the NPG with lower processing gains. We are also considering various statistical optimization techniques in order to solve the various game problems.

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TABLE I

THE VALUES OF PARAMETERS USED IN THE SIMULATIONS.

| | |
|---|---|
| $L$, number of information bits | 64 |
| $M$ length of the codeword | 80 |
| $W$, spread spectrum bandwidth | $10^6$, $10^7$ $Hz$ |
| $R$, data rate | $10^4$ bits/sec |
| $\sigma^2$, AWGN power at the BS | $5 \times 10^{-15}$ |
| $N$, number of users in the cell | 9 |
| $s^2$, specular component | 1 |
| $W/R$, spreading gain | $100,\ 1000$ |

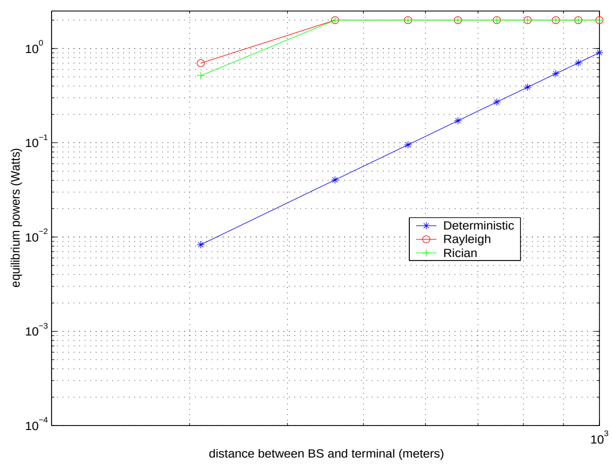


Fig. 1. Equilibrium powers of NPG for Rician flat fading channel gain (+), Rayleigh flat fading channel gain (o), and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 100$.

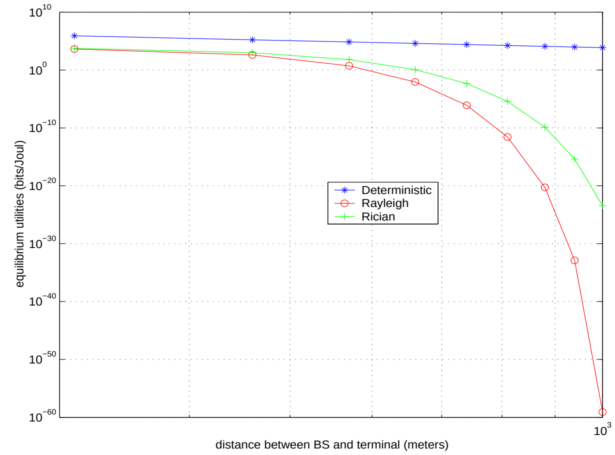


Fig. 2. Equilibrium utilities of NPG for Rician flat fading channel gain (+), Rayleigh flat fading channel gain (o), and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 100$.

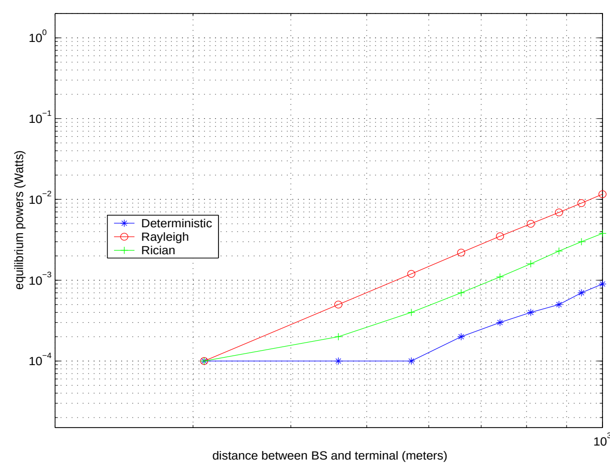


Fig. 3. Equilibrium powers of NPG for Rician flat fading channel gain (+), Rayleigh flat fading channel gain (o) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 1000$.

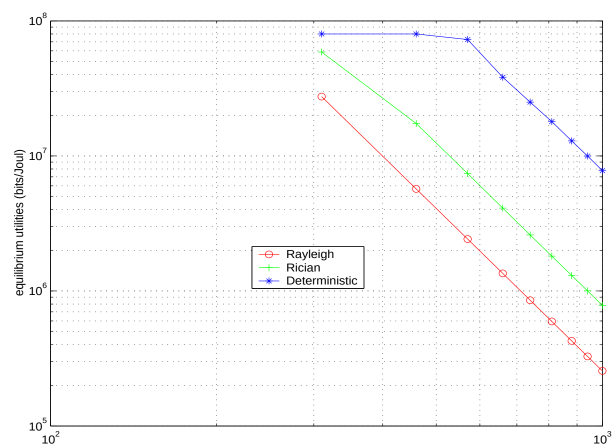


Fig. 4. Equilibrium utilities of NPG for Rician flat fading channel gain (+), Rayleigh flat fading channel gain (o), and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 1000$.


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