3-30-2012

Performance of Game Theoretic Power Control Algorithms for Wireless Data in Fading Channels

Chaouki T. Abdallah

M. Hayajneh

Performance of Game Theoretic Power Control Algorithms In Interference Limited Wireless Fading Channels

M. Hayajneh & C.T. Abdallah
Dept. of Electrical & Computer Engr., Univ. of New Mexico,
EECE Bldg., Albuquerque, NM 87131-1356, USA.
{hayajneh,chaouki}@ eece.unm.edu

Abstract

We consider a game-theoretic power control algorithm in interference limited fading channels, where we propose a distributed (non-cooperative) algorithm to optimize the induced fading outage probability by maximizing the certainty equivalent margin (CEM). We prove that the problem of maximizing CEM is the same (up to an upper bound) as minimizing the induced outage fading probability, and provide a distributed game theoretic power control algorithm.

I. Introduction

The mathematical theory of games was introduced by John Von Neumann and Oskar Morgenstern in 1944 [9]. In the late 1970's game theory became an important tool in the analyst's hand whenever he or she faces a situation in which a player's decision depends on what the other players did or will do. A core idea of game theory is the way strategic interactions between rational agents (players), generates outcomes according to the players' preferences [5],[10]. A player in a non-cooperative game responds to the actions of other players by choosing a strategy (from his strategy space) in an attempt to optimize a target function that quantifies the quality level, i.e. its level of satisfaction.

In a cellular communications system, users desire to have a high SIR (signal-to-interference ratio) at the BS (base station) coupled with the lowest possible transmit power. It is very important in such systems to have a high SIR, because this will be reflected in a low error rate, a more reliable system, and high channel capacity, which mean that users can send at higher bit rates [6],[7]. It is also important to decrease the transmit power because low power levels lead to longer battery life and helps alleviate the ever present near-far problem in CDMA systems [8].

In power control algorithms exploiting game theory, the tendency of each user is to increase his/her transmit power in response to other users' actions, leading to a sequence of power vectors that converges to a point where no user has incentive to increase his/her individual power. This operating point is called a Nash equilibrium. In many cases, and due to the lack of cooperation between the users (players), this point is not efficient, in the sense that it is not the most desirable social point [4]. The most desirable social point is called a Pareto optimal point, and may be viewed as the equilibrium point where no user can improve his level of satisfaction without harming at least one other user in the network.

The power control problem for wireless data CDMA systems was first addressed in the game theoretic framework in [3], then in a more detailed manner in [2] and [4]. In [1] the authors introduced a tight bound that relate the induced channel fading outage probability to the signal-to-interference margin where the signal power and the interference noise power were replaced by their mean. This enabled them to use Perron-Frobenius eigenvalue theory to allocate the power in order to obtain the optimal outage probability. For a general case, when there are bounds on the transmitted powers and other constraints, they showed that this problem can be posed as a geometric program. Using the geometric program framework enabled them to solve the optimization problem of outage probability efficiently and globally. The global solution for the optimization problem of the outage probability was based on the presence of a centralized controller or collector to collect data about path gains between the transmitters and the receivers in the cellular communications system.

In this paper the work in [1] to optimize the outage probability and the certainty-equivalent margin in a centralized fashion, which dealt with Rayleigh wireless fading channels, is considered by providing a non-centralized solution.

The remaining of this paper is organized as follows: In section II we present the system setup used in this paper. The tight relationship between the optimization of the outage probability and the optimization of the certainty-equivalent margin is emphasized in III. Non-cooperative power control game (NPG) is discussed in section IV. Simulation results are outlined in section V, and our conclusions are presented in section VI.

II. System Model

The system setup we are investigating is the same as that studied in [1], where the solutions for optimizing the system outage probability and the system certainty-equivalent margin were proposed based on a centralized power control algorithm. In this paper we propose a non-centralized power control game theoretic-algorithm for the same system. For convenience, we cast the system as follows: Suppose we have N transmitter/receiver pairs in a cellular mobile system. The ith transmitter is supposed to send messages at a power level p_i from his convex strategy space P_i to the ith receiver. A transmitter-receiver pair does not necessarily indicate physically separated transmitters and receivers [1]. The received power level at the ith receiver from the kth transmitter is given by:

$$G_{i,k}F_{i,k}p_k \tag{1}$$

where $G_{i,k} > 0$ is the path gain from the kth transmitter to the ith receiver. This gain may represent processing gain, cross correlation between codes in CDMA (code division multiple access) system. It can also, represent coding gain, log-normal shadowing and antenna gains. $F_{i,k},\;\; i,k = 1,2,...,N$ are exponentially iid (identical independent distributed) random variables with mean equal to 1 to represent the statistical power variation in a wireless Rayleigh flat fading channel. This means that the power received from the kth user at the ith receiver is exponentially distributed with expected value

$$\mathbb{E}\left\{G_{i,k}F_{i,k}p_k\right\} = G_{i,k}p_k$$

In interference-limited fading channels, the background additive white gaussian noise (AWGN) is assumed to be negligible compared to the interference power from the users. Henceforth, the signal-to-interference ratio of the ith user at the corresponding receiver is given by:

$$SIR_i = \frac{G_{i,i}F_{i,i}p_i}{\sum_{k\neq i}^{N} G_{i,k}F_{i,k}p_k} \tag{2}$$

SIR_i is thus a random variable, a ratio of an exponentially distributed random variable to a summation of independent exponentially distributed random variables with different means. The outage probability is defined as the probability that the SIR of an active user, i will go below a threshold SIR_{th}, so that for user i:

$$O_i = \mathbb{P}\{SIR_i \leq SIR_{th}\} = \mathbb{P}\{G_{i,i}F_{i,i}p_i \leq SIR_{th}\;\sum_{k\neq i}^{N} G_{i,k}F_{i,k}p_k\}\quad i = 1,2,...,N \tag{3}$$

This probability was evaluated in [1] and is given by:

$$O_i = 1 - \prod_{k\neq i}^{N} \frac{1}{1 + SIR_{th}\frac{G_{i,k}p_k}{G_{i,i}p_i}} \tag{4}$$

The outage probability of the system, O is defined as:

$$O = \max_{1\leq i\leq N}\; O_i \tag{5}$$

Where O plays a role of a figure of merit of the cellular system and the power control algorithm.

The certainty-equivalent margin of the ith user is defined as the ratio of his/her certainty-equivalent SIR to the corresponding threshold SIR. Mathematically,

$$CEM_i = \frac{SIR_i^{ce}}{SIR_{th}} \tag{6}$$

where, the certainty-equivalent SIR (SIR_i^{ce}) of the ith user is defined as the ratio of his/her mean received power at the corresponding receiver to the mean of the interference from the other users in the system, i.e.,

$$SIR_i^{ce} = \frac{G_{i,i}p_i}{\sum_{k\neq i}^{N} G_{i,k}p_k} \tag{7}$$

The certainty-equivalent SIR of the system SIR^{ce} is defined as:

$$SIR^{ce} = \min_{1\leq i\leq N} SIR_i^{ce} \tag{8}$$

Therefore, the certainty-equivalent margin of the system CEM is given by:

$$CEM = \min_{1\leq i\leq N} CEM_i \tag{9}$$

The CEM plays a role of yet another cellular system figure of merit for the power control algorithm and the complete system.

III. Relation Between Outage Probability and Certainty-Equivalent Margin

In this section we present the following proposition, which was initially stated as a comment in [1], and provide a simple proof.

Proposition 1: *The problems of minimizing the fading induced outage probability O_i and maximizing the certainty equivalent CEM_i of the ith user in a Rayleigh wireless fading channel are equivalent in terms of power allocation.*

Proof: First, recall that the problem under study is to minimize the outage probability of the system in a distributive fashion, i.e., each user transmits at a power level that minimizes his/her outage probability. Mathematically,

$$\min_{p_i\in P_i} O_i = \min_{p_i\in P_i} 1 - \prod_{k\neq i}^{N} \frac{1}{1 + SIR_{th}\frac{G_{i,k}p_k}{G_{i,i}p_i}} \tag{10}$$

and this in turn is equivalent to

$$\max_{p_i\in P_i} \prod_{k\neq i}^{N} \frac{1}{1 + SIR_{th}\frac{G_{i,k}p_k}{G_{i,i}p_i}} \tag{11}$$

or

$$\min_{p_i\in P_i} \prod_{k\neq i}^{N} \left(1 + SIR_{th}\frac{G_{i,k}p_k}{G_{i,i}p_i}\right) \tag{12}$$

Using the monotonicity of the Logarithmic function, optimizing (12) is equivalent to solving the following problem:

$$\min_{p_i\in P_i} \sum_{k\neq i}^{N} \log\left(1 + SIR_{th}\frac{G_{i,k}p_k}{G_{i,i}p_i}\right) \tag{13}$$

Now, using the inequality $\log(x) \leq x - 1$, (13) can be rewritten as:

$$\min_{p_i\in P_i} \sum_{k\neq i}^{N} \log\left(1 + SIR_{th}\frac{G_{i,k}p_k}{G_{i,i}p_i}\right) \leq \min_{p_i\in P_i} \sum_{k\neq i}^{N} SIR_{th}\frac{G_{i,k}p_k}{G_{i,i}p_i} \tag{14}$$

Finally, it is fairly simple to see that the right side of the inequality in (14) is exactly the same as:

$$\max_{p_i \in P_i} \frac{G_{i,i} p_i}{SIR_{th} \sum_{k \neq i}^{N} G_{i,k} p_k} = \max_{p_i \in P_i} CEM_i \tag{15}$$

■

In the next section we shall study the following non-cooperative power control algorithm (NPG) to maximize the system certainty-equivalent margin unilaterally:

$$\begin{aligned} NPG \quad : \quad & \max_{p_i \in P_i} CEM \\ & CEM = \frac{G_{i,i} p_i}{SIR_{th} \sum_{k \neq i}^{N} G_{i,k} p_k} \end{aligned} \tag{16}$$

which will result in minimizing the system outage probability as we proved in proposition 1.

IV. Power Control Algorithm to Optimize The Outage Probability

In this section we introduce a simple non-cooperative game-theoretic power control algorithm (NPG) which results in a Nash equilibrium point. Suppose $\mathcal{N} = \{1, 2, ..., N\}$ represent the index set of the users currently served in the cell and $\{P_j\}_{j \in \mathcal{N}}$ represents the set of strategy spaces of all users in the cell. Let $G = [\mathcal{N}, \{P_j\}, \{CEM_j\}]$ denote a noncooperative game, where each user chooses its power level from a convex set $P_j = [p_{j-min}, p_{j-max}]$ and where p_{j-min} and p_{j-max} are the minimum and the maximum power levels in the jth user strategy space, respectively. Initially, the goal of this algorithm was for all users to target a constant certainty-equivalent margin, say $CEM_j = CEM = constant$ for $j = 1, 2, ..., N$. In this way, all users will be satisfied with an SIR greater than their corresponding threshold SIR. This will allow all users to use their minimum possible transmit power level and while decreasing the interference from the other users and alleviating the near-far problem. Surprisingly, at the equilibrium point, all users achieved a higher certainty-equivalent margin than the targeted one, and the values of CEM_j and O_j for all $j = 1, 2, ..., N$ were very close to the results of the centralized algorithm in [1]. Moreover, we noticed that if we increase the target CEM (equal target for all users), we ended up almost with the same values of CEM and outage probability O_i but at higher power allocation.

Assume user j updates its power level at time instances that belong to a set T_j, where $T_j = \{t_{j_1}, t_{j_2}, ...\}$, with $t_{j_k} < t_{j_{k+1}}$ and $t_{j_0} = 0$ for all $j \in \mathcal{N}$. Let $T = \{t_1, t_2, ...\}$ where $T = T_1 \bigcup T_2 \bigcup ... \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{p}$ to be the smallest power vector in the total strategy space P $= P_1 \bigcup P_2 \bigcup ... \bigcup P_N$.

Algorithm 1: *Consider NPG as given in (16) and generate a sequence of power vectors as follows:*

1) *For al $j \in \mathcal{N}$, set $CEM_j = CEM = 1$*
2) *Set the power vector at time $t = 0$: $p(0) = \underline{p}$, let $k = 1$*
3) *For all $j \in \mathcal{N}$, such that $t_k \in T_j$:*
   a) *Given $p(t_{k-1})$, let the transmit power $p_j(t_k) = \frac{CEM\, SIR_{th}}{SIR_j^{ce}(t_{k-1})}\, p_j(t_{k-1})$*
4) *If $p(t_k) = p(t_{k-1})$ stop and declare the Nash equilibrium power vector as $p(t_k)$, else let $k := k + 1$ and go to 3.*

In the above algorithm we assume that the BS informs the users about their corresponding SIRs, and that the channel is known to the receiver.

V. Simulation Results

In this paper we simulated a non-cooperative power control algorithm defined in (1) for the setup studied in [1]. The cellular system is assumed to have $N = 50$ receiver/transmitter pairs. The path gains $G_{i,k}$ were generated according to a uniform distribution on the interval $[0, 0.001]$ for all $i \neq k = 1, 2, ..., N$ and $G_{i,i} = 1 \ \forall i = 1, 2, ..., N$. The algorithm (1) was run for different values of the threshold signal-to-noise ratios (SIR_{th}) in the interval $[1, 20]$.

In Fig.1 we show the system certainty-equivalent margin values (CEM) resulting from Algorithm (1) versus the threshold signal-to-noise ratios, SIR_{th}. While in Fig.2, we present the resulting system outage probability, O $(*)$

TABLE I

EQUILIBRIUM VALUES OF CEM_i AND O_i FOR THE FIRST 10 USERS USING PERRON-FROBENIUS THEOREM AND THE NPG INTRODUCED IN THIS PAPER AT $SIR_{th} = 3$

Results using Perron-Frobenius theorem [1]				**NPG**		
i	p_i	CEM_i	O_i	p_i	CEM_i	O_i
1	0.1292	13.7107	0.0703	0.0100	14.9809	0.0645
2	0.1162	13.7107	0.0703	0.0100	16.6629	0.0582
3	0.1476	13.7107	0.0703	0.0100	13.0747	0.0736
4	0.1482	13.7107	0.0703	0.0100	12.9549	0.0742
5	0.1290	13.7107	0.0703	0.0100	14.9275	0.0647
6	0.1192	13.7107	0.0703	0.0100	16.3274	0.0594
7	0.1297	13.7107	0.0703	0.0100	15.0442	0.0643
8	0.1312	13.7107	0.0703	0.0100	14.6970	0.0657
9	0.1327	13.7107	0.0703	0.0100	14.4091	0.0670
10	0.1361	13.7107	0.0703	0.0100	14.2906	0.0675
average		13.7107	0.0703	**average**	13.7823	0.0704

versus the threshold SIR (SIR_{th}) compared to the minimum bound $1/(1+CEM)$ (solid line) and the upper bound $1-e^{-1/CEM}$ (dashed line) derived in [1]. In this figure, as one can see, the upper bound overlaps with the equilibrium outage probability which is the output of Algorithm (1). The results shown in Fig.1 and Fig.2 happened to be very close to the results obtained in [1] at a lower power allocation. This is obvious as one see from table I and table II. In these tables we are showing CEM_i and O_i for the first 10 users as evaluated by the Perron-Frobenius theorem in [1] and as equilibrium outcomes of the NPG. The averages of CEM_i and O_i presented in the tables are calculated for all users in the system. We observed that the average value of CEM obtained through NPG was higher than that obtained by the Perron-Frobenius theorem for all values of SIR_{th}. The average value of O obtained through NPG was sometimes less and sometimes higher than that obtained by Perron-Frobenius theorem. We need to keep in mind that using Perron-Frobenius theorem requires a central controller or collector to collect data about the gains from the transmitters to the receivers, $G_{i,k},\ \ i,k = 1,2,...,N$, while in NPG we just need the receiver in the BS to inform the users of the certain-equivalent SIR (SIR^{ce}).

VI. CONCLUSIONS

We proved the tight relationship between the two problems: minimizing the system outage probability and maximizing the system certainty-equivalent margin. We then proposed an asynchronous distributed non-cooperative power control game-theoretic algorithm to optimize the system certainty-equivalent margin and the system outage probability. Power was more effectively and more simply allocated according to this proposed non-centralized algorithm than the centralized algorithm in [1].

REFERENCES

[1] Sunil Kandukuri and Stephen Boyd, “Optimal power control in interfernce limited fading wireless channels with outage probability specifications.” *IEEE Trans. Wireless Comm.*, VOL. 1, NO. 1, pp. 46-55, Jan. 2002.

[2] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, “Efficient power control via pricing in wireless data networks.” *IEEE Tras. Comm.*, VOL. 50, NO. 2, pp. 291- 303, Feb. 2002

[3] N. Shah, N. B. Mandayam, and D. J. Goodman. “Power control for wireless data based on utility and pricing.” In *Proceedings of PIMRC*, pp. 1427-1432, 1998. *IEEE Trans. Vehic. Tech.*, 41(3):305-311, Aug. 1992.

[4] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, “Pricing and power control in multicell wireless data network,” *IEEE JSAC*, VOL. 19, NO. 10, pp. 1883- 1892, Oct. 2001

[5] D. Fudenberg and J. Tirole. *Game Theory*, The MIT Press, 1991.

TABLE II

EQUILIBRIUM VALUES OF CEM_i AND O_i FOR THE FIRST 10 USERS USING PERRON-FROBENIUS THEOREM AND THE NPG INTRODUCED IN THIS PAPER AT $SIR_{th} = 10$

Results using Perron-Frobenius theorem [1]				**NPG**		
i	p_i	CEM_i	O_i	p_i	CEM_i	O_i
1	0.1439	4.0985	0.2159	0.0100	4.0243	0.2194
2	0.1541	4.0985	0.2159	0.0100	3.7254	0.2347
3	0.1266	4.0985	0.2159	0.0100	4.5414	0.1971
4	0.1497	4.0985	0.2159	0.0100	3.8603	0.2275
5	0.1375	4.0985	0.2159	0.0100	4.1930	0.2116
6	0.1378	4.0985	0.2159	0.0100	4.1877	0.2118
7	0.1096	4.0985	0.2159	0.0100	5.3148	0.1711
8	0.1514	4.0985	0.2159	0.0100	3.8264	0.2293
9	0.1398	4.0985	0.2159	0.0100	4.1414	0.2139
10	0.1384	4.0985	0.2159	0.0100	4.1889	0.2118
average		4.0985	0.2159	**average**	4.1255	0.2156

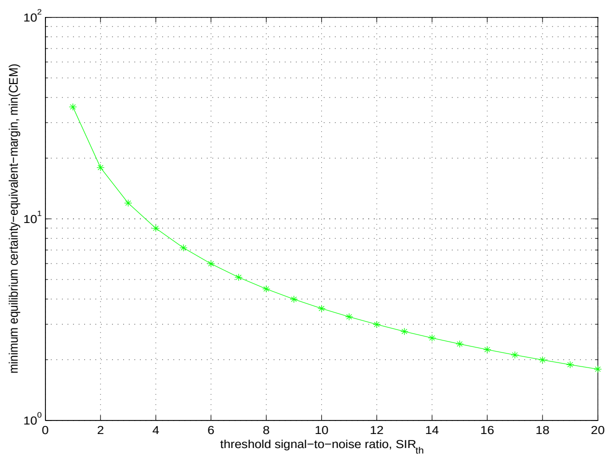

Fig. 1. Minimum equilibrium certainty-equivalent-margin versus the threshold signal-to-noise ratio.

[6] J. Zander. "Distributed cochannel interference control in cellular radio systems,"*IEEE Tran. Veh. Tchnol.*, VOL. 41, pp. 305-311, Aug. 1992

[7] J. G. Proakis, *Digital Communications*, The McGraw Hill Press 1221 Avenue of the Americas, New York, NY 10020, 2000.

[8] Roger L. Peterson, Rodger E. Ziemer and David E. Borth, *Introduction to Spread Spectrum Communications*, Prentice Hall, Upper Saddle River, NJ , 1995.

[9] J. V. Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, , Princeton, 1944.

[10] Don Ross, *What People Want: The concept of utility from Bentham to game theory*, University of Cape Town Press, South Africa, 1999.

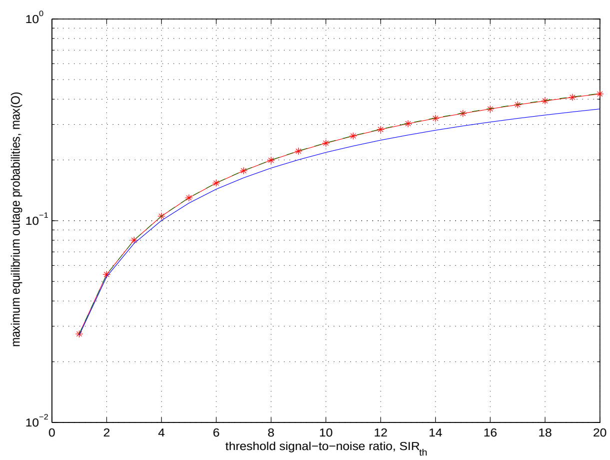

Fig. 2. Maximum equilibrium fading induced outage probability ($*$), the lower bound of the outage probability $\frac{1}{1+CEM}$ (solid line) and the upper bound $1 - e^{-1/CEM}$ (dashed line) versus the threshold signal-to-noise ratio.