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# Network Management Systems for Data Communications

A. C. Salazar  
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Today's mixed vendor environment has created a number of challenges for both customer and vendor.

Significant improvement in network operation is often the reason cited in justifying a network management system. More specifically, such a system can lead to reduction in the technical staff required to service equipment, since testing and network restoral can occur without major logistics problems. Network downtime can be reduced by using network restoral equipment at critical sites. Increased diagnostic capability of network management systems can facilitate the development of preventive maintenance techniques that spot troubles before they reach crisis proportions. Finally, planning for growth or more optimal network efficiency can occur on a more orderly basis if performance characteristics can be gathered over a period of time to spot trends that adversely affect network operation. In this paper we will briefly summarize the five major aspects of network management: namely, monitoring, control, trouble management, resource tracking, and network planning. In a later section, a list of factors will be discussed that are currently driving the development of network management systems today.

Network management is discussed from the viewpoint of a telecommunications customer, rather than from the viewpoint of a vendor. Today's mixed vendor environment has created a number of challenges for both customer and vendor. Issues related to regulatory and organizational constraints are not covered, nor are management services that a vendor might be expected to provide to a customer.

## Five Aspects of Network Management

### *Network Monitoring*

Network monitoring involves the gathering of performance and diagnostic data, including alarms from the network, while the individual network components are in service. This data is expected to yield insight into developing problems that can be dealt with before they seriously affect network operation. Parameters in analog modem networks such as receive signal level and phase jitter level could indicate degrading line conditions if measurements are taken and analyzed periodically. Raw data from the network monitoring activity can also be used for trending analysis in the long-range network planning process.

Since network components can number as high as hundreds (if not thousands) of units, the gathering of the network data must be planned carefully. The protocol used to communicate with the diverse segments of a network must have the addressing capability and efficiency to handle heavy data traffic commensurate with network size. The volume of expected network status information must match real-time data acquisition capability of the network management machine. Hence, raw data should be carefully screened before being logged into a database. Mechanized entry into a database can cause performance problems if the transaction involves many index computations and file accesses.

A major problem in the monitoring of diverse network elements occurs when both hardware interfaces are different and disparity exists in the diagnostic data format. A network management user needs to access all

network data from one console rather than have several control consoles for various network segments.

Automation of specific parametric measurements collected from network elements can be done on a user-programmed basis. Measurements taken while the network is on-line can yield one level of performance knowledge. Time of measurements and measure interval between replications are part of the automatic monitoring activity. A more detailed level of network performance measurements can be realized if links can be taken off-line and communication channels analyzed more thoroughly. These disruptive tests can be run during off-hours so as not to disrupt normal network operation.

## Network Control

Upon determination that a network problem has occurred through the receipt of an alarm condition, the private network user will then wish to enter the network control mode of his equipment. In this mode, he hopes to further isolate or identify the individual network elements which may be causing the problem. This is done by performing disruptive tests such as end-to-end error runs or disabling a terminal due to its streaming behavior. Restoral techniques such as using dial-backup lines in order to reinstate failed communication links, or activating hot-spare equipment in order to bypass failed units are an important part of network control (Fig. 1).

As discussed previously, the monitoring aspect of network management is highly automated and, in general, constitutes the mechanized accumulation of data from the network. The control aspect of network management relates to the human interactive response to the receipt of monitored data. Typical responses include running disruptive tests on a communications link in order to further characterize a reported fault and activation of service restoral equipment to work around the fault. These responses cause commands and

in some cases formatted data to flow from the control site into the network. It is essential that the user be able to react quickly and precisely to reported network faults. This real-time response capability is the essence of network control.

## Trouble or Problem Management

Large networks often have a significant number of reported problems which need to be recorded and tracked before final resolution. The keyboard entry of this information occurs at a trouble desk where dispatching of personnel to resolve the problem is often done. Summary or status reports need to be generated periodically to ensure that the number of problems has not gotten out of hand. Data concerning reported network problems can be used to generate time-to-repair or out-of-service averages. These parameters are often closely monitored for trending information. The trouble desk also needs to have the ability to associate different levels of criticality to problems not only at the time of first report but later when non-resolution may require escalation of attention.

Since keyboard data entry and report generation constitute the major functions of trouble management, the user interface for executing these functions becomes important. System response time for entering data and creating reports is a critical parameter. The data base used for this function of network management must have the tools necessary to generate customized reports which relate one set of data fields in one application differently from a set of fields in another report application. Figure 2 illustrates the different activities associated with trouble management.

## Resource Tracking

A private network consists of a large number of elements which differ widely in type, vendor, location, service level and functional criticality. There is a recognized need to inventory all the network elements for the purpose of asset control in addition to expediting maintenance and service restoral procedures. This need for asset control has increased dramatically in recent years as private network managers have been driven to maintaining and servicing their own networks. Self reliance in network management often requires knowing where equipment spares are located and who is able to complete the repair procedure. The diversity of network equipment which often parallels an equal diversity of suppliers or service bureaus has led many network managers to require resource tracking capability from their network command center.

Since the resource tracking function of network management is again data base intensive, keyboard data entry and report generation again become important user interface issues. Data base updates should be done quickly with minimum system transaction delay. As in trouble management, customized reports are an important feature to users whose needs for data relationships change from time to time.

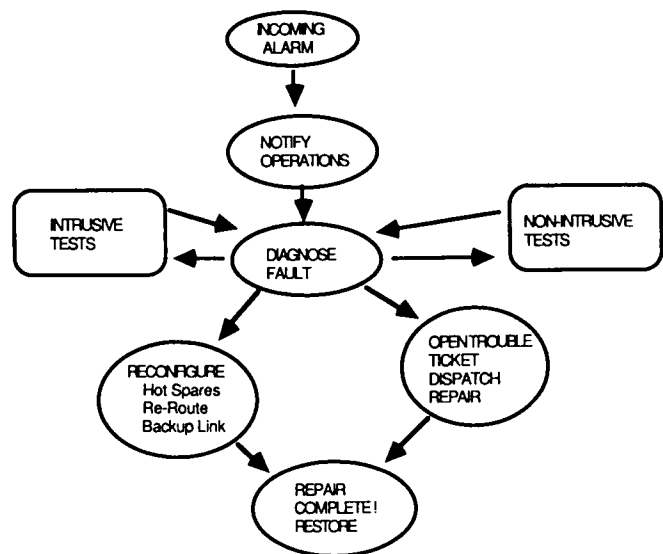


Fig. 1. Alarm Handling.

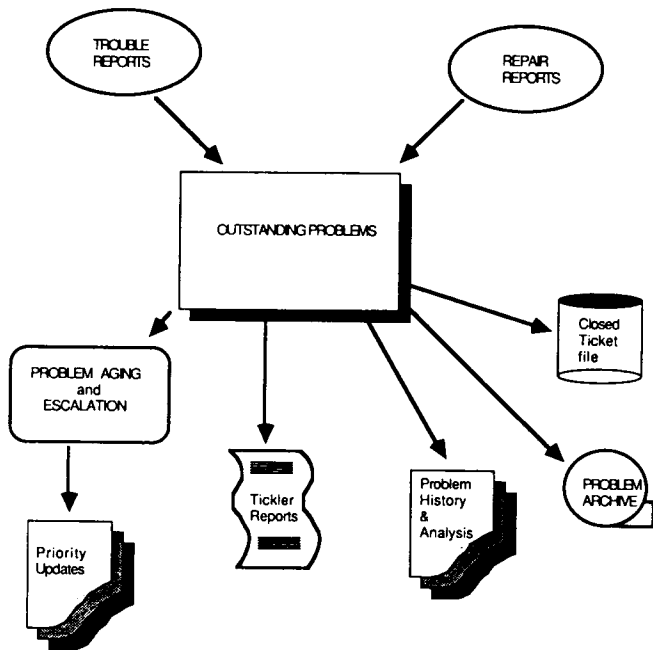


Fig. 2. Problem Management.

### Network Planning

Every network of data communications equipment has a set of performance measurement parameters whose values provide the network manager a network service index. These parameters include measurements such as number of terminals out of service for more than an hour, mean time to repair for last ten service calls, etc. In the end, downtime costs money as does restoral equipment. Hence, tradeoffs need to be made on the basis of (a) historical trouble data (b) inventory and service costs (c) business changes (d) line tariff changes. Network design can proceed on an optimized basis only if reliable data can be kept on the network performance parameters. Network managers find that flexible summary reports on resources and trouble tickets are invaluable in the network design process.

### Trends in Network Management Systems

#### Integration—The New Network Management Problem

In today's networks the underlying communications equipment has added another large layer of complexity. Networks previously were internally uniform. They were mostly analog networks, predominantly serving one major application, with one control center, and with one major network services provider; AT&T. (Fig. 3) Today's larger networks (Fig. 4) will involve:

- multiple network service vendors for the same technology
- multiple transmission technologies:
  - analog facilities
  - digital facilities

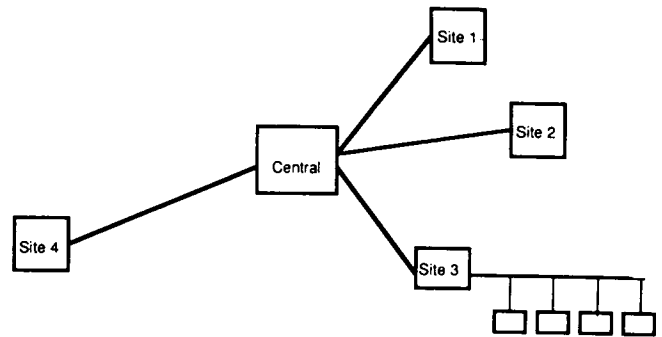


Fig. 3. Typical '60s Telecommunications Network.

- T1 and microwave facilities
- Satellite links
- Local Area Networks
- additional transmission processing functionality:
  - Multiplexors
  - packet-switching
  - remote polling
- mixed purpose systems—voice and data combined
  - ISDN

Each of these facilities may have control capabilities built in by their manufacturer. This leads to a wild proliferation of consoles at the network control center. Network managers now have to juggle several terminals and use the capabilities of several vendors' equipment to isolate and solve a single problem. A central control system that evaluates and diagnoses the entire network often does not exist. In its place are many separate and uncoordinated control systems for various network segments.

The control center is also being asked to do more than find and fix line problems. Now its job includes monitoring remote terminal equipment and maintaining system performance standards. These jobs add still more to the list of tasks and measurements to be taken and evaluated. The load for this is substantial. For example, just recording the performance statistics for

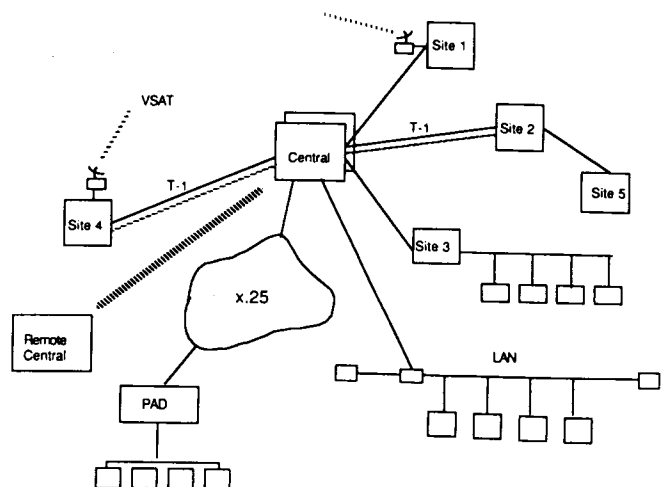


Fig. 4. Typical '80s Telecommunications Network.

500 terminals on an hourly basis can consume between 500,000 and 1,000,000 bytes per day. All this data requires processing to collect, to transmit, to evaluate, and to store and retrieve.

Transmission of data is no longer "free" either. With the shared facilities like packet switching every byte sent costs money. When the network was built of leased lines and the costs were traffic independent it was reasonable to send everything back to the center. Now the cost may be substantial when some of the links are priced on a traffic basis.

Very large networks also are evolving with multiple control centers. Some corporations find it preferable to have regional control centers which have responsibility for some portion of the network. This may be limited to only trouble-shooting related control access, or it may be a complete backup control center that is ready to step in and take over control in the event of a major failure at the primary center. Whichever organization is used, this means that the communications equipment in the network must now be designed to have several masters. It must keep track of which site(s) should be sent data and alarms, and still be responsible to any authorized controller for testing.

Another major problem today deals with the potential need to integrate local area networks (LAN's) into the wide area Network Management System. Many of today's LAN's have been expanding steadily since their introduction a few years ago. As they expand, network management becomes an important consideration to controlling them. A network with several hundred nodes offers many of the same problems to the network manager of a wide area network. Alarms, network performance, and configuration control are but a few of the common problems presented.

Unfortunately, most of our LAN's today have not been designed with network management in mind. Consequently, little or no diagnostic information is available to signal points of failure or potential bottlenecks in the network.

Configuration changes must be better controlled and regulated as networks grow. The addition or deletion of nodes on the network must be known to the network controller to maintain proper system availability.

In short, we have a number of new network management problems confronting today's network management designers. The "integration" battle represents one of the major new challenges ahead.

## Second Generation Systems

Many of today's second generation Network Management Systems are products of the minicomputer architectures of the last decade. Predecessors to the new 32-bit super-micro computers, these 16-bit computers delivered compute speeds of .5 to 1 MIP. Yet despite their rated performance, many of these first and second generation systems are a poor match to the real time demands of today's larger telecommunications network. Even super-minis like the VAX 11/780, rated at about 1 MIP, are underpowered for today's large hybrid network.

User complaints about system performance and the inherent lack of flexibility provided by a single processor system have limited the success of many of these systems. A single processor system (Figure 5) also offers limited upgrade potential for the growing telecommunications network. Oftentimes the only upgrade path is one requiring a wholesale exchange of the system. Both capital and data investments are lost.

Beyond the flexibility issue, the absence of industry standards for network control diagnostics has had a major effect on these early system designs. Proprietary system architectures have been adopted by most data communication vendors. Understandably, their primary focus has been on the development of data communication products and not system based products. The result is decreased flexibility for both network planners and operators.

With proprietary hardware or designs which were based on a computer vendor's closed system concept, it has become increasingly difficult for data communication vendors to take advantage of the tremendous strides in computer systems technology. Many systems have applications software which is not easily transportable to these cheaper and faster devices.

New processors like the Motorola's 68020 and Intel's 80386 already offer computational speeds in excess of 2MIPS. This is better than twice the speed of these early minicomputer systems and supermini systems like the VAX 11/780. Yet they are available from multiple vendors today at only a fraction of the cost of their super-mini counterparts. Lower cost memory, mass storage, and high speed local area networks have also changed the design strategies of network system designers, as more distributed architectures become a cost-effective alternative.

## Next Generation System Architecture

Network management is not structured in conformance with the ISO reference model. It is an application and is structured to meet its internal needs. However, since it is managing a data communications network which probably does match the ISO structure, it will be very strongly influenced by the ISO reference model.

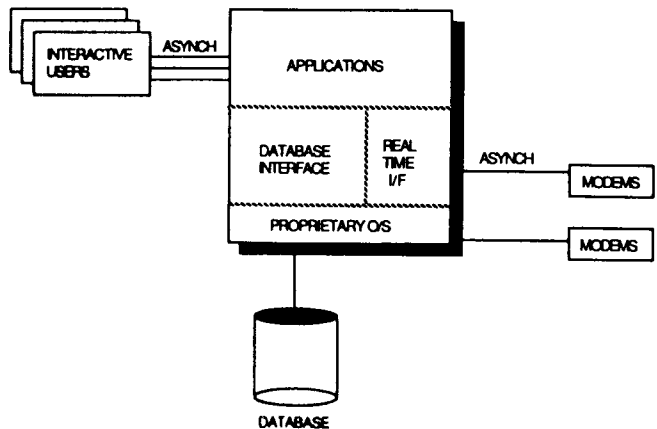


Fig. 5. Single Processor System.

The network management architecture is shown in Figure 6. There are three major components:

- The management layer, which focuses on the external and internal organizational issues of
  - Trouble or problem management
  - Resource tracking
  - Network planning
 These tend to be highly interrelated and usually share a common database.
- The network control layer
- The network monitoring layer

Network monitoring has interfaces to each of the layers of the data communications network. In a fully configured monitoring system there would be a monitoring interface at every termination of each layer. This means there must be a measurement point at each access point into a physical media, each link access point, each network termination, etc. Costs may restrict the monitoring to just the critical points.

The monitoring layer is connected to the control layer via the application layer services of some support network. This is usually necessary because monitoring points occur throughout the network. One serious problem that results from this configuration is that the support network may also be part of the network under control. In this situation application level commands may temporarily disrupt their own underlying network. This imposes the extra constraint on network design that an application layer connection survive

complete (temporary) loss of the lower layers. Some networks cannot do this, and for these the support network must be separate from the network being controlled.

The nature of different telecommunications technologies controls the internal structure of the monitoring layer. The kinds of monitoring and controlling activities that are appropriate to a LAN differ from those for a WAN. Different technologies lead to domains where each individual domain corresponds to a particular set of layers and interfaces that are tightly coupled. For example, 802 LAN's, X.25 networks, and T1 networks would each be a unique domain even though they provide services that appear in the same ISO layer. The monitoring and control interfaces to these domains are the focus of much of the current standardization efforts in network management.

The network layer becomes a collection of individual domain monitoring modules, each focused on only a few layers and supporting a specific technology. These modules may co-exist in the same processor, but they operate autonomously.

The network control layer comprises two sublayers. The lower sublayer is a collection of domain control modules in a one to one relationship with the monitoring domains. Each of these handles the control actions that are appropriate for their associated technology. The upper sublayer handles control actions that require coordination of multiple domains. This layer is only rarely implemented in current systems. It is usually handled directly by the network control personnel.

The network management layer comprises three different views of an integrated network management database. These correspond to each of the major management functions. The control layer provides continuous updates to the database to maintain a current view of the network. These updates are used by the network managers to determine both reconfiguration and organizational actions. Network managers also update the database to track and control their organizational activities.

In Figure 6, both the network management and control are shown located on a single host. This has the advantages of rapid interaction between management and control, and the advantage of sharing a single network database. It has the disadvantages of imposing maximal traffic loads on the support network, introducing network delays for control to monitoring layer interactions, and requiring a large central facility. A further disadvantage is that it now becomes a single site failure risk for the entire network. These problems can be reversed by shifting the control modules to the other side of the support network and placing both the control and monitoring activities for a domain into a domain processor. This has advantages when there are serious network problems, because local control and monitoring facilities will survive even serious network failures.

The management layer may also reside on either a single host or a distributed system. The tight coupling of the user tasks and the database make distributed operation more difficult. Reliable performance is most

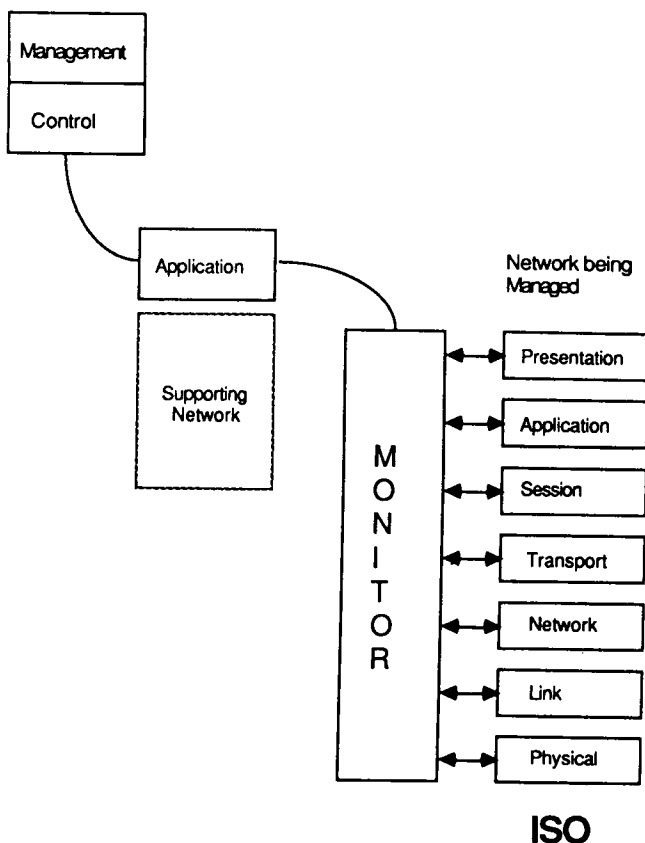


Fig. 6. Network Management Architecture.

critical when the network is failing, and this is the environment that poses the most problems for current distributed databases.

### Next Generation Designs

A third generation network management system design can exploit the economies available in today's technology and utilize a multi-processor approach versus a single processor architecture. This provides both greater flexibility and system performance over time.

Conformance to emerging computer industry standards is also a must today. As computer vendors move towards a more "open systems" philosophy to protect their own investments, it becomes important for third party developers to take advantage of the new flexibility and performance enhancements which emerge from these architectures.

New chip technologies, parallel processors, new RISC architectures will provide even more impressive price-performance ratios in the future. Applications software which is designed to take advantage of these changes will emerge ahead of the pack.

Systems designed around de facto industry standards like UNIX, Ethernet, TCP/IP, NFS may offer greater capability and provide the greatest protection against product obsolescence. In fact, in the data communications arena, a de facto standard may be more important than an official standard committee approval, since installed base may be the significant design consideration.

### Distributed Network Architecture

A distributed network architecture which utilizes several processors working in concert can deliver superior performance and greater flexibility. Horsepower can be added where needed rather than require the replacement of an entire central processor. The ability to couple intelligent user or network processors with a network management processor on a high speed LAN backbone provides an incremental approach to upgradeability (Figure 7).

One example of a distributed architecture design might have a processor which serves as a "database"

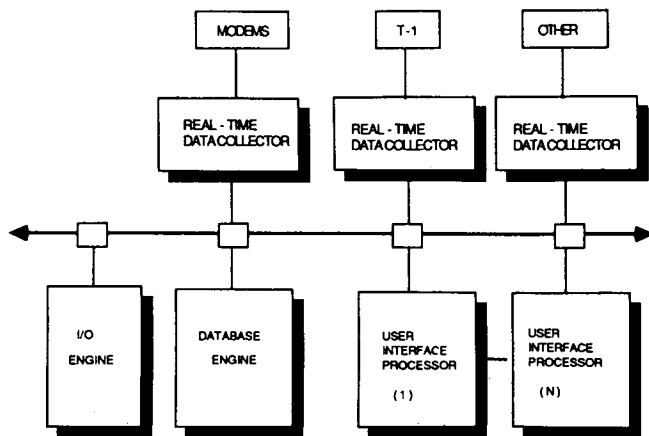


Fig. 7. Multi-Processor System.

engine or central file server on an ethernet backbone with several client "user" processors and "network interface" processors. Common information or files could be shared across this network yet the network management database itself would remain centralized under the control of the database engine. The user's investment in network data, problem management history, or other historical information can be preserved.

Each of the "engines" could be sized to the application requirements or added to the network as system requirements change. A separate dedicated network interface processor would be assigned to each class of data collection device on a hybrid network. This allows the network planner to plan network service expansion by simply adding a new front end data collector when needed.

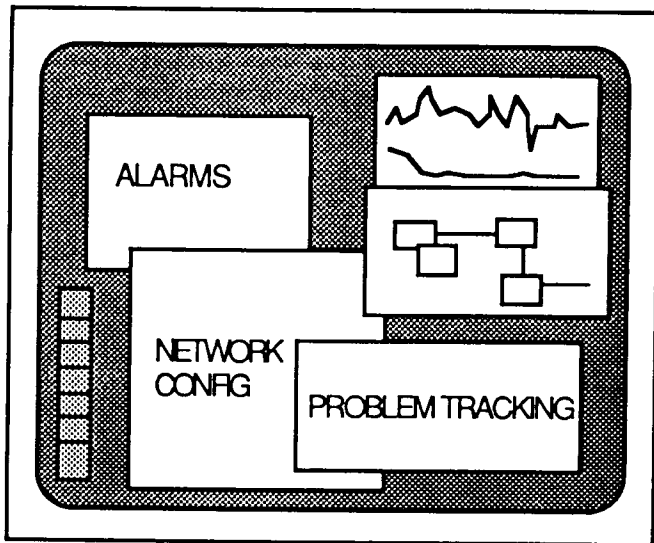
Complex networks supporting multiple network domains, i.e., analog and digital devices, private X.25 or T-1 products could then be more easily integrated into a single network management system. Software modifications would be limited to those front end data collection requirements of the new device alone.

Finally, as application and user demands increase, intelligent workstation processors can be added to the network to provide more sophisticated network control capabilities. Applications requiring high speed graphics processing or rapid command and control response can be better served by an intelligent workstation processor able to operate independently from the central data manager. These user processors, which are connected to a high speed ethernet backbone rather than medium speed asynchronous ports, provide broader user access, far better system response, and far greater system expandability.

### User Interface Trends

The use of workstation technologies is changing the user interface from a text and forms orientation to a multi-window, graphics environment. The problem analysis activity calls for examination of results from multiple tests and often from multiple domains. The windowing environments permit the simultaneous presentation of this information. There remain some areas where text is the appropriate format. Configuration details, field service addresses and phone numbers, and exact measurement results need the precision of text. But with the addition of graphics for displaying interconnections, trends of measurements and status monitoring the added power of human pattern recognition can be exploited. The new workstation technologies provide the processing and display power needed to present both the graphics and text in a timely and usable fashion.

The resulting systems present a much more dynamic image to the user. Routine surveillance is very graphically oriented, using color keys to highlight problems. Problem diagnosis and problem tracking activities take place in windows so that key surveillance monitoring can continue in parallel. (Figure 8) The network controllers have a wealth of information available on command, presented in the format most suitable for problem management. This information is called up



SYSTEM SURVEILLANCE

TEST RESULTS

PROBLEM TRACKING

NETWORK CONFIG

SPECIAL FUNCTION  
MENUS

Fig. 8. Multi-Window Environment.

and dismissed from the screen as the operator commands. These are all geared to reduce the problem diagnosis and repair time.

Early examples of this can be found in the new products being delivered by network control vendors. These have not yet made a significant penetration into the market and they will be modified based on the feedback received from the customers of these early shipments. The concept of using graphics is widely accepted. There is no consensus yet on what presentations are most effective.

### Expert Systems in Network Control

The computers themselves are also taking a more active role in network management. The greatest progress has been in:

1. Problem diagnosis—Expert systems are in use to speed the recognition and analysis of network faults. Instead of simply recognizing the fault and relying on the operator for diagnosis, systems are

analyzing the fault indications and network status. They can then present not only the fault, but also a composite fault diagnosis and then suggest corrective action.

2. Network configuration—The great many elements and rules associated with configuring communications lines, modems, multiplexors, etc. presents a major burden to the network designer. Expert systems are also being employed to aid in the configuring of complex networks. There will be increasing use of expert systems and artificial intelligence techniques to handle the routine surveillance and diagnosis activities and to assist in other network management activities.

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Since 1981 he has been with Infinet, Inc., a data communications systems manufacturer, where he is vice president of engineering. Active in IEEE and ACM sponsored conferences where he has participated as an officer, session organizer or author Dr. Salazar has also published numerous journal articles and is the editor of the IEEE Press book, *Digital Signal Computers and Processors*.

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From 1972-1981 he was with Environmental Research and Technology (a subsidiary of COMSAT) where he served as General Manager, Software Engineering Consulting Division. His work at ERT included management of numerous software consulting programs specializing in the area of data communications, data acquisition, emergency response, and computer modeling.

In 1981 he joined Microcom Inc as Vice President of Software Development where he directed the development of several communications software programs and their proprietary communications protocol (MNP).

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**Robert J. Horn** received his B.S. (Mathematics, Meteorology) degree from Massachusetts Institute of Technology in 1973. He was with Environmental Research and Technology (a subsidiary of COM-SAT) from 1973-1983 where his work included data communications, data acquisition, process control and radar systems design. From 1983-85 he was technical project manager for workstation based CAD/CAM systems at Computervision.

Since 1985, he has been with Infinet as manager of systems architecture. ■



is to pre-empt information bits and insert ones when necessary, and thus introduce errors into the pulse stream. The analysis of the resultant error rates for classes of digital and analog implementations of a PCM monitor is accomplished via theoretical consideration of certain stationary Markov processes.

## I. INTRODUCTION

Practical considerations place certain requirements on the binary data sequence transmitted through PCM (pulse code modulation) systems. An important requirement stems from a voltage threshold associated with the normal operating characteristics of a PCM digital repeater. Generally, a segment of a waveform corresponding to a particular bit sequence must have a minimum number of ones to preserve timing and regenerative capability at each repeater. In practice, transmitter data streams can be monitored to adapt the source bit sequences to meet a given engineering requirement, but the monitoring action introduces uncorrectable errors into the bit stream. The rate at which errors are inserted depends on the particular monitor implementation.

To investigate two possible PCM monitor designs, we present a pair of mathematical models representing distinct implementations of the monitoring function and determine the error rate corresponding to each implementation. The analysis of the models has led to the formulation and solution of a class of problems in the theory of stationary Markov processes.

One monitor implementation is digital in form and consists of using a minimum ones density coder at the transmitter. The coder action guarantees that the transmitted sequence contains a minimum number of ones,  $M$ , in any  $L$  consecutive bits ( $0 < M < L$ ). It is evident that the pair  $(M, L)$  can be chosen *a priori* by consideration of the analog waveform that a conditioned bit stream would produce. That is, a corrected stream would have enough variation in the transmitted analog waveform for proper repeater operation. We can visualize the digital coder action as the sliding of an  $L - 1$ -bit window along the transmitted stream (left to right) in 1-bit increments where the bit following the rightmost window bit is set to one whenever the within window sum is  $M - 1$ . We will see that the discrete nature of this algorithm lends itself to analysis by identifying words at the coder output as states of a Markov chain. The inserted error rate then corresponds to the average percentage of time that the coder spends doing corrective action.

Next we shall examine the second monitor implementation form, which is on the one hand straightforward in terms of the motivation which underlies it, but on the other hand vague insofar as its implications on the allowable bit patterns. Namely, the specification is that a certain signal (to be explained in Section III) derived from the PCM line signal must not drive a repeater tank voltage below a critical threshold (floor). A coupler network can enforce this specification by an analog monitoring of the signal, and changing certain zeros to ones if the voltage drops below the critical level. This second implementation would be applicable to PCM systems where an analog implementation of the monitor is more economical than digital implementation. The particulars of the monitor design and the motivation for the type of design chosen are not within the scope of this paper. Rather, we shall present a mathematical model for the monitor voltage process which simulates the repeater tank voltage. Our domain is probabilistic analysis of this threshold crossing from which we deduce an upper bound for the error rate. The samples of the analog waveform processed by the monitor form a stationary Markov sequence of random geometric series.

## II. A MINIMUM ONES DENSITY CODER—DIGITAL MONITOR

We begin our error rate analysis of the first model of a PCM monitor by observing its operation in terms of  $L$ -bit word (or

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### Markov Process Analysis of PCM Line Monitors

G. J. FOSCHINI, A. C. SALAZAR, MEMBER, IEEE, AND  
J. W. SMITH, MEMBER, IEEE

**Abstract**—In PCM transmission systems, ensuring repeater timing and regenerative capabilities requires monitored enforcement of the ones density of the line waveform. One possible corrective monitor action

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state) transitions. The Markovian nature of the digital PCM monitor will be apparent upon the labeling of states and identifying the transition matrix. We will set aside, for the moment, the particulars of the initialization of the monitor since an initial state distribution would have to be specified. As we shall see presently, since the Markov chain describing the monitor function is irreducible, the specification of an initial distribution is irrelevant.

#### A. Description of Operation

The coding of incoming data  $a_j$  ( $a_j = 0, 1$ ) into transmitted data  $b_j$  proceeds by choosing  $b_j = a_j$  if  $W\{b_k\}_{j-L+1}^{j-1} \geq M$  or  $b_j = 1$  if  $W\{b_k\}_{j-L+1}^{j-1} = M - 1$  where  $W\{b_k\}_{j-L+1}^{j-1}$  is the number of ones (weight) in the previous  $L - 1$  bits. We note that since all blocks of length  $L$  in the transmitted stream contain at least  $M$  ones, there are exactly  $2^L - \sum_{r=0}^{M-1} \binom{L}{r}$  permissible words or states at the coder output. We can index each state by using the binary word it represents, e.g.,  $(0, 1, 1, \dots, 1)$  corresponds to the  $2^{L-1}$ -th state. We note that each state belongs to one of two categories.

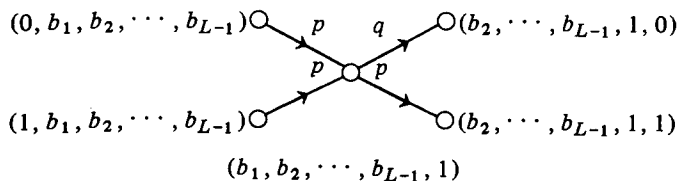
1) Those of the form  $b_{j-L+1} = 1$  and  $W\{b_k\}_{j-L+2}^j = M - 1$  (i.e., a one followed by  $M - 1$  ones in the next  $L - 1$  bits) in which an error can occur in the transition to the next state.

2) The rest of the states in which no error will occur in the transition to the next state.

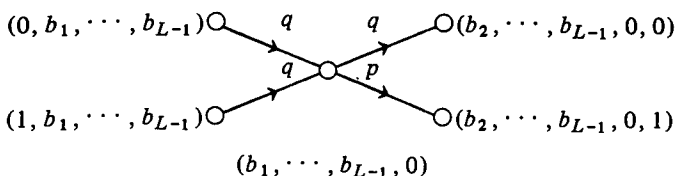
Note that there are  $\binom{L-1}{M-1}$  states of the first type in which errors can occur in shifting out the  $b_{j-L+1}$  bit and bringing in the  $b_{j+1}$  bit (which will be a one independent of  $a_{j+1}$ ).

We now assume that the incoming data stream consists of independent binary symbols 1 and 0 occurring with probabilities  $p$  and  $q$ , respectively.<sup>1</sup> Hence we can complete the description of the coder action in terms of a Markov chain by illustrating the possible state transitions.

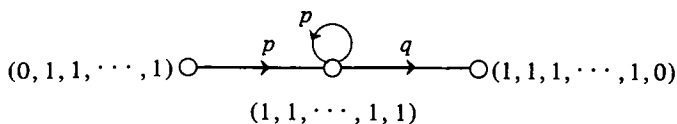
If we now examine the transition probabilities into and out of each type 2 state we have



or



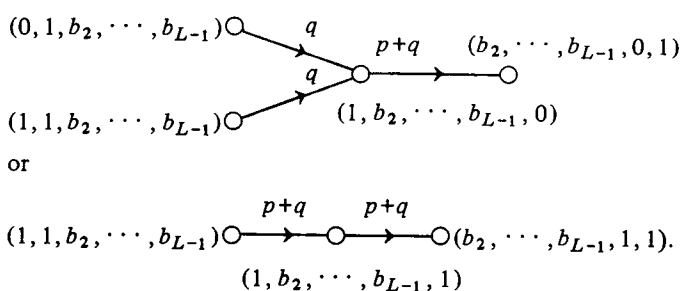
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where the states are identified by the transmitted sequence given in the parentheses.

Similarly, for each type 1 state we have

<sup>1</sup> The main case of interest is  $p = q = 1/2$ , but the analysis can be made more general easily enough, so we proceed with  $p, q$  unspecified, but  $pq \neq 0$ .



It is evident that the above-illustrated transitions supply the entries of the stochastic matrix describing the state transitions of a Markov chain. It is easy to see that the chain is irreducible, that is, each state is eventually reachable from any other state because the only closed class is the set of all states.<sup>2</sup> By a theorem in finite Markov chains [1, pp. 335-356] an irreducible chain has a unique stationary limiting distribution which is independent of the initial distribution. We can now begin our search for this limiting distribution for two important cases.

#### B. Special Case; $p = q = 1/2$

Because of its simplicity, we treat a special case of great practical importance first. Letting  $p = q = 1/2$ , we get a probability equation for each possible state transition of the form

$$x_A = \frac{1}{2}x_B + \frac{1}{2}x_C$$

or

$$x_D = x_E$$

where  $x_A$  is the steady-state probability of being in state  $A$ . The obvious solution for these equations is for each  $x$  to be equal to the inverse of the number of states. Therefore

$$x_A = x_B = \dots = \frac{1}{2^L - \sum_{r=0}^{M-1} \binom{L}{r}}$$

Then the probability of error is the probability of being in a type 1 state times the probability that the next customer bit is a zero or

$$p_e = \frac{q \binom{L-1}{M-1}}{2^L - \sum_{r=0}^{M-1} \binom{L}{r}} = \frac{\frac{1}{2} \binom{L-1}{M-1}}{2^L - \sum_{r=0}^{M-1} \binom{L}{r}} \quad (1)$$

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$$x_{2k+1} = px_k + px_{k+2L-1}$$

or

$$x_{2k} = qx_k + qx_{k+2L-1}$$

for  $k < 2^{L-1}$ . Here the index denotes the binary representation of the monitor's contents. For the type 1 states we have

$$x_{2j} = qx_j + qx_{j+2L-1}$$

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$$x_{2i+1} = x_{i+2L-1}$$

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state) transitions. The Markovian nature of the digital PCM monitor will be apparent upon the labeling of states and identifying the transition matrix. We will set aside, for the moment, the particulars of the initialization of the monitor since an initial state distribution would have to be specified. As we shall see presently, since the Markov chain describing the monitor function is irreducible, the specification of an initial distribution is irrelevant.

#### A. Description of Operation

The coding of incoming data  $a_j$  ( $a_j = 0, 1$ ) into transmitted data  $b_j$  proceeds by choosing  $b_j = a_j$  if  $W\{b_k\}_{j-L+1}^{j-1} \geq M$  or  $b_j = 1$  if  $W\{b_k\}_{j-L+1}^{j-1} = M - 1$  where  $W\{b_k\}_{j-L+1}^{j-1}$  is the number of ones (weight) in the previous  $L - 1$  bits. We note that since all blocks of length  $L$  in the transmitted stream contain at least  $M$  ones, there are exactly  $2^L - \sum_{r=0}^{M-1} \binom{L}{r}$  permissible words or states at the coder output. We can index each state by using the binary word it represents, e.g.,  $(0, 1, 1, \dots, 1)$  corresponds to the  $2^{L-1}$ -th state. We note that each state belongs to one of two categories.

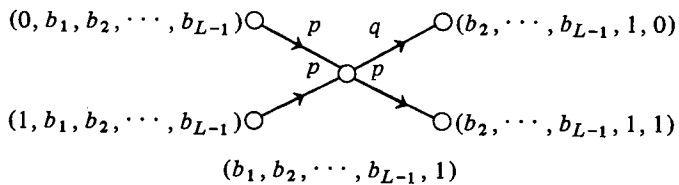
1) Those of the form  $b_{j-L+1} = 1$  and  $W\{b_k\}_{j-L+2}^j = M - 1$  (i.e., a one followed by  $M - 1$  ones in the next  $L - 1$  bits) in which an error can occur in the transition to the next state.

2) The rest of the states in which no error will occur in the transition to the next state.

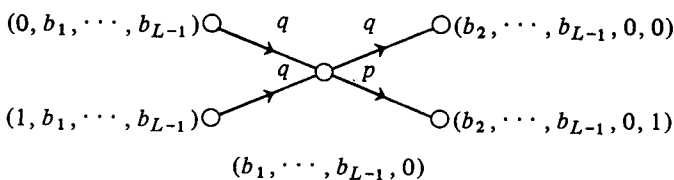
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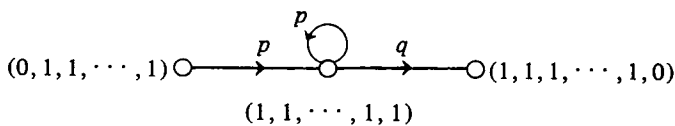
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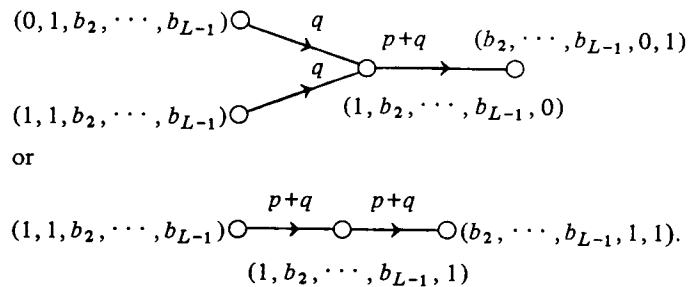
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for  $i, j < 2^{L-1}$  and  $W(2i + 1) = W(2j) = M$  where  $W(j)$  is the number of ones in the binary representation of  $j$ . Note that for  $k < 2^{L-1}$

$$W(2k + 1) = W(k + 2^{L-1}) = 1 + W(k) = 1 + W(2k). \quad (2)$$

Because of the symmetries involved in the state equations, we hypothesize that the solutions are of the form

$$x_k = y_{W(k)},$$

that is, all states with the same weight have the same probability of occurrence. Then, using (2), the state equations reduce to

$$y_{W(2k+1)}(1 - p) = qy_{W(2k+1)} = py_{W(k)} \quad (3)$$

$$y_{W(2k)}(1 - q) = py_{W(2k)} = qy_{W(2k+1)} \quad (4)$$

and since  $W(k) = W(2k)$ , it is apparent that the hypothesized form is indeed the solution.

There are exactly  $\binom{L}{N}$  states with  $N$  ones in  $L$  bits (weight  $N$ ), so we have

$$1 = \binom{L}{M} y_M + \binom{L}{M+1} y_{M+1} + \binom{L}{M+2} y_{M+2} + \dots + \binom{L}{L} y_L \quad (5)$$

since there are no states of weight less than  $M$ . From (3) or (4) and (2)

$$y_{N+1} = \frac{p}{q} y_N$$

and (5) becomes

$$p^M q^{L-M} = \left[ \binom{L}{M} q^{L-M} p^M + \binom{L}{M+1} q^{L-M-1} p^{M+1} + \dots + \binom{L}{L} p^L \right] y_M$$

or

$$y_M = \frac{p^M q^{L-M}}{1 - \sum_{r=0}^{M-1} \binom{L}{r} q^{L-r} p^r} \quad (6)$$

The error probability is then

$$p_e = \binom{L-1}{M-1} q y_M = \frac{\binom{L-1}{M-1} p^M q^{L-M+1}}{1 - \sum_{r=0}^{M-1} \binom{L}{r} q^{L-r} p^r} \quad (7)$$

We note briefly here that preliminary work, employing techniques associated with the strong law of large numbers, had realized fairly tight upper and lower bounds on the probability of error given by (7). The form of the bounds suggested the solution to the state equations for the Markov chain development reported here.

#### D. Some Calculations

It is obvious that (1) and (7) are identical for  $p = q = 1/2$ . For some cases of potential interest in PCM channels  $p = q = 1/2, 1 \gg \sum_{r=0}^{M-1} \binom{L}{r} q^{L-r} p^r$  and  $p_e \approx \binom{L-1}{M-1} / 2^{L+1}$

$$L = 16, M = 2: p_e \approx 15 \times 2^{-17} = 1.1 \times 10^{-4}$$

$$L = 32, M = 4: p_e \approx 4495 \times 2^{-33} = 5.2 \times 10^{-7}$$

$$L = 48, M = 6: p_e \approx 1.53 \times 10^6 \times 2^{-49} = 2.7 \times 10^{-9}$$

$$L = 64, M = 8: p_e \approx 5.53 \times 10^8 \times 2^{-65} = 1.5 \times 10^{-11}$$

The results are quite sensitive to  $q$ . For example, with  $q = 0.6$  and  $p = 0.4$ ,

$$L = 16, M = 2: p_e \approx 1.13 \times 10^{-3}$$

$$L = 16, M = 8: p_e \approx 8.1 \times 10^{-8}$$

The error probability in the  $L = 64, M = 8$  case is degraded by a factor of  $5.4 \times 10^3$  if  $q$  is in fact 0.6 instead of 0.5.

Interest in error pairs comes from the fact that there are PCM line usages where an occurrence of a pair of consecutive errors is essentially no more degrading to performance than a singleton error. We note that the only states which can lead to consecutive errors are those of the form

$$(1, 1, M - 2 \text{ ones in the next } L - 2 \text{ bits}).$$

Thus the probability of two consecutive errors is

$$p_{2e} = \binom{L-2}{M-2} q^2 y_M = \frac{(M-1)q}{(L-1)} p_e,$$

which usually amounts to a negligible fraction of the total error rate  $p_e$ .

### III. ANALOG MONITORING

We now give a mathematical development of the PCM line derived voltage process. The monitor uses this voltage to track what the repeater tank voltage would be if left unmonitored. Then we shall analyze this line derived voltage process to bound the percentage of time it spends below a floor level  $T$  expressed as a (small) percentage of its peak (all ones) level. The monitor pre-empts information bits with a dotting pattern<sup>3</sup> for the duration of the time for which consecutive voltage samples are below  $T$ . This pre-emption clearly results in the insertion of errors.

#### A. Error Rate Bound

Consider the random pulse train  $v(t)$  obtained by setting the voltage level on each interval  $(n, n + 1)$  to 1 or 0, depending on whether an independent flip of a fair coin results in a head or a tail, respectively. (For simplicity we have scaled the time slots to unity.) Apply the signal  $v(t)$  across a series resistor-capacitor connection and let  $V_c(t)$  denote the voltage across the capacitor. Let  $\tau$  denote the circuit time constant and set  $r = e^{-\tau^{-1}}$ . Sampling the capacitor voltage at the integers yields  $\{V_c(n)\}_{-\infty}^{+\infty}$ . Consider the problem of determining the percentage of samples for which the sampled output is below a threshold  $T$ . Letting  $\chi_{r,T}(x) = \begin{cases} 0 & x \geq T \\ 1 & x < T \end{cases}$  we pose the problem as seeking

$$\lim_{N \rightarrow \infty} \frac{1}{2N+1} \left\{ \sum_{-N}^{+N} \chi_{r,T}(V_c(n)) \right\} \quad (8)$$

To establish the existence of this limit notice

$$\{V_c(n) = (1 - r)(a_n + a_{n-1}r + a_{n-2}r^2 + \dots)\}_{-\infty}^{+\infty}$$

where  $\{a_n\}_{-\infty}^{+\infty}$  are independent and identically distributed Bernoulli variates with 0 and 1 equiprobable. The stationarity of  $\{V_c(n)\}_{-\infty}^{+\infty}$  is apparent. Now letting  $E$  stand for expectation, we have

$$E\{V_c(l) V_c(m)\} = \frac{(1-r)}{1+r} r^{|l-m|} \rightarrow 0 \quad (|l-m| \rightarrow \infty).$$

<sup>3</sup> A dotting pattern is preferred to an all ones sequence because in practice the dotting pattern corresponds to a spectral power distribution which is superior from a crosstalk standpoint.

Thus we meet the hypothesis of a theorem in [2, p. 382] which is a variant of the Birkhoff-Khinchine ergodic theorem. We find (8) exists and equals  $E(\chi_{r,T}(V_c(0)))$  with probability one.

Now  $E\{\chi_{r,T}(V_c(0))\}$  is merely the distribution function  $F(r, T)$  of  $V_c(0)$ . Clearly, this distribution function vanishes for  $T < 0$  and is 1 for  $T > 1$ . The determination of the distribution function  $F(r, T)$  of such a random geometric series is a famous unsolved problem in probability theory [3]. However,  $F(2^{-k^{-1}}, T)$  can be easily found for  $T \leq (2^{k^{-1}} - 1)$  for any positive integer  $k$ . To accomplish this, rearrange the absolutely convergent series for  $V_c(0)$  to get

$$V_0(0) = (1 - 2^{-k^{-1}}) \{ (a_0 + a_{-k} 2^{-1} + a_{-2k} 2^{-2} + \dots) + 2^{-k^{-1}} (a_1 + a_{-k-1} 2^{-1} + a_{-2k-2} 2^{-2} + \dots) + \dots + 2^{-(k-1)k^{-1}} (a_{-k+1} + a_{-2k+1} 2^{-2} + \dots) \}.$$

The random sums on each of the lines above are independent, and the  $i$ th sum is uniformly distributed on

$$[0, (1 - 2^{-k^{-1}}) \times 2^{1-ik^{-1}}] \quad [0 \leq i \leq k - 1].$$

So the distribution function of  $V_c(0)$  is seen to be the resultant of a  $k$ -fold convolution of uniforms. Observe that on  $[0, 2^{k^{-1}} - 1]$  the constancy of the uniforms gives that the repeated convolution reduces to the repeated integration

$$\left(\frac{1}{1 - 2^{-k^{-1}}}\right)^k \left(2 - \frac{1}{k} \int_0^x dx_{k-2}\right) \dots \left(2^{2k^{-1}-1} \int_0^{x_2} dx_1\right) \cdot \left(2^{k^{-1}-1} \int_0^{x_1} dx_0\right) 2^{-1}.$$

Hence

$$F(2^{-k^{-1}}, T) = \frac{1}{2^{(k+1)/2} (1 - 2^{-k^{-1}})^k} \frac{T^k}{k!} \quad (9)$$

Since (9) would not be changed if the return to zero point of each of the convolutants were increased by any positive number, we have that the right-hand side of (9) serves as an upper bound on  $F(2^{-k^{-1}}, T)$  for all  $T > 0$ .

**B. Example**

Consider the following hypothetical situation. For a PCM system with a bit rate of 2 mb/s and with a nominal error rate of  $10^{-6}$ , a monitor enforcing a 7 percent floor on the line-derived signal is to be examined. The time constant is nominally  $100/\pi$ ; however, a  $\pm 23$  percent variability is anticipated. The impact of the enforcement of this specification is by no means clear. While understanding the necessity for corrective monitoring of prohibited bit streams stemming from irregularities such as disconnects, concern has been expressed over the possibility of a deleterious effect on error rate during normal information transmission. However, it was felt that in the hypothetical situation outlined above, the threshold specification would not degrade the error rate significantly.

Employing (9) we can clarify the situation and defuse the issue concerning the corrective level monitoring by demonstrating that the monitor's contribution to the nominal line error rate is insignificant.

Transforming  $r$  to  $k$  and allowing for a 23 percent sensitivity analysis, the interval  $17 < k < 27$  is of interest. By Stirling's approximation, the right-hand side of (9) increases as  $k$  decreases for  $0 \leq T \leq 0.07$ . So

$$\frac{(0.07)^{17}}{2^9 (1 - 2^{-1/17})^{17} 17!} < 10^{-13}$$

is an upper bound on the probability of error for even the smallest of time constants of the form  $2^{-1/k}$  occurring in practice.

For completeness we next consider those values of  $r$  in the interval of interest which are not of the form  $2^{-1/k}$ . For convenience we unnormalize, seeking to upper-bound the 7 percent (of maximum) point of the distribution of  $V_c(0)/(1 - r)$ . Using primes to denote the unnormalized distribution functions, our objective is to upper-bound  $F'(r, 0.07 \cdot (1/(1 - k)))$  for  $r \in [2^{-1/17}, 2^{-1/27}]$ . Notice that for each sequence of outcomes,  $\{a_n\}_0^\infty V_c(0)/1 - r$  is a monotone increasing function of  $r$ . So for each real  $x$

$$F'(r^{**}, x) \leq F'(r^*, x) \quad (r^* < r^{**}).$$

Since  $F'$  is a distribution function

$$F'\left(r^*, 0.07 \frac{1}{1 - r^*}\right) < F'\left(r^*, 0.07 \frac{1}{1 - r^{**}}\right) \quad (r^* < r^{**}).$$

Combining the above two inequalities, we have that for

$$r \in [2^{-(1/k)}, 2^{-(1/k+1)}], \quad F'\left(r, 0.07 \frac{1}{1 - r}\right) < F'\left(2^{-(1/k)}, 0.07 \frac{1}{1 - 2^{-(1/k+1)}}\right). \quad (10)$$

Now we can revert to normalized form and consider the 7 percent point of  $V_c(0)$  directly (rather than  $V_c(0)/(1 - r)$ ). Normalizing  $V_c(0)/1 - 2^{-1/k}$ ,  $[0, 1/(1 - 2^{-1/k})] \rightarrow [0, 1]$ ; thus  $0.7 \cdot (1/(1 - 2^{-(1/k+1)})) \rightarrow 0.07 (1 - 2^{-(1/k)})/(1 - 2^{-(1/k+1)})$  and so corresponding to

$$F'\left(2^{-(1/k)}, 0.07 \frac{1}{1 - 2^{-(1/k+1)}}\right)$$

we have

$$F\left(2^{-(1/k)}, 0.07 \cdot \frac{1 - 2^{-(1/k)}}{1 - 2^{-(1/k+1)}}\right).$$

Substituting  $T = 0.07 \cdot (1 - 2^{-(1/k)})/(1 - 2^{-(1/k+1)})$  into (9) we get

$$F(r, 0.07) < \frac{(0.07)^k}{2^{(k+1/2)} (1 - 2^{-(1/k+1)})^k k!}, \quad 2^{-(1/k)} < r < 2^{-(1/k+1)}.$$

Numerical work shows  $F(r, 0.07) < 10^{-12}$  for  $2^{-(1/17)} < r < 2^{-(1/27)}$ .

Indeed the corrective level monitoring of a maxentropic source contributes insignificantly to the nominal line error rate.

**ACKNOWLEDGMENT**

The authors are grateful to R. D. Fracassi and R. C. Gifford for alerting them to the matters investigated here and for several enlightening discussions. Thanks are due to J. Salz for suggesting the Markov chain approach for analyzing the digital implementation of the monitor. Concerning the analog monitor, the authors have learned that S. O. Rice (in an unpublished 1957 document) pioneered the application of the

theory of random geometric series to estimate the distribution of the response of a resonant circuit to a random pulse train.

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