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An Analysis of the Monostable Cathode-Coupled Multivibrator

Joseph C. Connell

Connell — Monostable Cathode-Coupled Multivibrator

AN ANALYSIS OF THE MONOSTABLE CATHODE-COUPLED MULTIVIBRATOR

By

Joseph C. Connell

A Thesis

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The investigator wishes to acknowledge his indebtedness to Dr. Thomas L. Martin, Jr., of the University of New Mexico Electrical Engineering Department, whose lectures aroused an interest in the methods of analysis employed in this study. The work of Chapter II closely follows an analysis which appears in Dr. Martin's book, <u>Electronic Circuits</u>, to be published by Prentice-Hall, Inc., in 1954.

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CHAPTER I

A BRIEF HISTORY OF THE MULTIVIBRATOR

The multivibrator was first suggested in 1918 by Abraham and Block.[1] It would seem that a circuit of such ancient vintage should have been analyzed satisfactorily and completely long ago. This is not the case, however, for while a number of papers had appeared prior to 1940, it has only been during the past decade that satisfactory analyses appeared in the literature. Prior to this time the results given by various authors differed by factors of as much as two to one, and hence, were practically useless.[2] A probable reason for the lack of advancement during the first twenty years was the fact that there were few uses for such a circuit outside of the laboratory. The advent of World War II, with its concentration on the entire field of electronics and particularly on circuitry associated with radar, played a big part in the increased understanding of multivibrator operation. The commercial development of television has also contributed to recent interest in multivibrator analysis. Even though the present state of knowledge is fairly advanced, the design of variations of the basic multivibrator or the design of the basic multivibrator to produce certain characteristics is a problem which frequently demands ingenuity and patience;

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[1] Abraham, H., and Block, E., Ministére de la guerre Pub. 27, April, 1918.

[2] Richter, W., Fundamentals of Industrial Electronic Circuits (New York: McGraw-Hill Book Company, Inc., 1947), p. 397.

CHAPTER I

THE HISTORY OF THE MULTIVIBRATOR

The multivibrator was first suggested in 1918 by Abraham and Bloch.[1] It would seem that a circuit of [illegible] have been analyzed satisfactorily [illegible] the case, however, for [illegible] 1940, it has only been during the past decade that [illegible] appeared in the literature. Prior to this time the [illegible] given by various authors differed [illegible] were practically useless.[2] A probable reason for the lack of advancement during the first twenty years [illegible] uses for such a circuit outside of the laboratory [illegible] War II, with its concentration on the radar field [illegible] particularly on [illegible] the increased understanding of multivibrator operation. The commercial development of television has also contributed [illegible] multivibrator analysis. Even though the present [illegible] fairly advanced, the design of [illegible] or the design of the basic multivibrator to produce certain characteristics is a problem which frequently [illegible]

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[2] Richter, W., [illegible] (New York: McGraw-Hill Book Company, Inc., [illegible]), p. [illegible]

usually the method of trial and error is used in achieving a successful circuit.

I. THE MONOSTABLE CATHODE-COUPLED MULTIVIBRATOR

Present status of analysis. The monostable cathode-coupled multivibrator was used extensively in radar applications during World War II. A quite complete treatment of its general operation was used in the training of radar personnel for the Armed Forces and was later published.[3] This analysis is partly graphical and partly analytical, although few equations are given explicitly. Two recent publications give quite thorough analyses of multivibrator circuits.[4,5] The equations are derived by the use of equivalent circuits in both of these works. The first treatment does not work out the equations for the cathode-coupled case, but leaves this as an exercise for the reader; the second treatment includes several variations of the cathode-coupled multivibrator and examines each in some detail. Thus it is evident that several satisfactory analyses of the multivibrator circuit do exist at the present time.

Viewpoint of these analyses. Until quite recently, the schematic diagram of the multivibrator circuit was drawn as a two stage RC

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[3] M. I. T. Radar School Staff, Principles of Radar (New York: McGraw-Hill Book Company, Inc., 2nd. Ed., 1946), p. 2-53.

[4] Martin, Thomas L. Jr., Ultrahigh Frequency Engineering (New York: Prentice-Hall, Inc., 1950), p. 71.

[5] Seely, Samuel, Electron-tube Circuits (New York: McGraw-Hill Book Company, Inc., 1950), p. 416.

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I. THE MONOSTABLE CATHODE-COUPLED MULTIVIBRATOR

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[4] Martin, Thomas L. Jr., Electronic Circuits (New York: Prentice-Hall, Inc., 1955), p. [illegible].

[5] Seely, Samuel, Electron-tube Circuits (New York: McGraw-Hill Book Company, Inc., 1950), p. 414.

amplifier with capacitive coupling between the plate of the second tube and the grid of the first tube.[6] Despite this similarity to the RC amplifier, the various papers on multivibrators have treated them purely as trigger circuits and have made no attempt to show the multivibrator as a special case of the RC amplifier with degenerative feedback. Possibly such treatment is due to the phraseology used in one of the first treatises which analyzed the multivibrator in what has now become the standard fashion. The authors state that:

> . . . Because the wave forms encountered in these circuits are not sinusoidal, and because the grid-voltage variations are so great that the tube seldom acts as a linear amplifier, conventional methods of circuit analysis are of little value . . .[7]

It is true that by conventional methods the authors meant steady-state alternating current methods as opposed to transient analysis, yet the impression persists that the multivibrator is of a completely different nature than the RC amplifier. In practice, this viewpoint is of small importance since, save for the idea of pedagogical unity, it matters little how results are obtained if they are accurate.

Incompleteness of analyses. It is interesting to note that several of the analyses mentioned call attention to the fact that an important feature of the cathode-coupled multivibrator lies in the

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[6] Terman, Frederick E., Radio Engineering (New York: McGraw-Hill Book Company, Inc., 3rd. Ed. 1947), p. 588.

[7] Kiebert, Martin V., and Inglis, Andrew F., "Multivibrator Circuits," Proc. IRE, 33:534, August, 1945.

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[6] Terman, Frederick E., Radio Engineering (New York: McGraw-Hill Book Company, Inc., 3rd ed. 1947), p. 5[illegible].

[7] Kiebert, Martin V., and Inglis, Andrew F., "Multivibrator Circuits," Proc. IRE, 33:534, August, 1945.

nearly linear relationship, between pulse duration and grid voltage, which can be obtained by proper choice of plate and cathode resistors. Qualitative explanations of this phenomena have been given in the majority of published articles, although two treatments do justify this by means of analytical expressions.[8,9] The first of these does not extend the analysis to the relationships between the circuit parameters, but does give a circuit which was used extensively at the M. I. T. Radiation Laboratory. The duration of the pulse obtained from this circuit is a linear function of the applied grid voltage within $\pm$ 0.25 per cent over the range from about 8 to 150 microseconds. The second treatment gives an equation which is said to give values which are close to the ideal values. A comparison of the values calculated from this equation with the values used in the M. I. T. circuit shows a difference which is greater than this investigator would expect.

The magnitude of this difference, which is of the order of two to one, raises a question as to whether the theoretical treatment is completely satisfactory, or whether further analysis might reconcile the apparent deviation between theory and practice. At any rate, the present status of the theory does leave much to be desired insofar as the linearity phenomena is concerned.

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[8] Seely, op. cit., pp. 424-427.

[9] Glegg, Keith, "Cathode-Coupled Multivibrator Operation, Proc. IRE, 38:655, June, 1950.

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[8] Seely, op. cit., pp. [illegible].

[9] Clapp, Smith, [illegible], Proc. IRE, [illegible], June, 1950.

II. PURPOSE OF THESIS

The foregoing comments pointed out the facts that: 1) little has been done to show the similiarity between the multivibrator and the RC amplifier with degenerative feedback; 2) the existing treatment of the conditions under which a linear relationship between pulse duration and applied grid voltage will exist does not give results which are in very good agreement with the values used in a practical circuit. It is the purpose of this study to fill in these gaps by showing that the equations resulting from an analysis of the cathode-coupled multivibrator are, in general, those of the degenerative amplifier multiplied by an additional factor which compensates for the heavily biased operation, and that the conditions for a nearly linear relation between pulse duration and applied grid voltage depend upon a rather complicated function of the circuit parameters which does not allow a general solution, but which does allow values to be computed for any particular case.

The foregoing [illegible]

has been done to show the [illegible]

RC amplifier with de[illegible]

conditions under which a linear [illegible]

applied grid voltage will exist [illegible]

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CHAPTER II

ANALYSIS OF THE MONOSTABLE CATHODE-COUPLED MULTIVIBRATOR

I. GENERAL OPERATION

A circuit diagram of the monostable cathode-coupled multivibrator is shown in Figure 1. The symbols shown there are the ones which will be used throughout this study. It is seen from this figure that the grid of V_2 is connected to the supply voltage E_{bb} through a resistor R_g while the grid of V_1 is connected to a source of lower voltage E_{cc}. The higher grid voltage of V_2 makes it logical to suppose that if plate current flow is limited to one of the tubes it would flow in V_2. The question then arises as to whether plate current is flowing in V_1 during this time.

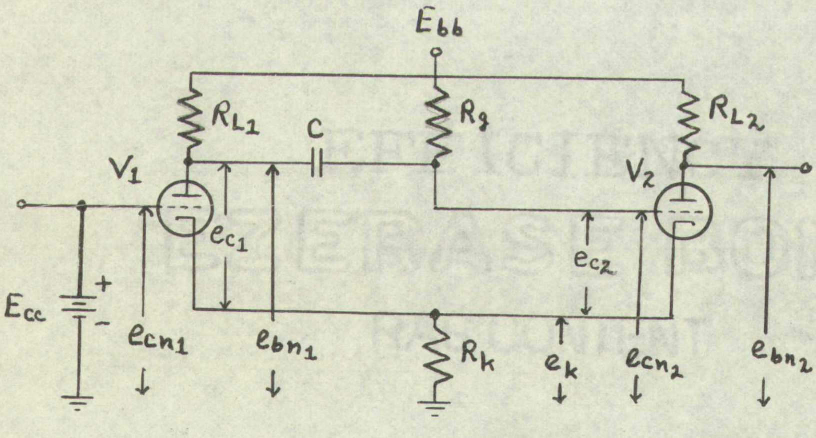

FIGURE I

CIRCUIT OF THE MONOSTABLE CATHODE-COUPLED MULTIVIBRATOR

Evidently the value of e_{c1} determines the answer to this question; if e_{c1} is greater than the cutoff voltage of V_1, plate current will flow. For operation as a monostable circuit, it is necessary for V_1 to be cutoff during the normal state. This gives a first condition for operation, that e_{c1} is less than the cutoff voltage e_{co1} of tube V_1, or

$$E_{cc} - E_k < e_{co1} \tag{1}$$

where e_{co1} is a negative quantity.

Assuming now that inequality (1) is satisfied, and that V_2 has been conducting a sufficient length of time for any transients to have virtually disappeared, we see that a positive pulse, of large enough amplitude to raise the grid of V_1 above cutoff, will, on being applied to V_1, cause plate current to flow. The resulting IR drop in R_{L1} will be transmitted through C to the grid of V_2 thus causing the grid voltage to decrease. The resulting decrease of total current in R_k causes the voltage E_k to decrease thus reversing inequality (1) and thereby aiding the transition. The circuit ends up in a quasi-stable or timing state with V_1 conducting steadily and with the grid of V_2 relatively far below its value of grid cutoff voltage. The capacitor C now discharges through the resistor R_g, thus allowing the grid voltage of V_2 to climb toward a less negative value along an exponential curve. When the cutoff value is reached, V_2 conducts, the voltage E_k increases, thus decreasing the grid voltage of V_1, and the reverse transition takes place. Following this switching action, the circuit is in the original stable condition. It is evident that, except for the short time during which switching is taking place, the multivibrator acts as

a one tube circuit since the non-conducting tube behaves like an open circuit. Thus, by neglecting the transition state which will not be considered in this investigation, the multivibrator can be thought of as being composed of two one-tube circuits which can be studied individually.

II. THE STABLE STATE

Since V_1 is cutoff and all transients have disappeared, the left side of the diagram of Figure 1 can be considered as an open circuit. It is then evident from the figure that

$$E_{bb} - E_k = i_{g_2}(\bar{r}_g + R_g) \tag{2}$$

where $\bar{r}_g$ is the static grid resistance of tube V_2.

The grid voltage e_{c_2} can be expressed as

$$e_{c_2} = (E_{bb} - E_k)\frac{\bar{r}_g}{R_g + \bar{r}_g} \tag{3}$$

which is approximately zero since $\bar{r}_g$ is about 1000 ohms and R_g is a megohm or more. We now replace the tube V_2 by its equivalent plate circuit thus obtaining the circuit of Figure 2. The tube has been represented as a generator of voltage μe_{c_2} in series with the dynamic plate resistance of the tube and a battery of voltage E_o.[1] Numerical values of these quantities can easily be found from the characteristic curves of the particular tube which is used in the multivibrator circuit.

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[1] Richter, Walther, Fundamentals of Industrial Electronic Circuits (New York: McGraw-Hill Book Company, Inc., 1947), pp. 70-77.

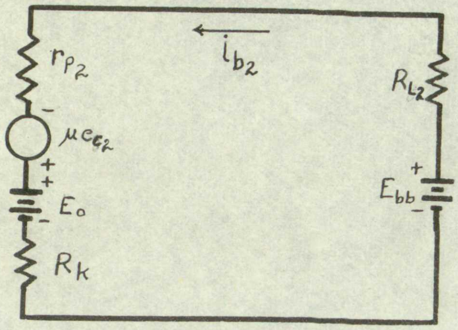

FIGURE 2

EQUIVALENT CIRCUIT FOR STABLE STATE

For triodes such as the 6J5 and 6SN7, representative values are r_p = 7,500 ohms and $E_o \cong 20$ volts, for zero grid voltage.

Since e_{c2} was found to be approximately zero, the current in the circuit of Figure 2 is readily computed to be

$$i_{b_2} = (E_{bb} - E_o)/(r_{p_2} + R_k + R_{L_2}). \tag{4}$$

From equation (4), the values which the various wave-forms have during the stable state can be computed readily. Reference to Figure 1 shows the values to be given by

$$E_k = (i_{b_2} + i_{g_2})R_k \simeq i_{b_2}R_k, \tag{5}$$

$$e_{bn_2} = E_{bb} - i_{b_2}R_{L_2}, \tag{6}$$

$$e_{cn_2} = E_k + i_{g_2}r_g \simeq E_k, \tag{7}$$

$$e_{bn_1} = E_{bb}. \tag{8}$$

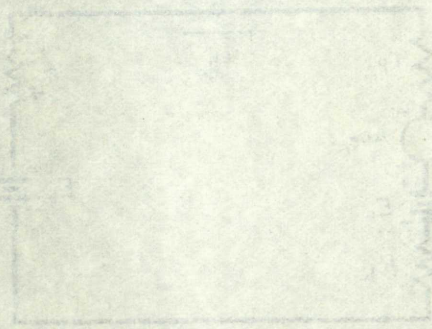

FIGURE 2

EQUIVALENT CIRCUIT FOR STEADY STATE

For triodes such as 6J5, 6C5 and 6J7, representative values are r_p = 7,500 ohms and μ = 20 [illegible].

Since e_g was found to be approximately zero, the current in the circuit of Figure 2 is readily computed to be

$$i_b = (\mu e_c - E_b)/(r_p + R_k + R_L) \quad (4)$$

From equation (4), [illegible] the [illegible] wave-forms have [illegible] [illegible] [illegible] [illegible]. Figure 1 [illegible]

$$[illegible] \quad (5)$$

$$[illegible] \quad (6)$$

$$[illegible] \quad (7)$$

$$[illegible] \quad (8)$$

The charge γ_1 on the condenser is given by

$$\gamma_1 = E_{bb} - e_{cn_2}. \qquad (9)$$

Calculation of the approximations of equations (5) and (7), which result from neglecting i_{g_2}, indicate that an error of one per cent or less is involved.

The analysis of the stable state is now complete, and we are ready to proceed with the second part of the analysis.

III. THE TIMING STATE

Since, in this state, V_2 is cutoff and V_1 is the conducting tube, the circuit of Figure 1 can be redrawn by replacing V_2 by an open circuit. When V_1 is replaced by its equivalent generator in series with the plate resistance and the constant voltage battery, the equivalent circuit will be that shown in Figure 3. A rearrangement of the circuit elements, and the use

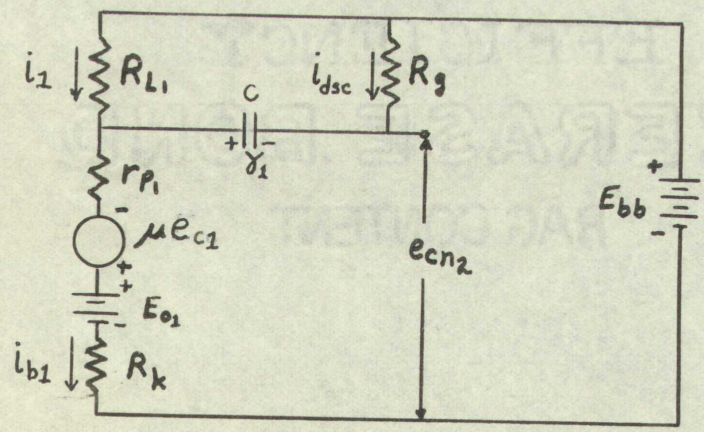

FIGURE 3

EQUIVALENT CIRCUIT FOR THE TIMING STATE

of a second plate supply battery of voltage E_{bb} transforms the circuit to that of Figure 4. The use of an additional battery allows the circuit to be redrawn in such a way that the method of attack becomes almost self-evident. It is legitimate since the internal impedance of the power supply is negligible and, hence, the voltage seen by either the R_g or the R_{L_1} branch is independent of the current flowing in the other branch. The equivalent generator has a voltage μe_{c_1} which, because of the cathode resistor R_k, is dependent on the tube current i_{b_1}. As shown in Figure 3, this current is the sum of i_1 and the capacitor discharge current i_{dsc} since it is assumed that no grid current flows. An inspection of Figure 1 shows that

$$E_{cc} = e_{c_1} + i_{b_1} R_k \tag{10}$$

and consequently

$$e_{c_1} = E_{cc} - i_{b_1} R_k \tag{11}$$

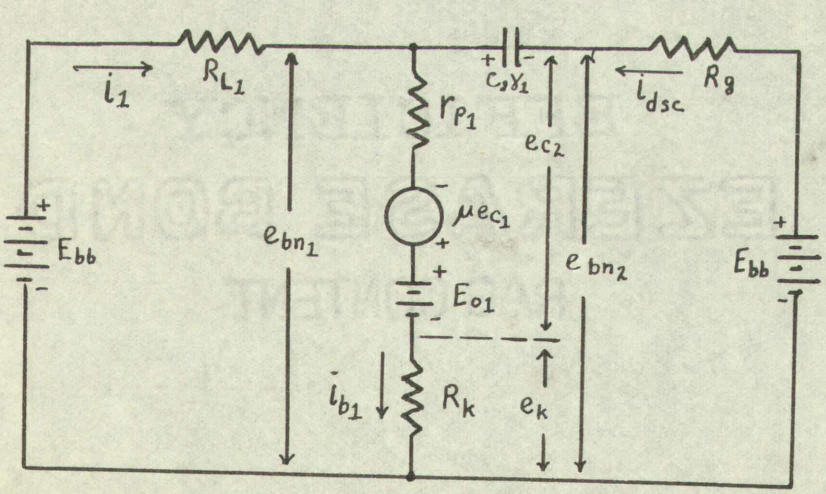

FIGURE 4

ALTERNATE FORM OF EQUIVALENT CIRCUIT

The voltage $\mu e_{c_1} = \mu E_{cc} - i_{b_1}\mu R_k$ contains a term which is similiar to the voltage drop in the cathode resistor. It is evident that exactly the same results will be obtained by dropping this term from the equivalent generator and adding a resistance μR_k to the cathode resistor. Doing this and redrawing the circuit gives a final form of the equivalent circuit as shown in Figure 5.

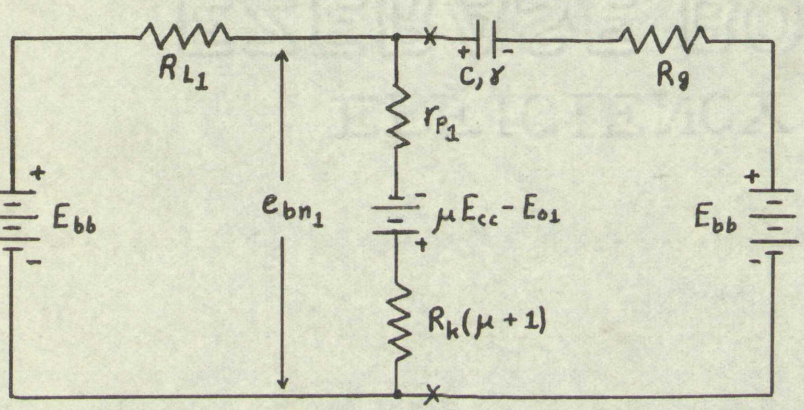

FIGURE 5

FINAL FORM OF EQUIVALENT CIRCUIT

The equivalent generator, being a constant voltage, has been replaced by a battery which also includes the voltage E_{o_1}. By applying Thevenin's theorem to the circuit to the left of the points marked by x's, we obtain the simple series circuit given in Figure 6, the voltage E_e and the resistance R_e being

$$E_e = E_{bb} - \frac{E_{bb} + \mu E_{cc} - E_{o_1}}{R_{L_1} + r_{p_1} + R_k(\mu + 1)} R_{L_1}, \tag{12}$$

$$R_e = \frac{R_{L_1}\left[r_{p_1} + (\mu + 1)R_k\right]}{R_{L_1} + r_{p_1} + R_k(\mu + 1)}.$$

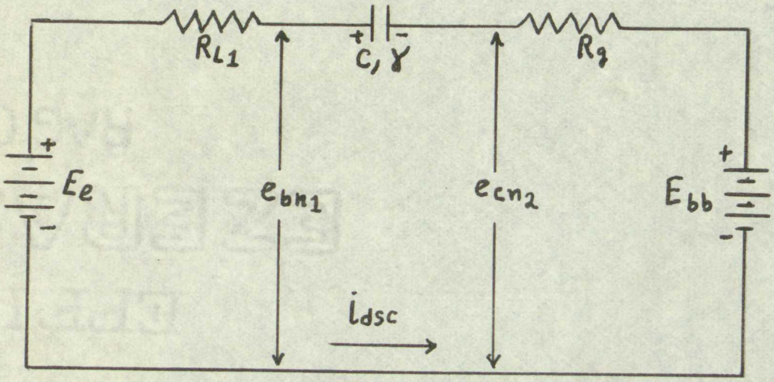

FIGURE 6

THEVENIN FORM OF CIRCUIT

Equation (12) can be simplified somewhat by assuming that E_{o_1} is much smaller than $(E_{bb} - E_{cc})$, a fair approximation for a triode as used in this circuit. By manipulation of the third term of equation (12), we obtain

$$\frac{(E_{bb}/r_{p_1} + \mu E_{cc}/r_{p_1})(R_{L_1} r_{p_1})/(R_{L_1} + r_{p_1})}{1 + R_k(\mu+1)/(R_{L_1} + r_{p_1})} = \frac{(E_{bb}\mu/\mu r_{p_1} + E_{cc}\mu/r_{p_1})(R_{L_1} r_{p_1})/(R_{L_1} + r_{p_1})}{1 + R_k(\mu+1)/(R_{L_1} + r_{p_1})}$$

which, since $\mu/r_p = g_m$, can be written as

$$\frac{(E_{bb}/\mu + E_{cc})(g_m R_{L_1} r_{p_1})/(R_{L_1} + r_{p_1})}{1 + R_k(\mu + 1)/(R_{L_1} + r_{p_1})}. \qquad (14)$$

It will be remembered that, when R_g is large in comparison with R_L and r_p, the expression

$$\frac{g_m R_L r_p}{R_L + r_p}$$

is the mid-frequency, or real, gain of the RC coupled amplifier and is denoted by G_r. Division of G_r by the quantity $[1 + R_k(\mu + 1)/(R_L + r_p)]$ gives the real gain G_r' for the case where cathode degeneration is present. Making these substitutions in equation (14), we obtain

$$(E_{bb}/\mu + E_{cc})G_r' \qquad (15)$$

and hence

$$E_e = E_{bb} - (E_{bb}/\mu + E_{cc})G_r'. \qquad (16)$$

An examination of Figure 6 shows that

$$e_{bn_1} = E_e - i_{dsc}R_e \qquad (17)$$

and that

$$i_{dsc} = \frac{E_{bb} + \gamma_1 - E_k - \left[E_{bb} - (E_{bb}/\mu + E_{cc})G_r'\right]}{R_e + R_g}\, e^{-\frac{t}{(R_e + R_g)}} \qquad (18)$$

where the value of γ_1 is given by equation (9). On inserting this value in equation (18) and simplifying, we find that at the instant of switch-over, i.e., at the time t = 0, which will be denoted by writing the value as a function of zero, $i_{dsc}(0)$ is

$$i_{dsc}(0) = \frac{E_{bb} - E_k + (E_{bb}/\mu + E_{cc})G_r'}{R_e + R_g}. \qquad (19)$$

On substituting equations (12) and (19) into (17), there results

$$e_{bn_1} = E_{bb} - \left(\frac{E_{bb}}{\mu} + E_{cc}\right)G_r' + \frac{\left[E_{bb} - E_k + \left(\frac{E_{bb}}{\mu} + E_{cc}\right)G_r'\right]R_e}{R_e + R_g}$$

$$= E_{bb}\left[1 + \frac{R_e}{R_e + R_g}\right] - \left[\left(\frac{E_{bb}}{\mu} + E_{cc}\right)G_r'\right]\left[1 - \frac{R_e}{R_e + R_g}\right] - \frac{E_k R_e}{R_e + R_g}. \qquad (20)$$

Since R_e is much smaller than R_g we can assume that R_e/R_g and $R_e/(R_e + R_g)$ are very much less than 1 and hence can be neglected in equation (20). Using this approximation, we find that

$$e_{bn_1}(0) = E_{bb} - (E_{bb}/\mu + E_{cc})G_r', \tag{21}$$

and from Figure 6 it is readily seen that

$$\begin{aligned} e_{cn_2}(0) &= e_{bn_1}(0) - \gamma_1 \\ &= E_k - (E_{bb}/\mu + E_{cc})G_r'. \end{aligned} \tag{22}$$

From Figure 5 we find an alternate expression for $e_{bn_1}(0)$

$$e_{bn_1}(0) = E_{bb} - i_1 R_{L_1}. \tag{23}$$

Combining these two equations and solving for i_1 gives

$$i_1 = (E_{bb}/\mu + E_{cc})G_r'/R_{L_1}. \tag{24}$$

As noted above, i_{b_1} is the sum of i_1 and i_{dsc}; however, i_{dsc} is limited by R_g and so has a value which is negligible in comparison with i_1. Hence, to a good approximation $i_{b_1} = i_1$ and, from Figure 4,

$$e_k(0) = i_1 R_k = (E_{bb}/\mu + E_{cc})G_r' R_k/R_{L_1}. \tag{25}$$

It should be noted that, since i_{dsc} is negligible, e_k will not vary with time but remains constant at the value given by equation (25). Referring to Figure 1, we see that

$$e_{c_1} = E_{cc} - e_k. \tag{26}$$

This is also a constant voltage. Thus, for all practical purposes, the only change which takes place during the timing cycle will be in the voltage across the capacitor. It is this voltage which is holding the grid of V_2 below cutoff and as this voltage decreases exponentially the grid voltage of V_2 will rise until it reaches its cutoff value.

At this time V_2 begins to conduct and the timing cycle is completed.

The value of cutoff voltage, E_{co_2}, can be found from the tube characteristic curves using the value of plate voltage on V_2. This voltage is

$$e_{b_2} = E_{bb} - e_k. \tag{27}$$

Since, in Figure 4

$$e_{cn_2} = e_{c_2} + e_k \tag{28}$$

we have the initial value of grid voltage on V_2 as

$$e_{c_2} = e_{cn_2} - e_k \tag{29}$$

where e_{cn_2} is given by equation (7).

The initial values of all waveform points have now been given in equations (21), (22), (25), (26), and (28). Of these values only e_{c_1} and e_{cn_1} change in value during the timing cycle so all others can be sketched for the entire timing cycle from their initial values.

The pulse duration, T, will evidently be the time required for the grid of V_2 to rise from the voltage e_{c_2}, with which the timing cycle starts, to the cutoff voltage, E_{co_2}. Referring to Figure 4, we see that

$$e_{c_2} = E_{bb} - \left[i_{dsc}(0)R_g\right] e^{-\frac{t}{RC}} - e_k$$

or

$$e_{c_2} = E_{bb} - e_k - \left[E_{bb} - E_k + (E_{bb}/ + E_{cc})G_r'\right] e^{-\frac{t}{RC}} \tag{30}$$

where $R = (r_{p_1} + R_k + R_g)$ which, since R_g is much greater than $(r_{p_1} + R_k)$, is very nearly equal to R_g. Using this approximation and substituting the value of cutoff voltage, E_{co_2}, in the preceding equation, we solve

for T obtaining

$$T = R_g C \ln \frac{E_{bb} - E_k + (E_{bb}/\mu + E_{cc})G_r'}{E_{bb} - E_{co_2} - e_k}. \tag{31}$$

The value of T is readily computed since all of these values are known or have been derived previously.

So far this analysis has been based upon the fulfillment of two conditions as follows: 1) that V_1 was cutoff during the stable state, and 2) that grid current does not flow in V_1 during the timing state. These conditions can be expressed by the inequalities

$$E_{cc} - E_k < e_{co_1} \tag{32}$$

where e_{co_1} is a negative quantity and

$$E_{cc} < i_{b_1}R_k = (E_{bb}/\mu + E_{cc})G_r'R_k/R_{L_1} . \tag{33}$$

Since e_{co_1} is approximately equal in magnitude to $(E_{bb} - E_k)/\mu$, inequality (32) can be written in expanded form by using the previously determined expression for E_k as

$$E_{cc} < E_{bb}R_k/R_2 \left[1 + \frac{1}{\mu} - \frac{E_o}{E_{bb}} - \frac{E_o}{E_{bb}\mu} - \frac{R_2}{R_k\mu}\right]. \tag{34}$$

The terms $1/\mu$ and E_o/E_{bb} being of the same order and of opposite sign effectively cancel each other; the fourth term in the brackets is small in comparison with the remaining terms and can be neglected. A good approximation for equation (34) is

$$E_{cc} < E_{bb}R_k/R_2 \left[1 - R_2/\mu R_k\right] \tag{35}$$

which can be expressed as an equality by

$$E_{cc} = kE_{bb}R_k/R_2, \quad k < 1 - R_2/\mu R_k. \tag{36}$$

In all the above equations R_2 denotes $(R_{L_2} + r_{p_2} + R_k)$.

Inserting this value of E_{cc} in inequality (33) and solving for R_{L_1}, we obtain, after some manipulation,

$$R_{L_1} < \frac{R_{L_2} + r_{p_2} + R_k(1 - k) - kr_{p_1}}{k}. \qquad (37)$$

Equations (36) and (37) are the basic equations for the design of a cathode-coupled multivibrator, once the choice of tubes and plate supply voltage has been made. Considerable latitude in the choice of R_k, R_{L_2}, and R_{L_1} is possible so that some control of the magnitude of the pulses is available. The value of R_gC can easily be determined for a desired pulse duration from equation (31), keeping in mind that the value of R_g should be one megohm or more.

The plate voltage of V_1 does not reach the plate supply value when V_1 is cutoff since the capacitor charging current flows through R_{L_1} giving an exponential shape to the rise of plate voltage. The equivalent circuit for this period is shown in Figure 7.

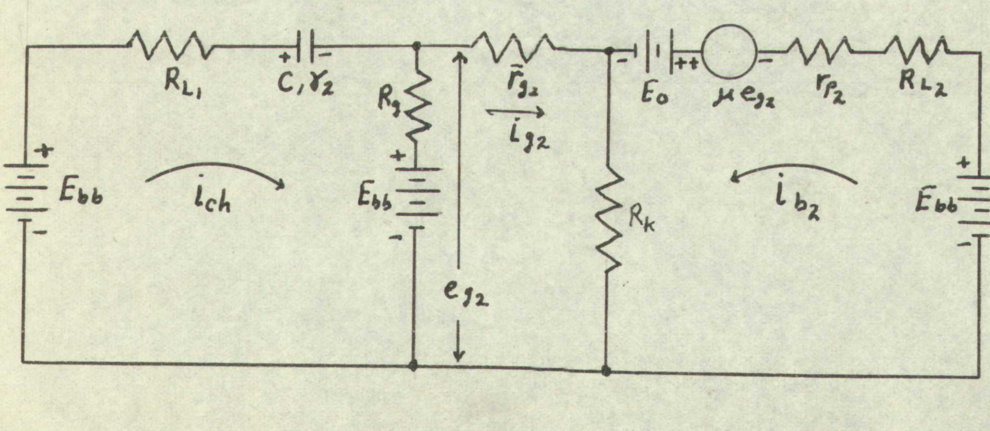

FIGURE 7

EQUIVALENT CIRCUIT IMMEDIATELY

AFTER TIMING CYCLE

Since R_g is of a much greater value than any other resistor or combination of resistors in the circuit and since E_o, for positive grid voltages, is very nearly zero, the circuit may be simplified with little loss of accuracy by neglecting these parameters. Figure 8 shows the simplified circuit.

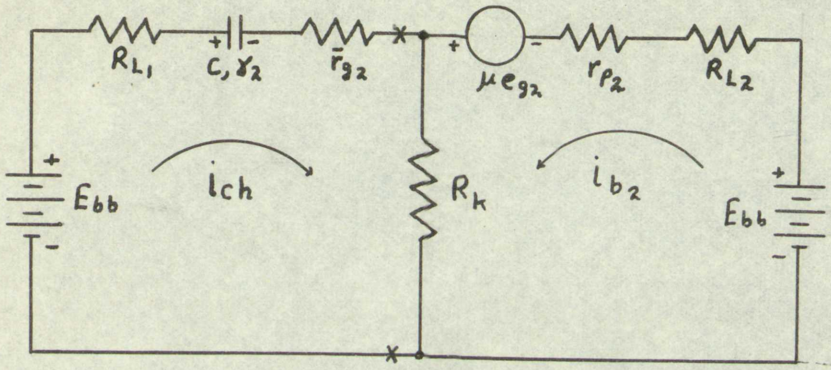

FIGURE 8

SIMPLIFIED CIRCUIT AFTER TIMING CYCLE

Applying Thevenin's theorem to the circuit to the right of the points marked by x's, we obtain the circuit indicated in Figure 9 where

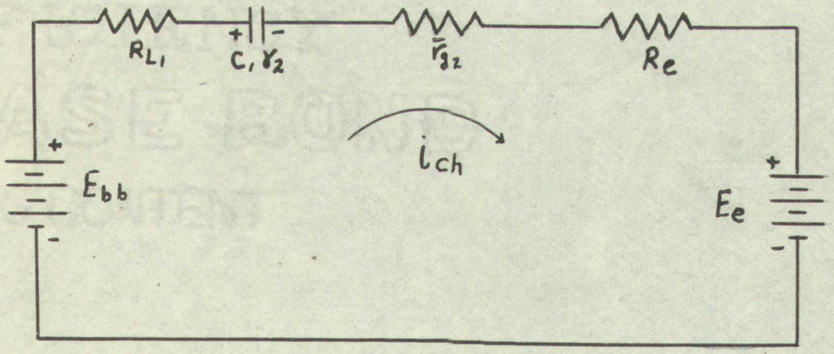

FIGURE 9

THEVENIN FORM OF FIGURE 8

the values of R_e and E_e are given by

$$R_e = R_k(r_{p_2} + R_{L_2})/(R_k + r_{p_2} + R_{L_2}) \tag{38}$$

$$E_e = (E_{bb} + \mu e_g)R_k/(R_k + r_{p_2} + R_{L_2}). \tag{39}$$

Solving the circuit of Figure 9 for the current gives

$$i_{ch} = (E_{bb} - E_e - \gamma_2)/(R_{L_1} + \bar{r}_{g_2} + R_e) \tag{40}$$

where $R_t = (R_{L_1} + \bar{r}_{g_2} + R_e)$. If we now let R_1 represent $(R_k + r_{p_2} + R_{L_2})$ and substitute equations (38) and (39) in equation (40), remembering that $e_g = i_{ch}\bar{r}_{g_2}$, we obtain, after some manipulation

$$i_{ch} = \frac{\frac{1}{R_t}\left[E_{bb}(1 - R_k/R_1) - \gamma_2\right] e^{-\frac{t}{R_tC}}}{1 - (\mu \bar{r}_{g_2} R_k/R_1R_t)\, e^{-\frac{t}{R_tC}}}\ . \tag{41}$$

We consider the zero time reference to be the instant the grid voltage of V_2 reaches cutoff and immediately jumps to a positive value. Since the switching transient takes a small, but finite, time for completion, this assumption while adequate for our purpose, is not physically exact. The value of i_{ch} for $t = 0$ reduces to

$$i_{ch}(0) = \frac{E_{bb}(1 - R_k/R_1) - \gamma_2}{R_t + \mu\bar{r}_{g_2}R_k}. \tag{42}$$

The maximum value of $e_{g_2}(0)$ is given by

$$e_{g_2}(0) = i_{ch}(0)\bar{r}_{g_2} \tag{43}$$

The corresponding value of plate current, $i_{b_2}(0)$, could be determined by inserting the correct values in equation (4). However, since the value of r_{p_2} must be obtained from the characteristic curves, it is

simpler to read the current value from the intersection of the $(R_k + R_{L_2})$ load-line and the $e_{g_2}(0)$ line. The $t = 0$ values of plate and cathode voltage of V_2 are given by

$$e_{bn_2}(0) = E_{bb} - i_{b_2}(0)R_{L_2}, \tag{44}$$

$$E_k(0) = \left[i_{ch}(0) + i_{b_2}(0)\right] R_k. \tag{45}$$

As would be expected, these equations reduce to equations (5) and (6) as t becomes indefinitely large. It should be noticed, from equation (41), that i_{ch} does not decrease as a pure exponential. The initial decay is faster than the pure exponential decay although, since $(\mu \bar{r}_{g2} R_k)/(R_1 R_t) < 1$, the difference is slight after $2R_tC$ seconds. Hence, for sketching the waveform, we may use an exponential curve of time constant R_tC without appreciable error.

After the charging current has decreased to zero, the circuit remains in the steady-state condition until the next trigger pulse initiates the cycle of operation.

IV. THE COMPLETE CYCLE

The two preceding sections have furnished the values needed to make a sketch of the waveforms for a complete cycle of operation of the monostable cathode-coupled multivibrator. For convenience, a tabulation of equation numbers and short comments are given in Table I. It remains only to sketch the stable state values on the time axis, following this with the timing state values. The comments of Table I give further information.

TABLE I

EQUATIONS GIVING WAVEFORM VALUES

WAVEFORM	EQUATION NUMBER		COMMENTS
	Stable state	Timing* state	
e_{bn_1}	(8)	(21)	Constant voltage for each state.
e_{bn_2}	(6)	(-)	Constant voltage for stable state. E_{bb} for timing state.
e_{cn_1}	(-)	(-)	Constant voltage of E_{cc} except for trigger pulses.
e_{cn_2}	(7)	(22)	Constant voltage for stable state. Exponential rise for timing state.
e_k	(5)	(25)	Constant voltage for stable state. Exponential rise for timing state

* After the timing state is completed, the trailing edge of the e_{bn_1}, e_{bn_2}, e_{cn_2}, and e_k waveforms have exponential shapes for a short time before the stable state values are reached. These may be approximated from equations (42) through (45) by using R_tC as the time constant.

V. COMPARISON OF RESULTS WITH RESULTS FROM ANOTHER ANALYSIS

The preceding analysis employed a number of approximations, and neglected terms in equations when these terms were small in comparison with the remaining terms. The justification for so doing lies in the fact that the difference between exact and approximate values is comparatively small while the savings in mathematical labor is usually considerable. The errors arising from such a treatment are of the order of a few per cent and thus are too small to be of any practical importance. Since the D.C. voltage term E_o, which has usually been neglected in previous analyses, has been included in the calculations, it would not seem too surprising if the results obtained from this analysis were not far different from the results obtained by using graphical methods and the tube characteristic curves. A comparison of the results given by these methods is shown in Table II; The circuit of Principles of Radar, M.I.T. Radar School Staff, 2nd Ed. page 2-58, is used as a standard. Values of the tube parameters, as obtained from the published values and characteristic curve are: g_m = 2600 micromhos, r_{p_1} = 7700 ohms, r_{p_2} = 7500 ohms. The percentage difference in results obtained by the two methods is also given.

An examination of the percentage differences given in Table II shows that results obtained from the two methods of analysis are quite comparable and well within slide rule accuracy. The largest difference, that of e_{cn_2}, is due mainly to the method of computation—taking the difference between two numbers of the same magnitude—but the accuracy

TABLE II

COMPARISON OF RESULTS WITH RESULTS OF ANOTHER METHOD

Item	This Analysis	M.I.T. Analysis	% Difference
		Stable State	
e_{bn_1}	300	300	0
e_{bn_2}	199	198.5	0.25
e_{cn_1}	70	70	0
e_{cn_2}	102	101.5	0.5
e_k	102	101.5	0.5
		Timing State	
e_{bn_1}	156.8	158.4	1.0
e_{bn_2}	300	300	0
e_{cn_1}	70	70	0
e_{cn_2}	-41.2	-40.1	2.5
e_k	71.6	70.8	1.1
T	68.1	67.5	0.9

is sufficient for most practical cases. This comparison gives a check on the accuracy of this method of analysis, and gives confidence that the approximations used will not greatly distort the resulting values.

VI. DESIGN CONSIDERATIONS

While the basic design equations were given on pages 16 and 17 there remain questions as to the order of magnitude of the remaining parameters. Several of the following considerations are of a general nature and are intended to appear reasonable but not to constitute a proof.

It will be remembered that, for a triode, the value of the amplification factor falls off at the lower values of plate current, and that in this region g_m and r_p change quite rapidly. Thus, there is a lower limit to the value of plate current which permits operation in the region of fairly constant r_p. For the triodes normally used this would be a plate current of about 10 milliamperes. A fairly standard practice is to choose R_{L_2} approximately equal to R_k. Assuming this to be the case, a rough center value for R_{L_2} and R_k can be found from the equation

$$R_{L_2} = R_k = E_{bb}/2i_{b_2} - r_p/2, \qquad (45)$$

which, for a plate current of 10 milliamperes becomes

$$R_{L_2} = 50E_{bb} - r_p/2.$$

A similiar range of values for these resistors might have been computed from a consideration of the circuit as a wide band amplifier. A wide latitude of choice is available as to the values used.

The variation of g_m and r_p with the operating point of the tube may cause some uncertainty as to which values of these parameters to use in the calculation of G_r', which appears in so many of the equations. It can be shown by taking the total differential that G_r' is relatively insensitive to variations of g_m and r_p and, hence, that satisfactory results are obtained by using the values published under the tube typical operating characteristics.

As was mentioned earlier, some control of the pulse magnitude is possible through the choice of resistance values. When equation (45) is used as a starting point, the resistance values depend to some extent on the supply voltage; in this case a rough rule-of-thumb is that the upper limit of the e_{bn_1} pulse magnitude is $E_{bb}/2$, and the upper limit of the e_{bn_2} pulse magnitude is $E_{bb}/3$.

The bias voltage E_{cc} has been represented by a battery in the foregoing analysis. In practice, the bias would be secured by means of a variable resistance voltage divider network. The resistance values should be large enough to limit V_1 grid current to safe values if large positive trigger pulses are used. To avoid over shoot on the leading edges of the timing cycle waveforms, the value of the trigger pulse should be just sufficient to raise the grid of V_1 above cutoff.

The variations in tube parameters from tube to tube, usually an important factor in the design of an electronic circuit, do not have a great effect on the operation of this circuit since the degenerative feedback action minimizes the effect of these variations. Variations in the value of static grid resistance, r_g, of V_2 due to differences

in electrode spacing, is not counteracted by the feedback and, hence, allows considerable differences to exist in the stable state operation for different tubes. The use of a clamping diode has been suggested as a remedy.[2]

Deviations in the values of the resistors used can, under the worst conditions, lead to results differing from the calculated results by more than twice the percentage deviation of the resistors. This should be kept in mind during the design of this circuit and if a number of multivibrators are to be built, either 1 % resistors or matched sets should be used to ensure similar characteristics.

VII. SUMMARY

This chapter has presented a qualitative explanation of the operation of the monostable cathode-coupled multivibrator, followed by a quantitative analysis of the two separate systems into which the circuit can be divided. For the analysis of the timing state, the procedure closely followed that which would be used in analyzing an RC coupled amplifier, the only difference being a slight increase in complexity. The two equations, (22) and (25), from which most other timing state waveform values can be derived, contain the factor G_r', the effective gain of the RC coupled amplifier with cathode degeneration, thus showing that from analytical considerations, the circuit can be considered

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[2] Chance, Britton, et al., Waveforms (New York: McGraw-Hill Book Company, Inc., 1949), p. 168.

as an offshoot of the degenerative amplifier.

Next, a comparison of values obtained using the equations derived in this analysis was made with values obtained by another method. For the particular case used, the two methods gave quite comparable results.

The basic design equations were pointed out, and, from general considerations, a method of choosing values of R_k and R_{L_2} was given by equation (45). This equation can readily be extended to ratios of R_k/R_{L_2} other than the one used above.

Finally, some discussion was given to the effects of the variability of tubes and resistors on the operation of the circuit.

From the analytical aspect, the first purpose of this investigation has been accomplished.

as an offshoot of the [illegible] [illegible].

Next, a comparison of [illegible] [illegible] using the equations derived in this analysis was made with [illegible] [illegible] [illegible]. For the particular case used, the two methods gave quite [illegible] results.

The basic design equations were derived [illegible], and, [illegible] considerations, a method of choosing values of [illegible] and [illegible] [illegible] equation (45). This equation can readily be extended to ratios of R_k/R_{k_2} other than the one used above.

Finally, some discussion was given to the effects of the variability of tubes and resistors on the operation of the circuit.

From the analytical aspect, the first purpose of this investigation has been accomplished.

CHAPTER III

CONDITIONS WHICH GIVE NEARLY LINEAR RELATIONSHIP BETWEEN PULSE DURATION AND APPLIED GRID VOLTAGE

In the preceding chapter no mention was made of the fact that a very nearly linear relation between the bias voltage on tube V_1 and the duration of the timing state pulse can be achieved by proper choice of the circuit parameters. It will be shown in the following section that this phenomena has an analytical explanation. The means whereby this condition may be achieved will be investigated.

I. ESTABLISHMENT OF LINEAR RELATION

The pulse duration, T, of the monostable cathode-coupled multivibrator was given by equation (31) in Chapter II. For convenience this equation is repeated.

$$T = R_g C \ln \frac{E_{bb} - E_k + \left(\frac{E_{bb}}{\mu} + E_{cc}\right) G_r'}{E_{bb} - E_{co_2} - e_k} \qquad (31)$$

On replacing e_k by the value given in equation (25), this becomes

$$T = R_g C \ln \frac{E_{bb}\left(1 + \frac{G_r'}{\mu}\right) - E_k + \left(\frac{E_{bb}}{\mu} + E_{cc}\right) G_r'}{E_{bb} - E_{co_2} - \left(\frac{E_{bb}}{\mu} + E_{cc}\right) G_r' R_k / R_{L_1}} \qquad (46)$$

This equation can be written in a form more suitable for algebraic manipulation as

$$T = R_g C \ln \frac{a + b E_{cc}}{d + e E_{cc}} \qquad (47)$$

where

$$a = E_{bb}(1 + G_r'/\mu) - E_k \tag{47}$$

$$b = G_r' \tag{49}$$

$$d = E_{bb}(1 - G_r'R_k/\mu R_{L_1}) - E_{co_2} \tag{50}$$

$$e = G_r'R_k/R_{L_1}. \tag{51}$$

Equation (47) can be rewritten in the form

$$T = R_gC \ln \left[\frac{a}{d}\left(\frac{1 + \frac{b}{a}E_{cc}}{1 - \frac{e}{d}E_{cc}}\right)\right] \tag{52}$$

from which, by expanding the logarithm, we obtain

$$T = R_gC \left[\ln \frac{a}{d} + \ln (1 + \tfrac{b}{a}E_{cc}) - \ln (1 - \tfrac{e}{d}E_{cc})\right]. \tag{53}$$

Now, from calculus it will be remembered that the function $\ln (1 + x)$ can be expanded in a Maclaurin's series which converges to the function for all values of x in the interval -1 to +1. This expansion is

$$\ln (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots. \tag{54}$$

If x is much smaller than 1, all but the first term of this series can be neglected since the higher order terms will be correspondingly smaller and will have little effect upon the value given by the series. When x is positive, the series alternates and the error made by stopping at any term is numerically less than that term. This property gives an easy method of determining the error introduced into the calculations by replacing the second logarithmic term in equation (53) by the first term of the corresponding series. For negative x, the series no longer alternates and another method of estimating the error must be used. Another method for so doing does exist, and fortunately, is quite

simple to apply to this series.[1] The error is given by the Lagrangian form of the remainder after n terms of the series, where the remainder is denoted by

$$R_n = f^{(n)}(\xi)\, x^n/n!, \quad 0 < \xi < x, \tag{55}$$

where $f^{(n)}(\xi)$ denotes the nth derivative of f, evaluated at $x = \xi$.

Using this method for the case n = 2, we find the error to be numerically less than $\frac{1}{2}x^2/(1 - x)^2$, when the function ln (1 - x) is replaced by the first term of the series (54).

If, for the present, we assume that the quantities bE_{cc}/a and eE_{cc}/d are sufficiently small to permit using the first term of the corresponding series without causing excessive error, we can write equation (53) in the form

$$T = R_g C \left[\ln \frac{a}{d} + \frac{b}{a}E_{cc} + \frac{e}{d}E_{cc}\right] \tag{56}$$

or

$$T = R_g C \left[\ln \frac{a}{d} + \left(\frac{b}{a} + \frac{e}{d}\right) E_{cc}\right]. \tag{57}$$

It is evident from equation (57) that T is a linear function of E_{cc}.

II. ATTAINMENT OF LINEAR CONDITIONS

The derivation of equation (57) was made possible by the assumption that bE_{cc}/a and eE_{cc}/d were small in relation to unity. A close inspection of equations (48) through (51) will show that, unless R_k is

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[1] Sokolnikoff, Ivan S., and Sokolnikoff, Elizabeth S., Higher Mathematics for Engineers and Physicists (New York: McGraw-Hill Book Company, Inc., 1941), p. 36.

simple to apply to this series.[1] The error is given by the Lagrange form of the remainder after n terms of the series, where the remainder is denoted by

$$R_n = f^{(n)}(\xi)\, x^n / n!, \quad 0 < \xi < x, \qquad \text{[illegible]}$$

where $f^{(n)}(\xi)$ denotes the nth derivative of f, evaluated when $x = \xi$.

Using this method for the case n = 2, we find the error to be numerically less than $x^2/(1 - x)^2$, when the function ln (1 + x) is replaced by the first term of the series (54).

If, for the present, we assume that the quantities $\delta I_m/I$ and $\delta E_{cc}/d$ are sufficiently small to permit using the first term for the corresponding series without causing excessive error, we can write equation (53) in the form

$$\text{[illegible]} \qquad (56)$$

or

$$\text{[illegible]} \qquad (57)$$

It is evident from equation (57) that T is a linear function of E_{cc}.

11. TREATMENT OF [illegible]

The derivation of equation (57) was made under the assumption that $\delta E_{cc}/a$ and $\delta I_m/I$ were small in relation to unity. A close inspection of equations (48) through (51) shows that, unless E_{cc} is

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[1] Sokolnikoff, Ivan S., and Elizabeth S. Sokolnikoff, Higher Mathematics for Engineers and Physicists (New York: McGraw-Hill Book Company, Inc., 1941), p. 35.

considerably greater than R_{L_1}, b/a will be greater than e/d so that, in general, if the inequality

$$bE_{cc}/a \ll 1 \tag{58}$$

is satisfied, the inequality

$$eE_{cc}/d \ll 1 \tag{59}$$

will also be satisfied. Writing equation (58) in its complete form and setting it equal to a constant A, we obtain

$$\frac{b}{a} E_{cc} = \frac{G_r' E_{cc}}{E_{bb}(1 + \frac{G_r'}{\mu}) - E_k} = A, \; A \ll 1 \tag{60}$$

Multiplying this equation through by the denominator and collecting terms in G_r', we obtain

$$G_r' \left[E_{cc} - (A/\mu) E_{bb} \right] = A(E_{bb} - E_k). \tag{61}$$

We let $E_{cc}/E_{bb} = B$, and substitute the value of E_k as given by equations (4) and (5) in equation (61) obtaining

$$G_r' \left[BE_{bb} - AE_{bb}/\mu \right] = A \left[E_{bb} - \frac{E_{bb} - E_o}{R_k + R_{L_2} + r_{p_2}} R_k \right]. \tag{62}$$

The inequality

$$E_{bb} \gg E_o \tag{63}$$

is generally fulfilled. Hence the term containing E_o can be neglected and equation (62) becomes, after some manipulation

$$G_r' \left[B - A/\mu \right] = \frac{A(R_{L_2} + r_{p_2})}{R_k + R_{L_2} + r_{p_2}}. \tag{64}$$

Letting $(R_{L_2} + r_{p_2})/(R_k + R_{L_2} + r_{p_2})$ be represented by R_2, we can simplify this last expression to

$$G_r'(B - A/\mu) = AR_2. \tag{65}$$

Since

$$G_r' = (\mu R_{L_1})/\left[R_{L_1} + r_{p_1} + R_k(\mu + 1)\right] = (\mu R_{L_1})/(R_{L_1} + R_1) \tag{66}$$

where R_1 represents $\left[r_{p_1} + R_k(\mu + 1)\right]$, equation (65) can be written as

$$(\mu R_{L_1})/(R_{L_1} + R_1) = (AR_2)/(B - A/\mu) \tag{67}$$

which, when solved for R_{L_1} gives

$$R_{L_1} = (AR_1R_2)/(B\mu - AR_2 - A). \tag{68}$$

This can be rewritten as

$$R_{L_1} = (R_1R_2)/\left[\mu C - (1 + R_2)\right], \tag{69}$$

where $C = B/A$.

Once R_{L_1} has been determined, it is advisable to compute the value of eE_{cc}/d to assure that it is less than or equal to bE_{cc}/a. If this is not found to be true, the ratio of R_k/R_{L_2} should be changed to allow this condition to be fulfilled.

It might be thought that the value of R_{L_1} could be determined by setting eE_{cc}/d equal to A and solving for R_{L_1}; however, on expanding eE_{cc}/d and examining the result, we will see that its value changes little for large changes of R_{L_1}, and consequently, this equation is too insensitive to use as a means of computing the load resistance.

In using the first term of the series expansion of equation (54), we obtain a result which is larger than the correct value. The corresponding value for ln (1 - x) is smaller than the correct value. The combination of these two terms in equation (57) is such that these

$$G_F'(B - A/n) = nG_2. \tag{65}$$

Since

$$G_F' = (\mu R_{L_1})/\left[R_{L_1} + r_{p_1} + R_{k_1}(\mu + 1)\right] = (\mu R_{L_1})/(R_{L_1} + R_1) \tag{66}$$

where R_1 represents $\left[r_{p_1} + R_{k_1}(\mu + 1)\right]$, equation (65) can be written as

$$(\mu R_{L_1})/(R_{L_1} + R_1) = (nG_2)/(B - A/n) \tag{67}$$

which, when solved for R_{L_1} gives

$$R_{L_1} = (nR_1G_2)/(\mu B - nG_2 - A). \tag{68}$$

This can be rewritten as

$$R_{L_1} = (R_1G_2)/\left[\mu G - (1 + G_2)\right], \tag{69}$$

where $G = B/A$.

Once R_{L_1} has been determined, it is advisable to compute the value of eF_{co}/d to assure that it is less than or equal to b_{co}/A. If this is not found to be true, the ratio of R_1/R_{L_1} should be changed to allow this condition to be fulfilled.

It might be thought that the value of R_{L_1} could be determined by setting eF_{co}/d equal to A and solving for R_{L_1}; however, on expanding eF_{co}/d and examining the result, we will see that its value changes little for large changes of R_{L_1}, and consequently, this equation is too insensitive to use as a means of computing the load resistance.

In using the first term of the series expansion of equation (54), we obtain a result which is larger than the correct value. The corresponding value for ln (1 - x) is smaller than the correct value. The combination of these two terms in equation (57) is such that their two errors partially cancel each other, thus aiding the approach to a linear relationship. Since this is the case, it would seem reasonable to design the circuit so that bE_{cc}/a and eE_{cc}/d were equal in the hope that the errors would cancel completely. To obtain this relation, we equate the values

$$\frac{G_r' E_{cc}}{E_{bb}(1 + G_r'/\mu) - E_k} = \frac{G_r' R_k E_{cc}/R_{L_1}}{E_{bb}(1 - G_r' R_k/\mu R_{L_1}) - E_{co_2}} . \tag{70}$$

Assuming that the terms in the denominators which contain G_r' are small compared to unity and neglecting the E_{co_2} term, we find, after substituting values for the remaining terms, that R_{L_1} is given approximately by

$$R_{L_1} = \frac{R_k(r_{p_2} + R_{L_2})}{R_k + r_{p_2} + R_{L_2}} \tag{71}$$

Equation (71) gives results which are quite close to those which would be obtained by using the equation given by Glegg in his analysis of this circuit.[2] This is not surprising since his method, basically, consisted of setting b/a equal to e/d and solving the resulting equation for R_{L_1}.

Unfortunately, one drawback to this method lies in the fact that the error term $\frac{1}{2}x^2/(1 - x)^2$ grows more rapidly for increasing values of E_{cc} than does the corresponding error term $\frac{1}{2}x^2$. The result is that in the higher grid voltage ranges the first error term is predominant and causes a deviation from linearity. From these considerations

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[2] Glegg, Keith, "Cathode-Coupled Multivibrator Operation," Proc. IRE, 38:655, June, 1950

two errors partially [illegible] each other, [illegible] and [illegible] linear relationship. Since [illegible] the same, it would [illegible] to design the circuit so that [illegible] that the errors would cancel completely. To [illegible] relation, we equate the values

$$[illegible] = [illegible] \quad (70)$$

Assuming that the terms in the denominators which contain $C_{\ }'$ [illegible] compared to unity and neglecting the $R_{\ }$ term, we find, after [illegible] ing values for the remaining terms, [illegible] is given approximately by

$$[illegible] = [illegible] \quad (71)$$

Equation (71) gives results which are quite close to those which would be obtained by using the equation given by [illegible] of this circuit.[2] This is not surprising since his method consisted of setting [illegible] equal to [illegible] and solving the resulting equation for R_{1}.

Unfortunately, one drawback to this method lies in the fact that the error term [illegible] $/(1 - x)^2$ grows more rapidly for increasing values of R_{oc} than does the corresponding error term [illegible]. The result is that in the higher grid voltage ranges the first error term is predominant and causes a deviation from linearity. [illegible]

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[2] [illegible], "[illegible]-Control [illegible]," Proc. IRE, 30:[illegible], June, 19[illegible]

it will be seen that the first method, that of having e/d less than b/a, will give more linear results when the range of pulse duration requires a large change in applied grid voltage. Table III shows a comparison of both methods for typical operating values. The table shows the percentage error between the actual values and the approximate values as given by equation (57). It will be noted that in each case the error is considerably smaller than the error between the actual value of the logarithmic terms and their approximate value. Over the ordinary operating range, approximately from 30 volts to 90 volts, the percentage error for the case b/a = e/d is about $\pm$3.5 per cent; for the case e/d = 0.6 b/a, the error is about $\pm$0.75 per cent; for the case e/d = 0.8 b/a, the error is about $\pm$ 1 per cent. While it is fairly obvious that an optimum relation between b/a and e/d can be found by trial and error, it should be noted that this relation would not hold for other values of b/a.

Two methods have been presented for finding values of R_{L_1} which will result in a close approach to linearity between the pulse duration and the applied grid voltage. It will be instructive to apply the first method to the circuit given by Seely, using his values of R_{L_2} and R_k.[3] Values of 300, 7700, 7500, and 20 are used for E_{bb}, r_{p_1}, r_{p_2}, and respectively. We choose B equal to 0.2, or E_{cc} equal to 60 volts for the maximum grid voltage, A equal to 0.3, or b/a(60) = 0.3. Applying

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[3] Seely, Samuel, Electron-tube Circuits (New York: McGraw-Hill Book Company, Inc., 1950), p. 427.

it will be seen that the first method, that of holding c/d [illegible] b/a, will give more linear results when the range of pulse duration requires a large change in applied grid voltage. Table [illegible] shows a comparison of both methods for typical operating values. The table shows the percentage error between the actual value [illegible] and the value as given by equation (57). It will be noted that in both cases the error is considerably smaller [illegible] logarithmic terms and their approximated values [illegible] operating range, approximately from 50 volts to 90 volts, the percentage error for the case c/d = b/a is about ±.9 per cent; for the case c/d = 0.6 b/a, the error is about ±.7 per cent; for the case c/d = 0.4 b/a, the error is about ± 1 per cent. While it is fairly obvious that an optimum relation between b/a and c/d can be found by trial and error, it should be noted[3] that this relation would not hold for other values of b/a.

The methods have been presented for finding values of [illegible] which will result in a close approach to [illegible] between the pulse duration and the applied grid voltage. [illegible] the first method to the circuit given by Seely, using the values [illegible] Values of 330, 7100, 7500, and 91 are used for [illegible], respectively. We choose b equal to 0.3, [illegible] equal to 60 volts for the maximum grid voltage, [illegible]

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[3] Seely, Samuel, *Electron-Tube Circuits*, New York: McGraw-Hill Book Company, Inc., 1950, p. [illegible].

TABLE III

COMPARISON OF EFFECT OF GRID VOLTAGE CHANGE

ON ERROR FOR TWO METHODS OF DESIGN

b/a	e/d	E_{cc}	x_1	x_2	$\ln(1 + x_1)$	$\ln(1 - x_2)$	% Error
.005	.005	10	.05	-.05	.04879016	-.05129329	-0.09
		30	.15	-.15	.13976194	-.16251893	-0.76
		60	.30	-.30	.26236426	-.35667494	-3.3
		90	.45	-.45	.37156356	-.59783700	-7.7
		120	.60	-.60	.47000363	-.91629073	-15.0
.005	.003	10	.05	-.03	.04879016	-.03045921	+0.94
		30	.15	-.09	.13976194	-.09431068	+2.5
		60	.30	-.18	.26236426	-.19845094	+4.0
		90	.45	-.27	.37156356	-.31471074	+4.0
		120	.60	-.36	.47000363	-.44628710	+4.4
.005	.004	10	.05	-.04	.04879016	-.04082199	+0.43
		30	.15	-.12	.13976194	-.12783337	+0.9
		60	.30	-.24	.26236426	-.27443684	+0.57
		90	.45	-.36	.37156356	-.44628710	-0.97
		120	.60	-.48	.47000363	-.65392647	-4.0

the equations we have derived in this section, we find R_{L_1} is given as 5060 ohms, in quite good agreement with Seely's value of 5000 ohms. Rounding this to 5000 ohms and continuing the analysis, we find the following values:

$$G_r' = 1.29,\ b/a = 0.00488,\ e/d = 0.003433,$$
$$a/d = 0.88.$$

The pulse duration for Ecc = 60 volts is 113 microseconds with an error of $+0.25\%$ from the exact value; for Ecc = 20 volts, it is 11.5 microseconds with an error of $+1.1\%$. The error over this range, expressed bilaterally, is $\pm 0.44\%$. These results are not greatly different from Seely's. Better agreement of the higher value of pulse duration could have been attained by the choice of a larger value for B. The values used for A and B were obtained partly by guess from Seely's circuit and partly from the experience gained from this study. Here, again, the agreement gives a check on the accuracy of the method used, and gives confidence in its validity. This concludes the theoretical treatment of the cathode-coupled multivibrator.

the equations to more closely [illegible]. In this case, [illegible] R_1 is shown as 5060 ohms, in quite good agreement with Sealy's value of 5000 ohms. Rounding this to 5000 ohms and continuing the analysis [illegible] the following values:

$$Q_1' = 1.39, \quad c/a = [illegible], \quad d = [illegible]$$

$$\mu = [illegible]$$

The pulse duration for E_b = [illegible] volts is [illegible] micro-seconds with an error of $\pm$0.25% from the exact value. For E_b = [illegible] volts, it is [illegible] micro-seconds with an error of $\pm$[illegible]. The error over this range, [illegible] bilaterally, is $\pm$[illegible]. These results [illegible] Sealy's. Better agreement of the [illegible] value [illegible] would have been obtained by the choice of a larger value for R. The values used for A and B were [illegible] partly by guess [illegible] Sealy's [illegible] and partly from the experience gained from this study. Here, again, the agreement gives a check on the accuracy of the method [illegible] and [illegible] confidence in its validity. This concludes the theoretical treatment of the cathode-coupled [illegible].

CHAPTER IV

EXPERIMENTAL WORK

The comparisons made in Chapter II indicate that the results obtained in this analysis are comparable to the results of other theoretical studies. In the practical application of theory, however, the final results may, and often do, differ quite radically from the computed values. This difference is generally due to the fact that the values of the tube parameters may differ from the average values given in the tube handbooks by as much as 20 per cent. Experimental multivibrators were constructed to obtain information as to the effect of such variations when the design equations of this analysis were used. The design of these multivibrators and the results obtained from them are the subjects of this chapter.

I. APPLICATION OF BASIC ANALYSIS

The equations derived in Chapter II were used to design a multivibrator which would produce a rectangular pulse having a duration of 335 microseconds. The plate load resistor R_{L_2} and the cathode resistor R_k were set equal, and a 6SN7 tube, operating at a plate current value of 10 milliamperes, was chosen. From this information the circuit parameters were computed to be

$$R_{L_2} = 11{,}240 \text{ ohms}, \qquad R_k = 11{,}240 \text{ ohms}, \qquad R_{L_1} < 26{,}000 \text{ ohms},$$

These values were rounded off to the preferred values

$$R_{L_2} = 11{,}000 \text{ ohms}, \qquad R_k = 11{,}000 \text{ ohms}, \qquad R_{L_1} = 24{,}000 \text{ ohms}.$$

CHAPTER IV

EXPERIMENTAL WORK

The computations made in Chapter II indicate that the results obtained in this analysis are comparable to those obtained in other theoretical studies. In the practical application of theory, however, the final results may, and often do, differ quite markedly from the computed values. This difference is generally due to the fact that the values of the tube parameters may differ from the average values given in the tube handbooks by as much as 20 per cent. Two multivibrators were constructed to obtain information on the effect of such variations when the design equations of this analysis were used. The design of these multivibrators and the results obtained from them are the subjects of this chapter.

I. APPLICATION OF DESIGN ANALYSIS

The equations derived in Chapter II were used to design a multivibrator which would produce a rectangular pulse having a duration of 335 microseconds. The plate load resistor R_{L_2} and the cathode resistor R_k were set equal, and a 6SN7 tube, operating at a plate current value of 10 milliamperes, was chosen. From this information the circuit parameters were computed to be:

R_{L_2} = 11,240 ohms, R_k = 11,240 ohms, R_{g_1} = 26,000 ohms.

These values were rounded off to the nearest preferred values:

R_{L_2} = 11,000 ohms, R_k = 11,000 ohms, R_{g_1} = 24,000 ohms.

For a grid voltage of +70 volts, the R_gC time constant was computed to be 975×10^{-6}. To obtain this value, R_g was chosen as 0.975 megohms and C as 0.001 microfarad. Since only 5 per cent resistors were available, their values were measured thus reducing this variation to about 2 per cent, the meter accuracy. The final values, which were used in the remaining calculations, were

$$R_{L_2} = 10{,}500 \text{ ohms}, R_k = 11{,}000 \text{ ohms}, R_{L_1} = 24{,}200 \text{ ohms},$$
$$R_g = 0.975 \text{ megohms}, C = 0.001 \text{ microfarad.}$$

The waveforms produced by the circuit were displayed on a Du Mont Type 274-A Cathode-ray Oscillograph and measurements of the pulse heights and pulse duration were made. Observations were taken using three different 6SN7 tubes to show the effect of variations from tube to tube. A comparison of computed and measured values is given in Table IV.

The agreement between the computed and measured values is quite good except for the e_{cn_2} value during the timing state. In the worst case this difference is 40 per cent of the computed value. It was mentioned in Chapter II that the equation used in computing this value could give rise to large percentage errors although the individual terms of the equation were in error by only a relatively small amount. A calculation made with the values used in this experiment shows that an error of 5 per cent in each of the terms of the equation could cause a final error of 38 per cent. The close agreement of all the other values indicates that the method of computation is the cause of the large percentage error.

For a grid voltage of -20 volts, the $R_L C$ time constant was calculated to be 975×10^{-6}. To obtain this value, R_g was chosen as 0.975 megohms and C as 0.001 microfarad. Since the 5 per cent resistors were available, their values were measured thus showing only variation to within 2 per cent, the meter accuracy. The final values, which were used in the remaining calculations, were

R_{L_3} = 10,500 ohms, R_L = 11,000 ohms, R_k = [illegible] ohms,

R_g = 0.975 megohms, C = 0.001 microfarad.

The waveforms produced by the circuit were checked on a Du Mont Type 224-A Cathode-ray Oscillograph and measurements of the pulse heights and pulse duration were made. These data were taken using three different 6SN7 tubes to show the effect of variation from tube to tube. A comparison of computed and measured values is given in Table IV.

The agreement between the computed and measured values is quite good except for the e_{cm_1} value during the [illegible] state. In the worst case this difference is 10 per cent of the measured value. It was mentioned in Chapter II that the equation used in computing this value could give rise to large percentage errors although the individual terms of the equation were in error by only a relatively small amount. A calculation made with the values used in this experiment shows that an error of 5 per cent in each of the terms of the equation would cause a final error of 38 per cent. The closer agreement in all the other values indicates that the method of computation is the cause of the large percentage error.

TABLE IV

COMPARISON OF COMPUTED AND MEASURED WAVEFORM VALUES

Item	Computed Value	Measured Value		
	Stable State	Stable State		
		Tube 1	Tube 2	Tube 3
		Not Triggered		
E_{bn_1}	300	299	300	300
E_{bn_2}	198.8	192	192	197
E_k	106.1	107.5	109.8	104.1
E_{cc}	70	70	70.5	70
E_{cn_2}	106.1	107.8	110	104.2
		Triggered		
E_{bn_1}	300	300	300	300
E_{bn_2}	198.8	199	191.3	199
E_k	106.1	107.5	109.8	104.1
E_{cc}	70	-	-	-
E_{cn_2}	106.1	107.8	110	104.2
	Timing State	Timing State		
e_{bn_1}	143.6	147	151.7	152
e_{bn_2}	300	300	300	300
e_k	70.1	69.3	71.6	68.4
e_{cn_2}	-50.3	-31.3	-27.8	-33.3
T (sec.)	334	344	344	337

The values of the resistors used in this experiment are fairly close to the values used in the computations which resulted in Table I. A comparison of those values with the measured values of Table IV shows that the waveform magnitudes are relatively stable for variations of the circuit components.

The experimental results show the equations of this analysis to be well suited for use in the design of a cathode coupled multivibrator which will give pulses of a predetermined duration.

II. APPLICATION OF LINEARITY ANALYSIS

In the design of the multivibrator to produce a pulse with a duration which varied linearly with the applied grid voltage, the following relations were decided upon:

$$R_k = 0.7\ R_{L_2}, \quad A = 0.2, \quad B = 0.3,$$
$$E_{cc(max)} = 60 \text{ volts}, \; E_{bb} = 300 \text{ volts},$$
$$i_{b_2} = 15 \text{ milliamperes.}$$

These and the 6SN7 tube-parameter values were used in the design equations of Chapter III, resulting in the following component values:

$$R_{L_2} = 7240 \text{ ohms}, \; R_k = 5060 \text{ ohms}, \; R_{L_1} < 7340 \text{ ohms.}$$

The preferred values which were used were:

$$R_{L_2} = 6800 \text{ ohms}, \; R_k = 5100 \text{ ohms}, \; R_{L_1} = 6800 \text{ ohms.}$$

The corresponding values of a, b, d, and e were then computed giving

$$a = 235.2, \; b = 1.093, \; d = 300, \; e = 0.82,$$

the resulting equation for the pulse duration being

$$T = R_g C \left[0.0074 E_{cc} - 0.246 \right].$$

The values of the [illegible] used in the experiment are fairly close to the values used in the circuit when [illegible]. A comparison of these values with the [illegible] values of Table IV shows that the waveform conditions are relatively stable for variations of circuit components.

The experimental results show the usefulness of this analysis to be well suited for use in the design of a bootstrap multivibrator which will give pulses of a predetermined duration.

IV. APPLICATION OF LINEARITY ANALYSIS

In the design of the multivibrator to produce a pulse with a duration which varied linearly with the control grid voltage, the following relations were decided upon:

$$a = 0.7\ R_{g1},\quad b = [illegible],\quad d = [illegible]$$

$$E_{c2(max)} = 68\ \text{volts},\quad E_{bb} = 300\ \text{volts}.$$

$$t_{d1} = 25\ \text{milliseconds}.$$

These and the 6SN7 tube-parameter values were used in the equations of Chapter III, resulting in the following component values:

$$R_{g2} = 7200\ \text{ohms};\ R_k = 5000\ \text{ohms},\ R_{L1} = [illegible].$$

The preferred values which were used were:

$$R_{g2} = 6800\ \text{ohms},\ R_k = 5100\ \text{ohms},\ R_{L1} = [illegible].$$

The corresponding values of a, b, d, and e were then computed to be:

a = [illegible], b = [illegible], d = [illegible], e = [illegible].

The resulting equation for the pulse duration was

$$T = R_k C[1.0052\ E_{c2} - [illegible]].$$

For the time constant, R_g and C were chosen as 1.5 megohms and 0.001 microfarad, respectively. The multivibrator was constructed using components within 2 per cent of these values, with the exception of R_g and C which were 5 per cent components. Only measurements of the applied grid voltage and pulse duration were made during this experiment. The results of computed and measured pulse durations are given in Table V.

A plot of the measured values of T against E_{cc} shows that T is a linear function of E_{cc} from 225 to 315 microseconds. The accuracy of the measurements, however, was not sufficient to detect deviations smaller than a few per cent. It will be noticed that there is a slow increase in the difference between the measured and computed values as the value of E_{cc} increases, indicating that the straight lines showing pulse duration as a function of E_{cc} have different slopes as well as different intercepts. Both of these differences can be attributed to a difference between the actual value of R_gC and the value used in the computations, as well as to inaccuracies of the experimental measurements.

The results show that the design equations gave a fair prediction of the time durations which resulted, and that, within the limits of the experimental errors, the linear relationship was attained.

For the time constants R_gC, ... [illegible] ... 0.001 and 0.005 microfarad, respectively. The pulse duration was constant within the [illegible] within 3 per cent [illegible] for the [illegible] values of R_g and C which were [illegible] each measurement. [illegible] grid voltage and pulse duration [illegible] during this measurement. The results of computed and measured [illegible] are shown in Table V.

A plot of the measured values of [illegible] R_{gc} shows that it is a linear function of R_{gc} [illegible] is [illegible]. The accuracy of the measurements, however, was not sufficient to detect variations smaller than a few per cent. It will be noticed from Table V that the increase in the difference between the measured and computed values as the value of R_{gc} increases, indicating that the plots of the measured pulse duration as a function of R_{gc} have different slopes as [illegible] different instruments. Each of these differences can be explained by a difference between the actual value of R_gC and the value used in the computations, as well as by inaccuracies of the experimental measurements.

The results show that the design should permit [illegible] tion of the time durations which required, and that, within the limits of the experimental errors, the linear relationship was attained.

TABLE V

COMPARISON OF COMPUTED AND MEASURED PULSE DURATIONS

Grid Voltage	Computed T	Measured T
52	208	224
56	238	269
58	274	291.5
60	297	314
62	318	359
64	340	390
66	363	412
68	386.5	426

CHAPTER V

SUMMARY AND CONCLUSIONS

This final chapter contains a summary of the preceding chapters, and states the conclusions which were reached from both the theoretical and experimental work. Emphasis is placed upon the important results and the results which are original. A short enumeration of several aspects of the linearity problem, which were not considered in this treatment is given as a guide for possible future investigations.

I. SUMMARY

A short history of the development of the multivibrator was given pointing out the fact that trial and error methods are necessary in a number of design applications. The discussion was then specialized to the monostable cathode-coupled multivibrator, it being pointed out that while adequate analyses for the general operation of this circuit do exist they attack the problem from the trigger circuit viewpoint, thus neglecting to show the close relationship between the cathode-coupled multivibrator and the degenerative RC amplifier. The failure of these analyses to provide design information for achieving a pulse duration which is a linear function of the grid bias voltage was also pointed out.

An analysis of the monostable cathode-coupled multivibrator was then made in which it was shown that the resulting equations are those of the degenerative RC amplifier multiplied by factors which compensate

for the overdriven conditions of operation. The use of these equations showed a very good agreement with results obtained by graphical methods. The important design equations were pointed out and general design suggestions were given. The effect of tube and component variability on the circuit operation was also discussed.

Next, the equations which had been developed were used to show that the pulse duration is a linear function of the applied grid voltage under certain conditions. These conditions were examined and relations between the circuit parameters were established which fulfill these conditions. These relations were given in the form of equations which are useful in the design of such a circuit. An analysis of a circuit of known characteristics, by the methods developed here, gave good agreement between predicted and actual operation. To this investigator's knowledge, no prior work has been published giving a complete analysis of the linearity phenomena.

Experimental circuits were then constructed in accordance with the design equations of this analysis. The measured results were in good agreement with the predicted operation.

II. CONCLUSIONS

As a result of the theoretical and experimental work, it is concluded that

1. The monostable cathode-coupled multivibrator is closely related to the degenerative RC amplifier.

2. The analysis presented in Chapter II is well fitted for use

for the overdrive conditions of operation. The use of the n equations showed a very good agreement with results obtained by graphical methods. The important design equations were pointed out and general design suggestions were given. The effect of tube and component variability on the circuit operation was also discussed.

Next, the conditions which had been advanced were used to show that the pulse duration is a linear function of the applied grid voltage under certain conditions. These conditions were examined and relations between the circuit parameters were established which fulfill these conditions. These relations were given in the form of equations which are useful in the design of such a circuit. An analysis of a circuit of known characteristics, by the methods developed here, gave good agreement between predicted and actual operation. To this investigator's knowledge, no prior work has been published giving a graphical analysis of the linearity phenomena.

Experimental circuits were then constructed in accordance with the design equations of this analysis. The measured results were in good agreement with the predicted operation.

II. CONCLUSIONS

As a result of the theoretical and experimental work, it is concluded that:

1. The monostable cathode-coupled multivibrator is closely related to the regenerative amplifier.

2. The analysis presented in Chapter II is valid for the

in the design of practical circuits and will give a good prediction of the actual operation.

3. The analysis of the linear relation of pulse duration to grid voltage gives an analytic explanation which is capable of furnishing quantitative data for the circuit operation.

4. The application of the linearity analysis to design problems gives a fair prediction of the actual operation.

III. RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

There are several topics in the linearity analysis which have not been completely answered in this investigation. Of these the statement of the final error from exact linearity in terms of the error introduced by using only the first term of the series expansions would be of considerable value. It may be that further examination of this question would result in such a statement. A second field for investigation lies in the extension of the linearity range in both directions while keeping the sensitivity to grid voltages changes within reasonable limits. Still a third, and comparatively untouched, field lies in the application of an analysis such as the present one to pentode tubes.

In the experimental field, considerable work can be done to illustrate the effects of various values of A and B as defined in Chapter III, a study which was beyond the power of the analytic methods of this analysis.

in the design of practical circuits and will give a good description of the actual operation.

3. The analysis of the linear effects of pulses superimposed on grid voltage sweeps in sawtooth configuration which is capable of furnishing quantitative data for the circuit operation.

4. The application of the linearity analysis to a noise problem gives a fair prediction of the actual results.

III. RECOMMENDATIONS FOR FURTHER INVESTIGATION

There are several topics in the linearity analysis which have not been completely answered in this investigation. Of these the statement of the final error from exact linearity in terms of the error introduced by using only the first term of the series expansion would be of considerable value. It may be that further examination of this problem would result in such a statement. A second field for investigation lies in the extension of the linearity range in both directions and to maintain the sensitivity to grid voltage changes within reasonable limits. Still a third, and comparatively untouched, field lies in the application of an analysis such as the present one to pentode tubes.

In the experimental field, considerable work may be done to illustrate the effects of various values of a and b as defined in Chapter III, a study which was beyond the scope of the available methods of the analysis.

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