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# An Analysis of the Monostable Cathode-Coupled Multivibrator

Joseph C. Connell

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MORRISON J. COFFIN  
—  
COMPLETELY REVISED EDITION  
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NEW YORK: HARVARD UNIVERSITY PRESS  
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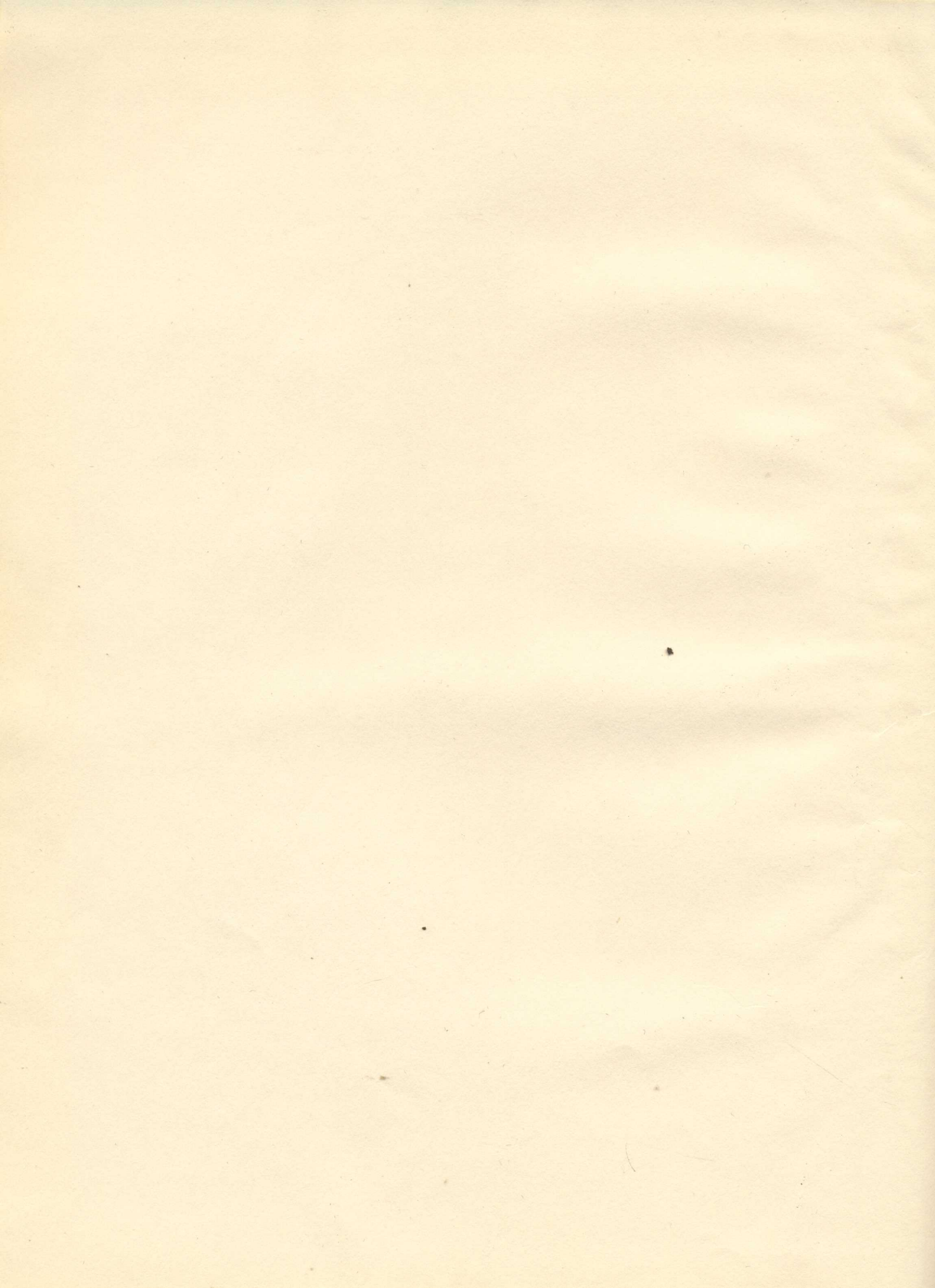














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AN ANALYSIS OF THE MONOSTABLE  
CATHODE-COUPLED MULTIVIBRATOR

By

Joseph C. Connell

A Thesis

In partial fulfillment of the  
Requirements for the Degree of  
Master of Science in Electrical Engineering

The University of New Mexico  
1953



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DATE

Thesis committee

Thomas F. Martin Jr.  
CHAIRMAN

Robert A. Hessemer Jr.

Ralph W. Tapp



This check is made and signed by the undersigned  
in full payment of the account of the  
University of New Mexico for the year  
ending on the day of

1922

*[Faint handwritten signature]*

*[Faint handwritten signature]*

Witness my hand

*[Faint handwritten signature]*

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#### ACKNOWLEDGMENT

The investigator wishes to acknowledge his indebtedness to Dr. Thomas L. Martin, Jr., of the University of New Mexico Electrical Engineering Department, whose lectures aroused an interest in the methods of analysis employed in this study. The work of Chapter II closely follows an analysis which appears in Dr. Martin's book, Electronic Circuits, to be published by Prentice-Hall, Inc., in 1954.



1931  
10  
100



MEMORANDUM

The investigation was conducted by the following persons:  
Thomas L. ... of the Division of ...  
Engineering Department, ...  
Methods of analysis employed in this report ...  
closely follow the ...  
... to be published by ...



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# BONE

FIGURE

1.	Structure of the bone
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## CHAPTER I

### A BRIEF HISTORY OF THE MULTIVIBRATOR

The multivibrator was first suggested in 1918 by Abraham and Block.<sup>1</sup> It would seem that a circuit of such ancient vintage should have been analyzed satisfactorily and completely long ago. This is not the case, however, for while a number of papers had appeared prior to 1940, it has only been during the past decade that satisfactory analyses appeared in the literature. Prior to this time the results given by various authors differed by factors of as much as two to one, and hence, were practically useless.<sup>2</sup> A probable reason for the lack of advancement during the first twenty years was the fact that there were few uses for such a circuit outside of the laboratory. The advent of World War II, with its concentration on the entire field of electronics and particularly on circuitry associated with radar, played a big part in the increased understanding of multivibrator operation. The commercial development of television has also contributed to recent interest in multivibrator analysis. Even though the present state of knowledge is fairly advanced, the design of variations of the basic multivibrator or the design of the basic multivibrator to produce certain characteristics is a problem which frequently demands ingenuity and patience;

---

<sup>1</sup> Abraham, H., and Block, E., Ministère de la guerre Pub. 27, April, 1918.

<sup>2</sup> Richter, W., Fundamentals of Industrial Electronic Circuits (New York: McGraw-Hill Book Company, Inc., 1947), p. 397.







usually the method of trial and error is used in achieving a successful circuit.

### I. THE MONOSTABLE CATHODE-COUPLED MULTIVIBRATOR

Present status of analysis. The monostable cathode-coupled multivibrator was used extensively in radar applications during World War II. A quite complete treatment of its general operation was used in the training of radar personnel for the Armed Forces and was later published.<sup>3</sup> This analysis is partly graphical and partly analytical, although few equations are given explicitly. Two recent publications give quite thorough analyses of multivibrator circuits.<sup>4,5</sup> The equations are derived by the use of equivalent circuits in both of these works. The first treatment does not work out the equations for the cathode-coupled case, but leaves this as an exercise for the reader; the second treatment includes several variations of the cathode-coupled multivibrator and examines each in some detail. Thus it is evident that several satisfactory analyses of the multivibrator circuit do exist at the present time.

Viewpoint of these analyses. Until quite recently, the schematic diagram of the multivibrator circuit was drawn as a two stage RC

---

<sup>3</sup> M. I. T. Radar School Staff, Principles of Radar (New York: McGraw-Hill Book Company, Inc., 2nd, Ed., 1946), p. 2-53.

<sup>4</sup> Martin, Thomas L. Jr., Ultrahigh Frequency Engineering (New York: Prentice-Hall, Inc., 1950), p. 71.

<sup>5</sup> Seely, Samuel, Electron-tube Circuits (New York: McGraw-Hill Book Company, Inc., 1950), p. 416.



usually the method of choice for the analysis of complex  
circuits.

## II. THE METHOD OF THE SUPERPOSITION PRINCIPLE

Principle of superposition. The principle of superposition  
states that in a linear circuit containing several  
independent sources, the response (voltage or current) in  
any branch is the algebraic sum of the responses that  
would be produced by each independent source acting  
alone, with all other independent sources replaced by  
their internal impedances. This principle is particularly  
useful in the analysis of circuits containing both  
independent and dependent sources. It allows the  
analysis to be broken down into simpler parts, each  
of which can be analyzed separately. The responses  
obtained for each part are then combined to find the  
total response. This method is especially useful for  
finding the response of a circuit to a single source  
when there are multiple sources present. It is also  
useful for finding the response of a circuit to a  
complex source, such as a periodic waveform, by  
decomposing the source into its constituent sinusoidal  
components and analyzing the circuit for each component  
separately. The responses for each component are then  
combined to find the total response. The principle of  
superposition is a powerful tool for the analysis of  
linear circuits and is widely used in the design and  
analysis of electronic systems.

Statement of the principle. Let  $V$  be the voltage  
across a branch of a linear circuit containing several  
independent sources. Let  $V_1, V_2, \dots, V_n$  be the  
voltages across the same branch when each source is  
acting alone, with all other sources replaced by their  
internal impedances. Then the voltage  $V$  is given by  
 $V = V_1 + V_2 + \dots + V_n$ .

J. N. I. V. Principle of Superposition, *Journal of  
Applied Electronics*, Vol. 1, No. 1, 1955, p. 1-10.  
A. S. S. Principle of Superposition, *Journal of  
Applied Electronics*, Vol. 1, No. 1, 1955, p. 1-10.  
S. S. S. Principle of Superposition, *Journal of  
Applied Electronics*, Vol. 1, No. 1, 1955, p. 1-10.



amplifier with capacitive coupling between the plate of the second tube and the grid of the first tube.<sup>6</sup> Despite this similarity to the RC amplifier, the various papers on multivibrators have treated them purely as trigger circuits and have made no attempt to show the multivibrator as a special case of the RC amplifier with degenerative feedback. Possibly such treatment is due to the phraseology used in one of the first treatises which analyzed the multivibrator in what has now become the standard fashion. The authors state that:

. . . Because the wave forms encountered in these circuits are not sinusoidal, and because the grid-voltage variations are so great that the tube seldom acts as a linear amplifier, conventional methods of circuit analysis are of little value . . .<sup>7</sup>

It is true that by conventional methods the authors meant steady-state alternating current methods as opposed to transient analysis, yet the impression persists that the multivibrator is of a completely different nature than the RC amplifier. In practice, this viewpoint is of small importance since, save for the idea of pedagogical unity, it matters little how results are obtained if they are accurate.

Incompleteness of analyses. It is interesting to note that several of the analyses mentioned call attention to the fact that an important feature of the cathode-coupled multivibrator lies in the

---

<sup>6</sup> Terman, Frederick E., Radio Engineering (New York: McGraw-Hill Book Company, Inc., 3rd. Ed. 1947), p. 588.

<sup>7</sup> Kiebert, Martin V., and Inglis, Andrew F., "Multivibrator Circuits," Proc. IRE, 33:534, August, 1945.



amplifier with the input terminals connected to the output of the preceding stage and the grid of the first tube. The output of the amplifier is taken from the output terminals of the amplifier, the various modes of operation being obtained by changing the connections of the input and output terminals. The amplifier is used as a special case of the general amplifier circuit. Possibly such treatment is due to the fact that the amplifier is a special case of the general amplifier circuit. The amplifier is used as a special case of the general amplifier circuit.

### the standard amplifier circuit

... However, the same basic circuit can be used for other purposes and not necessarily for amplification. The circuit is very simple and is not too different from the circuit of a vacuum tube amplifier. The circuit is very simple and is not too different from the circuit of a vacuum tube amplifier.

It is true that the amplifier circuit is a special case of the general amplifier circuit. The amplifier is used as a special case of the general amplifier circuit. The amplifier is used as a special case of the general amplifier circuit. The amplifier is used as a special case of the general amplifier circuit. The amplifier is used as a special case of the general amplifier circuit.

### Interconnection of amplifiers

Several of the amplifiers mentioned can be connected in series to form a multi-stage amplifier. The amplifier is used as a special case of the general amplifier circuit. The amplifier is used as a special case of the general amplifier circuit.

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6. Electronic Circuits, by R. M. Frenkel, McGraw-Hill Book Company, Inc., New York, N. Y., 1957.

7. Electronic Circuits, by R. M. Frenkel, McGraw-Hill Book Company, Inc., New York, N. Y., 1957.



nearly linear relationship, between pulse duration and grid voltage, which can be obtained by proper choice of plate and cathode resistors. Qualitative explanations of this phenomena have been given in the majority of published articles, although two treatments do justify this by means of analytical expressions.<sup>8,9</sup> The first of these does not extend the analysis to the relationships between the circuit parameters, but does give a circuit which was used extensively at the M. I. T. Radiation Laboratory. The duration of the pulse obtained from this circuit is a linear function of the applied grid voltage within  $\pm 0.25$  per cent over the range from about 8 to 150 microseconds. The second treatment gives an equation which is said to give values which are close to the ideal values. A comparison of the values calculated from this equation with the values used in the M. I. T. circuit shows a difference which is greater than this investigator would expect.

The magnitude of this difference, which is of the order of two to one, raises a question as to whether the theoretical treatment is completely satisfactory, or whether further analysis might reconcile the apparent deviation between theory and practice. At any rate, the present status of the theory does leave much to be desired insofar as the linearity phenomena is concerned.

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<sup>8</sup> Seely, op. cit., pp. 424-427.

<sup>9</sup> Glegg, Keith, "Cathode-Coupled Multivibrator Operation, Proc. IRE, 38:655, June, 1950.



... nearly linear relationship... which can be obtained by... Qualitative... justify or... by means of... extend the... but does give... Habitual... directly... per cent... treatment... close to... this equation... difference... The... two to one... in... also the... the present... as the linearly...

Scott, W. H., 1911, p. 112-113

From the...  
Scott, W. H., 1911, p. 112-113



## II. PURPOSE OF THESIS

The foregoing comments pointed out the facts that: 1) little has been done to show the similiarity between the multivibrator and the RC amplifier with degenerative feedback; 2) the existing treatment of the conditions under which a linear relationship between pulse duration and applied grid voltage will exist does not give results which are in very good agreement with the values used in a practical circuit. It is the purpose of this study to fill in these gaps by showing that the equations resulting from an analysis of the cathode-coupled multivibrator are, in general, those of the degenerative amplifier multiplied by an additional factor which compensates for the heavily biased operation, and that the conditions for a nearly linear relation between pulse duration and applied grid voltage depend upon a rather complicated function of the circuit parameters which does not allow a general solution, but which does allow values to be computed for any particular case.



The following comments apply to the tests described.

It has been found that the results of the tests are very similar and the

80 amplifier with its special feedback circuit is a suitable means of

BACKGROUND

EXPERIMENTAL

RESULTS

those resulting from an analysis of the results of the tests.

are, in general, those of an amplifier with a feedback circuit.

additional factors which contribute to the results of the tests.

and that the conditions of the tests are very similar to those of

function and applied with a feedback circuit.

function of the circuit is a function of the results of the tests.

input, but which does not seem to be a function of the results.

case,



CHAPTER II

ANALYSIS OF THE MONOSTABLE CATHODE-COUPLED MULTIVIBRATOR

I. GENERAL OPERATION

A circuit diagram of the monostable cathode-coupled multivibrator is shown in Figure 1. The symbols shown there are the ones which will be used throughout this study. It is seen from this figure that the grid of  $V_2$  is connected to the supply voltage  $E_{bb}$  through a resistor  $R_g$  while the grid of  $V_1$  is connected to a source of lower voltage  $E_{cc}$ . The higher grid voltage of  $V_2$  makes it logical to suppose that if plate current flow is limited to one of the tubes it would flow in  $V_2$ . The question then arises as to whether plate current is flowing in  $V_1$  during this time.

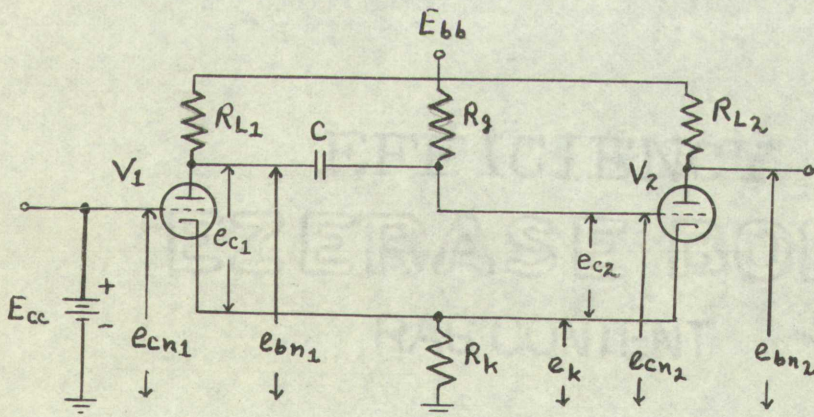


FIGURE I

CIRCUIT OF THE MONOSTABLE CATHODE-COUPLED  
MULTIVIBRATOR



ANALYSIS OF THE MONOSTABLE CATHODE-COUPLED MULTIVIBRATOR

1. GENERAL DEFINITION

A circuit diagram of the monostable cathode-coupled multivibrator is shown in Figure 1. The symbols shown there are the same which will be used throughout this study. It is seen from this figure that the grid of  $V_1$  is connected to the supply voltage  $E_{cc}$  through a resistor  $R_{L1}$  while the grid of  $V_2$  is connected to a source of lower voltage  $E_{cc}$ . The trigger grid voltage of  $V_2$  when it is equal to the voltage  $E_{cc}$  plate current flow is limited to one of the tubes as would flow in  $V_1$ . The question then arises as to whether plate current is flowing in  $V_1$  during this time.

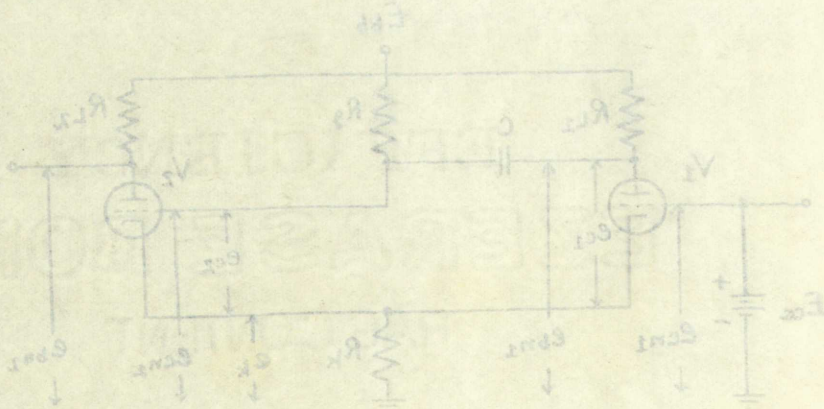


FIGURE 1

CIRCUIT OF THE MONOSTABLE CATHODE-COUPLED

MULTIVIBRATOR



Evidently the value of  $e_{c1}$  determines the answer to this question; if  $e_{c1}$  is greater than the cutoff voltage of  $V_1$ , plate current will flow. For operation as a monostable circuit, it is necessary for  $V_1$  to be cut-off during the normal state. This gives a first condition for operation, that  $e_{c1}$  is less than the cutoff voltage  $e_{col}$  of tube  $V_1$ , or

$$E_{cc} - E_k < e_{col} \quad (1)$$

where  $e_{col}$  is a negative quantity.

Assuming now that inequality (1) is satisfied, and that  $V_2$  has been conducting a sufficient length of time for any transients to have virtually disappeared, we see that a positive pulse, of large enough amplitude to raise the grid of  $V_1$  above cutoff, will, on being applied to  $V_1$ , cause plate current to flow. The resulting IR drop in  $R_{L1}$  will be transmitted through  $C$  to the grid of  $V_2$  thus causing the grid voltage to decrease. The resulting decrease of total current in  $R_k$  causes the voltage  $E_k$  to decrease thus reversing inequality (1) and thereby aiding the transition. The circuit ends up in a quasi-stable or timing state with  $V_1$  conducting steadily and with the grid of  $V_2$  relatively far below its value of grid cutoff voltage. The capacitor  $C$  now discharges through the resistor  $R_g$ , thus allowing the grid voltage of  $V_2$  to climb toward a less negative value along an exponential curve. When the cutoff value is reached,  $V_2$  conducts, the voltage  $E_k$  increases, thus decreasing the grid voltage of  $V_1$ , and the reverse transition takes place. Following this switching action, the circuit is in the original stable condition. It is evident that, except for the short time during which switching is taking place, the multivibrator acts as



Evidently the value of  $e_{c1}$  determines the answer to this question. If  $e_{c1}$  is greater than the cutoff voltage of  $V_1$ , plate current will flow. For operation as a monostable circuit, it is necessary for  $V_1$  to be cut-off during the normal state. This gives a first condition for operation, that  $e_{c1}$  is less than the cutoff voltage  $e_{c1c}$  of tube  $V_1$ , or

$$(1) \quad e_{c1} < e_{c1c}$$

where  $e_{c1c}$  is a negative quantity.

Assuming now that inequality (1) is satisfied, and that  $V_1$  has been conducting a sufficient length of time for any capacitance to have actually disappeared, we see that a positive pulse, of large enough amplitude to raise the grid of  $V_1$  above cutoff, will, on being applied to  $V_1$ , cause plate current to flow. The resulting IR drop in  $R_1$  will be transmitted through C to the grid of  $V_2$ , thus causing the grid voltage to increase. The resulting decrease of total current in  $R_2$  causes the voltage  $E_2$  to decrease thus reversing inequality (1) and thereby starting the transition. The circuit ends up in a quasi-stable or steady state with  $V_1$  conducting steadily and with the grid of  $V_2$  relatively far below its value of grid cutoff voltage. The capacitor C now discharges through the resistor  $R_2$ , thus allowing the grid voltage of  $V_2$  to climb toward a less negative value along an exponential curve. When the cutoff value is reached,  $V_2$  conducts, the voltage  $E_2$  increases, thus decreasing the grid voltage of  $V_1$ , and the reverse transition takes place. Following this switching action, the circuit is in the original stable condition. It is evident that, except for the short time during which switching is taking place, the multiplier acts as



a one tube circuit since the non-conducting tube behaves like an open circuit. Thus, by neglecting the transition state which will not be considered in this investigation, the multivibrator can be thought of as being composed of two one-tube circuits which can be studied individually.

## II. THE STABLE STATE

Since  $V_1$  is cutoff and all transients have disappeared, the left side of the diagram of Figure 1 can be considered as an open circuit. It is then evident from the figure that

$$E_{bb} - E_k = i_{g2} (\bar{R}_g \parallel R_g) \quad (2)$$

where  $\bar{R}_g$  is the static grid resistance of tube  $V_2$ .

The grid voltage  $e_{c2}$  can be expressed as

$$e_{c2} = (E_{bb} - E_k) \frac{\bar{R}_g}{R_g \parallel \bar{R}_g} \quad (3)$$

which is approximately zero since  $\bar{R}_g$  is about 1000 ohms and  $R_g$  is a megohm or more. We now replace the tube  $V_2$  by its equivalent plate circuit thus obtaining the circuit of Figure 2. The tube has been represented as a generator of voltage  $\mu e_{c2}$  in series with the dynamic plate resistance of the tube and a battery of voltage  $E_o$ .<sup>1</sup> Numerical values of these quantities can easily be found from the characteristic curves of the particular tube which is used in the multivibrator circuit.

---

<sup>1</sup> Richter, Walther, Fundamentals of Industrial Electronic Circuits (New York: McGraw-Hill Book Company, Inc., 1947), pp. 70-77.



a one tube circuit since the non-conducting tube behaves like an open circuit. Thus, by neglecting the transmission space which will not be considered in this investigation, the multiplier can be thought of as being composed of two one-tube circuits which can be treated individually.

### II. THE STABLE STATE

Since  $\bar{V}_B$  is small and all transients have disappeared, the left side of the diagram of Figure 1 can be considered as an open circuit. It is then evident from the figure that

$$(2) \quad \bar{V}_B - R_k \bar{I}_k = \bar{V}_B \left( \frac{1}{R_B} + \frac{1}{R_k} \right)$$

where  $\bar{V}_B$  is the static grid voltage of tube  $V_2$ .

The grid voltage  $\bar{V}_B$  can be expressed as

$$(3) \quad \bar{V}_B = \frac{\bar{V}_B}{\frac{1}{R_B} + \frac{1}{R_k}} = \bar{V}_B \left( \frac{R_B R_k}{R_B + R_k} \right)$$

which is approximately zero since  $\bar{V}_B$  is about 1000 ohms and  $R_k$  is a megohm or more. We now replace the tube  $V_2$  by its equivalent plate circuit thus obtaining the circuit of Figure 2. The tube has been represented as a generator of voltage  $\bar{V}_B$  in series with the dynamic plate resistance of the tube and a battery of voltage  $E_0$ . Theoretical values of these quantities can easily be found from the characteristic curves of the particular tube which is used in the multiplier circuit.

cont.

Radio Engineering Handbook, 2nd Edition, McGraw-Hill Book Company, Inc., 1947, pp. 70-71.



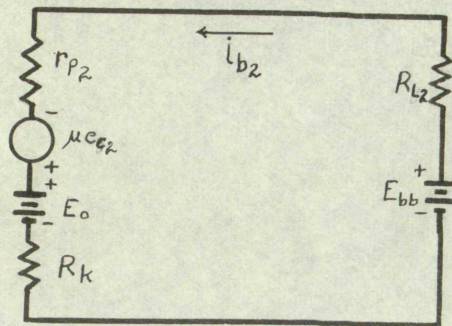


FIGURE 2

## EQUIVALENT CIRCUIT FOR STABLE STATE

For triodes such as the 6J5 and 6SN7, representative values are  $r_p = 7,500$  ohms and  $E_0 = 20$  volts, for zero grid voltage.

Since  $e_{c2}$  was found to be approximately zero, the current in the circuit of Figure 2 is readily computed to be

$$i_{b2} = (E_{bb} - E_0) / (r_{p2} + R_k + R_{L2}). \quad (4)$$

From equation (4), the values which the various wave-forms have during the stable state can be computed readily. Reference to Figure 1 shows the values to be given by

$$E_k = (i_{b2} + i_{g2})R_k \approx i_{b2}R_k, \quad (5)$$

$$e_{bn2} = E_{bb} - i_{b2}R_{L2}, \quad (6)$$

$$e_{cn2} = E_k + i_{g2}F_g \approx E_k, \quad (7)$$

$$e_{bn1} = E_{bb}. \quad (8)$$



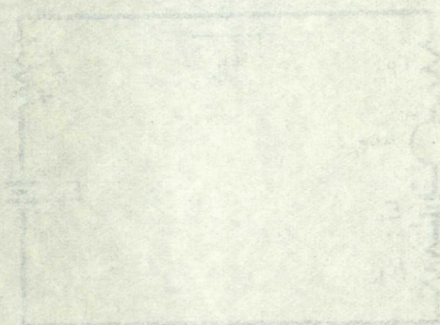


Figure 1

For problem 1, let  $R_1 = 10 \Omega$ ,  $R_2 = 20 \Omega$ ,  $V_1 = 100 \text{ V}$ , and  $V_2 = 100 \text{ V}$ .

1,200 ohms and  $V_2 = 100 \text{ V}$ , the two sources are

Find the current  $i_1$  and  $i_2$  in the circuit shown in Figure 1.

the circuit in Figure 1 is used to determine

$$(a) \quad i_1 = \frac{V_1}{R_1} = \frac{100}{10} = 10 \text{ A}$$

From equation (1), the current  $i_1$  is

have obtained the value of  $i_1$  and  $i_2$  in the circuit.

Figure 1

$$(b) \quad i_2 = \frac{V_2}{R_2} = \frac{100}{20} = 5 \text{ A}$$

$$(c) \quad i_3 = \frac{V_1}{R_1 + R_2} = \frac{100}{10 + 20} = 3.33 \text{ A}$$

$$(d) \quad i_4 = \frac{V_2}{R_1 + R_2} = \frac{100}{10 + 20} = 3.33 \text{ A}$$

$$(e) \quad i_5 = \frac{V_1 + V_2}{R_1 + R_2} = \frac{200}{10 + 20} = 6.67 \text{ A}$$



The charge  $\gamma_1$  on the condenser is given by

$$\gamma_1 = E_{bb} - e_{cn2} \quad (9)$$

Calculation of the approximations of equations (5) and (7), which result from neglecting  $i_{g2}$ , indicate that an error of one per cent or less is involved.

The analysis of the stable state is now complete, and we are ready to proceed with the second part of the analysis.

### III. THE TIMING STATE

Since, in this state,  $V_2$  is cutoff and  $V_1$  is the conducting tube, the circuit of Figure 1 can be redrawn, by replacing  $V_2$  by an open circuit. When  $V_1$  is replaced by its equivalent generator in series with the plate resistance and the constant voltage battery, the equivalent circuit will be that shown in Figure 3. A rearrangement of the circuit elements, and the use

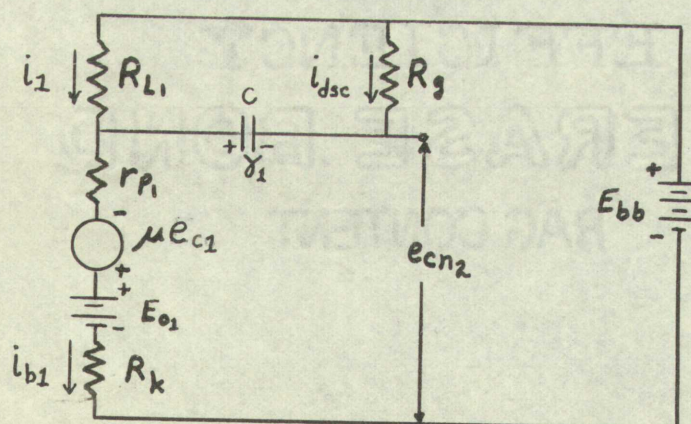


FIGURE 3

EQUIVALENT CIRCUIT FOR THE TIMING STATE



The change  $\Delta V_1$  on the cathode is given by

$$\Delta V_1 = I_{c1} R_{c1} - I_{p1} R_{p1} \quad (8)$$

Calculation of the space-charge of equations (7) and (8), which are  
with from neglecting  $I_{c1}$ , indicates that the error of our first ap-  
proximation is negligible.

The analysis of the circuit is now complete, and we are  
ready to proceed with the second part of the analysis.

### III. THE TIMING STATE

Since, in this state,  $V_1$  is small and  $V_2$  is the controlling  
tube, the circuit of Figure 1 can be reduced by replacing  $V_1$  by an  
open circuit. When  $V_1$  is replaced by its equivalent generator in series  
with the plate resistance and the constant voltage source, the equiv-  
alent circuit will be that shown in Figure 2. A rearrangement of the  
circuit elements, but the use

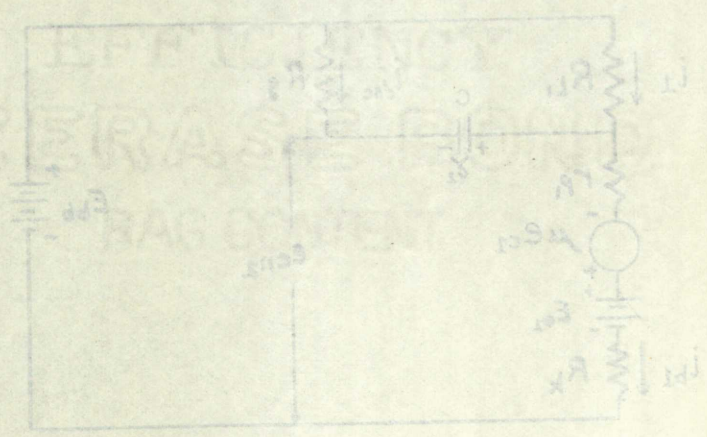


FIGURE 2

EQUIVALENT CIRCUIT FOR THE TIMING STATE



of a second plate supply battery of voltage  $E_{bb}$  transforms the circuit to that of Figure 4. The use of an additional battery allows the circuit to be redrawn in such a way that the method of attack becomes almost self-evident. It is legitimate since the internal impedance of the power supply is negligible and, hence, the voltage seen by either the  $R_g$  or the  $R_{L1}$  branch is independent of the current flowing in the other branch. The equivalent generator has a voltage  $\mu e_{c1}$  which, because of the cathode resistor  $R_k$ , is dependent on the tube current  $i_{b1}$ . As shown in Figure 3, this current is the sum of  $i_1$  and the capacitor discharge current  $i_{dsc}$  since it is assumed that no grid current flows. An inspection of Figure 1 shows that

$$E_{cc} = e_{c1} + i_{b1} R_k \tag{10}$$

and consequently

$$e_{c1} = E_{cc} - i_{b1} R_k \tag{11}$$

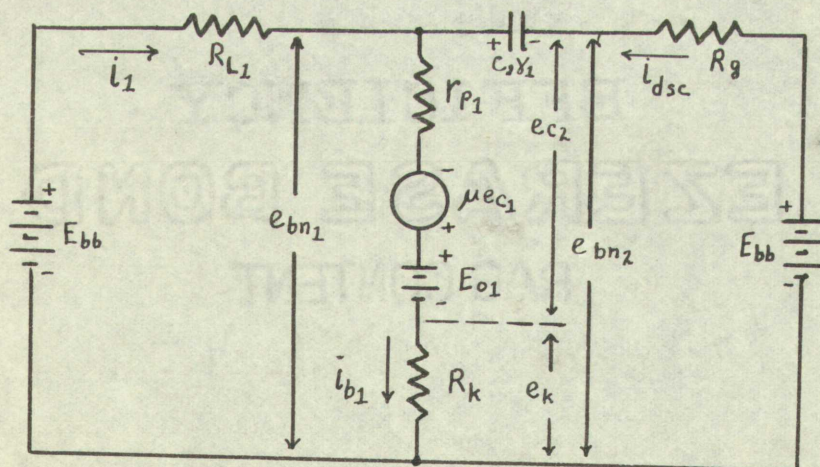


FIGURE 4

ALTERNATE FORM OF EQUIVALENT CIRCUIT



of a second plate supply battery of voltage  $E_{pp}$  transforms the circuit to that of Figure 4. The use of an additional battery allows the circuit to be redrawn in such a way that the method of attack becomes almost self-evident. It is legitimate since the internal impedances of the power supply is negligible and, hence, the voltage seen by either the  $R_{c1}$  or the  $R_{p1}$  branch is independent of the current flowing in the other branch. The equivalent generator has a voltage  $V_{oc1}$  which, because of the cathode resistor  $R_{c1}$ , is dependent on the tube current  $i_{p1}$ . As shown in Figure 3, this current is the sum of  $i_{b1}$  and the capacitor discharge current  $i_{dsc}$  since it is assumed that no grid current flows. An inspection of Figure 1 shows that

(10)

$$V_{oc1} = E_{oc1} - i_{p1} R_{c1}$$

and consequently

(11)

$$E_{oc1} = V_{oc1} + i_{p1} R_{c1}$$

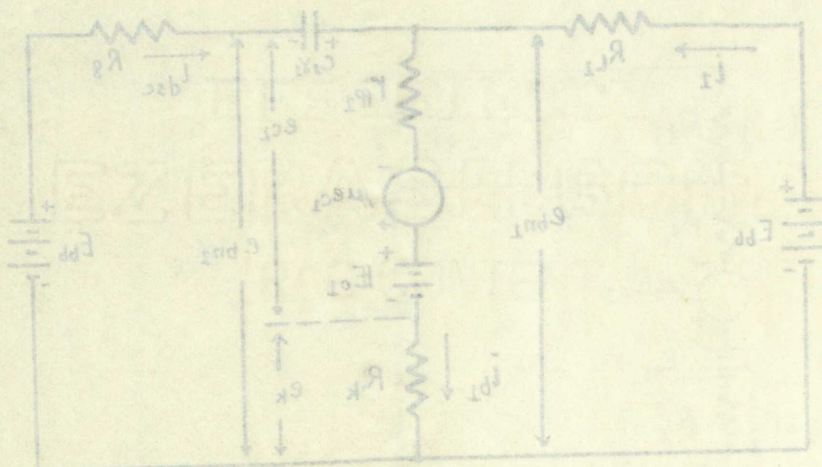


FIGURE 4

ALTERNATE FORM OF EQUIVALENT CIRCUIT



The voltage  $\mu e_{c_1} = \mu E_{cc} - i_{b_1} \mu R_k$  contains a term which is similar to the voltage drop in the cathode resistor. It is evident that exactly the same results will be obtained by dropping this term from the equivalent generator and adding a resistance  $\mu R_k$  to the cathode resistor. Doing this and redrawing the circuit gives a final form of the equivalent circuit as shown in Figure 5.

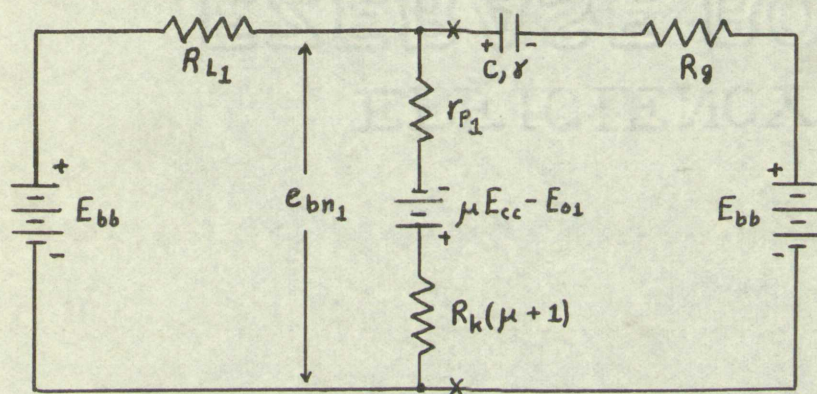


FIGURE 5

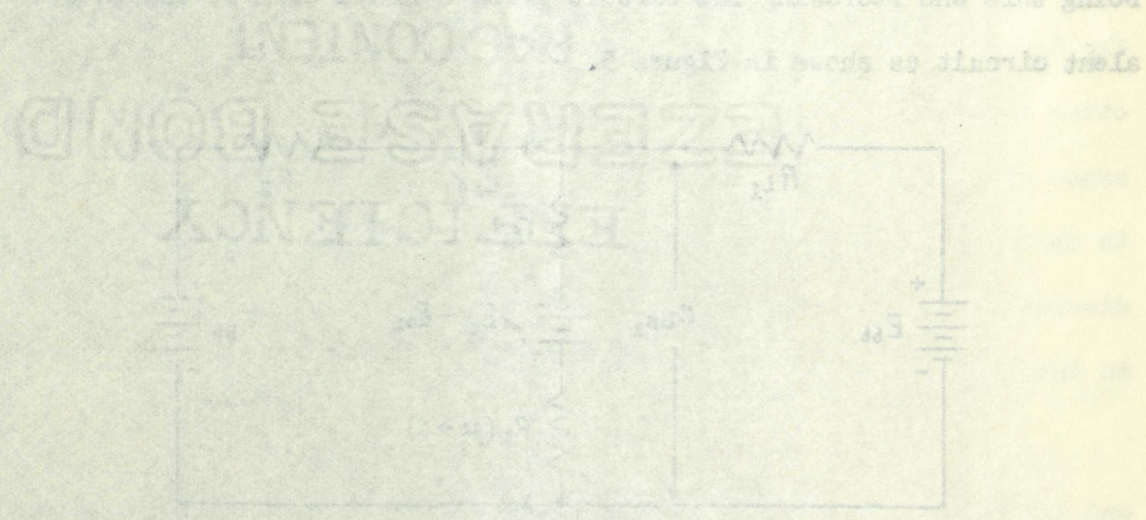
## FINAL FORM OF EQUIVALENT CIRCUIT

The equivalent generator, being a constant voltage, has been replaced by a battery which also includes the voltage  $E_{o_1}$ . By applying Thevenin's theorem to the circuit to the left of the points marked by x's, we obtain the simple series circuit given in Figure 6, the voltage  $E_e$  and the resistance  $R_e$  being

$$E_e = E_{bb} - \frac{E_{bb} / \mu E_{cc} - E_{o_1}}{R_{L_1} / r_{p_1} / R_k(\mu / 1)} R_{L_1}, \quad (12)$$



The voltage  $V_{oc} = E_{th} - I_{sc} R_{th}$  is the voltage across the load resistor  $R_L$  when the switch is closed. The voltage  $V_{oc}$  is the open-circuit voltage of the network. The current  $I_{sc}$  is the short-circuit current of the network. The voltage  $V_{oc}$  is the voltage across the load resistor  $R_L$  when the switch is closed. The current  $I_{sc}$  is the current through the load resistor  $R_L$  when the switch is closed.



The equivalent generator, battery, and resistor network, is shown in the diagram. The battery  $E_{th}$  and the resistor  $R_{th}$  are in series. The load resistor  $R_L$  is connected in parallel with the series combination of  $E_{th}$  and  $R_{th}$ . The current  $I$  is the current through the load resistor  $R_L$ .

$$I = \frac{E_{th}}{R_{th} + R_L}$$



$$R_e = \frac{R_{L1} [r_{p1} + (\mu + 1)R_k]}{R_{L1} + r_{p1} + R_k(\mu + 1)}$$

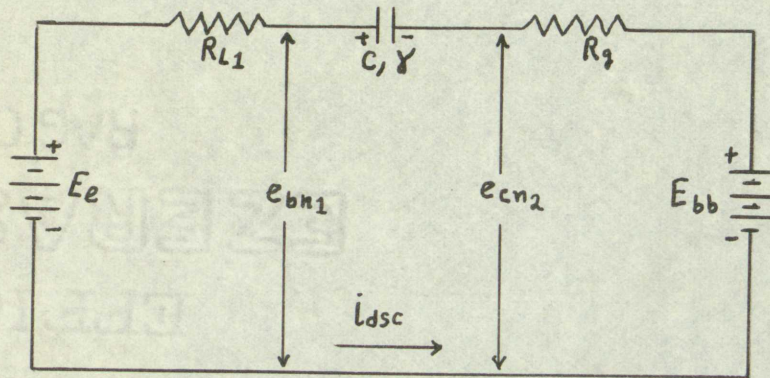


FIGURE 6

## THEVENIN FORM OF CIRCUIT

Equation (12) can be simplified somewhat by assuming that  $E_{o1}$  is much smaller than  $(E_{bb} - E_{cc})$ , a fair approximation for a triode as used in this circuit. By manipulation of the third term of equation (12), we obtain

$$\frac{(E_{bb}/r_p + \mu E_{cc}/r_{p1})(R_{L1} r_{p1}) / (R_{L1} + r_{p1})}{1 + R_k(\mu + 1) / (R_{L1} + r_{p1})} = \frac{(E_{bb}\mu/\mu r_{p1} + E_{cc}\mu/r_{p1})(R_{L1} r_{p1}) / (R_{L1} + r_{p1})}{1 + R_k(\mu + 1) / (R_{L1} + r_{p1})}$$

which, since  $\mu/r_p = g_m$ , can be written as

$$\frac{(E_{bb}/\mu + E_{cc})(g_m R_{L1} r_{p1}) / (R_{L1} + r_{p1})}{1 + R_k(\mu + 1) / (R_{L1} + r_{p1})} \quad (14)$$

It will be remembered that, when  $R_g$  is large in comparison with  $R_L$  and  $r_p$ , the expression



$$I = \frac{E}{R_1 + R_2 + R_3}$$

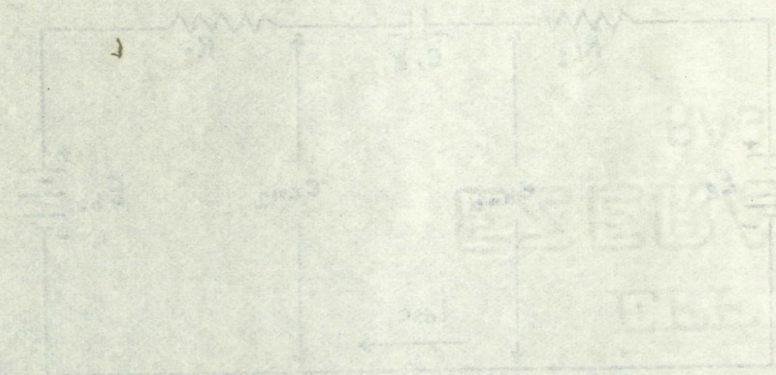


FIGURE 1  
CIRCUIT WITH A CAPACITOR

Equation (2) can be simplified somewhat by assuming that the capacitor is initially uncharged. In this case, the voltage across the capacitor is zero, and the circuit is equivalent to a simple series circuit. By application of the loop rule to equation (2), we obtain

$$E - IR_1 - IR_2 - IR_3 = 0$$

which, since \$I = q/C\$, can be written as

$$E - R_1 \frac{dq}{dt} - R_2 \frac{dq}{dt} - R_3 \frac{dq}{dt} = 0$$

It will be assumed that \$R\_1 = R\_2 = R\_3 = R\$, in which case the equation becomes



$$\frac{g_m R_L r_p}{R_L + r_p}$$

is the mid-frequency, or real, gain of the RC coupled amplifier and is denoted by  $G_r$ . Division of  $G_r$  by the quantity  $[1 + R_k(\mu + 1)/(R_L + r_p)]$  gives the real gain  $G_r'$  for the case where cathode degeneration is present. Making these substitutions in equation (14), we obtain

$$(E_{bb}/\mu + E_{cc})G_r' \quad (15)$$

and hence

$$E_e = E_{bb} - (E_{bb}/\mu + E_{cc})G_r' \quad (16)$$

An examination of Figure 6 shows that

$$e_{bn_1} = E_e - i_{dsc} R_e \quad (17)$$

and that

$$i_{dsc} = \frac{E_{bb} + Y_1 - E_k - [E_{bb} - (E_{bb}/\mu + E_{cc})G_r']}{R_e + R_g} e^{-\frac{t}{(R_e + R_g)}} \quad (18)$$

where the value of  $Y_1$  is given by equation (9). On inserting this value in equation (18) and simplifying, we find that at the instant of switch-over, i.e., at the time  $t = 0$ , which will be denoted by writing the value as a function of zero,  $i_{dsc}(0)$  is

$$i_{dsc}(0) = \frac{E_{bb} - E_k + (E_{bb}/\mu + E_{cc})G_r'}{R_e + R_g} \quad (19)$$

On substituting equations (12) and (19) into (17), there results

$$\begin{aligned} e_{bn_1} &= E_{bb} - \left(\frac{E_{bb}}{\mu} + E_{cc}\right)G_r' + \frac{[E_{bb} - E_k + (\frac{E_{bb}}{\mu} + E_{cc})G_r'] R_e}{R_e + R_g} \\ &= E_{bb} \left[1 + \frac{R_e}{R_e + R_g}\right] - \left[\left(\frac{E_{bb}}{\mu} + E_{cc}\right)G_r'\right] \left[1 - \frac{R_e}{R_e + R_g}\right] - \frac{E_k R_e}{R_e + R_g} \end{aligned} \quad (20)$$



is the mid-frequency, or  $\omega = 1$ , then in the case of a single pole the  
 defined by  $\dots$  the value of  $\dots$  gives the value of  $\dots$   
 process. Substituting these values in the equation  $\dots$

$$\dots = \dots$$

and hence

$$\dots = \dots$$

and that

$$\dots = \dots$$

where the value of  $\dots$  is given by the value of  $\dots$   
 in equation (13) and substituting the value of  $\dots$  in equation (13)

### EFFICIENCY

## ERASE-BOND

### PAG CONTENT

On substituting equation (13) in equation (12) we get

$$\dots = \dots$$

$$\dots = \dots$$



Since  $R_e$  is much smaller than  $R_g$  we can assume that  $R_e/R_g$  and  $R_e/(R_e + R_g)$  are very much less than 1 and hence can be neglected in equation (20).

Using this approximation, we find that

$$e_{bn_1}(0) = E_{bb} - (E_{bb}/\mu + E_{cc})G_r', \quad (21)$$

and from Figure 6 it is readily seen that

$$\begin{aligned} e_{cn_2}(0) &= e_{bn_1}(0) - \gamma_1 \\ &= E_k - (E_{bb}/\mu + E_{cc})G_r'. \end{aligned} \quad (22)$$

From Figure 5 we find an alternate expression for  $e_{bn_1}(0)$

$$e_{bn_1}(0) = E_{bb} - i_1 R_{L_1}. \quad (23)$$

Combining these two equations and solving for  $i_1$  gives

$$i_1 = (E_{bb}/\mu + E_{cc})G_r' / R_{L_1}. \quad (24)$$

As noted above,  $i_{b_1}$  is the sum of  $i_1$  and  $i_{dsc}$ ; however,  $i_{dsc}$  is limited by  $R_g$  and so has a value which is negligible in comparison with  $i_1$ . Hence, to a good approximation  $i_{b_1} = i_1$  and, from Figure 4,

$$e_k(0) = i_1 R_k = (E_{bb}/\mu + E_{cc})G_r' R_k / R_{L_1}. \quad (25)$$

It should be noted that, since  $i_{dsc}$  is negligible,  $e_k$  will not vary with time but remains constant at the value given by equation (25).

Referring to Figure 1, we see that

$$e_{c_1} = E_{cc} - e_k. \quad (26)$$

This is also a constant voltage. Thus, for all practical purposes, the only change which takes place during the timing cycle will be in the voltage across the capacitor. It is this voltage which is holding the grid of  $V_2$  below cutoff and as this voltage decreases exponentially the grid voltage of  $V_2$  will rise until it reaches its cutoff value.



Since  $R_w$  is much smaller than  $R_p$  we can assume that  $R_w \ll R_p$  and  $R_w \ll R_c$  (21) are very much less than 1 and hence can be neglected in equation (22).

Using this approximation, we find that

$$e_{c1}(0) = R_{cp} - (R_{cp} \sqrt{1 + R_{cp}^2 / R_c^2}) \quad (21)$$

and from Figure 5 it is readily seen that

$$e_{c2}(0) = e_{c1}(0) - \frac{1}{2} R_c \quad (22)$$

$$= R_c - (R_{cp} \sqrt{1 + R_{cp}^2 / R_c^2}) \quad (23)$$

From Figure 5 we find an alternate expression for  $e_{c1}(0)$

$$e_{c1}(0) = R_{cp} - \frac{1}{2} R_c \quad (24)$$

Combining these two equations and solving for  $i_1$  gives

$$i_1 = (R_{cp} \sqrt{1 + R_{cp}^2 / R_c^2}) / R_c \quad (25)$$

As noted above,  $i_1$  is the sum of  $i_1$  and  $i_{1c}$ ; however,  $i_{1c}$  is limited by  $R_c$  and so has a value which is negligible in comparison with  $i_1$ . Hence, to a good approximation  $i_1 = i_1$  and, from Figure 5,

$$e_c(0) = i_1 R_c = (R_{cp} \sqrt{1 + R_{cp}^2 / R_c^2}) \quad (26)$$

It should be noted that, since  $i_{1c}$  is negligible,  $e_c$  will not vary with time but remains constant at the value given by equation (26).

Referring to Figure 1, we see that

$$e_{c1} = R_{cp} - e_c \quad (27)$$

This is also a constant voltage. Thus, for all practical purposes, the only change which takes place during the firing cycle will be in the voltage across the capacitor. If in this voltage which is holding the grid of  $V_2$  below cutoff and as this voltage decreases exponentially the grid voltage of  $V_2$  will rise until it reaches the cutoff value.



At this time  $V_2$  begins to conduct and the timing cycle is completed.

The value of cutoff voltage,  $E_{co2}$ , can be found from the tube characteristic curves using the value of plate voltage on  $V_2$ . This voltage is

$$e_{b2} = E_{bb} - e_k \quad (27)$$

Since, in Figure 4

$$e_{cn2} = e_{c2} + e_k \quad (28)$$

we have the initial value of grid voltage on  $V_2$  as

$$e_{c2} = e_{cn2} - e_k \quad (29)$$

where  $e_{cn2}$  is given by equation (7).

The initial values of all waveform points have now been given in equations (21), (22), (25), (26), and (28). Of these values only  $e_{c1}$  and  $e_{cn1}$  change in value during the timing cycle so all others can be sketched for the entire timing cycle from their initial values.

The pulse duration,  $T$ , will evidently be the time required for the grid of  $V_2$  to rise from the voltage  $e_{c2}$ , with which the timing cycle starts, to the cutoff voltage,  $E_{co2}$ . Referring to Figure 4, we see that

$$e_{c2} = E_{bb} - [i_{dsc}(0)R_g] e^{-\frac{t}{RC}} - e_k$$

or

$$e_{c2} = E_{bb} - e_k - [E_{bb} - E_k + (E_{bb}/ + E_{cc})G_r] e^{-\frac{t}{RC}} \quad (30)$$

where  $R = (r_{p1} + R_k + R_g)$  which, since  $R_g$  is much greater than  $(r_{p1} + R_k)$ , is very nearly equal to  $R_g$ . Using this approximation and substituting the value of cutoff voltage,  $E_{co2}$ , in the preceding equation, we solve



At this time  $V_2$  begins to conduct and the timing cycle is completed. The value of cutoff voltage,  $V_{CO2}$ , can be found from the characteristic curves using the value of plate voltage on  $V_2$ . This voltage is

$$(27) \quad V_{CO2} = V_{PB} - V_{CO1}$$

Since, in Figure 4

$$(28) \quad V_{CO1} = V_{CO2} + V_{CO1}$$

we have the initial value of grid voltage on  $V_2$  as

$$(29) \quad V_{CO2} = V_{CO1} - V_{CO1}$$

where  $V_{CO2}$  is given by equation (7).

The initial values of all waveform points have now been given

in equations (21), (22), (23), (24), (25), (26), and (29). Of these values only

$V_{CO1}$  and  $V_{CO2}$  change in value during the timing cycle as all other values are allocated for the entire timing cycle from their initial values.

The pulse duration,  $T$ , will evidently be the time required for

the grid of  $V_2$  to rise from the voltage  $V_{CO2}$  with which the timing cycle starts, to the cutoff voltage,  $V_{CO1}$ . Referring to Figure 4, we see that

$$V_{CO2} = V_{PB} - \left[ \frac{1}{RC} \int_0^T V_{CO1} dt \right] e^{-\frac{t}{RC}}$$

or

$$(30) \quad V_{CO2} = V_{PB} - \left[ \frac{1}{RC} \int_0^T V_{CO1} dt \right] e^{-\frac{t}{RC}}$$

where  $R = (r_{p1} + R_1 + R_2)$  which, since  $R_2$  is much greater than  $(r_{p1} + R_1)$ ,

is very nearly equal to  $R_2$ . Using this approximation and substituting

the value of cutoff voltage,  $V_{CO1}$ , in the preceding equation, we solve



for T obtaining

$$T = R_g C \ln \frac{E_{bb} - E_k + (E_{bb}/\mu + E_{cc})G_r'}{E_{bb} - E_{co2} - e_k} \quad (31)$$

The value of T is readily computed since all of these values are known or have been derived previously.

So far this analysis has been based upon the fulfillment of two conditions as follows: 1) that  $V_1$  was cutoff during the stable state, and 2) that grid current does not flow in  $V_1$  during the timing state. These conditions can be expressed by the inequalities

$$E_{cc} - E_k < e_{co1} \quad (32)$$

where  $e_{co1}$  is a negative quantity and

$$E_{cc} < i_{b1} R_k = (E_{bb}/\mu + E_{cc})G_r' R_k / R_{L1} \quad (33)$$

Since  $e_{co1}$  is approximately equal in magnitude to  $(E_{bb} - E_k)/\mu$ , inequality (32) can be written in expanded form by using the previously determined expression for  $E_k$  as

$$E_{cc} < E_{bb} R_k / R_2 \left[ 1 + \frac{1}{\mu} - \frac{E_o}{E_{bb}} - \frac{E_o}{E_{bb} \mu} - \frac{R_2}{R_k \mu} \right] \quad (34)$$

The terms  $1/\mu$  and  $E_o/E_{bb}$  being of the same order and of opposite sign effectively cancel each other; the fourth term in the brackets is small in comparison with the remaining terms and can be neglected. A good approximation for equation (34) is

$$E_{cc} < E_{bb} R_k / R_2 \left[ 1 - R_2 / \mu R_k \right] \quad (35)$$

which can be expressed as an equality by

$$E_{cc} = k E_{bb} R_k / R_2, \quad k < 1 - R_2 / \mu R_k \quad (36)$$



for T obtaining

$$(32) \quad T = \frac{R_{cc} \ln \frac{R_{cc} + R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}}}{R_{cc} - R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}}}}{R_{cc} - R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}}}$$

The value of T is readily computed since all of these values are known or have been derived previously.

So far this analysis has been based upon the fulfillment of

two conditions as follows: 1) that  $W_1$  was small during the initial state, and 2) that grid current does not flow in  $W_2$  during the initial state. These conditions can be expressed by the inequalities

$$(33) \quad R_{cc} < R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}}$$

where  $R_{co}$  is a negative quantity and

$$(34) \quad R_{cc} < \frac{1}{2} R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}} = \frac{R_{cc} R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}}}{2}$$

Since  $R_{co}$  is approximately equal in magnitude to  $R_{cc}$ , the inequality (34) can be written in expanded form by using the previously

determined expression for  $R_{cc}$  as

$$(35) \quad R_{cc} < \frac{1}{2} R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}} \left[ 1 + \frac{1}{M} - \frac{R_{cc}^2}{R_{co}^2} - \frac{R_{cc}^2}{R_{co}^2} - \frac{R_{cc}^2}{R_{co}^2} \right]$$

The terms  $1/M$  and  $R_{cc}^2/R_{co}^2$  being of the same order and of opposite sign effectively cancel each other; the fourth term in the bracket is

small in comparison with the remaining terms and can be neglected. A good approximation for equation (35) is

$$(36) \quad R_{cc} < \frac{1}{2} R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}} \left[ 1 - \frac{R_{cc}^2}{R_{co}^2} \right]$$

which can be expressed as an equality by

$$(37) \quad R_{cc} = \frac{1}{2} R_{co} \sqrt{1 + \frac{R_{cc}^2}{R_{co}^2}} \left[ 1 - \frac{R_{cc}^2}{R_{co}^2} \right]$$



In all the above equations  $R_2$  denotes  $(R_{L2} \neq r_{p2} \neq R_k)$ .

Inserting this value of  $E_{cc}$  in inequality (33) and solving for  $R_{L1}$ , we obtain, after some manipulation,

$$R_{L1} < \frac{R_{L2} \neq r_{p2} \neq R_k(1 - k) - kr_{p1}}{k} \quad (37)$$

Equations (36) and (37) are the basic equations for the design of a cathode-coupled multivibrator, once the choice of tubes and plate supply voltage has been made. Considerable latitude in the choice of  $R_k$ ,  $R_{L2}$ , and  $R_{L1}$  is possible so that some control of the magnitude of the pulses is available. The value of  $R_g C$  can easily be determined for a desired pulse duration from equation (31), keeping in mind that the value of  $R_g$  should be one megohm or more.

The plate voltage of  $V_1$  does not reach the plate supply value when  $V_1$  is cutoff since the capacitor charging current flows through  $R_{L1}$  giving an exponential shape to the rise of plate voltage. The equivalent circuit for this period is shown in Figure 7.

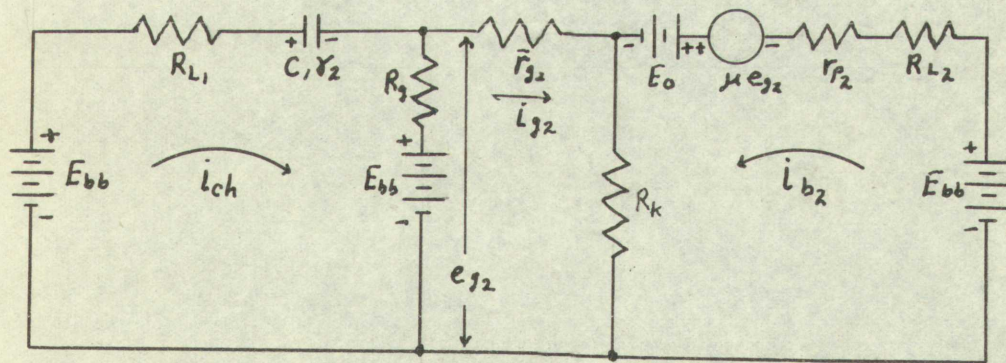


FIGURE 7

EQUIVALENT CIRCUIT IMMEDIATELY

AFTER TIMING CYCLE



In all the above equations  $R_2$  denotes  $(R_{12} + R_{13} + R_{14})$ .

Inserting this value of  $R_{10}$  in inequality (25) and solving for

$R_{11}$ , we obtain, after some simplification,

$$R_{11} > \frac{R_{12} + R_{13} + R_{14} + R_{15}(1 - k) - R_{16}}{k} \quad (27)$$

Equations (26) and (27) are the basic equations for the design of a

cathode-coupled multivibrator, once the choice of tubes and plate supply

voltage has been made. Considerable latitude in the choice of  $R_{11}$ ,  $R_{12}$

and  $R_{13}$  is possible so that some control of the magnitude of the pulse

is available. The value of  $R_{10}$  can easily be determined for a desired

pulse duration from equation (21), keeping in mind that the value of  $R_{10}$

should be one negative or more.

The plate voltage of  $V_1$  does not reach the plate supply value

when  $V_1$  is cutoff since the capacitor charges current flows through

$R_{11}$  giving an exponential shape to the rise of plate voltage. The

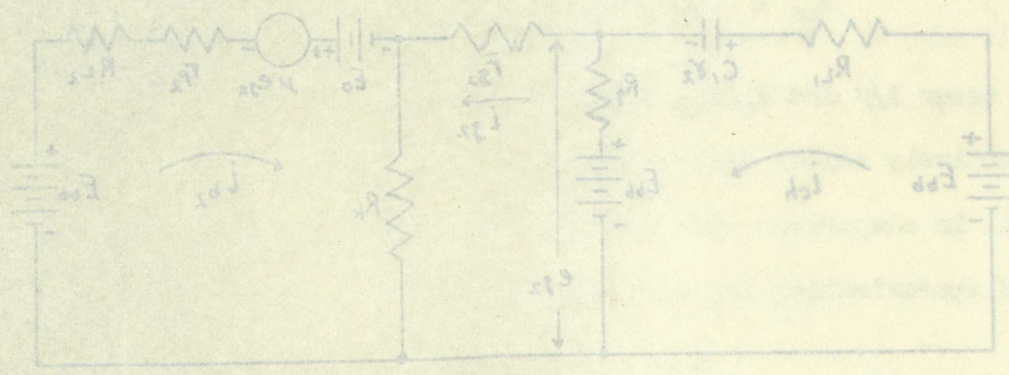


FIGURE 7  
EQUIVALENT CIRCUIT IMMEDIATELY  
AFTER TIMING CYCLE



Since  $R_g$  is of a much greater value than any other resistor or combination of resistors in the circuit and since  $E_o$ , for positive grid voltages, is very nearly zero, the circuit may be simplified with little loss of accuracy by neglecting these parameters. Figure 8 shows the simplified circuit.

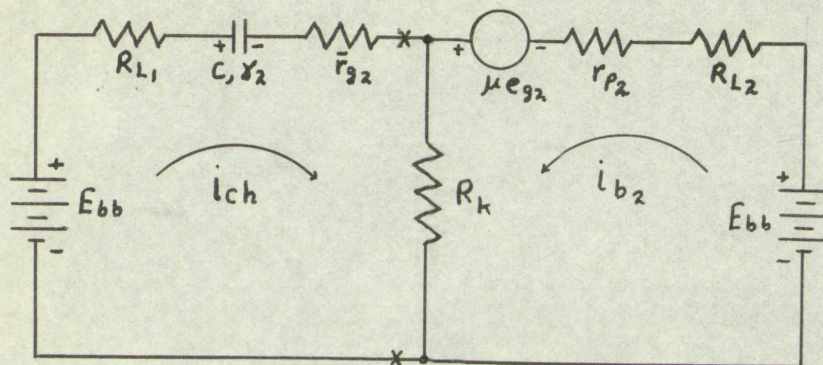


FIGURE 8

SIMPLIFIED CIRCUIT AFTER TIMING CYCLE

Applying Thevenin's theorem to the circuit to the right of the points marked by x's, we obtain the circuit indicated in Figure 9 where

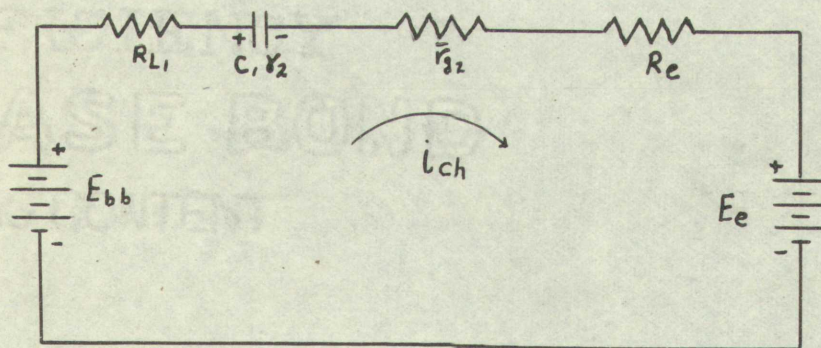


FIGURE 9

THEVENIN FORM OF FIGURE 8



Since  $R_B$  is of a much greater value than any other resistor or constant of resistance in the circuit and since  $E_{01}$  for positive grid voltages, is very nearly zero, the circuit may be simplified with little loss of accuracy by neglecting these parameters. Figure 8 shows the simplified circuit.

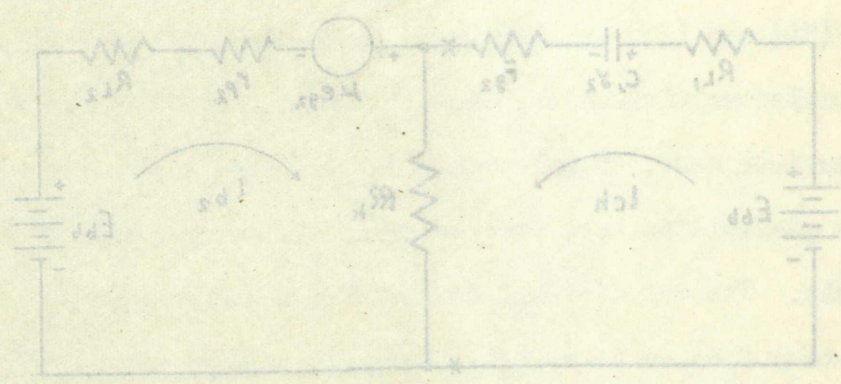


FIGURE 8

SIMPLIFIED CIRCUIT AFTER TUNING CYCLE

Applying Thevenin's theorem to the circuit to the right of the points marked by x's, we obtain the circuit indicated in Figure 9 where

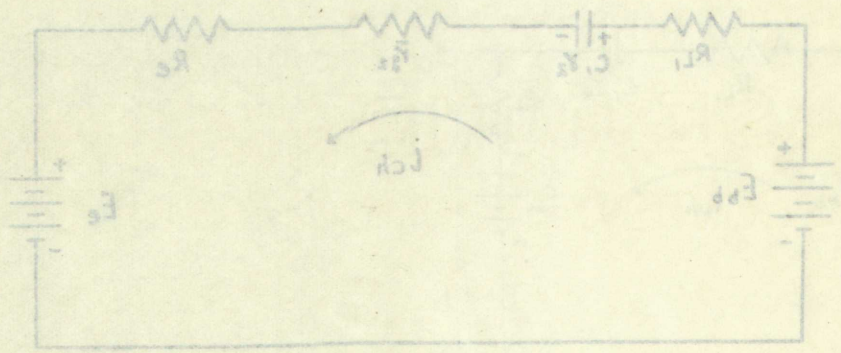


FIGURE 9

THEVENIN FORM OF FIGURE 8



the values of  $R_e$  and  $E_e$  are given by

$$R_e = R_k(r_{p2} \parallel R_{L2}) / (R_k \parallel r_{p2} \parallel R_{L2}) \quad (38)$$

$$E_e = (E_{bb} \parallel \mu e_g) R_k / (R_k \parallel r_{p2} \parallel R_{L2}). \quad (39)$$

Solving the circuit of Figure 9 for the current gives

$$i_{ch} = (E_{bb} - E_e - Y_2) / (R_{L1} \parallel \bar{r}_{g2} \parallel R_e) \quad (40)$$

where  $R_t = (R_{L1} \parallel \bar{r}_{g2} \parallel R_e)$ . If we now let  $R_1$  represent  $(R_k \parallel r_{p2} \parallel R_{L2})$  and substitute equations (38) and (39) in equation (40), remembering

that  $e_g = i_{ch} \bar{r}_{g2}$ , we obtain, after some manipulation

$$i_{ch} = \frac{\frac{1}{R_t} [E_{bb} (1 - R_k/R_1) - Y_2] e^{-\frac{t}{R_t C}}}{1 - (\mu \bar{r}_{g2} R_k / R_1 R_t) e^{-\frac{t}{R_t C}}} \quad (41)$$

We consider the zero time reference to be the instant the grid voltage of  $V_2$  reaches cutoff and immediately jumps to a positive value. Since the switching transient takes a small, but finite, time for completion, this assumption while adequate for our purpose, is not physically exact.

The value of  $i_{ch}$  for  $t = 0$  reduces to

$$i_{ch}(0) = \frac{E_{bb} (1 - R_k/R_1) - Y_2}{R_t \parallel \mu \bar{r}_{g2} R_k} \quad (42)$$

The maximum value of  $e_{g2}(0)$  is given by

$$e_{g2}(0) = i_{ch}(0) \bar{r}_{g2} \quad (43)$$

The corresponding value of plate current,  $i_{b2}(0)$ , could be determined by inserting the correct values in equation (4). However, since the value of  $r_{p2}$  must be obtained from the characteristic curves, it is



The values of  $R_1$  and  $R_2$  are given by

$$R_1 = R_1 \sqrt{1 + \frac{R_1}{R_2}} \quad (28)$$

$$R_2 = (R_2 + \frac{R_1}{R_2}) \sqrt{1 + \frac{R_1}{R_2}} \quad (29)$$

Solving the circuit of figure 2 for the current gives

$$i_{ch} = (R_2 - R_1 - \gamma_2) \sqrt{1 + \frac{R_1}{R_2}} \quad (30)$$

where  $R_1 = (R_1 + \frac{R_1}{R_2})$ . If we now let  $R_1$  represent  $(R_1 + \frac{R_1}{R_2})$  and substitute equation (28) and (29) in equation (30), remembering

that  $\frac{R_1}{R_2} = \frac{1}{\gamma_2}$ , we obtain, after some manipulation

$$i_{ch} = \frac{\frac{1}{R_1} [E_{ch} (1 - R_1 \sqrt{R_2}) - \gamma_2] e^{-\frac{t}{R_1 C}}}{1 - (\frac{R_1}{R_2}) \sqrt{R_2} e^{-\frac{t}{R_1 C}}} \quad (31)$$

We consider the case in which reference is to be made to the grid voltage of  $V_2$  reaches cutoff and immediately jumps to a positive value. Since the switching transient takes a small, but finite, time for completion, this assumption will be adequate for our purpose, is not physically exact.

The value of  $i_{ch}$  for  $t = 0$  reduces to

$$i_{ch}(0) = \frac{E_{ch} (1 - R_1 \sqrt{R_2}) - \gamma_2}{R_1 + \frac{R_1}{R_2}} \quad (32)$$

The maximum value of  $i_{ch}(0)$  is given by

$$i_{ch}^{\max}(0) = \frac{E_{ch}}{R_1 + \frac{R_1}{R_2}} \quad (33)$$

The corresponding value of plate current,  $i_{pb}(0)$ , could be determined by inserting the correct values in equation (1). However, since the value of  $\gamma_2$  must be obtained from the characteristic curves, it is



simpler to read the current value from the intersection of the  $(R_k \neq R_{L2})$  load-line and the  $e_{g2}(0)$  line. The  $t = 0$  values of plate and cathode voltage of  $V_2$  are given by

$$e_{bn2}(0) = E_{bb} - i_{b2}(0)R_{L2}, \quad (44)$$

$$E_k(0) = [i_{ch}(0) \neq i_{b2}(0)]R_k. \quad (45)$$

As would be expected, these equations reduce to equations (5) and (6) as  $t$  becomes indefinitely large. It should be noticed, from equation (41), that  $i_{ch}$  does not decrease as a pure exponential. The initial decay is faster than the pure exponential decay although, since  $(\mu \bar{F}_{g2} R_k) / (R_1 R_t) < 1$ , the difference is slight after  $2R_1 C$  seconds. Hence, for sketching the waveform, we may use an exponential curve of time constant  $R_1 C$  without appreciable error.

After the charging current has decreased to zero, the circuit remains in the steady-state condition until the next trigger pulse initiates the cycle of operation.

#### IV. THE COMPLETE CYCLE

The two preceding sections have furnished the values needed to make a sketch of the waveforms for a complete cycle of operation of the monostable cathode-coupled multivibrator. For convenience, a tabulation of equation numbers and short comments are given in Table I. It remains only to sketch the stable state values on the time axis, following this with the timing state values. The comments of Table I give further information.



slightly to read the current value from the intersection of the  $i_c$  and  $i_a$  load-line and the  $e_c(0)$  line. The  $t = 0$  values of  $i_c$  and  $i_a$  and voltage of  $V_c$  are given by

$$i_c(0) = I_{c0} - I_{c1} e^{-t/\tau_c} \quad (1)$$

$$i_a(0) = I_{a0} + I_{a1} e^{-t/\tau_a} \quad (2)$$

As would be expected, these equations reduce to equations (1) and (2) as  $t$  becomes infinitesimally large. It should be noted, from equation (1), that  $i_c$  does not decrease as a pure exponential, the initial decay is faster than the pure exponential decay because  $i_c$  is slightly above  $i_a$  at  $t = 0$ , the difference in slight above  $i_a$  is  $(V_{c0} - V_{a0}) / (R_c + R_a) > 0$ , for sketching the waveforms, we may use an exponential decay of  $i_c$  as a guide  $R_c$  without appreciable error. After the charging current has decreased to zero, the circuit remains in the steady-state condition until the next trigger pulse initiates the cycle of operation.

#### IV. THE QUANTILE CIRCUIT

The two preceding sections have furnished the values needed to make a sketch of the waveforms for a complete cycle of operation of the monostable cathode-coupled multivibrator. The waveforms, a tabulation of equation numbers and circuit constants are given in Table I. It remains only to sketch the steady state values of the circuit following this with the timing state values. The comments of Table I give further information.



TABLE I

## EQUATIONS GIVING WAVEFORM VALUES

WAVEFORM	EQUATION NUMBER		COMMENTS
	Stable state	Timing* state	
$e_{bn1}$	(8)	(21)	Constant voltage for each state.
$e_{bn2}$	(6)	(-)	Constant voltage for stable state. $E_{bb}$ for timing state.
$e_{cn1}$	(-)	(-)	Constant voltage of $E_{cc}$ except for trigger pulses.
$e_{cn2}$	(7)	(22)	Constant voltage for stable state. Exponential rise for timing state.
$e_k$	(5)	(25)	Constant voltage for stable state. Exponential rise for timing state

\* After the timing state is completed, the trailing edge of the  $e_{bn1}$ ,  $e_{bn2}$ ,  $e_{cn2}$ , and  $e_k$  waveforms have exponential shapes for a short-time before the stable state values are reached. These may be approximated from equations (42) through (45) by using  $R_t C$  as the time constant.



TABLE I

EQUATIONS GIVING WAVEFORM VALUES

COMMENTS	EQUATION NUMBER		WAVEFORM
	Timing state	Logic state	
Constant voltage for each state.	(21)	(8)	$V_{out}$
Constant voltage for stable state. $V_{out}$ for timing state.	(-)	(6)	$V_{out}$
Constant voltage of $V_{cc}$ except for output pulses.	(-)	(-)	$V_{out}$
Constant voltage for stable state. $V_{out}$ potential rise for timing state.	(22)	(7)	$V_{out}$
Constant voltage for stable state. $V_{out}$ potential rise for timing state.	(23)	(2)	$V_{out}$

\* After the timing state is completed, the trailing edge of the  $V_{out}$  pulse and a waveform have exponential decay for a short time before the stable state values are reached. These may be approximated from equations (22) through (23) by using  $V_{out}$  as the time constant.



V. COMPARISON OF RESULTS WITH RESULTS FROM  
ANOTHER ANALYSIS

The preceding analysis employed a number of approximations, and neglected terms in equations when these terms were small in comparison with the remaining terms. The justification for so doing lies in the fact that the difference between exact and approximate values is comparatively small while the savings in mathematical labor is usually considerable. The errors arising from such a treatment are of the order of a few per cent and thus are too small to be of any practical importance. Since the D.C. voltage term  $E_0$ , which has usually been neglected in previous analyses, has been included in the calculations, it would not seem too surprising if the results obtained from this analysis were not far different from the results obtained by using graphical methods and the tube characteristic curves. A comparison of the results given by these methods is shown in Table II; The circuit of Principles of Radar, M.I.T. Radar School Staff, 2nd Ed. page 2-58, is used as a standard. Values of the tube parameters, as obtained from the published values and characteristic curve are:  $g_m = 2600$  micromhos,  $r_{p1} = 7700$  ohms,  $r_{p2} = 7500$  ohms. The percentage difference in results obtained by the two methods is also given.

An examination of the percentage differences given in Table II shows that results obtained from the two methods of analysis are quite comparable and well within slide rule accuracy. The largest difference, that of  $e_{cn2}$ , is due mainly to the method of computation---taking the difference between two numbers of the same magnitude---but the accuracy



V. COMPARISON OF RESULTS WITH RESULTS FROM

ANOTHER ANALYSIS

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In examination of the percentage differences given in Table II shows that results obtained from the two methods of analysis are quite comparable and well within this accuracy. The largest difference that of  $\epsilon_{02}$  is due mainly to the method of computation--taking the difference between two numbers of the same magnitude--but the accuracy



TABLE II  
 COMPARISON OF RESULTS WITH RESULTS OF  
 ANOTHER METHOD

Item	This Analysis	M.I.T. Analysis	% Difference
Stable State			
$e_{bn1}$	300	300	0
$e_{bn2}$	199	198.5	0.25
$e_{cn1}$	70	70	0
$e_{cn2}$	102	101.5	0.5
$e_k$	102	101.5	0.5
Timing State			
$e_{bn1}$	156.8	158.4	1.0
$e_{bn2}$	300	300	0
$e_{cn1}$	70	70	0
$e_{cn2}$	-41.2	-40.1	2.5
$e_k$	71.6	70.8	1.1
T	68.1	67.5	0.9



TABLE II

COMPARISON OF ANALYSES OF THE

SOLUBLE FRACTIONS

Sample No.	Analysis	Reference
100	100	100
101	101	101
102	102	102
103	103	103
104	104	104
105	105	105
106	106	106
107	107	107
108	108	108
109	109	109
110	110	110
111	111	111
112	112	112
113	113	113
114	114	114
115	115	115
116	116	116
117	117	117
118	118	118
119	119	119
120	120	120



is sufficient for most practical cases. This comparison gives a check on the accuracy of this method of analysis, and gives confidence that the approximations used will not greatly distort the resulting values.

## VI. DESIGN CONSIDERATIONS

While the basic design equations were given on pages 16 and 17 there remain questions as to the order of magnitude of the remaining parameters. Several of the following considerations are of a general nature and are intended to appear reasonable but not to constitute a proof.

It will be remembered that, for a triode, the value of the amplification factor falls off at the lower values of plate current, and that in this region  $g_m$  and  $r_p$  change quite rapidly. Thus, there is a lower limit to the value of plate current which permits operation in the region of fairly constant  $r_p$ . For the triodes normally used this would be a plate current of about 10 milliamperes. A fairly standard practice is to choose  $R_{L2}$  approximately equal to  $R_k$ . Assuming this to be the case, a rough center value for  $R_{L2}$  and  $R_k$  can be found from the equation

$$R_{L2} = R_k = E_{bb}/21b_2 - r_p/2, \quad (45)$$

which, for a plate current of 10 milliamperes becomes

$$R_{L2} = 50E_{bb} - r_p/2.$$

A similar range of values for these resistors might have been computed from a consideration of the circuit as a wide band amplifier. A wide latitude of choice is available as to the values used.



is sufficient for most practical cases. This expression gives a check on the accuracy of this method of analysis, and gives confidence that the approximations used will not greatly distort the resulting values.

### VI. DESIGN CONSIDERATIONS

While the basic design equations were given on pages 16 and 17 there remain questions as to the order of magnitude of the resulting parameters. Several of the following considerations are of a general nature and are intended to appear reasonably but not to constitute a proof.

It will be remembered that for a definite, the value of the amplification factor falls off as the lower values of plate current, and that in this region  $\mu_p$  and  $r_p$  change quite rapidly. Thus, there is a lower limit to the value of plate current which permits operation in the region of fairly constant  $r_p$ . For the tubes normally used this would be a plate current of about 10 milliamperes. A fairly good and practice is to choose  $I_p$  approximately equal to  $I_{p0}$ , remaining this to be the case, a rough center value for  $I_p$  and  $r_p$  can be found from the equation

$$r_p = \frac{2000}{I_p} \quad \text{which, for a plate current of 10 milliamperes becomes}$$
$$r_p = 200 \quad \text{ohms}$$

A similar range of values for linear resistors might have been suggested from a consideration of the circuit as a whole. A wide latitude of choice is available as to the values used.

PROCEEDINGS  
CASE BOARD  
CONTENT



The variation of  $g_m$  and  $r_p$  with the operating point of the tube may cause some uncertainty as to which values of these parameters to use in the calculation of  $G_r'$ , which appears in so many of the equations. It can be shown by taking the total differential that  $G_r'$  is relatively insensitive to variations of  $g_m$  and  $r_p$  and, hence, that satisfactory results are obtained by using the values published under the tube typical operating characteristics.

As was mentioned earlier, some control of the pulse magnitude is possible through the choice of resistance values. When equation (45) is used as a starting point, the resistance values depend to some extent on the supply voltage; in this case a rough rule-of-thumb is that the upper limit of the  $e_{bn_1}$  pulse magnitude is  $E_{bb}/2$ , and the upper limit of the  $e_{bn_2}$  pulse magnitude is  $E_{bb}/3$ .

The bias voltage  $E_{cc}$  has been represented by a battery in the foregoing analysis. In practice, the bias would be secured by means of a variable resistance voltage divider network. The resistance values should be large enough to limit  $V_1$  grid current to safe values if large positive trigger pulses are used. To avoid over shoot on the leading edges of the timing cycle waveforms, the value of the trigger pulse should be just sufficient to raise the grid of  $V_1$  above cutoff.

The variations in tube parameters from tube to tube, usually an important factor in the design of an electronic circuit, do not have a great effect on the operation of this circuit since the degenerative feedback action minimizes the effect of these variations. Variations in the value of static grid resistance,  $\bar{r}_g$ , of  $V_2$  due to differences



The variation of  $k_{eff}$  and  $r_p$  with the operating point of the tube may cause some uncertainty as to which values of these parameters to use in the calculation of  $\theta_{eff}$ , which appears in so many of the equations. It can be shown by taking the total differential that  $\theta_{eff}$  is relatively insensitive to variations of  $k_{eff}$  and  $r_p$  and, hence, that satisfactory results are obtained by using the values furnished under the type and operating characteristics.

As was mentioned earlier, close control of the pulse magnitude is possible through the choice of resistance values. When equation (4) is used as a starting point, the resistance values depend to some extent on the supply voltage; in this case a rough rule-of-thumb is that the upper limit of the pulse magnitude is  $E_{dc}/\sqrt{2}$ , and the upper limit of the pulse magnitude is  $E_{dc}/\sqrt{2}$ .

The bias voltage  $E_{dc}$  has been represented by a battery in the foregoing analysis. In practice, the bias would be secured by means of a variable resistance voltage divider network. The resistance values should be large enough to limit grid current to safe values if large positive trigger pulses are used. To avoid over shoot on the leading edges of the timing cycle waveforms, the value of the trigger pulse should be just sufficient to raise the grid to  $V_1$  above cutoff.

The variations in tube parameters from tube to tube, usually an important factor in the design of an electronic circuit, do not have a great effect on the operation of this circuit since the degenerative feedback action minimizes the effect of these variations. Variation in the value of static grid resistance,  $r_g$ , or  $V_1$  has no effect.



in electrode spacing, is not counteracted by the feedback and, hence, allows considerable differences to exist in the stable state operation for different tubes. The use of a clamping diode has been suggested as a remedy.<sup>2</sup>

Deviations in the values of the resistors used can, under the worst conditions, lead to results differing from the calculated results by more than twice the percentage deviation of the resistors. This should be kept in mind during the design of this circuit and if a number of multivibrators are to be built, either 1 % resistors or matched sets should be used to ensure similar characteristics.

#### VII. SUMMARY

This chapter has presented a qualitative explanation of the operation of the monostable cathode-coupled multivibrator, followed by a quantitative analysis of the two separate systems into which the circuit can be divided. For the analysis of the timing state, the procedure closely followed that which would be used in analyzing an RC coupled amplifier, the only difference being a slight increase in complexity. The two equations, (22) and (25), from which most other timing state waveform values can be derived, contain the factor  $G_r'$ , the effective gain of the RC coupled amplifier with cathode degeneration, thus showing that from analytical considerations, the circuit can be considered

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<sup>2</sup> Chance, Britton, *et al.*, Waveforms (New York: McGraw-Hill Book Company, Inc., 1949), p. 168.



in electrode spacing, is not counteracted by the feedback and, hence, allows considerable differences to exist in the stable state operation for different tubes. The use of a changing grid has been suggested as a remedy.<sup>2</sup>

Deviations in the values of the resistors used can, under the worst conditions, lead to results differing from the calculated results by more than twice the percentage deviation of the resistors. This should be kept in mind during the design of this circuit and if a number of resistors are to be built, either 1% resistors or matched sets should be used to ensure similar characteristics.

VII. SUMMARY

This chapter has presented a qualitative explanation of the operation of the nonstable cathode-coupled triode, followed by a quantitative analysis of the two separate systems into which the circuit can be divided. For the analysis of the triode stage, the procedure closely followed that which would be used in analyzing an RC coupled amplifier, the only difference being a slight increase in complexity. The two equations, (22) and (23), from which most other results were derived, contain the factor  $\beta$ , the effective gain of the RC coupled amplifier with cathode degeneration, thus showing that from analytical considerations, the circuit can be considered

<sup>2</sup> Chance, Bittner, et al., Waveforms (New York: McGraw-Hill Book Company, Inc., 1952), p. 168.



as an offshoot of the degenerative amplifier.

Next, a comparison of values obtained using the equations derived in this analysis was made with values obtained by another method. For the particular case used, the two methods gave quite comparable results.

The basic design equations were pointed out, and, from general considerations, a method of choosing values of  $R_k$  and  $R_{L_2}$  was given by equation (45). This equation can readily be extended to ratios of  $R_k/R_{L_2}$  other than the one used above.

Finally, some discussion was given to the effects of the variability of tubes and resistors on the operation of the circuit.

From the analytical aspect, the first purpose of this investigation has been accomplished.



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as an offshoot of the...  
Next, a report...  
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CHAPTER III

CONDITIONS WHICH GIVE NEARLY LINEAR  
RELATIONSHIP BETWEEN PULSE DURATION  
AND APPLIED GRID VOLTAGE

In the preceding chapter no mention was made of the fact that a very nearly linear relation between the bias voltage on tube  $V_1$  and the duration of the timing state pulse can be achieved by proper choice of the circuit parameters. It will be shown in the following section that this phenomena has an analytical explanation. The means whereby this condition may be achieved will be investigated.

I. ESTABLISHMENT OF LINEAR RELATION

The pulse duration,  $T$ , of the monostable cathode-coupled multi-vibrator was given by equation (31) in Chapter II. For convenience this equation is repeated.

$$T = R_g C \ln \frac{E_{bb} - E_k + \left(\frac{E_{bb}}{\mu} + E_{cc}\right) G_r'}{E_{bb} - E_{co_2} - e_k} \quad (31)$$

On replacing  $e_k$  by the value given in equation (25), this becomes

$$T = R_g C \ln \frac{E_{bb} \left(1 + \frac{G_r'}{\mu}\right) - E_k + \left(\frac{E_{bb}}{\mu} + E_{cc}\right) G_r'}{E_{bb} - E_{co_2} - \left(\frac{E_{bb}}{\mu} + E_{cc}\right) G_r' R_k / R_{L1}} \quad (46)$$

This equation can be written in a form more suitable for algebraic manipulation as

$$T = R_g C \ln \frac{a + b E_{cc}}{d + e E_{cc}} \quad (47)$$



RELATIONSHIP BETWEEN RATE OF REACTION  
AND INITIAL CONCENTRATION

RAC CONTENT

IN THE PRESENCE OF A...  
A VERY NEARLY LINEAR...  
THE DURATION OF THE...  
OF THE CIRCULAR...  
THAT THIS PHENOMENON...  
THIS CONDITION MAY BE...  
1. INVESTIGATION OF...  
THE REACTION...  
VARIABLE WAS GIVEN BY...  
THIS EQUATION IS...  
(1) 
$$r = \frac{k_1 [A] [B]}{k_2 + [B]}$$
  
ON REARRANGING...  
(2) 
$$r = \frac{k_1 [A] [B]}{k_2 + [B]}$$
  
THIS EQUATION CAN BE...  
MANIPULATION AS  
(3) 
$$r = \frac{k_1 [A] [B]}{k_2 + [B]}$$



where

$$a = E_{bb}(1 - G_T'/\mu) - E_k \quad (47)$$

$$b = G_T' \quad (49)$$

$$d = E_{bb}(1 - G_T'R_k/\mu R_{L1}) - E_{CO2} \quad (50)$$

$$e = G_T'R_k/R_{L1} \quad (51)$$

Equation (47) can be rewritten in the form

$$T = R_g C \ln \left[ \frac{a}{d} \left( \frac{1 + \frac{b}{a} E_{cc}}{1 - \frac{e}{d} E_{cc}} \right) \right] \quad (52)$$

from which, by expanding the logarithm, we obtain

$$T = R_g C \left[ \ln \frac{a}{d} - \ln \left( 1 - \frac{b}{a} E_{cc} \right) - \ln \left( 1 - \frac{e}{d} E_{cc} \right) \right]. \quad (53)$$

Now, from calculus it will be remembered that the function  $\ln(1 - x)$  can be expanded in a Maclaurin's series which converges to the function for all values of  $x$  in the interval  $-1$  to  $1$ . This expansion is

$$\ln(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots \quad (54)$$

If  $x$  is much smaller than 1, all but the first term of this series can be neglected since the higher order terms will be correspondingly smaller and will have little effect upon the value given by the series. When  $x$  is positive, the series alternates and the error made by stopping at any term is numerically less than that term. This property gives an easy method of determining the error introduced into the calculations by replacing the second logarithmic term in equation (53) by the first term of the corresponding series. For negative  $x$ , the series no longer alternates and another method of estimating the error must be used. Another method for so doing does exist, and fortunately, is quite



where

(17)

$$a = E_{00}(1 + \frac{1}{2} \alpha^2) - R_x$$

(18)

$$b = U_x$$

(19)

$$d = E_{00}(1 - \frac{1}{2} \alpha^2) - R_{00}$$

(20)

$$e = E_x \sqrt{R_{00}}$$

Equation (17) can be written in the form

(21)

$$T = R_0 \ln \left[ \frac{1 + \frac{1}{2} \alpha^2 E_{00}}{1 - \frac{1}{2} \alpha^2 E_{00}} \right]$$

from which, by expanding the logarithm, we obtain

(22)

$$T = R_0 \left[ \frac{1}{3} \alpha^2 E_{00} - \frac{1}{45} \alpha^4 E_{00}^2 + \dots \right]$$

Now, from calculus it will be remembered that the function

$\ln(1+x)$  can be expanded in a MacLaurin's series which converges to

the function for all values of  $x$  in the interval  $-1 < x < 1$ . This ex-

pression is

(23)

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

If  $x$  is much smaller than 1, all but the first term of this series can

be neglected since the higher order terms will be correspondingly

smaller and will have little effect upon the value given by the series.

When  $x$  is positive, the series alternates and the error made by stop-

ping at any term is numerically less than that term. This property

gives an easy method of determining the error introduced into the cal-

culations by replacing the second logarithmic term in equation (21) by

the first term of the corresponding series. For negative  $x$ , the error

no longer alternates and another method of estimating the error must be

used. Another method for so doing does exist, and fortunately, is quite



simple to apply to this series.<sup>1</sup> The error is given by the Lagrangian form of the remainder after  $n$  terms of the series, where the remainder is denoted by

$$R_n = f^{(n)}(\xi) x^n/n!, \quad 0 < \xi < x, \quad (55)$$

where  $f^{(n)}(\xi)$  denotes the  $n$ th derivative of  $f$ , evaluated at  $x = \xi$ .

Using this method for the case  $n = 2$ , we find the error to be numerically less than  $\frac{1}{2}x^2/(1-x)^2$ , when the function  $\ln(1-x)$  is replaced by the first term of the series (54).

If, for the present, we assume that the quantities  $bE_{cc}/a$  and  $eE_{cc}/d$  are sufficiently small to permit using the first term of the corresponding series without causing excessive error, we can write equation (53) in the form

$$T = R_g C \left[ \ln \frac{a}{d} + \frac{b}{a} E_{cc} + \frac{e}{d} E_{cc} \right] \quad (56)$$

or

$$T = R_g C \left[ \ln \frac{a}{d} + \left( \frac{b}{a} + \frac{e}{d} \right) E_{cc} \right]. \quad (57)$$

It is evident from equation (57) that  $T$  is a linear function of  $E_{cc}$ .

## II. ATTAINMENT OF LINEAR CONDITIONS

The derivation of equation (57) was made possible by the assumption that  $bE_{cc}/a$  and  $eE_{cc}/d$  were small in relation to unity. A close inspection of equations (48) through (51) will show that, unless  $R_k$  is

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<sup>1</sup> Sokolnikoff, Ivan S., and Sokolnikoff, Elizabeth S., Higher Mathematics for Engineers and Physicists (New York: McGraw-Hill Book Company, Inc., 1941), p. 36.



able to apply to the case of the remainder of the series, where the remainder is denoted by

$$R_n = \sum_{k=n}^{\infty} a_k x^k$$

where  $\epsilon^{(n)}$  denotes the error term in (2) and  $\epsilon^{(n)} = O(x^n)$ . Using this notation we may write (2) as

$$f(x) = \sum_{k=0}^{n-1} a_k x^k + R_n(x) \quad (2')$$

It is now known that the remainder  $R_n(x)$  is  $O(x^n)$  and  $\epsilon^{(n)}$  are sufficiently small to permit writing the first term of the corresponding series which carries the error term as  $\epsilon^{(n)}$  then (2') in the form

$$f(x) = \sum_{k=0}^{n-1} a_k x^k + \epsilon^{(n)}$$

or  

$$f(x) = \sum_{k=0}^{n-1} a_k x^k + \epsilon^{(n)}$$
 It is evident from equation (2') that  $\epsilon^{(n)}$  is a small number of

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The derivation of equation (2') was made by the method of inspection of equation (2) through (2') for each value of  $n$  such that  $\epsilon^{(n)}$  and  $\epsilon^{(n)}$  are both in parentheses with  $\epsilon^{(n)}$ .

<sup>1</sup> See also, "The Theory of Functions of a Real Variable", by G. B. Folland, Jr., McGraw-Hill, New York, 1930.  
 Reprinted from "The Theory of Functions of a Real Variable", by G. B. Folland, Jr., McGraw-Hill, New York, 1930.  
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considerably greater than  $R_{L_1}$ ,  $b/a$  will be greater than  $e/d$  so that, in general, if the inequality

$$bE_{cc}/a \ll 1 \quad (58)$$

is satisfied, the inequality

$$eE_{cc}/d \ll 1 \quad (59)$$

will also be satisfied. Writing equation (58) in its complete form and setting it equal to a constant  $A$ , we obtain

$$\frac{b}{a} E_{cc} = \frac{G_r' E_{cc}}{E_{bb} \left(1 + \frac{G_r'}{\mu}\right) - E_k} = A, \quad A \ll 1 \quad (60)$$

Multiplying this equation through by the denominator and collecting terms in  $G_r'$ , we obtain

$$G_r' \left[ E_{cc} - (A/\mu) E_{bb} \right] = A (E_{bb} - E_k). \quad (61)$$

We let  $E_{cc}/E_{bb} = B$ , and substitute the value of  $E_k$  as given by equations (4) and (5) in equation (61) obtaining

$$G_r' \left[ B E_{bb} - A E_{bb}/\mu \right] = A \left[ E_{bb} - \frac{E_{bb} - E_0}{R_k \neq R_{L_2} \neq r_{p_2}} R_k \right]. \quad (62)$$

The inequality

$$E_{bb} \gg E_0 \quad (63)$$

is generally fulfilled. Hence the term containing  $E_0$  can be neglected and equation (62) becomes, after some manipulation

$$G_r' \left[ B - A/\mu \right] = \frac{A (R_{L_2} \neq r_{p_2})}{R_k \neq R_{L_2} \neq r_{p_2}}. \quad (64)$$

Letting  $(R_{L_2} \neq r_{p_2}) / (R_k \neq R_{L_2} \neq r_{p_2})$  be represented by  $R_2$ , we can simplify this last expression to



considerably greater than  $\frac{1}{2}$ , it will be seen that in general, if the inequality

$$\frac{1}{2} < \frac{1}{\sqrt{1 - \frac{1}{2} \frac{d}{d_0}}} < 1$$

is satisfied, the inequality

$$\frac{1}{2} < \frac{1}{\sqrt{1 - \frac{1}{2} \frac{d}{d_0}}} < 1$$

will also be satisfied. In this case, the inequality (10) in the preceding section and

setting it equal to a constant  $\frac{1}{2}$ , we obtain

$$\frac{d}{d_0} = \frac{1}{\sqrt{1 - \frac{1}{2} \frac{d}{d_0}}} - 1 = \frac{1}{\sqrt{1 - \frac{1}{2} \frac{d}{d_0}}} - 1$$

Multiplying this equation through by the denominator and rearranging

terms in  $d$ , we obtain

$$d^2 - \frac{1}{2} d = \frac{1}{2} d_0^2$$

We let  $\frac{d}{d_0} = x$ , and substituting the value of  $x$  in equation (10)

(A) and (2) in equation (12) obtaining

$$d^2 \left[ \frac{1}{2} x^2 - \frac{1}{2} x \right] = \frac{1}{2} d_0^2$$

The inequality

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$$\frac{1}{2} x^2 - \frac{1}{2} x = \frac{1}{2} \frac{d_0^2}{d^2}$$

Setting  $\frac{1}{2} x^2 - \frac{1}{2} x = \frac{1}{2} \frac{d_0^2}{d^2}$  or rearranging by  $\frac{1}{2} x^2 - \frac{1}{2} x - \frac{1}{2} \frac{d_0^2}{d^2} = 0$

gives this last expression as



$$G_r' (B - A/\mu) = AR_2. \quad (65)$$

Since

$$G_r' = (\mu R_{L_1}) / [R_{L_1} \neq r_{P_1} \neq R_k(\mu \neq 1)] = (\mu R_{L_1}) / (R_{L_1} \neq R_1) \quad (66)$$

where  $R_1$  represents  $[r_{P_1} \neq R_k(\mu \neq 1)]$ , equation (65) can be written as

$$(\mu R_{L_1}) / (R_{L_1} \neq R_1) = (AR_2) / (B - A/\mu) \quad (67)$$

which, when solved for  $R_{L_1}$  gives

$$R_{L_1} = (AR_1 R_2) / (B\mu - AR_2 - A). \quad (68)$$

This can be rewritten as

$$R_{L_1} = (R_1 R_2) / [\mu C - (1 \neq R_2)], \quad (69)$$

where  $C = B/A$ .

Once  $R_{L_1}$  has been determined, it is advisable to compute the value of  $eE_{cc}/d$  to assure that it is less than or equal to  $bE_{cc}/a$ . If this is not found to be true, the ratio of  $R_k/R_{L_2}$  should be changed to allow this condition to be fulfilled.

It might be thought that the value of  $R_{L_1}$  could be determined by setting  $eE_{cc}/d$  equal to  $A$  and solving for  $R_{L_1}$ ; however, on expanding  $eE_{cc}/d$  and examining the result, we will see that its value changes little for large changes of  $R_{L_1}$ , and consequently, this equation is too insensitive to use as a means of computing the load resistance.

In using the first term of the series expansion of equation (54), we obtain a result which is larger than the correct value. The corresponding value for  $\ln(1 - x)$  is smaller than the correct value. The combination of these two terms in equation (57) is such that these



(22)

$$G_{11}^{-1} = \frac{1}{\Delta} \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$$

Since

(23)

$$G_{11}^{-1} = \frac{1}{\Delta} \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$$

where  $R_{11}$  represents  $\begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$ , equation (23) can be written as

(24)

$$(R_{11}^{-1})_{11} = \frac{1}{\Delta} \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$$

which, when solved for  $R_{11}^{-1}$  gives

(25)

$$R_{11}^{-1} = \frac{1}{\Delta} \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$$

This can be rewritten as

(26)

$$R_{11}^{-1} = \frac{1}{\Delta} \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$$

where  $\Delta = \Delta$ .

Once  $R_{11}^{-1}$  has been determined, it is advisable to compute the value of  $\Delta$  to assure that it is non-zero or equal to  $\Delta$ . This is not found to be true, the ratio of  $\Delta$  to  $\Delta$  should be checked to allow this condition to be fulfilled.

It might be thought that the value of  $R_{11}^{-1}$  could be determined by setting  $\Delta$  equal to 1 and solving for  $R_{11}^{-1}$ ; however, to expand the  $\Delta$  and expanding the results, as will be seen, the value changes little for large changes of  $R_{11}^{-1}$ , and consequently, this equation is too insensitive to use as a means of computing the load resistance.

In using the first term of the series expansion of equation (24), we obtain a result which is larger than the correct value. The corresponding value for  $\ln(1-x)$  is smaller than the correct value. The combination of these two terms in equation (24) is such that they

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two errors partially cancel each other, thus aiding the approach to a linear relationship. Since this is the case, it would seem reasonable to design the circuit so that  $bE_{cc}/a$  and  $eE_{cc}/d$  were equal in the hope that the errors would cancel completely. To obtain this relation, we equate the values

$$\frac{G_r' E_{cc}}{E_{bb}(1 + G_r'/\mu) - E_k} = \frac{G_r' R_k E_{cc}/R_{L1}}{E_{bb}(1 - G_r' R_k/\mu R_{L1}) - E_{co2}} \quad (70)$$

Assuming that the terms in the denominators which contain  $G_r'$  are small compared to unity and neglecting the  $E_{co2}$  term, we find, after substituting values for the remaining terms, that  $R_{L1}$  is given approximately by

$$R_{L1} = \frac{R_k(r_{p2} + R_{L2})}{R_k + r_{p2} + R_{L2}} \quad (71)$$

Equation (71) gives results which are quite close to those which would be obtained by using the equation given by Glegg in his analysis of this circuit.<sup>2</sup> This is not surprising since his method, basically, consisted of setting  $b/a$  equal to  $e/d$  and solving the resulting equation for  $R_{L1}$ .

Unfortunately, one drawback to this method lies in the fact that the error term  $\frac{1}{2}x^2/(1-x)^2$  grows more rapidly for increasing values of  $E_{cc}$  than does the corresponding error term  $\frac{1}{2}x^2$ . The result is that in the higher grid voltage ranges the first error term is predominant and causes a deviation from linearity. From these considerations

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<sup>2</sup> Glegg, Keith, "Cathode-Coupled Multivibrator Operation," Proc. IRE, 38:655, June, 1950







it will be seen that the first method, that of having  $e/d$  less than  $b/a$ , will give more linear results when the range of pulse duration requires a large change in applied grid voltage. Table III shows a comparison of both methods for typical operating values. The table shows the percentage error between the actual values and the approximate values as given by equation (57). It will be noted that in each case the error is considerably smaller than the error between the actual value of the logarithmic terms and their approximate value. Over the ordinary operating range, approximately from 30 volts to 90 volts, the percentage error for the case  $b/a = e/d$  is about  $\pm 3.5$  per cent; for the case  $e/d = 0.6 b/a$ , the error is about  $\pm 0.75$  per cent; for the case  $e/d = 0.8 b/a$ , the error is about  $\pm 1$  per cent. While it is fairly obvious that an optimum relation between  $b/a$  and  $e/d$  can be found by trial and error, it should be noted that this relation would not hold for other values of  $b/a$ .

Two methods have been presented for finding values of  $R_{L1}$  which will result in a close approach to linearity between the pulse duration and the applied grid voltage. It will be instructive to apply the first method to the circuit given by Seely, using his values of  $R_{L2}$  and  $R_k$ .<sup>3</sup> Values of 300, 7700, 7500, and 20 are used for  $E_{bb}$ ,  $r_{p1}$ ,  $r_{p2}$ , and respectively. We choose  $B$  equal to 0.2, or  $E_{cc}$  equal to 60 volts for the maximum grid voltage,  $A$  equal to 0.3, or  $b/a(60) = 0.3$ . Applying

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<sup>3</sup> Seely, Samuel, Electron-tube Circuits (New York: McGraw-Hill Book Company, Inc., 1950), p. 427.







TABLE III

COMPARISON OF EFFECT OF GRID VOLTAGE CHANGE  
ON ERROR FOR TWO METHODS OF DESIGN

b/a	e/d	$E_{cc}$	$x_1$	$x_2$	$\ln(1 + x_1)$	$\ln(1 - x_2)$	% Error
.005	.005	10	.05	-.05	.04879016	-.05129329	-0.09
		30	.15	-.15	.13976194	-.16251893	-0.76
		60	.30	-.30	.26236426	-.35667494	-3.3
		90	.45	-.45	.37156356	-.59783700	-7.7
		120	.60	-.60	.47000363	-.91629073	-15.0
.005	.003	10	.05	-.03	.04879016	-.03045921	+0.94
		30	.15	-.09	.13976194	-.09431068	+2.5
		60	.30	-.18	.26236426	-.19845094	+4.0
		90	.45	-.27	.37156356	-.31471074	+4.0
		120	.60	-.36	.47000363	-.44628710	+4.4
.005	.004	10	.05	-.04	.04879016	-.04082199	+0.43
		30	.15	-.12	.13976194	-.12783337	+0.9
		60	.30	-.24	.26236426	-.27443684	+0.57
		90	.45	-.36	.37156356	-.44628710	-0.97
		120	.60	-.48	.47000363	-.65392647	-4.0



TABLE III

COMPARISON OF STATION OF LINES Y AND Z

IN YEAR 1, 1950

TABLE CONTENT

Line	Station	Year	Value	Value	Value
100	100	1950	100	100	100
90	90	1950	90	90	90
80	80	1950	80	80	80
70	70	1950	70	70	70
60	60	1950	60	60	60
50	50	1950	50	50	50
40	40	1950	40	40	40
30	30	1950	30	30	30
20	20	1950	20	20	20
10	10	1950	10	10	10



the equations we have derived in this section, we find  $R_{L_1}$  is given as 5060 ohms, in quite good agreement with Seely's value of 5000 ohms. Rounding this to 5000 ohms and continuing the analysis, we find the following values:

$$G_p' = 1.29, b/a = 0.00488, e/d = 0.003433,$$

$$a/d = 0.88.$$

The pulse duration for  $E_{cc} = 60$  volts is 113 microseconds with an error of  $\pm 0.25\%$  from the exact value; for  $E_{cc} = 20$  volts, it is 11.5 microseconds with an error of  $\pm 1.1\%$ . The error over this range, expressed bilaterally, is  $\pm 0.44\%$ . These results are not greatly different from Seely's. Better agreement of the higher value of pulse duration could have been attained by the choice of a larger value for B. The values used for A and B were obtained partly by guess from Seely's circuit and partly from the experience gained from this study. Here, again, the agreement gives a check on the accuracy of the method used, and gives confidence in its validity. This concludes the theoretical treatment of the cathode-coupled multivibrator.



the equations were solved in a similar manner to those in the  
3000 cases. In order to obtain a more accurate solution  
rounding this to 32 bits and maintaining the accuracy of the  
following values:

$$E_1 = 1.28 \times 10^{-10} \text{ sec}^{-1} \text{ at } 25^\circ \text{C}$$
$$E_2 = 1.28 \times 10^{-10} \text{ sec}^{-1} \text{ at } 25^\circ \text{C}$$

The pulse duration for  $E_1 = 1.28 \times 10^{-10} \text{ sec}^{-1}$  at 25°C was  
of 40.25% from the total value for  $E_1 = 1.28 \times 10^{-10} \text{ sec}^{-1}$   
seconds with an error of 1.1%. The error was very small  
discrepancy in  $E_1$ . These results are in good agreement  
Bell's. Better agreement of the  $E_1$  values with a constant  
have been obtained by the method of a larger value for  $E_1$  and values  
used for  $E_2$  were obtained by using the values of  $E_1$  and  $E_2$   
partly from the experiment which were also used for  $E_1$  and  $E_2$   
agreement gives a check on the accuracy in the method used and  
confidence in the results. The results are in good agreement  
of the constants and the values of  $E_1$  and  $E_2$ .

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CHAPTER IV

EXPERIMENTAL WORK

The comparisons made in Chapter II indicate that the results obtained in this analysis are comparable to the results of other theoretical studies. In the practical application of theory, however, the final results may, and often do, differ quite radically from the computed values. This difference is generally due to the fact that the values of the tube parameters may differ from the average values given in the tube handbooks by as much as 20 per cent. Experimental multivibrators were constructed to obtain information as to the effect of such variations when the design equations of this analysis were used. The design of these multivibrators and the results obtained from them are the subjects of this chapter.

I. APPLICATION OF BASIC ANALYSIS

The equations derived in Chapter II were used to design a multivibrator which would produce a rectangular pulse having a duration of 335 microseconds. The plate load resistor  $R_{L2}$  and the cathode resistor  $R_k$  were set equal, and a 6SN7 tube, operating at a plate current value of 10 milliamperes, was chosen. From this information the circuit parameters were computed to be

$$R_{L2} = 11,240 \text{ ohms}, \quad R_k = 11,240 \text{ ohms}, \quad R_{L1} < 26,000 \text{ ohms},$$

These values were rounded off to the preferred values

$$R_{L2} = 11,000 \text{ ohms}, \quad R_k = 11,000 \text{ ohms}, \quad R_{L1} = 24,000 \text{ ohms}.$$



The comparison made in the present investigation was between the results obtained in this analysis and those obtained in the previous physical studies. In the present study, the final results were obtained after the comparison of the measured values. This difference is probably due to the fact that the values of the parameters were taken from the same set of data in the two papers. The results of the present study are in good agreement with those reported in the literature. The results of the present study are in good agreement with those reported in the literature. The results of the present study are in good agreement with those reported in the literature.

### 1. APPARATUS AND METHOD

The apparatus used in this study is similar to that used in the previous study. The vibrator which would produce a constant amplitude sinusoidal vibration with a frequency of 333 micro-cycles per second. The plate was supported by a spring and a damper. The plate was supported by a spring and a damper. The plate was supported by a spring and a damper. The plate was supported by a spring and a damper. The plate was supported by a spring and a damper.

$$R_{12} = 11,200 \text{ ohms}, R_{13} = 11,200 \text{ ohms}, R_{14} = 11,200 \text{ ohms}$$

$$R_{23} = 11,200 \text{ ohms}, R_{24} = 11,200 \text{ ohms}, R_{34} = 11,200 \text{ ohms}$$



For a grid voltage of  $\neq 70$  volts, the  $R_g C$  time constant was computed to be  $975 \times 10^{-6}$ . To obtain this value,  $R_g$  was chosen as 0.975 megohms and  $C$  as 0.001 microfarad. Since only 5 per cent resistors were available, their values were measured thus reducing this variation to about 2 per cent, the meter accuracy. The final values, which were used in the remaining calculations, were

$$R_{L_2} = 10,500 \text{ ohms}, R_k = 11,000 \text{ ohms}, R_{L_1} = 24,200 \text{ ohms}, \\ R_g = 0.975 \text{ megohms}, C = 0.001 \text{ microfarad}.$$

The waveforms produced by the circuit were displayed on a Du Mont Type 274-A Cathode-ray Oscillograph and measurements of the pulse heights and pulse duration were made. Observations were taken using three different 6SN7 tubes to show the effect of variations from tube to tube. A comparison of computed and measured values is given in Table IV.

The agreement between the computed and measured values is quite good except for the  $e_{cn_2}$  value during the timing state. In the worst case this difference is 40 per cent of the computed value. It was mentioned in Chapter II that the equation used in computing this value could give rise to large percentage errors although the individual terms of the equation were in error by only a relatively small amount. A calculation made with the values used in this experiment shows that an error of 5 per cent in each of the terms of the equation could cause a final error of 38 per cent. The close agreement of all the other values indicates that the method of computation is the cause of the large percentage error.



For a grid voltage of 400 volts, the maximum value of the  
of  $10^{-6}$ . The maximum value of  $\beta$  was found to be 100  
and 0 on 0.001 millimeter. These values are very low, and  
this, their values were not used for the purpose of the  
2 per cent, the maximum error, the first value, and the  
the maximum value, was

$$E_1 = 10,000 \text{ volts, } E_2 = 11,000 \text{ volts, } E_3 = 12,000 \text{ volts,}$$
$$E_4 = 0.001 \text{ millimeter, } E_5 = 0.001 \text{ millimeter.}$$

The maximum values of the direct and indirect  
in both (Type 212) technology and measurements of the  
pulse heights and other features. The values are  
using three different methods to find the effect of  
tube to tube, the maximum of indirect and direct values is given  
in Table II.

The agreement between the measured and calculated values is quite  
good except for the 0.001 millimeter value. This is due to  
easy to find values to 10 per cent of the maximum value. It is  
shown in Chapter II that the measured and calculated values  
could give rise to large percentage errors although the individual  
of the equation was in error by only a relatively small amount.  
calculation made with the values used in this treatment shows that  
error of 2 per cent in each of the terms of the equation would result  
final error of 10 per cent. This also indicates that the values used here  
indicates that the method of computation in this case is well adapted

centage error.



TABLE IV

## COMPARISON OF COMPUTED AND MEASURED WAVEFORM VALUES

Item	Computed Value	Measured Value		
		Stable State		
		Tube 1	Tube 2	Tube 3
		Not Triggered		
$E_{bn1}$	300	299	300	300
$E_{bn2}$	198.8	192	192	197
$E_k$	106.1	107.5	109.8	104.1
$E_{cc}$	70	70	70.5	70
$E_{cn2}$	106.1	107.8	110	104.2
		Triggered		
$E_{bn1}$	300	300	300	300
$E_{bn2}$	198.8	199	191.3	199
$E_k$	106.1	107.5	109.8	104.1
$E_{cc}$	70	-	-	-
$E_{cn2}$	106.1	107.8	110	104.2
		Timing State		
$e_{bn1}$	143.6	147	151.7	152
$e_{bn2}$	300	300	300	300
$e_k$	70.1	69.3	71.6	68.4
$e_{cn2}$	-50.3	-31.3	-27.8	-33.3
T ( sec.)	334	344	344	337



COMPARISON OF WORKING AND NON-WORKING DAYS

Item	Company	State	Working Days	Non-Working Days	Total Days
100	100	100	100	100	100
101	101	101	101	101	101
102	102	102	102	102	102
103	103	103	103	103	103
104	104	104	104	104	104
105	105	105	105	105	105
106	106	106	106	106	106
107	107	107	107	107	107
108	108	108	108	108	108
109	109	109	109	109	109
110	110	110	110	110	110
111	111	111	111	111	111
112	112	112	112	112	112
113	113	113	113	113	113
114	114	114	114	114	114
115	115	115	115	115	115
116	116	116	116	116	116
117	117	117	117	117	117
118	118	118	118	118	118
119	119	119	119	119	119
120	120	120	120	120	120

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The values of the resistors used in this experiment are fairly close to the values used in the computations which resulted in Table I. A comparison of those values with the measured values of Table IV shows that the waveform magnitudes are relatively stable for variations of the circuit components.

The experimental results show the equations of this analysis to be well suited for use in the design of a cathode coupled multivibrator which will give pulses of a predetermined duration.

## II. APPLICATION OF LINEARITY ANALYSIS

In the design of the multivibrator to produce a pulse with a duration which varied linearly with the applied grid voltage, the following relations were decided upon:

$$R_k = 0.7 R_{L_2}, \quad A = 0.2, \quad B = 0.3,$$

$$E_{cc(max)} = 60 \text{ volts}, \quad E_{bb} = 300 \text{ volts},$$

$$i_{b_2} = 15 \text{ milliamperes.}$$

These and the 6SN7 tube-parameter values were used in the design equations of Chapter III, resulting in the following component values:

$$R_{L_2} = 7240 \text{ ohms}, \quad R_k = 5060 \text{ ohms}, \quad R_{L_1} < 7340 \text{ ohms.}$$

The preferred values which were used were:

$$R_{L_2} = 6800 \text{ ohms}, \quad R_k = 5100 \text{ ohms}, \quad R_{L_1} = 6800 \text{ ohms.}$$

The corresponding values of  $a$ ,  $b$ ,  $d$ , and  $e$  were then computed giving

$$a = 235.2, \quad b = 1.093, \quad d = 300, \quad e = 0.82,$$

the resulting equation for the pulse duration being

$$T = R_g C [0.0074 E_{cc} - 0.246].$$



The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I. A comparison of these values with the values of  $\eta_{sp}/c$  for the polymer solutions is given in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I.

The experimental results show that the values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I.

### DISCUSSION

In the study of the effect of the concentration of the polymer solutions on the values of  $\eta_{sp}/c$ , it was found that the values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I.

The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I.

These and the other data are listed in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I. The values of  $\eta_{sp}/c$  for the polymer solutions are listed in Table I.

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For the time constant,  $R_g$  and  $C$  were chosen as 1.5 megohms and 0.001 microfarad, respectively. The multivibrator was constructed using components within 2 per cent of these values, with the exception of  $R_g$  and  $C$  which were 5 per cent components. Only measurements of the applied grid voltage and pulse duration were made during this experiment. The results of computed and measured pulse durations are given in Table V.

A plot of the measured values of  $T$  against  $E_{cc}$  shows that  $T$  is a linear function of  $E_{cc}$  from 225 to 315 microseconds. The accuracy of the measurements, however, was not sufficient to detect deviations smaller than a few per cent. It will be noticed that there is a slow increase in the difference between the measured and computed values as the value of  $E_{cc}$  increases, indicating that the straight lines showing pulse duration as a function of  $E_{cc}$  have different slopes as well as different intercepts. Both of these differences can be attributed to a difference between the actual value of  $R_g C$  and the value used in the computations, as well as to inaccuracies of the experimental measurements.

The results show that the design equations gave a fair prediction of the time durations which resulted, and that, within the limits of the experimental errors, the linear relationship was attained.







TABLE V

COMPARISON OF COMPUTED AND MEASURED PULSE DURATIONS

Grid Voltage	Computed T	Measured T
52	208	224
56	238	269
58	274	291.5
60	297	314
62	318	359
64	340	390
66	363	412
68	386.5	426



TABLE 7

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Year	Value	Percentage
1917	100	100
1918	105	105
1919	110	110
1920	115	115
1921	120	120
1922	125	125
1923	130	130
1924	135	135
1925	140	140
1926	145	145
1927	150	150



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## CHAPTER V

### SUMMARY AND CONCLUSIONS

This final chapter contains a summary of the preceding chapters, and states the conclusions which were reached from both the theoretical and experimental work. Emphasis is placed upon the important results and the results which are original. A short enumeration of several aspects of the linearity problem, which were not considered in this treatment is given as a guide for possible future investigations.

#### I. SUMMARY

A short history of the development of the multivibrator was given pointing out the fact that trial and error methods are necessary in a number of design applications. The discussion was then specialized to the monostable cathode-coupled multivibrator, it being pointed out that while adequate analyses for the general operation of this circuit do exist they attack the problem from the trigger circuit viewpoint, thus neglecting to show the close relationship between the cathode-coupled multivibrator and the degenerative RC amplifier. The failure of these analyses to provide design information for achieving a pulse duration which is a linear function of the grid bias voltage was also pointed out.

An analysis of the monostable cathode-coupled multivibrator was then made in which it was shown that the resulting equations are those of the degenerative RC amplifier multiplied by factors which compensate



EXPERIMENTAL WORK

This report contains a summary of the experimental work and general conclusions which were reached from the results and experimental work. The results of the experimental work and the results which are obtained from the analysis of the points of the linearly plotted, which were not considered in this report, are given in a table for general reference.

DISCUSSION

A short history of the development of the theory of the given points and the fact that the first series are generally in a number of higher applications. The discussion was then related to the general dielectric constant which is related to the

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that will describe the case for the general case. The results are given after the results are given. The results are given in this report to show the clear relationship between the dielectric constant and the degree of the dielectric constant.

of these results to provide data for the dielectric constant and the dielectric constant of the dielectric constant of the dielectric constant and the dielectric constant of the dielectric constant.

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for the overdriven conditions of operation. The use of these equations showed a very good agreement with results obtained by graphical methods. The important design equations were pointed out and general design suggestions were given. The effect of tube and component variability on the circuit operation was also discussed.

Next, the equations which had been developed were used to show that the pulse duration is a linear function of the applied grid voltage under certain conditions. These conditions were examined and relations between the circuit parameters were established which fulfill these conditions. These relations were given in the form of equations which are useful in the design of such a circuit. An analysis of a circuit of known characteristics, by the methods developed here, gave good agreement between predicted and actual operation. To this investigator's knowledge, no prior work has been published giving a complete analysis of the linearity phenomena.

Experimental circuits were then constructed in accordance with the design equations of this analysis. The measured results were in good agreement with the predicted operation.

## II. CONCLUSIONS

As a result of the theoretical and experimental work, it is concluded that

1. The monostable cathode-coupled multivibrator is closely related to the degenerative RC amplifier.
2. The analysis presented in Chapter II is well fitted for use



for the overvoltage condition of the system. The use of the overvoltage  
shows a very good agreement with results obtained by previous methods.  
The important feature of the method is that it is very simple and  
sections were drawn. The effect of the overvoltage is that it  
the electric operation was also discussed.

Next, the relations which hold between the overvoltage and the  
that the pulse duration is a linear function of the overvoltage and  
under certain conditions. These conditions are that the overvoltage  
between the capacitor and the inductor is small compared with the  
ditions. These relations were given in the form of equations which are

useful in the design of such a circuit. It is noted that the  
known characteristics of the capacitor and inductor are not  
must however be satisfied in order to obtain the results which are  
known, no prior work has been published which contains a  
of the linearly increasing.

Experimental circuits were then constructed in accordance with  
the design equations of the circuit. The measured results were in  
good agreement with the results of the analysis.

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faced to the linearly increasing pulse.
2. The results obtained in this paper will be useful for the  
design of the overvoltage protection circuit.



in the design of practical circuits and will give a good prediction of the actual operation.

3. The analysis of the linear relation of pulse duration to grid voltage gives an analytic explanation which is capable of furnishing quantitative data for the circuit operation.

4. The application of the linearity analysis to design problems gives a fair prediction of the actual operation.

### III. RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

There are several topics in the linearity analysis which have not been completely answered in this investigation. Of these the statement of the final error from exact linearity in terms of the error introduced by using only the first term of the series expansions would be of considerable value. It may be that further examination of this question would result in such a statement. A second field for investigation lies in the extension of the linearity range in both directions while keeping the sensitivity to grid voltages changes within reasonable limits. Still a third, and comparatively untouched, field lies in the application of an analysis such as the present one to pentode tubes.

In the experimental field, considerable work can be done to illustrate the effects of various values of A and B as defined in Chapter III, a study which was beyond the power of the analytic methods of this analysis.







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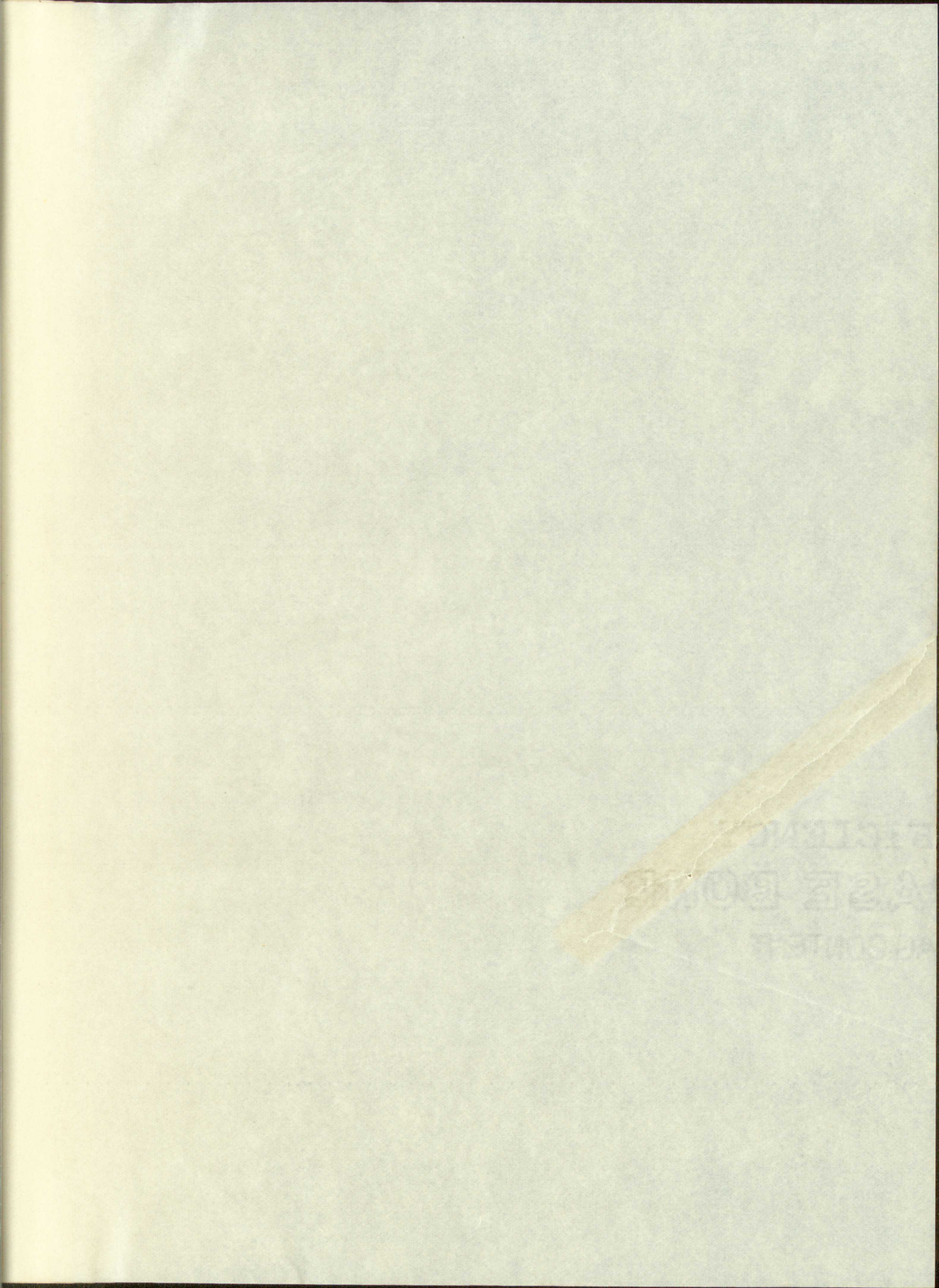


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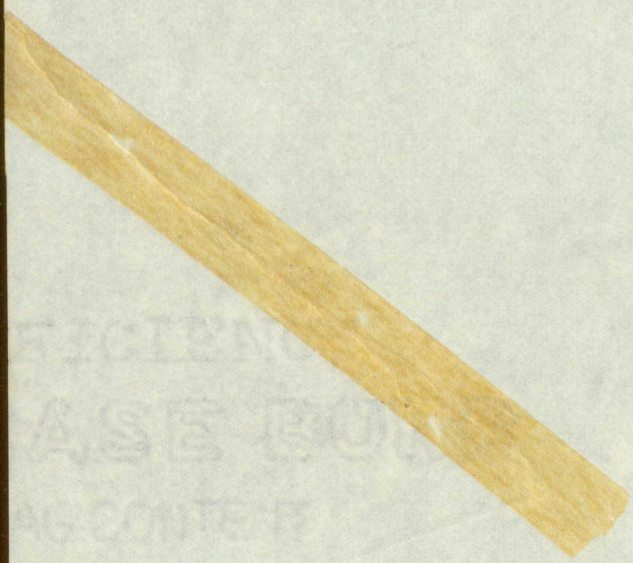


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