Enhancement of quantum nondemolition measurements with an electro-optic feed-forward amplifier

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Methods for the enhancement of optical quantum nondemolition (QND) measurements are discussed. We review the use of meter squeezing as a QND enhancement tool and present a method of QND enhancement using an electro-optic feed-forward amplifier. By applying a linearized theory it is shown that these techniques work very well together. The combined effect of these enhancement methods is modeled for two QND systems, a squeezed light beam splitter and an optical parametric amplifier. We also discuss the conflict between the normal QND criteria and QND systems that involve noiseless amplification. We use an additional parameter to quantify the problem. A method for correcting the effects of noiseless amplification is discussed and modeled. We also discuss a special case of QND that eliminates the optical interaction between the meter and signal input beams. This system is shown to be a very effective QND device. [S1050-2947(99)06411-2]

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I. INTRODUCTION

If one wishes to accurately measure the position of a free particle, Heisenberg's uncertainty principle necessarily implies an unpredictable momentum. Subsequent measurements of the position will therefore be affected by the previous measurement through the increase of the uncertainty in the momentum. Alternatively, one could measure the momentum of the particle. Although there would be an increase in the uncertainty in the position, this does not impinge upon further measurements of the momentum. For the free particle Hamiltonian, momentum is therefore a ''quantum nondemolition'' (QND) variable. Multiple measurements may be made of the momentum with no error on the nth measurement due to the $(n-1)$th measurement. A general condition for a variable to be QND is that it commutes with the system Hamiltonian. It was Braginsky *et al.* who first wrote on the possibility of such a measurement [1]. Braginsky, and later Thorne *et al.* [2] considered the possibility of using a QND readout scheme in a gravity wave detection scheme. Largely due to the relative ease of optical experimentation over mechanical systems, most implementations of QND have involved making measurements of quadratures of the electromagnetic field [3–5].

The efficiency of a given QND system depends on the internal dynamics of the machine and the environment in which it is placed. Often it may be more practical to manipulate the environment to enhance the performance of a QND device since the internal dynamics are not always accessible. Methods of QND enhancement are the focus of this paper. One technique for improving QND is the use of a squeezed meter input. This method was suggested theoretically in 1980 [6] and has since been demonstrated experimentally [4,5]. The introduction of the meter squeezing occurs at the input to the QND machine and we will refer to this as ''pre-enhancement.'' The idea presented in this paper is the use of electro-optic feedforward as a tool for QND enhancement. Feedforward has previously shown its usefulness as a noiseless amplifier [7–9]. By placing a QND machine inside a feedforward loop the signal-to-noise ratio of the signal output may be improved. This occurs at the output of a QND machine and we will therefore describe it as ''postenhancement.''

The use of a noiseless amplifier in this way highlights a challenge to the validity of the standard QND criteria. Strong noiseless amplification moves any signal to a level well above the quantum noise. The signal satisfies all the regular QND criteria, yet the signal is obviously different from the original since it is now very robust to optical attenuation. This problem has been discussed previously by Levenson *et al.* [10]. We use a parameter called ''sensitivity'' [7] to quantify the effect. We show that the sensitivity of an amplified signal can be recovered by mixing the signal output with a bright squeezed beam.

The layout of the paper is as follows. In Sec. II we introduce the standard QND criteria as developed by other authors [11,12]. Section III is a discussion of enhancement techniques. First, we review meter squeezing pre-enhancement. Second, a theory of feed-forward postenhancement is developed. This theory is used to model two QND machines; the squeezed light beam splitter and an optical parametric amplifier. In both cases we show that significant gains can be made using postenhancement, especially when used in conjunction with pre-enhancement. Lastly, in Sec. IV we discuss the sensitivity problem and its solution using a bright squeezed beam.

II. QND CRITERIA: AN OVERVIEW

We begin by examining the working of QND via the linearized input/output formalism developed by Collett and Gardiner [13]. For the optical systems considered here, signals will be encoded on a quadrature of the quantized electromagnetic field. If we have a field described by the time domain boson operator $\hat{S}$, we can consider making a nondestructive measurement of the general quadrature $\hat{X}_s^{\text{in}}$ given by

$$\hat{X}_s^{\text{in}}=e^{-\imath\theta}\hat{S}+e^{\imath\theta}\hat{S}^{\dagger}. \tag{1}$$

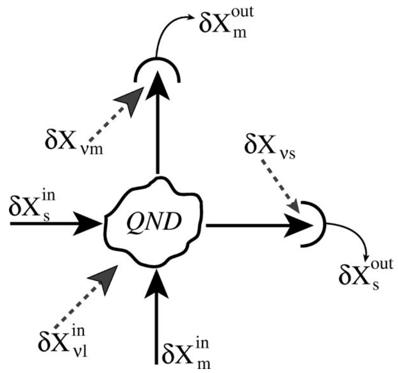

FIG. 1. The inputs and outputs of a general QND system. $\delta X_s^{\rm in}$ is the signal input, $\delta X_m^{\rm in}$ is the meter input, $\delta X_{\nu l}^{\rm in}$ is quantum noise due to internal loss, $\delta X_{\nu m}$ and $\delta X_{\nu m}$ are quantum noise due to detector inefficiency, and $\delta X_m^{\rm out}$ and $\delta X_s^{\rm out}$ are the meter and signal output photocurrents, respectively.

For small signals only fluctuations about the steady-state expectation value of $\hat{X}_s$ will be important. For this reason we will express our QND theory in terms of the fluctuation operators denoted by a ''δ;'' for example, the fluctuations of $\hat{X}_s$ about the steady state are given by $\delta\hat{X}_s$. The final step is to consider the frequency domain version of this fluctuation operator $\delta\tilde{X}_s(\omega)$, where the ''$\tilde{}$'' indicates the Fourier domain fluctuation operator. This operator is used to derive expressions for the spectral variances of the field that are the commonly measured quantity in optical QND. The spectral variance $V_s(\omega)$ is given by

$$\delta(\omega-\omega')V_s(\omega)=\langle\delta\tilde{X}_s(\omega)\delta\tilde{X}_s^*(\omega')\rangle. \tag{2}$$

We will refer to $\delta\tilde{X}_s(\omega)$ and $V_s(\omega)$ as the more compact $\delta\tilde{X}_s$ and V_s, respectively.

A general QND scheme is shown in Fig. 1. We begin with a signal $\delta\tilde{X}_s^{\rm in}$, which we wish to measure nondestructively using a meter input $\delta\tilde{X}_m^{\rm in}$. An internal loss term $\delta\tilde{X}_{\nu l}$ is also considered. The signal and meter outputs of the system are detected with efficiencies η_s and η_m, respectively. These efficiencies are associated with the additional vacuum fluctuations $\delta\tilde{X}_{\nu s}$ and $\delta\tilde{X}_{\nu m}$. In order to evaluate the success of the QND system, we compare the signal and meter photocurrents, which have statistics given by the operators $\delta\tilde{X}_s^{\rm out}$ and $\delta\tilde{X}_m^{\rm out}$. These operators may be expressed using the matrix equation

$$\begin{pmatrix}\delta\tilde{X}_s^{\rm out}\\ \delta\tilde{X}_m^{\rm out}\end{pmatrix}=\begin{pmatrix}\sqrt{\eta_s} & 0\\ 0 & \sqrt{\eta_m}\end{pmatrix}\begin{pmatrix}a & b & c\\ d & e & f\end{pmatrix}\begin{pmatrix}\delta\tilde{X}_s^{\rm in}\\ \delta\tilde{X}_m^{\rm in}\\ \delta\tilde{X}_{\nu l}\end{pmatrix}+\begin{pmatrix}\sqrt{1-\eta_s} & 0\\ 0 & \sqrt{1-\eta_m}\end{pmatrix}\begin{pmatrix}\delta\tilde{X}_{\nu s}\\ \delta\tilde{X}_{\nu m}\end{pmatrix}. \tag{3}$$

The matrix of coefficients $a\ldots f$ in Eq. (3) are parameters determined by the internal dynamics of a given QND device. In particular, the coefficients a, b, d, and e define the internal strength of coupling between the meter/signal inputs and meter/signal outputs. A machine that introduces extra quantum noise due to internal loss will have c and f nonzero.

The aim of QND is to both measure and avoid destruction of the input signal. For ideal nondemolition, we require $\delta\tilde{X}_s^{\rm out}=\delta\tilde{X}_s^{\rm in}$. On the other hand, an ideal measurement will be made when $\delta\tilde{X}_m^{\rm out}=G\,\delta\tilde{X}_s^{\rm in}$ where G is a known constant.[1] These two conditions are always satisfied when $d=G$, $a,\eta_s,\eta_m=1$, and $b,c,e,f=0$. A system that could produce such a result is a perfect QND machine. For any meter and signal input, the meter output contains an exact copy of the signal input (with a known amplification G), and the signal output is undisturbed. Since it makes a perfect nondestructive measurement of a quadrature, it necessarily implies an infinite variance in the complementary quadrature. Unsurprisingly, such a system has not been developed. Instead, we must content ourselves with nonideal QND systems, where a, η_m, and η_s are not unity, and b,c,e,f are nonzero. Under these conditions some signal is lost, the meter is not a perfect copy of the input signal, and extra noise may be added due to internal loss.

To evaluate the nonideal performance of a QND device we use two parameters [11,12]. The first is the signal transfer. For an ideal measurement the signal-to-noise ratio ($\mathcal{R}$) on the meter output is identical to the signal-to-noise of the signal input. We define T_m as

$$T_m=\frac{\mathcal{R}_m^{\rm out}}{\mathcal{R}_s^{\rm in}}, \tag{4}$$

so that in the case of an ideal measurement, $T_m=1$. We also require that the signal-to-noise ratio of the signal output be not degraded by the measurement process. The ratio T_s is therefore defined as

$$T_s=\frac{\mathcal{R}_s^{\rm out}}{\mathcal{R}_s^{\rm in}}, \tag{5}$$

so that for ideal preservation of the signal $T_s=1$. The first parameter used to evaluate QND measurement is the sum of these signal transfer ratios,

$$T_{s+m}=T_s+T_m. \tag{6}$$

If the signal-to-noise ratio of the meter and signal outputs is identical to that of the signal input, then the QND system is behaving in an ideal fashion and we have $T_{s+m}=2$. A classical measurement system has $T_{s+m}\leq 1$, so for the system to have some QND properties we require

$$T_{s+m}>1. \tag{7}$$

Expressions for T_s and T_m may be derived using Eq. (3) to give

[1]Note that the presence of G allows for possible amplification of the meter output. This is acceptable since we do not require the meter output to be identical in size to the signal input, just that it contain a faithful copy of the information of the signal input.

$$T_s=\frac{\eta_s|a|^2}{\eta_s[|a|^2+|b|^2V_m^{\rm in}+|c|^2]+1-\eta_s}, \tag{8}$$

$$T_m=\frac{\eta_m|d|^2}{\eta_m[|d|^2+|e|^2V_m^{\rm in}+|f|^2]+1-\eta_m}. \tag{9}$$

The second measure of a QND system is the conditional variance $V_{s|m}$. This measures the correlation of the meter and signal output and is defined as

$$V_{s|m}=V_s^{\rm out}-\frac{\langle|\delta\tilde{X}_s^{\rm out}\delta\tilde{X}_m^{\rm out}|^2\rangle}{V_m^{\rm out}}. \tag{10}$$

A QND device requires $V_{s|m}<1$, and in the limit of a perfect QND device, we find $V_{s|m}=0$. Starting from Eq. (3) we find

$$V_{s|m}=\eta_s[|a|^2V_s^{\rm in}+|b|^2V_m^{\rm in}+|c|^2]+1-\eta_s - \frac{\eta_s\eta_m|adV_s^{\rm in}+beV_m^{\rm in}+cf|^2}{\eta_m[|d|^2V_s^{\rm in}+|e|^2V_m^{\rm in}+|f|^2]+1-\eta_m}. \tag{11}$$

As a simple example of a potential QND device, we can consider a beam splitter. For a 50/50 beam splitter with coherent signal and meter inputs and ideal detectors, we have $a=-b=d=e=1/\sqrt{2}$, $c=f=0$, $V_m^{\rm in}=V_s^{\rm in}=1$, and $\eta_s=\eta_m=1$. The signal transfer ratio and conditional variance may be evaluated using Eqs. (8), (9), and (11). We find that $T_{s+m}=1$ and $V_{s|m}=1$. This device is clearly an imperfect QND device in fact, it has no QND properties at all according to the above definitions. We will now examine some methods that can be applied to any nonideal QND device to enhance its functionality.

III. QND ENHANCEMENT

A. Pre-enhancement

When the QND system is letting you down and no further advantage can be gained by improving the internal workings of the device, other ways of improving the system need to be considered. (This is particularly true for a beam splitter where there are no moving parts at all.)

If the parameters b and e of Eq. (3) cannot be reduced, then their effect can be minimized by suppressing the fluctuations on the meter beam. This is done by squeezing $\delta\tilde{X}_m^{\rm in}$. By preparing the meter beam in this way, any imperfect QND device can be enhanced. This idea has been discussed previously by various authors. Shapiro [6] and Holland [11] discuss using a beam splitter with a squeezed meter as a QND device. This idea has since been realized experimentally by Bruckmeier *et al.* [4]. With a 3.7-dB squeezed meter beam, a signal transfer $T_{s+m}=1.29$ and conditional variance $V_{s|m}=0.73$ were measured. The enhancement achievable by this method is limited only by the amount of squeezing, since $V_m\rightarrow 0$ implies $T_{s+m}\rightarrow 2$ and $V_{s|m}\rightarrow 0$. The best vacuum squeezing reported to date is 7 dB [14], which would produce $T_{s+m}=1.53$ and $V_{s|m}=0.38$ with a 50/50 beam splitter and 95% efficient detection.

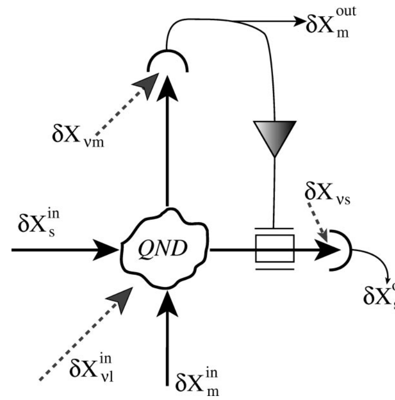

FIG. 2. The inputs and outputs of a general QND system with feedforward. Parameters are the same as those shown in Fig. 1 with the exception that $\delta\tilde{X}_{\nu s}$ is now the quantum noise for the combined effect of modulator loss and signal detector inefficiency.

A squeezed meter has also been used to improve the performance of an already functional QND device. Bruckmeier *et al.* [5] used a 3.4-dB squeezed meter to improve the performance of a QND system that used an optical parametric amplifier (OPA). In this system, the meter and signal beams are injected into the OPA and the nonlinear interaction between the two form a QND coupling. The OPA was run using a vacuum meter input and $T_{s+m}=1.05$ and $V_{s|m}=0.56$ were measured. The meter was then replaced with a squeezed-vacuum input with the result that T_{s+m} was increased to 1.12, and $V_{s|m}$ reduced to 0.53.

These examples demonstrate the utility of this form of QND enhancement. We will now consider a second complementary strategy for QND improvement.

B. Postenhancement with feed-forward

A standard linear amplifier has a 3-dB noise penalty associated with high-gain amplification [15]. It has been shown that electro-optic feedforward can be used as a noiseless amplifier [8,9] with no such noise penalty. A feedforward loop works by tapping off some of the signal and detecting it. The photocurrent may then be used to control a modulator in the signal beam down stream from the tap-off point. Quantitatively, the noise penalty associated with amplification may be expressed in terms of the signal transfer coefficient. A standard linear amplifier has $T_s=1/2$ (for high gain), whereas a feedforward amplifier can attain $T_s=1$ in the limit of ideal in-loop photodetection. Equation (6) makes it apparent that bringing T_s closer to 1 will allow superior values of T_{s+m} to be achieved.

The use of feedforward in QND enhancement is shown in Fig. 2. The meter output is used to modulate the signal output. In this way some of the signal degradation due to the measurement process can be compensated by careful use of the information on the meter beam. We emphasize that no extra manipulation of the meter beam is required to implement feedforward. The effect of feedforward is modeled by modifying Eq. (3) to include coupling between the meter output and the signal output. The new equation for the QND device is

$$\begin{pmatrix} \delta\tilde{X}_{sf}^{\rm out} \\ \delta\tilde{X}_{m}^{\rm out} \end{pmatrix} = \begin{pmatrix} \sqrt{\eta_s} & K\sqrt{\eta_m\eta_s} \\ 0 & \sqrt{\eta_m} \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} \delta\tilde{X}_{s}^{\rm in} \\ \delta\tilde{X}_{m}^{\rm in} \\ \delta\tilde{X}_{\nu l} \end{pmatrix} + \begin{pmatrix} \sqrt{1-\eta_s} & K\sqrt{\eta_s(1-\eta_m)} \\ 0 & \sqrt{1-\eta_m} \end{pmatrix} \begin{pmatrix} \delta\tilde{X}_{\nu s} \\ \delta\tilde{X}_{\nu m} \end{pmatrix}. \tag{12}$$

The quantity K is the gain of the feed-forward loop. In general, it is complex and a function of frequency. Having terms dependent on K in the off-diagonal elements of Eq. (12) has the effect of coupling the signal output to the meter output as required. We note that the addition of a modulator causes some attenuation of the signal beam prior to detection. This is accounted for theoretically by modifying the value of the signal detection efficiency η_s to include loss due to the modulator.

Equation (12) clarifies the mechanism by which feedforward aids QND. It can be used to cancel the effect of the noise terms in the signal output. For example, by satisfying $b\sqrt{\eta_s}+K\sqrt{\eta_s\eta_m}e=0$, we can make the signal output $\delta\tilde{X}_s^{\rm out}$ independent of the meter noise $\delta\tilde{X}_m^{\rm in}$. The system requires efficient detection of the meter beam; otherwise, the additional noise due to that detection efficiency may ruin any benefit derived from the elimination of the meter noise. In fact, this effect has important implications for the optimization of a feedforward loop. In the presence of poor meter detection, the optimum gain is below that which gives perfect cancellation of the noise term. There is a trade-off between the reduced noise from the cancellation and the increased noise from the detection loss.

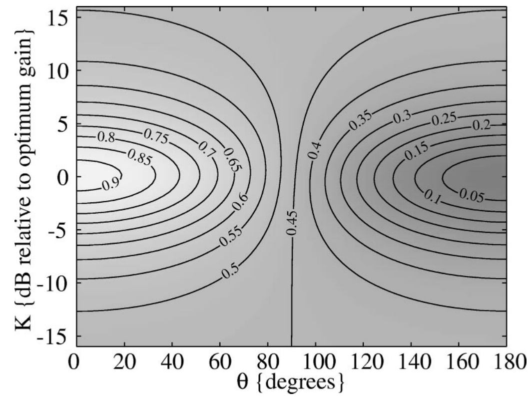

FIG. 3. The effect of feedforward on T_s as a function of gain magnitude K and feedforward phase θ for a nonsqueezed vacuum at the beam splitter. The values of T_s are shown as contours in intervals of 0.05. The gain has been normalized to the optimum value so that the point $K=0$ dB and $\theta=0$ represents gives the optimum T_s of 0.925. Parameters used for the plot are $\varepsilon=0.5$, $\eta_m=0.95$, $\eta_s=0.9$, and $V_m=1$.

Feedforward has no effect on T_m, since the feedforward all occurs downstream of the meter detection. Equation (9) is therefore still used to calculate T_m. The effect on the conditional variance is small. $V_{s|m}$ is actually independent of the gain K. The only impact on $V_{s|m}$ comes from vacuum noise added by the extra attenuation due to the modulator in the signal beam. Under typical experimental conditions, the loss in an amplitude modulator may be as low as 5%. With feedforward applied, Eq. (11) may still be used to evaluate $V_{s|m}$ although η_s now includes this extra attenuation due to the modulator. Feedforward greatly modifies T_s. Starting from Eq. (12) we obtain

$$T_s=\frac{\eta_s|a+K\sqrt{\eta_m}d|^2}{\eta_s[|a+K\sqrt{\eta_m}d|^2+|b+K\sqrt{\eta_m}e|^2V_m^{\rm in}+|c+K\sqrt{\eta_m}f|^2+|K|^2(1-\eta_m)]+1-\eta_s}. \tag{13}$$

Using this result, we can model the effect of feedforward on QND for some specific systems.

1. Example 1: The beam splitter

If we consider a beam splitter QND device with transmission ε, the matrix $a,\ldots,f$ in Eq. (12) becomes

$$\begin{pmatrix} \sqrt{\varepsilon} & -\sqrt{1-\varepsilon} & 0 \\ \sqrt{1-\varepsilon} & \sqrt{\varepsilon} & 0 \end{pmatrix}. \tag{14}$$

Before showing the effect of feedforward on QND, we will consider the effect of feedforward gain K on the value of T_s. Figure 3 shows contours of T_s mapped as a function of the magnitude and the phase of K for a vacuum meter input ($V_m^{\rm in}=1$). The best performance of the system is for a phase of $0\,°$. The magnitude of K has been normalized by the optimum value so that $K=0$ dB corresponds to the optimum gain magnitude which, as discussed above, depends on the meter detection loss. The plot shows the design requirements of a feed-forward loop under typical operating conditions with a 50/50 beam splitter. The gain must be controlled to within $20\,°$ of the optimum phase and the magnitude to within 2 dB of the optimum to maintain $T_s>0.9$. This diagram also demonstrates the operation of a feedforward loop as a noise eater. When the phase is at $180\,°$ the feedforward loop can cancel all of the signal on the signal output beam and T_s drops to 0.

Equations (9), (11), and (13) may be used to evaluate the QND performance of the system as a function of the input vacuum squeezing V_m. In Fig. 4 we consider values of meter squeezing between 0 and 7 dB with no feedforward and optimized feedforward. For a 50/50 beam splitter (lines i and i_f), we see the expected increase in T_{s+m} with feedforward. Also demonstrated is the minor degradation of $V_{s|m}$ by the modulator loss. However, it is apparent that we can do better.

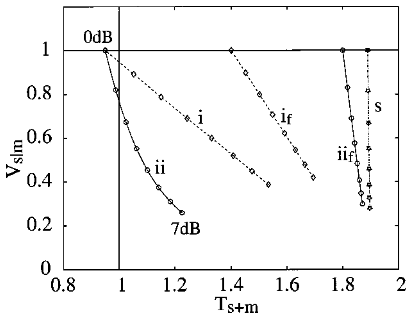

FIG. 4. Beam-splitter QND with varying amounts of meter squeezing. Lines i and i_f show the results for a 50/50 beam splitter ($\varepsilon=0.5$) without and with feedforward, respectively. Lines ii and ii_f show results for $\varepsilon=0.1$ without and with feedforward, respectively. Other parameters used are $\eta_m=\eta_s=0.95$ with no feedforward. With feedforward we modify $\eta_s=0.9$ to account for 5% loss in the modulator. K is optimized to give maximum T_s. Line s shows the special case of $\varepsilon=0$. Here the parameters are the same as the other cases, except for K, which is made very large to maximize T_s, in this case $K=100$.

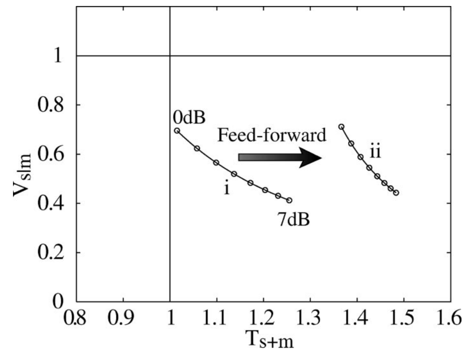

FIG. 5. OPA QND with varying amounts of meter squeezing. With no feedforward (line i) $\eta_m=\eta_s=0.95$. With feedforward (line ii) we have $\eta_s=0.9$ to account for 5% loss in the modulator. The passive loss r^2 is 0.179. K is optimized to maximize T_s.

Feedforward improves T_s, leaving T_m unchanged. Figure 3 shows that with a 50/50 beam-splitter ratio T_s may be improved to over 0.9. Unfortunately T_m will remain stuck at ≈ 0.5, limiting the maximum T_{s+m}.

The way around this is to reduce the beam-splitter ratio ε. For example, with $\varepsilon=0.1$ 90% of the light is detected by the feedforward detector, so that T_m is now increased to ≈ 0.9. These conditions are shown by lines ii and ii_f. Without feedforward, T_{s+m} is reduced compared to the case of the 50/50 beam splitter (line i compared to line ii). This is because the gains in the value of T_m are more than cancelled out by the degradation of T_s. With feedforward T_s is recovered while simultaneously taking advantage of the improved T_m. The end result is that with $\varepsilon=0.1$ and $V_m=-7$ dB feedforward can improve the T_{s+m} of the beam splitter from 1.18 to 1.87. Another advantage of this setup is an improvement in $V_{s|m}$, which is 0.29 for $\varepsilon=0.5$ (with $V_m=-7$dB) compared to 0.2 for $\varepsilon=0.1$.

There is an interesting limit to this system when ε goes to 0. This means that the signal input is completely detected. This is equivalent to removing the beam splitter altogether and eliminating any optical interaction between the meter and signal beams. For the feedforward to be meaningful we would require some power in the squeezed meter beam to ensure a signal output beam with coherent amplitude. The complete detection of the signal beam by the meter detection system ensures a T_m equal to the meter detection efficiency. For the signal transfer T_s, there is no longer an optimum feedforward gain point. In the limit of high gain, the signal imposed by the modulator will far exceed the noise of the squeezed beam therefore making T_s at best equal to T_m. The value of $V_{s|m}$ is restricted, as before, by the amount of squeezing on the meter beam. The performance of the system at high gain and $\varepsilon=0$ is shown by line s on Fig. 4. By the QND criteria presented thus far, this system clearly achieves the best QND measurement. In the sense that we completely measure the signal of interest and then recreate it on a separate beam, the $\varepsilon=0$ case is analogous to the work of Roch *et al.* [16] and Goobar *et al.* [17] who fully detected the signal beam then used that signal to drive a light-emitting diode (LED) or laser diode. The difference between our scheme and the laser diode work is that the signal output of our system may be coherent with the signal input. This may be achieved by generating the squeezed source using an OPA that has been seeded with a portion of the original laser beam. Our scheme also allows QND on the phase or amplitude quadratures, whereas the diode work is restricted to amplitude.

2. *Example 2: The OPA*

The system may also be applied to the OPA QND system of Bruckmeier *et al.* For this system the matrix $a, \ldots, f$ in Eq. (12) is shown to be [5]

$$\begin{pmatrix} 1-r & -\sqrt{r} & -r^{3/2} \\ \sqrt{r} & -r^2/2 & r \end{pmatrix}, \tag{15}$$

where r^2 is the ratio of the passive loss per round-trip of the OPA and the parametric deamplification. For the experiment by Bruckmeier this parameter was determined to be $r^2=0.179$. This system may now be modeled as for the beam splitter using Eqs. (9), (11), and (13). Figure 5 shows the effect of feedforward and meter squeezing on the OPA system. Again we see the effectiveness of feedforward in improving the value of T_{s+m}.

IV. THE SENSITIVITY

Despite the credible performance of the feedforward loop as described by the QND parameters, there is a significant objection to the description of the above examples as QND. As described by Lam *et al.* [8], a feedforward loop is a noiseless signal amplifier. Although we have shown that the output of a QND machine with feedforward postenhancement has all the right signal transfer and correlation characteristics according to the regular QND parameters (as defined in Sec. II), the output of the feedforward loop is actually amplified well above the quantum noise. It is therefore more robust to loss than the input state. This could be seen as an advantage in systems where optical loss is a problem. However, it is a little imprecise to describe the whole process as QND if the input and output states have different properties with respect to optical attenuation. If the output of the QND machine were required for a quantum-noise-limited measurement a robust, amplified signal would not do. For this reason another parameter may be appropriate to distinguish the amplified signals from their originals. We therefore define the ''sensitivity'' S as [7]

$$S=\frac{V_s^{\rm in}}{V_s^{\rm out}}. \tag{16}$$

Under this definition, an ideal QND machine has $S=1$; i.e., the signal input variance is at 0 dB relative to the size of the signal output variance. A system that requires a quantum-noise-limited beam from the output of a QND machine will not function if $S<0$ dB. By way of comparison, the sensitivity of the squeezed light beam splitter with feedforward is -11 dB, assuming a beam-splitter ratio of $\varepsilon=0.1$, which gave the best QND results in Sec. III B 1.

This sensitivity problem is by no means confined to feed-forward. Any QND system that has noiseless amplification properties is equally prone to poor sensitivity. Levenson *et al.* [10] investigated the use of an OPA as a noiseless amplifier and found that it can satisfy the QND criteria. Yet they also suggest that this is not really QND because of the amplification. Their experiment records a 9-dB amplification that gives a sensitivity of -9 dB. Similarly, the OPA modeled in Sec. III B 2 has a sensitivity of -3.7 dB. Another QND scheme is to fully detect the signal and then reemit using a diode laser or LED. The noiseless amplification, and therefore sensitivity, in such experiments is rather extreme. In the work of Roch *et al.* [16] the sensitivity was around -20 dB, and in that of Goobar *et al.* [17], $S=-8$ dB.

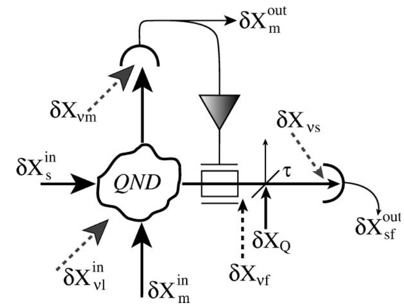

FIG. 6. Scheme for improving the sensitivity of the QND system. A bright squeezed beam with fluctuation operator δX_Q is incident on a beam splitter with transmission τ. Other parameters are the following: $\delta X_{\nu f}$ is the vacuum noise due to the modulator attenuation; $\delta X_s^{\rm in}$ is the signal input; $\delta X_m^{\rm in}$ is the meter input; $\delta X_{\nu l}^{\rm in}$ is quantum noise due to internal loss; $\delta X_{\nu m}$ is the quantum noise due to meter detector efficiency; $\delta X_{\nu s}$ is the quantum noise due to the signal detector efficiency; and $\delta X_m^{\rm out}$ and $\delta X_s^{\rm out}$ are the meter and signal output photocurrents, respectively.

The sensitivity may be rectified by using a second stage of postenhancement. This consists of a bright squeezed beam incident on a second beam splitter with transmission τ, as shown in Fig. 6. The power in the bright beam may be chosen to bring the intensity of the output state back to that of the original. The squeezing of this bright beam pulls the amplified signal back onto the quantum-noise floor without adding any additional noise; i.e., the squeezed light beam splitter is a noiseless deamplifier. This extra feature may be incorporated into the linearized description of the system by writing

$$\begin{pmatrix}\delta\tilde{X}_{sf}^{\rm out}\\ \delta\tilde{X}_m^{\rm out}\end{pmatrix}=\begin{pmatrix}\sqrt{\eta_f\eta_s\tau} & K\sqrt{\eta_f\eta_s\eta_m\tau}\\ 0 & \sqrt{\eta_m}\end{pmatrix}\begin{pmatrix}a & b & c\\ d & e & f\end{pmatrix}\begin{pmatrix}\delta\tilde{X}_s^{\rm in}\\ \delta\tilde{X}_m^{\rm in}\\ \delta\tilde{X}_{\nu l}\end{pmatrix}+\begin{pmatrix}\sqrt{1-\eta_s} & K\sqrt{\eta_f\eta_s\tau(1-\eta_m)} & \sqrt{\eta_s\tau(1-\eta_f)} & \sqrt{\eta_s(1-\tau)}\\ 0 & \sqrt{1-\eta_m} & 0 & 0\end{pmatrix}\begin{pmatrix}\delta\tilde{X}_{\nu s}\\ \delta\tilde{X}_{\nu m}\\ \delta\tilde{X}_{\nu f}\\ \delta\tilde{X}_Q\end{pmatrix}. \tag{17}$$

With the inclusion of a squeezed source between the modulator and detector in the signal beam we cannot incorporate the modulator attenuation into the signal detection efficiency, as we did when deriving Eq. (2). Instead, we explicitly include the vacuum noise $\delta\tilde{X}_{\nu f}$ due to the nonunity modulator transmission η_f.

The improvement of the sensitivity due to the addition of the bright squeezed source is shown in Fig. 7. The dashed line shows the value of S for the feed-forward system as modeled in Sec. III B 1 with 7-dB meter squeezing and $\varepsilon=0.1$. The solid line shows the variation of S as a function of the beam-splitter ratio τ with $V_Q=-7$ dB. At $\tau=0.06$ the sensitivity is shown to be brought back to 0 dB.

The performance of the sensitivity compensated system may also be investigated in terms of the parameters T_{s+m} and $V_{s|m}$. From Eq. (17) we obtain new equations for T_s and $V_{s|m}$. These are

$$\begin{aligned}T_s=&\{\eta_s\eta_f\tau|a+K\sqrt{\eta_m}d|^2\}/\{\eta_s\eta_f\tau[|a+K\sqrt{\eta_m}d|^2\\&+|b+K\sqrt{\eta_m}e|^2V_m^{\rm in}+|c+K\sqrt{\eta_m}f|^2+|K|^2(1-\eta_m)]\\&-\eta_s\eta_f\tau-\eta_s(1-\tau)(1-V_Q)+1\}\end{aligned} \tag{18}$$

and

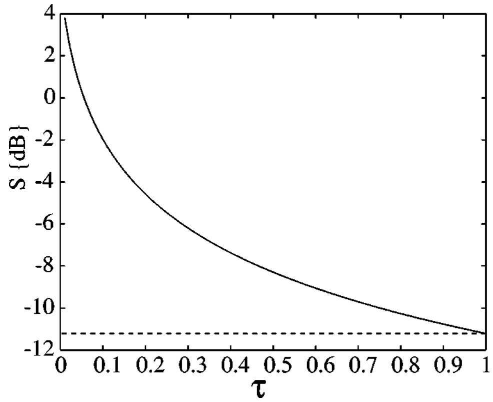

FIG. 7. Comparison of the sensitivity for an $\varepsilon=0.1$ beam-splitter feedforward system with and without an additional squeezed source. The dashed line shows the sensitivity level with of the system analyzed in Sec. III B 1. The solid line shows the variation of the sensitivity as a function of the the beam-splitter ratio τ. Parameters used are $\eta_m=0.95$, $\eta_s=0.95$, $\eta_f=0.95$, $V_m=-7$ dB, and $V_Q=-7$ dB. K is optimized to maximize T_s.

$$V_{s|m}=\eta_s\eta_f\tau[|a|^2V_s^{\text{in}}+|b|^2V_m^{\text{in}}+|c|^2]-\eta_s\eta_f\tau-\eta_s(1-\tau)(1-V_Q)+1-\frac{\eta_s\eta_f\eta_m\tau|adV_s^{\text{in}}+beV_m^{\text{in}}+cf|^2}{\eta_m[|d|^2V_s^{\text{in}}+|e|^2V_m^{\text{in}}+|f|^2]+1-\eta_m}. \quad (19)$$

Using Eqs. (18) and (19) we model the effect of sensitivity correction on the beam-splitter QND system with $\varepsilon=0.1$. Line *i* of Fig. 8 shows the QND system with feedforward and no sensitivity correction. This is identical to line ii_f of Fig. 4. As before, the circles represent meter squeezing in intervals of 1 dB. The performance of the compensated system is shown in two parts. Line *ii* shows the performance of the system with no meter squeezing ($V_m=0$ dB) and varying amounts of bright squeezing V_Q. The squares are in 1-dB steps. The results of line *ii* show that we may make an efficient QND system with $S=0$ dB by using only feedforward and sensitivity postenhancement. We do not require meter squeezing pre-enhancement to perform QND with a beam splitter. This system is similar to that described by Ralph [7]. The value of τ required to obtain $S=0$ dB varies as a function of V_Q. With $V_Q=0$ dB in line ii,τ is close to 0, while at $V_Q=-7$ dB, $\tau=0.07$. The addition of meter squeezing V_m takes the performance from the end of line *ii* along line *iii*. As V_m is changed in line iii, τ decreases to 0.06. This due to the higher feedforward gain used as the squeezing V_m is increased.

The sensitivity corrected system is shown to have worse T_{s+m} characteristics than the uncompensated case. This is due to the small amount of extra noise introduced by V_Q. The conditional variance, on the other hand, is improved in the compensated system due to the overall reduction in the signal noise level.

For the particular case of the beam-splitter with feedforward, the the sensitivity becomes poor due to the gain (K) of the feedforward loop. The amount of gain was chosen initially to optimize the signal transfer T_s. Instead, we may sacrifice some of the signal transfer and choose a gain that

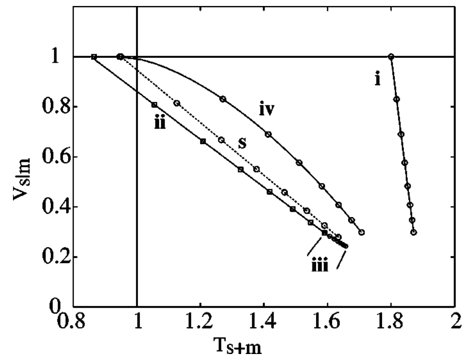

FIG. 8. Comparison of the standard QND parameters, with and without the addition of sensitivity correction, for an $\varepsilon=0.1$ beam-splitter QND system. Line *i* is the performance of the system as previously shown in line ii_f of Fig. 4 with no sensitivity correction and V_m varying from 0 to -7 dB. The circles again represent 1-dB intervals of V_m. Line *ii* shows the effect of sensitivity correction with $V_Q=-7$ dB, $V_m=0$ dB, and τ chosen to give $S=0$ dB. The squares are 1-dB intervals of V_Q. Line *iii* shows the effect of turning on the meter squeezing V_m with V_Q held constant at -7 dB. The circles represent 1-dB intervals of V_m. τ is again chosen to give $S=0$ dB. For lines i, ii, and iii K is chosen to optimize T_s. Lines iv and s show the retrieval of the sensitivity by reducing the gain of the feedforward instead of using an additional bright squeezed source. In both cases the squeezing V_m is varied from 0 to -7 dB. Line iv shows the $\varepsilon=0.1$ case and line s is the special case of $\varepsilon=0$. Other parameters for lines ii, iii, iv, and s are $\eta_m=0.95$, $\eta_s=0.95$, and $\eta_f=0.95$.

gives a sensitivity of 0 dB. Doing this for the case of $\varepsilon=0.1$ gives the line iv of Fig. 8. The results of this method are seen to be better than those obtained with the extra squeezed beam. Reducing the feedforward gain is not, however, a general method for sensitivity correction. It is only applicable when the feedforward is the reason for the loss of sensitivity, as is the case for the beam splitter. If the QND device were an OPA, the signal gain that ruins the sensitivity would be present without feedforward amplification. In this case an additional squeezed beam would be required to recover the sensitivity.

The special case of $\varepsilon=0$ may also be investigated in terms of the sensitivity. As for $\varepsilon=0.1$, the sensitivity is easily retrieved by reducing the gain. This situation is shown by line s of Fig. 8. This system is not as effective as the $\varepsilon=0.1$ case. The reason is that the transfer of signal onto the signal output beam with $\varepsilon=0$ relies solely on the feedforward. With a reduction in gain the signal drops proportionally. In the case of $\varepsilon=0.1$ not all the signal travels through the feedforward. Some of the signal still leaks straight through the beam splitter into the signal output. Reducing the gain therefore has a less drastic effect. We may conclude that the special case of QND with no interaction at the beam splitter is better if the sensitivity is regarded as unimportant (as shown in Fig. 4); however, with sensitivity correction achieved by altering the gain of the feedforward loop it is better to have some signal passing through the beam splitter.

V. CONCLUSION

We have shown the utility of feedforward as a postenhancement tool for a general QND scheme. Examples of the beam splitter and optical parametric amplifier were considered and the potential benefits under realistic experimental conditions were shown to be significant. Feedforward may also be used in conjunction with squeezed meter pre-enhancement to further improve a range of QND devices. The sensitivity of the output state from a feedforward loop was shown to be well below that required for quantum-noise-limited measurements. This problem may be overcome for any QND system with the use of a bright squeezed light beam splitter to deamplify the output state back to its original size.

For the particular case of QND performed with a squeezed-vacuum beam splitter and feedforward, where the only source of signal amplification is the feedforward, the sensitivity may be more easily corrected by reducing the feedforward gain. We have also examined a limit of the beam-splitter system where the signal and meter input beams do not interact optically. This system is shown to have an advantage over the normal beam-splitter QND arrangement, provided the sensitivity is not important.

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