Noiseless phase quadrature amplification via an electro-optic feed-forward technique

Ben C. Buchler, Elanor H. Huntington, and Timothy C. Ralph

Department of Physics, Faculty of Science, The Australian National University, Canberra, Australian Capital Territory 0200, Australia

Theoretical results are presented that show that noiseless phase quadrature amplification is possible, and limited experimentally only by the efficiency of the phase-detection system. Experimental results obtained using a neodymium-doped yttrium aluminum garnet laser show a signal gain of 10 dB and a signal transfer ratio of $T_s = 0.9$. This result easily exceeds the standard quantum limit for signal transfer. The results also explicitly demonstrate the phase-sensitive nature of the amplification process. [S1050-2947(99)01107-5]

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I. INTRODUCTION

The transmission of small optical signals carried by coherent light is fraught with difficulty. Attenuation of the optical power causes degradation of the signal-to-noise ratio as the signal recedes into the quantum noise of the beam. A simple method to rectify this problem would be to amplify the signal. However, this is not a trivial exercise as phase-insensitive amplifiers, such as laser amplifiers, introduce excess noise [1]. In the limit of high gain, the noise penalty for a phase-insensitive amplifier is 3 dB, i.e., a halving of the signal-to-noise ratio. This is known as the standard quantum limit (SQL) for phase-insensitive amplifiers. To overcome this problem, phase-sensitive amplification is required [2]. This can entail a nonlinear process, as in the case of the optical parametric amplifier, where one observable is amplified while the conjugate is deamplified [3].

A far simpler method has been demonstrated by Lam *et al.* [4] using positive electro-optic feed forward. In this scheme, a part of the input beam is tapped off using a beamsplitter and detected. The signal is then added back to a modulator further downstream. Using this method with amplitude signals, Lam *et al.* achieved a signal transfer ratio of $T_s = 0.88$.

In this paper we use an analogous feed-forward network to show better-than-SQL amplification of phase quadrature signals. To our knowledge, this is the first demonstration of phase signal amplification superior to the amplifier SQL.

In *any* experiment that generates phase quadrature signals close to the quantum noise limit (QNL), our system could be used to make this signal robust optical loss. Specific suggestions for the application of phase feed forward include quantum nondemolition measurements [5] and continuous variable teleportation [7,8].

II. THEORY

We will model the system shown in Fig. 1. At the input to the system is a beam containing some phase signal, some of which is tapped off to the homodyne detection system ($H1$) by a beamsplitter of transmissivity ε. The remaining light is passed through an electro-optic phase modulator, (EOM), which is controlled by the signal detected in $H1$.

A theoretical model of the behavior of this system will be generated using linearized operators. This is the same approach used previously in [4] and [5]. The time domain annihilation operator for the input beam to our system, $\hat{A}_{in}$, will be written as

$$\hat{A}_{in} = \bar{A}_{in} + \delta\hat{A}_{in}, \tag{1}$$

where $\bar{A}$ is the mean value of the field amplitude and $\delta\hat{A}$ is the time-dependent component of the field with an expectation value of 0. Embedded in this time-dependent component is the signal information and quantum noise carried by the beam. After traversing the beam path through the beamsplitter and EOM, the field operator $\hat{A}_f$ for the output can be written as

$$\hat{A}_f = \sqrt{\varepsilon}\,\bar{A}_{in} + \sqrt{\varepsilon}\,\delta\hat{A}_{in} - \sqrt{1-\varepsilon}\,\delta\hat{\nu}_b + \iota\,\delta\hat{r}, \tag{2}$$

where $\delta\hat{\nu}_b$ is the vacuum input due to the beamsplitter and $\delta\hat{r}$ is a modulation imposed on the beam by the EOM. The operator $\delta\hat{r}$ will be a function of fluctuations detected in the homodyne detection system $H1$.

A homodyne system, such as that shown in Fig. 1, measures a particular quadrature amplitude of a low power beam by mixing it with a much more intense local oscillator beam on a 50/50 beamsplitter [6]. The phase difference between the local oscillator and the signal beam θ determines which quadrature of the signal beam is measured. The output of the system is the difference of the photocurrents from the two photo diodes. The form of this current may be calculated by
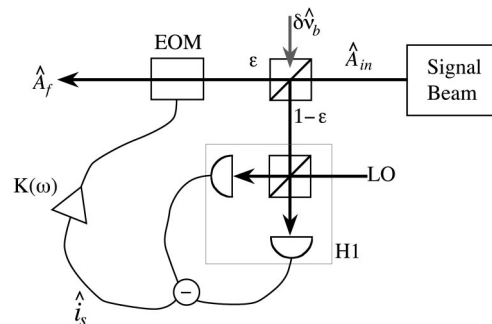

FIG. 1. A schematic diagram showing the components required for noiseless phase quadrature amplification using phase feed forward.

finding the difference of the photon number operators (i.e., $\hat{A}^{\dagger}\hat{A}$) for the fields incident on each of the detectors. To simplify matters we will linearize the equations by neglecting all terms greater than first-order in the fluctuation operators. The form of the subtracted photocurrent is found to be

$$\hat{i}_s = 2\sqrt{\eta_d \eta_h}\bar{A}_s \cos\theta + \sqrt{\eta_h \eta_d (1-\varepsilon)}\, \delta\hat{X}_{A_{\text{in}}}^{\theta}$$
$$+ \sqrt{\eta_d}(\sqrt{\eta_h \varepsilon}\, \delta\hat{X}_{\nu_b}^{\theta} + \sqrt{1-\eta_h}\, \delta\hat{X}_{\nu_h}^{\theta})$$
$$+ \frac{\sqrt{1-\eta_d}}{\sqrt{2}}(\delta\hat{X}_{\nu_{d1}} + \delta\hat{X}_{\nu_{d2}}), \qquad (3)$$

where we have defined the general quadrature of an operator $\hat{z}$ to be

$$\delta\hat{X}_z^{\theta} = e^{-\iota\theta}\hat{z} + e^{\iota\theta}\hat{z}^{\dagger}. \qquad (4)$$

For convenience of notation, the amplitude quadrature ($\theta = 0$) will be written with no superscript, while the phase quadrature ($\theta = \pi/2$) will be written as $\hat{X}_z^-$. The quantum noise terms $\delta\hat{X}_{\nu_{d1,d2}}$ result from the efficiency η_d of the two photodetectors in the homodyne system. The mode-matching efficiency of the homodyne system η_h also gives rise to a source of quantum noise, namely $\delta\hat{X}_h^{\theta}$. The mode-matching efficiency η_h is given by the square of the fringe visibility in the homodyne system [9], i.e.,

$$\text{Homodyne efficiency} = \eta_h = \left(\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}\right)^2, \qquad (5)$$

where $I_{\min/\max}$ is the minimum/maximum power of the output of one arm of the homodyne system as measured with equal power in the signal and local oscillator arms. To measure the phase quadrature with homodyne detection, we require that $\theta = \pi/2$. This means that the dc component of the current in Eq. 3 is zero. Experimentally, this provides a means for locking the homodyne system to the phase quadrature.

The form of the modulation $\delta\hat{r}$ imposed by the EOM will be given by a convolution of $\delta\hat{i}_s$, the time-dependent part of the current $\hat{i}_s$, and the time response of the feed-forward electronics $k(t)$. We may, therefore, write

$$\delta\hat{r}(t) = \int_0^t k(u)\delta\hat{i}_s(t-u)du. \qquad (6)$$

Combining Eqs. (2), (3), and (6), as well as choosing the phase of the homodyne detection system to be $\pi/2$ so that the phase quadrature is being fed forward, the final form of the fluctuations in the output beam of the EOM will be given by

$$\delta\hat{A}_f = \sqrt{\varepsilon}\, \delta\hat{A}_{\text{in}} - \sqrt{1-\varepsilon}\, \delta\hat{\nu}_b + \iota\int_0^t k(u)$$
$$\times\left\{ \sqrt{\eta_h \eta_d (1-\varepsilon)}\, \delta\hat{X}_{A_{\text{in}}}^-(t-u) \right.$$
$$+ \sqrt{\eta_d}[\sqrt{\eta_h \varepsilon}\, \delta\hat{X}_{\nu_b}^-(t-u) + \sqrt{1-\eta_h}\, \delta\hat{X}_{\nu_h}^-(t-u)]$$
$$\left. + \frac{\sqrt{1-\eta_d}}{\sqrt{2}}[\delta\hat{X}_{\nu_{d1}}(t-u) + \delta\hat{X}_{\nu_{d2}}(t-u)] \right\}du. \qquad (7)$$

Considering the general quadrature ϕ of $\delta\hat{A}_f$ and taking the Fourier transform of the operators, we obtain

$$\delta\widetilde{X}_{A_f}^{\phi} = [K(\omega)\sin\phi\sqrt{\eta_h \eta_d (1-\varepsilon)}\, \delta\widetilde{X}_{A_{\text{in}}}^- + \sqrt{\varepsilon}\, \delta\widetilde{X}_{A_{\text{in}}}^{\phi}]$$
$$+ [K(\omega)\sin\phi\sqrt{\eta_d \eta_h \varepsilon}\, \delta\widetilde{X}_{\nu_b}^- - \sqrt{1-\varepsilon}\, \delta\widetilde{X}_{\nu_b}^{\phi}]$$
$$+ K(\omega)\sin\phi\left[\sqrt{\eta_d}\sqrt{1-\eta_h}\, \delta\widetilde{X}_{\nu_h}^- + \frac{\sqrt{1-\eta_d}}{\sqrt{2}}\right.$$
$$\left.\times(\delta\widetilde{X}_{\nu_{d1}} + \delta\widetilde{X}_{\nu_{d2}})\right], \qquad (8)$$

where $\widetilde{X}_z$ is the Fourier transform of $\hat{X}_z$. The spectrum of $\hat{A}_f$, normalized to the (QNL), can now be obtained for arbitrary quadrature phase angle by evaluating $V_{Af}^{\phi} = \langle|\delta\widetilde{X}_{A_f}^{\phi}|^2\rangle$. Using the result that the spectra of the quantum noise sources is just 1, we obtain

$$V_{Af}^{\phi} = \sqrt{\frac{V_{A_{\text{in}}}^{-2}}{\sin^2\alpha + V_{A_{\text{in}}}^{-2}\cos^2\alpha}}$$
$$\times[\varepsilon\cos^2\phi + |\sqrt{\varepsilon} + K(\omega)\sqrt{\eta_h \eta_d (1-\varepsilon)}|^2\sin^2\phi]$$
$$+ |K(\omega)\sqrt{\eta_d \eta_h \varepsilon} - \sqrt{1-\varepsilon}|^2\sin^2\phi + (1-\varepsilon)\cos^2\phi$$
$$+ |K(\omega)|^2(1-\eta_d \eta_h)\sin^2\phi \qquad (9)$$

where the angle α is given by

$$\tan\alpha = \left(1 + \frac{K(\omega)\sqrt{\eta_h \eta_d (1-\varepsilon)}}{\sqrt{\varepsilon}}\right)\tan\phi.$$

It has also been assumed that the amplitude noise of the input beam $\hat{A}_{\text{in}}$ is at the quantum limit. To investigate the action of the feed-forward system on the phase quadrature, we take $\phi = \pi/2$, so that

$$V_{Af}^- = |\sqrt{\varepsilon} + K(\omega)\sqrt{\eta_h \eta_d (1-\varepsilon)}|^2 V_{A_{\text{in}}}^- + |K(\omega)\sqrt{\eta_d \eta_h \varepsilon}$$
$$- \sqrt{1-\varepsilon}|^2 + |K(\omega)|^2(1-\eta_d \eta_h). \qquad (10)$$

From Eq. (10) it is clear that a value of the electronic gain may be chosen such that the second term in Eq. (10) becomes 0. Physically this may explained by the division of the beamsplitter vacuum fluctuations $\delta\hat{\nu}_b$. The component of $\delta\hat{\nu}_b$ imposed on the signal beam (i.e., the beam that goes

through the EOM) is anticorrelated with the component of $\delta\hat{\nu}_b$ imposed on the light passed to the homodyne system $H1$. The result is that that for a unique value of *positive* feed-forward gain, the EOM will be driven such that the signal it imposes exactly cancels the beamsplitter vacuum fluctuations originally introduced to the signal beam. If we assume ideal homodyne and detector efficiency, the value of the gain required for cancellation is

$$K(\omega) = \sqrt{\frac{1-\varepsilon}{\varepsilon}}. \tag{11}$$

With this gain, the phase noise of the beam at the output of the feed-forward modulator is given by

$$V_{A_f}^{-} = \frac{V_{A_{in}}^{-}}{\varepsilon}. \tag{12}$$

Assuming perfect in-loop detection, the system can now been seen to behave as a noiseless amplifier with signal amplification of ε^{-1}.

When detection losses are considered, matters become a little more complex. The optimum gain is no longer that which gives total cancellation of the beamsplitter vacuum noise. This is because the extra noise due to detection scales with feed-forward gain. The result is that the optimum gain level is less than that found for ideal detection. To evaluate the performance of this system with imperfect detection we define a signal-transfer ratio T_s. This is given by the ratio of the signal-to-noise ratio (SNR) at the output of the system to the signal-to-noise at the input of the system, i.e.,

$$T_s = \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}}. \tag{13}$$

For a signal with measured variance V_s, which exists in the presence of a noise floor V_n, the SNR is given by

$$\text{SNR} = \frac{V_s - V_n}{V_n} = \frac{V_s}{V_n} - 1. \tag{14}$$

The variance of the noise floor must be subtracted from the signal variance in Eq. (14) because the measured signal is the sum of the signal you wish to quantify and the noise floor that surrounds it.

The optimum value of T_s occurs at a gain of

$$K(\omega) = \sqrt{\frac{\eta_h \eta_d (1-\varepsilon)}{\varepsilon}}, \tag{15}$$

for which

$$T_s = \varepsilon(1 - \eta_h \eta_d) + \eta_h \eta_d. \tag{16}$$

We note that with ideal detection we obtain $T_s = 1$ indicating perfect signal transfer.

When using a feed-forward system as a signal amplifier the optical power of the signal beam is attenuated by a factor of ε. This power loss can be overcome by using an injection locking system [10]. The final output beam can then be an optical state with identical power to the original state, but with an amplified signal.
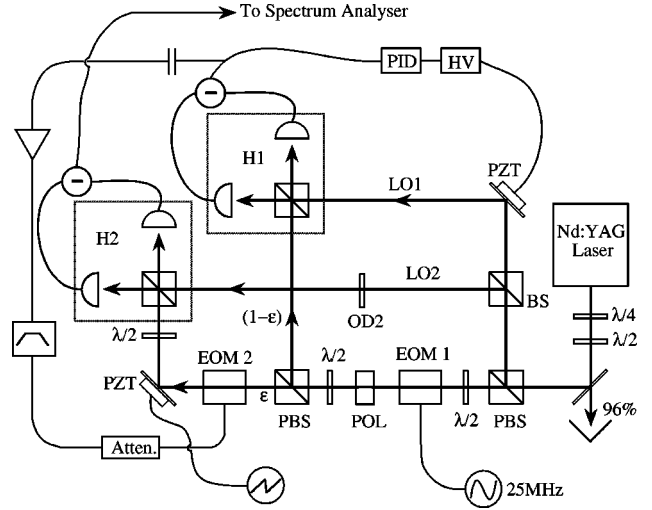

FIG. 2. The layout of our phase feed-forward experiment.

III. THE EXPERIMENT

An experiment was set up as shown in Fig. 2. The light source was a single mode Nd:YAG laser pumped by a diode laser array. The output power of the laser was 300 mW at 1064 nm. Most of the light was dumped at the 96% beamsplitter. Of the remaining light, 99.75% was tapped off for the local oscillator (LO) beams LO1 and LO2. The remaining light formed the signal beam for the phase feed forward. The electronics used for the feed forward consisted of two high gain amplifiers and a bandpass filter to ensure sufficient gain at the signal frequency. A variable electronic attenuator was used to adjust the overall gain of the loop.

A phase signal was imposed on the beam using the phase modulator EOM 1. A frequency of 25 MHz was chosen because the laser was at the QNL at this frequency. The signal was measured using the homodyne detection systems $H1$ and $H2$. The homodyne detector $H1$ consisted of two identical low-noise detectors with a quantum efficiency of $\eta_d = 0.91 \pm 0.02$ [11]. The homodyne efficiency was measured to be $\eta_h = 0.94 \pm 0.02$. System $H2$ contained similar detectors also with a quantum efficiency of $\eta_d = 0.91 \pm 0.02$. The homodyne efficiency of $H2$ was 0.88 ± 0.02.

The homodyne system $H1$ was locked to the phase quadrature using the dc voltage from the subtraction of the photocurrents of the homodyne system as an error signal. As discussed previously in relation to Eq. (3), when the DC voltage from $H1$ is 0, the detection system measures the phase quadrature.

The phase modulation imposed by EOM 1 is shown in Fig. 3, as measured using a spectrum analyzer (Hewlett-Packard 3589 A). The upper trace shows the signal level with 100% of the signal beam directed into the locked homodyne system $H1$. The signal is observed to be 8.0 ± 0.4 dB above the quantum noise, shown at 0 dB. Taking detection efficiency into account means that the inferred signal level is 8.6 ± 0.4 dB above the QNL.

We then altered our system so that 80% of the light was directed to $H1$ while the remaining 20% was passed through EOM 2 to the homodyne detector $H2$. This corresponds to $\varepsilon = 0.2$ in the above theory. The signal measured by $H1$ was used to drive EOM 2 through the feed-forward loop. The
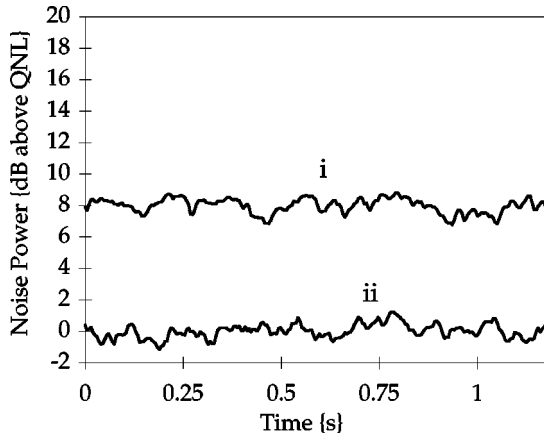
FIG. 3. Trace i shows a plot of the input phase signal at 25 MHz. This measurement was made with 100% of the signal beam directed into the homodyne detector $H1$ and is, therefore, a measurement of $V_{A_{in}}$. Trace ii show the quantum noise. The spectrum analyser was set on a resolution bandwidth of 100 Hz and a video bandwidth of 3 Hz.
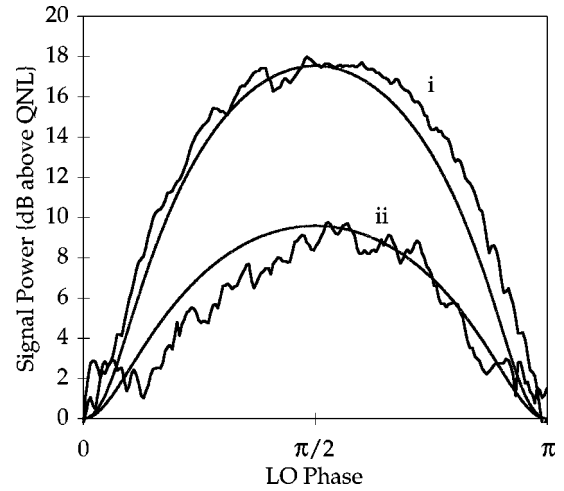

FIG. 4. Trace i shows a plot of the phase signal measured at 25 MHz with 80% of the light directed into $H1$ and the feed forward optimized. Trace ii show the noise level at 25.0003 MHz. The theoretical fit to the data was obtained using Eq. (9). The spectrum analyser was set on a resolution bandwidth of 100 Hz and video bandwidth of 3 Hz. This plot is an average of three sweeps.

result is shown in Fig. 4. Homodyne system $H2$ was swept and signals are therefore plotted as a function of the phase of LO2. Having a swept homodyne system allows us to demonstrate the phase-sensitive nature of the feed-forward system. Trace i shows the signal power at 25 MHz on the phase quadrature to be $17.6 \pm 0.2 dB$ above the QNL. The signal gain of the system is therefore ≈ 10 dB.

The signal level measured after feed forward must be compared to the noise observed at a frequency slightly removed from the signal frequency. Trace ii of Fig. 4 shows the phase quadrature noise power as measured at 25.0003 MHz to be at 9.5 ± 0.4 dB above the QNL. The difference between traces i and ii when the phase of LO2 is $\pi/2$ shows the fed-forward signal to be 8.1 ± 0.4 dB above the noise floor.

A theoretical fit of this data may be obtained using Eq. (9). This is also shown in Fig. 4. The only free parameter in this fit is the gain $K(\omega)$, which we take to be a constant K at a fixed frequency of 25 MHz. The value of K used to give this fit is 3.2, which is 70% (or 2.4 dB) larger than the calculated optimum. However, with high signal gain, the system is insensitive to detuning from optimum K [4]. For example, in the limit of large K the T_s of our system approaches 0.68. To achieve a T_s of 0.7 requires electronic gain 13 dB above the optimum level. This insensitivity means good signal transfer is achievable even if the system is not perfectly optimized.

Importantly, Fig. 4 demonstrates that the system is truly phase sensitive. The amplitude quadrature of the signal beam, as measured at phase angles of 0 and π, is shown to be quantum noise limited.

From an operational point of view, the system is very successful. We measure an input signal of 8 ± 0.4 dB above the noise, and retrieve a signal 8.1 ± 0.4 dB above the noise. This signal is almost 18 dB above the QNL instead of 8 dB and, therefore, more immune to attenuation. To rigorously determine how our system rates as a noiseless amplifier, we employ the signal-transfer ratio defined in Eq. (13). In this equation, SNR_{in} is the inferred signal-to-noise ratio (not the

detected) of the input beam and SNR_{out} is the inferred signal to noise of the output beam. Considering the homodyne and detection efficiency of $H1$, we can infer an input signal-to-noise ratio of $SNR_{in} = 6.2 \pm 0.7$. From the efficiency of the homodyne system $H2$ we find the inferred SNR_{out} to be 5.6 ± 0.6, therefore giving $T_s = 0.90 \pm 0.14$. The uncertainty in this result is largely due to the swept operation of $H2$. We expect that a locked homodyne system would allow more accurate determination of T_s.

Using Eq. (16) we find that the detector and homodyne efficiencies of $H1$ limit the maximum achievable T_s to 0.88, which is in agreement with our experimental results. The maximum transfer coefficient achievable with a phase insensitive amplifier (PIA) with a 10 dB signal gain is 0.53, which our system easily exceeds.

IV. CONCLUSION

We have developed a theoretical model of a phase feed-forward network and shown that it can behave as a noiseless phase quadrature amplifier in the limit of ideal phase homodyne phase signal detection. Experimental results with a neodymium-doped yttrium aluminum garnet (Nd:YAG) laser source demonstrate the practicality of the system. A signal transfer ratio of $T_s = 0.9 \pm 0.14$ with a signal gain of 10 dB was measured.

The relative simplicity of this system and the demonstrated insensitivity to nonoptimum gain make this system an ideal addon to any experiment where close to QNL phase signals need to be made robust to optical loss. Finally, these results indicate that quantum nondemolition and continuous variable teleportation experiments relying on electro-optic control of the phase quadrature are feasible.

ACKNOWLEDGMENTS

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