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Abstract

"Social Choice: Recent Developments"

Walter Bossert and John A. Weymark

In the past quarter century, there has been a dramatic shift of focus in social choice theory, with structured sets of alternatives and restricted domains of the sort encountered in economic problems coming to the fore. This article provides an overview of some of the recent contributions to four topics in normative social choice theory in which economic modelling has played a prominent role: Arrovian social choice theory on economic domains, variable-population social choice, strategy-proof social choice, and axiomatic models of resource allocation.

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1. Introduction

With the exception of the research on single-peaked preferences and their multidimensional generalizations, for the most part, the early literature on social choice theory dealt with abstract sets of alternatives and domains of preferences and feasible sets that exhibited little structure. In the past quarter century, there has been a dramatic shift of focus, with structured sets of alternatives and restricted domains coming to the fore. In particular, a great deal of attention has been directed towards the kinds of concrete problems that arise in economics, with alternatives being allocations of goods and preferences and feasible sets satisfying the kinds of restrictions encountered in economic models. In this article, we provide an overview of some of the recent contributions to four topics in normative social choice theory in which economic modelling has played a prominent role: Arrovian social choice theory on economic domains, variable-population social choice, strategy-proof social choice, and axiomatic models of resource allocation. Structured environments have also been considered in positive social choice theory, notably in the political economy literature. See Austen-Smith and Banks (2005) for an introduction to this literature. Other areas of social choice theory have been active as well in recent years. See Arrow, Sen, and Suzumura (2002, 2006) for recent surveys of these topics.

2. Arrovian social choice on economic domains

Arrow's Theorem (see Arrow, 1963) is concerned with the aggregation of profiles of individual preference orderings into a social ordering of a set of alternatives X. Let $\mathcal{R}$ denote the set of all orderings of X. In Arrow's Theorem, there is a finite set of individuals $N = \{1, \ldots, n\}$ with $n \geq 2$, each of whom has a weak preference ordering R_i on X. An *(Arrovian) social welfare function* f assigns a social ordering $R = f(\mathbf{R})$ of X to each profile $\mathbf{R} = (R_1, \ldots, R_n)$ of individual preference orderings in some domain $\mathcal{D}$ of profiles. Arrow's Theorem demonstrates that it is impossible for a social welfare function to satisfy Independence of Irrelevant Alternatives, henceforth IIA (the social ranking of a pair of alternatives only depends on the individual rankings of these alternatives), Weak Pareto (if everyone strictly prefers one alternative to a second, then so does society), and Nondictatorship (nobody's strict preferences are always respected) if the domain is unrestricted ($\mathcal{D} = \mathcal{R}^n$) and $|X| \geq 3$.

Arrow's Theorem is not directly applicable to economic problems. In economic problems, both the social alternatives and the individual preferences exhibit considerable structure and, therefore, a social welfare function only needs to be defined on a restricted domain of preference profiles. For a comprehensive survey of the literature on Arrovian social choice on economic domains, see Le Breton and Weymark (2006).

When X is a subset of the real line $\mathbb{R}$, a preference R_i is *single-peaked* if there is a unique best alternative $\pi(R_i)$ in X, the *peak*, and alternatives on the same side of the peak are worse the further away from the peak they are. Let $\mathcal{S}$ denote the set of all single-peaked preferences on X. If the alternatives in X are different levels of a single public good, it is natural to expect individual preferences to be single-peaked. Black (1948) has shown that ranking pairs of alternatives by majority rule produces a social ordering if the individuals have single-peaked preferences when n is odd. More generally, it follows from results in Moulin (1980) that on $\mathcal{S}^n$, any *generalized median social welfare function* satisfies all of the Arrow axioms except his domain assumption with Nondictatorship strengthened to Anonymity (permuting preferences leaves the social ordering invariant). These functions are defined by first fixing single-peaked preferences for $n-1$ phantom voters and then applying majority rule to profiles consisting of the preferences of the n real individuals and $n-1$ phantoms. Note that the number of individuals, both real and phantom, is odd, so Black's Theorem applies. Recently, Ehlers and Storcken (2002) have characterized all of the social welfare functions on this domain that satisfy IIA and Weak Pareto.

A domain $\mathcal{D}$ of preference profiles is *Arrow inconsistent* if no social welfare function satisfying Arrow's three non-domain axioms exists on $\mathcal{D}$. In a seminal article, Kalai, Muller, and Satterthwaite (1979) identified a sufficient condition for $\mathcal{D}$ to be Arrow inconsistent when $\mathcal{D}$ is the Cartesian product of individual preference domains $\mathcal{D}_i$. A set of alternatives is *free* if preference profiles are unrestricted on this set. A domain is *saturating* if (i) there are at least two free pairs, (ii) any two free pairs of alternatives can be connected to each other by means of a series of overlapping free triples, and (iii) any other pair of alternatives is *trivial* in the sense that there is only one way in which any individual ranks these alternatives. When each of the individual preference domains $\mathcal{D}_i$ is the same, saturating preference domains are Arrow inconsistent. Because a free pair is part of a free triple when the domain is saturating, Arrow's Theorem implies that there is a dictator on this pair when IIA and Weak Pareto are satisfied. The same person must be a dictator on all free pairs because adjacent free triples in the connection procedure have two alternatives in common. On trival pairs, by Weak Pareto, everyone is a dictator. This method of showing that a domain is Arrow inconsistent is known as the *local approach*.

Kalai, Muller, and Satterthwaite (1979) have also shown that when $X = \mathbb{R}^m_+$, interpreted as the set of all allocations of m divisible public goods, the domain of all profiles of *classical public-goods preferences* (i.e., continuous, monotonic, and convex preferences) is saturating and, hence, is Arrow inconsistent when $m \geq 2$. Other examples of saturating domains include the set of all expected utility preferences on the set of lotteries on three or more certain outcomes (Le Breton, 1986) and the set of Euclidean spatial preferences on $\mathbb{R}^m_+$ or $\mathbb{R}^m$; i.e., preferences for which there is a global best alternative and alternatives are ranked by the negative of their distance from this alternative (Le Breton and Weymark, 2002). The Arrow inconsistency of the spatial preference domain was originally shown by Border (1984) using a different proof strategy.

When alternatives are allocations of private goods and individuals only care about their own consumption, Bordes and Le Breton (1989) have identified a strengthening of the concept of a saturating domain that implies that the domain is Arrow inconsistent. If X consists of all the allocations of two or more divisible private goods in which everyone is guaranteed to receive a positive amount of some good, then the domain satisfies this condition if individuals can have any *classical private-goods preference*; i.e., a preference that is continuous, monotonic, ancd convex over own consumption. (Some of the results concerning classical private-goods preferences discussed below require some additional minor restrictions on preferences that are not stated explicitly here.) See also Maskin (1976) and Border (1983).

The examples considered so far all have the feature that the set of alternatives has a Cartesian structure. If X incorporates feasiblity constraints, this is not the case. Using a modification of the local approach, Bordes, Campbell, and Le Breton (1995) have shown that the domain of classical private-goods preferences are Arrow inconsistent if the set of alternatives is the set of feasible allocations with positive consumptions of all goods for an exchange economy with two or more divisible private goods. Bordes and Le Breton (1990) have also adapted the local approach to analyze Arrow consistency in assignment, matching, and pairing problems. In an *assignment problem*, one of n indivisible objects is assigned to each of the n individuals. In a *matching problem*, there are two groups of n individuals with each person from one group matched to one person from the other group. In a *pairing problem*, an even number n of individuals are grouped in pairs.

If the preference domains in these problems are such that individuals only care about which individual or good they are matched, paired, or assigned to, but are otherwise unrestricted, then the domain is Arrow inconsistent when $n \geq 4$.

The preceding discussion suggests that economic domain restrictions do not provide a satisfactory way of circumventing Arrow's social welfare function impossibility theorem when the set of alternatives is not one-dimensional. This conclusion is reinforced by the results in Redekop (1995) that show that in order for a subset of a domain of Arrow-inconsistent economic preferences to be Arrow consistent, the subdomain must be topologically small. Roughly speaking, this requirement severely limits the amount of preference diversity that can be present in the domain.

Arrow's Theorem can also be formulated in terms of a social choice correspondence. For each preference profile $\mathbf{R}$ in its preference domain $\mathcal{D}$, a *social choice correspondence* C specifies the socially optimal alternatives $C(A, \mathbf{R})$ in each agenda A (feasible subset of X) in its agenda domain $\mathcal{A}$. In its choice-theoretic formulation, the Arrow axioms are Arrow's Choice Axiom (for a fixed preference profile, if agenda A is a subset of agenda B, then the alternatives chosen in A consists of the restriction to A of the set of alternatives chosen from B when this restriction is nonempty), Independence of Infeasible Alternatives (the alternatives chosen from an agenda only depend on the preferences for alternatives in this agenda), Pareto Optimality (only Pareto optimal alternatives are chosen), and Nondictatorship (the chosen alternatives are not always a subset of one individual's best feasible alternatives). Arrow's Theorem shows that these conditions are inconsistent if the preference domain is unrestricted and the agenda domain consists of all the finite subsets of X. When the agenda domain is closed under finite unions (as is the case in the choice-theoretic version of Arrow's Theorem), Arrow's Choice Axiom is necessary and sufficient for the chosen alternatives in each admissible agenda to be generated by maximizing a profile-dependent social ordering of X (see Hansson, 1968).

In some economic applications, the ability to restrict the agenda domain, not just the preference domain, has weakened the constraints on the admissible social choice correspondences sufficiently so that the Arrovian axioms are consistent. This observation was first made by Bailey (1979), who noted that the set of feasible allocations in an exchange economy does not contain a finite number of alternatives, and so does not satisfy Arrow's agenda domain assumption. While the example Bailey used to show the consistency of the Arrow axioms is problematic, as Donaldson and Weymark (1988) have shown, if each agenda in the agenda domain is the set of feasible allocations for an exchange economy with divisible private goods (different aggegate endowments yield different agendas) and the preference domain only includes classical private-goods preferences, then the Arrow axioms are consistent. For example, the *Equal-Division Walrasian social choice correspondence* satisfies these axioms. For each exchange economy, this correspondence selects the set of Walrasian (competitive) equilibrium allocations using an equal division of the aggregate endowment as each individual's endowment vector.

When production is possible, an agenda is the set of feasible allocations given the aggregate resource endowment and the production technologies. Possible restrictions on agendas include compactness, comprehensiveness (i.e., they satisfy free disposal), and convexity. When there are only public goods, Le Breton and Weymark (2002) have shown that the Arrow axioms are consistent if the preference domain only includes Euclidean spatial preferences on $\mathbb{R}^m_+$ with $m \geq 2$ and the agenda domain only includes compact sets with nonempty interiors. With these domain assumptions, a social choice correspondence satisfying the Arrow axioms can be constructing by fixing a utility representation for each preference and then using an individualistic Bergson–Samuelson social welfare function to choose the best alternatives from each agenda for each preference profile.

In both of these examples, one of the Arrow axioms is vacuous. In the exchange economy example, it is Arrow's Choice Axiom, whereas in the spatial example, it is Independence of Infeasible Alternatives. For a public goods economy with at least two divisible goods, none of the Arrow axioms are vacuous if the agenda domain includes only compact comprehensive sets with nonempty interiors and the preference domain only includes classical public-goods preferences. By means of an example, Donaldson and Weymark (1988) have shown that the Arrow axioms are consistent with these domain assumptions. However, their example exhibits dictatorial features and it is not known if the axioms are still consistent if Nondictatorship is replaced with Anonymity (permuting preferences for a given agenda does not change the set of chosen alternatives). Donaldson and Weymark have also established a private-goods version of this possibility theorem.

Arrovian impossibility results have also been obtained using the social choice correspondence framework. For example, for public goods economies, Duggan (1996) has shown that Arrow's axioms are inconsistent if $X = \mathbb{R}^m_+$ with $m \geq 3$, the agenda domain consists of all the compact, comprehensive, and convex subsets of X, and the preference domain is the set of all profiles of classical public-goods preferences.

Unlike with the local approach used to analyze social welfare functions, no unifying methodology has been developed to investigate the consistency of Arrow's choice-theoretic axioms, with the consequence that little is yet known about where the the boundary between possibility and impossibility for social choice correspondences lies.

3. Variable-population social choice

The Arrovian framework is based on ordinal preferences that are interpersonally non-comparable and, hence, any social decision rule that makes use of interpersonal utility comparisons, such as classical utilitarianism, is ruled out from the outset. Sen (1974) has argued that this informational poverty plays a fundamental role in precipitating Arrovian impossibilities and has proposed a generalization of the concept of an Arrovian social welfare function called a social welfare functional to allow for interpersonal utility comparisons. Each individual i is assumed to have a utility function U_i on the set of alternatives X_i in which he is alive and a *social welfare functional* maps each admissible profile of individual utility functions into a social ordering of the set of all alternatives X. In fixed-population social choice, $X_i = X$ for all i. There is an extensive literature that has investigated the implications for the functional form of these functionals of combining different assumptions concerning the measurability and interpersonal comparability of utility with various normative criteria, including analogues of the Arrovian axioms, when there is a fixed population. See Bossert and Weymark (2004) for a survey. In this section, we provide an introduction to the main issues that arise in selecting appropriate social objective functions when the population size is not fixed. A detailed treatment of this topic and further references may be found in Blackorby, Bossert, and Donaldson (2005).

Population ethics has established itself as an important branch of moral philosophy over the last three decades. Parfit (1984) has been particularly influential in bringing this issue to the attention of philosophers and, more generally, to scholars in various disciplines interested in applied ethics. An up-to-date account of variable-population issues in moral philosophy is given by Broome (2004). Although there are many economic applications of variable-population social choice, such as the design of aid packages to developing countries that may have population consequences, the choice of budgets devoted to prenatal care, and policies affecting the intergenerational allocation of resources, the economics literature, with few exceptions, did not pay much attention to this topic until recently. Much of the recent interest in these issues can be traced to the influential article by Blackorby and Donaldson (1984), who extended the welfarist model of social choice to allow for a variable population.

In this setting, each alternative $x \in X$ is a complete description of the relevant state of affairs including the size and composition of the population. Furthermore, alternatives are interpreted as full histories of the world, from the remote past to the distant future. Thus, the set of those alive in x contains everyone who has ever lived in this alternative and not merely those who are alive in a given period. This assumption is important to avoid counter-intuitive conclusions regarding the termination of lives. As a consequence, ending someone's life does not change population size; it affects the lifetime and, possibly, the lifetime utility of the person in question.

For each $x \in X$, $u_i = U_i(x)$ is the lifetime well-being (utility) of any individual i alive in x and $U(x)$ is the vector of utilities of these individuals. The standard convention is to assign a lifetime-utility level of zero to a *neutral* life. A life, taken as a whole, is a neutral life from the viewpoint of the individual leading it if it is as good as a life without any experiences (a state of permanent unconsciousness). Note that it is not necessary to invoke states of non-existence of an individual in order to define the notion of neutrality. In particular, it is not claimed that an individual can gain or lose by being brought into existence. Therefore, an existing person's life is worth living if the individual's lifetime utility is positive.

Welfarism is the principle that the only features of an alternative that are socially relevant are the utilities of the individuals alive in this alternative. Welfarism implies that the social ordering of X for any profile of individual utility functions in the domain of the social welfare functional can be determined by a single *social welfare ordering* of all possible vectors of individual utilities $\mathcal{U} = \cup_{n \in \mathbb{N}} \mathbb{R}^n$. That is, if a social welfare functional is welfarist, there exists an ordering R on $\mathcal{U}$ such that alternative $x \in X$ is at least as good as alternative $y \in X$ for the profile of utility functions U if and only if $U(x) R U(y)$. The set of individuals alive in x and y need not be the same. Thus, given welfarism, the problem of variable-population social evaluation can be reduced to the problem of establishing a social welfare ordering R on the set $\mathcal{U}$ of all utility vectors (of varying dimension). If there are $n \in \mathbb{N}$ individuals alive in an alternative, without loss of generality they can be labelled $1, \ldots, n$ provided that the standard anonymity property is satisfied. A representation of the restriction of R to fixed-population comparisons is an individualistic Bergson–Samuelson social welfare function.

The most commonly-discussed examples of variable-population social welfare orderings are extensions of utilitarianism. According to *average utilitarianism* (AU) (resp. *classical utilitarianism* (CU)), average (resp. total) utilities are used as the criterion to compare any two utility vectors. Formally, for all $n, m \in \mathbb{N}$, all $u \in \mathbb{R}^n$, and all $v \in \mathbb{R}^m$, $uR_{\mathrm{AU}}v$ if and only if $\frac{1}{n}\sum_{i=1}^{n} u_i \geq \frac{1}{m}\sum_{i=1}^{m} v_i$ (resp. $uR_{\mathrm{CU}}v$ if and only if $\sum_{i=1}^{n} u_i \geq \sum_{i=1}^{m} v_i$). Clearly, fixed-population comparisons are the same according to R_{AC} and R_{CU}, but this is not necessarily the case if n and m differ.

Average utilitarianism is rejected by most contributors to this area. Its fundamental problem is that the value of adding a person, ceteris paribus, depends on the utilities of those alive. This has rather unfortunate consequences. Suppose, for example, that everyone is extremely well-off in an alternative and we consider the addition of an individual who, if brought into existence, would have a lifetime utility just slightly below the average of the existing population and no one else's utility is affected. According to AU, this person should not be brought into existence. The dual of this example at the other end of the spectrum can be considered even more disturbing. Consider a society in which everyone is extremely miserable by all standards (and well below neutrality). AU recommends the ceteris paribus addition of anyone with a lifetime utility slightly above the average, even if this utility level is well below neutrality.

Classical utilitarianism suffers from what Parfit (1984) calls the *repugnant conclusion.* A variable-population social welfare ordering R implies the repugnant conclusion if, for any population size n, for any positive level of utility ξ (no matter how high), and for any level of utility $\varepsilon \in (0, \xi)$ (no matter how close to zero), there exists a population size $m > n$ such that a population with n people in which everyone has a lifetime utility of ξ is considered inferior to a population of m individuals each of which has a lifetime utility of ε. That is, for any situation in which everyone alive has an arbitrarily high level of well-being, there is always a situation of mass poverty (with everyone arbitrarily close to neutrality) that is considered superior.

In order to avoid the repugnant conclusion and, at the same time, the counter-intuitive implications of average utilitarianism, Blackorby and Donaldson (1984) have proposed *critical-level utilitarianism* (CLU) with a positive critical level as an alternative criterion. CLU employs a parameter $\alpha \in \mathbb{R}$ (the critical level) and is defined by letting, for all $n, m \in \mathbb{N}$, all $u \in \mathbb{R}^n$, and all $v \in \mathbb{R}^m$, $uR_{\mathrm{CLU}}v$ if and only if $\sum_{i=1}^{n}[u_i - \alpha] \geq \sum_{i=1}^{m}[v_i - \alpha]$. The special case corresponding to $\alpha = 0$ is CU. The parameter α has an intuitive interpretation: it is the level of utility that, if experienced by an additional person, makes the alternative resulting from the ceteris paribus addition of such a person to any given society as good as the original. Because the critical level is constant, the problems of AU alluded to above are avoided. If, moreover, α is positive, the repugnant conclusion is avoided because there is a positive difference between the critical level and the level of utility representing neutrality.

In addition to providing a thorough analysis of critical-level utilitarianism and its main alternatives, Blackorby, Bossert, and Donaldson (2005) have discussed several extensions of the basic model. For example, the critical-level utilitarian orderings can be generalized by considering transformed utilities rather than the utilities themselves. If the transformation is chosen to be strictly concave, the corresponding social ordering represents inequality aversion in utilities. Furthermore, they have considered orderings that use non-welfare information such as birth dates and lifetimes in addition to lifetime utilities, as well as variants that incorporate uncertainty. Moreover, they have analyzed variable-population choice problems and applications.

4. Strategy-proof social choice

A *social choice function* g chooses one alternative from the set of alternatives X for each preference profile in the domain $\mathcal{D}$. If it is only known that the true profile is in $\mathcal{D}$, in order to implement the desired choice $g(\mathbf{R})$ when the profile is $\mathbf{R}$, individuals must have an incentive to truthfully report their preferences. *Strategy-proofness* is the requirement that nobody can can obtain a preferred outcome by reporting a false preference regardless of what the preferences of the other individuals are. Strategy-proofness places severe constraints on the kinds of social choice functions that can be considered and, on some domains, conflicts with other social desiderata. For introductions to recent developments in strategy-proof social choice theory, see Sprumont (1995) and Barberà (2001).

The classic result on strategy-proofness is the Gibbard (1973)–Satterthwaite (1975) Theorem, which shows that no social choice function g can satisfy both Nondictatorship and Pareto Optimality when $\mathcal{D} = \mathcal{R}^n$ if $|X| \geq 3$. The same conclusion follows if Pareto Optimality is replaced with *Unanimity*, the requirement that an alternative is chosen if everybody agrees that it is uniquely best. Either of these conditions imply that the range $\text{rg}(g)$ of g is all of X when the domain is unrestricted. A variant of the Gibbard–Satterthwaite Theorem states that on an unrestricted domain, if $|\text{rg}(g)| \geq 3$, then strategy-proofness implies that someone must be a dictator on $\text{rg}(g)$ (i.e., g always chooses one of this person's best alternatives on $\mathrm{rg}(g)$).

More positive results are obtained if it is known that preferences are single-peaked. Moulin (1980) has shown that if $X \subseteq \mathbb{R}$, $\mathcal{D} = \mathcal{S}^n$, and the social choice function g only depends on the peaks of the individual preferences, then g satisfies Strategy-proofness if and only if it is a minmax social choice function and it satisfies Strategy-proofness, Pareto Optimality, and Anonymity if and only if it is a generalized median social choice function. A *minmax* social choice function g is defined by specifying an alternative x_S in the closure of X for each coalition of individuals with $x_T \leq x_S$ if $S \subseteq T$ and setting

$$g(\mathbf{R}) = \min_{S \subseteq N} \left\{ \max_{i \in S} \{\pi(R_i), x_S\} \right\}, \quad \forall \mathbf{R} \in \mathcal{S}^n.$$

For each $\mathbf{R} \in \mathcal{S}^n$, a *generalized median social choice function* chooses the median of the actual individual preference peaks and the fixed peaks of $n-1$ phantom voters. These functions are minmax rules in which the alternatives x_S are the same for coalitions of the same size. Barberà, Gul, and Stacchetti (1993) have provided an alternative characterization of minmax rules in terms of winning coalitions that has proved to be quite useful. If, as is the case with minmax rules, the chosen alternative for each profile only depends on each person's most-preferred alternative(s) on the range, the social choice function satisfies the *tops-only* property. On the domain $\mathcal{S}^n$, Barberà and Jackson (1994) have shown that the tops-only property assumed by Moulin (1980) is implied by Strategy-proofness if either $\mathrm{rg}(g)$ is an interval or g satisfies Pareto Optimality.

The original strategies used to prove the Gibbard–Satterthwaite Theorem cannot be adopted to analyze strategy-proofness when preferences are continuous. The problem is that these proofs alter profiles by moving two alternatives to the top two positions in a person's preference, but this is not possible with continuous preferences if X is a connected set, as there can be no second-ranked alternative. This difficulty was overcome by Barberà and Peleg (1990) who established a version of the Gibbard–Satterthwaite Theorem for continuous preferences on a metric space of alternatives using the option set methodolgy introduced by Laffond (1980), Satterthwaite and Sonnenschein (1981), and Barberà (1983). An *option set* identifies the set of outcomes that are feasible given the preferences of a subgroup of individuals for some admissible reported preferences of the rest of the population. For example, when there is a dictator d, the option set generated by R_d consists of the best alternatives on the range for this preference and the option set generated by any other person's preference is the whole range. The option set methodology proceeds by identifying the structure imposed on option sets by the properties that one wants the social choice function to satisfy.

In order for a social choice function to be strategy-proof, it must ignore most of the information about individual preferences. On many domains in which all admissible preferences have unique best alternatives on the range, strategy-proofness implies the tops-only property provided that the range of the social choice function satisfies some regularity condition. Weymark (2004) has proposed a proof strategy for establishing the tops-only property that avoids the model specificity of earlier proofs.

A social choice function defined on a domain of profiles of separable preferences on a product set of alternatives is *decomposable* if the value chosen for a component only depends on the individual marginal preferences for that component. The first decomposability results were established by Border and Jordan (1983) who, for example, showed that for the domain of all profiles of separable quadratic preferences on a multidimensional Euclidean space, a social choice function satisfies Strategy-proofness and Unanimity if and only if it decomposes into strategy-proof, unanimous social choice functions on each component. Furthermore, these one-dimensional mechanisms can be any member of Moulin's class of minmax social choice functions. Since the development of the option set methodology, decomposibility results for strategy-proof social choice functions have been established for a number of other domains of separable preferences. For example, Barberà, Gul, and Stacchetti (1993) have shown that if X is a discrete product set in a Euclidean space and individuals can have any separable peference that satisfies a multidimensional analogue of single-peakedness, then the conclusions of Border and Jordan's theorem hold if the range of the social choice function is all of X. Whether Strategy-proofness and auxiliary conditions such as Unanimity imply decomposability depends on how much preference variability is present in the domain. Establishing a decomposability theorem typically involves first showing that the tops-only property is satisfied, as in Barberà, Gul, and Stacchetti (1993). Much of the literature on this issue has been synthesized and extended by Le Breton and Sen (1995, 1999).

If X is a product set, but only a subset Z of X is feasible, decomposability results are still possible, but not every combination of the corresponding one-dimensional social choice functions is admissible. For example, using the model in Barberà, Gul, and Stacchetti (1993) with the best alternative for each preference required to be in Z, Barberà, Massó, and Neme (1997) have shown that any social choice function g that is strategy-proof and whose range is Z is decomposable into one-dimensional minmax rules on each component, but in order for a combination of such minmax rules to always produce a feasible outcome, g must satisfy a rather complicated condition called the *intersection property*.

In the preceding discussion, everyone has the same set of admissible preferences, and so it is possible that they might agree on what is best. When there are private goods and individuals only care about their own consumption, one generally expects there to be distributional conflicts. We illustrate the implications of strategy-proofness with private goods in two problems: the allotment problem and the exchange of divisible goods.

In an *allotment problem*, there is a fixed amount Ω of a divisible good to allocate. If individuals only care about own consumption, each person's preference is defined on $X = [0, \Omega]$. If these preferences are single-peaked, a prominent solution to this problem is the *uniform rule* (see Benassy, 1982) which, for each admissible profile $\mathbf{R} \in \mathcal{S}^n$, chooses the unique allocation $x = (x_1, \ldots, x_n) \in X^n$ (x_i is person i's allocation) for which (i) if $\Omega \leq \sum_{i=1}^n \pi(R_i)$, there exists $\lambda \in \mathbb{R}_+$ such that, for all $i \in N$, $x_i = \min\{\pi(R_i), \lambda\}$ and (ii) if $\Omega \geq \sum_{i=1}^n \pi(R_i)$, there exists $\lambda \in \mathbb{R}_+$ such that, for all $i \in N$, $x_i = \max\{\pi(R_i), \lambda\}$. Sprumont (1991) has shown that if the domain is the set of all profiles of continuous single-peaked preferences on X, then a social choice function satisfies Strategy-proofness, Pareto Optimality, and Private-Goods Anonymity (permuting preferences results in the same permutation of the individual allocations) if and only if it is the uniform rule. Sprumont's article also includes the first explicit theorem about the tops-only property in the strategy-proofness literature.

When X is the set of allocations of an exchange economy with two or more divisible private goods, the general conclusion is that Strategy-proofness and Pareto Optimality conflict with other desirable properties for a social choice function on a sufficiently rich domain of classical private-goods preference profiles. If the aggregate endowment is privately owned and participation in the collective choice procedure is voluntary, the social choice function must satisfy Individually Rationality; i.e., each person is guaranteed a consumption bundle weakly preferred to his endowment. Hurwicz (1972) has shown that Strategy-proofness, Pareto Optimality, and Individual Rationality are inconsistent for two-person, two-good exchange economies on such a preference domain. This impossibility theorem has only recently been extended to the general n-person, m-good case by Serizawa (2002).

With monotonic preferences, a dictator in an exchange economy always receives the whole endowment. For the domain of classic private-goods preference profiles, Zhou (1991) has shown that Strategy-proofness, Pareto Optimality, and Nondictatorship are inconsistent when there are at least two goods, but only two individuals. When there are at least three individuals, Satterthwaite and Sonnenschein (1981) have shown by example how to construct Pareto optimal, strategy-proof, nondictatorial social choice functions for this domain. In their example, someone is *bossy* (i.e., there is an individual who can change the consumption bundle of someone else by reporting a different preference without affecting his or her own consumption bundle) and, for each profile, one of two individuals receives all of the endowment. It is generally agreed that bossy mechanisms are unsatisfactory. Serizawa and Weymark (2003) have shown that any social choice function that satisfies Strategy-proofness and Pareto Optimality cannot guarantee everyone a consumption bundle bounded away from the origin on a rich domain of classical private-goods preferences.

Given that any strategy-proof and Pareto optimal social choice function g must fail even minimal distributional desiderata on such domains, Barberà and Jackson (1995) have explored the implications of abandoning Pareto Optimality. For private ownership exchange economies with classical private-goods preferences, they have shown that if g is strategy-proof, nonbossy, and satisfies some other auxiliary conditions, then trade must be restricted to occur in a limited set of fixed proportions with possibly upper limits on the amounts that can be exchanged. In the case of two goods and two individuals, if g satisfies Strategy-proofness and Individual Rationality, there are only two such proportions, and the choice procedure resembles the fixed-price trading rules studied by Benassy (1982) with different buying and selling prices for each good.

5. Axiomatic models of resource allocation

The recent literature on axiomatic models of resource allocation has a close affinity to the literature on Arrovian social choice on economic domains. As is the case with Arrovian social choice when there are multiple agendas, the research on axiomatic models of resource allocation investigates the implications of normative criteria (axioms) when both individual preferences and the set of feasible agendas satisfy the kinds of restrictions found in economic models. What distinguishes this literature is the set of axioms considered, many of which rely on the special structure provided by economic models for their definition. In this section, we present a very selective introduction to the models and axioms considered in this literature and describe a few of the theorems that have been obtained. For a comprehensive survey of this literature, see Thomson (2006).

In an *allocation problem*, there is an aggregate social endowment $\Omega \in \mathbb{R}^m_{++}$ of m private goods that are to be allocated among $n \geq 2$ individuals based on their preferences. In the canonical allocation problem, $m \geq 2$ and all goods are divisible. An *economy* is then described by a pair $E = (\mathbf{R}, \Omega)$, where $\mathbf{R}$ is a profile of classical private-goods preferences. Let $\mathcal{E}$ denote the set of all such economies. Given the endowment Ω, the corresponding agenda $A(\Omega)$ is the set of feasible allocations $x = (x_1, \ldots, x_n)$ that exhaust Ω, where $x_i \in \mathbb{R}^m_+$ is person i's consumption bundle.

A *solution* is a mapping that selects a subset of the feasible allocations for each economy in $\mathcal{E}$. Note that a solution φ can be identified with a social choice correspondence C by setting $C(A(\Omega), \mathbf{R}) = \varphi(E)$ for all $E \in \mathcal{E}$. A solution satisfies *Efficiency* if it always chooses Pareto optimal allocations and it satisfies *No Envy* if, at any selected allocation, nobody strictly prefers anyone else's allocated consumption bundle. No Envy, which was independently introduced by Tinbergen (1953), Foley (1967), and Kolm (1972), is the fundamental fairness condition considered in this literature. An example of a solution satisfying both Efficiency and No Envy for this class of economies is the *Equal-Division Walrasian solution* φ^{W}, which is defined from the Equal-Division Walrasian social choice correspondence in the manner described above.

The recent literature on fair allocation has expanded the scope of the canonical model in several respects. For example, economies with varying populations or with production have been considered. In addition, variations of this model have been explored, for example, by allowing for indivisibilities or, when there is only one good, preference restrictions such as single-peakedness.

An alternative setup with public goods has also been examined. The existence of solutions satisfying Efficiency and No Envy in public-goods environments is a more complex matter than in the private-goods case, largely because arguments involving pure exchange cannot be invoked when there are public goods. Furthermore, the technology that permits us to transform private goods into public goods is also important. With some additional assumptions, however, Efficiency and No Envy can be satisfied; see Diamantaras (1991), for example.

Prominent among the new axioms that have been introduced and used in characterizations of existing and new solutions is *Consistency*, which is discussed in detail in Thomson (1990). In order to define Consistency, the notion of a solution must be extended to include economies with different numbers of individuals. Let x be an allocation that is selected by such a solution for an $(n+1)$-person economy. Now suppose that person k leaves the economy with the consumption x_k. Define a *reduced* n-person economy by removing k and subtracting x_k from the total endowment. Consistency demands that the allocation $(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_{n+1})$ is selected in the reduced economy.

Other important properties include monotonicity conditions with respect to the quantities of the resources available, with respect to the technology, or with respect to the population. A solution φ satisfies *Resource Monotonicity* if, whenever the social endowment expands, no-one becomes worse off in any chosen allocation. In private-goods models with production, an axiom similar in spirit to resource monotonicity is *Technology Monotonicity*. It requires that if the only difference between two economies is that the technology of one dominates that of the other, then everyone should be at least as well-off in any allocation chosen for the former economy than in any allocation chosen for the latter. *Population Monotonicity* is a solidarity axiom. As is the case for Consistency, it applies in models with variable population. Suppose that the population is expanded, but the total endowment is unchanged. Population Monotonicity demands that the burden imposed on the existing population by the presence of the additional individuals is shared by all of its members; no-one who is present before the population expansion is better off as a consequence of the population augmentation.

If there is only one divisible good, each economy $E = (\mathbf{R}, \Omega)$ defines an allotment problem, as in the preceding section. When all preference profiles are single-peaked, the *uniform solution* simply applies the uniform rule for the allotment problem to each economy in the domain. In addition to the characterization of this solution presented in the preceding section, there have been axiomatizations of the uniform solution using No Envy, Consistency, and variants of either Resource Monotonicity or Population Montonicity, among other axioms. See Thomson (2006).

If some of the goods to be allocated are indivisible, much of the theory developed in the context of perfectly divisible goods still applies. Due to the specific nature of the problem of allocating indivisible objects, some interesting additional results can be obtained. As an illustration, consider an assignment problem in which n indivisible objects are to be allocated to n individuals and there is also a perfectly divisible good ("money") that can be consumed in any amount, positive or negative. See Thomson (2006) for references to contributions that permit the number of goods and individuals to differ. A commodity bundle for person i is now a pair $(t_i, j) \in \mathbb{R} \times N$, where t_i (resp. j) is the amount of money (resp. object) allocated to i. It is assumed that i's preference R_i on $\mathbb{R} \times N$ is strictly monotonic in money and that money can be used to compensate for the receipt of a less desirable good in the sense that, for all $t_i \in \mathbb{R}$ and all $j, k \in N$, there exists $s_i \in \mathbb{R}$ such that i is indifferent between (s_i, k) and (t_i, j). An economy now consists of a preference profile $\mathbf{R}$ with the properties introduced above, an aggregate endowment of money $T \in \mathbb{R}$, and the n indivisible objects. Because the the set of objects is fixed, an economy can be characterized by a pair $E = (\mathbf{R}, T)$. A feasible allocation for E is a pair (t, ρ), where $t \in \mathbb{R}^n$ is a vector of *balanced* monetary allocations (i.e., $\sum_{i=1}^{n} t_i = T$) and $\rho\colon N \to N$ is a permutation with $\rho(i)$ specifying the object allocated to person i.

Solutions and the Efficiency and No Envy axioms are defined in the usual manner. If the monetary allocations are restricted to be non-negative, it is clear that solutions satisfying No Envy may not exist. For example, if $T = 0$ and everyone regards the same object as being uniquely best regardless of the amount of money received, whoever is allocated this object is envied by everyone else because no monetary compensations are possible. Sufficient conditions for the existence of solutions satisfying No Envy with non-negative monetary allocations are discussed in Thomson (2006). As is to be expected, these conditions ensure that there is a sufficient amount of money available to carry out the requisite compensation payments.

In the case of perfectly divisible goods, we have noted that the Equal-Division Walrasian solution φ^{W} satisfies Efficiency and No Envy. Interestingly, in the indivisible-good model considered here, the allocations generated by an adaptation of this Walrasian solution to the present framework are the *only* allocations satisfying No Envy (see Svensson, 1983). Moreover, Efficiency *follows* as a consequence of No Envy. In this model, for an economy E, if everyone is provided with the same endowment $t_0 \in \mathbb{R}_+$ of money, a *Walrasian equilibrium* is a feasible allocation (t, ρ) and a price $p_k \in \mathbb{R}_+$ for each good $k \in N$ such that the bundle $(t_i, \rho(i))$ is weakly preferred by individual i among all bundles that have values no more than t_0. The solution φ^{W} is then defined by letting $\varphi^{\mathrm{W}}(E)$ be the set of Walrasian equilibrium allocations that can be obtained in this way.

A number of fairness principles besides No Envy have been considered in the literature. See Fleurbaey and Maniquet (2006) and Thomson (2006). Particularly notable among them is *Egalitarian Equivalence*, which is due to Pazner and Schmeidler (1978). In the canonical allocation problem, Egalitarian Equivalence requires that, for each economy $E = (\mathbf{R}, \Omega)$, each selected allocation x has the property that there exists a consumption bundle $z_0 \in \mathbb{R}_+^m$ that everyone is indifferent to. Pazner and Schmeidler (1978) have shown that on the domain of economies $\mathcal{E}$ for this problem, solutions exist that satisfy both Egalitarian Equivalence and Pareto Optimality. However, Egalitarian Equivalence need not satisfy Independence of Infeasible Alternatives, as the egalitarian allocation $(z_0, \ldots, z_0)$ associated with x need not be feasible.

There is now an extensive literature that employs the framework and many of the axioms described in this section to re-examine the foundations of egalitarian theories. If individuals are held responsible in part for the outcomes they receive, conditional versions of egalitarianism demand that individual differences caused by factors beyond the individuals' control should be compensated for, whereas inequities that can be attributed to choices for which an individual is responsible do not attract that kind of equalization. Variants of this theory have been advocated by, for example, Roemer (1993). See Fleurbaey and Maniquet (2006) for a detailed survey of this literature.

6. Concluding remarks

As noted above, the response of Sen (1974) to Arrovian social welfare function impossibilities was to abandon the ordinal noncomparability of individual utilities built into the Arrow framework. However, he maintained the spirit of IIA by assuming that the social ranking of any two alternatives should only depend on the individual utilities obtained with them. This independence assumption is the cornerstone of the welfarist approach employed in the literatures on social choice with interpersonal utility comparisons and on variable-population social choice.

A different resolution of the Arrovian dilemma has been described and defended in Fleurbaey (2002). Rather than abandoning ordinal noncomparability, Arrow's IIA assumption is relaxed so as (i) to allow the social ranking of a pair of alternatives to depend on how these alternatives are ranked relative to some other alternatives and (ii) to incorporate some principle of fairness. This proposal has been explored in a series of articles by Fleurbaey and various co-authors. For example, Fleurbaey, Suzumura, and Tadenuma (2005) have shown that when the alternatives are the set of all allocations of $m \geq 2$ divisible private goods and the domain of the social welfare function is the set of profiles of classical private-goods preferences, then Weak Pareto and a private-goods version of Anonymity are compatible with an independence condition that incorporates fairness considerations of the sort embodied in Egalitarian Equivalence. Independence conditions such as this or ones based on envy-freeness employ non-local information about preferences, including information about alternatives that may not be feasible if resource constraints are taken into account. If a social welfare function that satisfies one of these fairness-based independence conditions is used to choose alternatives out of different agendas, Independence of Infeasible Alternatives will be violated. No consensus has yet emerged as to whether the benefits of the ordinal fairness-based approach to social choice theory are sufficient to overcome objections that it is based on comparisons that may involve non-feasible alternatives.

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