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Nonlinear Monetary Policy Rules: Some New Evidence for the U.S.*

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Abstract

This paper derives optimal monetary policy rules in setups where certainty equivalence does not hold because either central bank preferences are not quadratic, and/or the aggregate supply relation is nonlinear. Analytical results show that these features lead to sign and size asymmetries, and nonlinearities in the policy rule. Reduced-form estimates indicate that US monetary policy can be characterized by a nonlinear policy rule after 1983, but not before 1979. This finding is consistent with the view that the Fed's inflation preferences during the Volcker-Greenspan regime differ considerably from the ones during the Burns-Miller regime.

JEL Classification: E52
Key Words: nonlinear Taylor rules, inflation targets, asymmetric preferences, nonlinear Phillips curve, monetary policy

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Résumé

Dans cet article, nous dérivons les règles de politique monétaire optimales dans des cadres où "l'équivalence certaine" n'est pas satisfaite. On trouve des cas de ce type lorsque, par exemple, les préférences de la banque centrale ne sont pas quadratiques ou lorsque la relation d'offre agrégée n'est pas linéaire. Les résultats théoriques semblent montrer qu'ils peuvent conduire à des asymétries de signe et de niveau et à des nonlinéarités dans la règle de politique. À partir d'estimations de la forme réduite sur données américaines, nous trouvons que la politique monétaire des États-Unis peut être caractérisée par une règle décisionnelle non linéaire après 1983, mais que ce n'est pas le cas avant 1979. Ces résultats reflètent bien la nette différence d'attitude de la Réserve Fédérale face à l'inflation durant les périodes Volcker-Greenspan et Burns-Miller.

Mots clés : règles de Taylor non linéaires, objectifs d'inflation, préférences asymétriques, courbe de Phillips non linéaire, politique monétaire

1 Introduction

This paper derives and estimates optimal monetary policy rules in a setup where certainty equivalence does not hold. In particular, our approach combines two different strands of the literature on monetary policy rules that depart from the standard linear-quadratic framework because either (*i*) central bank preferences are not quadratic, or (*ii*) the aggregate supply relation is nonlinear.

As it is well known [see, for example, Svensson (1997) and Clarida, Gali, and Gertler (1999, 2000)], the combination of a quadratic loss function and a linear aggregate supply constraint leads to a linear reaction function, or Taylor rule, by the central bank. The optimal policy rule implies that the nominal short-term interest rate under the central bank's control is a linear function of the inflation and output gap deviations from their respective targets. Depending on the backward or forward nature of wage and price setting, and on assumptions regarding the information available to the central bank, both variables appear in the rule either in current terms or as expectations of their future values. Because they provide a reasonably good description of policy, linear rules have become a key element of diagnosis in the toolkit of monetary-policy analysts.

Recently, there have been a number of studies that seek to extend this traditional setup. The generalizations fall in two groups. *First*, Nobay and Peel (1998), Cukierman (1999), Gerlach (1999), and Ruge-Murcia (2002a, 2002b) relax the assumption of a quadratic central bank loss function and adopt instead asymmetric preference specifications. Their functional forms allow different weights for positive and negative inflation and/or output deviations from their target. Asymmetric preferences modify some of the results previously derived in the linear-quadratic framework. For example, Cukierman (1999) shows that when the central bank is more concerned about under- than over-employment and there is uncertainty regarding future realizations of inflation and unemployment, an inflation bias can arise even if the unemployment target is the natural rate. Cukierman's proposition is examined empirically by Ruge-Murcia (2002a, 2002b) using cross-section data from OECD countries and time series data from G-7 countries.

Second, Schaling (1999) and Dolado, María-Dolores, and Naveira (2003) study models where the aggregate supply curve is not linear, but convex. In particular, the difference between realized and expected inflation is a convex function of the output gap. The underlying idea behind this specification goes back to the traditional Keynesian assumption that nominal wages are flexible upwards but rigid downwards, implying that inflation is a decreasing and convex function of the unemployment rate. This implies that an increase in unemployment will drive inflation down by much less when unemployment is high than when it is low [see, for example, Layard, Nickell, and Jackman (1991), and Álvarez-Lois (2001)]. If unemployment and the output gap are related through Okun's law, then a convex relationship between inflation and the output gap is a natural generalization of the linear aggregate supply. Combined with a quadratic loss function, the optimally-derived Taylor rule has nonlinear features: it implies that the central bank will increase interest rates by a larger amount when inflation is above target than it will reduce them when inflation is below target.

The goal of this paper is to construct and estimate a more general model that combines both asymmetric central bank preferences and a non-linear Phillips curve. This is important for several reasons. *First,* it allows the joint analysis of two departures from the linear-quadratic setup that until now have been studied separately in the literature. *Second,* it permits us to trace back nonlinearities and asymmetries in the nominal interest rate to either central bank preferences, nonlinearities in the supply curve, or both. *Finally,* parameter estimates will indicate the relative importance of these two elements in monetary policy making.

The contributions of this paper are twofold. From an analytical viewpoint, we construct a model of inflation targeting where the central bank's preferences are asymmetric and the aggregate supply curve is nonlinear. Preferences are asymmetric in the sense that positive deviations from the inflation target can be weighted more (or less) severely than negative deviations in the central bank's loss function. The aggregate supply curve is an increasing and convex function of the output gap. In this manner, we are able to derive a Taylor rule in a nonlinear framework that generalizes the usual specification in the literature where the objective function is quadratic and constraints are linear.

From an empirical viewpoint, we confront the new Taylor rule with data on short-term interest rate interventions by the U.S. Federal Reserve. Reduced-form estimates indicate that U.S. monetary policy can be characterized by a nonlinear rule after 1983, but not before 1979. Although we do not find evidence in favor of a convex aggregate supply curve, we do find evidence consistent with asymmetric inflation preferences on the part of the U.S. Federal Reserve after 1983. This suggests that the Fed's inflation preferences during the Volcker-Greenspan regime differ considerably from the ones during the Burns-Miller regime. When we compare our results with those of Clarida, Gali, and Gertler (2000) using a linear Taylor rule for the post-1982 period, we do not find evidence that the response of the short–term interest rate to inflation was larger than unity once asymmetric preferences are allowed for. The reason for this result is that under asymmetric preferences, the targeted interest rate depends on the conditional variance of inflation, that in turn depends nonlinearly on lagged inflation. The response of the interest rate to inflation depends on a linear part and a nonlinear part such that the overall response is stabilizing, as suggested by Clarida, Gali, and Gertler. However, the interpretation of how stabilization was achieved in the Volcker-Greenspan era is different in both models.

The rest of the paper is structured as follows. Section 2 derives the form of the nonlinear policy rule under the general case of asymmetric preferences and a convex aggregate supply curve, and compares it to several relevant subcases. Section 3 estimates the nonlinear rule for the U.S., distinguishing between the two relevant subperiods and using a wide array of alternative specification to check the robustness of the results. Finally, Section 4 concludes. An Appendix contains a detailed derivation of the nonlinear monetary policy rule.

2 A Simple Model

In order to fix ideas, it is helpful to consider a simple model of optimal monetary policy. The model follows closely the one proposed by Svensson (1997), but generalizes the specification of the central bank preferences and aggregate supply curve in a manner to be made precise below. Although Section 3 reports estimates of the policy rule obtained using this model, it also shows that the main finding of this paper is robust to the precise form of the rule (for example, whether forward or backward looking).

Assume that monetary policy is conducted by a central bank that chooses the sequence of short-term interest rates that minimizes the present discounted value of its loss function. The loss function depends on the distance between realized inflation and its socially optimal rate. Formally, the central bank's problem is[1]

$$\underset{\{i_{t+s}\}_{s=0}^{\infty}}{Min} \qquad E_t \sum_{s=0}^{\infty} \beta^s L(\pi_{t+s} - \pi^*), \tag{1}$$

where i_t is the nominal interest rate, $0 < \beta < 1$ is the discount rate, π_t is the inflation rate, π^* is socially-optimal inflation rate, and the loss function $L(\cdot)$ takes the form:

$$L(\pi_t - \pi^*) = \frac{\exp(\gamma(\pi_t - \pi^*)) - \gamma(\pi_t - \pi^*) - 1)}{\gamma^2}.$$

This loss function corresponds to the linex function, originally proposed by Varian (1974). This function has several important properties. *First,* it permits different weights for positive and negative inflation deviations from π^*. Consider the case where $\gamma > 0$. For inflation

[1] In preliminary work, we considered adding an output stabilization term to the objective function. However, we were not able to obtain a closed-form solution for the more general case with asymmetric preferences and a nonlinear Phillips curve. See the Appendix in Dolado, María-Dolores, and Ruge-Murcia (2002). For the linear case, Svensson (1997) shows that allowing an output stabilization term does not change the form of the policy function, although it reduces the interest-rate response by the central bank to inflation and the output gap.

rates below π^*, the linear term becomes progressively more important as inflation decreases and, consequently, the loss rises linearly. For inflation rates above π^*, it is the exponential term that eventually dominates and the loss associated with a positive deviation rises exponentially. Hence, positive deviations from π^* are weighted more severely than negative ones in the central banker's loss function. *Second,* it predicts that both the size and sign of a deviation affect the central banker's loss. In contrast, under quadratic preferences, the loss is completely determined by the size of the deviation. *Third,* it relaxes certainty equivalence and allows a prudence motive on the part of the central banker. Then, moments of higher order than the mean might play a role in the formulation of monetary policy. For this loss function, the coefficient of relative prudence [see Kimball (1990)] is $\gamma(\pi_t - \pi^*)$, that is directly proportional to the inflation deviation from its target and increasing in γ. *Finally,* it nests the quadratic function commonly used in previous literature as a special case when the preference parameter γ tends to zero. This result is important because it suggests that the hypothesis that the central banker's preferences are quadratic over inflation could be evaluated by testing whether γ is statistically different from zero.

The central bank takes as given the behavior of the private sector, that is summarized by:

$$y_{t+1} = \delta y_t - r_t + \eta x_t, \tag{2}$$

$$\pi_{t+1} = \pi_t + f(y_t) + u_{t+1}, \tag{3}$$

where

$$f(y_t) = \alpha y_t/(1 - \alpha\phi y_t), \tag{4}$$

$$x_{t+1} = \psi x_t + e_{t+1}, \tag{5}$$

$$i_t = r_t + E_t\pi_{t+1}, \tag{6}$$

y_t is the output gap, r_t is the real interest rate, x_t is an exogenous variable that follows the AR(1) process in (5), u_t and e_t are normally and independently distributed shocks with zero mean and variances σ_u^2 and σ_e^2, respectively, and the remaining parameters satisfy $0 < \psi, \delta < 1$, $\alpha > 0$, and $\phi \geq 0$. Note that although we assume constant unconditional variances for u_t and e_t, we allow the possibility that these shocks are conditionally heteroskedastic. Equation (2) is an IS relationship where the output gap depends on the lagged output gap, the real interest rate, and the exogenous variable, x_t. Equation (3) is a backward-looking AS relationship where inflation depends on lagged inflation and output gap, the latter appearing in a (possibly) non-linear way. The nonlinearity of the AS curve is represented using the functional form (4). This form has been used previously by Schaling (1999) and Dolado, María-Dolores, and Naveira (2003) and includes the cases of a linear AS curve when $\phi = 0$ and a convex one when $\phi > 0$. Notice that under this specification, y_{t+1} is predetermined at time t and, consequently, forms part of the agents' information set at time t. Finally, equation (6) is the Fisher relation. Although this model is a highly stylized description of the economy, it is representative of the type of models used by the literature on monetary policy rules.

Since the interest rate affects inflation with a two-period lag, without any effects in t and $t+1$, the central bank can find the optimal interest rate at time t as the solution to the simpler period-by-period problem [see Svensson (1997)]:

$$\underset{\{i_t\}}{Min} \quad E_t \; \beta^2 L(\pi_{t+2} - \pi^*). \tag{7}$$

The Appendix shows that the first-order conditions for minimizing (7) subject to the constraints (2) and (3), yields the following Taylor rule for the nominal interest rate:

$$i_t = \pi_t + f(y_t) + \delta y_t + \frac{(1/\alpha)(\pi_t - \pi^* + \gamma\sigma_{\pi,t}^2/2 + f(y_t))}{1 - \phi(\pi_t - \pi^* + \gamma\sigma_{\pi,t}^2/2 + f(y_t))} + \eta x_t, \tag{8}$$

where $\sigma_{\pi,t}^2 = \sigma_{u,t}^2$ denotes the conditional variance of the inflation rate. The subscript t indicates that this conditional variance might change over time.

The Taylor rule (8) is general in that it nests the cases where the central bank's preferences are quadratic ($\gamma \to 0$), the AS schedule is linear ($\phi = 0$), or both. The latter case corresponds to the linear monetary policy rules examined by previous literature. In order to gain intuition regarding this policy rule, the following sections examine three special cases contained in (8).

2.1 Case I: Linear Aggregate Supply Schedule ($\phi = 0$)

When $\phi = 0$, the function $f(\cdot)$ becomes $f(y_t) = \alpha y_t$ and the AS curve is linear. In this case the only nonstandard feature of the model is the asymmetry in central bank preferences, and the nonlinear Taylor rule simplifies to

$$i_t = \pi_t + (1 + \alpha + \delta)y_t + (1/\alpha)(\pi_t - \pi^* + \gamma\sigma_{\pi,t}^2/2) + \eta x_t. \tag{9}$$

Under asymmetric preferences, the conditional variance of inflation, $\sigma_{\pi,t}^2$ (along with the inflation rate and the output gap) is one of the main determinants of the interest rate target. If $\sigma_{\pi,t}^2$ depends on lagged inflation and output (for example, as in ARCH-type models), the Taylor rule will be nonlinear in lagged inflation and output.

Consider the situation where $\gamma > 0$, meaning that the central bank weights more severely positive than negative inflation deviations from its optimal rate. Since $\alpha > 0$, an increase in inflation volatility (as measured by $\sigma^2_{\pi,t}$), leads to an increase in the nominal interest rate, even if the level of inflation and the output gap remain unchanged. The increase is directly proportional to γ because the central bank's prudence increases with γ.[2] The increase is inversely proportional to α for the following reason: when α is large, the central bank needs to increase the nominal and real interest rates by less because a given decrease in the output gap leads to a proportionally larger decrease in inflation when the AS curve is steep.

2.2 Case II: Quadratic Loss Function ($\gamma \to 0$)

When $\gamma \to 0$, the central bank preferences become quadratic in inflation and there is no longer a prudence motive in the implementation of monetary policy. However, if $\phi > 0$, the AS curve is convex and the Taylor rule takes the nonlinear form

$$i_t = \pi_t + f(y_t) + \delta y_t + \frac{(1/\alpha)(\pi_t - \pi^* + f(y_t))}{1 - \phi(\pi_t - \pi^* + f(y_t))} + \eta x_t. \tag{10}$$

In this case y_t, will not appear in a linear way but through the $f(\cdot)$ transformation. As a result of the second-to-last term in (10), the nominal interest rate will depend nonlinearly on inflation and the output gap, but this nonlinearity is conceptually and functionally different from the one in Case I above. It is shown below that when the AS curve is convex, interest rate changes in response to inflation/output deviations from their target are subject to sign and size asymmetries.

2.3 Case III: Linear Rule

The case where both $\gamma \to 0$ and $\phi = 0$ corresponds to the usual model with quadratic preferences and linear constraints. In this case, the optimal reaction function is linear in inflation and output:

$$i_t = \pi_t + (1 + \alpha + \delta) y_t + (1/\alpha)(\pi_t - \pi^*) + \eta x_t. \tag{11}$$

We will see below, in this case changes in the short-term nominal interest rate are symmetric, proportional, and history-independent. Put differently, for linear models, the impulse-response associated with a shock of size 1 (standard deviation) would be the mirror image of the response to a shock of size -1, one-half the response of shock size 2, and independent of the moment the shock is assumed to take place [see Koop, Pesaran, and Potter (1996)].

[2] A comparable result can be found in the literature on precautionary savings. When the assumption of quadratic utility is relaxed and labor-income risk in nondiversifiable, then uncertainty increases the expected marginal utility of future consumption. To satisfy the Euler condition, prudent households decrease current consumption compared to future consumption and increase their savings.

2.4 Implications

As we have seen above, the combination of asymmetric central bank preferences and a nonlinear AS curve has nontrivial implications for the interest-rate response to inflation and output gap deviations from their desired values. This section explores in more detail some of these implications and the interaction between the two main features of the model.

When the AS curve is linear, a marginal change in the current inflation rate leads the central bank to change the nominal interest rate by $\partial i_t/\partial \pi_t = 1 + 1/\alpha$. In this case, the change in i_t is independent of the current output gap and inflation rate and is symmetric, meaning that if inflation increases (decreases) by 1 percent, the nominal interest rate increases (decreases) by $1 + 1/\alpha$ percent.

In contrast, under the general Taylor rule (8) (where the AS curve is nonlinear), the change in i_t is $\partial i_t/\partial \pi_t = 1 + (1/\alpha)(1 - \phi(\pi_t - \pi^* + \gamma\sigma^2_{\pi,t}/2 + f(y_t))^{-2}$. The nonlinear interest rate rule gives rise to sign and size asymmetries. The *sign* asymmetry refers to the fact that under the nonlinear Taylor rule, the response to an increase in inflation is larger than the response to a decrease, even if both are of the same magnitude. As an illustration, assume that inflation is exactly the optimal rate, the output gap is zero, and $\alpha = 2, \delta = 0.1, \phi = 0.2$ and $\gamma\sigma^2_{\pi,t}/2 = 0.4$. Then, $\Delta\pi_t = +1$ induces $\Delta i_t = +1.76$ but $\Delta\pi_t = -1$ induces $\Delta i_t = -1.48$.

The *size* asymmetry refers to the fact that the interest rate response does not change linearly with the change in the inflation rate. For example, taking the same parameter values above, $\Delta\pi_t = +1$ induces $\Delta i_t = +1.76$ but $\Delta\pi_t = +2$ induces $\Delta i_t = +4.09$. Although $\Delta\pi_t = +2$ is twice $\Delta\pi_t = +1$, the interest rate response $+4.09$ is more than twice $+1.76$. On the other hand, while $\Delta\pi_t = -1$ induces $\Delta i_t = -1.48$, $\Delta\pi_t = -2$ induces $\Delta i_t = -2.82$, that is less than twice -1.48. The sign and size asymmetries that arise when the AS curve is convex follow directly from the fact that the interest rate response with respect to inflation $(\partial i_t/\partial \pi_t)$ is convex on the rate of inflation.

Note that when $\gamma = 0$ and, consequently, $\gamma\sigma^2_{\pi,t}/2 = 0$, the corresponding interest rate responses to $\Delta\pi_t = +1$ and -1 under the nonlinear Taylor rule would be $\Delta i_t = +1.63$ and -1.42, respectively, whereas for $\Delta\pi_t = \pm 2$ they would be $\Delta i_t = +3.67$ and -2.71. Hence, asymmetric preferences appear to reduce the size of both the sign and size asymmetries. The reason is that the interest rate response to inflation is less convex on inflation as $\gamma\sigma^2_{\pi,t}/2$ decreases.

Similar results regarding sign and size asymmetries arise when considering the interest rate response to a change in the output gap. When the AS curve is linear, $\partial i_t\ /\partial y_t = 1+\alpha+\delta$, but under the nonlinear Taylor rule, $\partial i_t/\partial y_t = \delta + f'(y_t)(1+(1-\phi(\pi_t - \pi^* + \gamma\sigma^2_{\pi,t}/2 + f(y_t))^{-2})$

where $f'(y_t) = \partial f(y_t)/\partial y_t = \alpha(1 - \alpha\phi y_t)^{-2}$. The interest rate depends nonlinearly on the current output gap giving rise to asymmetric responses on the part of the central bank. For the parameter values above, the interest rate response to $\Delta y_t = \pm 0.1$ is $\Delta i_t = 0.35$ and -0.31. The response to $\Delta y_t = \pm 0.2$ is 0.74 and -0.59. Hence, as before, the response depends on the sign of the output gap deviation from its target and is nonlinearly related to the size of the deviation. This implication is in line with research by Bec, Salem, and Collard (2002), who find that the state of the business cycle (measured by the output gap) is important for U.S. monetary policy.

In summary, a convex AS curve leads an optimizing central bank to respond asymmetrically, in both sign and size, to changes in the output gap and inflation rate. Asymmetric preferences leads to prudent behavior whereby the central bank responds to the conditional variance of inflation. When both features are present, asymmetric preferences appear to reduce the sign and size asymmetries that arise due to the nonlinearity of the supply curve. Since asymmetric preferences and a nonlinear AS schedule lead to different types of nonlinearity in the interest rate response by the central bank, it might be possible to assess empirically relative importance of these two elements in monetary policy making.

3 Empirical Evidence

3.1 Data

The nonlinear Taylor rule is estimated using U.S. data on inflation, the output gap, and the federal funds rate. Previous literature employs both monthly and quarterly data frequencies to estimate monetary policy rules. We report results using both data frequencies and show that the main result of the paper is robust to whether one uses monthly or quarterly data in estimation. At the monthly frequency, inflation is measured by the annual percentage change in the Consumer Price Index (CPI). Output is measured by the seasonally-adjusted Industrial Production Index (IPI). The natural output level is the Hodrick-Prescott (HP) trend of the logged IPI. The output gap is then computed as the difference between the logged IPI and its HP trend. We also consider a second measure of the output gap constructed as minus the difference between the seasonally-adjusted unemployment rate and its HP trend. The sample period is 1970:01 to 2000:12, but we focus on the subsamples 1970:01 to 1979:06 and 1983:01 to 2000:12. The first subsample corresponds (roughly) to the chairmanships of Arthur Burns and William Miller. The second subsample corresponds to the chairmanships of Paul Volcker and Alan Greenspan, but excludes the period when the Federal Reserve targeted nonborrowed reserves, rather than short-term interest rates.

At the quarterly frequency, inflation is measured by the annualized quarterly percentage change in the Implicit GDP Deflator. Two measures of the output gap are constructed as explained above, except that the quarterly observation of the IPI and unemployment rate is the arithmetic average of the three observations in each quarter. Since at the quarterly frequency the number of observations in the first subsample 1970:I to 1979:II is too small to yield reliable results, we follow Clarida, Gali, and Gertler (2000) in starting the quarterly sample in 1960:I.

3.2 Preliminary Analysis

The estimation of the non-linear Taylor-rule is carried out using a two-step procedure. First, the conditional variance of inflation is estimated from the aggregate supply relation. Then, $\sigma^2_{\pi,t}$ is replaced in the Taylor rule and the rule is estimated by Generalized Method of Moments (GMM). However, some issues need to be addressed prior to estimation. First, the precise form of the non-linear Taylor rule depends on whether the aggregate supply relation is linear or not. Recall that the AS curve is linear when $\phi = 0$ and convex when $\phi > 0$. Hence, it is important to test whether ϕ is statistically different from zero in our data set. Second, the prediction that the conditional variance of inflation is a component of the policy rule can be examined in a time series setup only if inflation is conditionally heteroskedastic. That is, if $\sigma^2_{\pi,t}$ varies over time. Otherwise, if $\sigma^2_{\pi,t}$ is constant, its coefficient might not be identified. Hence, one must also test whether the conditional variance of inflation is indeed time-varying.

In order to address these two issues, we estimate the aggregate supply relation (3) by nonlinear least squares treating the disturbance term u_t is conditionally homoskedastic. We then test the null hypothesis $\phi = 0$ using a t-test, and the null hypothesis of no conditional heteroskedasticity using a LM test for neglected ARCH. The LM statistics were calculated as the product of the number of observations and the uncentered R^2 of the OLS regression of the squared unemployment residual on a constant and six of its lags. Under the null hypothesis of no conditional heteroskedasticity, the statistic is distributed chi-square with as many degrees of freedom as the number of lagged squared residuals included in the regression.

Results in Panels A and B of Table 1, support the notion of an upward sloping AS curve (as predicted by the theory), but results using quarterly data are somewhat weaker than the ones using monthly data. This result might be explained by the fact that the econometrician has more data points to estimate α when using monthly than quarterly data. In all cases the hypothesis $\phi = 0$ cannot be rejected at standard levels. Hence, for these sample periods and data frequencies, it would appear that the U.S. aggregate supply curve is well approximated by a linear relation.[3] Results of the LM tests for neglected ARCH are reported in the first row of Table 2. Note that the hypothesis of no conditional heteroskedasticity is rejected for both frequencies and output gap measures.

In light of these results, we estimate a linear (in mean) AS curve with conditionally heteroskedastic errors. The parameter ϕ is constrained to be zero and the conditional variance of inflation is parameterized using a GARCH(1,1) model. These results are reported in Panel C of Table 1. The terms ψ_0, ψ_1, and ψ_2 denote the constant, the coefficient of the lagged square residual, and the coefficient of the lagged conditional variance, respectively. Note that in all cases their estimates are significant and suggest a persistent process for $\sigma^2_{\pi,t}$.

Since the conditional variance is estimated using inflation and output data, $\sigma^2_{\pi,t}$ is a generated regressor for the second step of the estimation procedure. The implications of generated regressors in estimation and inference have been examined by Pagan (1984) and Pagan and Ullah (1988). Generated regressors can be problematic because they measure with noise the true, but unobserved, regressor. In the case of models where a conditional variance is one of the explanatory variables, estimates can be biased and inconsistent if the ARCH-type model employed is misspecified. Pagan and Ullah (p. 99) suggest specification tests to assess whether the chosen ARCH model is valid. A standard misspecification test for ARCH models is the same LM test for neglected ARCH described above, but applied to the standardized residuals. If the ARCH model is correctly specified, then the residuals corrected for heteroskedasticity and squared should be serially uncorrelated. The second row in Table 2 reports these LM statistics. Since all statistics are below the 5 percent critical value of the appropriate distribution, the null hypothesis of no autocorrelation cannot be rejected. Hence, it would appear that the parsimonious GARCH(1,1) model employed here adequately captures the conditional heteroskedasticity present in the U.S. inflation data.

3.3 Estimation

Following Clarida, Gali, and Gertler (2000), the observed smoothing of interest rates is represented by a partial adjustment model whereby lagged values of the interest rate are also included as explanatory variables. The optimally determined interest rate is interpreted as the desired rate towards which the current interest rate sluggishly adjusts. That is,

$$i_t = \rho(L)i_{t-1} + (1-\rho)i_t^* + \xi_t, \tag{12}$$

[3] Similar results are reported by Gordon (1997) and Dolado, María-Dolores, and Naveira (2003). The latter authors report evidence consistent with a nonlinear Phillips curve for the main countries of the Euro-zone, but cannot reject the hypothesis of linearity for the U.S.. Blinder (1999, p.19) points out that for the U.S. a "linear Phillips curve fits the data extremely well."

where $\rho(L) = \rho_1 + \rho_2 L + \cdots + \rho_{n+1} L^n$, $\rho \equiv \rho(1)$, and i_t^* is given by the right hand side of equation (9). Substituting (9) into (12), the estimated model is

$$i_t = a + \rho(L) i_{t-1} + (1 - \rho)(b y_t + c \pi_t + d \sigma_{\pi,t}^2 + \eta x_t) + \xi_t, \tag{13}$$

where a is an intercept term, $b = 1 + \alpha + \delta$, $c = 1 + 1/\alpha$, and $d = \gamma/2\alpha$.

If the current values of inflation and the output gap are taken to be unknown by the central bank when setting interest rates, then equation (13) can be estimated by Instrumental Variables (IV), using lagged values of the variables as instruments. As an additional check on the robustness of the results, we also estimate two forward-looking versions of (13) where the current values of $\pi_t(y_t)$ are replaced by expectations of future variables $k(q)$ periods ahead, $E_t \pi_{t+k}$ and $E_t y_{t+q}$, and a backward-looking version where they are replaced by $\pi_{t-1}(y_{t-1})$.

The partial adjustment models were estimated by *Generalized Method of Moments* (GMM).[4] Denoting by $\mathbf{z}_t$ a vector of m instruments, GMM exploits the set of orthogonality conditions $E(\xi_t | \mathbf{z}_t) = 0$ to estimate the relevant parameters. The validity of the $(m - p)$ overidentification restrictions can be assessed through the J test that is asymptotically distributed as a chi-square with $(m - p)$ degrees of freedom.

The estimated nonlinear rules are reported in Tables 3 and 4, for the periods 1970:01 to 1979:06 and 1983:01 to 2000:12, respectively. The basic difference between both set of results is that the coefficient on the conditional variance of inflation (d) is not statistically significant in the first subsample, but it is always positive and significant in the second one. This result is robust to both forward and backward-looking specifications of the Taylor rule [see columns (3) to (5)]. Notice that in most cases for the second subsample, the rate of inflation is no longer statistically significant once one introduces the conditional variance as a regressor.[5] In all cases, the overidentification restrictions of the model are not rejected by the data at standard significant levels.

These findings suggest the following. *First,* monetary policy in the United States could be well approximated by a linear Taylor rule prior to 1979. *Second,* the Fed's inflation preferences could be described as symmetric with respect to inflation in the period prior 1979. More precisely, the hypothesis that preferences are quadratic ($\gamma = 0$) would not be rejected by the data against the alternative of asymmetric preferences ($\gamma \neq 0$). *Third,* after 1983, a nonlinear Taylor rule seems to provide a more accurate characterization of U.S. monetary policy than a linear rule. In particular, the Federal Funds rate appears to react

[4]In the case of forward-looking rules, we replace expectations of future variables by their realized values. Then, by construction, the error term in the estimable form will be a $MA(h)$ process with $h = max\{k, q\} - 1$ implying that an optimal weighting matrix that allows for serial correlation ought to be used.

[5]Note that the conditional variance employed here is the one estimated using the full sample. However, using estimates of $\sigma_{\pi,t}^2$ for each subperiod yields qualitatively the same results as those reported.

more strongly to the volatility than to the level of inflation after 1983. *Fourth,* since the coefficient on the conditional variance of inflation is positive and statistically significant after 1983, this suggest that the Fed's inflation preferences during the Volcker-Greenspan might be asymmetric. In particular, positive deviations of inflation from its target appear to be weighted more severely than negative ones, even if they are of the same magnitude.

3.4 Comparison with Clarida, Gali, and Gertler (2000)

The results above parallel somewhat the evidence in Clarida, Gali, and Gertler (2000), where it is reported that the coefficient on inflation in a forward-looking version of the Taylor rule is substantially different in the pre-Volcker and Volcker-Greenspan eras. In order to make this comparison more direct, consider results in Tables 5 and 6 that report the estimated rules using quarterly data for the periods 1960:I to 1979:II and 1983:I to 2000:IV. As before, the second sample excludes the period when the Federal Reserve targeted nonborrowed reserves, rather than short-term interest rates. However, starting the second subsample in 1979:III yields the same results as reported here.[6]

Column (1) in both tables illustrates Clarida, Gali, and Gertler's main result, namely that the reaction with respect to inflation (c), is smaller than unity prior to 1979 but larger than unity during the tenure of chairmen Volcker and Greenspan. This result is robust to the measure of the output gap. However, note in Table 5 that this result does not hold completely once we allow for asymmetric inflation preferences on the part of the central bank. Although results are sensitive to the form of the rule, there are specifications for which the point estimate of c is larger than one prior to 1979, though one would not be able to reject the null hypothesis that the true value is less than one. For example, column (3) in Table 5 correspond to the baseline model reported by Clarida, Gali, and Gertler (p. 157) but includes the conditional variance of inflation as one of the regressors. The point estimates of the inflation coefficient are 1.14(0.12) and 1.04(0.12) depending on the output gap measure employed. Also, notice that in certain cases, the coefficient on σ_π^2 is negative and statistically different from zero. The intuition for this result is explained below.

Regarding the post-1982 data, Table 6 shows that the inflation response is considerably smaller when we allow asymmetric preferences. For some specifications, $\hat{c}$ is smaller than one, though one would not be able to reject the null hypothesis that the true value is larger than one. The reason for this result is straightforward: since the conditional variance of inflation depends on lagged squared inflation, the inflation response consists of a linear part,

[6]Results when the second subsample is 1979:III to 2000:IV are available from the corresponding author upon request.

with coefficient c, and a nonlinear part, with coefficient d. The overall response with respect to inflation is stabilizing, as suggested by Clarida, Gali, and Gertler. The contribution of Table 6 is to show that the nonlinear reaction to the conditional variance of inflation is a quantitative and, in most cases, statistically important component of Fed's reaction function after 1982.

3.5 What Drives the Results?

In order to understand the empirical results reported in this paper, it is instructive to consider the relation between the real interest rate and the conditional variance of inflation in both subsamples. Although the policy rule is defined in terms of the nominal interest rate, one can think of the central bank as implicitly targeting a measure of the real interest rate, that in turn affects output through the IS curve. Figures 1 and 2 plot the relation between the two variables at the quarterly frequency, and the fitted values of an OLS regression of the real rate on σ_π^2. The estimated parameters of these regressions are reported in columns (3) and (4) in Table 7. (Results using monthly data are reported in columns (1) and (2).) Notice that in the first subsample, the real rate is negatively but mildly related to the conditional variance. The coefficient is statistically different from zero, but given the large variability of inflation during this period, the real interest rate response is considerably muted. In contrast, in the second subsample, there is a strong positive relationship between the two variables. The result is striking in that inflation is much less volatile in the second than in the first subsample. The positive relation between the real interest rate and the conditional variance of inflation is consistent with asymmetric inflation preferences because this specification predicts a prudence motive in the implementation of monetary policy.

4 Conclusions

This paper contributes to the literature on optimal monetary policy rules by considering setups where certainty equivalence does not hold because either central bank preferences are not quadratic and/or the aggregate supply schedule is convex. Under some simplifying assumptions, albeit not restrictive ones, it is possible to derive a nonlinear Taylor rule incorporating both features. This rules is general in that it nests the cases where either feature is present or where none is and, consequently, the monetary policy rule is linear.

In order to examine how relevant nonlinear monetary policy rules are in practice, we estimate the rule using U.S. data during the Burns-Miller (pre-1979) and Volcker-Greenspan (post-1982) regimes at the U.S. Federal Reserve. Although, there is no evidence against a linear aggregate supply schedule in either regime, we find fairly robust evidence in favor of the view that the central bank preferences are considerably different in both regimes. In particular, the Fed's inflation preferences during the Volcker-Greenspan regime appear to be asymmetric, in the sense that positive inflation deviations from its target are weighted more heavily than negative ones, even if they are of the same magnitude. In contrast, it is not possible to reject the null hypothesis of quadratic inflation preferences during the Burns-Miller regime. Under asymmetric preferences, the fact that certainty equivalence does not hold, means that a prudence motive can arise in the conduct of monetary policy and interest rates respond not only to inflation changes but also to its variability.

A final interesting result of this paper is that, in contrast to Clarida, Gali, and Gertler (2000) who report that interest rate policy in the Volcker-Greenspan period appears to have been much more sensitive to changes in expected inflation than in the pre-Volcker period, we do not find the response of interest rates to inflation to be larger than unity in the Volcker-Greenspan period. However, once the additional effect stemming from the conditional variance of inflation is considered, the rule in the Volcker-Greenspan era is found to be stabilizing as well.

Table 1. Estimated Aggregate Supply Schedules

	Monthly Data		Quarterly Data	
	IPI	(−)Unemp.	IPI	(−)Unemp.
Coefficients	(1)	(2)	(3)	(4)
		A. Linear		
$\hat{\alpha}$	0.06*	0.24*	0.05†	0.21†
	(0.006)	(0.03)	(0.029)	(0.13)
		B. Nonlinear with No ARCH		
$\hat{\alpha}$	0.06*	0.24*	0.04	0.06
	(0.007)	(0.03)	(0.03)	(0.05)
$\widehat{\phi}$	0.31	0.12	−0.79	47.92
	(0.47)	(0.59)	(2.47)	(37.87)
		C. Linear with GARCH(1,1)		
$\hat{\alpha}$	0.05*	0.22*	0.015	0.08
	(0.006)	(0.03)	(0.03)	(0.15)
$\widehat{\phi}$	0	0	0	0
	–	–	–	–
$\widehat{\psi}_0$	0.005*	0.004*	0.07†	0.07†
	(0.0025)	(0.0019)	(0.04)	(0.04)
$\widehat{\psi}_1$	0.21*	0.17*	0.08†	0.08†
	(0.05)	(0.05)	(0.05)	(0.05)
$\widehat{\psi}_2$	0.75*	0.80*	0.87*	0.87*
	(0.06)	0.05	(0.06)	(0.06)

Notes: The figures in parenthesis are standard errors. The superscripts * and † denote the rejection of the hypothesis that the true coefficient is zero at the 5 percent and 10 percent significance levels, respectively.

Table 2. LM Test for Neglected ARCH

	Monthly Data		Quarterly Data	
	IPI	(−)Unemp.	IPI	(−)Unemp.
Residuals	(1)	(2)	(3)	(4)
Original	34.54*	25.96*	15.72*	24.29*
Standardized	4.50	3.34	9.83	9.93

Notes: The LM statistics were calculated as the product of the number of observations and the uncentered R^2 of the OLS regression of the squared unemployment residual on a constant and six of its lags. The superscripts * and † denote the rejection of the null hypothesis of conditional homoskedasticity at the 5 percent and 10 percent significance levels, respectively.

Table 3. Estimated Reaction Functions
Monthly Data
Pre-Volcker

		Nonlinear			
			Forward (q,k)		
	Linear	Baseline	$(3,6)$	$(6,6)$	Backward
Coefficient	(1)	(2)	(3)	(4)	(5)
		A. Using IPI Gap			
$\hat{a}$	−0.10	0.38*	0.39*	0.32*	0.36*
	(0.33)	(0.15)	(0.08)	(0.09)	(0.17)
$\hat{b}$	0.57†	0.49*	0.08	0.12†	0.43*
	(0.31)	(0.13)	(0.06)	(0.11)	(0.10)
$\hat{c}$	0.84*	0.38†	0.73*	0.80*	0.40†
	(0.31)	(0.22)	(0.07)	(0.09)	(0.23)
$\hat{d}$	0	4.27	−5.19	−5.94	4.52
	–	(6.81)	(3.57)	(4.31)	(7.41)
J statistic	4.98	6.76	6.20	7.15	7.14
d.f.	8	13	13	13	13
		B. Using (minus) Unemployment Gap			
$\hat{a}$	0.34	0.71*	0.78*	0.80*	0.27
	(0.28)	(0.17)	(0.16)	(0.16)	(0.22)
$\hat{b}$	1.93*	4.33*	0.29*	0.37*	2.04*
	(0.50)	(1.30)	(0.14)	(0.17)	(0.33)
$\hat{c}$	0.75*	0.10	0.70*	0.70*	0.72*
	(0.13)	(0.29)	(0.04)	(0.04)	(0.10)
$\hat{d}$	–	3.03	−3.35	−3.93†	4.02
		(4.92)	(2.05)	(2.05)	(5.09)
J statistic	7.02	6.43	6.63	6.17	7.93
d.f.	8	13	13	13	13

Notes: The figures in parenthesis are standard errors. The instruments are a constant and six lags of the variables in the estimated rule. d.f. stands for degrees of freedom. The superscripts * and † denote the rejection of the hypothesis that the true coefficient is zero at the 5 percent and 10 percent significance levels, respectively.

Table 4. Estimated Reaction Functions
Monthly Data
Volcker-Greenspan

		Nonlinear			
			Forward (q,k)		
	Linear	Baseline	$(3,6)$	$(6,6)$	Backward
Coefficient	(1)	(2)	(3)	(4)	(5)
		A. Using IPI Gap			
$\hat{a}$	0.50^{*}	0.38^{*}	0.28^{*}	0.08	0.26^{*}
	(0.23)	(0.15)	(0.10)	(0.05)	(0.12)
$\hat{b}$	$0.40^{\dagger}$	0.88^{*}	1.98^{*}	0.65	0.90^{*}
	(0.23)	(0.29)	(0.81)	(1.02)	(0.35)
$\hat{c}$	0.89^{*}	0.73^{*}	-0.02	0.46	0.55
	(0.19)	(0.26)	(0.59)	(0.66)	(0.37)
$\hat{d}$	0	10.17^{*}	21.05^{*}	$29.10^{\dagger}$	17.87^{*}
	–	(3.00)	(7.34)	(15.07)	(6.21)
J statistic	6.00	10.76	10.17	12.67	11.69
d.f.	8	13	13	13	13
		B. Using (minus) Unemployment Gap			
$\hat{a}$	0.52^{*}	$0.13^{\dagger}$	$0.15^{\dagger}$	0.13	$0.15^{\dagger}$
	(0.23)	(0.08)	(0.08)	(0.08)	(0.08)
$\hat{b}$	2.75^{*}	4.06^{*}	4.52^{*}	6.74^{*}	3.40^{*}
	(1.38)	(1.97)	(2.16)	(3.24)	(1.45)
$\hat{c}$	0.74^{*}	0.43	0.72	0.59	0.57
	(0.24)	(0.46)	(0.46)	(0.52)	(0.36)
$\hat{d}$	–	17.38^{*}	17.85^{*}	22.41^{*}	16.86^{*}
		(7.10)	(7.10)	(9.74)	(6.07)
J statistic	7.05	9.87	10.20	9.43	9.90
d.f.	8	13	13	13	13

Notes: The figures in parenthesis are standard errors. The instruments are a constant and six lags of the variables in the estimated rule. d.f. stands for degrees of freedom. The superscripts * and † denote the rejection of the hypothesis that the true coefficient is zero at the 5 percent and 10 percent significance levels, respectively.

Table 5. Estimated Reaction Functions
Quarterly Data
Pre-Volcker

		Nonlinear			
			Forward (q,k)		
	Linear	Baseline	$(1,1)$	$(1,2)$	Backward
Coefficient	(1)	(2)	(3)	(4)	(5)
		A. Using IPI Gap			
$\hat{a}$	0.58*	0.43	0.99*	1.00*	0.72*
	(0.15)	(0.29)	(0.24)	(0.27)	(0.22)
$\hat{b}$	1.07	2.97	0.31*	0.29†	0.94*
	(0.66)	(6.56)	(0.11)	(0.16)	(0.30)
$\hat{c}$	0.49†	−0.77	1.14*	1.21*	0.36
	(0.28)	(4.49)	(0.12)	(0.20)	(0.27)
$\hat{d}$	–	1.41	−1.46*	−1.87*	0.59
		(5.99)	(0.43)	(0.73)	(0.74)
J statistic	6.59	8.31	4.09	5.51	7.22
d.f.	4	7	7	7	7
		B. Using (minus) Unemployment Gap			
$\hat{a}$	0.68*	0.73*	−0.99*	1.08*	0.87*
	(0.16)	(0.21)	(0.21)	(0.27)	(0.26)
$\hat{b}$	3.21*	3.82*	1.55*	1.57*	3.52*
	(0.91)	(1.48)	(0.43)	(0.77)	(0.65)
$\hat{c}$	0.71*	0.63*	1.04*	1.14*	0.46*
	(0.11)	(0.26)	(0.12)	(0.20)	(0.15)
$\hat{d}$	–	0.05	−1.04*	−1.59*	0.64
		(0.57)	(0.39)	(0.66)	(0.48)
J statistic	7.03	6.96	4.60	4.56	6.26
d.f.	4	7	7	7	7

Notes: The figures in parenthesis are standard errors. The instruments are a constant and four lags of the variables in the estimated rule. d.f. stands for degrees of freedom. The superscripts * and † denote the rejection of the hypothesis that the true coefficient is zero at the 5 percent and 10 percent significance levels, respectively.

Table 6. Estimated Reaction Functions
Quarterly Data
Volcker-Greenspan

		Nonlinear			
			Forward (q,k)		
	Linear	Baseline	$(1,1)$	$(1,2)$	Backward
Coefficient	(1)	(2)	(3)	(4)	(5)
		A. Using IPI Gap			
$\hat{a}$	-0.12	$-0.99^{\dagger}$	-0.96^{*}	$-0.78^{\dagger}$	$0.62^{\dagger}$
	(0.20)	(0.56)	(0.44)	(0.40)	(0.33)
$\hat{b}$	$0.76^{\dagger}$	0.51^{*}	0.72^{*}	1.03^{*}	0.51^{*}
	(0.44)	(0.11)	(0.19)	(0.24)	(0.15)
$\hat{c}$	2.96^{*}	1.18^{*}	1.22^{*}	0.90^{*}	0.73^{*}
	(0.75)	(0.23)	(0.27)	(0.39)	(0.23)
$\hat{d}$	–	5.44^{*}	5.43^{*}	6.25^{*}	1.00
		(1.13)	(1.53)	(2.06)	(2.20)
J statistic	5.72	3.89	5.05	8.22	8.91
d.f.	4	7	7	7	7
		B. Using (minus) Unemployment Gap			
$\hat{a}$	-0.19^{*}	-0.44	-0.87	-1.42^{*}	0.36
	(0.26)	(0.39)	(0.63	(0.55)	(0.33)
$\hat{b}$	$2.36^{\dagger}$	2.91^{*}	2.86^{*}	2.74^{*}	2.17^{*}
	(1.37)	(0.76)	(0.80)	(0.66)	(0.58)
$\hat{c}$	3.08^{*}	1.14^{*}	0.89^{*}	$1.03^{\dagger}$	0.37
	(0.76)	(0.35)	(0.22)	(0.52)	(0.28)
$\hat{d}$	–	4.81^{*}	5.39^{*}	6.88^{*}	3.18
		(1.57)	(1.13)	(2.05)	(2.19)
J statistic	4.35	5.82	4.20	7.78	8.87
d.f.	4	7	7	7	7

Notes: The figures in parenthesis are standard errors. The instruments are a constant and four lags of the variables in the estimated rule. d.f. stands for degrees of freedom. The superscripts * and † denote the rejection of the hypothesis that the true coefficient is zero at the 5 percent and 10 percent significance levels, respectively.

Table 7. Results OLS Regression

	Monthly Data		Quarterly Data	
	Pre-Volcker	Volcker-Greenspan	Pre-Volcker	Volcker-Greenspan
Coefficient on	(1)	(2)	(3)	(4)
Constant	0.63*	2.62*	2.32*	−0.58
	(0.30)	(0.19)	(0.33)	(1.13)
σ_π^2	−3.83	7.03*	−0.94*	4.15*
	(2.34)	(1.92)	(0.17)	(1.06)

Notes: The figures in parenthesis are standard errors. The superscripts * and † denote the rejection of the hypothesis that the true coefficient is zero at the 5 percent and 10 percent significance levels, respectively.

A Appendix

This Appendix derives the optimal monetary policy rule when preferences are asymmetric and the supply curve is nonlinear. The dynamic problem of the central bank [see eq. (1) in text] can be decomposed into the sequence of period-by-period problems:

$$\underset{\{i_t\}}{Min} \quad E_t \beta^2 L(\pi_{t+2} - \pi^*),$$

subject to

$$\begin{aligned} y_{t+1} &= \delta y_t - r_t + \eta x_t, \\ \pi_{t+1} &= \pi_t + f(y_t) + u_{t+1}, \end{aligned}$$

where $f(y_t) = \alpha y_t/(1 - \alpha\phi y_t), x_{t+1} = \psi x_t + e_{t+1}$, and $i_t = r_t + E_t \pi_{t+1}$, with all notation as defined in the text. In order to derive the first-order condition, apply the chain rule and note that y_{t+1} is predetermined at time t:

$$\begin{aligned} E_t((\partial L_{t+2}/\partial \pi_{t+2})(\partial \pi_{t+2}/\partial y_{t+1})(\partial y_{t+1}/\partial r_t)(\partial r_t/\partial i_t)) &= 0, \\ -\alpha(\exp(\gamma E_t(\pi_{t+2} - \pi^*) + (\gamma^2/2)\sigma^2_{\pi,t}) - 1)/(\gamma(1 - \alpha\phi y_{t+1})^2) &= 0. \end{aligned}$$

This condition is satisfied if and only if:

$$E_t \pi_{t+2} = \pi^* - \gamma \sigma^2_{\pi,t}/2.$$

But, from the aggregate supply relation

$$\begin{aligned} E_t \pi_{t+2} &= E_t(\pi_{t+1} + f(y_{t+1}) + u_{t+2}), \\ &= E_t(\pi_t + f(y_t) + f(y_{t+1}) + u_{t+2} + u_{t+1}), \\ &= \pi_t + \alpha y_t/(1 - \alpha\phi y_t) + \alpha(\delta y_t - r_t + \eta x_t)/(1 - \alpha\phi(\delta y_t - r_t)). \end{aligned}$$

Hence, it must be the case that

$$\pi^* - \gamma \sigma^2_{\pi,t}/2 = \pi_t + \alpha y_t/(1 - \alpha\phi y_t) + \alpha(\delta y_t - r_t + \eta x_t)/(1 - \alpha\phi(\delta y_t - r_t)). \tag{14}$$

Solving for r_t :

$$r_t = \delta y_t + \frac{(\pi_t - \pi^* + \gamma \sigma^2_{\pi,t}/2 + f(y_t))}{\alpha(1 - \phi(\pi_t - \pi^* + \gamma \sigma^2_{\pi,t}/2 + f(y_t)))} + \eta x_t. \tag{15}$$

Substituting into the Fisher equation, using $E_t \pi_{t+1} = \pi_t + f(y_t)$, and simplifying:

$$i_t = \pi_t + f(y_t) + \delta y_t + \frac{(1/\alpha)(\pi_t - \pi^* + \gamma \sigma^2_{\pi,t}/2 + f(y_t))}{1 - \phi(\pi_t - \pi^* + \gamma \sigma^2_{\pi,t}/2 + f(y_t))} + \eta x_t,$$

that corresponds to expression (8) reported in the text.

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Fig. 1. Relation between the Real Interest Rate and the Conditional Variance of Inflation 1960:I to 1979:II

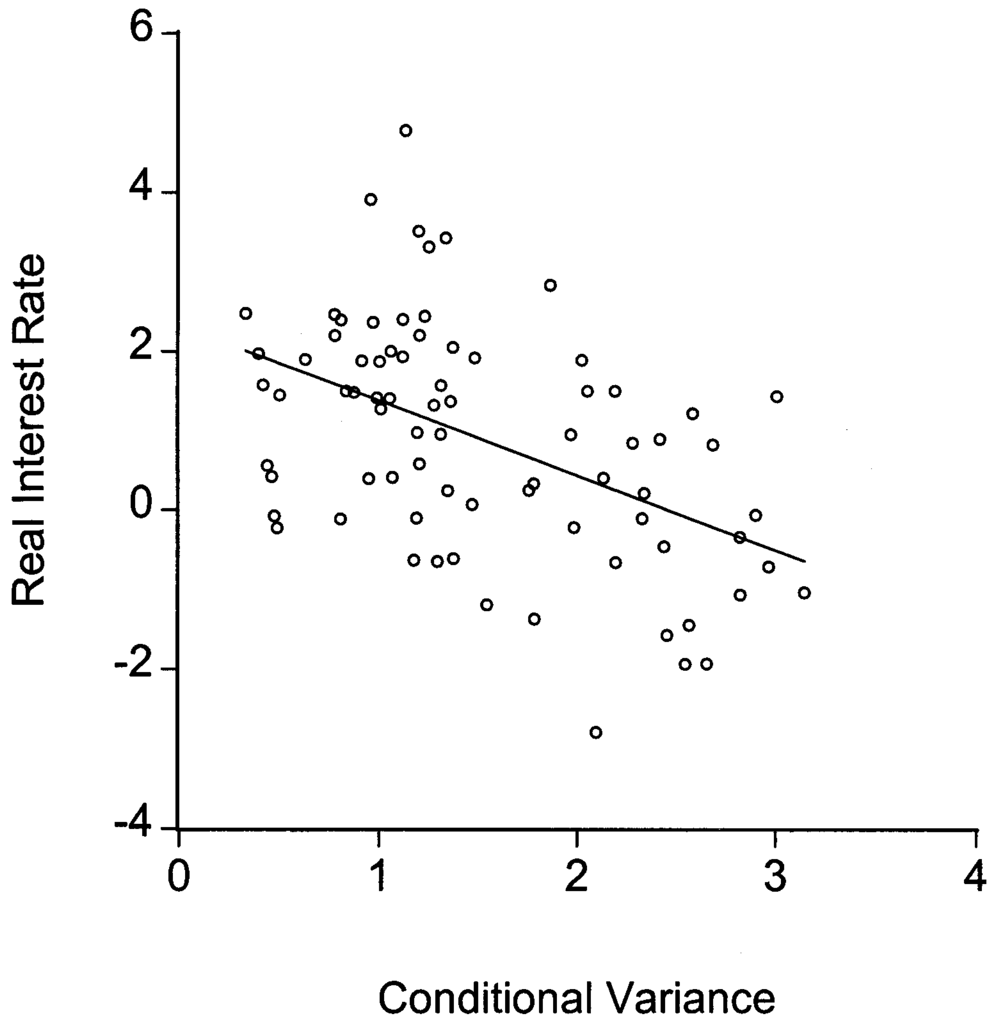

Fig. 2. Relation between the Real Interest Rate and the Conditional Variance of Inflation
1983:I to 2000:IV