Vortex Formation by Merging of Multiple Trapped Bose-Einstein Condensates

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We report observations of vortex formation by merging and interfering multiple ^{87}Rb Bose-Einstein condensates (BECs) in a confining potential. In this experiment, a single harmonic potential well is partitioned into three sections by a barrier, enabling the simultaneous formation of three independent, uncorrelated BECs. The BECs may either automatically merge together during their growth, or for high-energy barriers, the BECs can be merged together by barrier removal after their formation. Either process may instigate vortex formation in the resulting BEC, depending on the initially indeterminate relative phases of the condensates and the merging rate.

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In superfluids, long-range quantum phase coherence regulates the formation and dynamics of quantized vortices [1,2]. In a dilute-gas Bose-Einstein condensate (BEC), for example, vortices can be created using direct manipulation of the quantum phase profile of the BEC [3,4]. Vortices in BECs have also been created using methods more analogous to those of classical fluids [5], namely, through rotating traps [6–9], turbulence [10], and dynamical instabilities [11,12]. Yet in contrast with classical fluids, vortex generation via the mixing of initially isolated superfluids remains experimentally unexplored. Because of the relative ease of microscopic manipulation and detection techniques, BECs are well suited to answer open questions related to superfluid mixing and vortex generation.

In this Letter, we describe our experiments demonstrating that merging together three condensates in a trap can lead to the formation of quantized vortices in the merged BEC. We ascribe the vortex generation mechanism to matter-wave interference between the initially isolated BECs, and show that vortices may be induced for both slow and fast merging rates. While it is now well known that matter-wave interference may occur between BECs [13], and that condensates can be gradually merged together into one larger BEC [14], our experiment demonstrates a physical link between condensate merging, interference, and vortex generation, providing a new paradigm for vortex formation in superfluids. We emphasize that no stirring or BEC phase engineering steps are involved in our work; the vortex formation process is stochastic and uncontrollable, and partially depends on relative quantum phases that are indeterminate prior to condensate merging. This vortex formation mechanism may be particularly relevant for developing further understanding of the roles of potential-well defects, roughness, and disorder on establishing a superfluid state. Furthermore, this work may be viewed as a model for studies of spontaneous symmetry breaking and topological defect formation during phase transitions [15,16].

To illustrate the basic concept underlying our experiment, we first describe our atom trap, which is formed by the addition of a time-averaged orbiting potential (TOP) trap [17] and a central repulsive barrier of axially (vertically) propagating blue-detuned laser light shaped to segment the harmonic oscillator potential well into three local potential minima. Figure 1(a) shows an example of potential-energy contours of our triple-well potential. We will assume throughout the ensuing discussions that the energy of the central barrier is low enough that it has negligible effect on the thermal atom cloud, as in our experiments, but high enough for independent condensates to begin forming in the three local potential minima from the single thermal cloud. There are two important regimes in this range of barrier energies: (1) if the central barrier is weak, condensates with repulsive interatomic interactions will grow and merge together during evaporative cooling; and (2) if the barrier is strong, the condensates will remain independent, but may be merged together by lowering the barrier while keeping the atoms trapped [18]. We have examined both scenarios.

Depending on the relative phases of the three condensates and the rate at which they merge together (via either process), the final merged BEC may have acquired nonzero net angular momentum about the trap axis. To demonstrate

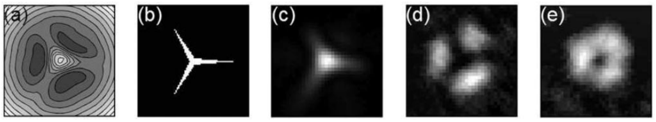

FIG. 1. (a) Potential-energy contours of a horizontal slice through the center of our triple-well trap, representing the addition of the potential-energy profiles of our TOP trap and barrier beam. (b) The binary transmission mask used to create the optical barrier. (c) An image of the optical barrier. (d),(e) Phase-contrast images of trapped condensates as viewed along the trap axis. Each shows an area of 85 μm per side, as do (a) and (c). In (d), three condensates are created in the presence of a strong barrier beam with 170 μW. (e) With 45 μW in the beam, the initial three condensates merge together during evaporative cooling. A hole in the BEC is formed by the barrier beam displacing atoms from the trap center.

this, we first envision two condensates in two potential minima merged slowly enough that although interference occurs between the condensate pair, interference *fringes* do not. As merging begins, above-barrier fluid flow between the pair is established, with the initial flow direction depending on the sine of the phase difference between the overlapping states (as also occurs in the Josephson Effect [1,19] for the case of tunneling). Recalling that the relative phase between two independent condensates is indeterminate until it is measured via interference, the relative phase and hence fluid flow direction will vary randomly upon repeated realizations of the experiment [20].

When the three condensates of our experiment gradually merge, a net fluid flow over the barrier arms may occur that is either clockwise, counter-clockwise, or neither, relative to the trap center. For ease of this discussion, and keeping in mind that only relative phases carry physical meaning, we imagine that the condensates formed in the three local minima can be labeled with phases ϕ_j, where the indices $j = 1$, 2, and 3 identify the condensates in a clockwise order, respectively. Upon merging, if the relative phases happen to be (say) $\phi_2 - \phi_1 = 0.7\pi$ and $\phi_3 - \phi_2 = 0.8\pi$, thus necessarily $\phi_1 - \phi_3 = 0.5\pi$, then clockwise fluid flow will be established for the fluid. More generally, if the three merging condensates happen to show relative phases $\phi_2 - \phi_1$, $\phi_3 - \phi_2$, and $\phi_1 - \phi_3$ that are each simultaneously between 0 and π, or each between π and 2π, the resulting BEC will have acquired nonzero net angular momentum after the merger, which will be manifest as a vortex within the BEC [21]. By examining the full range of phase difference possibilities, the total probability P_v for a net fluid flow to be established in either azimuthal direction is found to be $P_v = 0.25$, given random phase differences for each experimental run. P_v is thus the probability for a vortex to form as the three condensates merge together. This relationship between vortex trapping and relative phases is an application of the so-called geodesic rule [22]. Related work includes a theoretical investigation of three Josephson-coupled BECs [23], and spontaneous defect trapping in liquid crystals [24].

For yet faster merging rates and correspondingly steeper phase gradients, interference *fringes* may indeed develop as the condensates merge. To estimate the longest time scale τ_f over which two merging condensates can support a single dark interference fringe, we envision two condensates that are initially atomic point sources separated by a distance d, and that each expands to a radius of d in time τ_f such that the condensates overlap in the intervening region. The condensate expansion speed $v \sim d/\tau_f$ corresponds to a phase gradient at the side of each condensate of $\nabla\phi = \frac{vm}{\hbar} \sim \frac{dm}{\tau_f \hbar}$, with m the atomic mass. To create a single full interference fringe in the overlap region, $\nabla\phi \sim \pi/d$. With $d \sim 35$ μm, appropriate for our experiment, $\tau_f \sim 550$ ms; shorter merging times would produce more interference fringes, while longer times correspond to slow merging and no fringes. Each dark fringe will be subject to the same dynamical instabilities as dark solitons and decay to vortices, antivortices, and possibly vortex rings over times on the order of 50 ms [11,12,25]. Similar decay has been seen in recent numerical simulations [26]. For condensates merged together over times of τ_f or shorter, we may thus expect to find multiple vortex cores in a BEC, or to find a value of P_v exceeding 0.25.

Our basic single BEC creation technique involves the following steps. We first cool a thermal gas of $|F = 1, m_F = -1\rangle$ ^{87}Rb atoms to just above the BEC critical temperature in an axially symmetric TOP trap with radial and axial trapping frequencies of 40 and 110 Hz, respectively. We then ramp the TOP trap magnetic fields such that the final trap oscillation frequencies are 7.4 Hz (radially) and 14.1 Hz (axially). A final 10-sec stage of radio-frequency forced evaporative cooling produces a condensate of $\sim 4 \times 10^5$ atoms, with a condensate fraction near 65% and a thermal cloud temperature of $\sim$22 nK. The BEC chemical potential is $k_B \times 8$ nK, where k_B is Boltzmann's constant.

To study vortex formation induced by merging together three condensates formed independently in a triple-well potential, we modify the above procedure by ramping on the three-armed optical barrier immediately *before* the final BEC-producing 10-sec stage of evaporative cooling. The barrier itself is formed by illuminating a binary mask, illustrated in Fig. 1(b), with a focused blue-detuned Gaussian laser beam of wavelength 660 nm. After passing through the mask and a lens to image the mask onto the atom trap, the beam enters our vacuum chamber along the trap axis. Because of diffraction, the beam has an intensity profile as shown in Fig. 1(c), with a maximum intensity and thus barrier energy aligned with the center of the TOP trap. The barrier's potential energy decreases to zero over $\sim$35 μm radially along the three barrier arms separated by azimuthal angles of 120°. With 170 μW in the beam, corresponding to a maximum barrier energy of $k_B \times$ 26 nK, three condensates are created without merging together during their growth [18]; a set of three such BECs is shown in Fig. 1(d). With, instead, 45 μW in the beam, corresponding to a maximum barrier energy of $k_B \times$ 7 nK, three independent condensates also *initially* form, but as the condensates grow in atom number, they gain enough interaction energy to flow over the barrier arms. The three condensates then naturally merge together into one BEC during evaporative cooling, as shown in Fig. 1(e). We stress that in neither case is a *single* BEC formed that is then split into three sections.

In our first study, three spatially isolated condensates were created in the presence of a strong barrier of maximum potential energy $k_B \times 26$ nK, and were then merged together by ramping down the strength of the barrier to zero over a variable time τ. Since vortex cores are too small to be directly observed in the trapped BEC, we suddenly removed the trapping potential after merging and viewed the atom cloud using absorption imaging along the trap axis after 56 ms of ballistic expansion. This process was repeated between 5 and 11 times for each of 6 different barrier ramp-down times τ between 50 ms and 5 sec.

In a significant fraction of our merged BECs, one or more vortex cores were visible, indicating that condensate merging can indeed induce vortex formation. The spatial density distributions varied from shot to shot, as would be expected with indeterminate phase differences between the initial condensates, while many images were absent of vortices. Example images of expanded BECs in Figs. 2(a)–2(d) show the presence of vortex cores after various barrier ramp-down times. An analysis of vortex observation statistics is given in Fig. 2(e) for the different values of τ examined. Here we define a vortex observation fraction F_v as the fraction of images, for each value of τ, that show at least one vortex core. The error bars reflect our uncertainty in determining whether or not an image shows at least one vortex. For example, corelike features at the edge of the BEC, or features obscured by imaging noise, may lead to uncertainty in our counting statistics and determination of F_v. As the plot shows, F_v reaches a maximum of $\sim$0.6 for the smaller τ values, and drops to $\sim$0.25 for long ramp-down times. We expect that with a large number of images, F_v should approximate P_v for each τ. Thus our results are consistent with our conceptual expectations, where $P_v > 0.25$ for fast merging times, and $P_v = 0.25$ for slow merging according to the geodesic rule for random initial phase differences. We note that τ is an overestimate of the *actual* merging time, since the condensates are merged before the barrier is completely removed.

For $\tau \leq 1$ sec, multiple cores were often observed, perhaps signifying the creation of both vortices and antivortices. Although we are unable to determine the direction of fluid circulation around the vortex cores, we checked this interpretation by ramping off the barrier in 200 ms, thus forming multiple vortex cores with a high probability. By inserting additional time to hold the final BEC in the unperturbed harmonic trap before our expansion imaging step, the probability of observing multiple cores dropped dramatically: for no extra hold time, we observed an average of 2.1 vortex cores per image, whereas this number dropped to 0.7 for an extra 100-ms hold time, suggestive of either vortex-antivortex combination on the 100-ms time scale, or other dynamical processes by which vortices leave the BEC. However, single vortices were observed even after 5 sec of extra hold time in our trap following the barrier ramp-down, indicating relatively long single-vortex lifetimes in the harmonic trap.

In our second main investigation, we differed from the above experiment by using a weaker barrier with a maximum energy of $k_B \times 7$ nK such that three condensates initially formed but naturally merged together into one BEC during evaporative cooling. Here, merging is due solely to the increasing condensate chemical potentials exceeding the potential energy of the barrier arms; the barrier strength remained constant throughout condensate growth and merging when vortices could form. After evaporative cooling produced a single (merged) BEC in the weakly perturbed harmonic trap, we removed the weak barrier over 100 ms and released the atoms from the trap for observation. Under these conditions, our vortex observation fraction was $F_v = 0.56 \pm 0.06$ in a set of 16 images, with examples shown in Figs. 3(a) and 3(b). By adding an extra 500-ms hold time after BEC formation but *before* the barrier and trap removal, F_v decreased to 0.28 ± 0.14, perhaps again due to vortex-antivortex combination. From this we can conclude that with low barrier energies, vortices are formed during the BEC creation process in the perturbed TOP trap, rather than during removal of the weak barrier, consistent with phase-contrast images of trapped BECs that show a continuous final density distribution as in Fig. 1(e).

By using various barrier strengths and barrier ramp-down rates, up to at least four clearly defined vortex cores have been observed upon condensate merging, as the examples of Figs. 3(c)–3(f) show. Density defects other than clear vortex cores have also been observed, as in the upper left of Fig. 3(g), where a "gash" may be an indicator of vortex-antivortex combination. Most often, however, no vortices were observed, as in Fig. 3(h). For comparison, a BEC created in a trap without a barrier is shown in Fig. 3(i).

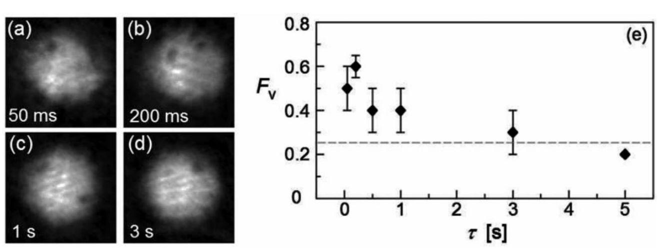

FIG. 2. (a)–(d) 170-μm wide images showing vortices in condensates created as a strong ($k_B \times 26$ nK) barrier was ramped off over the time τ indicated. (e) Vortex observation fraction F_v vs τ. The data for τ values of 50 ms, 200 ms, 500 ms, 1 sec, 3 sec, and 5 sec, consisted of 5, 11, 10, 10, 5, and 5 images, respectively. For clarity, statistical uncertainties due to finite sample sizes are not shown, but they generally exceed our counting uncertainties. The expected lower limit of $F_v \sim P_v = 0.25$ is represented by a dashed line.

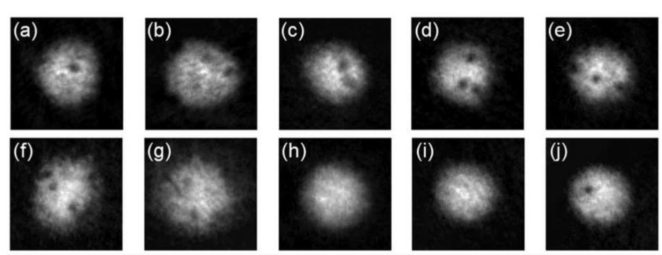

FIG. 3. (a),(b) 170-μm wide images showing vortices naturally occurring in condensates created in a trap with a $k_B \times 7$ nK barrier. (c)–(h) Images obtained using various barrier energies. (i),(j) BECs created without an optical barrier.

As a check on our results and analysis, we used a split-step method to solve the Gross-Pitaevskii equation in simulations of three merging two-dimensional condensates. Details of the simulations will be deferred to a future publication; however, we mention that the simulations display features qualitatively similar to those seen in our experiment, namely: (1) arbitrarily slow merging gives a 25% probability for vortex formation, given random initial phases, and without formation of any interference fringes (solitons); (2) rapid merging leads to interference fringes that decay to multiple vortices and antivortices, which may annihilate each other in the BEC; and (3) as merging times decrease, P_v increases. Our simulations have shown two additional features: (1) slightly asymmetric or off-center barriers, or unequal numbers of atoms in the three wells, can also lead to vortex formation upon merging; and (2) a vortex core may migrate to and be pinned at the center of the barrier where the energy cost of displacing fluid is low; this may help explain why a weak barrier does not appear to readily destroy all BEC angular momentum. We also emphasize that to generate vortices by the mechanisms described here, it is important for three reasons that condensates merge and interfere *while trapped*. First, in a trapped BEC, the nonlinear dynamics due to interatomic interactions play a key role in the structural decay of interference fringes. Second, arbitrarily slow merging times can be studied. Finally, a gas confined in an asymmetric potential well can acquire angular momentum from the trap [21].

We finally note that in a related test, for our basic single BEC creation procedure outlined initially and *without a segmenting barrier ever turned on*, we have observed spontaneous formation of single vortices in about 10% of our images. An example is shown in Fig. 3(j). These observations appear to indicate spontaneous topological defect formation [16] during cooling through the BEC transition, as has been predicted [15]. A full description of this experiment will be given in a future publication.

In summary, we have demonstrated vortex generation by merging isolated and initially uncorrelated condensates into one BEC. Our main results are that (1) subsequent vortex observations are consistent with a conceptual analysis regarding merging rates and indeterminate phase differences between the initial condensates, and (2) BECs created in the presence of weak trapping potential defects or perturbations, such as our weak optical barrier, may naturally acquire vorticity during BEC creation. This second result challenges the notion that a BEC *necessarily* forms with no angular momentum in the lowest energy state of a trapping potential; rather, the shape of a static confining potential may be sufficient to induce vortex formation during BEC growth, a concept important to current and future BEC experiments and perhaps to experiments with other superfluids.

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