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Models of Modelling: Genres, Purposes or Perspectives

Peter Galbraith
University of Queensland
p.galbraith@uq.edu.au

Abstract

The number of papers and research reports addressing the theory and/or practice of mathematical modelling with some form of connection to education is growing astronomically. Small wonder then that educational publications featuring articles emerging from this field, present such a plethora of views that even those experienced in the field can become disoriented, let alone those feeling their way in a new area. This paper joins a conversation that concerns itself with meanings, approaches, priorities, and intentions associated with the use of the term 'mathematical modelling' as it occurs in education. For example it will be argued that there are essentially two generic approaches to modelling within education: modelling that acts primarily as a 'vehicle' for the attainment of other curricular priorities, and modelling as 'content' that seeks first to nurture and enhance the ability of students to solve authentic real world or life-like problems. Within these approaches there are particular purposes and perspectives, but the latter are just that – they are not (as sometimes suggested) additional modelling genres. The paper visits areas of relevance to its theme: such as stated priorities of educational authorities in curriculum statements; types of activity that make up the two modelling genres; a selection of writings that canvass a rich array of issues, challenges, and research foci that are currently engaging interest and activity within the field; and the implications of criticisms of modelling, both appropriate and misplaced.

Keywords: modelling genre; curricular mathematics; modelling as content

1. Introduction

As we are well aware the number of papers and research reports addressing the theory and/or practice of mathematical modelling with some form of connection to education has been growing astronomically. A short web-search, combining 'mathematical modelling' (both spellings) with 'education' elicited about 23 000 references; if model* was used in place of modelling the number increased to about 14.5 million. Small wonder then that educational publications have been increasingly featuring articles emerging from this field, even less that such a plethora of views are presented that even those experienced in the field can become disoriented, let alone those feeling their way in a new area. This paper joins a conversation that concerns itself with meanings, approaches, priorities, and intentions associated with the use of the term 'mathematical modelling' as it occurs in education. It cannot expect to solve problems, or meet with everyone's approval - what it will try to do is simplify discussion by disentangling some elements that seem to have become confused. For example it will be argued that there are essentially two generic approaches to modelling within education and that that within these approaches there are particular purposes and perspectives. The latter are just that – they are not (as sometimes suggested) additional modelling genres.

That two distinct genres can be identified with respect to mathematical modelling within educational settings is made clear through the following quotations:

> "The curricular context of schooling in our country does not readily admit the opportunity to make mathematical modeling an explicit topic in the K-12 mathematics curriculum. The primary goal of including mathematical modeling activities in students' mathematics experiences within our schools typically is to provide an alternative – and supposedly engaging – setting in which students learn mathematics without the primary goal of becoming proficient modelers. We refer to the mathematics to be learned in these classrooms as "curricular mathematics" to emphasize that this mathematics is the mathematics valued in these schools and does not include mathematical modeling as an explicit area of study. Acknowledging this curricular context, we recognize that extensive student engagement in classroom modeling activities is essential in mathematics

> instruction only if modeling provides our students with significant opportunities to develop deeper and stronger understanding of curricular mathematics." (Zbiek and Conner, 2005).

Julie (2002) and elsewhere uses the term 'modelling as vehicle' to describe the above approach, and contrasts it with 'modelling as content' as illustrated specifically in the following.

> "It needs to be borne in mind, however, that whatever claims are made to embedding mathematics in context the purpose of this embeddedness is not the construction of mathematical models but rather the use of context and sometimes mathematical models as vehicles for the learning of mathematical concepts, procedures and at times justifications. Mathematical modelling should not only be a vehicle for these mathematical ideas. Remaining at this level conceals the "behind-the-scene" work and intricacies involved in the construction of a mathematical model… In trying to come to grips with the behind the - scene work and intricacies involved in mathematical model construction, it is necessary that mathematical modelling should also be experienced as content. Mathematical modelling as content entails the construction of mathematical models for natural and social phenomena without the prescription that certain mathematical concepts or procedures should be the outcome of the model building process. It also entails the scrutiny, dissection, critique, extension and adaptation of existing models with the view to come to grips with the underlying mechanisms of mathematical model construction." (Julie, 2002).

In the above terms mathematical models in education are constructed for one of two curricular reasons. Either to use as a 'vehicle' for purposes of introducing other curricular material and associated priorities, or to enable students to learn and apply modelling skills to solve real problems relevant to their world – modelling as 'content'. And it is fair to allow that both genres may be included in some educational settings to achieve complementary goals.

The different alternative approaches have different motivations and different global purposes. The rationale for the 'modelling as vehicle' approach has been articulated clearly in the first of the above quotations – here modelling acts a servant to other curricular needs or educational purposes – while 'modelling as content' gives primacy to modelling activity, and has a dual purpose. On the one hand, as Julie stresses, it provides students with genuine real world related problem solving experience; but additionally it aims to help them develop a mental 'modelling infrastructure' so that they can become users of their mathematical knowledge in the sense of being able to independently address problems in their world. This adds a particular learning purpose beyond the goal of obtaining solutions to individual problems, one whose motivation stems from a belief that it is unacceptable (if not shameful) for students to spend ten, twelve, and even fifteen years in mathematics curriculum subjects yet be unable to access their store of mathematical knowledge for purposes other than undertaking coursework or assessment items. This dual emphasis elevates 'modelling as content' beyond the pragmatic view that an end product is the primary or sole purpose of a problem solving venture, to a focus on what goes on in the modelling process, and how this impacts on students, teachers, and the problem itself both with respect to the problem at hand, and with respect to developing modelling expertise over time.

Recognition that such real world problem solving expertise is a foremost educational goal continues to be reinforced internationally, at least officially, through documents that set specific educational goals for the learning of mathematics – as in the following:

> "Mathematical literacy is defined in PISA as the capacity to identify, understand and engage in mathematics, and to make well-founded judgements about the role that mathematics plays in an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen." (OECD 2001, p.22 cited in Kaiser, 2005).

> "Students develop numeracy, reasoning, thinking skills, and problem solving skills through the learning and application of mathematics. These are valued not only in science and technology, but also in everyday living and in the workplace." (Curriculum Planning and Development Division, Ministry of Education, Singapore, 2006)

> "It (Australian Mathematics Curriculum) develops the numeracy capabilities that all students need in their personal, work and civic life, and provides the fundamentals on which

> mathematical specialties and professional applications of mathematics are built. These capabilities enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently." (Australian Curriculum Assessment and Reporting Authority, 2010).

However such abilities can only develop if mathematical experiences are drawn genuinely from these same areas of personal, vocational, and civic contexts. Much work carried out under the banner of mathematical applications or modelling skates over the aspect most crucial to developing modelling expertise – formulating a mathematical problem from a messy real world context, and doubts remain whether authors of such official statements really appreciate the kind of educational support that is needed for schools, teachers, and students to successfully engage with 'modelling as content' if such intentions are to become reality.

While as emphasised above, the 'content' and 'vehicle' approaches differ in key respects, and their respective goals are distinct, they should not be viewed as necessarily antagonistic. In seeking to solve genuine problems the need for new mathematical content may emerge, while real-world contexts can provide legitimate vehicles for the introduction of desired mathematics.

2. Versions of modelling as vehicle

Below are summarised, approaches that have used the term 'modelling' in some sense to describe what they set out to achieve. With one exception they are better classified as some form of application, since the fundamental attribute of formulating a mathematical model from a real or genuinely lifelike situation is effectively absent as a serious component of the enterprise. Certainly they share the aim of using contextualised mathematics for the purpose of pursuing other content related purposes, rather than the advancement of modelling abilities as such.

2.1. Using contextualised examples to motivate the study of mathematics.

Pierce and Stacey (2006) explored reasons why some teachers chose problems set in real world contexts, and the teaching priorities that influenced the way in which such problems were subsequently treated. In particular they were interested in the extent to which the 'halo effect' - interpreted to mean the achievement of positive feelings towards mathematics through the use of problem contexts that appealed to the students' interests - influenced pedagogical choices. It emerged that the goals were overwhelmingly affective rather than cognitive, so that mathematical learning was frequently subservient, more weight being attached to how the students were viewing a task, and their associated feelings about mathematics at large, than to exploiting its potential for enhancing mathematical understanding and power. Such an approach renders terms like mathematical modelling inappropriate to describe the intention and practice of the classroom activity, and the existence of such practices illustrates that the mere use of a real-world context in presenting mathematics does not necessarily mean that mathematical modelling or applications work is being conducted in some significant way.

2.2. Using real problem situations as a preliminary basis for abstraction.

Bardini, Pierce, and Stacey (2004) and Bardini and Stacey (2006) describe studies into the teaching of linear functions to junior secondary school students (years 9 and 8 respectively) using problems based in real-world contexts. A main purpose was to develop the function rule $y = mx + c$ through examples in which 'm' and 'c' had real world meanings, such as services that comprise a flat call charge plus a labour charge that accumulates with time. Symbolic, numerical, and graphical representations of the relationship were considered, so that intercepts, slopes, points of intersection, and intervals required interpretation in context, across a variety of problem settings. Of primary interest was the level of understanding and facility with algebra that students developed during the five-weeks of the study. Students learned to write algebraic rules in conventional formats, were comfortable selecting symbols that made sense in terms of the problem settings, and showed understanding of the function property of expressing one variable quantity in terms of another.

Concerns include the seemingly excessive time (five weeks) taken to accomplish some rather basic goals, and the way the mode of instruction impacted on the authenticity of the approach if viewed as a modelling enterprise. In both studies direct teaching was employed with the ‘effectiveness’ of learning assessed by means of pre and post testing. This compromised the reality of the approach – for example students were expected to make decisions about contextualised problems on their own, whereas in reality, for example, a decision about which plumber to hire would usually be taken by a couple or family group, after examining, and perhaps debating, the merits of respective quotes. Confusion was exhibited by students, as in giving competing interpretations to the parameters ‘m’ and ‘c’ in a practical context, precisely the kind of situation where individual understanding can be clarified by the type of peer discussion promoted in team approaches to modelling tasks.

2.3. Emergent Modelling

Emergent modelling (Gravemeijer, 2007; Doorman and Gravemeijer, 2009) is an instructional design heuristic, developed as a component of a domain-specific instruction theory generated within the Reality in Mathematics Education framework in the Netherlands. ‘Emergent’ refers both to the nature of the process by which models emerge from students’ experience, and to the process by which these models support the emergence of formal mathematical ways of knowing - that are no longer dependent on the support of the original models. That is, there is emphasis on a search for models that can be developed into entities of their own, and subsequently into models for mathematical reasoning. Gravemeijer (2007) summarises the process as one of “abstraction-as-construction” in which mathematical knowledge is grounded in earlier experiences that are meaningful and applicable. In that they are familiarised with a mathematical take on everyday life situations in the process, students are incidentally prepared for more serious application and modelling adventures in the future - indeed Gravemeijer has usefully referred to emergent modelling as a precursor to mathematical modelling. This is consistent with the earlier views of Freudenthal that mathematical models are only found at the lowest levels of mathematising that has its roots in an extra-mathematical situation. This also focuses attention on the distinction consistently emphasised between modelling and use of models – the former incorporating also what is described by Freudenthal as mathematising, and frequently involving several of the latter. Emergent modelling can be viewed as an organised and theorised approach to the use of models for systematically developing mathematical concepts and understanding. By contrast the approach described in the previous section involved one-off attempts to use contextualised mathematics to motivate and attain proficiency with a particular mathematical relationship, without a clear instructional rationale to guide the teaching.

2.4. Modelling as curve fitting

This approach has become increasingly significant with the availability of regression menus in software and graphical calculators. A model generated by this means can become a purely technical artefact whose parameters vary with the particular data set, and which can be generated in complete ignorance of the principles underlying the real situation – indeed undertaken without knowledge of where a table of data comes from. It raises a profound theoretical issue – the relative authority of disembodied data as such, versus the theoretical structure underpinning its generation In one example curves were fitted to population data by using successively the full suite of regression choices available on a graphical calculator – with no apparent realisation that data generated by births deaths and migration should have an underlying exponential pattern. Curve fitting remains an important activity within the modelling enterprise, but when used mindlessly it creates a dangerous aberration of the modelling concept. Riede (2003) demonstrates good modelling practice when relating weightlifting records to weights of athletes. An inverted parabola was postulated to model the data, on the grounds that weight lifted at first increases with body weight, but ultimately (beyond the super heavyweight class) begins to decrease as body weight impairs the ability to lift. The subsequent fit was excellent. Technology is an indispensible aid in progressing modelling capability, but the ultimate authority must be the integrity of the modelling enterprise, not the seductive attractions of menu options.

2.5. Word problems

Verschaffel (e.g. Greer and Verschaffel, 2007; Verschaffel and Van Dooren, 2010), has been writing and researching insightfully, for many years on the subject of student approaches to word problems. Studies in a variety of countries have consistently demonstrated the propensity of students to ignore contextual factors, and apply (often incorrect) actions based on perceptions of what school mathematics is about – such as being divorced from reality. His work with colleagues has included a focus on the suspension of sense making by students while working on word problems, so that aberrations are produced that the same students would never contemplate in their real lives outside the classroom. Various intervention studies to identify problems and stimulate improvement have been designed and implemented, with varied outcomes (see Vershaffel and Van Dooren (2010) for a summary of some of these). Attention is drawn to the impact of classroom culture in seeking change, for not only are different types of problems needed, but improvement “would also imply a classroom culture radically different from that which typically exists in many mathematics classes.” The difficulty of producing change may well be compounded by the types of intervention materials proposed – more realistic word problems in text books do not address the cultural issues that learning from text books in this area themselves reinforce. If the medium remains the same a different message is difficult to promote.

With the exception of the first, all of the above types of activity have the intention of enhancing the mathematical performance of students, and it is not the intention to decry the contributions they can genuinely make to curricular mathematics. It is important that incisive work continues, to increase the effectiveness of their respective implementations. It is simply pointed out that taken alone they are not equipped to produce students who are effective modellers, or to foster abilities such as those implied in the national curriculum statements referenced above.

3. Characteristics of modelling as content

Mathematical modelling as content had its origins in the advocacy of individuals such as Henry Pollak, who brought their experience as modellers in industry to Education. At ICME-3 in 1976 Pollak, who had been advocating an integration of applications and modelling into mathematics teaching for some time (e.g., Pollak 1969) brought applications and modelling to the fore through his lecture on "The Interaction between Mathematics and Other School Subjects" (Pollak 1979). Thus began a symbiotic relationship between mathematicians and mathematics educators which continues to enrich the field to this day. At roughly the same time the ICTMA group held its first meetings, and its presence as an Affiliated Study Group of ICMI is an important influence in the continued advocacy of the importance of ‘mathematical modelling as content’ as a genre within educational contexts. The importance of the contribution of expert modellers who ply their trade outside education cannot be measured, but it remains a significant referent in maintaining the authenticity of modelling activities. Central to its influence is the representation (often diagrammatically) of a cyclic process that captures and distils the essential characteristics of modelling activity. Pedley (2005) in his role as President of the IMA (Institute of Mathematics and its Applications) describes, with examples what an applied mathematician does in approaching problems, and a diagrammatic interpretation of his verbal description is shown in Figure 1.

1. Understand the real problem situation.
2. Frame an appropriate mathematical question
3. Formulate a model, using simplifying assumptions etc
4. Analyse the model
5. Compare mathematical outcomes with reality
6. Modify and repeat until an adequate solution has been found.

Figure 1. Modelling Process (after Pedley, 2005)

The arrows on the left display the ultimate pathway from problem setting to solution, while those on the right indicate that iterative back tracking may occur repeatedly between any phases of the modelling cycle - whenever such a need is identified. This is a compact version of the modelling framework that over the years has appeared in various forms (often with seven stages) in many sources, such as (Edwards and Hamson, 1996). It is reasonable to expect that readers of this journal are familiar with such representations. A seventh stage which is not shown in the above involves communicating the results of the modelling enterprise.

3.1. Two functions of modelling frameworks

Frameworks (such as Figure 1) have been used for many years within modelling programs, on the grounds firstly that they accurately reflect the problem solving process undertaken by modellers. For example the construction of Figure 1 emerged from the description (Pedley 2005) of how an applied mathematician goes about applying mathematics to address real world problems.

Their second purpose is didactical in providing mental infrastructure or scaffolding for students learning the modelling process. However in terms of a theoretical framework for teaching modelling, historically a missing link existed between the use of such heuristic devices and pedagogical theory, a link that can be supplied using elements from two complementary theories, information processing, and learning as theorised by Vygotsky (1978). The approach is embedded within a sociocultural orientation to learning, which develops the connection between classroom activity and that of a mathematical community of practice, supported by concepts of scaffolded learning within zones of proximal development. To the extent that the modelling framework is consistent with a mathematician's approach to real-world problem solving it creates a ZPD that can contribute actively to supporting the development of student mathematical expertise. The scaffolding notion of the zone of proximal development (Vygotsky, 1978; Palincsar, 1986) refers to the development of individual expertise through appropriate guidance from more capable others, including peers, or teacher. So the role of the modelling framework is elevated beyond the representation of the modelling process for which purpose it was first conceived, to that of an essential participant in the learning process as a continuing referent, whereby it provides metacognitive support to assist the development and monitoring of associated mathematical learning. But Vygotskian principles do not stop here. To complement the role of the modelling diagram in scaffolding the learning of individuals, collaborative activity, such as occurs when students form teams to develop models, is supported by another interpretation of the zone of proximal development - its application in egalitarian partnerships. Unlike the scaffolding notion that is based on different levels of expertise, this view of the ZPD accesses the learning potential in peer groups, where students have incomplete but complementary expertise – each partner possessing some knowledge and skill but requiring the others' contribution in order to enhance progress. Compared to the expert-novice situation, the co-production of the task is likely to involve more explicit contest and trial-and-error as the partners begin to appreciate the perspectives of others and coordinate their incomplete competence (cf Forman and McPhail, 1993). Students are provided with the opportunity to own the ideas they are constructing, and to experience themselves and their partners as active participants in creating and testing personal mathematical insights. However, it is important to recognise that not all student constructions are equally valid, and hence the teacher, as a more experienced knower in the field, has an essential role to play in identifying student ideas that are fruitful to pursue, and in asking students to justify their conjectures, strategies, and solutions in the context of the problem being addressed.

The information-processing component emerges through the contribution of the modelling framework to reducing the load on working memory during problem solving. Use of a visual modelling aid effectively reduces the cognitive load by structuring the extended task, so enabling students to focus their resources at each stage on the associated aspect of modelling. This interpretation is consistent with the behaviour and development of students in the two-year mathematical modelling program reported by Galbraith and Clatworthy (1989; 1990). In an evaluation of that program students rated the "seven box diagram" as the most useful assistance they received with respect to developing modelling expertise. The use of the framework was very explicit and visually obvious in the early stages, but that by the end of the course it had been internalised to the degree that its only overt appearance occurred in student reports where its categories re-emerged as convenient headings. That is, the visual representation effectively acted as a scaffolding prop, with a "use by" date decided by students on an individual basis. What happened was the development over time of a mental infrastructure for modelling, a structure honed by its use on a range of modelling problems. In the early stages students typically placed considerable overt reliance on the framework, which assisted materially with both model development and report structuring. With experience the students sublimated the structure as part of their mathematical and modelling know-how, whereby overt use of the visual framework gave way to the smooth movement between phases in the modelling process without recourse to any visible form of procedural representation.

In summary 'mathematical modelling as content' is anchored in processes used in solving real world problems, and learning and teaching skills of modelling requires attention to criteria that are both internal and external to education. Approaches within the 'vehicle' conception can achieve a variety of important instructional purposes within mathematics curricula, including some aspects that 'modelling as content' is not equipped to address as directly. However constrained curricular purposes dictate their ultimate design and purpose, so that if conflicting priorities arise, the necessary sacrifices involve those attributes (sometimes called modelling competencies) that are distinctive for producing students competent to consistently address real life problems relevant to their world.

Here it is also appropriate to confront a criticism that is sometimes levelled at the 'content' approach, but which contains a logical fallacy. The fallacy is that A implies B, means also that not-A implies not-B, and a typical argument runs as follows. "A process used by professional modellers (e.g. as represented in Figure 1) is not applicable for use in education because students not being professional modellers have different interests and abilities."

While there are differences of course, what both groups need for success are modelling competencies that can be applied effectively and sensitively, including the ability to work productively both as individuals and as team members. The following questions are relevant for both groups. Is it important to be able to: Define a problem from a real-world setting? Formulate and defend an appropriate mathematical model to address it? Solve the mathematics involved in the model? Interpret the mathematical results in terms of their real world meanings and implications? Evaluate and report the outcomes of the model both for mathematical validity, and in terms of their relevance to the original question? Revisit and challenge material produced within any part of the modelling process in the interests of improved outcomes? Can any of these 'stages' be omitted from a seriously constructed modelling endeavour? Is the ordering of the stages arbitrary? If as we contend the answer to every question except the last two is "yes", and to those two is "no", we have a process that characterises essential modelling activity that is as relevant to learners as it is to those doing modelling professionally or for personal reasons. And this recognition is significant also in the selection of contexts that provide authentic settings for the exercise of modelling activity.

4. Classifying purposes and perspectives

The paper by Kaiser and Sriraman (2006) has provided a significant service by highlighting the issue that there are many voices within the field of applications and modelling in education, and that they speak with a variety of dialects. The intention here is to revisit the idea of classification from a simplified perspective. My choice of the two recognizably global approaches, 'modelling as vehicle' and 'modelling as content' as fundamental archetypes stems from reasoning such as the following. Assigning names such as 'realistic modelling', 'contextual modelling', 'socio-critical modelling' etc creates a plethora of alternatives suggestive of a multitude of distinct modelling genres, rather than drawing attention to the alternative purposes and priorities that a given modelling activity might engage. For example 'context' is an essential component of 'realistic modelling' as it is described, where the purpose is to solve a problem located in a real world setting, and of course 'context' is of central importance when matters of socio-critical substance are the focus of attention. Rather than present perspectives as different types of modelling, the approach will be to relate them respectively to the structure of the two principal archetypes.

To do this reference will be made selectively to existing literature for purposes of illustration, as was the approach within Kaiser and Sriraman (2006). Beginning with the reference made to some of our own work (Galbraith and Stillman, 2006) in that paper, we see our approach as strongly within the modelling as content genre, since the overriding purpose was to help students apply their mathematics to solve real world problems of which the chosen example was one. That our work had an educational component was correctly identified, but this related to purpose not the approach to modelling as such. As well as this didactical purpose a research focus was present in the priority given to identifying and describing the types of blockages that interfered with the students' progress. Within the present discussion this could be described as a cognitive purpose. To support the research focus the basic modelling framework was elaborated into what for purposes of distinction is called a 'modelling diagram'. Figure 2, developed from a corresponding diagram in (Galbraith and Stillman, 2006) is designed to support research description, analysis, and discussion, rather than to support students learning the art and science of modelling - although it does contain an embedded version of the traditional 'modelling cycle' - stages A to G (with transitions 1 – 7), linked clockwise by the heavy single headed arrows.

It serves to define - and via its structure and the attached box identify - key foci for research with respect to individuals learning mathematical modelling, and pressure points for those teaching within the field. For example, the kinds of mental activity that individuals engage in as modellers attempting to make the transition from one modelling stage to the next, and which provide key foci for research, are given by the broad descriptors of cognitive activity, (boxed) 1 to 7 in Fig 2.

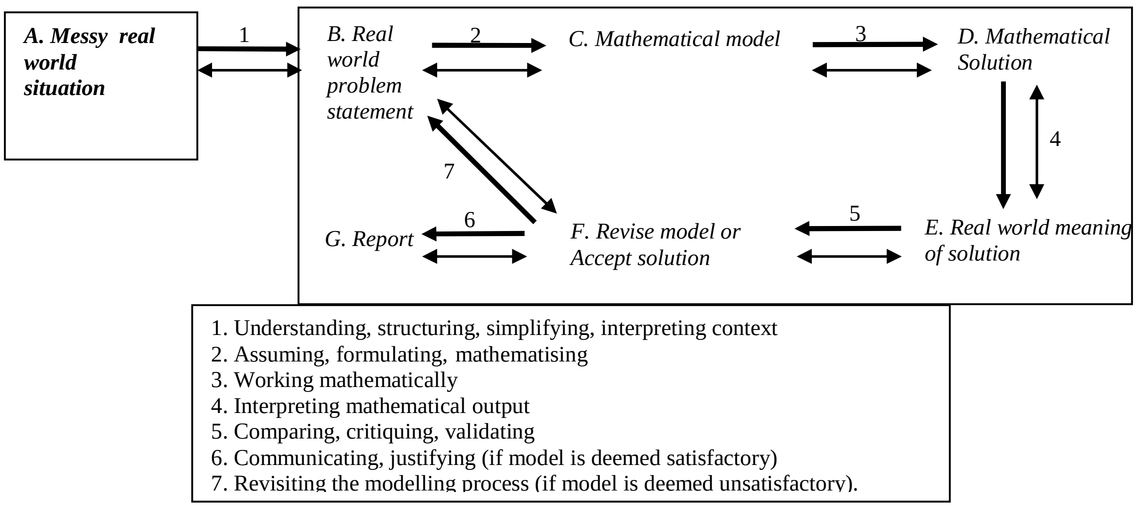

Figure 2. Modelling Process (after Galbraith and Stillman 2006)

The light double-headed arrows emphasise that thinking within the modelling process is far from linear (Borromeo Ferri, 2006) and indicate the presence of reflective metacognitive activity as articulated by many researchers (e.g., Maaß, 2007). Such reflective activity can look both forwards and backwards with respect to stages in the modelling process[1].

Moving to other considerations, socio-critical aspects associated with modelling (e.g., Julie, 2002; Barbosa 2006) are representative of purposes and priorities, but again do not need a separate genre to provide for them. The sensibilities involved in using modelling to address problems of inequality and disadvantage are located precisely at the point of assumptions made to articulate a problem statement from a given real world issue, and in then constructing a model to address it. As noted above Cyril Julie has been taking this approach for years in addressing issues of disadvantage in the South African context (e.g., Julie and Mudaly, 2007).

With respect to modelling competencies Maaß (2010) importantly indicates that what are viewed as competencies will depend on the nature of the tasks embedded in lesson material, and hence on the approach to modelling that is enacted in a particular classroom. Thus if modelling is regarded as a servant (vehicle) used primarily or only for the development of conventional curricular material the view of associated competencies will be likewise limited. That is, how modelling is fundamentally approached will dictate how views of competencies are developed and valued.

[1] The double- headed arrows in Figure 3 are indicative rather than exhaustive. In theory they connect every pair of stages e.g. (C, E), but diagrammatic clarity precludes the inclusion of them all.

Michelsen (2006) invokes both vehicle and content attributes (not in those terms), in considering ways that school mathematics and science might be better integrated. As a paper linking mathematics with other school based academic disciplines his focus is an important one that is underrepresented in the modelling literature at large. Arguments are supported with illustrative examples that anchor arguments for useful contributions that might be achieved through both modelling approaches, with the interdisciplinary focus a defining feature.

Borromeo-Ferri (2006) has surfaced a substantial number of issues in her focus on different modelling cycles and different structures within phases of the modelling process. The introduction of real model and situation model as intermediate structures within mathematical model formulation are efforts to represent more completely outcomes of mental procedures that occur within the phase of modelling (as content) long recognised as one of the most demanding parts of the whole process. To what extent these are present (perhaps subliminally) in all modelling examples, to what extent they should be afforded formal status within modelling structure, or to what extent they appeal to some individuals as helpful heuristic devices, remain active avenues for exploration. On this theme arguments for the reduction of phases in the modelling cycle, due to reported difficulties (Maaß, 2004 cited in Borromeo Ferri, 2006), that it was difficult for seventh graders to distinguish between ‘real model’ and ‘mathematical model’ might be reason to cast doubt on the essentiality of the former, rather than remove phases from a cycle whose completeness is essential for authenticity? It is harder to be sanguine about indications that different modelling cycles are being produced by various individuals for selective purposes – such as teaching and research specifics. The creation of idiosyncratic cycles to serve a variety of purposes, such as portraying cognitive processes employed by solvers, or for describing approaches to the solution of word problems, raises concern that the concept of ‘modelling cycle’ might be reduced to an internal second level construct that services the preferences of researchers, rather than a core structure that fundamentally represents an authentic modelling process. It serves to draw attention to the important distinction between a modelling cycle (or framework) and a modelling diagram. The former (illustrated in Figure 1 above) contains a description of the modelling process as a summary of steps involved in real world problem solving. The process is cyclic, although the pathway of individual solvers may vary depending on the amount of backtracking and revising that occurs. Modelling diagrams are many, and while typically containing some embedding of the modelling process, will also contain elaborations that focus on the purposes and perspectives of their particular creators. For example Figure 2 (above) while containing an embedded representation of the modelling cycle, contains additional structure specific to its research purpose, of identifying blockages that occur at transition points between phases of the modelling process, and investigating the metacognitive ramifications of these. In summary it is important that the integrity of the modelling cycle is tested and preserved as an externally warranted process consistent with the recorded activity of professional modellers, and that this is kept distinct from research or other purposes which may be legitimately represented by various additional diagrammatic elaborations where appropriate. In these terms the research approach which stands to produce the most authentic elaborations or challenges is likely to be Grounded Theory (e.g., Corbin and Strauss, 2008).

Barquero et.al., (2009) and Schmidt (2009) address issues related to curricular implications arising at tertiary level if mathematical modelling is to be a serious inclusion in coursework. The former in considering the ‘ecology’ of mathematical modelling at university level review constraints noted previously (Burkhardt, 2006; Blomhoj and Kjeldsen, 2006; Kaiser, 2006). In asking what limitations and constraints work against a wider implementation in tertiary courses we need to ask how profound is the impact of various versions all claiming mathematical modelling as part of their rationale. The authors capture precisely the tensions existing when a prevailing approach to service other content needs (vehicle), comes face to face with a desire to enhance the problem solving capabilities of students through the incorporation of modelling (content). Similarly Schmidt (2009) has reminded us of classroom challenges related to modelling, pointing to organisational, student related, and teacher related obstacles. Again we are reminded of difficulties that emerge when a mix of purposes, priorities, and beliefs about what modelling means meet in the classroom. This author recalls this situation as it emerged in a senior school classroom a number of years ago (Galbraith and Clatworthy, 1989; 1990). The problem was addressed by developing two strands that ran in parallel throughout the two years of the program. These were respectively a ‘concepts and techniques strand’ known as the toolbox strand, and a mathematical modelling strand. The goals and purposes of the strands were kept conceptually distinct – the former to learn and practice new mathematics, and the latter to learn how to solve real world problems. Of course as the program of study progressed, elements from the first strand were imported as required into the second strand. However, keeping the goals and approaches of the complementary strands delineated seemed to be influential in the success of the program.

In summary it is argued that while a variety of purposes and priorities motivate different approaches within modelling programs, these are ably provided for as alternative emphases (particularly related to modelling assumptions), and do not warrant the creation of specialised modelling genres to provide for them.

5. Modelling Critique

A consequence of the way in which 'modelling' has been appropriated to describe a variety of different activities and emphases is that utmost care is needed when critiquing aspects of its theory and practice. This does not matter so much when the audience is well informed about modelling, as may be assumed of most readers accessing this journal. It matters a great deal when the audience is a general one within mathematics education, one liable to believe the claim of sources purporting to speak representatively concerning the modelling field. In particular, damage stands to be done by claims that are inaccurate or unrepresentative. Poor modelling practice exists and needs to be exposed, and informed critique is important in adding to this dimension. However it is essential that such examples are presented in context, and not erroneously as if representative of the field as a whole, and those who work within it. Some examples follow.

Jablonka and Gellert (2007) argued that there is no straightforward way to move from a real problem context to a mathematical model, because it is virtually impossible to quantify non-mathematical characteristics, and relate them mathematically in one step. There seems to be confusion here between a procedure (step) and a phase in the modelling process – the latter may contain several steps and will vary in complexity with the sophistication of the problem. They further argue that there can be no validation because a result is not put back into a 'real' real situation. This seems certain to be true of some situations and is therefore important to record, but it is demonstrably false as a generalisation, notably where modelling involves actions both inside and outside the classroom. For example (Osawa, 2002) describes a project in which the goal was to optimise the baton changing practice of relay teams. Activity took place alternately in a classroom, and on the running track which acted as the laboratory within which theoretical results were tested. Neither place alone would have sufficed to carry out the complementary theoretical and experimental activities that successive improvements demanded. Equally striking are the actions of individuals who have applied their school learning in modelling to address problems outside school that were real to them. Examples include the successful case prepared by a 12 year old girl (in terms of both finance and time utilisation) to convince her parents that she could both care for, and provide for the ongoing upkeep of a much wanted pony, and a mature age student who used the cyclic modelling process to redesign the culture he used for growing tomatoes hydroponically.

A very recent paper by the same authors (Jablonka and Gellert, 2011) begins with the statement that "Modelling approaches are propagated to enhance the quality of the outcomes of mathematics education by providing students with generic competencies and thereby creating a flexible work force". This is an overstated generalisation, as motivations are various, and include centrally that of student empowerment, as in: "… for students to spend years learning mathematics without any sense of how to apply it in the world around them, is inappropriate." (Stillman, et.al., 2010).

It is further alleged that modelling conceptions do not see associated competencies as 'culture bound and value driven", yet Niss, et.al., (2007) point out that "the best route for a new freeway", implies that "best" must be interpreted, and this implies not only considerations such as "most direct", or "cheapest", but also "least disruptive to communities". Note also the extensive work undertaken by Julie referred to previously that has specifically used mathematical modelling to address problems associated with disadvantaged individuals and communities in South Africa.

Again it is asserted that "…contextuality of all knowledge is (mis)interpreted in a way that leads to the contention that mathematical concepts can be meaningfully learned only within a 'real life' context". Compare with:

> "neither the content nor vehicle approach argues in some abstract sense that all mathematical curricular content must be justified in terms of relevance - mathematical modelling has a role to play in meeting certain important goals, but other significant mathematical skills and purposes are important as well." (Stillman et.al., 2008).

The paper raises some important issues concerning equity, but a drawback seems to be a dependency on selections chosen seemingly to support the ideology of the critique, rather than choosing examples from the modelling field that illustrate both the legitimacy of an issue raised and its inappropriateness as a generalisation. The examples chosen reinforce the cautionary tale that there are many versions of modelling out there that cover the full range of good, bad, and indifferent implementations. But it is imperative that the theory of mathematical modelling, its purposes and possibilities, are kept conceptually separate from poor implementations, and abuses. There is no question that the latter exist, but they must not be allowed to undermine arguments for what is possible when the best is undertaken.

6. Summary

This paper began by referring to the numerous sources that link mathematical modelling with education in some form. We have noted that curriculum statements from many national contexts (e.g., OECD, Singapore, Australia, and elsewhere of course) continue to affirm the importance of students being able to apply the mathematics they learn to workplace situations, in their personal lives, and in their role as citizens. We also note that while mathematical modelling is supremely suited to address these goals, the uptake in schools and universities continues to fall short of expectations (Blum, et.al., 2007; Burkhardt, 2006). The position taken in this paper is that we as the modelling community in education need to accept responsibility for aspects of this gap between theory and practice, because of the confusing variety of voices with which we speak. Confusing voices stand to compromise progress at any level, for if those working in a field give mixed messages, why should others listen to their advocacy. This is to say that we should be publicly clear about what the presence of modelling in education sets out to do and what it can uniquely achieve – in no way does it argue against the necessity for vigorous internal debate within the field The position taken here is that there are fundamentally two genres that describe how modelling has been employed within educational settings. When used as a 'vehicle', some parts of a modelling process, or aspects related to modelling, are used to enhance the learning of mathematical concepts or processes that form part of the curricular mathematics included in syllabuses. When included as 'content', mathematical modelling sets out to enable students to use their mathematical knowledge to solve real problems, and to continue to develop this ability over time. This involves much more than trying to find opportunity to infiltrate a modelling related activity into a crowded schedule. The two modelling genres have different goals, and classroom priorities will be different depending on which is being pursued at a given time. One difficulty has been that traditionally the goals of mathematics teaching have been overwhelmingly aligned with the presentation of curricular mathematics, so that as noted above mathematical modelling as 'content', and the potential for student empowerment associated with it, has suffered. This tension is behind the reluctance of teachers to undertake modelling that seems to be so different from what has conventionally become accepted as classroom mathematics - at both school and university. For this to change, an additional and specific requirement, for students to be able to demonstrate abilities to formulate and solve problems located in their world using their store of mathematical knowledge, needs to be made as explicit as goals long associated with the learning of curricular mathematics which of course remain important. This would give legitimacy to the coexistence of two complementary strands in mathematics programs, so that modelling goals can have an integrity that is independent of that associated with the learning of conventional material. It does not mean that the strands need have equal weight in terms of time, but it does recognise fundamental ontological distinctions between the respective purposes of modelling as 'content', and of conventional curricular mathematics. It goes far beyond 'feel good' statements about the importance of students being able to apply their mathematical knowledge, for the belief that modelling can be somehow integrated as one competency strand among many curricular imperatives has profound implications for what can be pursued as modelling competencies. In particular such a belief creates pressure for an atomistic approach to competencies at the expense of a holistic approach based around the use of authentic life-related modelling problems. Educational traditions have led to a privileging of a certain conception of what school mathematics is about, and what mathematics teaching and classrooms are allowed to be – and these traditions impact severely when modelling initiatives are required to fit the stereotype and be subject to associated practices. By contrast, what modelling properly conducted can do, is to challenge some of those norms, assumptions, and stereotypes - mathematical, situational, and pedagogical. And in that modelling as 'content' involves intersections between the values and methods of more than one community of practice, (applied mathematicians and those applying quantitative methods in their professions, and mathematics educators) it challenges the boundaries of the existing education industry.

Goals for students to become skilled at using their mathematics to address problems arising in work, personal, and civic contexts require the development of abilities fostered by experiencing mathematical modelling as 'content', but it is doubtful if the national authorities responsible for publishing such laudable goals, are aware of the resulting implications for classroom support and curricular priorities. However it provides the modelling community with rare opportunities to make our voice heard – provided we can give a prioritised and unfragmented message, which will require good scholarship, good relationships, and goodwill to achieve.

Of course as practitioners and researchers we have a great variety of legitimate individual interests and priorities (see e.g., Kaiser and Sriraman, 2006) in striving to identify problems, highlight issues of importance and enhance the effectiveness of learning and teaching in the modelling field. It is argued that these are more appropriately addressed as purposes, priorities, and perspectives than in terms of additional modelling genres. Such an approach also stands to help in the aforementioned dealings with education authorities, since the nature of modelling can first be portrayed with clarity, and its immense richness as a source of diverse classroom activity in terms of perspectives and purposes, then promoted as a consequence of its presence.

Note has been made that published criticisms of modelling practice can both enhance and damage the field. Indeed, poor implementations need to be exposed as such, but inaccurate generalisations or misleading comments need to be identified and addressed. Otherwise curriculum authorities, teachers, lecturers, researchers, and particularly those who may be attracted to the field, stand to be misinformed at great cost to the future of the field and its future students. This cannot be allowed to happen.

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