Bilevel Optimization Approach to Design of Network of Bike Lanes

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A bike lane is an effective way to improve cycling safety and to decrease greenhouse gas emissions with the promotion of cycling. Improvements include high-quality off-road facilities and on-road bike lanes. Whereas construction of off-road lanes is not always possible because of urban land constraints and construction costs, on-road lanes can be a cost-effective alternative. An optimization framework for the design of a network of bike lanes in an urban road network was proposed. This framework identified links on which a bike lane could be introduced. Allocation of a lane to cyclists would increase the use of cycling, although it could disadvantage auto traffic. The presented approach balances the effects of a bike lane for all stakeholders. A bilevel optimization was proposed to encompass the benefits of cyclists and car users at the upper level and a model for traffic and bike demand assignment at the lower level. The objective function was defined by a weighted sum of a measure for private car users (total travel time) versus a measure for bike users (total travel distance on bike lanes). A genetic algorithm was developed to solve the bilevel formulation, which included introduction of a special crossover technique and a mutation technique. The proposed optimization will help transport authorities at the planning stage to quantify the outcomes of various strategies for active transport.

Cycling is a sustainable mode of travel and can improve the health and well-being of riders. Cycling can benefit other road users by reducing carbon emissions and relieving congestion in the transport network. A lack of cycling facilities is a major barrier (*1*). Although off-street bike paths offer a safe and comfortable riding environment, they are little used in urban areas because of the lack of suitable space and the high costs of construction.

Studies have shown that the built environment influences cycling rates. Surveys in the United States indicate that bicycle commuters have a high preference for bicycle lanes (*2*). A positive correlation between levels of commuting by bicycle and density of bicycle lanes was identified in a study of large cities in the United States (*3*). Adding bike lanes increases the likelihood of cycling (*4*). Evidence also shows that cyclists adjust their routes to use bicycle facilities (*5*).

Transport authorities in Australia and around the world have included walking and cycling as a priority for future development. For instance, an active transport target is to double mode share by cycling (to reach 11%) in South East Queensland by 2031 (*6*). However, no methodology has been proposed for designing an integrated network of bike lanes. The recently released *Victorian Cycling Strategy* includes building networks to connect communities, reducing conflicts and risks for cyclists, integrating cycling with public transport, and integrating the needs of cyclists with land use planning and the built environment (*7*). The strategy acknowledges the need for cycling networks to provide continuous quality connections to major destinations and public transport hubs. On-street bike lanes can reduce conflicts between motor vehicles and bikes. However, principal bicycle networks in many Australian cities are not well developed.

Although the reviewed studies have different emphases on proposed bike lanes, all studies evaluate bike lane alternatives (BLAs). Despite the level of detail in some studies, the evaluations reveal only whether a BLA (i.e., a set of bike lanes) should be implemented; this does not mean that the given BLA is the best possible or the optimal BLA for the network. Therefore, an optimization method is needed for finding the best set of links for installed bike lanes.

This paper outlines a methodology for finding the optimal BLA. The optimal BLA determines the links in the transport network on which a bike lane should be introduced. The methodology can be applied to medium and large networks.

BILEVEL OPTIMIZATION

Two levels of decision making are proposed for finding the optimal BLA. At the upper level, the transport authority would propose a BLA. Given this BLA, system users at the lower level would choose a strategy to maximize their own benefit under prevailing conditions. Again, the transport authorities would modify the initial BLA on the basis of the behavior of users, and the cycle would continue. This problem can be modeled as a Stackelberg competition, in which the transport authority is the leader firm and system users are the follower firms (*8*). The optimal BLA is chosen in equilibrium conditions when neither the transport authority nor users can improve their benefits. The Stackelberg competition can thus be modeled as a bilevel optimization problem.

The upper level is articulated in accordance with the transport authority's point of view. Therefore, a system optimum is formulated in this paper for the upper level. The transport authority takes into account the total travel time by car as well as a performance measure for a bike system, such as travel distance on bike lanes. There can also be a series of practical constraints for a priority scheme formulated in the constraints of the upper level. The output of the upper level is the set of decision variables that define the location of the bike lanes.

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User response to the decision made by transport authorities is modeled at the lower level by applying a traditional four-step modeling approach. In this study, it is assumed that the travel demand in the network is not changed by introduction of a BLA. It is also assumed that the two modes of private car and bikes use the network, and mode shift is negligible. Thus, the demand of each mode is known. In the last step of the four-step model, car and bike demand should be assigned to the network links. At the lower level for private cars and bikes, models for a car demand assignment and a bike demand assignment, respectively, are used. Although the BLA is determined at the upper level, the objective function is calculated in the lower level.

NOTATION

The following notation is used in this paper:

- A = set of all links in the network, $A = A_1 \cup A_2$;
- A_1 = set of links in the network where provision of priority is possible;
- A_2 = set of links in the network where provision of priority is impossible;
- bdg = available budget;
- e_{am} = cost of bike lane type *m* for length on link (*a*);
- $f_k^{c,rs}$ = car flow on path *k* connecting origin node *r* to destination node *s;*
- $f_k^{b,rs}$ = bike flow on path *k* connecting origin node *r* to destination node *s;*
- l_a = length of link (*a*);
- q_{rs}^c = trip rate between *r* and *s* by car;
- q_{rs}^b = trip rate between *r* and *s* by bike;
- $t_a^c(x)$ = travel time on link (*a*) by car (*c*), which is a function of flow;
- x_a^c = motor vehicle flow on link (*a*);
- x_a^b = bike flow on link (*a*);
- α, β = weighting factors to convert the units and adjust the relative importance of each impact in the objective function, $\alpha, \beta \geq 0$;
- $\delta_{k,a}^{rs}$ = incident matrix that relates x_a^c to f_k^{rs}, where δ is 1 if link (*a*) is on path *k* for any origin–destination pair *rs* and 0 otherwise;
- φ_{am} = 1 if bike lane type *m* is introduced on link (*a*) and 0 otherwise;
- ϕ_a = 1 if any type of bike lane is introduced on link (*a*) and 0 otherwise; and
- Φ = vector of ϕ_a for all candidate links.

Upper-Level Formulation

The upper-level model is formulated as system optima from the transport authority's perspective. The goal for the objective function (*Z*) is to maximize the portion of bike travel on bike lanes; this goal is best achieved by defining a bike lane where feasible. However, each bike lane will take some road space from cars and allocate it to bikes. Therefore, the transport authority has to take into account the performance of cars. The performance measure used for cars is the total travel time of car users.

The upper level is proposed as follows:

$$\max Z = \alpha \sum_{a \in A_1} l_a x_a^b \phi_a(x) - (\beta) \sum_{a \in A} x_a^c t_a^c(x) \qquad (1)$$

subject to

$$\sum_{a \in A_1} l_a \left(\sum_m e_{am} \varphi_{am} \right) \leq \text{bdg} \qquad (2)$$

$$\phi_a \sum_m \varphi_{am} \leq 1 \qquad \forall a \in A_1 \qquad (3)$$

$$\varphi_{am} = 0 \text{ or } 1 \qquad \forall a \in A_1 \qquad (4)$$

connected graph (5)

The first term of the objective function is the total travel distance on bike lanes; the second term represents the total travel time by car. The first term accounts for the length of the bike lanes in the transport network as well as the volume of riders on each bike lane. Coefficients α, β can reflect different policies in the relative importance of each term. They also convert the units. As Equation 1 shows, the objective function is formed from a transport authority's perspective. The budget constraint is accounted for in Equation 2.

There are two types of links in the network. The first is the links that could have a bike lane (Set A_1). The second type is the links on which no lane can be dedicated to bikes (Set A_2). This classification could be the result of a road use hierarchy (*9*). Decision variables determine which type of bike lane would be introduced on potential links. Equation 3 ensures that only one type of bike lane is chosen for a link. The binary decision variable is defined in Equation 4.

Constraint 5 demonstrates an important practical consideration of continuity. The proposed network of bike lanes in a BLA should be connected. Connectivity is defined if there is a path on bike lanes from the end point of any link with a bike lane to one of travel destinations. It is important that the bike lanes form a continuous path instead of being provided only where possible. There could be separate but connected graphs since the continuity constraint means there should be a route only to one of the destinations. For instance, in the case of a network with two spatially sparse destinations, two connected graphs—one to each destination—satisfies the connectivity constraint. In graph theory, links with a bike lane should form a number of connected components that have at least one destination node (vertex). This constraint can be verified with graph theory methods such as breadth-first search or depth-first search (*10*) or, more specifically, with Dijkstra's shortest path algorithm (*11*). Computation of flow and travel time at the lower level is based on the set of decision variables in the upper level.

Lower-Level Formulation

When a BLA is determined, it is the users' turn to decide how they will use the provided facilities. In other words, models at the lower-level estimate user response to a given BLA. These models in the bilevel structure function as constraints to the optimization programming presented in the upper level. From these models, flow and travel time are obtained.

With the assumption of a constant travel demand, two models are involved in transport modeling:

1. Car demand assignment and
2. Bike demand assignment.

Traffic assignment is the first model at the lower level. With the introduction of a bike lane, the width of the general traffic lanes and therefore their capacity is reduced. As a result, drivers may choose alternative routes in the network. Traffic assignment is carried out to consider route choice behavior of car users. A static user equilibrium model is used for car demand assignment, a conventional model for strategic planning (*12*). This model uses an optimization approach to determine car flow and travel times in the network. The effect of the decision variables on the flow and travel time cannot explicitly be expressed; this is one of the reasons a bilevel approach is proposed. The user equilibrium formulation with objective function Y is as follows:

$$\min Y = \sum_{a \in A} \int_0^{x_a^c} t_a^c(x)\,dx \tag{6}$$

subject to

$$\sum_k f_k^{c,rs} = q_{rs}^c \qquad \forall r, s \tag{7}$$

$$f_k^{c,rs} \geq 0 \qquad \forall k, r, s \tag{8}$$

$$x_a^c = \sum_{rs} \sum_k f_k^{c,rs} \delta_{k,a}^{rs} \qquad \forall (i, j) \in A \tag{9}$$

Equations 7 and 8 are conservation of flow and nonnegativity constraints. The third constraint, Equation 9, defines the relationship between paths and links.

The second model assigns the bike demand to the transport network. Bike assignment is the other reason for which a bilevel approach is proposed. Many of the models proposed in the literature for traffic assignment can be applied in this framework. In very low levels of bike demand, an all-or-nothing assignment based on the shortest path could be appropriate. However, when the bike demand is considerable with possibility of congestion on bike lanes, bike congestion should be taken into account. In this paper, a model based on user equilibrium with objective function W is adapted.

$$\min W = \sum_{a \in A} \int_0^{x_a^b} t_a^b(x)\,dx \tag{10}$$

subject to

$$\sum_k f_k^{b,rs} = q_{rs}^b \qquad \forall r, s \tag{11}$$

$$f_k^{b,rs} \geq 0 \qquad \forall k, r, s \tag{12}$$

$$x_a^b = \sum_{rs} \sum_k f_k^{b,rs} \delta_{k,a}^{rs} \qquad \forall (i, j) \in A \tag{13}$$

Equations 11 and 12 are conservation of flow and nonnegativity constraints. The last constraint defines the relationship of paths to links.

SOLUTION ALGORITHM

A bilevel structure even with linear objective function and constraints at both levels is an NP-hard problem and is difficult to solve (*13, 14*). In this study, a heuristic approach based on a genetic algorithm (GA) is proposed in which new solutions are produced by combining two predecessor solutions (*15, 16*). Inspired by evolution theory, a GA starts with a feasible set of solutions called a population (Figure 1). Each individual answer in the population (called a chromosome) is assigned a survival probability that is based on the value of the objective function. Then, based on this probability, the algorithm selects individual chromosomes to breed the next generation of the population. The GA uses crossover and mutation operators to breed the next generation, which replaces the predecessor generation. The algorithm is repeated with the new generation until a convergence criterion is satisfied. Several studies have applied the GA to bilevel formulation (*17, 18*). Two recent examples are a transit network design problem that considers variable demand (*19*) and optimization of a bus lane network (*20*).

Here, a GA is used to optimize a bike lane network. To adapt a GA to this study, a genome is defined as the binary variable φ_{am}, a gene is defined to represent the binary variable ϕ_a, and a chromosome is the vector of genes (Φ). In this GA, a chromosome represents a BLA. A chromosome (or BLA) contains a feasible combination of links on which an exclusive lane may be introduced (set A_1). Therefore, the length of the chromosome is equal to the size of A_1. The algorithm starts with a feasible initial population. The chromosomes of the initial population are produced randomly.

Any produced chromosome could be feasible or infeasible according to constraint represented in Equations 2 through 5. In this study, a penalty function is used to ensure that the feasible answers will be given a greater chance in the reproduction process. The penalty is proportional to the amount that a constraint has been violated. A special crossover and mutation technique developed in the next subsection ensures that feasible solutions are produced. A generic crossover or mutation technique has very poor performance when used in this example because most of the generated answers would be disconnected. A disconnected answer is considered infeasible according to Constraint 5 at the upper level.

Once a chromosome population is produced, the upper-level objective function for all chromosomes should be determined. Each chromosome identifies the leader's decision vector for the network. Then the users at the lower level choose their route. Thus, for each chromosome, the lower-level models are carried out as depicted in Figure 1. Flow and travel time at the lower level are used to determine the objective function for the chromosome. The lower-level calculations are repeated for all chromosomes in the population (Figure 1).

The chromosomes with higher value of the objective function are assigned a higher survival probability. Then the GA operators of selection, crossover, and mutation are used to produce the next generation (set of BLAs). Similar to the process in the initial population, this process ensures the feasibility of the new generation. The new generation replaces the previous one and the calculations are repeated. To increase the convergence rate of the algorithm, the best chromosome of the previous population should be kept. The algorithm stops when either the number of iterations reaches the maximum or the best answer does not improve in a certain number of iterations. This cycle is demonstrated in Figure 1.

Crossover Technique

Development of a crossover technique is intended to produce new feasible solutions (BLAs). A generic crossover technique does not guarantee a feasible solution, however; in experiments by the

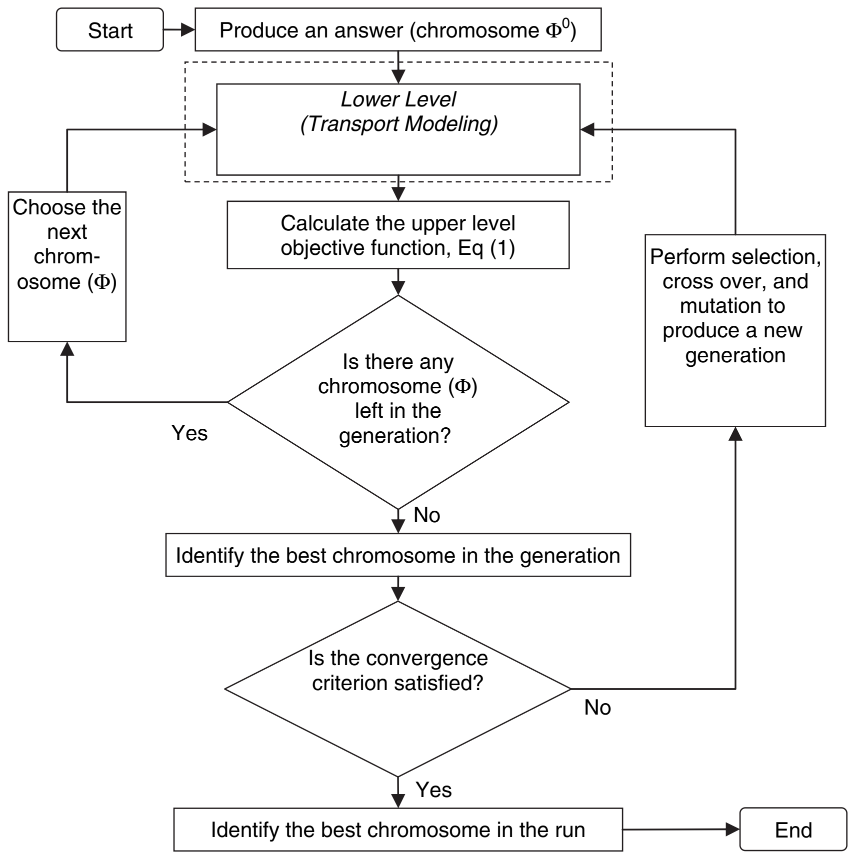

FIGURE 1 Flowchart of GA solution (Eq = equation).

authors, a generic crossover technique produced less than 5% feasible solutions in a medium-sized network. The overall efficiency of the GA is reduced because most of the produced solutions are not feasible. A crossover technique that produces feasible solutions is presented in this section.

For simplicity, assume there is only one type of bike lane. This reduces the feasibility constraints of the upper level to the budget constraint (Constraint 2) and the connectivity constraint (Constraint 5). Also assume the parent solutions are feasible. The developed crossover technique is shown in Figure 2. In the first stage, the given solutions are compared to determine the shared nodes by examining the topology of the solutions. The four points A through D are shared between the dashed (red) and dotted (blue) solutions. One of these shared points is chosen randomly for crossover (say, Point C). All links preceding the crossover point (Point C in this example) are exchanged between the solutions. This produces two children solutions that are connected. If no node is shared between the parent solutions, the crossover does not produce a new solution.

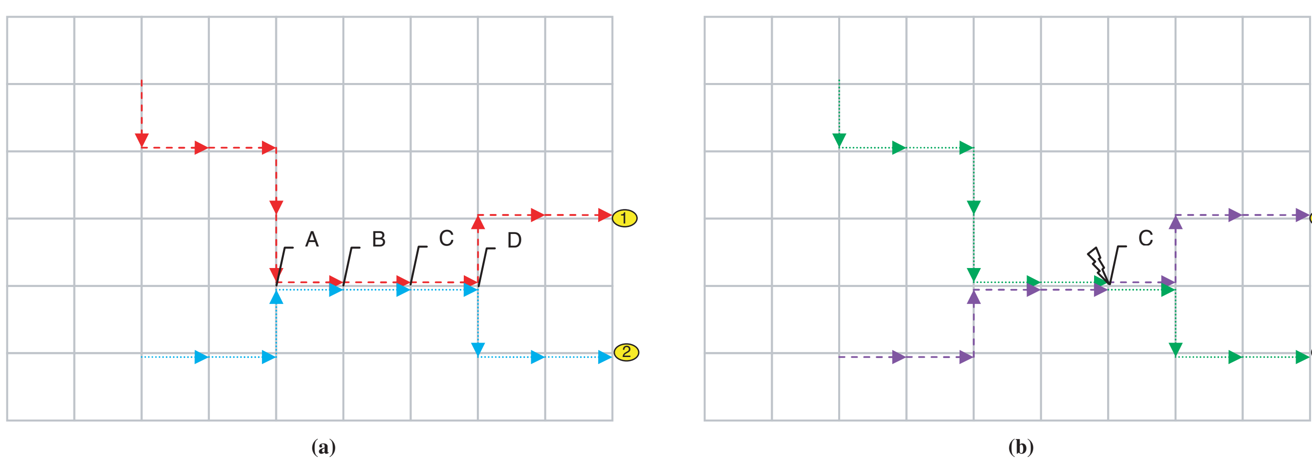

FIGURE 2 Crossover method while keeping solution connected: (*a*) parent solutions with possible crossover nodes and (*b*) children with crossover on Point C.

Although the children produced by the crossover satisfy the connectivity constraint, they could violate the budget constraint. Equation 2 is used to calculate the budget required for each child. Should the constraint be violated, links from the tail of the child solution are removed one by one until the solution becomes feasible. This procedure ensures that the produced children both are connected and cost less than the available budget.

Mutation Technique

A given solution is mutated in the GA. However, a generic mutation technique may introduce a bike lane on any of the links in the network. This can turn a connected network of bike lanes into a disconnected one. For instance, in Figure 3 a dotted (blue) arrow indicates a generic result of one mutation. As the figure shows, there is a high probability that the introduced link would cause a solution (solid arrows) to be disconnected. The proposed mutation technique involves two stages. In the first stage, the potential links to be added or removed are determined such that the network of bike lanes remains connected. Links can be added on any node where the given solution passes. The potential links that could be added to the current solution are shown in Figure 3*a,* and those that could be removed are shown in Figure 3*b.*

At Stage 2 of the mutation, one of the following actions is introduced:

1. Addition,
2. Removal, and
3. Replacement.

Addition means adding one of the potential links randomly. Removal means removing one of the possible links randomly. Replacement is a combination of addition and removal. Mutation is an important step of the GA, enriching the gene pool of the solutions. The proposed mutation technique provides a mean to move to undiscovered areas of the possible solution without violating the connectivity constraint.

NUMERICAL EXAMPLE

The proposed method is applied to an example network in this section. Figure 4 shows the layout of the network. This grid network consists of 42 nodes, 142 links, 15 origins, and two destinations. The 15 interior centroids are origins, and the two exterior centroids are destinations. A flat demand of 150 cars per hour and 15 bikes per hour travel from all origins to all destinations. The total demand for all 30 origin–destination pairs is 4,950 trips per hour.

Vertical and horizontal links are 800 m long with two lanes in each direction and a speed limit of 50 km/h. It is assumed that if a bike lane is introduced on a link, the opposite direction may or may not get an exclusive lane. A bike lane can be introduced on a total of 90 candidate links (one-directional) in the network shown in Figure 4. These links are highlighted in the figure by a thick dotted (green) line. The following cost functions are assumed for links with a bike lane (Equation 14) and without a bike lane (Equation 15):

$$t^c_{1,a} = t_{0,a}\left(1 + m\left(\frac{x^c_a}{\text{cap}^c_{1,a}}\right)^n\right) \qquad t^b_{1,a} = t_{0,a} \tag{14}$$

$$t^c_{0,a} = t_{0,a}\left(1 + m\left(\frac{x^c_a}{\text{cap}^c_{0,a}}\right)^n\right) \qquad t^b_{0,a} = t_{0,a} \tag{15}$$

where

t_0 = travel time with free-flow speed,
m and n = model parameters, and
cap = link capacity.

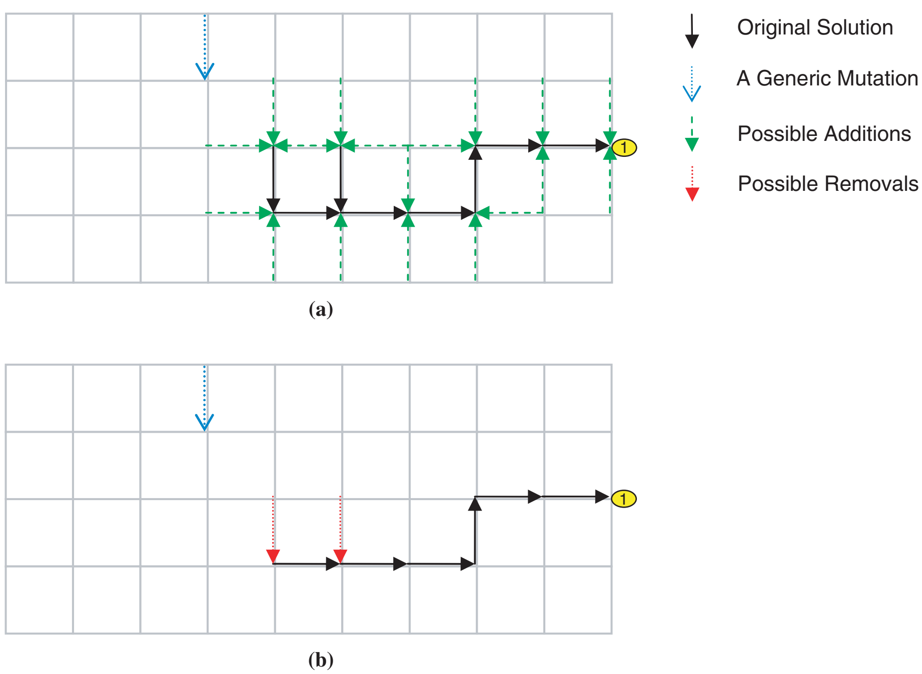

FIGURE 3 Mutation technique: (*a*) possible additions and (*b*) possible removals.

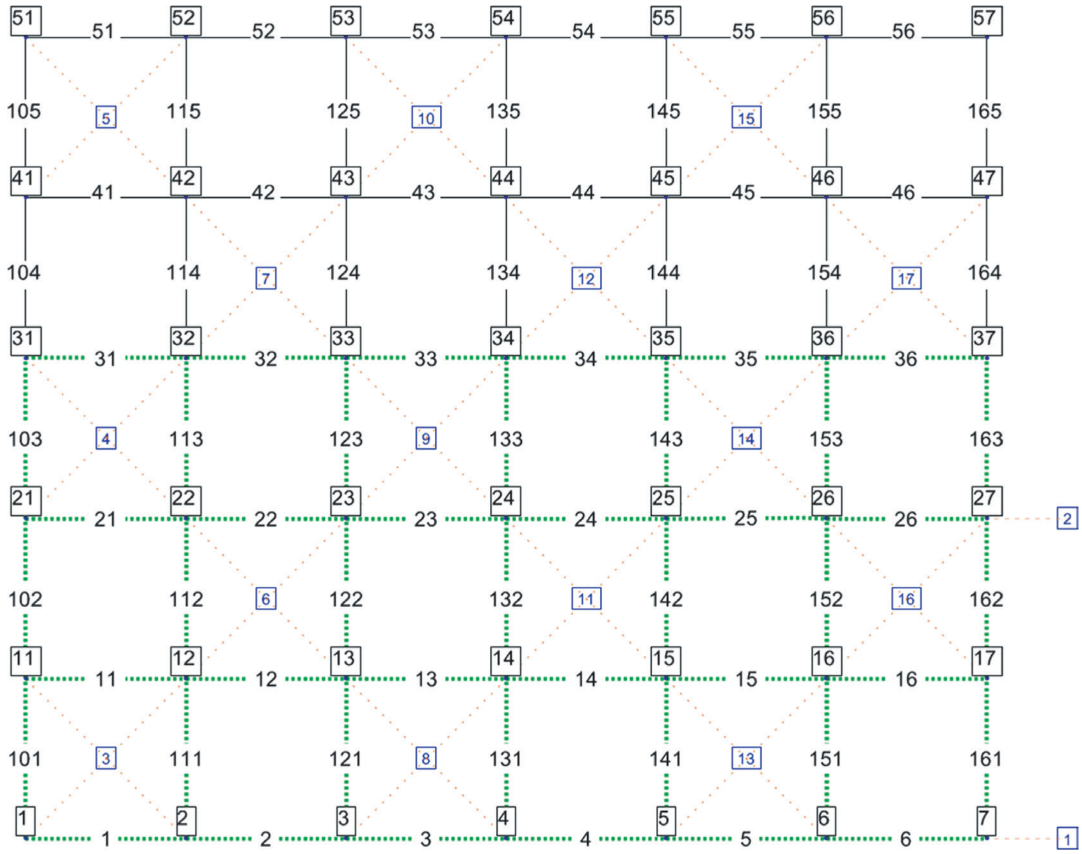

FIGURE 4 Example network with candidate links (dotted lines) and normal links (solid lines).

It is assumed that one type of on-road bike lane will be introduced in the network, which will reduce the car capacity from 1,800 to 1,500 vehicles per hour. It is assumed that each link has two car lanes and that $m = 1$, $n = 2$; $cap^c_{0,a} = 1{,}800$ vehicles per hour; and $cap^c_{1,a} = 1{,}500$ vehicles per hour.

Once the demand matrices are determined, the two user equilibrium models are used to assign car demand and bike demand. It is assumed that the number of bikes on a link does not affect the travel time. The lower-level transport model is implemented with the Visum modeling package (*21*).

In this example, weighting factors of the upper-level objective function are assumed to be 0.01 and 0.0001 for α and β, respectively. These factors may vary depending on their relative importance to transport authorities. The upper-level objective function includes total travel distance on bike lanes (vehicle kilometers) and total travel time by cars (vehicle seconds). The absolute value of the objective function therefore can be very large. To avoid numerical problems, the improvement of each term compared with a base case is considered instead of the absolute value of the term in the objective function. The base case is assumed to be the case in which no link is provided with an exclusive lane ($\Phi = 0$). Regarding the constraints, it is assumed that the budget allows for 10 bike lanes of a total of 90 candidate links to be constructed.

A common stopping criterion for the GA is number of generations. If the objective function does not improve for a considerable number of generations, calculations are terminated. In this example, a proper stopping criterion is investigated by increasing the number of generations to 600. Figure 5 shows the value of the objective function for two independent runs of the GA. The figure shows that the objective function did not improve after 300 generations, which can be introduced as the stopping criterion for this example.

Application of the proposed method to the network shown in Figure 4 resulted in the introduction of a bike lane on 10 links: 4, 5, 6, 22, 23, 24, 25, 26, 161, and 162.

This answer was anticipated because it includes all links close to the destinations. These links carry relatively more bikes than do those far from the destination centroids. In the objective function of Equation 1, the ratio of the weighting factors (α, β) determines the relative importance of the two objective function terms. A sensitivity analysis on the values of the upper-level weighting factors (α, β) shows that with a decrease in α, β, the relative importance of bike travel distance to car travel time decreases; thus, bike lanes will be introduced on links with lower traffic volumes.

Effect of Crossover Probability

The effect of crossover probability on the value of the objective function is investigated in this section. As Figure 6 shows, by changing the crossover probability from 0 to 1, the optimal answer found by the GA is slightly improved. By increasing the number of generations (300 generations in this example), a value close to optimum for the objective function value was eventually achieved by all experiments. Although the crossover probability was low for some experiments, a large number of generations provided enough chance for the evolutionary process to find a close-to-optimum answer in most of the experiments.

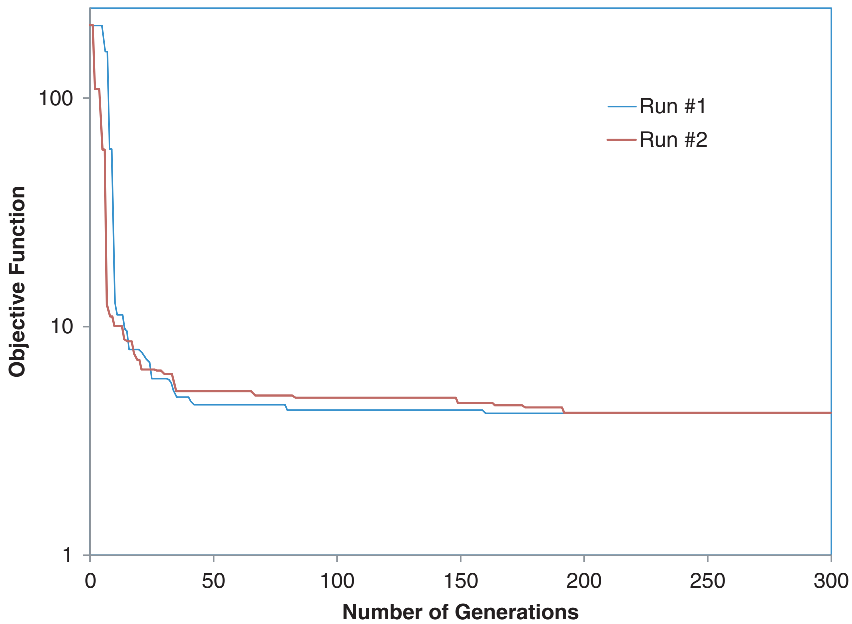

FIGURE 5 Effect of number of generations on value of objective function.

Effect of Mutation Probability

Low values of mutation probability do not provide enough opportunity for finding a better local optimum than the current one, and high values disturb the search direction so that it does not permit a local optimum to be found. To study this effect, mutation probability is changed from 0.05 to 0.20, as depicted in Figure 7. By increasing the number of generations (300 generations in this example), a close-to-optimum value for the objective function value was achieved by all experiments. However, the number of BLAs to be evaluated and therefore the computer execution time increase with an increase in the mutation probability (Figure 7). Thus, execution time can be saved by choosing a relatively small value of the mutation probability. For this example, a value of 0.05 to 0.10 is recommended by the results shown in Figure 7.

CONCLUSION

A bilevel formulation was proposed to optimize bike lane facilities from a network viewpoint. All previous approaches considered only a few alternatives for a bike lane project, whereas the presented approach considers all feasible combinations. In a bilevel programming formulation, the upper level is system optimal from a transport authority's perspective, and the lower level is adapted by using car demand and bike demand assignments to predict user behavior. The objective function of the upper level balances the benefits to private car users and cyclists. Two important constraints, budget and bike lane connectivity, were considered at the upper level. An efficient GA algorithm was suggested for solving the bilevel optimization problem. To ensure the challenging constraint of connectivity, special crossover and mutation techniques were suggested.

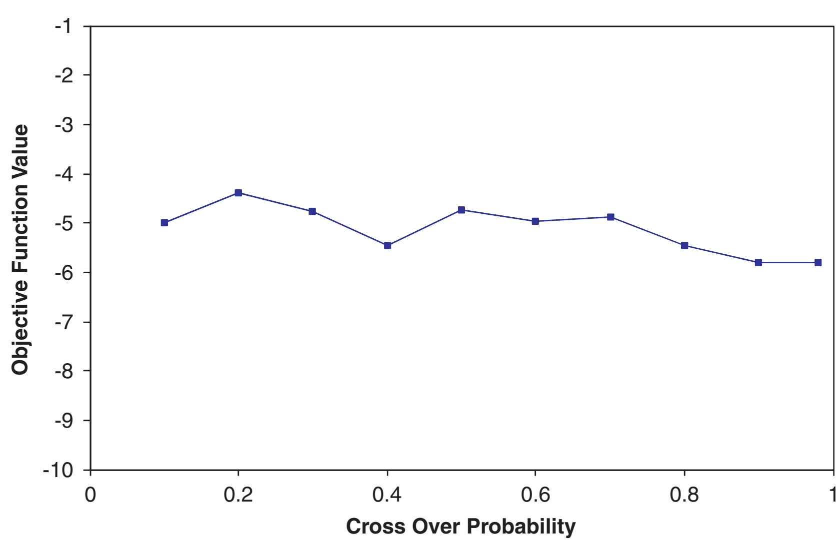

FIGURE 6 Effect of crossover probability on value of objective function.

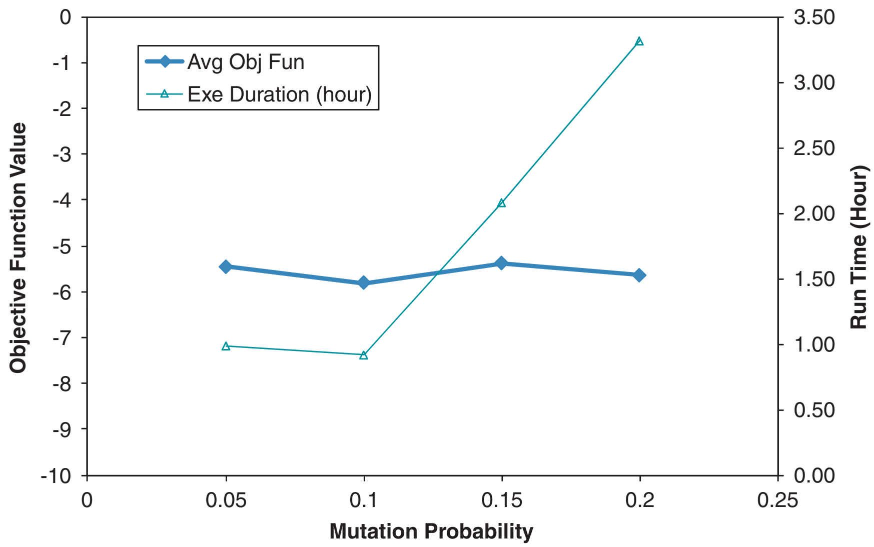

FIGURE 7 Effect of mutation probability on value of objective function (avg obj fun = average objective function; exe = execution).

The method was applied to a medium-sized example network and the results were presented. The proposed method should be tested on a real-scale network with additional practical constraints at the upper level.

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