EVALUATION OF EQUIVALENT POTENTIAL TEMPERATURE (EPT) FROM RADIOSONDE DATA

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ABSTRACT

An approximate expression is derived for the lifting condensation temperature in terms of the dry bulb and dew point temperatures. It is shown that the approximation used in the formula for EPT yields accurate results. The use of dew point temperature in place of the lifting condensation level temperature introduces errors less than 0.5°C in the computed values of EPT.

INTRODUCTION

THE pseudo-equivalent potential temperature (EPT) of a sample of air is the temperature it would attain by ascending pseudo-adiabatically till all the water vapour in it has been condensed and fallen out and then descending dry-adiabatically to the standard pressure level of 1000 mb. Equations for calculation of EPT are available in standard text-books[1-3]. Rao[4] discussed the topic at great length and gave extensive tables for evaluation of EPT.

The routine aerological data furnish values of dry bulb temperature (T) and dew point temperature (T_d) at various pressure levels in the atmosphere. The formula for the evaluation of EPT involves the adiabatic lifting condensation level temperature (T_c) of the moist air, the calculation of which is somewhat involved. In this contribution we show that an approximate expression can be derived for T_c in terms of T and T_d which can be made use of for evaluation of EPT to a high degree of accuracy. We also show that even the crude approximation of using T_d in place of T_c in the formula introduces errors in the computed values of EPT which are less than 0.5°C.

FORMULA FOR THE EVALUATION OF EPT

It is useful to recapitulate briefly the steps in the derivation of the formula for EPT. We start with unsaturated air with total pressure p, vapour pressure e and temperature T. The mixing ratio of the air is given by $x = \varepsilon e/(p - e)$ where $\varepsilon = 0.622$. The entropy (ϕ) of $(1 + x)$ grams of moist air is given by

$$\phi = C_p \log T - R \log (p - e) + (xc) \log T + (Lx)/T, \quad (1)$$

where C_p = specific heat of dry air at constant pressure, R = specific gas constant for dry air, c = specific heat of liquid water and L = latent heat of vaporisation of water at temperature T.

Let θ_d be the partial potential temperature of the dry air defined by

$$\theta_d = T [p_0/(p - e)]^k, \quad (2)$$

where $k = R/C_p = 0.286$ and p_0 stands for the standard pressure of 1000 mb. It is readily seen that

$$\phi = C_p \log \theta_d - R \log p_0 + (xc) \log T + (Lx)/T. \quad (3)$$

Let the moist air ascend adiabatically till it reaches the lifting condensation level (LCL) where the pressure, temperature and saturation vapour pressure are p_c, T_c and e_c respectively. The partial potential temperature of the dry air and the total entropy of $(1 + x)$ grams of air remain unchanged up to the LCL. At the LCL (3) takes the form

$$\phi = C_p \log \theta_d - R \log p_0 + (xc) \log T_c + (L_c x)/T_c, \quad (4)$$

where L_c is the latent heat of vaporisation at the temperature T_c. The variation of latent heat of vaporisation of water with temperature is given by

$$L = 597.3 - 0.566 (T - 273°) \text{ cal g}^{-1}.$$

In view of the occurrence of super-cooled water in the atmosphere up to $-40°C$ we may continue to use this equation even for temperatures below 273°K.

From the LCL the saturated air ascends pseudo-adiabatically till all the water has been condensed and removed from the system; thereafter the air descends dry-adiabatically to the standard pressure level p_0 attaining the temperature θ_e which is the EPT. The entropy of 1 g of the air at the standard pressure level is

$$\phi' = C_p \log \theta_e - R \log p_0. \quad (5)$$

During the pseudo-adiabatic process there is a small decrease in the entropy of the system due to the removal of the water. This is approximately equal to $(xc) \log T_c$. Omitting this term, we can equate the right

sides of (4) and (5) yielding

$$C_p \log \theta_e = C_p \log \theta_d + (L_c x)/T_c$$

or $\qquad \theta_e = \theta_d \operatorname{Exp}\left[(L_c x)/(C_p T_c)\right].$ (6)

This is the formula for the peudo-equivalent potential temperature (θ_e) originally due to Rossby[5].

To evaluate θ_e from radiosonde data we first compute e of the air. This is the saturation vapour pressure at the dew point temperature. In a computer program the Goff-Gratch formula[6] can be employed. Thereafter the computations of x and θ_d are direct. However, the computation of T_c is somewhat involved.

COMPUTATION OF T_c

Since the mixing ratio of the air remains constant up to the LCL, $e/p = e_c/p_c$. The following equations connecting pressure and temperature between the initial level and the LCL hold good for the adiabatic ascent of the unsaturated air:

$$T/p^k = T_c/p_c^k,$$ (7a)

$$T/(p-e)^k = T_c/(p_c - e_c)^k,$$ (7b)

$$T/e^k = T_c/e_c^k,$$ (7c)

where $k = R/C_p = 0.286$. In (7c) the value of the left side is known from the initial values of T and e. On the right side e_c is the saturation vapour pressure at T_c. Since the saturation vapour pressure is a unique function of temperature, the evaluation of T_c consists in locating by iterative procedure the appropriate temperature which satisfies the equality. A graphical method can also be adopted which, however, is not suitable for computer processing.

APPROXIMATE EXPRESSION FOR T_c IN TERMS OF T AND T_d

The specific entropy (ϕ) of dry air of pressure p and temperature T is

$$\phi = C_p \log T - R \log p + \text{constant}.$$ (8)

The *slope of the isobar* on the tephigram is

$$(\partial \phi / \partial T)_p = C_p/T.$$ (9)

Since T varies generally between 300° and 200°K under atmospheric conditions, the slope of an isobar changes only gradually and a small segment of an isobar can be regarded as a straight line to a high degree of approximation.

Along an isohygric the saturation mixing ratio (x_s) is

given by

$$x_s = \varepsilon e_s/(p - e_s) = \text{constant}.$$ (10)

From (10) the following relation follows:

$$p/e_s = 1 + \varepsilon/x_s,$$

or $\qquad \log p = \log e_s + \log(1 + \varepsilon/x_s).$ (11)

From the Clausius–Clapeyron equation

$$\log e_s = -(\varepsilon L)/(RT) + \text{constant}.$$ (12)

Combining (11) and (12) and substituting in (8)

$$\phi = C_p \log T + \varepsilon L/T - R \log(1 + \varepsilon/x_s) + \text{constant}.$$ (13)

From (13) the *slope of the isohygric* on the tephigram is obtained as

$$\left(\frac{\partial \phi}{\partial T}\right)_{x_s} = C_p/T - (\varepsilon L)/T^2 = \frac{1}{T}(C_p - \varepsilon L/T).$$ (14)

Taking $L \sim 600$ cal g^{-1} and $T \sim 300°\mathrm{K}$

$$\varepsilon L/T \sim 1.24 \text{ cal } g^{-1} \,°K^{-1}$$

$$C_p = 0.24 \text{ cal } g^{-1} \,°K^{-1}.$$

Hence $\qquad (\partial \phi / \partial T)_{x_s} \sim -1/T$ (15)

This is approximately the slope of the isohygric on the tephigram. Over a short interval, the isohygric can also be regarded as a straight line.
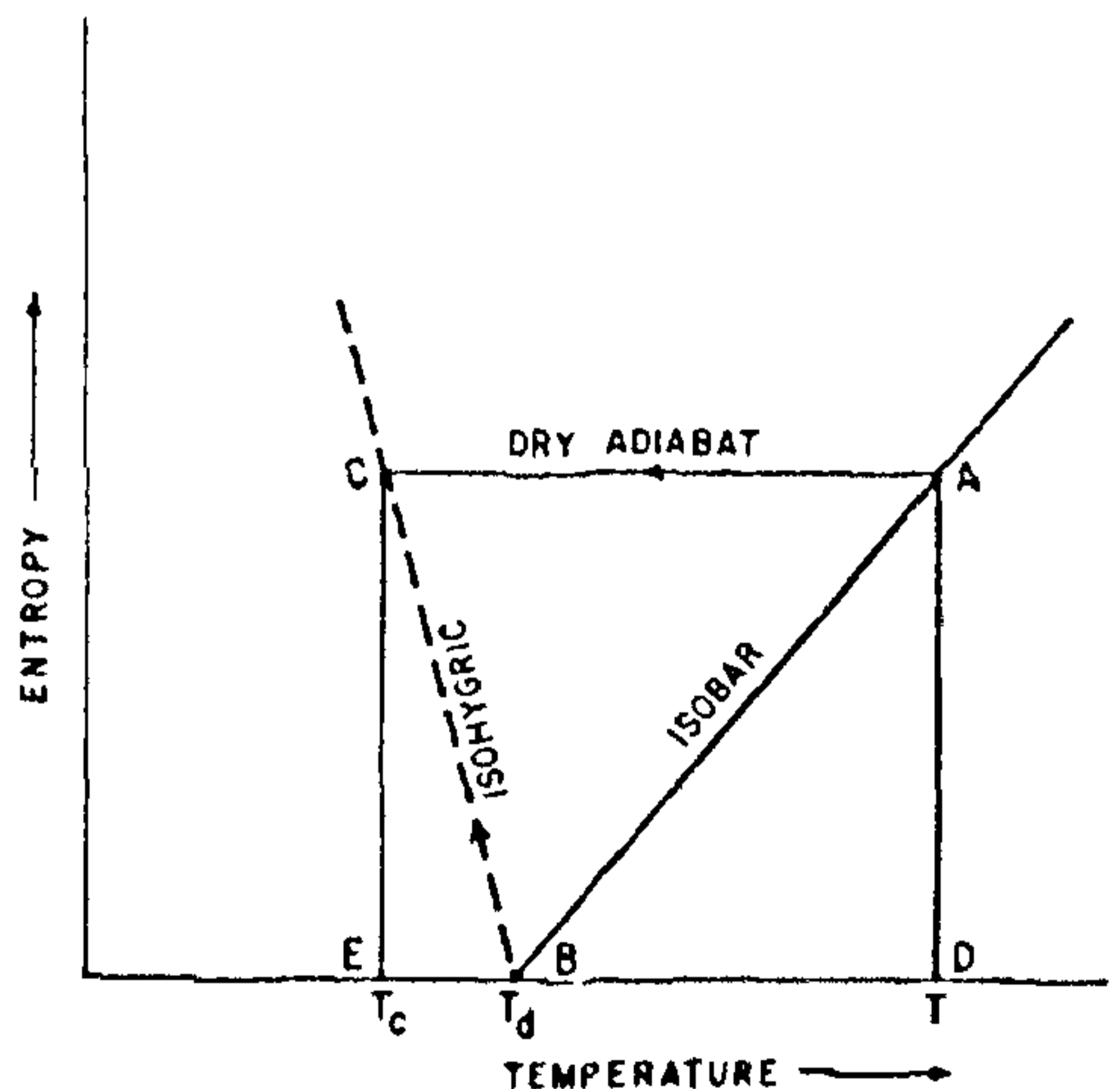

Figure 1.

Referring to figure 1, AB represents a small section of an isobar on the tephigram. The points A and B on the isobar represent the dry bulb temperature (T) and

the dew point temperature (T_d) respectively of a sample of air. BC is the isohygric through B which intersects the dry adiabat through A at the point C. This is the level of condensation of the air ascending from A.

Let T_c be the temperature corresponding to C. We assume that AB and BC are straight lines. From the geometrical construction shown in the diagram we note that the slope of the isobar $AB = C_p/T = AD/BD = AD/(T-T_d)$. Slope of the isohygric $BC = -1/T = -CE/BE = -CE/(T_d-T_c)$. Since $AD = CE$, it follows that

$$1/(T-T_d) = C_p/(T_d-T_c)$$

or

$$T_c = T_d - C_p(T-T_d)$$
$$= T_d - 0.24\,(T-T_d). \qquad (16)$$

This is the approximate equation connecting T_c with T and T_d.

The validity of the approximation can be seen from tables 1(a) and (b) in which are shown the results of computations for two cases. The first case relates to air at 1000 mb with temperature of 308°K and relative humidities varying from 100% to 5%. The dew points of the air samples range from 308° to 263°K and the condensation temperatures from 308° to 254°K. The values of the condensation temperature calculated by the exact formula (7c) are shown against T_c and those obtained from the approximate formula (16) are

shown against T'_c. Note that for relative humidities up to 30% the two values differ by less than 0.5°C. For very low humidities the two values differ by about 2 to 3°C.

The second case shown in table 1(b) relates to air at 700 mb with dry bulb temperature of 283°K and relative humidities varying from 100% to 5%. In this case the T_d values range from 283° to 245°K and the T_c values lie between 283° and 239°K. The two sets of T_c values show similar behaviour as in the first case.

PSEUDO-EQUIVALENT POTENTIAL TEMPERATURE (θ_e)

The last three rows in tables 1(a) and (b) give the values of EPT calculated using formula (6) in which the following values of T_c have been utilised:

(a) Row θ_e: the exact values T_c have been made use of,

(b) Row θ'_e: the approximate values T'_c have been made use of,

(c) Row θ''_e: the dew point temperature T_d have been made use of as a crude approximation.

Note that the values of θ_e and θ'_e differ by less than 0.1°C in all cases. The use of T_d in place of T_c introduces maximum deviation of about 0.6°C between θ_e and θ''_e under surface conditions when the relative humidity is around 50%. The error is much

Table 1 *Comparison of EPT calculated using T_c, T'_c and T_d (Temperatures in °K)*
(a) $P = 1000$ mb, $T = 308°K$ $(= 35°C)$

RH(%)	100	70	50	30	20	10	5
e (mb)	56.233	39.363	28.117	16.870	11.247	5.623	2.812
x (g/kg)	37.061	25.487	17.995	10.673	7.075	3.517	1.754
T_d	308.00	301.69	296.02	287.84	281.70	271.87	262.77
T_c	308.00	300.16	293.21	283.41	276.20	264.89	254.68
T'_c	308.00	300.18	293.14	283.00	275.39	263.20	251.93
θ_e	418.29	382.79	360.71	339.60	329.28	318.95	313.67
θ'_e	418.29	382.78	360.72	339.65	329.35	319.03	313.75
θ''_e	418.29	382.27	360.05	338.98	328.76	318.61	313.46

(b) $P = 700$ mb, $T = 283°K$ $(= 10°C)$

RH(%)	100	70	50	30	20	10	5
e(mb)	12.271	8.590	6.135	3.681	2.454	1.227	0.614
x(g/kg)	11.098	7.728	5.500	3.288	2.188	1.092	0.546
T_d	283.00	277.78	273.06	266.22	261.07	252.74	245.01
T_c	283.00	276.66	270.99	262.92	256.94	247.45	238.79
T'_c	283.00	276.53	270.67	262.19	255.81	245.48	235.89
θ_e	346.95	337.06	330.49	323.89	320.61	317.10	315.36
θ'_e	346.95	337.07	330.51	323.92	320.66	317.13	315.39
θ''_e	346.95	336.94	330.33	323.73	320.40	317.01	315.26

less for low and high values of relative humidity and also at upper levels where the moisture content of the air is less.

It is of interest to examine why the approximation of using T_d in place of T_c in formula (6) yields values of θ_e which are very close to the correct values. Keeping all the parameters except T_c constant in formula (6), the change in the value of θ_e for a change in T_c is

$$\delta\theta_e = -[(L_c\theta_e)/(C_pT_c^2)](x\delta T_c). \qquad (17)$$

Under atmospheric conditions the following values are typical for the parameters within the square brackets:

$$L_c \sim 600 \text{ cal g}^{-1},$$

$$C_p = 0.24 \text{ cal g}^{-1} \,^\circ K^{-1},$$

$$\theta_e \sim 300^\circ \text{ to } 400^\circ K,$$

$$T_c \sim 300^\circ \text{ to } 200^\circ K.$$

With these values, it is easily seen that the expression within the square bracket varies between 8 and 25. Let us assume that $\delta T_c = T_d - T_c$. From tables 1(a) and 1(b) it can be seen that the largest values of δT_c are of the order of 7 to 8°C when the relative humidity is 5%.

The error $\delta\theta_e$ depends on the product $(x\delta T_c)$. It may be noted that when the air is saturated $\delta T_c = 0$ and when the air is dry $x = 0$. Thus at both the extremes $(x\delta T_c) \rightarrow 0$. The largest value of this quantity occurs near relative humidity of 50%. This value is found to be about 0.05 in table 1(a) and .01 in table 1(b). With the values in table 1, the magnitude of the quantity inside the square brackets is found to be about 10. Thus the maximum value of $\delta\theta_e$ is about 0.5°C under

surface conditions and much less for conditions at higher levels.

CONCLUSION

The computation of EPT using the approximate value of lifting condensation level temperature in terms of T and T_d yields accurate values. This procedure is convenient for use in a computer program. The use of T_d in place of T_c introduces errors of less than 0.5°C in the computed values of EPT.

ACKNOWLEDGEMENTS

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NEWS

SYMPOSIUM ON MICROSTRUCTURE OF CEMENT AND CONCRETE

A two-day conference on Microstructure of Cement and Concrete is being organised by the Materials and Testing Group of The Institute of Physics jointly with the University of Leeds and The Institute of Metals at Tetley Hall, University of Leeds on 24 and 25 September 1985.

Five major themes have been identified— 1. Development of microstructure in traditional cement pastes, 2. The influence of cement replace-

ments *e.g.* PFA, BFS on microstructure development. 3. The influence of chemical admixtures on microstructure development. 4. Novel techniques for assessment of microstructure. 5. Degradation/enhancement of microstructure—Alkali aggregate reaction, corrosion cabonation.

For further information apply to The Meeting Officer, The Institute of Physics, 47, Belgrave Square, London SWIX 8QX. UK.