NUMERICAL SIMULATION OF CLOUD SEEDING

EXPERIMENTS IN MAHARASHTRA STATE, INDIA

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ABSTRACT

Two numerical cloud seeding simulation experiments of 5-, 8- and 10-years duration, with a double area cross-over design and area randomization, were performed using historic rainfall data for the Deccan plateau region in Maharashtra state.

The first numerical experiment (EXP-TR) used the simulation technique of Twomey and Roberstson (1973), second (EXP-MMM) used different simulation technique proposed in the present study. The results of the two numerical experiments have agreed closely. The EXP-MMM technique not only reduces computational time by an order of magnitude but also defines the exact lower limit for the double ration value which can be detected at 5 per cent level of significance.

The results of the numerical experiments suggest that 15 and 20 percent increases in rainfall due to seeding in Maharahtra could be detected, with 80 percent or more probability, in 5 years. In a 10-year experiment the probabilities of detecting 5 and 10 percent increases in rainfall due to seeding are 27 and 65 percent, respectively.

1. INTRODUCTION

Numerical simulation of cloud seeding experiments using the historic rainfall data of any region can indicate the chances of detecting a prescribed increase in rainfall due to seeding with a specified degree of confidence. Such experiments were carried out for selected areas in Australia (Twomey and Robertson, 1973; Smith and Shaw, 1976). They require a great deal of computer time even on high-speed modern computers. Such investigations have not been undertaken in India so far.

Two numerical simulation experiments for Maharashtra State (Figure 1) were performed using historic rainfall data for the period 1951-1960. The first experiment (EXP-TR) is based on the simulation technique of Twomey and Robertson (1973). The second experiment (EXP-MMM) uses the numerical methodology proposed in the present study, which reduces the computational time by about an order of magnitude. Also, EXP-MMM defines the lower limit of the double ratio value for the detection of the seeding effect at the
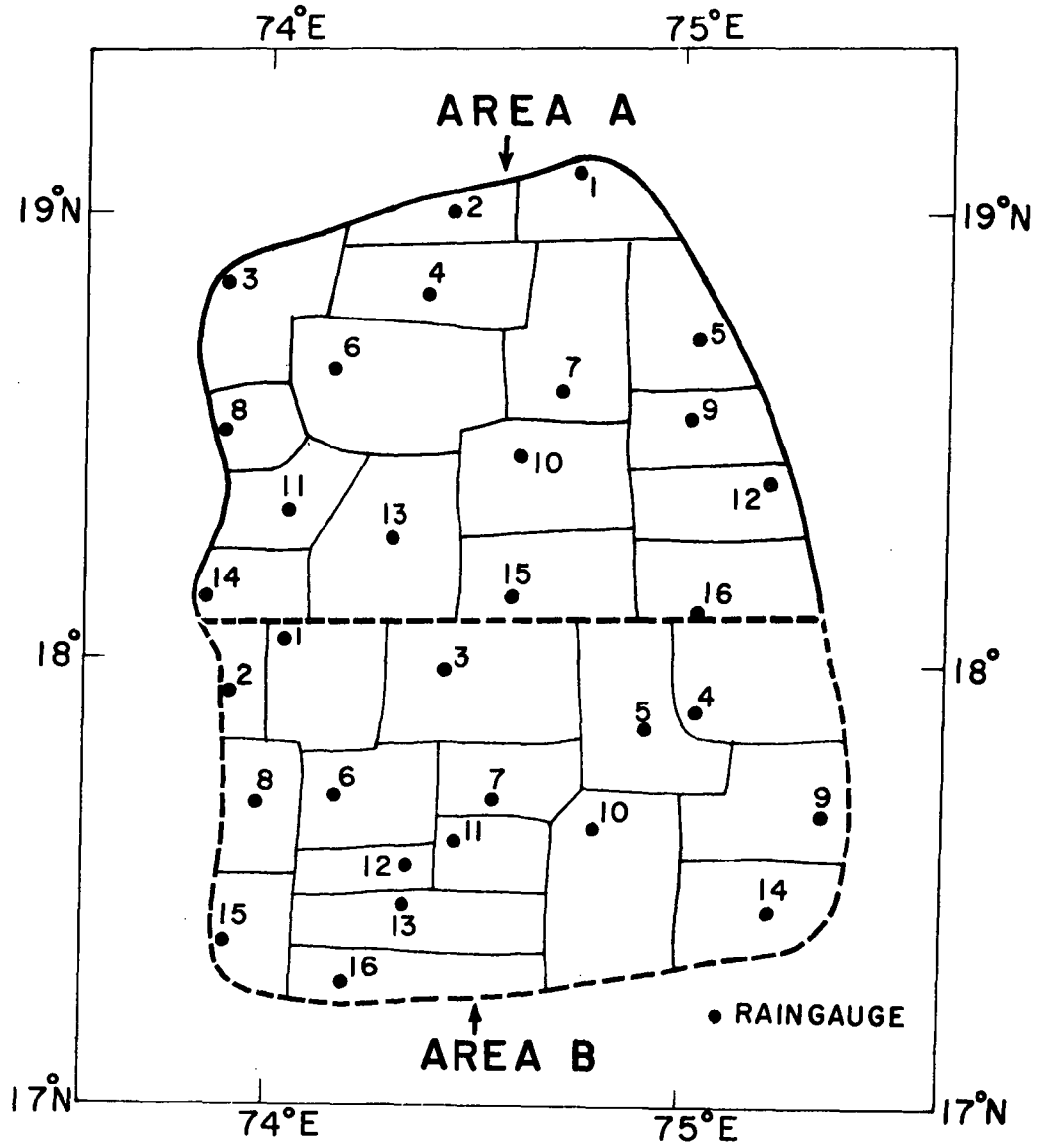
Figure 1 : Areas of the numerical experiment and the raingauge net-work

5 percent level of significance.

2. EXPERIMENTAL AREA AND RAIN GAUGE NETWORK

The area of the numerical experiment (the 32 rain gauge stations are shown in Figure 1) is on the lee side of the western ghats in Maharashtra State. The average elevation of the area is about 66 m A.S.L. The total area was divided into Areas A and B, with 16 rain gauges in each. The representative area considered for each rain gauge is also shown in the figure. Area-weighted average rainfall for the two areas was computed and used.

3. METEOROLOGICAL CONDITIONS

The characteristics of the summer monsoon circulation and monsoon rainfall have been reviewed by Ananthakrishnan, (1977). Westerly air flow in the lower troposhere during the summer monsoon (June to September) brings a large influx of moisture inland from the Arabian Sea. Synoptic and mesoscale disturbances leading to low level convergence, and vertical motion of the humid air give rise to continuous to intermittant rain from stratiform clouds. The annual rainfall in the area varies from 30 to 75cm, of which about 75 percent falls during the summer monsoon season.

4. NUMERICAL EXPERIMENTS

The two numerical experiments, EXP-TR and EXP-MMM were simulated for double-area cross-over design with area randomization and performed for 5-, 8- and 10-year durations using the Robotron EC-1040 computer. The simulated assigned increases in rainfall due to seeding were 5, 10, 15 and 20 percent.

The 5-year experiments (1951-1955) used 1-day and 7-day rainfall periods from the second week of June to the last week of September, adopting the numerical techniques of the EXP-TR and EXP-MMM. The 8-year experiments (1951-1958) used 7-day rainfall periods from the second week of June to the last week of September, adopting the numerical techniques of EXP-TR and EXP-MMM. The 10-year experiments (1951-60) used 1-month rainfall periods from June to September, adopting the numerical techniques of EXP-TR and EXP-MMM.

The details of the rainfall data used in the above experiments and the relevant statistical parameters of the rainfall distributions for the two areas A and B are given in Table 1. The effect of possible trends in rainfall records for different periods has not been examined in the present study.

Table 1

Details of rainfall data used in the numerical experiments

Experiment duration	Rainfall period	Total number of rainfall periods	Mean rainfall for each period in mm		Correlation coefficiant (r) between the rainfall of Area A and Area B
			Area A	Area B	
5-years (1951-55)	1-day	560	3.1 (5.1)	3.3 (4.6)	0.65
5-years (1951-55)	7-day	80	20.3 (20.7)	23.5 (18.9)	0.78
8-years (1951-58)	7-day	128	22.2 (22.6)	25.8 (21.5)	0.81
10-years (1951-60)	1-month	40	101.3 (52.9)	116.1 (54.1)	0.86

Foot-note : Figures in brackets indicate the standard deviations

5. NUMERICAL EXPERIMENT EXP-TR

EXP-TR is based on the numerical simulation technique of Twomey and Robertson (1973). In the main simulation experiment (main-experiment) the historic rainfall data corresponding to the model experiment (e.g. in respect to the assigned increase in rainfall due to seeding, period length etc.) were used. Each main-experiment consisted of 100 sub-experiments.

The simulation of the hypothesized effect of seeding in the main-experiment was done according to the following procedure:

(i) A random seeding sequence was generated by taking a random series of zeros and ones and alternating it with a second series by changing zeros to ones to zeros. This procedure was adopted for the simulation of equal number of seeded and non-seeded periods in the numerical experiment.

(ii) Area A or area B was designated as seeded or non-seeded for the 'kth' period according to whether the 'kth' number in the random seeding sequence was zero or one.

(iii) The hypothesized effect of seeding was simulated by altering the seeded area rainfall by an assigned percentage increase (PERC) according to the random seeding sequence. The increase in rainfall due to seeding was normally distributed among the raingauge stations in the area.

(iv) If the seeded area consisted of 'q' raingauge stations, each station was assigned a random increase in rainfall due to seeding such that the mean increase in the seeded area rainfall was equal to the assigned hypothesized percentage increase, with a standard deviation of 1 percent. The standard deviation of 1 percent was chosen assuming that in the real experiment the increase in rainfall would be uniform over a large area. This simple assumption may not hold good in a real experiment.

The change in rainfall due to seeding is estimated by the double ratio:

$$S_E = \left[\frac{\sum A_S}{\sum B_{NS}} \times \frac{\sum B_S}{\sum A_{NS}} \right]^{\frac{1}{2}} \quad (1)$$

where the subscript S refers to the seeded rainfall and N to the non-seeded rainfall for the two areas A and B.

The above procedure was repeated for each main-experiment using different random seeding sequences. The number of main-esperiments conducted was between 59 and 100 for 7-day and 1-month rainfall periods. However, for experiments performed using 1-day rainfall periods the number of main-experiments was restricted to 25 because of the enormous computer time involved.

The significance of the double ratio (S_E) values obtained from each main-experiment was then evaluated by performing 100 further sub-experiments. A re-randomization scheme was used in the sub-experiments.

For each sub-experiment the rainfall data, modified by the assigned seeding effect in the main-experiment described above, were used. Thus the data sample used in the sub-experiments consisted of the rainfalls in non-seeded periods together with the seeded periods' rainfalls as modified by the assigned seeding effects. Using the above rainfall data and a different random seeding sequence for each sub-experiment the double ratio values (S_{PERC}) were computed. The S_{PERC} values corresponding to the 5, 10, 15 and 20 percent assigned increase (PERC) in rainfall due to seeding are designated respectively as S_5, S_{10}, S_{15} and S_{20}.

The S_{PERC} distribution was then used to evaluate the probability of occurrence of S_E. The significance of S_E is given by the probability of occurrence of a S_{PERC} value to S_E.

For an increase in rainfall due to seeding (i.e., for $S_E > 1$) the significance of S_E is given by the total number of cases with $S_{PERC} \geqslant S_E$ in the 100 sub-experiments. Similarly for a decrease in rainfall due to seeding ($S_E < 1$) the significance of S_E is given by the number of cases when $S_{PERC} \leqslant S_E$ in the 100 sub-experiments.

Alternately the significance of S_E can be obtained from the cumulative percentage frequency of S_{PERC} distribution at $S_{PERC} = S_E$. In the case of an increase in rainfall due to seeding, the cumulative percentage frequency was evaluated starting from the highest value of S_{PERC}. This cumulative percentage frequency distribution is from hereafter designated as $S_{PERC(MSX)}$. Similarly in the case of a decrease in rainfall due to seeding, the cumulative percentage frequency was evaluated starting from the minimum value of S_{PERC}. This cumulative percentage frequency distribution is from hereafter designated as $S_{PERC(MIN)}$.

This method of evaluating the significance of S_E does not involve assumptions about the nature of the distribution of S_{PERC}, i.e. whether normal or otherwise (Twomey and Robertson, 1973).

Any increase in rainfall due to seeding in the main-experiment is counted as a detection if the S_E value is significant at 5 percent level or better. In the present numerical experiment (EXP-TR) the significance of any single value of S_E was obtained by performing 100 sub-experiments, The number of detections in 100 main-experiments will give the percentage probability of detection of any one model experiment of a given duration and any given

assigned increase in rainfall due to seeding. Thus the numerical experiment EXP-TR involves a great deal of computations requiring a total number of 100 x 100 sub-experiments for any one model experiment.

6. NUMERICAL EXPERIMENT EXP-MMM

As shown in the previous section EXP-TR involves a great deal of computer time. Therefore a simple method which can reduce the computer time without compromising the scientific value of the results has been developed and tested.

For an experiment with a duration of 'k' periods, let A_1, A_2, A_3 .... A_K and B_1, B_2, B_3 ....B_K represent the period total rainfalls for the areas A and B respectively. For any one particular random seeding sequence of length of 'k', a double ratio value S_N can be obtained as before:

$$S_N = \left[\frac{\Sigma A_S}{\Sigma B_{NS}} \times \frac{\Sigma B_S}{\Sigma A_{NS}} \right]^{\frac{1}{2}} \tag{2}$$

The distribution of S_N was obtained starting from the <u>historic</u> rainfall data without any assigned increase in rainfall due to seeding. In the present numerical experiment 1000 values of S_N were obtained using 1000 different random seeding sequences. From the distribution of S_N the cumulative percentage frequency distributions of $S_{N(MAX)}$ and $S_{N(MIN)}$ were obtained starting respectively from the maximum and minimum values of S_N.

The significance of S_E values of EXP-TR can be directly determined using the S_N distribution of EXP-MMM without performing the sub-experiments of EXP-TR. Since the significance of S_E values is determined by the cumulative percentage frequency distribution of S_{PERC}, the cumulative percentage frequency distribution of S_N and S_{PERC} distributions were compared. For this purpose the chi-square test (Spiegel, 1961) was applied to the following distributions:

(i) $S_{N(MAX)}$ distribution of EXP-MMM and $S_{PERC(MAX)}$ distribution of EXP-TR.

(ii) $S_{N(MIN)}$ distribution of EXP-MMM abd $S_{PERC(MIN)}$ distribution of EXP-TR.

The results of the chi-square test (Table 2) show that the above pairs of distributions are not statistically different. The comparison was made starting from a value of $S_{N(MAX)}$ and $S_{N(MIN)}$ for which the cumulative percentage frequency value was at least 5, since the chi-square test cannot be applied for smaller frequencies. This limitation does not alter significantly the percentage probability of detections (Table 4).

The percentage significance of S_E values as obtained from EXP-TR and as obtained from the S_N distribution of EXP-MMM for the 5-year model experiment performed using the 1-day rainfall periods are plotted in Figure 2. The values obtained from the two numerical experiments agree closely, particularly for the significance levels ranging from 5 to 0 percent which correspond to the range of significance levels for the detection of S_E values.

This statistical analysis shows that 100 sub-experiments are not required for evaluating the significance of any one value of S_E obtained in the main experiment of EXP-TR. The significance of any S_E value can be directly obtained from the $S_{N(MAX)}$ distribution for values of $S_E > 1$ and from $S_{N(MIN)}$ distribution for values of $S_E < 1$. The number of values of $S_E > 1$ significant at 5 percent level gives the percentage probability of detection for any assigned percentage increases in rainfall due to seeding.

Alternatively, the percentage probability of detections may also be evaluated by a further shorter method, in which the S_E values of EXP-TR can be directly obtained from the S_N values of EXP-MMM.

For any one particular random seeding sequence of length 'k' the value of S_N was obtained from Equation (2). The experiment was then repeated with the same random sequence but with the modified rainfall data due to the assigned increase in rainfall (PERC) due to seeding. The double ratio value was obtained as explained in the following example.

For a 10 percent assigned increase in rainfall (PERC) due to seeding, the double ratio can be expressed as:

$$S_E = \left[\frac{(1.1) \ \Sigma \ A_S}{\Sigma \ B_{NS}} \ x \ \frac{(1.1) \ \Sigma \ B_S}{\Sigma \ A_{NS}} \right]^{\frac{1}{2}} \tag{3}$$

Equation (3) can be expressed as $S_E = 1.1 \ S_N$.

Table   2

Comparison of cumulative percentage frequency distribution of S_N and S_{PERC}

Duration of Experiment	Range of double ratio (S_N)	Comparison of $S_{N(MAX)}$ with $S_{PERC(MAX)}$ Chi-Square value				Range of double ratio (S_N)	Comparison of $S_{N(MIN)}$ with $S_{PERC(MIN)}$ Chi-Square values			
		S_5 (MAX)	S_{10} (MAX)	S_{15} (MAX)	S_{20} (MAX)		S_5 (MIN)	S_{10} (MIN)	S_{15} (MIN)	S_{20} (MIN)
5-years (1951-55) with 1-day rainfall periods	1.10-0.85 (26)	0.08	0.24	0.70	1.44	0.91-1.00 (10)	0.12	0.38	1.02	1.58
	1.10-1.00 (11)	0.06	0.21	0.59	1.27					
5-years (1951-55) with 7-day rainfall periods	1.13-0.82 (32)	0.36	2.07	6.15	12.31	0.89-1.00 (12)	0.18	0.73	2.58	6.20
	1.13-1.00 (14)	0.30	1.87	5.66	11.04					
8-years (1951-58) with 7-day rainfall periods	1.11-0.85 (27)	0.15	0.48	1.55	2.69	0.91-1.00 (10)	0.34	1.62	3.35	4.28
	1.11-1.00 (12)	0.14	0.24	0.92	1.80					
10-years (1951-60) with 1-month rainfall periods	1.08-0.87 (22)	0.52	2.15	5.90	14.50	0.92-1.00 (9)	0.80	4.03	14.02	
	1.08-1.00 (9)	0.39	1.53	3.80	10.10	0.95-1.00 (6)				9.90

Foot-note :   Figures in brackets indicate the number of pairs for which the comparison was made

Table 3

Comparison of $S_{E(MAX)}$ distributions obtained from EXP-TR and EXP-MMM using Chi-Square test

| | Assigned percentage (PERC) increase in rainfall due to seeding | | | | | | | |
| | 5 per cent | | 10 per cent | | 15 per cent | | 20 per cent | |
	Range of the double ratio (S_E)	Chi-Square value	Range of the double ratio (S_E)	Chi-Square value	Range of the double ratio (S_E)	Chi-Square value	Range of the double ratio (S_E)	Chi-Square value
5-years (1951-55) with 1-day rainfall periods	1.15-0.97 (19)	11.16	1.19-1.02 (18)	21.02	1.24-1.06 (19)	20.02	1.29-1.11	25.90
5-years (1951-55) with 7-day rainfall periods	1.19-0.90 (30)	6.41	1.24-0.93 (32)	13.63	1.30-0.95 (36)	8.49	1.36-0.97 (40)	6.13
8-years (1951-58) with 7-day rainfall periods	1.15-0.95 (21)	25.87	1.21-0.99 (23)	20.20	1.26-1.03 (24)	23.11	1.32-1.08 (25)	22.87
10-years (1951-60) with 1-month rainfall periods	1.14-0.94 (21)	9.23	1.20-0.98 (23)	6.92	1.25-1.03 (23)	11.25	1.31-1.07 (25)	8.07

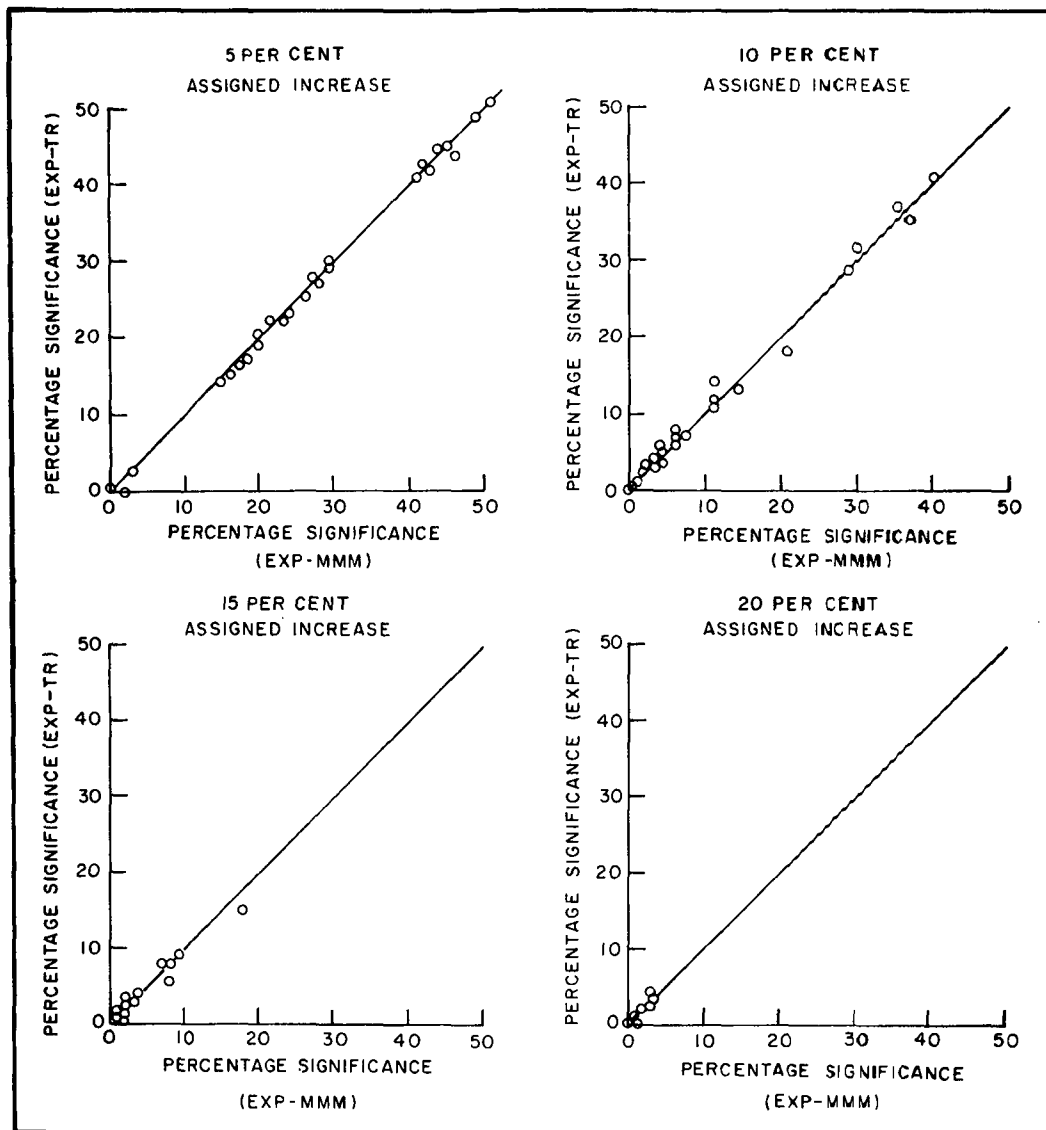Foot-note : Figures in brackets indicate the number of pairs for which the comparison was made

Figure 2 : Percentage significance of the double ratio
(S_E) values obtained using the numerical
simulation techniques of EXP-TR and EXP-MMM
for the 5-year (1951-55) experiments with
1-day rainfall periods (see Section 6.0 of
the text for full explanation)

Similarly for any assigned percentage increase in rainfall (PERC) due to seeding Equation (3) can be expressed as:

$$S_E = \left[1 + \frac{PERC}{100} \right] \qquad (4)$$

The $S_{E(MAX)}$ distribution can be derived from the $S_{N(MAX)}$ distribution using the Equation 4 as shown by an example in Section 8.0.

For an increase in rainfall due to seeding (i.e., for $S_E > 1$) the lowest value of S_E which can be detected at the 5 percent level of significance is denoted as S_D. The value of S_D can be readily obtained from the $S_{N(MAX)}$ distribution. The value of S_N for which the cumulative frequency is 5 will give the value of S_D (Figures 3 and 4).

From the $S_E(MAX)$ distribution the cumulative percentage frequency of S_E at $S_E = S_D$ will give the percentage probability of detection of the assibned percentage increase (PERC) in rainfall due to seeding.

Thus the numerical methodology of EXP-MMM will reduce the computation time required for EXP-TR, since it eliminates the need for 10^4 sub-experiments to obtain the significance of 100 values of S_E of the main-experiments of EXP-TR for any one model experiment.

7. COMPARISON OF RESULTS OF EXP-TR AND EXP-MMM

The distributions of (i) $S_{E(MAX)}$ obtained from EXP-TR, (ii) $S_{N(MAX)}$, $S_{N(MIN)}$ and $S_{E(MAX)}$ obtained from EXP-MMM of the 5-year (1951-55) and 10-year (1951-60) experiments are shown respectively in Figures 3 and 4.

In Figures 3 and 4 the right half of the peaked graph gives the $S_{N(MAX)}$ distribution and the left half of the peaked graph gives the $S_{N(MIN)}$ distribution. The peaked graph is symetrical on either side of the ordinate at $S_N = 1.0$. As mentioned in Section 6, the $S_{N(MAX)}$ and $S_{N(MIN)}$ obtained from EXP-MMM give the significance levels of values of $S_E > 1$ and of $S_E < 1$ respectively.

The crosses in Figures 3 and 4 represent the significance of S_E values as obtained from EXP-TR. The $S_{N(MAX)}$ and $S_{N(MIN)}$ curves appear to be a good fit for the crosses i.e., the significance levels of S_E as obtained from
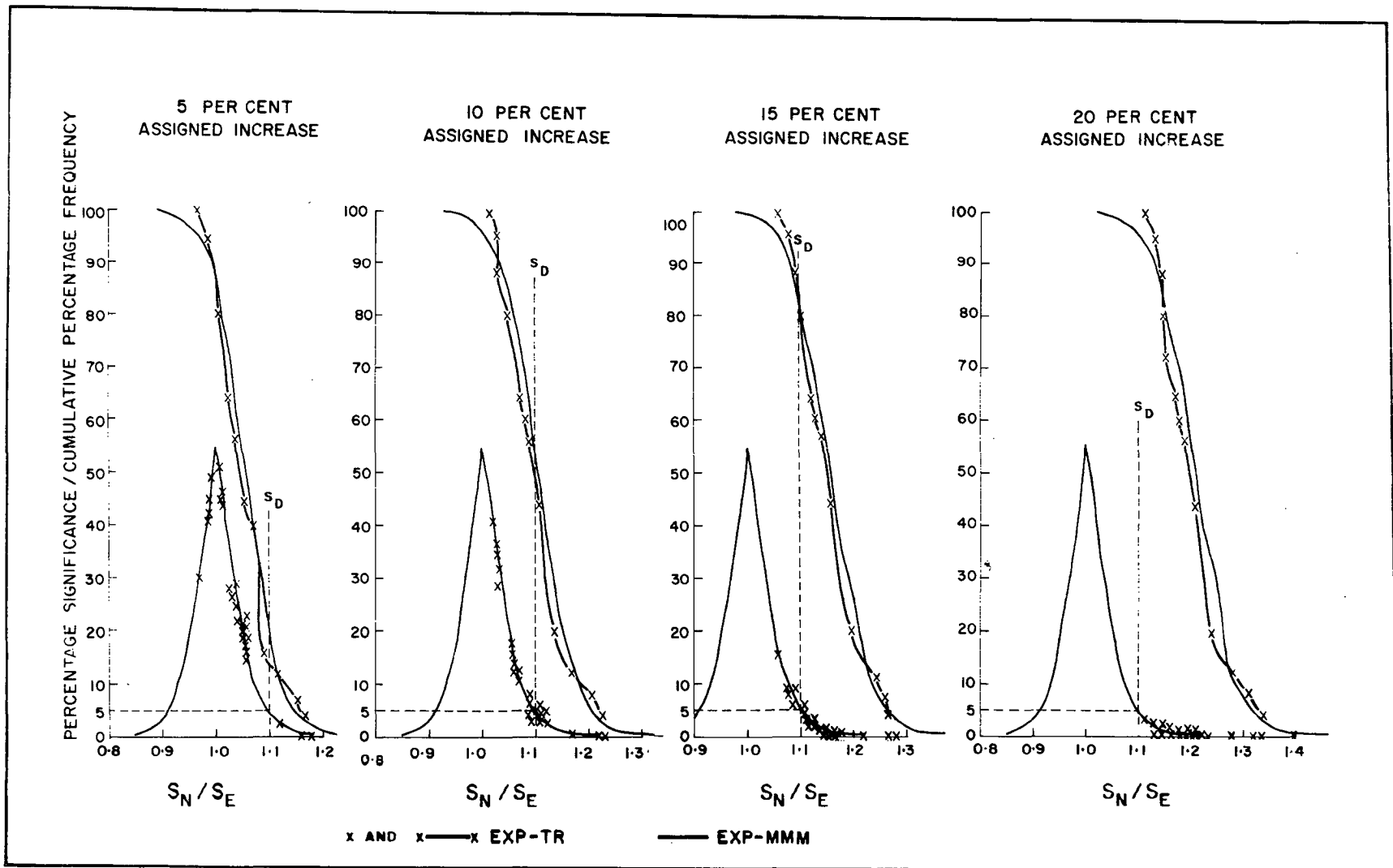
Figure 3 : Distributions of (i) $S_{E(MAX)}$ of EXP-TR, (ii) $S_{N(MAX)}$, $S_{N(MIN)}$ and $S_{E(MAX)}$ of EXP-MMM obtained from the 5-year (1951-55) experiments with 1-day rainfall periods. The right half and the left half of the peaked graph, which is symmetrical on either side of the ordinate at $S_N = 1.0$, represent the $S_{N(MAX)}$ and $S_{N(MIN)}$ distributions respectively. The crosses represent the significance of the S_E values of EXP-TR. The $S_{E(MAX)}$ distributions as obtained from EXP-TR and EXP-MMM are shown to the right of the peaked graph. S_D is the lower limit of the double ratio for detection at 5 per cent significance level. The standard deviations of $S_5 = 0.001 \times S_N$; $S_{10} = 0.002 \times S_N$; $S_{15} = 0.003 \times S_N$; $S_{20} = 0.004 \times S_N$ (see Section 7.0 of the text for full explanation)
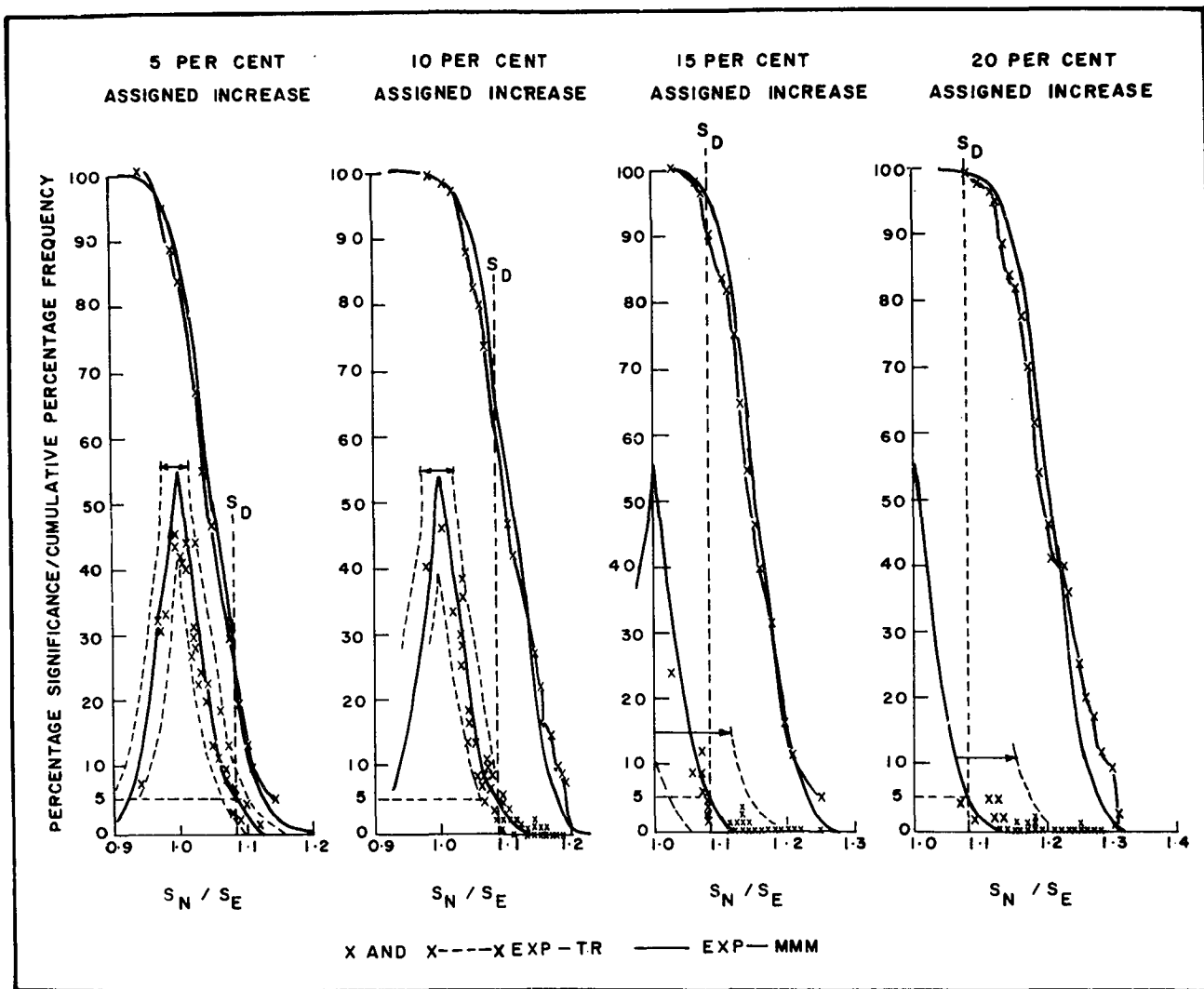
Figure 4 : Same as Figure 3 for the 10-year (1951-60) experiments with 1-month rainfall periods. Two standard deviations are shown on either side of S_N curve by arrows and dashed line

EXP-TR. Since there is a good fit between the results obtained from the two numerical experiments, it can be concluded that the significance of S_E values as obtained from EXP-MMM are not significantly different from those of EXP-TR.

The $S_{E(MAX)}$ distributions as obtained from EXP-TR and EXP-MMM are shown to the right of the peaked graph in Figures 3 and 4. The chi-square test was used to compare the $S_{E(MAX)}$ distributions obtained from the two numerical experiments. The results (Table 3) show that the two distributions are not significantly different.

The two distributions were compared starting from $S_{E(MAX)}$ for which the cumulative percentage frequency value was at least 5, since the chi-square test cannot be applied for smaller frequencies. Also, the percentage probability of detection in the present numerical experiments is always above this value and within the range for which the comparison had been made.

For an increase in rainfall due to seeding (i.e., for $S_E > 1$) the lowest value of S_E which can be counted as a detection (S_D) is also shown in Figures 3 and 4 by a dashed vertical line. The S_D value as explained in Section 6 is obtained from the value of $S_{N(MAX)}$ for which the cumulative percentage frequency is 5.

The value of S_D for any given case can be readily obtained, from Figures 3 and 4 by the ordinate through the point of inter-section of the $S_{N(MAX)}$ curve with the abscissa corresponding to the cumulative percentage frequency value of 5. The ordinate and the abscissa corresponding to S_D are shown by the dashed lines in Figures 3 and 4.

The results obtained from EXP-TR and EXP-MMM are given in Table 4. The percentage probability of detections as obtained from (i) EXP-TR, (ii) EXP-MMM using the S_D value and (iii) $S_{E(MAX)}$ of EXP-TR and S_D value are also given in the Table. The three sets of values agree well. In the case of the 5-year model experiments performed using the 1-day rainfall periods, the percentage probability of detections for any assigned increase in rainfall due to seeding was higher compared to the value obtained in the case of the 5-year model experiments performed with the 7-day rainfall periods. This is due to the large number of seeded and non-seeded periods in the case of the former experiments. This result is in agreement with that of Smith and Shaw (1976).

As already mentioned, for limiting the computer time, the percentage number of detections in the 5-year (1951-55) experiment (EXP-TR) with 1-day rainfall periods was inferred from 25 instead of the ideal number of 100 main-experiments. With this approximation, even the small deviations would be magnified especially when the number of detections was very small as in the cases of 5 and 10 percent increases in rainfall due to seeding (Table 4).

Table   4

Percentage number of detections obtained from EXP-TR and EXP-MMM
for different model experiments

Assigned percentage (PERC) increase in rainfall due to seeding/ S_D value	Percentage number of detections											
	5-year duration (1951-55) 1-day rainfall periods			5-year duration (1951-55) 7-day rainfall periods			8-year duration (1951-58) 7-day rainfall periods			10-year duration (1951-60) monthly rainfall periods		
	EXP-TR	EXP-MMM	EXP-TR*	EXP-TR	EXP-MMM	EXP-TR*	EXP-TR	EXP-MMM	EXP-TR*	EXP-TR	EXP-MMM	EXP-TR*
5	12 (25)	21	22	17 (59)	17	17	33 (100)	20	27	25 (60)	27	23
10	44 (25)	55	50	34 (59)	37	40	57 (100)	50	56	62 (76)	65	59
15	80 (25)	80	84	61 (59)	67	60	88 (100)	81	84	90 (60)	93	90
20	100 (25)	95	100	83 (59)	88	80	100 (100)	98	98	100 (76)	100	99
S_D value	1.100	1.100		1.125	1.125		1.110	1.110		1.085	1.085	

Foot-note : EXP-TR* :   The number of detections in this case has been evaluated using the cumulative percentage frequency distribution of S_E of EXP-TR and the S_D value obtained from EXP-MMM (See Sections 6.1 and 6.2 for full explanation). S_D is the lowest value of the double ratio (S_E which can be detected at 5 per cent level of significance)

This would give rise to differences in the percentage number of detections obtained from EXP-TR and EXP-MMM.

In other experiments, in majority of the cases, the percentage probability of detections as obtained from EXP-MMM appear to be slightly higher (up to a maximum of 7 percent) than those obtained from EXP-TR (Table 4). Possible reasons for the differences are discussed in the following section.

8. ESTIMATION OF THE SCATTER OF THE SIGNIFICANCE LEVELS OF S_E AS OBTAINED FROM EXP-MMM

Any particular S_E value obtained from EXP-TR has a range of significance values (Figures 3 and 4). This is due to the modified rainfall values used to determine the S_{PERC} values in the sub-experiments of EXP-TR (Section 5). Hence the significance levels as obtained from EXP-TR show a scatter around the $S_{N(MAX)}$ and $S_{N(MIN)}$ curves obtained from EXP-MMM. A numerical method for the evaluation of the range of the above scatter is now described:

A sample calculation for a model experiment with a duration of 12 rainfall periods and a 10 percent assigned increase in rainfall due to seeding is presented. Details of (i) the period totals of historic rainfalls for the two areas A and B for the above model experiment, (ii) the two random seeding sequence, one for the main-experiment and the other for the sub-experiment of EXP-TR and (iii) the modified rainfall values resulting from the main-experiment of EXP-TR are given in Table 5. The S_E value of the main-experiment of EXP-TR (Section 5) can be expressed as:

$$S_E = \left[\frac{(A_1+A_4+A_5+A_7+A_9+A_{12})\ (1.1)}{(B_1+B_4+B_5+B_7+B_9+B_{12})} \times \frac{(B_2+B_3+B_6+B_8+B_{10}+B_{11})\ (1.1)}{(A_2+A_3+A_6+A_8+A_{10}+A_{11})} \right]^{\frac{1}{2}} \quad (5)$$

The double ratio (S_N), using historic rainfall data without any superimposed assigned increase and using the same random seeding sequence of the main-experiment of EXP-TR, can be expressed as:

$$S_N = \left[\frac{(A_1+A_4+A_5+A_7+A_9+A_{12})}{(B_1+B_4+B_5+B_7+B_9+B_{12})} \times \frac{(B_2+B_3+B_6+B_8+B_{10}+B_{11})}{(A_2+A_3+A_6+A_8+A_{10}+A_{11})} \right]^{\frac{1}{2}} \quad (6)$$

Table 5

Simulation procedure of a model numerical experiment with 12 rainfall periods
and a 10 per cent assigned increase in rainfall due to seeding

Rainfall Period	Historic Rainfall of Area A	Historic Rainfall of Area B	Random seeding sequence used in the main-experiment of EXP-TR	Rainfall of Area A as modified in the main-experiment of EXP-TR	Rainfall of Area B as modified in the main-experiment of EXP-TR	Random seeding sequence used in the first sub-experiment
1	A_1	B_1	1	$(1.1)A_1$	B_1	0
2	A_2	B_2	0	A_2	$(1.1)B_2$	1
3	A_3	B_3	0	A_3	$(1.1)B_3$	1
4	A_4	B_4	1	$(1.1)A_4$	B_4	0
5	A_5	B_5	1	$(1.1)A_5$	B_5	1
6	A_6	B_6	0	A_6	$(1.1)B_6$	0
7	A_7	B_7	1	$(1.1)A_7$	B_7	1
8	A_8	B_8	0	A_8	$(1.1)B_8$	0
9	A_9	B_9	1	$(1.1)A_9$	B_9	0
10	A_{10}	B_{10}	0	A_{10}	$(1.1)B_{10}$	1
11	A_{11}	B_{11}	0	A_{11}	$(1.1)B_{11}$	1
12	A_{12}	B_{12}	1	$(1.1)A_{12}$	B_{12}	0

Foot-note : 1 : Seeding Indicator for the Area A

0 : Seeding Indicator for the Area B

Using this relation, Equation (5) can be expressed as

$$S_E = 1.1 \ S_N$$

In the following the S_{PERC} is expressed in terms of S_N. For the case of a 10 percent assigned increase in rainfall due to seeding the double ratio value S_{10} in the sub-experiment was evaluated using the modified rainfall data resulting from the main-experiment of EXP-TR. A different random seeding sequence was used for each of the sub-experiments. In the following example of the sub-experiment the double ratio (S_{10}) value was evaluated from the following relation:

$$S_{10} = \left[\frac{(A_2+A_3+1.1 \ A_5+1.1 \ A_7+A_{10}+A_{11})}{(1.1 \ B_2+1.1 \ B_3+B_5+B_7+1.1 \ B_{10}+1.1 \ B_{11})} \right.$$
$$\left. x \ \frac{(B_1+B_4+1.1 \ B_6+1.1 \ B_8+B_9+B_{12})}{(1.1 \ A_1+1.1 \ A_4+A_6+A_8+1.1 \ A_9+1.1 \ A_{12})} \right]^{\frac{1}{2}} \qquad (7)$$

Equation (7) can be re-written as:

$$S_{10} = \left[\frac{(A_2+A_3+A_5+A_7+A_{10}+A_{11}) + 0.1(A_5+A_7)}{(1.1)(B_2+B_3+B_5+B_7+B_{10}+B_{11}) - 0.1(B_5+B_7)} \right.$$
$$\left. x \ \frac{(B_1+B_4+B_6+B_8+B_9+B_{12}) + (0.1)(B_6+B_8)}{(1.1)(A_1+A_4+A_6+A_8+A_9+A_{12}) - (0.1)(A_6+A_8)} \right]^{\frac{1}{2}} \qquad (8)$$

If the total number of seeded periods for any area is N (in the present case N = 6) then the sum of any N_1 (N_1 < N) periods of historic rainfall, with

$$N_1/N_2 = Z, \quad \text{is assumed to be equal to Z X (the}$$
$$\text{total N periods of historic rainfall).}$$

Hence the total historic rainfall contribution due to any N_1 periods is assumed to proportional to N_1.

$$\frac{A_5 + A_7}{A_2 + A_3 + A_5 + A_7 + A_{10} + A_{11}} = \frac{2}{6} = Z_1 \qquad (9)$$

$$\frac{B_6 + B_8}{B_1 + B_4 + B_6 + B_8 + B_9 + B_{12}} = \frac{2}{6} = Z_1 \qquad (10)$$

$$\frac{B_5 + B_7}{B_2 + B_3 + B_5 + B_7 + B_{10} + B_{11}} = \frac{2}{6} = Z_2 \qquad (11)$$

$$\frac{A_6 + A_8}{A_1 + A_4 + A_6 + A_8 + A_9 + A_{12}} = \frac{2}{6} = Z_2 \qquad (12)$$

In Equations (9) and (10),

Z_1 = Ratio of the total number of modified rainfall periods to the total number of seed-designated rainfall periods in the sub-experiment of EXP-TR.

and in Equations (11) and (12),

Z_2 = Ratio of the total number of unaltered rainfall periods to the number of non-seeded rainfall periods in the sub-experiment of EXP-TR.

Hence $Z_1 = Z_2 = Z$(say)

Using the relation of Z Equation (8) can be expressed as:

$$S_{10} = \left[\frac{(A_2 + A_3 + A_5 + A_7 + A_{10} + A_{11})}{(B_2 + B_3 + B_5 + B_7 + B_{10} + B_{11})} \times \frac{(B_1 + B_4 + B_6 + B_8 + B_9 + B_{12})}{(A_1 + A_4 + A_6 + A_8 + A_9 + A_{12})} \right]^{1/2}$$
$$\times \left[\frac{(1 + 0.1Z)}{(1.1 - 0.1Z)} \times \frac{(1 + 0.1Z)}{(1.1 - 0.1Z)} \right]^{\frac{1}{2}} \qquad (13)$$

The S_N value using the historic rainfall data and same random seeding sequence used in the sub-experiment of EXP-TR is given by:

$$S_N = \left[\frac{(A_2 + A_3 + A_5 + A_7 + A_{10} + A_{11})}{(B_2 + B_3 + B_5 + B_7 + B_{10} + B_{11})} \times \frac{(B_1 + B_4 + B_6 + B_8 + B_9 + B_{12})}{(A_1 + A_4 + A_6 + A_8 + A_9 + A_{12})} \right]^{\frac{1}{2}} \qquad (14)$$

Using the relation Equation (13) can be expressed in terms of S_N and Z.

$$S_{10} = S_N \times \frac{1 + (0.1Z)}{1.1 - (0.1Z)} \tag{15}$$

$$\text{When } Z = 0.1, \quad S_{10} = S_N \times \frac{1.01}{1.09} \tag{16}$$

$$\text{When } Z = 0.9, \quad S_{10} = S_N \times \frac{1.09}{1.01} \tag{17}$$

Thus the value of S_{10} can be up to $\pm$ 8 percent of S_N. From the following it is seen that the probability of occurrence of $Z = 0.1$ and $Z = 0.9$ is the same. When $Z = 0.1$; then $N_1 = 0.1 N$. Where N_1 = Number of seed-designated periods in the main-experiment of EXP-Tr, and N = Total number of seed-designated periods in the sub-experiment of EXP-TR.

The number of possible combinations in which the N seed-designated periods of the sub-experiments of EXP-TR will contain N_1 seed-designated periods of the main-experiment of EXP-TR is the number of combinations of N objects taken N_1 at a time, which is the same as the numbered combinations of N objects taken $(N - N_1)$ at a time, or

$$\begin{matrix} N \\ N_1 \end{matrix} = \frac{N!}{N_1! \; (N-N_1)!} \tag{18}$$

Hence the probability of occurrence of the value of Z in each pair given below is the same.

(i) 0.1. 0.9; (ii) 0.2, 0.8; (iii) 0.3, 0.7; (iv) 0.4, 0.6

The percentage probability of occurrence of Z values ranging from 0.1 to 0.9 may be calculated from the total number of possible combinations of N Quantities among themselves: $(2^N) - 1$. The percentage probability (P) of occurrence of N_1 seed-designated periods in the main-experiment of EXP-TR among the total N seed-designated periods of the sub-experiment of EXP-TR will therefore be given by the following expression.

$$P = \frac{100 \; \begin{matrix} N \\ N_1 \end{matrix}}{2N - 1} \tag{19}$$

The percentage probability of Z values (i) 0.1, 0.9; (ii) 0.2, 0.8; (iii) o.3, 0.7; (iv) 0.4, 0.6 and (v) 0.5 are given in Table 6.

Table 6

Percentage probability of occurrence of Z values
(See Section 8.0 of the text for full explanation)
for different model experiments

Experiment	Z values				
	0.1 and 0.9	0.2 and 0.8	0.3 and 0.7	0.4 and 0.6	0.5
5-year duration (1951-55) with 1-day rainfall periods	0.000	0.000	0.000	0.017	4.764
5-year duration (1951-55) with 7-day rainfall periods	0.000	0.007	1.520	5.720	12.540
8-year duration (1951-58) with 7-day rainfall periods	0.000	0.000	0.047	3.300	9.900
10-year duration (1951-60) with 1-month rainfall periods	0.020	0.460	3.710	12.010	17.620

In general, for any percentage increase (PERC) in rainfall due to seeding, Equation (16) may be written as:

$$S_{PERC} = S_N \times \left[\frac{1 + \frac{PERC}{100} \times (Z)}{1 + \frac{PERC}{100} - \frac{PERC}{100} \times (Z)} \right] \qquad (20)$$

The double ratio (S_{PERC}) can be calculated from S_N using Equation (19) if the value of Z is known. For a particular S_N value all possible values of (S_{PERC}) can be evaluated for various values of Z ranging from 0.1 to 0.9. In other words the S_N values are modified according to Equation (19). Knowing the percentage probability of occurrence of Z the standard deviations of S_{PERC} around the mean S_N can be evaluated.

The S_{PERC} values corresponding to a particular S_N will have dispersion around the mean value of S_N due to various possible values of Z. Hence the standard deviation (S.D) of the scatter of the S_{PERC} corresponding to any particular S_N may be evaluated using the following expression.

$$S.D. = \left[\frac{\Sigma\{(S^2_{PERC}) \times (P)\}}{\Sigma P} - (\overline{S}_{PERC})^2 \right]^{\frac{1}{2}} \qquad (21)$$

where $\overline{S}_{PERC}$ is the mean value.

Twice the standard deviation of S_N calculated using Equation (20) is shown by the dotted line on either side of the peaked graph ($S_{N(MAX)}$ and $S_{N(MIN)}$) in Figures 3 and 4. The limits of scatter of the significant levels of S_E as obtained from the numerical experiment EXP-TR are well defined by this dotted line. However, this small scatter of the significance levels of S_E (Figures 3 and 4) would give rise to small differences in the number of detections obtained from the two numerical experiments (Table 4).

CONCLUSIONS

Two numerical cloud seeding simulation experiments performed, using different methodologies, for 5-, 8- and 10-year durations with a double-area cross-over design and area randomization using the historic rainfall data

for the Deccan plateau region suggest:

(i) The results of the first numerical experiment (EXP-TR) performed using the simulation technique of Twomey and Robertson (1973) agree closely with the results of the second numerical experiment (EXP-MMM) performed using the simulation technique proposed in the present study.

(ii) The second numerical experiment (EXP-MMM) not only reduces the computation time by an order of magnitude but also defines the exact lower limit for the double ratio value which can be detected at 5 percent level of significance.

(iii) Increases of 15 and 20 percent in rainfall due to seeding could be detected, with more than 80 percent probability, in 5-years duration. Increases of 5 and 10 percent in rainfall due to seeding could be detected respectively with 27 and 65 percent probability in 10-years duration.

(iv) The above results corroborate those of the preliminary investigation (Mary Selvam et al., 1978) undertaken to evaluate the chances of detecting prescribed changes in rainfall due to the salt seeding experiment (Krishna et al., 1976) conducted on the Deccan Plateau since 1973.

10. ACKNOWLEDGEMENTS

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