STUDIES OF TRENDS AND PERIODICITIES OF RAINFALL OVER MADHYA PRADESH

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Trends and periodicities in the annual rainfall of the meteorological sub-divisions of Madhya Pradesh (MP) (viz., East and West MP) have been studied utilizing the data for the 60-year period from 1901 to 1960. It is seen that the normal annual rainfall for the entire state of MP is about 120 cm and the frequency distribution of the annual rainfall of East and West MP is normal. Significant increase of 15 per cent of the mean annual rainfall per 30 years is observed in West MP. Quasi-biennial oscillation (QBO) and 11-year cycle are also noticed in both the sub-divisions of MP.

INTRODUCTION

Madhya Pradesh (MP) is located in the central parts of the country and consists of two meteorological sub divisions viz., East MP and West MP whose areas approximately are 2,01,800 and 2,11,900 sq. km respectively.

The southwest monsoon rainfall of the 25 sub-divisions of India was recently examined for trends by Rao and Jagannathan (1963). Their study, was restricted to monsoon season and the method of polynomials alone was used. In the present study the latest sub-divisional rainfall averages based on the existing state boundaries have been employed in order to examine the trends and periodicities using the following statistical techniques :

(*a*) Mann-Kendall rank statistic method, (*b*) Students' *t*-test, (*c*) Crammer's test, (*d*) Low-pass filter, (*e*) Correlogram analysis and (*f*) Power spectrum analysis.

DATA

Sub-divisional rainfall has been worked out based upon the data of all the State and India Meteorological Department (IMD) rainguages. Using the rainfall data of these rainguages for the period 1901 to 1960, the average monthly/annual rainfall, standard deviation and coefficient of variation for the two sub-divisions of MP (viz., East and West MP) have been worked out and the same are given in Table I.

Frequencies of excesses and deficiencies of rainfall from the normal have been shown in Table II. There have been various criteria adopted for judging years of excessive or deficient rainfall. Walker (1914) classified years of deficient rainfall in three different categories viz., (*i*) years with rainfall deficiency between 30 and 45 per cent of the normal annual rainfall, (*ii*) between 45 and 60 per cent and (*iii*) over 60 per cent. He called these as 'large', 'serious' and 'disastrous' years. It is observed

TABLE I

Rainfall statistics of East and West MP

Months	East MP				West MP				
	Average cms	Standard deviation cms	Coeff. of variation (%)	% of annual	Average cms	Standard deviation cms	Coeff. of variation (%)	% of annual	Average rainfall for entire MP. cms
January	1.7	2.08	120	1.2	1.4	1.45	103	1.4	1.5
February	2.3	2.22	96	1.7	1.0	1.07	112	1.0	1.6
March	1.8	2.12	121	1.3	0.8	1.06	129	0.8	1.3
April	1.6	1.94	123	1.2	0.4	0.62	143	0.4	1.0
May	1.8	1.81	103	1.3	1.0	1.30	129	1.0	1.4
June	17.8	8.54	48	13.0	12.0	6.11	51	11.6	14.8
July	41.1	9.20	22	30.0	33.9	9.42	28	32.8	37.4
August	39.6	9.05	23	28.9	30.0	10.28	34	29.0	34.7
September	21.6	7.53	35	15.7	17.5	9.31	53	16.9	19.5
October	6.0	4.86	82	4.4	3.3	3.65	111	3.2	4.6
November	1.3	1.88	147	0.9	1.5	2.59	178	1.4	1.4
December	0.5	0.86	180	0.4	0.6	1.00	162	0.5	0.5
Seasonal June to September	120.1	—	—	87.6	93.4	—	—	90.3	106.4
Annual	137.1	17.97	13	—	103.4	19.21	19	—	119.8

TABLE II

Frequency of rainfall departures from the normal

Departure from normal	East MP	West MP
	No. of occasions	No. of occasions
Serious excesses + 60% to + 45%	0	1
Large excesses + 45% to +30%	1	3
Normal + 30% to —30%	58	54
Large deficiency — 30% to — 45%	1	2
Serious deficiency — 45% to —60%	0	0

from Table II that out of 60 years' data there are one and two cases of large deficiency in East and West MP respectively. It is also observed that there are one and three cases of large excesses in respect of East and West MP respectively. However, it may be stated that on 90 per cent occasions, rainfall over East and West MP is normal i.e., lying between +30 to —30 per cent of the normal annual rainfall.

For the application of statistical tests for any time series it is essential to know the nature of frequency distribution. The nature of frequency distribution of annual rainfall series were tested for normality by using Fisher's statistics g_1 and g_2 and comparing these with their respective standard errors. Table III gives the values of these two parameters viz., $g_1/SE\,(g_1)$ and $g_2/SE\,(g_2)$. For 5 per cent level of significance the value of these two parameters should be less than 1.96. From Table III it is evident that these values are very much below the significant value mentioned above. Hence, the rainfall of the period 1901 to 1960 for the sub-divisions of East and West MP have been regarded as normal for all practical purposes.

TABLE III

Statistical parameters for East and West MP

	East MP	West MP
Mean cms	136.99	103.41
Standard deviation	17.97	19.28
$g_1/SE(g_1)$	— 0.495**	+ 0.899**
$g_2/SE(g_2)$	+ 0.217**	— 0.325**
Statistic τ	+ 0.116	+ 0.262**
Mean for 1901-30 $\bar{x}_1$ cms	134.25	95.71
Mean for 1931-60 $\bar{x}_2$ cms	139.73	111.10
Mean difference ($\bar{x}_2$—$\bar{x}_1$) per 30 years	5.48	15.39
Calculated student's t-test	— 1.173	— 3.314**
Percentage of change of mean for 30 years	+ 4.0%	+14.9%

**Significant at 99% level.

TREND ANALYSIS

The trend analysis of the annual rainfall for East and West MP has been worked out by various statistical methods which are described in succeeding sections:

Mann-Kendall Rank Statistic—Kendall and Stuart (1961) has suggested that this is a powerful test when the most likely alternative to randomness is linear or non-linear trend. In this case the statistic τ is computed by using the following empirical formula :

$$\tau = \frac{4 \Sigma n_i}{N(N-1)} = 1 \qquad (1)$$

where n_i is the number of values larger than the ith value in the series subsequent to its position in the time series. The statistic $(\tau)_t$ is given by Eq. 2 :

$$-(\tau)_t = \pm\, t_g \sqrt{\frac{4N+10}{9N(N-1)}} \qquad (2)$$

where t_g is the value of 't' at the probability point in the Gaussian distribution appropriate to the two-tailed test. Table III also shows the Mann-Kendall rank statistic values for both the sub-divisions of East and West MP. It is observed from this Table that trend is significant at 99 per cent for West MP and not significant for East MP.

Student's t-Test—WMO Commission on climatology has recommended the use of 30 years means i.e., 1901 to 1930 and 1931 to 1960 etc., for the interpretation of climatological normals (WMO 1967). In this study the rainfall series of the period 1901 to 1960 was broken into two equal periods (1901–30 and 1931–60). The significance of the difference of the mean between the first and second period was tested by the "Student's t-test" and the magnitude of the gradient ascertained. The means of the two periods (i.e., 1901–30 and 1931–60) their difference and calculated 't' values are also given in Table III. From this table it is noticed that difference between the two means for West MP is significant at 99 per cent level and gradient of mean is about 14.9 per cent. It is however seen in respect of East MP that 't' vague is not significant and the difference of means is 4.0 per cent.

TABLE IV

Statistical parameters for different decades

Decade	East MP				West MP			
	Mean cms	Std. deviation	Coeff. of var. in %	t_k	Mean cms	Std. deviation	Coeff. of var. in %	t_k
1901-10	126.48	13.74	10.8	—2.063†	91.70	11.55	12.6	—2.147†
1911-20	138.03	23.27	16.8	+0.196	96.72	25.54	26.4	—1.194
1921-30	138.25	15.63	11.3	+0.239	98.70	9.28	9.4	—0.835
1931-40	145.95	12.46	8.5	+1.735*	111.46	10.96	9.8	+1.448
1941-50	140.97	19.16	13.5	+0.757	114.63	18.44	16.0	+2.051†
1951-60	132.29	14.51	10.9	—0.896	107.22	22.14	20.6	+0.676

*Significant at 90% level
†Significant at 95% level

Crammer's Test—The decade averages have been examined in order to see whether the individual decade means differ from the mean of the entire period. This can also be regarded as another aspect of the study of variation of change in rainfall pattern with time. Table IV gives the following statistical parameters for each decade :

(*i*) Average,
(*ii*) Standard deviation,
(*iii*) Coefficient of variation and
(*iv*) $$t_k = \overline{M}_k \left[\frac{k(n-2)}{n-k-k\overline{M}_k^2} \right]^{\frac{1}{2}} \quad (3)$$

where $\overline{M}_k = \dfrac{\overline{x}_k - \overline{x}}{S}$

and $\bar{x}$ is the average of the whole series and $\bar{x}_k$ is the mean of corresponding decade k and S is the standard deviation of entire series. It is seen from the Table IV that the values for the decades 1901–10 and 1931–40 are significant for East MP whereas the values for the decades 1901–10 and 1941–50 are significant for West MP.

Low-pass Filter—To understand the nature of the trend, the series were subjected to a 'low-pass filter' (WMO 1966) in order to supress the high frequency oscillations. The weights used were nine ordinates of the Gaussian probability curve (0.01, 0.05, 0.12, 0.20, 0.24, 0.20, 0.12, 0.05 and 0.01). The response curve of the Gaussian low-pass filter has a response function that is equal to unity at infinite wave lengths and it tails off asymptotically to zero with decreasing wavelength. The response is approximately given by the following equation :—

$$R(f) = exp\ (-2\,\pi^2\,\sigma_g\,f^2) \qquad (4)$$

where σ_g is the appropriate standard deviation i.e., $6\,\sigma_g = 10$ years. The filtered series along with the unfiltered series and decadal mean are shown in Fig. 1. It is observed that the trend is not linear but oscillatory consisting of periods of length of 10 years or more. Some salient features of Fig. 1 are given below :—

(*a*) *East MP* — A general increase from 1901 to 1935 and thereafter a general decrease is noticed.

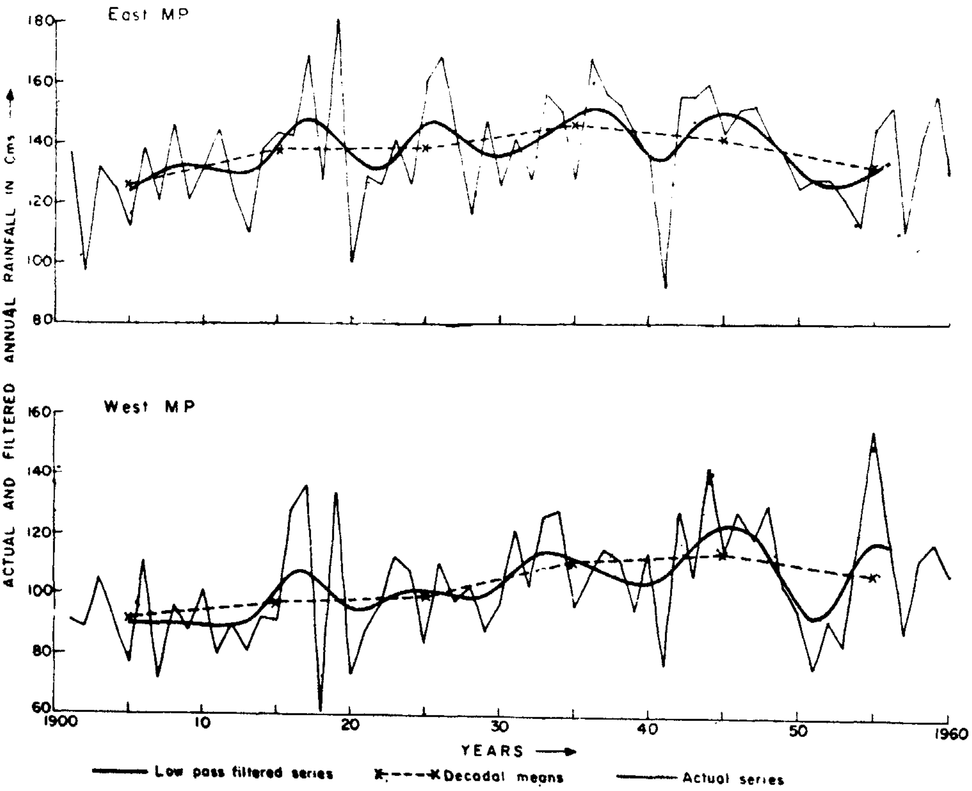

FIG. 1. Profiles of rainfall for East and West M. P. showing actual and filtered annual series.

(*b*) *West MP* : A general increase from 1901 to 1945 and there after a decreasing trend is noticed.

Periodicity Studies

The analysis for periodicities in the annual rainfall for east and west MP has been made by employing various methods which are described in succeeding sections :

Correlogram Analysis—Auto-correlation coefficient upto 22 lags have been calculated for both the sub-divisional rainfall series. The following formula was used :—

$$\gamma_j = \frac{(N-j)\sum_{i=1}^{N-j} x_i\, x_{i+j} - \left(\sum_{i=1}^{N-j} x_i\right)\left(\sum_{i=j}^{N} x_i\right)}{\sqrt{\left\{(N-j)\sum_{i=1}^{N-j} x_i^2 - \left(\sum_{i=1}^{N-j} x_i\right)^2\right\}\left\{(N-j)\sum_{i=j}^{N} x_i^2 - \left(\sum_{i=j}^{N} x_i\right)^2\right\}}} \quad (5)$$

Fig. 2 shows auto-correlation coefficients from lags 1 to 22 for East and West MP. Confining ourselves to high auto-correlation values it is seen that East and West Madhya Pradesh show significant correlation coefficient at lag 11, at 90 per cent and 99 per cent levels respectively. However, at lag 21 the significant level is 95 per cent for West MP and at lag 19 East MP is not significant even at 90 per cent level. It

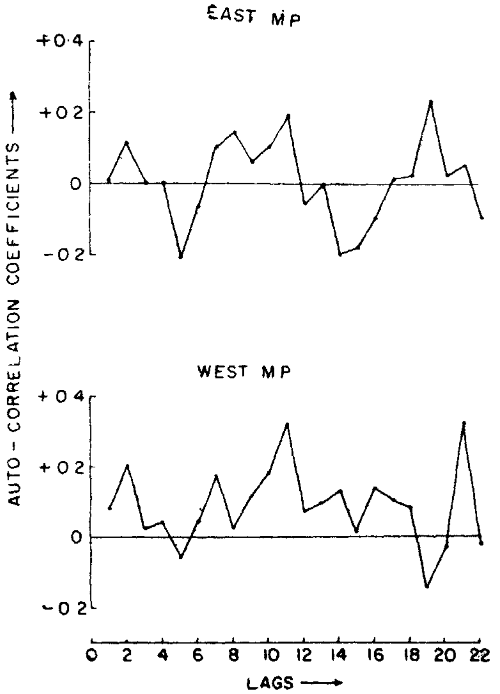

FIG. 2. Correlogram analysis of actual rainfall series (annual)

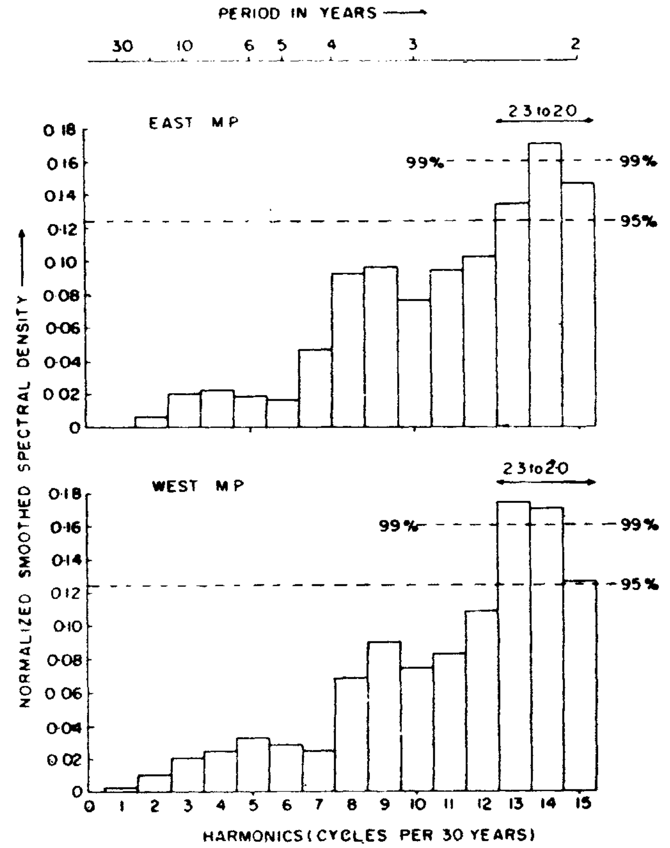

FIG. 3. Spectrum analysis of the annual rainfall series after removal of trend by 'difference filter.

can be inferred from the above analysis that a significant 11-year cycle is present so far as West MP is concerned and a weak cycle is observed in the case of East MP.

Power Spectrum Analysis—Time series of the mean annual rainfall were subjected to power spectrum analysis. Tukey's (1950) procedure for computing power spectra has been followed. From the trend analysis carried out earlier it has been observed that there is some trend in the series. This means that the power is contained in one or a few very narrow bands. Such bands can cause errors in the spectral estimates where there is less power. Hence it may be advantageous to filter the data digitally in order to improve the spectral estimates at these frequencies. In this study 'digital difference filter' has been employed to remove low frequency oscillations. A difference filter is defined by the equation

$$y_t = (x_t - x_{t-1}) \tag{6}$$

where y_t is the filtered series and x_t is the unfiltered series. The frequency response function is given by the equation :—

$$R(f) = 2\,j\,exp\,(-j\,\pi\,f)\,sin\,\pi\,f \tag{7}$$

where j is $\sqrt{-1}$ and f is the frequency. This filter acts as a "high-pass filter". The original annual rainfall series were filtered through this filter mentioned at equation (6) and the filtered series thus obtained were subjected to the power spectrum analysis. The power spectrum results are shown in Fig. 3 for both East and West MP. Significant QBO is observed in both the sub-divisions of MP at 99 per cent level.

Conclusions

From the foregoing the following are some of the salient features of trends and periodicities for the MP region :—

(1) The normal annual rainfall of the entire state of MP based upon the data of all the state and IMD rainguages numbering nearly 230 is about 120 cm.

(2) Frequency distribution of annual rainfall for the period of 1901 to 1960 of East and West MP is normal.

(3) In 60-year period it is noticed that there are one and two cases of large deficiency in East and West MP respectively.

(4) Increasing trend in annual rainfall of both the sub-divisions is noticed upto 1935. In the case of East MP there is a decreasing trend thereafter while in the case of West MP the increasing trend continues upto 1945 and thereafter the trend starts decreasing.

(5) Significant increase of 15 per cent of the mean annual rainfall per 30 years is noticed in West MP whereas the increase is 4 per cent of the mean annual rainfall in the case of East MP which is not statistically significant.

(6) Increasing mean annual rainfall is observed in the decade 1901-10 for both East and West MP while there is decreasing rainfall in the decades 1931-40 and 1941-50 for East and West MP respectively.

(7) It is seen that there is an eleven-year cycle in both the sub-divisions, and

(8) Highly significant QBO is also observed in East and West MP.

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