A Comparative Study of Fourier Analysis Procedure and Cressman's Method in Objective Analysis of the Wind Field

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ABSTRACT

The value of a meteorological parameter at a grid point can be derived from Fourier analysis of data at station observations around the grid point. But the inhomogeneity of data distribution does not warrant such an elaborate and rigorous approach. Cressman's method which is shown as a simplified version of local-point Fourier analysis is found to be more feasible from rms error statistics of wind field analyses by both methods.

1. Introduction

Most of the current operational methods used in the objective analysis of meteorological fields employ a predictor corrector approach to the problem. In such a procedure a preliminary guess field is first prescribed for the grid points, which may be a prognostic field or the result of a formal extrapolation of an analyzed field at a previous observational time or a suitable combination of persistence and climatology. One of the methods is known as the Cressman's (1959) method of objective analysis. This method was applied to the analysis of wind data over the Indian region by Sikka and Ramanathan (1970). Later, Ramanathan and Sikka (1971) tested a more general method of objective analysis of the wind field in which the corrections to the first guess were made by a Fourier analysis technique. The purpose of the present paper is to make a comparative study of the local-point Fourier and Cressman's method of objective wind analysis using identical observations.

In Section 2, a local-point Fourier analysis method of incorporating the corrections is briefly described and in Section 3 Cressman's method is shown as a simplified version of the Fourier technique. The last section deals with the evaluation of the results of analyses by both the methods on 90 synoptic situations of the 500-mb wind field in the Indian region.

2. Local-point Fourier analysis technique

The procedure for incorporating the corrections to the guess value at a grid point can be intuitively considered in the light of generalized Fourier transforms. Let the deviation of a field value at an observing station (R,θ) from some norm be defined as $s(R,\theta)$, where R and θ are the polar coordinates with the origin at the center which is the grid point to be corrected. From the deviations given at all such irregularly distributed observations around a grid point, the deviation at the grid point has to be computed.

For the existence of a Fourier transform of $s(R,\theta)$, a sufficient condition is that

$$\int_0^\infty \int_0^{2\pi} |s(R,\theta)| R dR d\theta \tag{2.1}$$

must be convergent. The idea motivating generalized Fourier analysis is to make such a convergence possible. This also means that we recognize a limit to the area of influence around a grid point so that information only at stations within this area is used to derive the grid point value.

To this end, we create a set of scaling functions $G(R,\theta)$, where R is any real number whose properties are:

(i)
$$\left.\begin{matrix} G(R,\theta) \rightarrow 0 \\ R \rightarrow \infty \end{matrix}\right\}; \tag{2.2}$$

as $R \rightarrow \infty$, $G(R,\theta)$ approaches zero with rapidity and the product $s(R,\theta)G(R,\theta)$ becomes convergent and integrable.

(ii)
$$\left.\begin{matrix} G(R,\theta) \rightarrow 1 \\ R \rightarrow 0 \end{matrix}\right\}. \tag{2.3}$$

One such scaling function satisfying the above two criteria is $\exp(-R^2/N^2)$, where R and N are real numbers.

In the local-point finite Fourier analysis in practical problems, we use a truncated version of the scaling function, as $G(R,\theta)$ must approach zero when $R\rightarrow N$, which is a finite limit.

We take R as the distance of an observation station from the grid point for which corrections are to be evaluated, and N as the scan limit within which the observations at stations are to be used for correcting the guess field. One such scaling function is $(N^2-R^2)/(N^2+R^2)$ which is incidentally the sum of the first two terms of the expansion $\exp[-2R^2/(R^2+N^2)]$. This function satisfies the two criteria mentioned above; it has been chosen since it is exactly the same as the weighting function in Cressman's method. We now proceed to compute the amplitudes

$$B(m,n)=\int_0^N\int_0^{2\pi} s(R,\theta)\left[\frac{N^2-R^2}{N^2+R^2}\right] \times e^{-iR(mk\cos\theta+nl\sin\theta)}RdRd\theta, \quad (2.4)$$

where m and n are the harmonic indices of the fundamental wavenumbers k and l in the zonal and meridional directions, respectively, defined for a limited space on some fundamental proportional to N. In practice, it is sufficient to calculate a limited number of harmonics indicated by the mesh length and the scales of motion represented; the value (c) at the grid point which is the origin for this local-point Fourier analysis is given by

$$c=\sum_{m=-MM}^{MM}\sum_{n=-NN}^{NN} B(m,n)\Delta m\Delta n, \quad (2.5)$$

where MM and NN are the limits for the number of harmonics given by the indices m and n.

3. Cressman's method

Cressman's method is now shown as a simplification of the procedure of local-point Fourier analysis discussed in Section 2.

The average (AV) of the weighted corrections at the station observations in the scan area is

$$AV=\frac{\int_0^N\int_0^{2\pi} s(R,\theta)\left[\frac{N^2-R^2}{N^2+R^2}\right]RdRd\theta}{\int_0^N\int_0^{2\pi} RdRd\theta}. \quad (3.1)$$

But if $m=n=0$ in Eq. (2.4)

$$B(0,0)=\int_0^N\int_0^{2\pi} s(R,\theta)\left[\frac{N^2-R^2}{N^2+R^2}\right]RdRd\theta, \quad (3.2)$$

so that

$$AV=\frac{B(0,0)}{\pi N^2}.$$

In Cressman's analysis, series of scans of decreasing radius are used which, according to Cressman, "allows the analysis of a spectrum of scales." In that method only the average of the weighted correction within this scan circle is calculated and incorporated as the guess value at the grid point. From the consideration of the Fourier analysis procedure [Eq. (3.2)] it can be said that the corrections are made with respect to $B(0,0)$ only. The sequential analysis by decreasing the scan length can be visualized as a practical way of incorporating the higher harmonics of the Fourier spectrum.

The following simplified treatment using the one-dimensional Fourier transform is the background of our considering such an interpretation of the Cressman's technique. Let

$$B(m)=\int_0^N\left(1-\frac{R^2}{N^2}\right)e^{-imkR}dR, \quad (3.3)$$

where $(1-R^2/N^2)$ is a scaling function similar to $(N^2-R^2)/(N^2+R^2)$ used by Cressman but much easier to tackle mathematically. The harmonics m take values $0, \pm1, \pm2, \ldots$ up to limits $\pm m$ in a finite Fourier analysis $k=2\pi/L$ where L is the fundamental wavelength in a finite space and R the distance of an observation point within the scan distance N. The sum of the amplitudes of all harmonics would be:

$$B(m)+B(-m)=2\int_0^N\left(1-\frac{R^2}{N^2}\right)\cos(mkR)dR. \quad (3.4)$$

By repeated integration by parts and applying limits, we can write (3.4) as

$$B(m)+B(-m)=\frac{2}{mk}\left[\sin(mkN)-\frac{2}{mkN}\cos(mkN) -\frac{2\sin(mkN)}{m^2k^2N^2}\right]. \quad (3.5)$$

In the tropics let us consider $L\approx3000$ km and the fundamental scan length $N\approx1000$ km. For $m=\pm1$, ±2, ±3 the value of $B(m)s$ from (3.5) is 550 km. If we reduce the scan length to $n=N\times h$ (say), where h is the constant of proportionality, we have

$$B_h(0)=\int_0^n\left(1-\frac{r^2}{n^2}\right)dr=\frac{2n}{3}. \quad (3.6)$$

Taking $h=\frac{1}{2}$ and $\frac{1}{3}$ successively we get $B_{\frac{1}{2}}(0)+B_{\frac{1}{3}}(0)=556$ km, which is about the same as the contributions from the harmonics ±1, ±2, ±3.

4. Discussion of the results of analyses

Wind observations at 500 mb (0000 GMT) for 28 stations over India were collected for 90 days from 1 December 1966 to 28 February 1967. Fig. 1 shows the

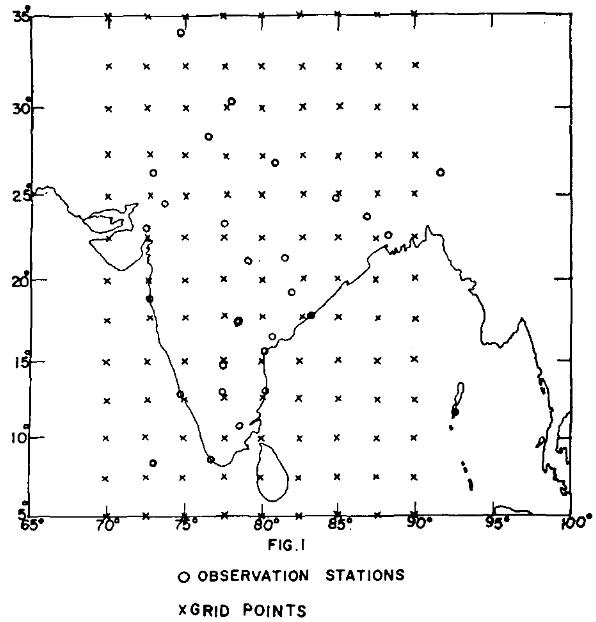

FIG. 1. Stations and grid points used in the 500-mb wind analyses.

distribution of stations and the grid points (2.5° latitude, longitude interval) within the area considered. All the synoptic situations fall within the winter season when the major synoptic-scale disturbances are troughs in the subtropical westerlies.

In the present study we have preferred to analyze the deviation of the wind components from their average value for the period under study. This has been done so as to keep homogeneity and isotropy of the field being analyzed as far as possible since these properties in the case of meteorological fields are known to be reasonably satisfied by the deviations from their normals. We have adopted the following analysis procedure:

1) Obtain for each day the deviation of the wind components (u' and v') from their average value at each observation station.

2) Compute the deviation value at each grid point from the station data by following both Cressman's and the local-point Fourier methods. Consistent with the discussions in Section 3, the scan lengths in Cressman's method are 10.0°, 5.0° and 3.5° and the fundamental wavelength in the Fourier method was taken as 3000 km. For a 2.5° mesh length adopted in the Indian tropics, the smallest scale that could be represented is the grid distance of $10° \approx 1000$ km of cyclone scale. For this, it is sufficient to calculate harmonics up to number 3 on the fundamental wavelength of 3000 km. Accordingly, 13 harmonic components as given in Table 1 were sequentially added.

3) Interpolate from the grid point deviation values obtained under step 2) the deviation value corresponding to each station.

4) Find out the difference between the interpolated value at each station and its correct initial value. The difference between the two values gives an estimate of the goodness of the analysis procedure.

5) Obtain the rms values of the differences computed under step 4).

Figs. 2 and 3 are the scatter diagrams for the rms values for u and v components, respectively. In these diagrams the ordinate refers to the Fourier value and the abscissa to the corresponding Cressman's value. If the errors from both methods had been the same for each day, all the points would have fallen on the central diagonal line shown in the corresponding figures. The

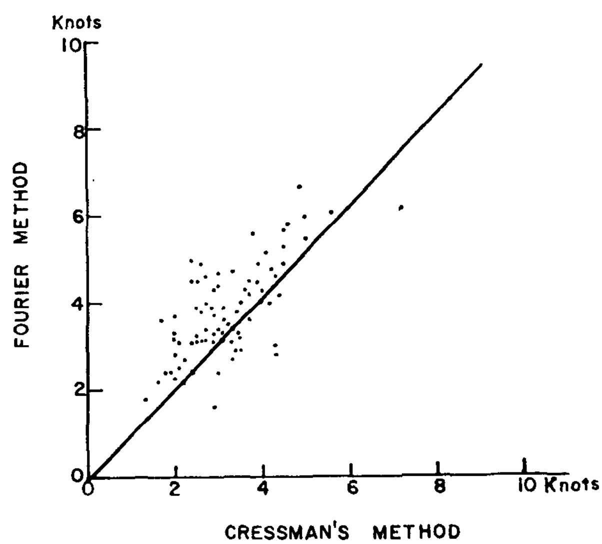

FIG. 2. rms errors (kt) in the deviation of the zonal wind components (u') from the mean at 28 stations.

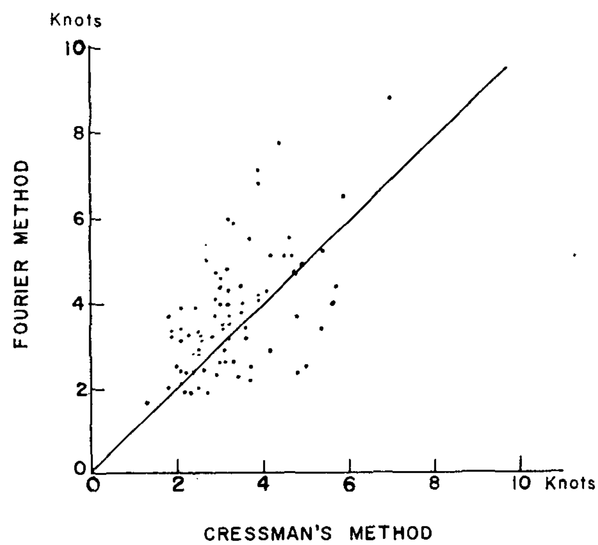

FIG. 3. rms errors (kt) in the deviation of the meridional wind components (v') from the mean at 28 stations.

figures show that most of the rms errors fall between 2–4 kt; that in the case of the u deviations most of the points fall to the left of the diagonal indicating that the errors by the Cressman's method are about 1 kt less than those from the corresponding Fourier method; and that in the case of the v deviations the scatter is equal on both sides of the diagonal.

It appears from this that the elaborate procedure required in the local-point Fourier analysis is not necessary, and a simplified Cressman's procedure is more than adequate for the following reasons:

1. The station observations are not uniformly distributed around the grid points over the Indian region where there are large data holes in the oceanic and the Himalayan areas.
2. There may not be even continuity of analysis from grid point to grid point if there are no common station observations in the scan areas around contiguous grid points.

5. Conclusion

Cressman's method of objective wind analysis, which is a simplified version of the local-point Fourier analysis method, has been found to compare favorably with the more rigorous and elaborate Fourier analysis procedure over the Indian region where the observational network is not uniformly distributed with respect to the grid points.

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REFERENCES

Cressman, G. P., 1959: An operational objective analysis scheme. *Mon. Wea. Rev.*, **87**, 367–374.

Ramanathan, Y., and D. R. Sikka, 1971: Fourier expansion techniques in objective analysis. *Indian J. Meteor. Geophys.*, **23**, No. 2.

Sikka, D. R., and Y. Ramanathan, 1970: Some experiments in the objective analysis of the wind field over India and neighbourhood. *Indian J. Meteor. Geophys.*, **21**, 237–244.