UDC 551.513.1:551.553.21:551.557.5(540)

ON THE MAINTENANCE OF THE MEAN MONSOON TROUGH OVER NORTH INDIA

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ABSTRACT

The different terms in the equation for the balance of standing eddy kinetic energy are estimated in the region of the monsoon trough over north India. The main contribution to the maintenance of the trough against frictional dissipation is from the work done by the horizontal pressure forces. The contribution by the mean and transient advection terms is of a smaller order of magnitude.

There is loss of standing eddy kinetic energy by the rising of cold air and sinking of warm air in the x-p plane in the lower troposphere. The circulation is forced by the flux of potential energy from outside, mainly from the bottom and southern boundaries.

1. INTRODUCTION

The Indian southwest monsoon circulation is one of the important stationary disturbances of the Tropics. The important parts of the anatomy of this circulation are 1) the mean zonal westerlies in the lower troposphere and the easterlies in the upper troposphere and 2) the mean monsoon trough running from West Pakistan to north Bay of Bengal (fig. 1). This trough extends from the sea level to 500 mb, with a southward slope with height. Keshavamurty (1968) studied the maintenance of the mean zonal motion in the Indian monsoon and found that it is largely maintained by the Coriolis term. It is the object of the present paper to attempt to study the maintenance of the kinetic energy of the time mean monsoon trough over north India.

Superposed on this mean monsoon flow are transient disturbances which may have interactions with the mean flow. Monsoon Lows and depressions form over north Bay of Bengal, move west-northwestward across north India, and dissipate over Northwest India-West Pakistan. Typical tracks of these monsoon depressions in the month of July are also shown in figure 1 (India Meteorological Department 1964). These are the main rain-producing systems in the Indian monsoon.

The kinetic energy transfer between different scales of motion in the atmosphere has been extensively studied by writing the equations in spectral form and computing the interactions between different wave numbers (Saltzman 1957; Saltzman and Fleisher 1960*a*, 1960*b*; Murakami and Tomatsu 1964; Wiin-Nielsen et al. 1963, 1964). Murakami (1963*a*, 1963*b*) studied the energy transfers between standing eddies, mean zonal motion, and transient motion and showed that the standing eddies drain their energy to transient disturbances as well as to mean zonal motion.

Most of the studies of energetics so far have been for closed regions. When the equations for the balance of the different forms of energy are integrated over a closed region, the flux terms integrate to zero, and we get expressions for the rate of conversion between different forms of energy. But while considering a limited region, the flux terms are, in general, nonzero. Also, in the case of the conversion functions arising from the nonlinear advection terms, we have the further difficulty that they are not unique for the limited regions.

2. EQUATION FOR THE BALANCE OF STANDING EDDY KINETIC ENERGY

As we are considering a limited region, we shall write the complete equation for the balance of standing eddy kinetic energy. We shall interpret the terms by the physical processes by which they occur and shall be careful in interpreting them as conversions between different forms of energy.

The equation of motion along the ϕ-direction in spherical coordination is

$$\frac{\partial v}{\partial t}=-u\frac{\partial v}{a\cos\varphi\partial\lambda}-v\frac{\partial v}{a\partial\varphi}-\omega\frac{\partial v}{\partial p}-fu-g\frac{\partial z}{a\partial\varphi}+Y. \quad (1)$$

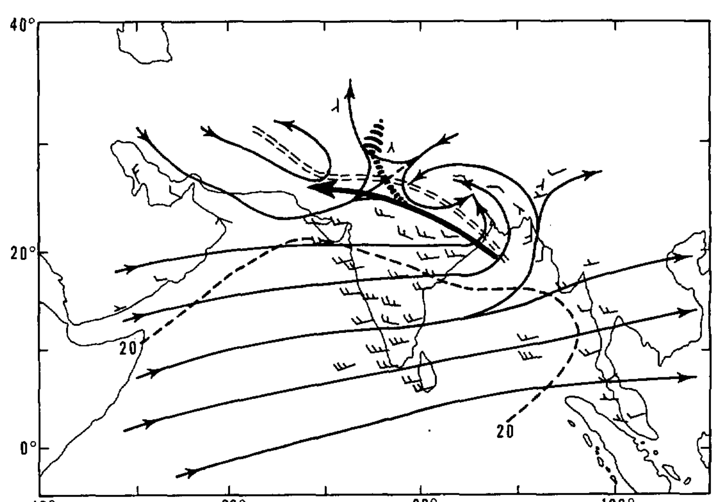

FIGURE 1.—Mean wind chart for July, 850 mb; the parallel dashed lines, the monsoon trough; the heavy solid line and its striped offshoot, the typical tracks of monsoon depressions.

The metric terms are small for the scales of motion we are considering. Adding

$$-v\left(\frac{\partial u}{a\cos\varphi\partial\lambda}+\frac{\partial v}{a\partial\varphi}+\frac{\partial\omega}{\partial p}\right)=0$$

to equation (1), we get

$$\frac{\partial v}{\partial t}=-\frac{\partial}{a\cos\varphi\partial\lambda}(uv)-\frac{\partial}{a\partial\varphi}(v^2)-\frac{\partial}{\partial p}(v\omega)-fu-g\frac{\partial z}{a\partial\varphi}+y. \tag{2}$$

Averaging equation (2) with respect to time, we get

$$\frac{\partial\bar{v}}{\partial t}=-\frac{\partial}{a\cos\varphi\partial\lambda}(\overline{uv})-\frac{\partial}{a\partial\varphi}\overline{v^2}-\frac{\partial}{\partial p}\overline{v\omega}-f\bar{u}-g\frac{\partial\bar{z}}{a\partial\varphi}+\bar{y} \tag{3}$$

where

$$\bar{x}=\frac{1}{T}\int_0^T x\,dt \text{ and } x'=x-\bar{x}.$$

Again averaging equation (3) along a latitude circle between two longitudes λ_1 and λ_2, we get

$$\frac{\partial[\bar{v}]}{\partial t}=-\left[\frac{\partial}{a\cos\varphi\partial\lambda}\overline{uv}\right]-\left[\frac{\partial}{a\partial\varphi}\overline{v^2}\right]-\left[\frac{\partial}{\partial p}\overline{v\omega}\right]-f[\bar{u}]-\left[\frac{\partial\bar{z}}{a\partial\varphi}\right]+[\bar{y}] \tag{4}$$

where

$$[x]=\frac{1}{\lambda_2-\lambda_1}\int_{\lambda_1}^{\lambda_2}x\,d\lambda \text{ and } x^*=x-[x].$$

Subtracting (4) from (3), we get

$$\frac{\partial\bar{v}^*}{\partial t}=-\left\{\left(\frac{\partial}{a\cos\varphi\partial\lambda}\overline{uv}\right)^*+\left(\frac{\partial}{a\partial\varphi}\overline{v^2}\right)^*+\left(\frac{\partial}{\partial p}\overline{v\omega}\right)^*\right\}$$
$$-\left\{\left(\frac{\partial}{a\cos\varphi\partial\lambda}\overline{u'v'}\right)^*+\left(\frac{\partial}{a\partial\varphi}\overline{v'^2}\right)^*+\left(\frac{\partial}{\partial p}\overline{v'\omega'}\right)^*\right\}$$
$$-f\bar{u}^*-g\left(\frac{\partial\bar{z}}{a\partial\varphi}\right)^*+\bar{y}^*. \tag{5}$$

Multiplying by $\bar{v}^*$, we get

$$\frac{\partial}{\partial t}\frac{\bar{v}^{*2}}{2}=-\bar{v}^*\left\{\left(\frac{\partial}{a\cos\varphi\partial\lambda}\overline{uv}\right)^*+\left(\frac{\partial}{a\partial\varphi}\overline{v^2}\right)^*+\left(\frac{\partial}{\partial p}\overline{v\omega}\right)^*\right\}$$
$$-\bar{v}^*\left\{\left(\frac{\partial}{a\cos\varphi\partial\lambda}\overline{u'v'}\right)^*+\left(\frac{\partial}{a\partial\varphi}\overline{v'^2}\right)^*+\left(\frac{\partial}{\partial p}\overline{v'\omega'}\right)^*\right\}$$
$$-f\bar{u}^*\bar{v}^*-g\bar{v}^*\left(\frac{\partial\bar{z}}{a\partial\varphi}\right)^*+\bar{v}^*\bar{y}^* \tag{6}$$

Adding a similar equation for the standing eddy u-motion, we get

$$\frac{\partial}{\partial t}\left(\frac{\bar{v}^{*2}+\bar{u}^{*2}}{2}\right)=-\left\{\bar{u}^*\left(\left(\frac{\partial}{a\cos\varphi\partial\lambda}\overline{uv}\right)^*+\left(\frac{\partial}{a\partial\varphi}\overline{v^2}\right)^*+\left(\frac{\partial}{\partial p}\overline{\omega v}\right)^*\right)+\bar{u}^*\left(\left(\frac{\partial}{a\cos\varphi\partial\lambda}\overline{u^2}\right)^*+\left(\frac{\partial}{a\partial\varphi}\overline{uv}\right)^*+\left(\frac{\partial}{\partial p}\overline{\omega u}\right)^*\right)\right\}\Bigg|\,L$$
$$-\left\{\bar{u}^*\left(\left(\frac{\partial}{a\cos\varphi\partial\lambda}\overline{u'v'}\right)^*+\left(\frac{\partial}{a\partial\varphi}\overline{v'^2}\right)^*+\left(\frac{\partial}{\partial p}\overline{v'\omega'}\right)^*\right)+\bar{u}^*\left(\left(\frac{\partial}{a\cos\varphi\partial\lambda}\overline{u'^2}\right)^*+\left(\frac{\partial}{a\partial\varphi}\overline{u'v'}\right)^*+\left(\frac{\partial}{\partial p}\overline{u'\omega'}\right)^*\right)\right\}\Bigg|\,M$$
$$-g\left\{\bar{u}^*\left(\frac{\partial\bar{z}}{a\cos\varphi\partial\lambda}\right)^*+\bar{v}^*\left(\frac{\partial\bar{z}}{a\partial\varphi}\right)^*\right\}\Bigg|\,N$$
$$+\underbrace{\left\{\bar{u}^*\bar{x}^*+\bar{v}^*\bar{y}^*\right\}}_{P}. \tag{7}$$

From scale considerations (Charney 1948 and Murakami 1963a), the vertical flux terms in L and M are small compared to the horizontal flux terms and can be neglected. Equation (7) is complete and is valid at a place. The term M gives the production of standing eddy kinetic energy at a place by the physical process of convergence of momentum accomplished by the transient motion. In the region of the monsoon trough, this is probably due to the migratory monsoon Lows and depressions. If this term is positive (negative), the transient motion reinforces (dissipates) the standing eddy. If this term were the dominant one, it would mean that the standing eddy is largely the statistical effect of transient perturbations. If this term were integrated over a closed region, this energy would have completely come from (gone into) the kinetic energy of transient motion. But in the present case, probably only a part of this energy comes from (goes into) transient motion. The term L represents the production of standing eddy kinetic energy by the convergence of momentum at a place accomplished by the time-mean motion itself. The term N represents the production by the work done by horizontal pressure forces by the flow of air from high pressure to low pressure across the isobars. When integrated over a region, this can be written as

$$-\int_M \bar{v}^*\cdot\nabla\bar{\varphi}^*\,dm$$
$$=-\underbrace{\int_M \nabla\cdot\bar{v}^*\bar{\varphi}^*\,dm}_{Q}-\underbrace{\int_M\frac{\partial}{\partial p}\bar{\omega}^*\bar{\varphi}^*\,dm}_{R}-\underbrace{\int_M\bar{\omega}^*\bar{\alpha}^*\,dm}_{S}. \tag{8}$$

The term S gives the conversion from standing eddy available potential energy by the physical process of the rising of warm air and sinking of cold air in the x-p plane. The terms Q and R give the flux of potential energy across the side walls and top-bottom boundaries, respectively. Palmén (1959) recommends the calculation of work terms for limited regions. For a closed region, of course, the terms Q and R will be zero, and the work terms can be interpreted as a conversion from potential energy (S). The term P gives the dissipation of standing eddy kinetic energy by friction.

3. DATA AND COMPUTATIONS

As the mean monsoon trough extends from the surface up to 500 mb only, we shall limit our computation to the

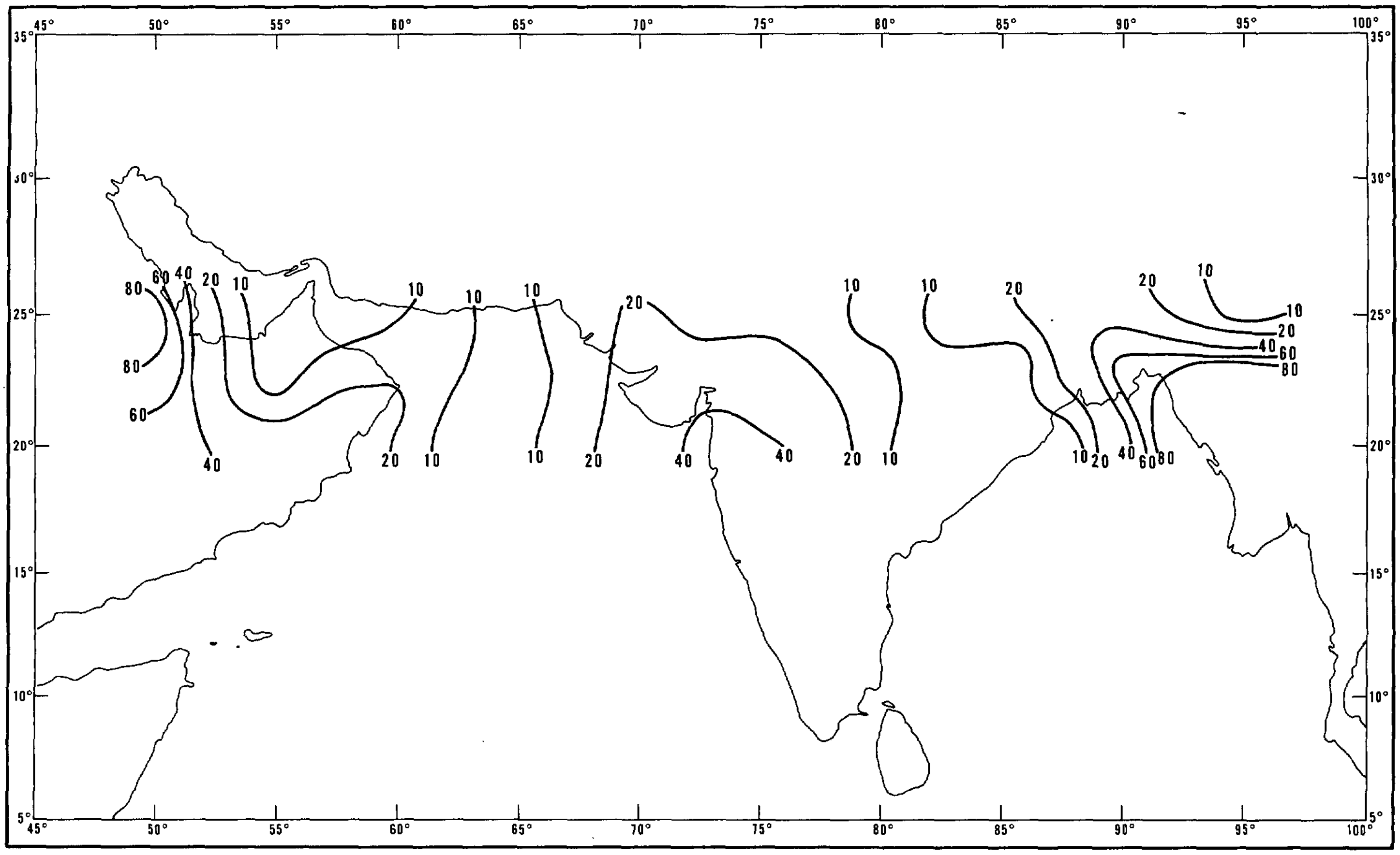

FIGURE 2.—July 850-mb standing kinetic energy (kt²).

lower troposphere. The trough is most marked at 850 and 700 mb. The terms L, M, and N of equation (7) are estimated for the region of the monsoon trough at 850 and 700 mb. The area for which the computations are made is from 20° to 25° N. and 50° to 95° E. The mean advection terms are computed using mean wind data at 850 and 700 mb. At 850 mb, pilot balloon winds were also used. The u- and v-component fields were analyzed, and the values taken at 2.5° grid points. The transient advection terms were computed from data of July and August of 1963 and of 1964. The fields of $\overline{u'v'}$ $\overline{u'^2}$ and $\overline{v'^2}$ were analyzed and grid-point values read off. The $\overline{v'^2}$ values for the period were taken from International Indian Ocean Expedition data printouts (1963–1964). For computing the work terms, the mean contour height fields were also analyzed at 850 and 700 mb. The work terms (N) and the mean advection terms (L) are based on mean data. The transient advection terms are based on data of 2 yr (1963–1964). As these were normal monsoon years, perhaps they are representative. The frictional dissipation (P) is not computed. As this term has to be negative, the sum of the other terms has to be positive, and we can estimate their relative importance in maintaining the trough against frictional dissipation.

4. DISCUSSION

Figure 2 shows the distribution of standing eddy kinetic energy at 850 mb.

TABLE 1.—*Rate of production of standing eddy kinetic energy; units, ergs sec⁻¹ cm⁻²*

Mean advection L	Transient advection M	Work done by horizontal pressure forces N	Horizontal flux Q	Vertical flux R	Conversion from potential energy S
25	5	320	800	2600	−2200

Table 1 shows the contribution of the various terms L, M, and N in equation (7) and other terms integrated in the vertical between 900 and 600 mb and averaged in the horizontal.

It is seen that the trough is maintained largely by the work done by the horizontal pressure forces. Starr (1951), Lorenz (1967), and Pisharoty (1954) stress the importance of this term in the production of the kinetic energy of the atmosphere. Kung (1967) estimated the work done by horizontal pressure forces from wind and geopotential data over the North American Continent and found that it largely balances the frictional dissipation of kinetic energy. The difficulties of computing this work term from data are well known (Wiin-Nielsen 1968). However, the region under consideration is one having a good data network; also as we are considering mean charts, many of the errors of observation are likely to have been smoothed out. Moreover, as we are considering only the lower levels, radiosonde height values are likely to be reliable.[1] The transient and mean advection terms are an order of magnitude smaller. Smagorinsky (1953) concluded that large-scale horizontal eddy transport does not appear to be a deciding factor in determining the quasi-stationary flow. As the contribution of the transient advection term is small, the short period of data for computing it is perhaps not a serious disadvantage. Though small, its distribution at 850 mb appears to be interesting (fig. 3). This term is negative over north Bay of Bengal and central India, and positive over regions contiguous to the heat Low over Northwest India-West Pakistan. This is in keeping with the fact that monsoon Lows and depressions form over north Bay of Bengal and dissipate or merge with the seasonal Low over West Pakistan. Petterssen (1950) also mentioned that the dissipation of the monsoon Lows over West Pakistan may also contribute to the maintenance of the heat Low.

As mentioned, it is difficult to interpret the terms L, M, and N as energy conversion functions. For instance, at 850 mb, at 25° N., $[\bar{u}^*\bar{v}^*]$ is -17.3 kt^2. The negative sign shows that the monsoon trough is transporting westerly momentum southward, that is, toward the westerly maximum or against the gradient of u. This sign of the momentum transport can also be inferred from the northwest-southeast tilt of the monsoon trough. If the system we are considering were a closed one (no flux at the boundaries), the monsoon trough would be losing kinetic energy to the mean zonal motion. However, we find that the mean advection terms contribute to the maintenance of the standing eddy.

To understand better the physical mechanism of the term N (which is the dominant one), we can split it as in equation (8) and estimate the contribution by the different terms Q, R, and S. We can estimate the conversion from standing eddy available potential energy, that is, $-[\bar{\omega}^*\bar{\alpha}^*]$; on a constant pressure chart, this is proportional to $-[\bar{\omega}^*\bar{T}^*]$. Das (1962) has computed the mean $\bar{\omega}$ field in the monsoon region. Das used the monthly mean geopotential field. As we are interested in the maintenance of the time-mean monsoon trough, we use his $\bar{\omega}$-values. At 900 mb, he gets a region of strong ascent over northeast India and a region of descent over the heat Low over Northwest India-West Pakistan. We can take Gauhati and New Delhi as representative of these two regions. Gauhati has lower mean virtual temperature than New Delhi in

[1] We shall be presently estimating the contribution from the different terms in equation (8) which comprise this work term.

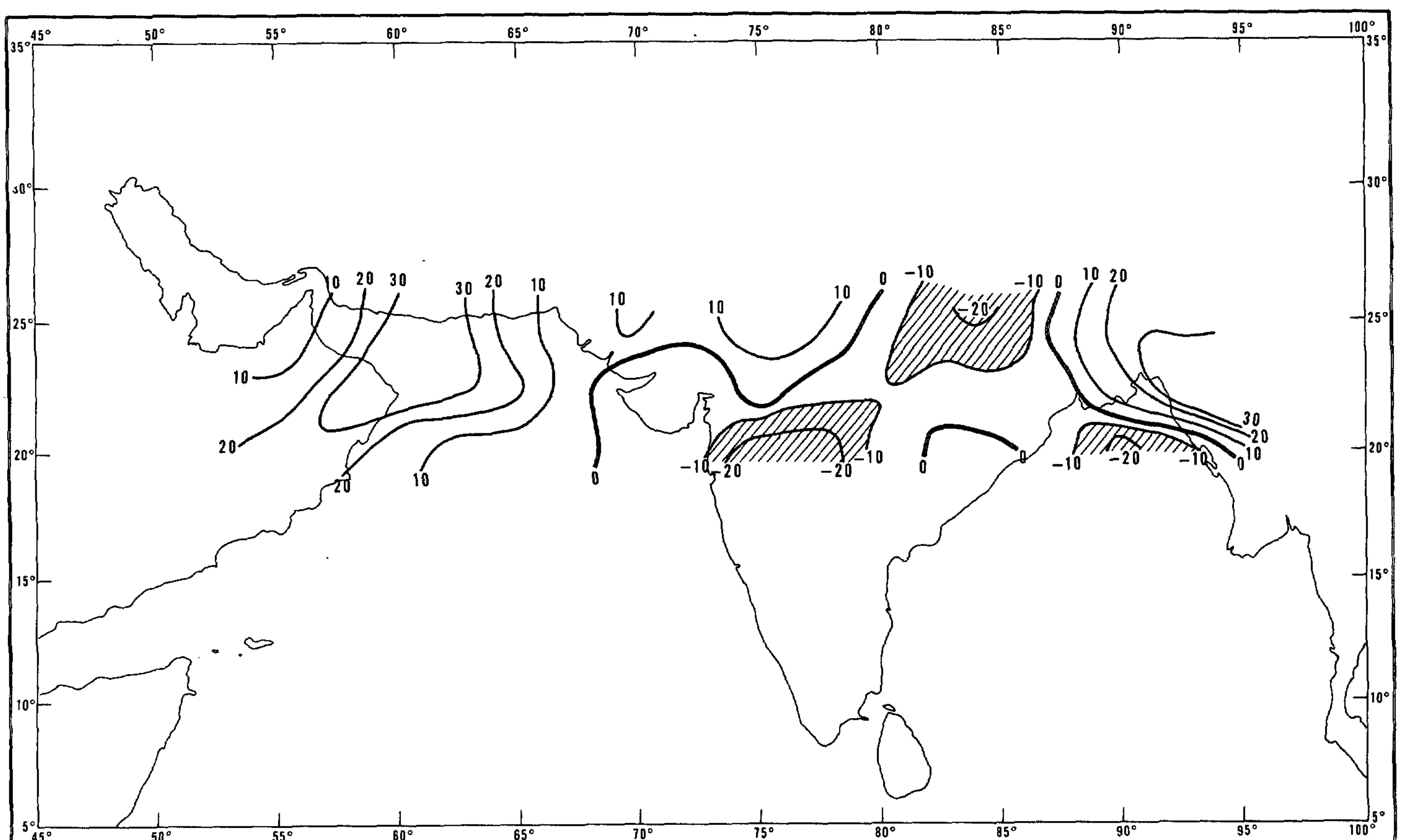

FIGURE 3.—Contribution from the transient motion to standing kinetic energy (kt^2 deg^{-1}) at 850 mb.

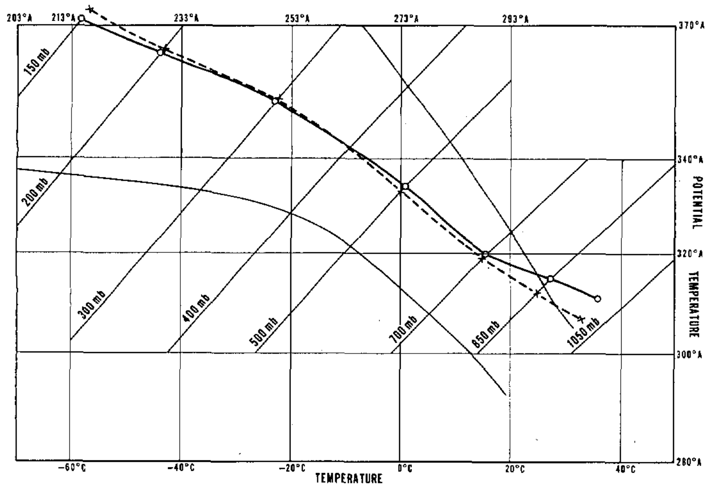

FIGURE 4.—Mean virtual temperature profile, the solid line, New Delhi, the dashed line, Gauhati.

the lower troposphere (fig. 4). So we get ascent of cold air and descent of warm air, which will give dissipation of standing eddy kinetic energy. The vertically integrated value of $-[\bar{\omega}^*\bar{T}^*]$ in the lower troposphere is negative. However, the temperature gradient reverses by about 400 mb, and we have production of standing eddy kinetic energy in the upper troposphere. The vertical motions and the temperature contrast in the upper tropospheres are much smaller than in the lower troposphere, so that the vertically integrated (whole troposphere) value of $-[\bar{\omega}^*\bar{T}^*]$ will still be negative. If we use $\bar{\omega}$-values derived by using the release of latent heat as an explicit forcing function, the production of standing eddy kinetic energy in the upper troposphere may become much larger. However, our primary object here is the study of the energetics of the monsoon trough, which is only in the lower troposphere. Considering the synoptic scale vertical motion, there is conversion of kinetic into potential energy in the lower troposphere.[2]

We have the term S as negative. However, we have obtained the work term as positive and as the main term contributing to the maintenance of the monsoon trough. Therefore, the net contribution from the other two terms, Q and R, should be positive, that is, there should be import of standing eddy potential energy from outside the region.[3] Therefore, the contributions from the terms Q and R have been estimated. For estimating Q, mean wind and contour charts were used. For estimating the vertical flux of geopotential (R), ω-values of Das (1962) were again used.

It is seen from table 1 that the loss of kinetic energy by conversion into potential energy by the indirect vertical circulation is largely offset by the vertical flux convergence (R). The vertical flux at the bottom is very large. In the region of upward motion over northeast India, the geopotential is higher than in the region of descent over Northwest India. The vertical flux at 500 mb is small, so that there is convergence of this vertical flux. The horizontal flux is also considerable. This is mainly accomplished by import from the monsoon westerlies at the southern boundary. The term N, that is the work done by horizontal pressure forces, is a small residue of these large terms.

The large values of the vertical flux convergence (R) and of the conversion into potential energy (S) are due to the large orographically induced vertical velocities used. Even if these vertical velocities were reduced to, say, 25 percent of their original values, the sign of these terms does not change, but magnitudes reduce to the order of the horizontal flux convergence (Q). The ω-values used do not explicitly include nonadiabatic effects. But the distribution of vertical velocities induced by heat source and sinks in the Indian region, again theoretically deduced by Das and Julka (1967), is similar to the one used in this study, that is, ascent over northeast India and descent over Northwest India-West Pakistan. Therefore, its inclusion will probably not affect the terms materially.

[2] However, in a discussion, Dr. T. N. Krishnamurti suggested that parameterization of the smaller scale cumulus convection is important and might affect the energetics. This aspect will be studied subsequently.

[3] This can be interpreted better as pressure work done at the boundaries.

5. CONCLUSIONS

1) The main contribution to the maintenance of the mean monsoon trough over north India is from the work done by horizontal pressure forces.

2) The trough is not a statistical result of transient perturbations.

3) The standing eddy loses its kinetic energy by the rising of cold air and sinking of warm air in the x-p plane. The circulation is an indirect one that is maintained by the flux of potential energy from outside, mainly from the bottom and southern boundaries.

ACKNOWLEDGMENTS

The authors wish to express their grateful thanks to Dr. R. Ananthakrishnan, Director, Institute of Tropical Meteorology, Poona, for his interest, encouragement, and many helpful discussions, to Dr. G. C. Asnani, Assistant Director (Training), Institute of Tropical Meteorology, Poona, for his interest, encouragement, many helpful discussions, and his lucid lectures on the "energetics of the atmosphere" at the Institute, to Dr. P. K. Das, Director, NHAC, for permission to use his $\bar{\omega}$-charts, to Dr. D. Sreeramamurthy, Mr. C. M. Dixit, and Mr. V. B. Rao for many helpful disscussions, to Mr. J. M. Pathan for assistance in computations, and to Mr. A. Girijavallabhan and Miss S. M. Bakre for typing.

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[*Received July 14, 1969; revised November 12, 1969*]