Intra-seasonal variations of kinetic energy of lower tropospheric zonal waves during northern summer monsoon

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Space spectral analysis of zonal (u) and meridional (v) components of wind and time spectral analysis of kinetic energy of zonal waves at 850 hPa during monsoon 1991 (1st June 1991 to 31st August 1991) for the global belt between equator and 40°N are investigated. Space spectral analysis shows that long waves (wavenumbers 1 and 2) dominate the energetics of Region 1 (equator to 20°N) while over Region 2 (20°N to 40°N) the kinetic energy of short waves (wavenumbers 3 to 10) is more than kinetic energy of long waves. It has been found that kinetic energy of long waves is dominated by zonal component while both (zonal and meridional) the components of wind have almost equal contribution in the kinetic energy of short waves.

Temporal variations of kinetic energy of wavenumber 2 over Region 1 and Region 2 are almost identical. The correlation matrix of different time series shows that (**i**) wavenumber 2 over Regions 1 and 2 might have the same energy source and (**ii**) there is a possibility of an exchange of kinetic energy between wavenumber 1 over Region 1 and short waves over Region 2. Wave to wave interactions indicate that short waves over Region 2 are the common source of kinetic energy to wavenumber 2 over Regions 1 and 2 and wavenumber 1 over Region 1. Time spectral analysis of kinetic energy of zonal waves indicates that wavenumber 1 is dominated by 30–45 day and bi-weekly oscillations while short waves are dominated by weekly and bi-weekly oscillations.

The correlation matrix, wave to wave interaction and time spectral analysis together suggest that short period oscillations of kinetic energy of wavenumber 1 might be one of the factors causing dominant weekly (5–9 day) and bi-weekly (10–18 day) oscillations in the kinetic energy of short waves.

1. Introduction

The Fourier technique is widely used to study the spectral characteristics of various atmospheric parameters in both space and time domain. The advantage of the Fourier technique is that the observed field gets decomposed into independent components. These components are called harmonics. Space harmonics are called zonal waves while time harmonics are called frequencies. Through space spectral analysis one can study the energetics of zonal waves and energy exchange among them. Time spectral analysis gives information regarding the variability (periodicity) of atmospheric parameters.

There are a number of studies where the Fourier technique has been used either for space or time domain. For example, works of Saltzman (1970); Krishnamurti and Kanamitsu (1981); Murakami (1981); Awade and Bawiskar (1982); Bawiskar *et al* (1989); Bawiskar and Singh (1992) are related to space spectral analysis while studies of Krishnamurti and Bhalme (1976); Lau and Chan (1988); Leite and Peixoto (1995) refer to time spectral analysis.

Shapiro and Fred (1960) examined the time-space spectrum of kinetic energy of the geostrophic meridional wind. Kao (1968) presented two dimensional (space and time) Fourier transform method. Dapradine (1978) has examined space-time spectra of the 200 mb motion field. Hayasi (1979b) presented a method of computing wavenumber frequency cross spectra.

In the present study, we propose to address the following points:

Keywords. Space spectral analysis; time spectral analysis.

Proc. Indian Acad. Sci. (Earth Planet. Sci.), **107**, No. 2, June 1998, pp. 121–126

- How the kinetic energy of long waves and short waves varies during the northern summer monsoon?
- Are the temporal variations of kinetic energy of long waves and short waves inter-related?

To study these aspects, first we have decomposed the observed u and v field at 850 hPa into a spectrum of zonal waves (space spectral analysis) and computed the kinetic energy. The time series of kinetic energy of long waves (wavenumbers 1st and 2) and short waves (wavenumbers 3–10) for 92 days (1st June 1991 to 31st August 1991) are prepared, intercompared and correlated. These time series are further decomposed into a spectrum of frequencies (time spectral analysis) so as to single out dominant oscillations during the northern summer monsoon.

2. Data

Daily u and v data at 850 hPa for the period from 1st June to 31st August 1991 were utilized for this study. The data were provided by the National Center for Medium Range Weather Forecasting (NCMRWF), New Delhi. The NCMRWF has a very sophisticated objective analysis and data assimilation system. The data contain objectively analysed gridded field cast $2.5^\circ \times 2.5^\circ$ latitude/longitude grid. We have considered the global area between the equator and 40°N.

The northern summer monsoon of 1991 has been selected because it is one of the normal monsoons of recent years and nearly 75% of Indian land mass received excess or normal rainfall. The broad circulation features of monsoon 1991 at 850 hPa are given in figure 1 which presents the stream function (Ψ) field. The Ψ values are computed at each grid point using time averaged (1st June to 31st August 1991) winds. The seasonal monsoon trough over India, southwesterlies over the Indian Ocean and seasonal high pressure cells over the Atlantic and Pacific Oceans are very well depicted (see figure 1) by the data set used for this study.

3. Methodology

Harmonic analysis technique for both space and time domain has been used. Bawiskar and Singh (1992) have presented a detailed procedure for the computation of kinetic energy of zonal waves in the space. In this study, we have considered the first ten waves. Wavenumbers 1 and 2 represent long waves and wavenumbers 3 to 10 represent short waves. The harmonic analysis technique for time domain has been used by many workers (e.g., Chapman and Lindzen 1970; Haurwitz and Cowley 1973; and Ananthakrishnan *et al* 1984). The harmonic analysis technique for time domain is briefly discussed below:

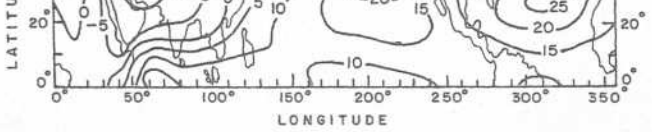
STREAM FUNCTION AT 850 hPa

Figure 1. Summer mean (1st June to 31st August 1991) stream function field at 850 hPa. Contour interval: $10^6\,\mathrm{m}^2\,\mathrm{s}^{-1}$.

Let $K(t)$ represent kinetic energy of a zonal wave. Here, t is time. $K(t)$ is expressed as:

$$K(t) = \sum_{\omega=0}^{T/2} a_\omega \cos\left(\frac{2\pi\omega t}{T}\right) + b_\omega \sin\left(\frac{2\pi\omega t}{T}\right), \qquad (1)$$

where T is total number of days. Although we have computed kinetic energy for 92 days (1st June to 31st August 1991), for convenience, we have considered the total period (T) of 90 days by omitting the first and last day of the time series. ω is harmonic number. $\omega/90$ gives the number of cycles per day. For example, ω equals to 3 represents 3/90 cycles per day, in other words, $\omega = 3$ means 30 day period.

Variance explained by an individual harmonic is given by

$$\sigma_\omega^2 = \tfrac{1}{2} R_\omega^2 \qquad (2)$$

where,

$$R_\omega = (a_\omega^2 + b_\omega^2)^{1/2} \qquad (3)$$

$$\%\sigma_\omega^2 = \frac{(\frac{1}{2} R_\omega^2)}{(\frac{1}{2}\sum_{\omega=1}^{T/2} R_\omega^2)} \times 100. \qquad (4)$$

Equation (4) gives the percentage variance explained by an individual harmonic ω. Since there is one observation in a day, the shortest period for which the spectrum can be estimated is two days.

4. Results and discussion

4.1 *Space spectral analysis (wave number domain)*

Figure 2 gives latitudinal variation of time averaged (1st June to 31st August 1991) kinetic energy for long waves (wavenumbers 1 and 2) and short waves (wavenumbers 3 to 10). The kinetic energy of long waves increases sharply from the equator to 12.5°N and then decreases northwards, whereas the kinetic energy of short waves gradually increases northwards. In the belt between the equator and 20°N, the kinetic energy of long waves dominates over short waves and in the belt between 20°N and 40°N, the kinetic energy of short waves is more as compared to the kinetic energy of long waves. Considering the above spatial variation of kinetic energy of long waves and short waves, we have

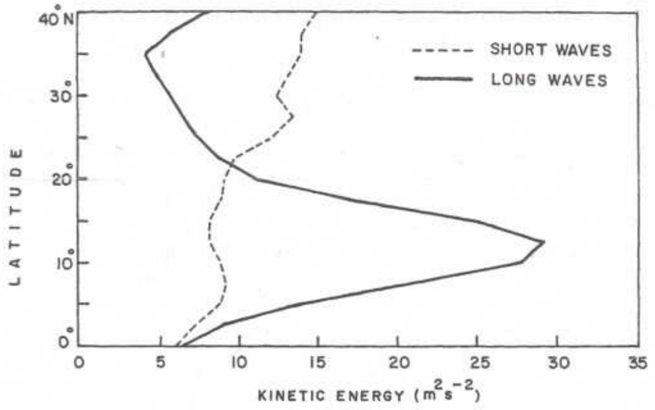

Figure 2. Latitudinal variation of time averaged (1st June to 31st August 1991) kinetic energy (unit: $m^2 s^{-2}$) of long waves (wavenumbers 1 and 2) and short waves (wavenumbers 3 to 10) at 850 hPa.

divided the global belt between the equator and 40°N into two regions; (**i**) from the equator to 20°N and (**ii**) from 20°N to 40°N. Hereafter, these two regions will be referred to as R1 and R2 respectively.

Table 1 gives contribution by zonal (u) and meridional (v) components of wind to the time averaged kinetic energy of long waves and short waves over R1 and R2. It can be seen from table 1 that the kinetic energy of long waves is dominated by zonal component while for short waves the contribution by both the components is almost equal (slightly higher for u). This shows that the meridional component of wind is equally important for short waves.

4.2 *Temporal variations of kinetic energy*

Figure 3 gives temporal variations of kinetic energy of wavenumbers 1, 2 and short waves over R1 and R2. The kinetic energy of wavenumber 1 (figure 3a) is substantially larger over R1 as compared to over R2. The day-to-day variation is totally different from one region to another and the variation about time mean is very large over R1. In case of wavenumber 2 (figure 3b) the pattern of variation over R1 and R2 is some what similar. The kinetic energy of short waves (figure 3c) is more over R2 as compared to over R1 for most of the days.

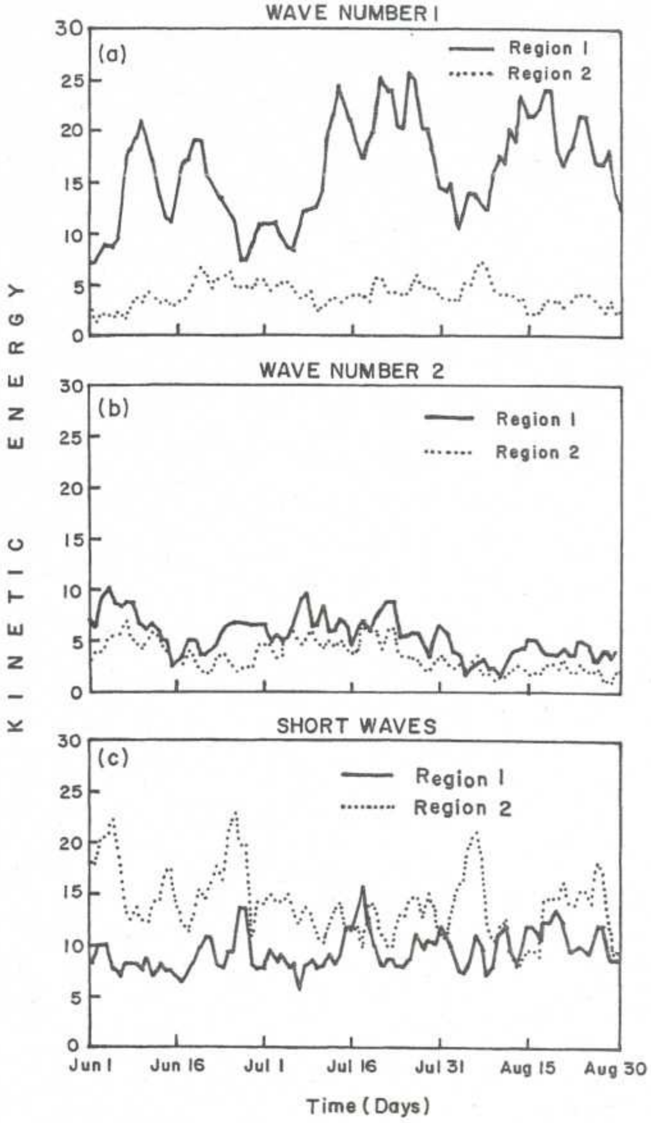

Figure 3. Temporal variations of kinetic energy (unit: $m^2 s^{-2}$) of (**a**) wavenumber 1, (**b**) wavenumber 2 and (**c**) wavenumbers 3 to 10 over Region 1 (equator to 20°N) and Region 2 (20°N to 40°N) at 850 hPa for the period from 1st June to 31 August 1991).

4.3 *Correlation coefficient between different time series*

To know the degree of dependency between any two time series, the correlation coefficient for all the possible combinations are computed and presented in

Table 1. *Contribution of zonal (u) and meridional (v) components of wind to the time averaged (1st June to 31st August 1991) kinetic energy (unit: $m^2 s^{-2}$) of long waves (wavenumbers 1 and 2) and short waves (wavenumbers 3 to 10) over Region 1 (equator to 20°N) and Region 2 (20°N to 40°N) at 850 hPa.*

Sr. No.	Waves	K.E. of u	K.E. of v
1.	Long waves over Region 1	20.12	1.47
2.	Long waves over Region 2	5.00	2.50
3.	Short waves over Region 1	5.09	4.36
4.	Short waves over Region 2	7.42	6.70

Table 2. *Correlation coefficient of different time series.*

Correlation coefficient	Wave 1 (R1)	Wave 2 (R1)	Waves 3–10 (R1)	Wave 1 (R2)	Wave 2 (R2)	Waves 3–10 (R2)
Wave 1 (R1)	–	−0.144	0.221	0.016	−0.069	**−0.499**
Wave 2 (R1)	−0.144	–	−0.150	−0.213	**0.723**	0.081
Waves 3–10 (R1)	0.221	−0.150	–	−0.046	−0.247	0.028
Wave 1 (R2)	0.016	−0.213	−0.046	–	−0.130	0.139
Wave 2 (R2)	−0.069	**0.723**	−0.247	−0.130	–	−0.008
Waves 3–10 (R2)	**−0.499**	0.081	0.028	0.139	−0.008	–

table 2. The most interesting feature is the correlation (0.723) between kinetic energy of wavenumber 2 over R1 and R2 which may imply that wavenumber 2 over R1 and R2 might have the same source of kinetic energy. The other significant correlation (−0.499) is between wavenumber 1 over R1 and short waves over R2 which implies that with the increase (decrease) of kinetic energy of wavenumber 1 over R1 there is decrease (increase) of kinetic energy of short waves over R2. A similar relationship exists between large scale (long waves) and synoptic disturbances (small waves) in a convective type of atmosphere. Colton (1973) showed that synoptic disturbances remove kinetic energy from planetary scale waves during their growing stage and feed energy back, while decaying. Similarly, we visualize the possibility of an exchange of kinetic energy between wavenumber 1 over R1 and short waves over R2. To verify the above possibility, we shall consider the exchange of kinetic energy among the waves in the next section.

4.4 *Exchange of kinetic energy*

Saltzman (1957) has derived spectral energy equations for measuring the rate of exchange of various forms of energy among the waves. Wave to wave interaction, $L(n)$ is one of the spectral equations which measures the rate of exchange of kinetic energy among the waves (please refer to Bawiskar and Singh 1992, for detail computational procedure of $L(n)$). A negative (positive) sign of $L(n)$, indicates, that wavenumber n is source (sink) of kinetic energy to other waves. It is seen from table 3 that wavenumbers 1, 2 and short waves over R1 and wavenumber 2 over R2 are sink of kinetic energy while wavenumber 1 and short waves over R2 are source of kinetic energy. The sum of all waves (wavenumbers 1 to 10) is positive (6.86 units) over R1 and negative (−2.37 units) over R2. Theoretically, the sum of $L(n)$ for all waves should vanish. But this is possible, only when the integration of $L(n)$, is considered over the entire globe. For a limited area (as in the present study) the sum need not necessarily be zero. Nonzero sum is called imbalance. Negative (positive) imbalance means surplus (deficit) kinetic energy over that region. According to Murakami (1981), a region of surplus kinetic energy transports the kinetic energy to the region of deficit kinetic energy. In our case R2 is a region of surplus kinetic energy and R1 is a region of deficit kinetic energy and therefore, waves over R2 will transport the surplus kinetic energy to waves over R1. Table 3 further indicates that shortwaves over R2 have a dominant contribution for having surplus kinetic energy over R2 and thus they play a major role in transporting the kinetic energy to waves over R1. In other words, shortwaves over R2 not only supply kinetic energy to wavenumber 2 over R2 but also transport and supply kinetic energy to wavenumbers 1, 2 and shortwaves over R1.

Table 3. *Wave to wave interaction, $L(n)$ at 850 hPa for the period June through August 1991 (unit: $10^{-6}\ m^2\ s^{-3}$).*

Wavenumber	Region 1 (Equator to 20°N)	Region 2 (20°N to 40°N)
1	2.20	−1.61
2	3.02	4.48
Short waves (waves 3–10)	1.64	−5.24
Total (waves 1–10)	6.86	−2.37

This explains why we are getting significant correlations (**i**) between wavenumber 1 over R1 and short waves over R2 and (**ii**) between wavenumber 2 over R1 and R2 in section 4.2. In the same section, we have also inferred that wavenumber 2 over R1 and R2 might have the same energy source. Here, we find that short waves over R2 is the common source of kinetic energy to wavenumber 2 over R1 and R2.

4.5 *Time spectral analysis (frequency domain)*

Spectral distribution of percentage variance (in time) of kinetic energy of wavenumbers 1, 2 and short waves (wavenumbers 3 to 10) over R1 and R2 is presented in table 4. As the contribution from higher harmonics is very small, the harmonics greater than 4 are grouped into three frequency bands as; (**i**) harmonics 5–9 (10–18 day oscillation), (**ii**) harmonics 10–18 (5–9 day oscillation) and (**iii**) harmonics 19–45 (2–4.7 day oscillation).

It is seen from table 4 that wavenumber 1 has a 30-day dominant oscillation over R1 and 45-day dominant oscillation over R2. In addition to the 30–45 day oscillations, the short period oscillations of

Table 4. *Spectral distribution of the percentage variance (in time) of the kinetic energy of zonal wavenumbers 1, 2 and 3–10 over Region 1 (equator to 20° N) and Region 2 (20° N to 40° N) at 850 hPa during the period from 2nd June 1991 to 30th August 1991.*

Harmonics	T Days	Wave 1 (R1)	Wave 2 (R1)	Waves 3–10 (R1)	Wave 1 (R2)	Wave 2 (R2)	Waves 3–10 (R2)
1	90	19.9	20.5	9.4	19.3	24.3	12.9
2	45	8.3	23.2	4.9	30.0	27.1	1.7
3	30	37.3	3.5	14.3	4.8	10.1	17.9
4	22.5	2.5	13.6	3.5	1.4	7.7	6.2
5–9	10–18	20.4	18.8	25.9	24.5	11.4	28.9
10–18	5–9	6.5	11.6	23.2	11.2	9.1	19.9
19–45	2–4.7	5.0	8.8	18.9	8.8	10.2	12.6

the 10–18 day period (bi-weekly oscillation) have also a significant contribution of about 20.4% over R1 and 24.5% over R2. Harmonic 2 (45 day oscillation) has the maximum contribution for wavenumber 2 over both the regions. In case of short waves, even though the first maxima occurs at harmonic 3 over both the regions, the major part of variance (more than 48%) is explained by weekly (5–9 day) and bi-weekly (10–18 day) oscillations. Krishnamurti *et al* (1976) detected quasi bi-weekly oscillations for many meteorological elements during the northern summer. We have also found dominant weekly and bi-weekly oscillations in the kinetic energy of short waves.

Thus, we find that long waves are dominated by the 30–45 day and bi-weekly oscillations and short waves are dominated by weekly and bi-weekly oscillations. But how far are they significant. A particular harmonic may be dominant but may not be statistically significant, or vice versa. To verify the significance, all the six series are subjected to power spectrum analysis and the results are discussed in the next section.

4.6 *Statistical significance test*

The computation of power spectra is based on the procedure given by Blackman and Tukey (1958). The Hamming method is used for smoothing the raw spectrum. Red noise spectrum and corresponding 95% confidence level (C.L.) are computed and presented in figure 4. The maximum lag used is 45 days but for convenience only results of 30 harmonics are presented. We find that harmonics 3 (30 day oscillation) and 8 (bi-weekly oscillation) for wavenumber 1 over R1 (figure 4a) are significant at 95% C.L. In the case of wavenumber 1 over R2 (figure 4b), harmonic 24 is significant. No harmonics are significant for

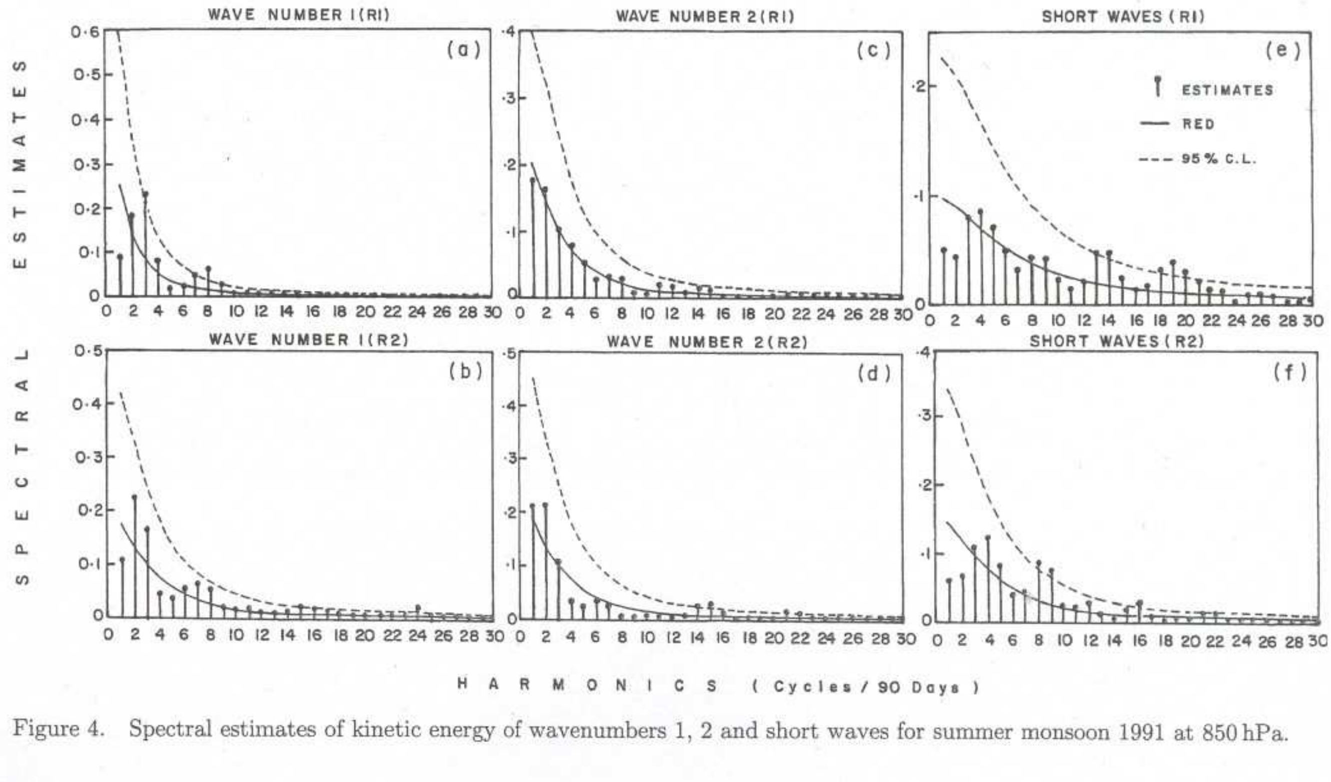

Figure 4. Spectral estimates of kinetic energy of wavenumbers 1, 2 and short waves for summer monsoon 1991 at 850 hPa.

wavenumber 2 over R1 (figure 4c). Harmonics 14, 15 (weekly oscillation) and 21, 22 (3–5 day oscillation) are significant for wavenumber 2 over R2 (figure 4d). In the case of short waves over R1 (figure 4e), harmonic 14 (weekly oscillation) and harmonics 18, 19, 20 (3–5 day oscillation) are significant. Harmonics 8 and 9 (bi-weekly oscillation) and harmonic 16 (weekly oscillation) are significant for short waves over R2 (figure 4f).

The results of harmonic analysis (table 4) and power spectrum analysis (figure 4) are somewhat identical. Most of the dominant harmonics are found to be significant at 95% confidence level. Only in the case of wavenumber 1 over R2, harmonic 2 (45-day oscillation) is dominant (table 4) but not significant (figure 4b).

All the results relating to wavenumber 1 over R1 and short waves over R2 lead to an interesting inference. Firstly, they are significantly correlated (−0.499), secondly, there is an exchange of kinetic energy between them and lastly, both have dominant (as well as significant) bi-weekly oscillations. On the basis of the above observations and results, we can infer that the short period oscillations of wavenumber 1 might be one of the factors causing dominant weekly and bi-weekly oscillations in the kinetic energy of short waves.

5. Conclusions

Even though the data sample is small to draw any definite conclusions, nevertheless the important points indicated in the present study are:

- Kinetic energy of long waves (wavenumbers 1 and 2) dominates over the kinetic energy of short waves (wavenumbers 3 to 10) over Region 1 (equator to 20°N) while the kinetic energy of short waves is more than the kinetic energy of long waves over Region 2 (20°N to 40°N).
- Kinetic energy of long waves is dominated by the zonal component of wind while both the components of wind have almost equal contribution to the kinetic energy of short waves.
- Temporal variations of kinetic energy of wavenumber 2 over Region 1 and Region 2 are almost identical.
- Short waves over Region 2 are the common source of kinetic energy to wavenumber 2 over Regions 1 and 2 and wavenumber 1 over Region 1.
- 30-day and bi-weekly oscillations are significant for wavenumber 1 and bi-weekly and weekly oscillations are significant for short waves.
- Short period oscillations of wavenumber 1 might be one of the factors causing dominant weekly and bi-weekly oscillations in kinetic energy of short waves.

Acknowledgement

The authors are thankful to the Director, Indian Institute of Tropical Meteorology, Pune, for his interest in the study. We are grateful to NCMRWF, New Delhi for having provided the grid point data for this study.

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MS received 12 May 1997; revised 3 October 1997