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Empirical algorithm to estimate the average cosine of underwater light field at 490 nm

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The average cosine of the underwater light field $\mu(\lambda)$, where λ is wavelength, is an apparent optical property that describes the angular distribution of radiance at a given point in water. Here we present a simple algorithm to determine the average cosine at 490 nm, $\mu(490)$, which was developed using the measured optical parameters from the eastern Arabian Sea and coastal waters off Goa. The algorithm is validated using measured optical parameters. This algorithm, based on a single optical parameter, performed better compared to other empirical algorithms to determine average cosine of underwater light field. The absorption coefficient at 490 nm, derived as an application of $\mu(490)$, compared well with the synthetic optical data and optical data measured from other regions.

1 Introduction

The average cosine of the underwater light field $\mu(\lambda)$, where λ is the wavelength, is an apparent optical property (AOP) that describes the angular distribution of radiance at a given point and is considered as the average cosine of the zenith angle of all photons at a particular point (Kirk 1994, Mobley 1994, Berwald *et al*. 1995). The average cosine is also the ratio of the net irradiance to the scalar irradiance and the spectral average cosine at a depth *z,* $\mu(\lambda)$ is given as

$$\mu\ (\lambda) = \frac{\left[E_{\mathrm{d}}(\lambda) - E_{\mathrm{u}}(\lambda)\right]}{E_{\mathrm{o}}(\lambda)} \tag{1}$$

where $E_d(\lambda)$, $E_u(\lambda)$, $E_d(\lambda)$ - $E_u(\lambda)$ and $E_o(\lambda)$ are the spectral downwelling irradiance, upwelling irradiance, net irradiance and scalar spectral irradiance at depth z. The value of $\mu(\lambda)$ varies between 0 and 1, with value of $\mu(\lambda) = 0$ indicating that light is uniformly distributed in the water, and all the light propagating vertically down when $\mu(\lambda) = 1$. The average cosine varies with water type, with values ranging from 0.1 to 0.3 for very turbid waters and 0.9 to 0.95 for very clear waters (Kirk 1994, Berwald *et al.* 1995, 1998, Haltrin 1998, Sokoletsky *et al.* 2003). The mean cosine is an apparent optical property so it depends on the incident solar light and the inherent optical properties such as absorption, scattering

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and the shape of volume scattering function (McCormic 1995). It is an important optical property that can be used to determine absorption coefficient using a relationship by Gershun 1939. Average cosine depends on the proportionality of absorption and scattering in water (Bannister 1992, Kirk 1981). Thus value of $\mu(\lambda)$ is comparatively higher in absorption dominated waters such as open ocean than in coastal waters where scattering dominates over absorption. It approaches an asymptotic value at great depths in homogenous waters (Preisendorfer 1959, Hojerslev and Zaneveld 1977).

Average cosine has been used for various applications such as primary productivity (Sathyendranath and Platt 1989, Morel 1991), chlorophyll distribution in water (McCormic 1995) and behavior of optical properties in deep waters (Preisendorfer 1959, Hojerslev and Zaneveld 1977). Average cosine has been modeled in terms of single scattering albedo (Timofeyeva 1971, Haltrin 1999) and in terms of the absorption and backscattering coefficient (Haltrin 1998).

A simple empirical algorithm based on a single optical parameter to determine $\mu(490)$ from the measured data and from the ocean color satellite data is discussed. Absorption coefficients of water derived using $\mu(490)$ were also evaluated comparing with the synthetic data and in-situ measured data from other regions.

2 Methodology

2.1 Data

The in-situ optical data were measured using an AC-9 instrument (Wetlabs Inc, Philomath, Oregon, USA) and hyperspectral profiling radiometer HyperOCR (Satlantic Inc., Halifax, Nova Scotia, Canada). AC-9 measures absorption and beam attenuation at nine wavelengths: 412, 440, 488, 510, 532, 555, 650, 676, 715 nm, and the HyperOCR measures upwelling radiance, downwelling irradiance and surface solar irradiance from 350 to 800 nm. These measurements were carried-out in the coastal waters off Goa and the estuaries of Mandovi and Zuari (15.40° N - 15.54° N, 73.70° E – 73.99° E) during 2010 and 2011 for 27 stations and during the cruise SSK-009 on *ORV Sidhu-Sankalp* in the eastern Arabian Sea (15.18° N - 16.88° N, 72.25° E – 73.58° E) for 9 stations. There were data from a total of 36 stations and of these data from 19 stations were used for the development of algorithms, while the remaining data were used for validation (Figure 1). The measurements were carried out between 11:00 to 13:00 hrs under fair weather conditions throughout the year except during rainy monsoon season (June to October). Most of the measurements were carried out with clear skies. Maximum water depth varied from 3 m to 20 m for coastal waters off Goa and the Mandovi and Zuari estuaries, and from 35 m to 250 m in the eastern Arabian Sea during the cruise SSK009. The Secchi depth, Z_{sd}, varied from 1.3 m to 20.5 m, diffuse attenuation coefficient at 490 nm, $K_d(490)$, varied from 0.0036 m^{-1} to 1.6 m^{-1} .

For the development of the algorithm, the optical data derived from radiative transfer simulations with the Hydrolight code Version 5.0 (Mobley 1994) were used. The inputs to the Hydrolight included measured inherent optical properties from AC-9, the surface solar irradiance available from the reference sensor of the hyperspectral radiometer, HyperOCR and the meteorological data.

Meteorological data of wind speed and relative humidity were obtained from the closest Automatic Weather Station at the National Institute of Oceanography, Goa (AWS-NIO) (http://inet.nio.org/index.html) for the coastal stations off Goa and from the meteorological station on the ship during the ship cruise. Considering that most of the stations were from coastal waters, the default volume scattering function of Petzold (1972) was used for the radiative transfer simulations. The reflectance measured close to the bottom was considered as the bottom reflectance for the measurements carried out in shallow waters.

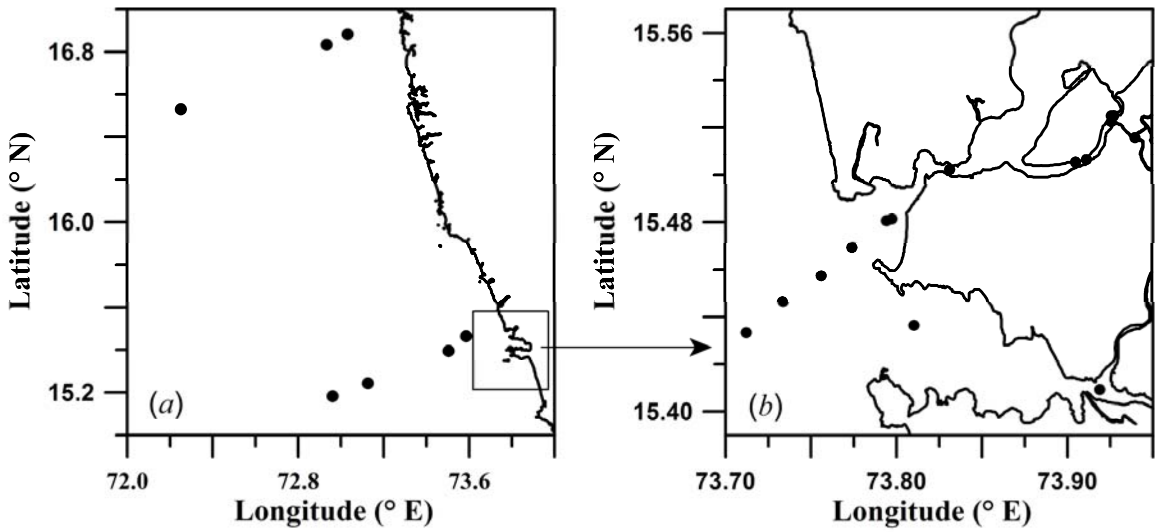

Figure 1 Station locations of the optical measurements carried out in (*a*) eastern Arabian sea and (*b*) coastal waters off Goa.

2.2 Algorithm for finding $\mu(490)$

From the measurements in the Indian Ocean, it has been found that the average cosine is highly correlated with the irradiance reflectance and shows an inverse relationship (Pelevin and Prokudina 1979, Kirk 1994, Stramska *et al*. 2000). Here the average cosine is modeled as a function of remote sensing reflectance at 490 nm, $R_{rs}(490)$, an optical parameter that can be obtained from ocean color satellite data. Since all the ocean color satellite sensors have satellite pass during noon, the influence of solar zenith is ignored in the algorithm. Using the $R_{rs}(490)$ and $\mu(490)$ from the simulation data a simple empirical relationship is formulated as

$$\mu(490) = p \times [R_{rs}(490)]^{q} \quad (2)$$

where p = 0.2035 and q = -0.223

Hereafter this algorithm is referred to as 'Mu'.

3 Assessment

μ(490) derived from Hydrolight simulations for various waters types was found to vary, with values above 0.7 for the waters with Secchi depth greater than 4.5 m and relatively lower values for turbid coastal waters (Figure 2).

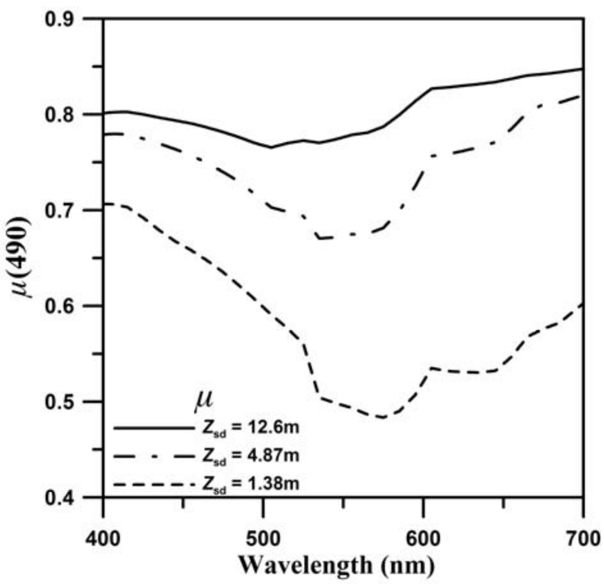

Figure 2. Average cosine, for various water types (Z_{sd} = Secchi depth)

3.1 Validation and evaluation of algorithm Mu

The Hydrolight simulations were carried out using the data reserved for validation. The algorithm Mu was validated using the μ(490) generated from Hydrolight simulations. The algorithm was also compared with two other algorithms of Timofeyeva (1971) and Haltrin (1998). The algorithm of Haltrin (1998) depends on the absorption and back scattering coefficient and algorithm of Timofeyeva (1971) depends on scattering and attenuation coefficients. The average cosine at 490 nm for the two algorithms of Timofeyeva and Haltrin were derived using the inherent optical properties available from the Hydrolight simulations. The average cosine derived from the algorithm Mu compared well with those obtained from the simulations and its performance was better compared to the algorithms of Timofeyeva and Haltrin. The error analyses was carried out using the statistical parameters average percentage deviation (APD), mean percentage deviation (MPD), root mean square error (RMSE) and coefficient of determination (R^2) (Suresh *et al*. 2011) (Table 1 and Figure 3).

Table 1. Comparisons of algorithms to determine $\mu(490)$

	Haltrin (1998)	Timofeyeva (1971)	Mu
4 RMSE	0.0489	0.0694	0.0497
APD	0.0512	0.0890	0.0603
MPD (%)	4.6217	0.2022	1.7771
$\boldsymbol{R^2}$	0.5869	0.6191	0.6870

RMSE: Root mean square error, APD- Average percentage deviation, MPD-Mean percentage deviation, R^2-coefficient of determination

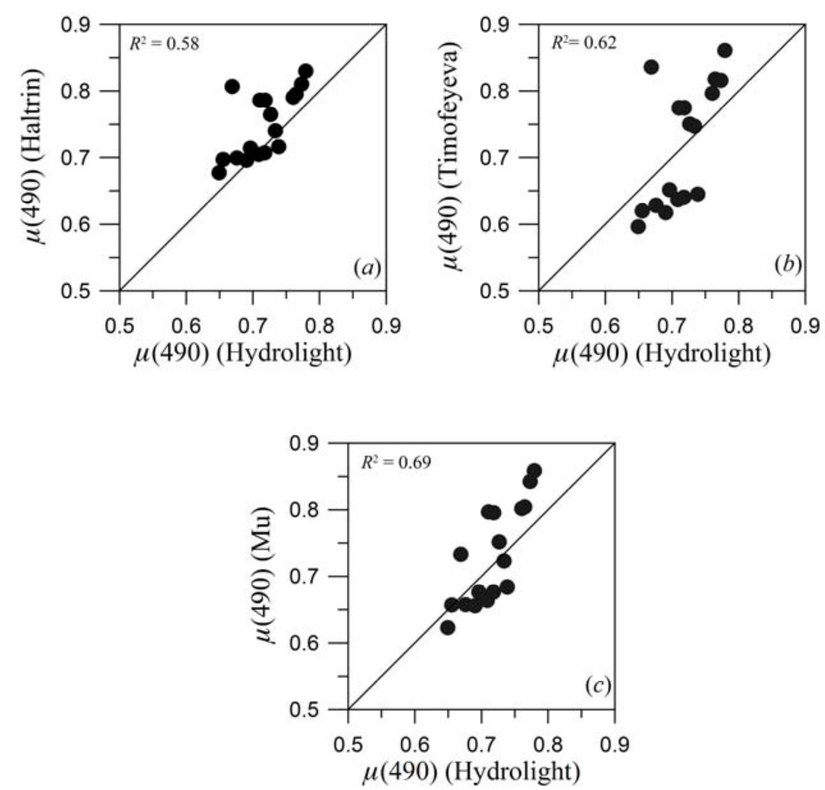

Figure 3. Comparison of algorithms to derive average cosine at 490nm using (*a*) Haltrin(1998), (*b*) Timofeyeva(1971) and (*c*) Mu.

4.1 Evaluation of a(490) algorithm with synthetic data

From the equation of Gershun (1939) the absorption coefficient of water is given as

$$\boldsymbol{a(\lambda) = \mu(\lambda) \times K_{\mathrm{E}}(\lambda)} \tag{3}$$

where $a(\lambda)$ is the absorption coefficient, an inherent optical property, and $K_E(\lambda)$ is the diffuse attenuation coefficient of net irradiance, $E_d(\lambda) - E_u(\lambda)$, an apparent optical property. The diffuse attenuation for vector irradiance $K_E(\lambda)$ is approximately equal to $K_d(\lambda)$, where $K_d(\lambda)$ is downwelling diffuse attenuation coefficient (Kirk 1994, McCormic 1995, Sokoletsky *et al*. 2004). The comparison of $K_E(\lambda)$ and $K_d(\lambda)$ data available from the Hydrolight simulations for various stations also supported this claim.

The absorption coefficient at 490 nm, $a(490)$ derived using $\mu(490)$ and diffuse attenuation coefficient at 490 nm, $K_d(490)$ as given in equation (3) were compared with the $a(490)$ available in the synthetic data of International Ocean Colour Coordinating Group (IOCCG) (IOCCG 2006). These optical data were generated using Hydrolight simulations for a wide range of bio-optical properties. These data are often considered for the development and evaluation of algorithms, as they are devoid of any measurement errors. The data can be downloaded from http://www.ioccg.org/groups/OCAG_data.html. The $\mu(490)$ was derived with the algorithm Mu, using $R_{rs}(490)$ from the synthetic data set. $a(490)$ was calculated using equation (3) with $K_d(490)$ available from the same data set and $\mu(490)$ derived from the algorithm. The $a(490)$ derived from the algorithm were compared with the $a(490)$ available in the synthetic data set. The $a(490)$ derived using the algorithm was found to closely match with the synthetic data over a wide range, with some underestimation. The performance of $a(490)$ derived using $\mu(490)$ was compared with $a(490)$ derived using the Quasi Analytical Algorithm, QAA (Lee *et al*. 2002, Lee and Carder 2004). The error analyses are given in Table 2 and Figure 4.

Table 2. Comparison of $a(490)$ obtained with QAA algorithm and $a(490)$ derived with $\mu(490)$ using the Mu algorithm with synthetic data.

	QAA	**$a(490)$ using $\mu(490)$**
RMSE	0.4651	0.0452
APD	0.2699	0.1546
MPD (%)	24.4485	-12.7822
R^2	0.6442	0.9816

RMSE: Root mean square error, APD- Average percentage deviation, MPD-Mean percentage deviation, R^2- coefficient of determination

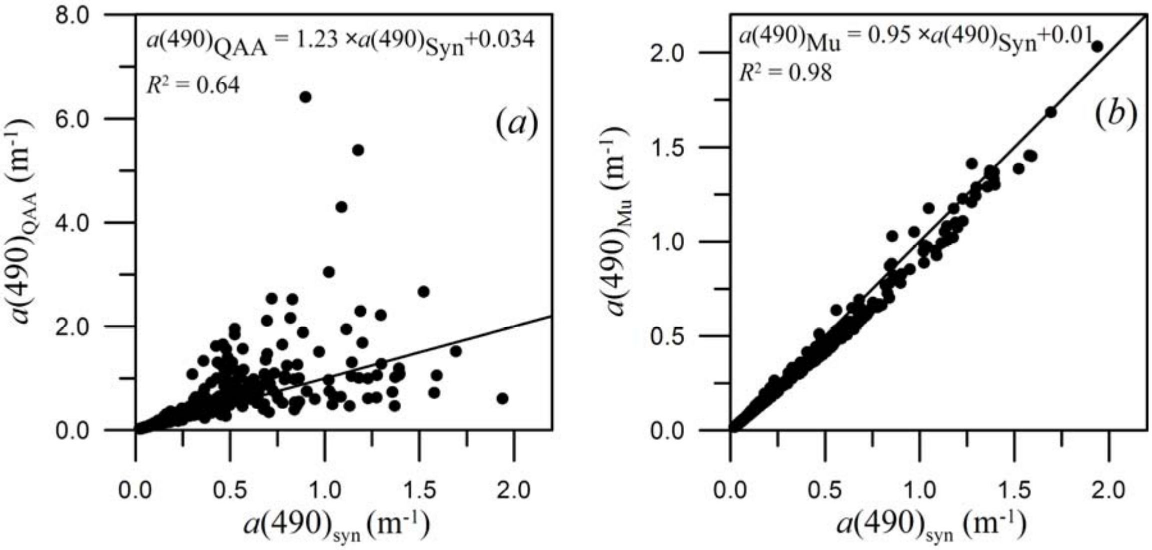

Figure 4. Comparison of $a(490)$ derived using synthetic data (IOCCG), $a(490)_{syn}$, with algorithms (*a*) QAA ($a(490)_{QAA}$) and (*b*) Mu ($a(490)_{Mu}$)

Another optical data set, NOMAD (NASA bio-Optical Marine Algorithm Data) Version 1.3, created on 1 September 2005 (Werdell and Bailey 2005) was also used for validation. This data set is a compilation of high quality in-situ data contributed by the ocean color research community from several regions and has been used for algorithm development and validations. The data can be downloaded from http://seabass.gsfc.nasa.gov/cgi/nomad.cgi. Similar procedure to evaluate $a(490)$ as followed for IOCCG synthetic data set was followed for the NOMAD data set (Table 3 and Figure 5).

Table 3. Error analysis for $a(490)$ derived using $\mu(490)$ derived using Mu algorithm with NOMAD data

RMSE	0.1290
APD	0.4109
MPD (%)	-18.1789
$\boldsymbol{R^2}$	0.9024

RMSE: Root mean square error, APD- Average percentage deviation, MPD-Mean percentage deviation, R^2-coefficient of determination

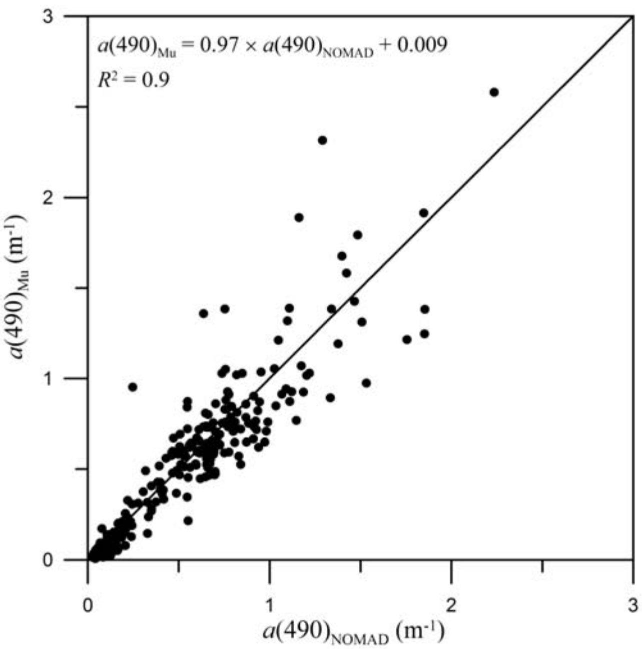

Figure 5. Comparison of $a(490)$ from NOMAD ($a(490)_{NOMAD}$) with $a(490)$ derived from NOMAD data using algorithm Mu ($a(490)_{Mu}$)

5 Conclusion

There are presently few algorithms available for finding the average cosine of underwater light field. The algorithm given here is simple and uses only a single optical parameter. The performance of this algorithm was found to be more accurate, compared to other algorithms.

Average cosine μ could be used with the diffuse attenuation coefficient to determine the much-required inherent optical property of spectral absorption coefficient of water. The absorption coefficients derived using this method compared well with the synthetic data and in-situ data measured over other regions. Presently there are empirical and semi-analytical bio-optical models, which are used for determining absorption and backscattering properties of the constituents in water, which need to be refined and improved (Shanmugham 2010). The absorption coefficient can be obtained from satellite data with only two parameters, $\mu(490)$ and $K_d(490)$ and algorithms for these require only remotely sensed reflectance, $R_{rs}(490)$. $K_d(490)$ can be retrieved from satellite data with less error with algorithms such as that of Ocean Biology Processing Group of NASA, OBPG (Werdell and Bailey 2005, Suresh *et al*. 2011). Since this algorithm can retrieve the absorption coefficient at 490 nm with higher accuracy, it can also improve the performance of empirical algorithms to obtain beam attenuation and scattering coefficients (Doron *et al*. 2006).

Presently the algorithm depends on the absolute value of the remote sensing reflectance, hence for satellite ocean color remote sensing applications, its accuracy will depend on the accuracy of the retrieved value of $R_{rs}(490)$. However the approach given here to model average cosine to determine absorption coefficient could be explored further and adopted for remote sensing and development of bio-optical models. Using this methodology with larger data set from measurements in various water types the algorithm could be further refined to provide spectral average cosine, which could be used to find spectral absorption coefficient of water.

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