Financial Market Imperfections and Aggregate Fluctuations

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Boston College Electronic Thesis or Dissertation, 2010

Boston College

The Graduate School of Arts and Sciences

Department of Economics

FINANCIAL MARKET IMPERFECTIONS AND AGGREGATE FLUCTUATIONS

a dissertation

by

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submitted in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

August 2010

DISSERTATION ABSTRACT

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This dissertation examines the fluctuations of the aggregate economy when frictions in financial markets are present. I focus on the the asymmetric information problems between creditors and debtors on the quality of debtor's projects and I analyze how these frictions cause the fluctuations in aggregate economy which is potentially inefficient.

The first chapter examines the interaction between the perverse incentives and the general equilibrium effects of misallocated bank credit. This essay is intended to elucidate the mechanism of zombie lending in Japan. By incorporating a soft budget problem into a neoclassical dynamic general equilibrium model, the model shows that an inefficient zombie lending regime can be selected as an equilibrium. In this equilibrium, the incentives and the general equilibrium effects are interdependent. The inefficient use of resources crowds out investment when banks have incentives to bail out insolvent firms. On the other hand, the general equilibrium effects give rise to the perverse incentives endogenously through the formation of the liquidation value and the continuation value of insolvent firms. In the

worst case, agents fail to resolve non-performing loan problems, and the model economy permanently falls into an inefficient regime.

The second chapter proposes a model that generate boom-and-bust cycles by securitization of subprime mortgages. I construct a dynamic housing choice model in which mortgages are financed by securitization and I assume that creditors have errors in measuring the default risks of subprime mortgages. With this setup, the resource availability for housing fluctuates endogenously and it causes the boom-and-bust cycles. Particularly, there are two channels that change the resource availability: the security design of the securitized assets and the evolution of house price inflation. I illustrate that subprime mortgages can be cheaply financed by securitization when creditors mismeasure the quality of the subprime mortgages. This ignites a boom in the model. However, the boom can be terminated as the profitability of securitization declines along with the decline in the expectation of house price inflation. This is because the house price inflation is tied with the liquidation value of the defaulted mortgages. As the expectation of the house price inflation slows down, the subprime mortgages become more risky and the securitization becomes less profitable. Eventually, issuers of securitized assets withdraw from the securitization market and the boom collapses.

The last chapter explores the transmission mechanisms of international business cycles when the borrowing capacity of multinational enterprises (MNEs) is limited. I embed MNEs that face borrowing constraints in a two-country international business cycle model. I show that the net worth of MNEs plays a significant role in generating the international business cycle co-movement: the wealth effect in response to the change in MNEs' net worth has a strong multiplier effect on domestic and foreign investment of MNEs. Output moves in the same direction between the two countries due to the synchronized investment. The model is also able to generate reasonable cross-country correlations in real estate price and consumption.

Acknowledgements

I am deeply grateful to Professor Basu for his sincere advising. His broad knowledge on economics guided my dissertation to the right direction. And his technical advice was fundamental for tuning up the details of my research. Also I really appreciate that he always accommodated himself to and make arrangement for the unusual situation that I have been placed because of the dual occupation. Without his advising and help, it would have been never possible to complete this dissertation.

I am also indebted to Professor Iacoviello and Professor Konishi. Professor Iacoviello's proficiency on the topics of my dissertation helped me greatly in shaping my research: many of the ideas and the details of my dissertation were drawn from his advising and teaching. The game-theoretical approach in my dissertation is inspired by the discussion with Professor Konishi. His advising gave me a lot of insights in incorporating a game in a macroeconomic model.

The support of Mr. Kato, who was my former boss in Bank of Japan, was tremendous and much appreciated. His experience and guidance were like beacon lamps for a sailor, me, on dark ocean.

I thank my classmates who were present in my informal seminars. I especially thank Samson Alva, Alessandro Barattieri, Isabella Blengini, Shoghik Hovhannisyan and Radoslav Raykov for their suggestions and comments. These suggestions and comments are beneficial in shaping ideas in early stages of my study.

Last, but not least, I would like to thank my wife Akiko for her invaluable support. Her support, which took in various forms, encouraged me when I struggled in my research. Her devotion and encouragement were, ultimately, what motivated me to pursue my study to the end.

Contents

Chapter 1

Incentives and Macroeconomic Effects of Zombie Lending in Japan

1.1 Introduction

This paper examines the perverse incentives of misallocated bank credit and its macroeconomic effects during the 1990s in Japan. The focus of this paper is the dynamic interaction between banks' incentives and the depressing macroeconomic effects of zombie lending. Here, I define zombie lending as the bailout of insolvent firms by extending additional credit. In the model, the incentives and the general equilibrium effects affect one another. Additionally, this interaction creates a coordination failure problem among savers, borrowers and financial intermediaries in resolving issues involving non-performing loans. I show that this coordination failure can induce the economy to select an inefficient zombie lending regime.

It has been widely argued that the prevalence of zombie lending is among the major causes of the decade-long economic downturn in Japan. Hoshi and Kashyap (2004, 2005) hypothesize that zombie lending caused the prolonged recession in Japan in the 1990s. Peek and Rosengren (2005) and Sekine, Kobayashi and Saita (2003) find evidence that is consistent with zombie lending. Moreover, Caballero, Hoshi and Kashap (2008), Barseghyan (2008) and Dekle and Kletzer (2005) develop theoretical foundations of depressive effects of zombie lending on businesses and economic growth. However, we still lack formal studies on the interaction between the incentives and the effects of zombie lending. The model in this paper addresses the following questions jointly: why do banks repeatedly bail out insolvent firms? How do the incentives for banks affect the performance of the Japanese economy? Does the poor performance of Japan's economy aggravate banks' incentives to rescue insolvent firms?

To answer these questions, I utilize the studies of soft budget constraints developed by Dewatripont and Maskin (1995) and Berglöf and Roland (1997, 1998) and others. The advantage of utilizing soft budget constraints is that the concept of soft budget constraints matches that of zombie lending and highlights the perverse incentives for banks to rescue insolvent firms.[1] The setup of soft budget constraints consist of lenders, good borrowers and poor borrowers. Poor borrowers have incentives to divert borrowing to enjoy private benefits, and lenders are incapable of distinguishing poor borrowers from good borrowers. If poor borrowers divert, they ask the lenders to refinance their projects. The key in this problem is that ex ante and ex post incentives for lenders are different due to the sunk nature of initial lending. If the liquidation value of the borrowers is small and the continuation value for which lenders do not consider the cost of initial lending is large, lenders ultimately refinance the debt of poor borrowers. Taking this into account, poor borrowers safely enjoy private benefits. This moral hazard problem is the essence of soft budget constraints.

The novelty of this paper is to incorporate the structure of soft budget constraints into a neoclassical-style dynamic general equilibrium model. This framework enables us to examine the interaction between the incentives and the general equilibrium effects of zombie lending. I endogenize the liquidation value and the continuation value of insolvent firms, thereby linking the outcome of general equilibrium with the incentives of bailout. The liquidation value consists of the value of land that borrowers own. This specification is used to follow banking practices in Japan, in which land is the chief means of collateral.

[1] Although the soft budget constraint was originally designed to examine the perverse incentives on credit allocation in socialist countries and transition economies, it can be applied to bad-loan problems in Japan. For example, see Hosono and Sakuragawa (2003).

I show that the lack of coordination among banks, savers and borrowers in resolving non-performing loan problems causes the economy to select inefficient refinancing equilibrium. To get an intuition, suppose that households (savers) and firm owners (borrowers, but land providers at the same time) anticipate that banks will refinance non-performing loans. The non-performing loans reduce the rates of return due to the inefficient use of saving. Households are thus discouraged from saving, and the capital stock declines. Firm owners lower the valuation of land due to the complementarity of land and capital. On the other hand, the collective behavior of households and firm owners forms the liquidation value and the continuation value of insolvent firms. These values can be consistent with banks' incentives to rescue insolvent firms. As a result, households' and firm owners' suppositions are valid, and the economy as a whole selects the inefficient zombie lending regime. This phenomenon is similar to the prisoner's dilemma in the sense the lack of cooperation to fight against low-return outcomes forces the agents to select low-return outcomes.

This paper is closely related to the theoretical studies of zombie lending conducted by Caballero, Hoshi and Kashayp (2008), Barseghyan (2008) and Dekle and Kletzer (2005). Caballero et al. (2008) point out that keeping unproductive firms alive creates congestion effects; costs of production rise because unprofitable firms, who normally exit from markets, soak up resources. Then productive firms reduce new businesses creation and increase existing businesses destruction which are inefficient. Barseghyan (2008) and Dekle and Kletzer (2005) yield qualitatively similar results. Their models predict that bailouts of banks crowd out investment because the government soaks up saving to bail out the banks. Effectively, my model yields the crowding out effect on investment. In this sense, the macroeconomic effects of zombie lending in this paper are not much different from those of the previous works. However, this paper goes one step further by endogenizing the incentives of the bailout.[2] The incentives and the general equilibrium effects are inter-

[2] To my knowledge, only Brandt and Zhu (2001) have utilized a soft budget constraint in a macroeconomic model. Their objective is to examine the macroeconomic effects of the incentive for the Chinese government to keep state-owned firms alive. However, they assume that the value of refinancing state-owned firms is large enough. In this sense, the emergence of a soft budget constraint is fated in their model.

dependent in my model; zombie lending raises loan interest rates due to the risk premium of loans, and it generates crowding-out effects on investment. At the same time, the allocation of resources and prices support the incentives of zombie lending. This paper shows that the interaction between the incentives and the general equilibrium effects can induce the economy to select an inefficient refinancing regime as an equilibrium.

This paper is organized as follows. Section 1.2 presents an overview of the zombie lending in Japan to set the context for this paper. Section 1.3 builds up the model. I describe the financial contract and the dynamic general equilibrium sequentially. Section 1.4 presents the main results of this paper. I present the dynamic properties of the model, the sources of inefficiencies in the zombie lending regime and a simulation result in that section. Section 1.5 conducts some policy analyses to show the desirable policy for restoring efficiency. Finally, section 1.6 concludes this paper.

1.2 An overview of zombie lending in Japan

During the 1990s, when Japan faced the so called "lost decade," the Japanese banking sector faced a rising amount of non-performing loans; the overall macroeconomic condition deteriorated in this decade, causing record numbers of borrowers becoming insolvent. Eventually, banks responded to this challenge by engaging in bailouts of insolvent firms, which is called zombie lending. According to Caballero, Hoshi and Kashyap (2008), the share of firms that received interest concessions from banks suddenly increased at the beginning of the 1990s and remained large throughout the decade.[3] Peek and Rosengren (2005) find that the likelihood that a firm obtains additional credit from a bank increases as the profitability of the firm deteriorates. They call it a perverse incentive of credit.

Sudden and large fall of commercial land prices at the onset of the 1990s is a factor

[3] Caballero et al. (2008) define zombie lending as the loans for which implied ex post loan interest rates, which are calculated from actual loan repayments, are lower than the minimum required interest rate, which is calculated from public data. The measure may not cover all of the bailout, as there would be other forms of bailout than interest concessions.

that triggered zombie lending. Commercial land was the main form of collateral of the bank loans at that time. In this economic environment, banks had trouble writing off non-performing loans: The low value of collateral would have significantly impaired the banks' capital if they had written off the non-performing loans. The effects of the plunge of commercial land prices were pronounced due to the fact that banks increased lending to the real estate sector during the 1980s, where it heavily utilized land as collateral (See Ueda (2001)). Eventually, banks extended additional credit to insolvent firms to keep them alive in the hope that these firms would recover. Ogawa and Kitasaka (2001) find evidence that is consistent with this view. They estimate the loan supply function of the Japanese banks and find that the change in land price has a positive and significant effect on the loan supply during the late 1980s. More interestingly, the effect becomes ambiguous after 1990. This is consistent with zombie lending; banks did not respond to the plunge of the value of main collateral in a systematic way. Some banks actually increased loan supplies, which may be due to zombie lending.

Another factor that contributed to zombie lending is so-called regulatory forbearance. Hoshi and Kashyap (2004, 2005) and Hosono and Sakuragawa (2003) point out that the loose standards of bank supervision caused and enabled banks to hide non-performing loans. Although the Japanese government followed the BIS standard on bank capital requirements, the government did not actively inspect the quality of the loan portfolios of the Japanese banks. In this circumstance, banks had incentives to hide non-performing loans to meet the capital requirements. Ultimately, banks disguised non-performing loans as good loans by extending additional credit to insolvent firms. This structure implies that the government bailed out banks because these loans were generating losses and were eventually rescued by deposit insurance. The deposit insurance in Japan guaranteed the full amount of deposits at that time.

Although there is plenty of evidence about zombie lending, its contribution to the macroeconomic condition is not so conclusive. First of all, zombie lending alone cannot fully account for the stagnation of the Japanese economy. The structural reduction of work hours enforced by a government regulation apparently contributed to the fall of output.[4] Moreover, Hayashi and Prescott (2002) argue that the financial breakdown was not a source of the economic downturn in Japan. Instead, they claim that the decline of total factor productivity (TFP) was the fundamental source. However, they remark that the financial distress might have caused the deterioration in their measure of TFP. Moreover, the way in which they measure the TFP, which is simple growth accounting, is too naive to support their points. A more rigid estimate of the TFP by Kawamoto (2005) finds that the growth of TFP did not slow down during the 1990s. More recent studies that utilize micro data suggest that zombie lending had significant effects on economic activities. For example, Caballero, Hoshi and Kashyap (2008) find evidence of congestion effects caused by zombie lending. They find that the level of investment and the growth in the employment of healthy firms decline in industries in which the share of zombie lending is large. To this end, it would be safe to say that zombie lending had at least some impact on the condition of the overall economy.

1.3 The model

This section presents the building blocks of the model to analyze the interaction between the incentives and the macroeconomic effects of zombie lending. The model has two ingredients, one-shot financial contracts and dynamic consumption-saving decisions. A simplified version of soft budget constraints (hereafter SBC) characterizes the financial contracts. Dynamic optimization is taken from neoclassical-style dynamic general equilibrium models. As we see later, the interaction between the outcome of the financial contracts and the dynamic optimization creates rich implications for the selection of equilibria.

In the model, a firm owner appoints a project manager who is the unique agent for

[4] Christiano and Fujiwara (2006) examine the general equilibrium effects of the structural work-hour reduction in the 1990s together with the economic boom in preceding years.

managing production. I define a firm as a composite of a firm owner and a project manager. Banks and firms are involved in the financial contracts. The outcome of the financial contracts is either a hard or a soft budget constraint. SBC results in bailouts of insolvent firms by banks, which matches the concept of zombie lending. Finally, households and firm owners make dynamic consumption-saving choices given the outcome of the financial contracts.

1.3.1 Financial contracts

A firm makes a financial contract with a bank. Before the arrangement of the financial contract, a firm owner appoints a project manager who is the unique agent for implementing production in the model. There are two types of project managers. One is good and the other is poor. The total population of firm managers is a unit mass and the share of good managers is fixed to $\eta \in [0, 1]$ at all periods. I assume that banks as well as firm owners cannot distinguish poor managers from good managers ex ante. Because a firm owner and a project manager are tied with a one-to-one relationship, I characterize a financial contract as an arrangement between a bank and a project manager (either good or poor). In the contracts, banks are lenders, while project managers are borrowers. All project managers die after the contracts end. I shall use "he" as a pronoun to indicate a bank and "she" to indicate a project manager (either good or poor) in order to avoid confusion.

All managers have two technologies: they convert the borrowing from banks into capital, and then they produce the consumption good by combining capital and other resources, which households and firm owners provide directly. The problem in this contract is that poor managers potentially divert a part of initial borrowings to enjoy private benefits. The capacity of the diversion is denoted by $\nu \in [0, 1]$, that is, poor managers can divert ν units of consumption good per unit of initial borrowing. In cases where a poor manager diverts, she asks the bank for additional credit to complete the project.[5]

[5] An implicit assumption is that the diversion alone is not sufficient for poor managers to enjoy private

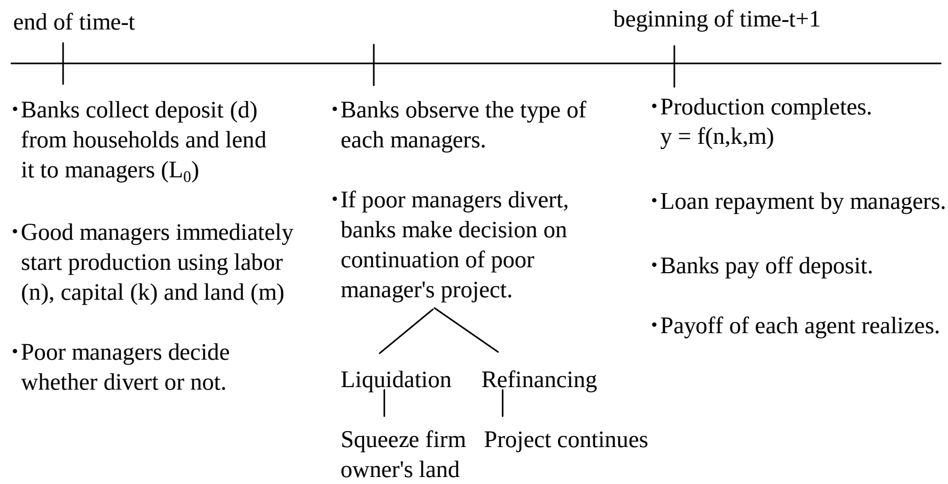

Figure 1-1: Flow of events in the financial contract

Good managers

Good managers act in good faith, and they start production as soon as they get financed from banks.[6] The production technologies of good managers are summarized below:

$$\text{capital creation:} \quad L_0 = k,$$

$$\text{consumption good production:} \quad y = A(n)^{\alpha_n}(k)^{\alpha_k}(m)^{1-\alpha_n-\alpha_k}.$$

The first line describes the capital creation function. Good managers convert borrowings into capital in one-to-one relation. The second line describes the production function of the consumption good: it is a Cobb-Douglas function of labor (n), capital (k) and land (m), and it has a constant return to scale property. A is the total factor productivity. α_n and α_k are the labor share and the capital share, respectively. Good managers hire labor from the labor market where households are the suppliers of labor. A firm owner directly supplies

benefits. Survival until the end of the period and repaying the contractual return to banks is necessary to enjoy private benefits.

[6]Alternatively, we can assume that good managers are not endowed with the capacity for diversion. In this case, the good managers have no choice but to behave even if they want to enjoy private benefits as poor managers.

land to a project manager when the firm owner appoint the project manager. Households and firm owners are not actively involved in the financial contracts.

Poor managers

Poor managers are endowed with the capacity to divert a part of their initial borrowings, and their objective is to pursue private benefits by the diversion: poor managers can divert borrowing by $\nu \in [0, 1]$ per unit of initial borrowing. They can convert it to private benefits if they obtain refinancing and keep their positions in the firms until the end of the financial contracts. The private benefits are proportional to the amount of diversion, νL_0. However, poor managers are not able to enjoy the private benefits if they lose their positions due to the liquidation of firms. Moreover, poor managers expense costs in this case, which is proportional to the amount of the diversion. This can be interpreted as, for example, legal costs brought about by a lawsuit for misappropriation. Poor managers have an outside option to prevent losses from unsuccessful diversions. The outside option is to begin production immediately after they get their initial borrowings. I assume that poor managers have the same technologies as good managers but that poor managers do not obtain private benefits by engaging in production.[7]

Poor managers' initial choice is whether to try to divert or not. Their payoff is described as follows:

$$PB = \left\{ \begin{array}{ll} \psi\nu L_0 & \text{if a poor manager diverts and the bank refinances} \\ 0 & \text{if a poor manager decides to produce immediately} \\ -\psi\nu L_0 & \text{if a poor manager diverts but the bank does not refinance} \end{array} \right\}.$$

I assume that the private benefit is proportional to the amount of diversion, νL_0, if it is successful. ψ is the coefficient that controls the magnitude of the benefit. I assume that ψ

[7] It would be feasible to assume that poor managers' technologies are inferior to those of good managers. However, the poor managers' incentives to divert rather than the technologies are what cause inefficiencies in my model. As a result, the difference in the technologies is non-essential.

is small so that the social planner will not prefer poor managers to divert in equilibrium. As noted above, their private benefit becomes negative if they fail to divert. This type of preference ranking is frequently assumed in SBC literatures. This also implies that poor managers conduct diversions only if refinancing is credible. It is ideal for banks to commit to liquidate poor managers' projects if they ask refinancing. However, I will show that such a commitment is not credible under certain conditions. I assume that poor managers are no longer capable of diversion after obtaining refinancing. Thus, poor managers finally start production after they obtain refinancing.

Banks

Banks collect deposits from households (which I introduce later) and lend them to project managers. The timing of deposit collection is twofold: first, a bank raises deposits at the beginning of contracts to finance all the project managers who apply to the bank for loans. Recall that the bank cannot tell poor managers from good managers ex ante. He thus cannot select borrowers according to the type of managers at this point.[8] Second, the bank raises additional deposits, if necessary, during the interim period of the contracts when he refinances poor managers' projects. I shall assume that banks can raise deposits as much as they want during the period under the financial contracts. (In the general equilibrium, households' choices regarding saving and firms' borrowing demand determine the amount of deposit available to banks.) Given the assumption of nonidentifiability, banks initially lend the same amount to all project managers who apply to the banks for loans. I denote the total amount of deposits raised by a bank as d and the total amount of lending as L. L can be divided into two parts, $L = L_0 + L_1$. Here, L_0 is the amount of initial lending and L_1 is the amount of refinancing, if any.

The objective of banks is to maximize profits from lending. The profit of banks is

[8]Deposits are made in the form of the consumption good which is non-storable. Also, there is no outside option for the usage of deposits. Banks thus have no option but to lend to all managers who apply for loans.

defined as:

$$\pi_{bank} = \tilde{R}\tilde{L} - Rd.$$

In this profit formula, R, $\tilde{R}$ and $\tilde{L}$ denote the gross deposit interest rate, the gross loan interest rate and the repaid portion of lending, respectively. $\tilde{L}$ can be L, the total amount of lending, or a part of L, depending on whether or not poor managers divert. As we will see later, $\tilde{L}$ is a constant fraction of d in any equilibrium. I assume that the banking sector is competitive and that banks take the interest rates as given. The competitive nature of the banking sector and the constant return to scale of the profit with respect to d imply that banks' profit is zero in equilibrium.

Now suppose that a poor manager diverts and asks the bank for refinancing, at which point he is able to observe the type of the manager. The bank has two options: refinancing her project or liquidating it. If the bank refinances the poor manager's project, he extends credit by the same amount as the diversion. That is, the amount of refinancing, L_1, is $(1-\eta)\nu \times L_0$.[9] In this case, the loans to good managers, the non-diverted portion of loans to poor managers and the refinancing loans are repaid. I assume that the poor manager is no longer capable of diversion after she obtains refinancing. She thus finally starts to produce the consumption good. If the bank liquidates the project, he then squeezes the assets of the firm, whose value is Q. Banks take the liquidation value as given in the financial contracts. I will discuss banks' incentives to refinance in section 1.3.1, and I will describe what consists of firms' liquidation value in section 1.3.2.

The incentive of refinancing – Soft budget constraint

To understand why banks rescue poor firms, it is important to distinguish between ex ante and ex post incentives due to the sunk nature of initial lending. At the time when

[9] Each poor manager obtains νL_0 and the population of poor managers are $(1-\eta)$. Because the production function is continuous in k, there can be other refinancing schemes. Indeed, as long as $\tilde{R} > R$, which is true in the refinancing regime, banks can obtain unbounded profit by setting $L_1 \to \infty$. However, profit maximizing firms never demand an infinite amount of borrowing and they demand just the same amount as poor managers divert given $\tilde{R}$. As a result, the options for banks are whether to refinance by νL_0 or liquidate the project.

banks realize that poor managers divert, the diverted part of initial lending is already sunk. Banks thus compare the net return from new lending with the liquidation value of the outstanding loan for the refinancing-liquidation choice. This leads us to the following refinancing condition.

Condition 1 *Banks are willing to refinance poor managers' projects ex post if and only if* $(\tilde{R} - \nu R)L_0 \geq Q$

The left-hand side of this condition is the net ex post return from refinancing. By compensating for the lost part of initial lending, banks obtain the gross return $\tilde{R}L_0$. Refinancing requires banks to raise additional νL_0 units of deposits, which costs $R\nu L_0$. The net ex post return is thus $(\tilde{R} - \nu R)L_0$.[10] Note banks do not consider the cost of initial lending when calculating the ex post return. This is because the cost of initial lending is already sunk. Note also that all banks have a common incentive, as R, $\tilde{R}$ and Q are given and identical among all banks.

Banks' ex ante incentive is different from their ex post incentive: banks do not have an ex ante incentive to refinance if $\tilde{R} < (1+\nu)R$, that is, if the rate of return of the investment is less than the cost of investment, including refinancing. If this condition is satisfied, banks, ex ante, have incentives to commit not to refinance. However, this commitment is not credible because poor managers know that banks are willing to refinance once the banks realize that poor managers diverted the borrowing.

Having viewed the incentive for banks, poor managers' incentive is obvious. Poor managers divert if and only if condition 1, banks' ex post incentive, is satisfied. Otherwise, poor managers immediately start production because they anticipate that diversion will not be successful. This implies the liquidation never occurs in equilibrium.

[10] In principle, banks can squeeze required repayments without refinancing due to the continuity of the production function: as long as total output less wage payments is more than the required loan repayment, banks can claim that part of the output because firm owners are the residual claimers in this model. However, the assumption that the firm owners severely punish poor managers implies that the poor managers would not produce output without refinancing. This is because poor managers would be unwilling to engage in production while they are punished.

Note that the amount of capital produced by poor managers is the same as that of good managers, regardless of whether poor managers behave or divert. Even if poor managers divert, banks cover the losses in equilibrium due to the incentive structure above. Then, the amount of borrowing that poor managers invest to produce capital is the same as that of good managers. The amount of loan repayment is also the same across all firms for the same reason. The amount is $\tilde{R}L_0$ in any case.

1.3.2 Dynamic general equilibrium

In this subsection, I describe dynamic consumption-saving choices and the concept of dynamic general equilibrium. Households and firm owners make the dynamic choices in my model. Households' savings go to banks, but firm owners have means of saving through land transactions. The main reason why I differentiate between the means of saving is to obtain analytical properties of the equilibrium. In calculating the allocation of the general equilibrium, the financial contracts above are built in period by period.[11] That is, once households make decisions regarding saving, banks take the savings as deposits and lend them to project managers. Thereafter, the decision mechanisms of banks and project managers are as described above

Households

There exists a unit mass of infinitely-lived households. They derive utility from consumption, c_h. They supply a fixed amount of labor, normalized to one, in each period. The means of saving for households are bank deposits. One unit of saving today yields R units of gross return tomorrow. Households are the fundamental savers in this model. Formally, households' problem is written in a simple way as:

[11] I can safely introduce the contracts period by period because firm managers die in one period. The histories of project managers become irrelevant, and banks have no way of identifying the type of managers at the beginning of each period.

$$\max \sum_{s=t}^{\infty} \beta^{s-t} \log c_{h,s},$$
$$s.t.\ c_{h,t} + d_{t+1} = w_t + R_t d_t.$$

β is the subjective discount rate of households, d is the saving of households and w is real wage rate. Solving this problem yields the following first-order conditions:

$$\frac{1}{c_{h,t}} = \frac{\beta R_t}{c_{h,t+1}}, \tag{1.1}$$
$$\lim_{T\to\infty} \beta^{T-t} \frac{1}{c_{h,T}} d_{T+1} = 0.$$

Firm owners

Firm owners own land and appoint project managers to utilize their land. A manager, either good or poor, is randomly assigned to a firm owner, given the assumption that firm owners are incapable of judging the types of project managers. Firm owners have unlimited liability; they compulsorily provide their wealth to banks when their firms default. This setup differs from the notion of a corporation, where firm owners (or shareholders) have limited liability. However, this case is more relevant to the bank lending structure in Japan: more than 70% of bank loans were originated to small-to-medium sized firms whose structures are characterized by unlimited liability. (See, for example, Ogawa and Kitasaka (2000)). Also, this specification is a convenient way of highlighting the role of land in the determination of liquidation value and equilibrium.

Firm owners live infinitely and derive utility from consumption, c_l. Their resources are the profit from the operation of firms, π_t, and the value of land, $q_t m_t$, which they own at the beginning of the period. q denotes the price of land, and I assume that the total supply of land in this economy is fixed to $\bar{m}$ for all periods. As I described in the beginning of section

1.3.2, excluding firm owners from the deposit market enables me to derive the analytical properties of the equilibrium. The firm owners' problem is written as follows:

$$\max \quad \sum_{s=t}^{\infty} \tilde{\beta}^{s-t} \log c_{l,s},$$
$$s.t. \quad c_{l,t} + q_t m_{t+1} = \pi_t + q_t m_t.$$
$$\pi_t = y_t + (1-\delta)k_t - w_t n_t - \tilde{R}_t L_{0,t}.$$

$\tilde{\beta}$ is the subjective discount rate of firm owners and δ is the depreciation rate of capital.[12] Firm owners are residual claimers in the sense that the profit, π_t, is gross output $(y_t + (1-\delta)k_t)$ less wage and loan payments.[13] Although firm owners face uncertainty in the assignment of project managers, there is no uncertainty in capital creation, production and loan repayment. Remember that regardless of whether poor managers behave or divert, they produce the same amount of capital as good managers. Poor managers also repay the same amount to banks as good managers. I can thus write the profit formula without uncertainty. Firm owners are the agents who determine the resource allocations of the firms, and project managers are the agents who implement the production and the financial transactions in this model. Capital creation as well as labor demand are thus relevant choice variables for firm owners.

Solving the firm owner's problem yields the following first-order conditions:

[12] Capital can be converted to the consumption good in a one-to-one relation by project managers.

[13] In equilibrium, this corresponds to the product of land.

$$\frac{q_t}{c_{l,t}} = \frac{\tilde{\beta}}{c_{l,t+1}}[(1-\alpha_k-\alpha_n)\frac{y_{t+1}}{m_{t+1}} + q_{t+1}], \tag{1.2}$$

$$w_t = \alpha_n \frac{y_t}{n_t},$$

$$\tilde{R}_t = \alpha_k \frac{y_t}{k_t} + (1-\delta),$$

$$\lim_{T\to\infty} \tilde{\beta}^{T-t} \frac{1}{c_{l,T}} q_T m_{T+1} = 0.$$

Other than the role of land as an input of production, the value of land, $q_t m_t$, functions as the liquidation value of firms. Remember that project managers themselves do not own any resources. When a poor manager defaults and the bank decides to liquidate the firm (though liquidation does not occur in equilibrium), the bank claims the firm owner's land because this represents the only asset that the firm possesses. In this sense, the liquidation value, Q, which is taken as given in the financial contract is the value of land, qm, and this value is determined endogenously in the general equilibrium.

It is worthwhile to note that the liquidation value of firms consists solely of the value of land. This implies that the economy is vulnerable to specific shocks to land value, as the fluctuation of the land price is crucial in determining banks' incentive and the entire regime of the economy.

It is an assumption that firm owners do not have access to the deposit market. The payoff of this assumption is that this enables us to obtain analytical solutions of firm owners' consumption and the land price. This comes from the fact that the net savings of firm owners are always zero in equilibrium ($q_t m_{t+1} = q_t m_t = q_t \bar{m}$).[14] On the other hand, this specification generates a gap in the rate of return between capital and land. In Appendix A, I construct a model in which I unify households and firm owners, thereby equating the rates of return. There, I show that the one-sector model has the same properties as the benchmark model.

[14]The homogeneity of firm owners and the fixed supply of land imply that each firm owner owns the same amount of land at every period.

Equilibrium

Because the production function of each manager exhibits a constant return to scale and identical technology, I can safely aggregate their production functions:

$$y_t = A_t (n_t)^{\alpha_n} (k_t)^{\alpha_k} (m_t)^{1-\alpha_n-\alpha_k}.$$

Here, n_t denotes the labor demand. In equilibrium, the labor demand is set equal to the labor supply, which is fixed to one. Thus prices satisfy the following conditions:

$$\begin{aligned} w_t &= \alpha_n y_t, \\ \tilde{R}_t &= \alpha_k \frac{y_t}{L_{0,t}} + (1-\delta). \\ q_t &= \tilde{\beta} \left(\frac{c_{l,t+1}}{c_{l,t}} \right)^{-1} \left[(1-\alpha_k-\alpha_n) \frac{y_{t+1}}{m_{t+1}} + q_{t+1} \right]. \end{aligned} \tag{1.3}$$

The first two conditions state that the rental rates of labor and capital are their marginal products. The last line tells that the land price today is the sum of the discounted value of the marginal product and the resale value of land tomorrow.

In equilibrium, this economy falls into one of the two regimes, depending on the ex post incentive for banks. I define the two regimes as follows:

Definition 1 *The economy is said to be in the normal regime at time s if*

$$\begin{aligned} (\tilde{R}_s - \nu R_s) L_{0,s} &< q_s \overline{m}, \\ d_s = L_{0,s} = k_s, &\qquad \tilde{R}_s = R_s. \end{aligned} \tag{1.4}$$

and it is said to be in the refinancing regime at time s if

$$\begin{aligned}(\tilde{R}_s - \nu R_s)L_{0,s} &\geq q_s\overline{m}, \\ \frac{1}{1+(1-\eta)\nu}d_s = L_{0,s} = k_s, &\qquad \tilde{R}_s = [1+(1-\eta)\nu]\,R_s.\end{aligned} \tag{1.5}$$

The equations in the second line of the refinancing regime hint at the inefficiencies of that regime. First, we can see that not all the savings are allocated to the creation of capital (indicated by the term $\frac{1}{1+(1-\eta)\nu}d_s$). Second, the loan interest rate is higher than the deposit interest rate (indicated by $\tilde{R}_s = [1+(1-\eta)\nu]\,R_s$). This is because banks need to squeeze more from the undiverted part of lending because the other part is not repaid.

Having defined the regime, I introduce the notion of a "regime path".

Definition 2 (Regime Path) *Let $\iota_s \in \{0,1\}$ be an indicator variable at time s where $\iota_s = 1$ if the economy is in the normal regime at time s and $\iota_s = 0$ otherwise. A regime path $\{\iota_s\}_{s=t}^{\infty}$ is a sequence of ι_s. Let I be the set of all regime path. Define $I^N \in I$ as the normal path where $\iota_s = 1$ for $\forall s \geq t$, and Define $I^R \in I$ as the refinancing path where $\iota_s = 0$ for $\forall s \geq t$.*

Now I am in a position to define the concept of an equilibrium.

Definition 3 (Equilibrium) *An equilibrium is a regime path $\{\iota_s\}_{s=t}^{\infty}$, an allocation $\{c_{h,s}, c_{l,s}, d_{s+1}, m_{s+1}\}_{s=t}^{\infty}$ and associated prices $\left\{w_s, R_s, \tilde{R}_s, q_s\right\}_{s=t}^{\infty}$ given d_t and m_t such that*

1. *Households' optimality condition (1.1) is satisfied*

2. *Firm owners' optimality condition (1.2) is satisfied. And each firm owner owns $m_s = \bar{m}$ for $\forall s \geq t$.*

3. *$\left\{w_s, \tilde{R}_s, q_s\right\}_{s=t}^{\infty}$ satisfies (1.3)*

4. *If* $\iota_s = 1$, *(1.4) is satisfied. In particular,* $(\tilde{R}_s - \nu R_s)d_s < q_s\overline{m}$. *The aggregate resource constraint is given by* $y_s = c_{h,s} + c_{l,s} + d_{s+1} - (1-\delta)d_s$.

5. *If* $\iota_s = 0$, *(1.5) is satisfied. In particular,* $(\tilde{R}_s - \nu R_s)\frac{d_s}{1+(1-\eta)\nu} \geq q_s\overline{m}$. *The aggregate resource constraint is given by* $y_s = c_{s,t} + c_{s,t} + d_{s+1} - \frac{(1-\delta)}{1+(1-\eta)\nu}d_s$.

For future reference, I call an equilibrium where $\{\iota_s\}_{s=t}^{\infty} = I^N$ and the associated allocation and the prices satisfy the definition above as the normal equilibrium. On the other hand, I call an equilibrium where $\{\iota_s\}_{s=t}^{\infty} = I^R$ and the associated allocation and the prices satisfy the definition above as the refinancing equilibrium.

1.4 The dynamics and the selection of equilibrium

I examine the dynamic properties and the equilibrium selection of the model in this section. I first show the analytical properties of the dynamics in two special equilibria, namely, the normal equilibrium and the refinancing equilibrium. I then show wedges incurred by zombie lending through the Euler equation of households' consumption. Finally, I conduct a simulation to illustrate the interaction between the incentive and the macroeconomic effects.

1.4.1 The dynamics

In the normal equilibrium, the equations that describe the dynamics of the economy are represented by the following system:

$$\begin{aligned}
\frac{1}{c_{h,t}} &= \frac{\beta}{c_{h,t+1}}\left[\alpha_k A_{t+1} d_{t+1}^{\alpha_k - 1}\bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta)\right], \\
c_{h,t} + d_{t+1} &= (\alpha_k + \alpha_n)A_t d_t^{\alpha_k}\bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta)d_t, \\
c_{l,t} &= (1-\alpha_k-\alpha_n)A_t d_t^{\alpha_k}\bar{m}^{1-\alpha_k-\alpha_n}, \\
q_t &= \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n)A_t d_t^{\alpha_k}\bar{m}^{-\alpha_k-\alpha_n}.
\end{aligned} \tag{1.6}$$

The latter two equations reveal that firm owners' consumption and the land price have analytical solutions: firm owners consume a constant fraction of output (remember that $k_t = d_t$ in this regime, so $y_t = A_t d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n}$.) and the land price is a constant fraction of the marginal product of land today. However, it does not mean that firm owners do not optimize dynamically. Instead, the assumption that firm owners are excluded from the deposit market induces them to consume a constant fraction of output in the equilibrium. Then, by substituting the consumption function into the land price formula (the third line in (1.3)), we are able to see that firm owners evaluate the future marginal product of land as a constant fraction, $\tilde{\beta}^s$, of today's marginal product of land. This dramatically simplifies the land price formula (See Appendix A for the derivation of (1.6)).

We can analyze household's consumption-saving decision separately by the first two equations in (1.6). (These equations derive from households' Euler equation and the flow of budget constraint.) Figure 1-2 shows the graphical intuition of the dynamics of households' consumption and savings. This is a similar diagram as in the one-sector neoclassical growth model (See, for example, Barro and Sala-i-Martin (2003)). The two loci in the graph correspond to the set of points in (d, c_h) space such that consumption does not change ($c_{h,t+1} - c_{h,t} = 0$) and savings do not change ($d_{t+1} - d_t = 0$). The $\Delta c_h = 0$ locus is increasing and concave in (d, c_h) space, and the $\Delta d = 0$ locus is concave and intersects twice with $c_h = 0$ axis, with one of these intersections at the origin. As the graphical intuition suggests, households' consumption-saving dynamics under the normal equilibrium are summarized as follows:

Proposition 1 *Let (d^N, c_h^N) be the intersection of the $\Delta c_h = 0$ locus and the $\Delta d = 0$ locus. Then, (d^N, c_h^N) is the unique steady state of the normal equilibrium. Moreover, it is globally saddle-point stable and globally monotone in the sense $\{c_{h,s}, d_s\}_{s=t}^{\infty}$ is an increasing sequence when $d_t < d^N$ and is a decreasing sequence when $d_t > d^N$. Finally, $\{c_{l,s}\}_{s=t}^{\infty}$ and $\{q_s\}_{s=t}^{\infty}$ are increasing sequences when $d_t < d^N$ and are decreasing sequences when $d_t > d^N$.*

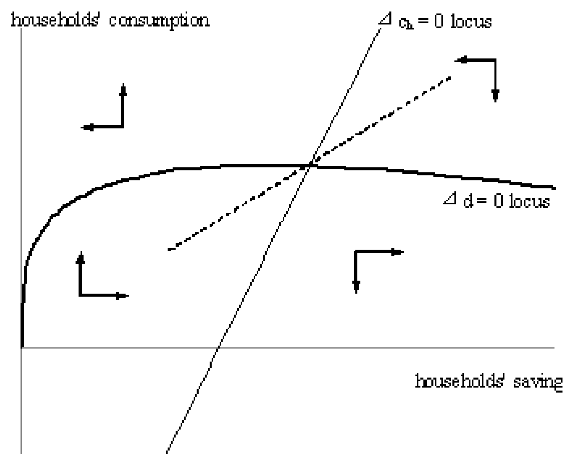

Figure 1-2: Phase diagram of households consumption and saving.

Proof. See Appendix A.■

The statement above is almost obvious from the phase diagram and the last two equations in (1.6). When households' savings are below the steady state, households accumulate savings and increase consumption over time by enjoying the return from savings. The speed of saving accumulation slows down as they become close to the steady state, and they eventually converge to the steady state. During this course, firm owners increase their consumption over time as their profit increases. Notice that the marginal unit of land yields more output as the economy accumulates more capital. Mathematically, the cross-derivative of the marginal product of land with respect to capital is positive. This implies that the land price, as a function of the marginal product of land, also rises over time. Finally, it must be true that banks do not have ex post incentives to refinance poor managers' projects on the course of the dynamics. Otherwise, the normal equilibrium is not sustainable.

We can obtain the corresponding system of equations, the phase-diagram and the dynamic properties of the refinancing equilibrium. The system of equations in the refinancing equilibrium is:

$$\begin{aligned}
\frac{1}{c_{h,t}} &= \frac{\beta}{c_{h,t+1}}\left\{\tfrac{1}{1+(1-\eta)\nu}\left[\alpha_k A_{t+1}\left(\tfrac{d_{t+1}}{1+(1-\eta)\nu}\right)^{a_k-1}\bar{m}^{1-\alpha_k-\alpha_n}+(1-\delta)\right]\right\},\\
c_{h,t}+d_{t+1} &= (\alpha_k+\alpha_n)A_t\left(\tfrac{d_t}{1+(1-\eta)\nu}\right)^{a_k}\bar{m}^{1-\alpha_k-\alpha_n}+\tfrac{(1-\delta)}{1+(1-\eta)\nu}d_t, \qquad (1.7)\\
c_{l,t} &= (1-\alpha_k-\alpha_n)A_t\left(\tfrac{d_t}{1+(1-\eta)\nu}\right)^{a_k}\bar{m}^{1-\alpha_k-\alpha_n},\\
q_t &= \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n)A_t\left(\tfrac{d_t}{1+(1-\eta)\nu}\right)^{a_k}\bar{m}^{-\alpha_k-\alpha_n}.
\end{aligned}$$

The phase diagram of the refinancing equilibrium is similar to that of the normal equilibrium. However, the $\Delta d = 0$ locus is always below that of the normal equilibrium, and the $\Delta c_h = 0$ locus intersects with the $\Delta d = 0$ locus at the point (d^R, c_h^R) at which $d^R < d^N$ and $c_h^R < c_h^N$. Remember that (d^N, c_h^N) is the steady state of the normal equilibrium. As a result, households' consumption and savings in the steady state of the refinancing equilibrium are less than those of the normal equilibrium. This implies that firm owners' consumption is less and that the land price is lower in the steady state of the refinancing equilibrium. The intuition is that households are discouraged from saving because of the low return of deposits under the refinancing regime. This comes from the fact that poor managers divert in the refinancing regime. I give a more detailed explanation of this mechanism in the next subsection.

1.4.2 Wedges in the refinancing regime.

Wedges arise in the refinancing regime precisely because poor managers divert their borrowings. We can see the wedges when we look at the Euler equation of households:

$$\frac{1}{c_{h,t}} = \frac{\beta}{c_{h,t+1}}\left\{\underbrace{\tfrac{1}{1+(1-\eta)\nu}}_{(b)}\left[\alpha_k A_{t+1}\underbrace{\left(\tfrac{d_{t+1}}{1+(1-\eta)\nu}\right)}_{(a)}^{a_k-1}\bar{m}^{1-\alpha_k-\alpha_n}+(1-\delta)\right]\right\}. \qquad (1.8)$$

Part (a) in the equation (1.8) represents the waste of savings. In the refinancing regime, poor managers divert some of their borrowings to enjoy private benefits. This impedes the full use of households' savings from the social point of view.[15] Moreover, this waste of savings forces banks to charge higher loan interest rates to meet the deposit obligation. Part (b) in (1.8) reflects this and it can be interpreted as the inverse of the risk premium of bank loans. Combining (a) and (b), I can show that the return from deposits in the refinancing regime is less than that of the normal regime given the level of savings.

The underlying story initially comes from the demand side of loans. The higher loan interest rate depresses the borrowing demands of firms. In equilibrium, the smaller borrowing demands lower the deposit interest rate, thereby reducing households' savings. This mechanism is qualitatively similar to the congestion effects studied by Caballero et al. (2008) and the crowding-out effects studied by Barseghyan (2008). In my model, all of the adjustments of investment take place through intensive margins. The mechanism in my model is thus closer to the crowding-out effects. In my model, banks' endogenous incentive effectively creates the crowding-out effects while they arise from the exogenous government bailout in Barseghyan (2008).

As for the wedges for firm owners, the loan risk premium (part (b)) does not take effect because the firm owners do not save in the form of deposits. However, the waste of savings still affects their welfare. The reduced rate of capital creation ($k = \frac{d}{1+(1-\eta)\nu}$) lowers their profit and thus it lowers the consumption of firm owners (See the third equation of (1.7)).

[15] As I assumed, the private benefits for firm managers are small relative to the utility of households and firm owners (ψ is small).

1.4.3 The interaction between the incentive and the macroeconomic effects

The Incentive in equilibrium

The most interesting feature of my model is that banks' incentive can entirely change the regime of the economy: the direction of the inequality $(\tilde{R} - Rv)L_0 \gtreqless q\overline{m}$ is important in determining an equilibrium. Before examining the nexus of the incentive and the general equilibrium effects, it is useful to introduce conditions on the determination of equilibrium regimes. By substituting the equilibrium conditions (1.6) and (1.7) into the incentive conditions in (1.4) and (1.5) (see details in Appendix A), I get the following statement:

Lemma 1 *At time s ($s \geq t$), the normal regime can be supported in an equilibrium if and only if*

$$(1-\nu)[\alpha_k + \tfrac{1-\delta}{A_s \overline{m}^{1-\alpha_k-\alpha_n}} d_s^{1-a_k}] < \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n). \tag{1.9}$$

Likewise, at time s ($s \geq t$), the refinancing regime can be supported in an equilibrium if and only if

$$\tfrac{1-\eta\nu}{1+(1-\eta)\nu}[\alpha_k + \tfrac{1-\delta}{A_s \overline{m}^{1-\alpha_k-\alpha_n}} \left(\frac{d_s}{1+(1-\eta)\nu}\right)^{1-a_k}] \geq \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n). \tag{1.10}$$

Proof. See Appendix A.■

The intuition of this lemma becomes clear when we set $\delta = 1$, when the capital depreciates fully in one period. In this case, only the parameters $(\alpha_k, \alpha_n, \tilde{\beta}, \nu, \eta)$ matter for the selection of the regimes. First, a higher capital share indicates a greater likelihood that the refinancing regime is supported. This is because the value of capital increases and the value of land decreases when α_k becomes large. An increase in $\tilde{\beta}$ makes the refinancing regime less likely to be supported as this implies that firm owners value land more highly. When ν rises, it becomes less likely that the refinancing regime is supported. An increase in η yields the same prediction. Increases in these parameters increase the cost of refinancing.

When $\delta \neq 1$ the level of savings affects the incentive condition and makes the refinancing regime more likely to hold. It is possible that both inequalities are simultaneously satisfied, implying that the model can have multiple equilibria. However, the following condition eliminates many possible equilibria.

Proposition 2 *Assume that total factor productivity, A, is constant over time. Let d^* satisfy the following equation:*

$$\max\left\{(1-\nu)[\alpha_k + \tfrac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}}(d^*)^{1-a_k}], \tfrac{1-\eta\nu}{1+(1-\eta)\nu}[\alpha_k + \tfrac{1-\delta}{A_s\bar{m}^{1-\alpha_k-\alpha_n}}\left(\frac{d^*}{1+(1-\eta)\nu}\right)^{1-a_k}]\right\}$$
$$= \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n).$$

Then only the refinancing regime is supported at time s in an equilibrium if the level of saving satisfies $d_s \geq d^*$.

Proof. See Appendix A.■

This condition still does not guarantee the global uniqueness of the solution. However, it narrows the set of possible equilibria. In particular, we have a following corollary about the feasibility of the refinancing equilibrium.[16] (I display a graphical intuition of the corollary in Figure 1-3.)

Corollary 1 *Assume that total factor productivity, A, is constant over time. The refinancing equilibrium can be supported as an equilibrium if $d_t \geq d^R$ and $d^* \leq d^R$ where d_t is the initial level of deposit, d^R is the steady-state level of deposit in the refinancing equilibrium and d^* is defined as in proposition 2.*

---

[16] I suspect that none but the refinancing equilibrium is the equilibrium in this case. The intuition is the following: the refinancing equilibrium is the worst equilibrium in the sense that it produces the least output compared to other equilibria where the normal regime is supported at some points of time. Even in this "worst" equilibrium, households do not reduce savings below d^R if the initial saving level is greater than d^R. Then for any equilibrium, it would not be optimal to have savings lower than d^R, as the marginal return of savings is at least as large as in the refinance equilibrium at any point of time.

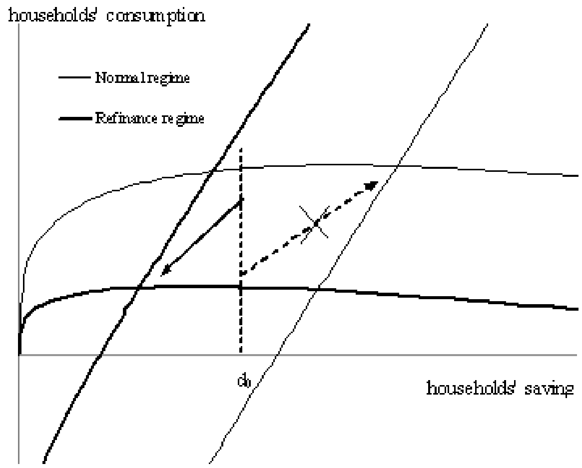

Figure 1-3: Graphical intuition of corollary 1

A simulation – factor specific shock

In this subsection, I conduct an experiment to show how the perverse incentive of zombie lending interacts with the general equilibrium effects. The interaction causes the economy to select the refinancing equilibrium under certain sets of parameters. I interpret this interaction as a coordination failure among households, firm owners and banks to resolve non-performing loan problems; they are not able to internalize the behavior of other parties, thereby acting with the expectation of inevitable bailouts. This materializes the welfare decreasing outcome.

I give a shock to the model that changes incentives for banks. The shock that hits in this economy is a permanent shock to α_k. This raises the marginal product of capital and lowers the marginal product of land under the parameter values in Table 1-1. Intuitively, this shock lowers the liquidation value while raising the net return from refinancing. Note that the focus of this simulation is the results of changes in banks' incentive. The capital share shock is a convenient way to change the incentive.

Table 1-1 shows the parameter values in pre-shock periods and post-shock periods. The time frequency is annual. The discount rate and the depreciation rate are consistent with many RBC studies. In principle $\tilde{\beta}$ can differ from β. However I assume $\beta = \tilde{\beta}$ in this

	Description	Pre-shock		Post-shock
$\beta, \tilde{\beta}$	Discount rates	0.975		0.975
δ	Depreciation rate	0.1		0.1
α_k	Capital share	0.28	$\rightarrow$	0.30
α_n	Labor share	0.67		0.67
$1-\eta$	The share of poor managers	0.15		0.15
ν	Diversion capacity	0.15		0.15

Table 1-1: Parameter values of the economy

simulation. I calibrate the share of poor managers $(1-\eta)$ to match the observations about zombie lending reported in Caballero et al. (2008). They estimate that the asset-weighted share of zombie firms[17] reached to 15% by the middle of the 1990s and that it remained around that level during the latter half of the 1990s. I thus set $1-\eta$ to be 0.15. The diversion parameter, ν, is difficult to calibrate directly. Instead, I calibrate this parameter to target the interest rate spread between the average lending rate and the average deposit rate. In this model, $(1-\eta)\times\nu$ represents this spread if the refinancing regime were supported. According to the data published by Bank of Japan, this spread fluctuated around 200bps during the 1990s. $\nu = 0.15$ is consistent with the data. I could neither find reliable estimates of the capital share nor those of the land share in Japan. Instead, I use values from a study by Valentinyi and Herrendorf (2008), who estimate these shares in the U.S. economy.

I assume that the economy is in the steady state of the normal equilibrium at pre-shock periods. Indeed, under the pre-shock parameters, the steady-state allocation of the normal equilibrium is supported as an equilibrium. The shock raises α_k from 0.28 to 0.30, a 7.1% increase; a large shock is required to change the regime of the economy.

Figure 1-4 plots the impulse responses to the shock. The solid line is the actual path of the economy. The economy follows the refinancing path because the condition in Corollary

[17] Zombie firms are defined as firms that obtain some form of interest payment concessions from banks. Caballero et al. (2008) note that these shares are sensitive to some extent to the manner of defining the minimum required interest payment. As a result, the calibrations in my model face uncertainty. I confirm that the equilibrium selection does not change in the simulation for small changes in the parameters.

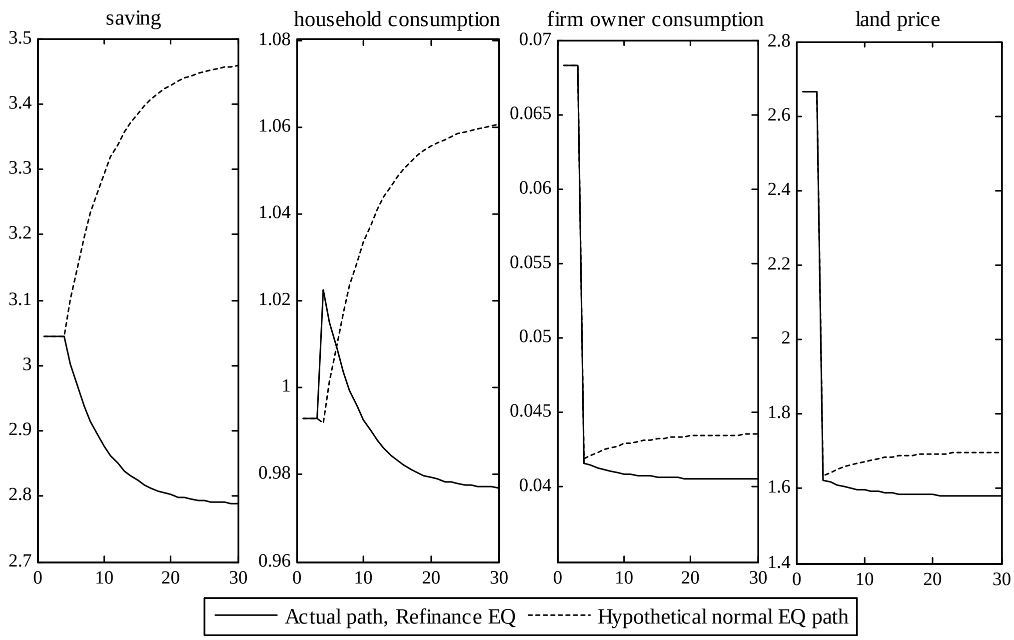

Figure 1-4: Impulse responses to one-time permanent shock to capital share (α_k)

1 is satisfied.[18] For comparison purposes, I show a hypothetical path of the economy with dashed lines. In this hypothetical path, I assume that the economy follows the normal path but that path is not supported with post-shock parameters.

On impact, the normal regime is no longer supported, and banks start to refinance poor projects. Households reduce savings on impact due to the low return of deposits. This contrasts with the hypothetical normal path along which households increase savings. Note that the shock itself enhances savings (The rise of α_k increases the marginal product of capital), but the prevalence of zombie firms lowers the rate of return. As a mirror image, households increase their consumption on impact by destocking savings. Moreover, they keep reducing the amount of saving over time because they anticipate that banks will engage in zombie lending in the future.

The land price plunges on impact. The sudden drop of the marginal product of land

[18] I assume that the economy switches to the refinancing equilibrium. It may be possible that the economy falls into other types of equilibria, where normal regimes are supported at some points of time. However households' deposits, d, must decline less than d^R to realize these types of equilibria under the calibrated parameters. I suspect this cannot happen due to the reason outlined in footnote 16.

lowers the valuation of land. More importantly, the land price continues to decline after the shock. This is an indirect effect of the decline of households' savings. Recall that the marginal product of land is increasing in capital. Anticipating that the stock of capital decline, which reduces the marginal product of land, the firm owners lower the valuation of land over time.

On the other hand, the path of the economy that results from the dynamic choices of households and firm owners actually justifies banks' incentive to engage in zombie lending: interest rates and the land price are consistent with the banks' ex post incentive to engage in zombie lending. Banks thus have no option but to refinance poor managers' projects. That is, the condition in proposition 2 is satisfied on the course of the dynamics.

To summarize, a lack of coordination among households, firm owners and banks to resolve non-performing loan problems forces the economy to select the refinancing equilibrium in this simulation. Because households and firm owners cannot control the outcome of the financial contracts, they are discouraged from saving if they anticipate that banks will engage in zombie lending. On the other hand, banks take the prices as given, and these prices are determined by the general equilibrium. If the prices are such that *condition 1* is satisfied, then banks have no choice but to refinance poor managers' projects. In the worst case scenario, the coordination failure lasts forever, and the economy is not able to exit the inefficient refinancing regime.

The interpretation of the results in relation with the Japanese experience

Land took an important role in Japanese banking: land was a common form of collateral of the bank loans. However, after the beginning of the 1990s, when land prices suddenly fell, banks were unable to recover loan losses by squeezing the collateral. This abrupt change in business environment is one of the factors that banks evergreened loans to insolvent firms. These banks' incentives were reinforced by loose government policy on non-performing loans, which I formalize in terms of my model in the next section. This phenomenon is essentially an SBC syndrome defined in Kornai et al. (2003).

One may suspect that the factor-specific shock in the simulation does not summarize the nature of shocks that hit Japan during the 1990s. That could be true, and I do not argue against this point in this chapter. Rather, my focus is on the aftermath of shocks that drop land prices. My model has rich implications in this respect, whatever the nature of the shocks[19]; the ineffective use of savings and the associated increase of the risk premium discourage households from saving and crowd out investment. Moreover, the interaction between the incentive and the general equilibrium effects under the lack of coordination among agents can cause the economy to select an inefficient equilibrium.

This model focuses on bailouts of insolvent firms whose prospects are still good when they default on initial debt. Remember that the net continuation value of poor firms, $\left(\tilde{R} - \nu R\right)$, is positive under the refinancing regime. To some extent, this understates the banking problem in Japan. It has been said that banks have continued to extend credit to firms whose net present value is even negative. As discussed by many economists, I need to take into account the regulatory issues specific to Japan to explain why banks bailed out seriously troubled firms. However, this chapter shows even a mild degree of zombie lending, rescuing firms whose net continuation value is positive, induces serious moral hazard problems. As we can see in Figure 1-4, the problem has economically significant impacts.

1.5 Policy analyses

In this section, I conduct policy exercises to hint at desirable policies to restore the efficiency of the economy. I focus on two policies, namely government bailouts and the strict enforcement of loan-loss provisions, and I show which policy the government should have taken to restore the efficient allocation in my model. As these policies are the measures

[19] In terms of my model, I can generate the same results when I lower $\tilde{\beta}$, the discount rate of firm owners. Effectively, this shock lowers firm owners' valuation of land. Alternatively, investment-specific technology shocks a la Greenwood, Hercowitz and Krusell (2000) also work in the same way.

that the Japanese government took, I am able to evaluate the effects of these policies in terms of my model.

1.5.1 The government bailout

The Japanese government continuously bailed out troubled banks during the 1990s. It injected capital to troubled banks several times. However, more importantly, it implicitly bailed out troubled banks by overlooking non-performing loans. As described in section 1.2, the bank supervision department in Japan did not actively inspect banks' voluntary reports on the quality of loan portfolios. Thus, banks could easily hide non-performing loans to meet minimum capital requirements. The implicit bailouts effectively took the form that banks accumulated claims to the deposit insurance system managed by the government. This is called regulatory forbearance.

Because the implicit bailouts are not fundamentally different from explicit bailouts in the sense that the government covers the losses of impaired loans, I define the government bailout in a simple way: the government grants $\theta \in [0, \nu]$ units of credit per unit of initial lending if banks refinance poor managers' projects. This means that banks have to raise only $(\nu - \theta)L_0$ units of additional deposits when they refinance poor managers' projects. Then, the ex post incentive condition such that banks are willing to refinance poor managers' projects changes, as:

$$\left[\tilde{R}_s - R_s(\nu - \theta)\right] L_{0,s} \geq q_s \overline{m}. \tag{1.11}$$

An instant look at (1.11) hints that it is less likely to recover efficiency when the government bails out troubled banks. This is because the cost of refinancing for banks shrinks to $R_s(\nu - \theta)$ per unit of initial lending.

The remaining modifications that need to be done are those to the government budget. I assume that the government runs a balanced budget and levies a lump-sum tax on households.[20] Let τ_t be the amount of the lump-sum tax. The following conditions must hold in equilibrium:

$$c_{h,t} + d_{t+1} = w_t + R_t d_t - \tau_t,$$
$$\tau_t = \left[\frac{(1-\eta)\theta}{1+(1-\eta)(\nu-\theta)} d_t\right](1-\iota_t).$$

The starting point in this exercise is to assume that the economy cannot attain the normal equilibrium, the first-best outcome, without government interventions. In particular, the normal regime is not supported at initial period, time t, given the initial level of savings $d_t < d^N$. I then obtain the following result:

Proposition 3 *Assume that $d_t < d^N$ and d_t satisfies the following condition:*

$$(1-\nu)[\alpha_k + \frac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}} d_t^{1-\alpha_k}] \geq \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n).$$

(This implies that the normal regime is not supported in any equilibrium at time t.) Assume further that A is constant. Then, for any $\theta \in [0,\nu]$, the normal path cannot be supported as an equilibrium. That is, the government bailout never restores the first-best outcome.

Proof. See Appendix A.■

In my model, the government bailout strengthens banks' incentive to refinance poor projects. It is never an optimal policy in terms of restoring efficiency in this economy. Regulatory forbearance, a policy that the Japanese government adopted up to the end of the 1990s, was welfare diminishing.[21]

[20]The Japanese government did not run a balanced budget, instead issuing government bonds. However, the assumption of a balanced budget yields the same result as long as a lump-sum tax is available and Ricardian equivalence holds, as in my model. Ricardian equivalence breaks in Barseghyan (2008), and a lump-sum tax is not available in Dekle and Kletzer (2005). These setups create additional dimensions of inefficiency.

[21]There are, however, certain welfare-improving aspects of the government bailout in my model. Allowing the economy to stay in the refinancing regime, the increase of the government bailout reduces the risk premium of loan interest rates. This is because the costs that banks pay for refinancing decreases as a result

I admit that my model may understate the benefits of government bailouts. In the real world, there may be some benefits from bailing out troubled banks: bailing out one bank may prevent chain reaction bankruptcies of multiple banks; the injection of capital may restore banks' ability to lend, thereby stimulating the economy; and the transfer of resources to firm sectors, through financial intermediaries, has a strong multiplier effect on investment when firms face borrowing limits. Bernanke, Gertler and Gilchrist (1998) and Kato (2007) and others have illustrated this mechanism. However, I can claim that the bailout policy alone is not a desirable policy package, as it can aggravate the moral hazard problem. The moral hazard problem is not trivial because shifts between equilibrium regimes induce large gaps in welfare.

1.5.2 The strict enforcement of loan-loss provisions

I define the strict enforcement of loan-loss provisions as a requirement for banks to accumulate loan-loss reserves. I assume that the government can observe the quality of loan portfolios costlessly at the time when banks make refinancing-liquidation decisions on poor managers' projects. Also, for simplicity, I assume that the government requires banks to accumulate $\gamma \in [0, \nu]$ units of loan-loss provisions per unit of initial lending if banks refinance the poor projects. This implies that the refinancing incurs an additional cost by γRL_0. As a result, banks' ex post incentive for engaging in zombie lending changes as follows:

$$\left[\tilde{R}_s - R_s(\nu + \gamma)\right] L_{0,s} \geq q_s \overline{m}. \tag{1.12}$$

By the amount of γRL_0, banks' incentive to refinance poor managers' projects is weakened. This has an immediate implication as follows:

of the bailout. In the limiting case of the full bailout, that is, $\theta = \nu$, the investment discouraging distortion in (1.8) disappears. However, the government spending wedge, τ, becomes large in this case. As a result, inefficiency never vanishes, and there would be an optimal degree of government bailout that balances the investment wedge and the government spending wedge.

Proposition 4 *Assume that $d_t < d^N$ and d_t satisfies the following condition:*

$$(1-\nu)[\alpha_k + \frac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}} d_t^{1-a_k}] \geq \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n).$$

Assume further that A is constant. Suppose there exists $0 \leq \gamma^* \leq \nu$ that satisfies the following equation:

$$[1-\nu-\gamma^*]\,[\alpha_k + \frac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}} \left(d^N\right)^{1-a_k}] = \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n).$$

Then the normal equilibrium is restored for $\forall \gamma \in (\gamma^*, \nu]$.

Proof. See Appendix A.■

This type of policy summarizes one of the measures that were introduced in the "Program for Financial Revival" in 2002: the Financial Services Agency of Japan finally decided to intensify the monitoring of banks and demanded that banks accumulate appropriate loan-loss reserves. The objective of this policy was to clean out the corruption of the banking sector and to bring back the correct incentives for banks to terminate zombie lending.[22]

As Aghion, Bolton and Fries (1999) shows, things are not so straightforward when the condition of loan portfolios is private knowledge to banks. In this case, the strict policy can postpone the revelation of non-performing loans because bank managers fear that the revelation puts the retention of their positions in the banks at risk.[23] However, what the Financial Services Agency did in the end was to make public the information on loan portfolios. In this case, bank managers would no longer be capable of hiding non-performing loans. Banks would then reveal the true condition of the loan portfolios and would refrain from zombie lending.

[22]The program also introduced a measure called the "due consideration to loans to small-and-medium-sized enterprises." This measure encouraged banks to lend to small-sized firms, in which zombie firms may be included. In this sense, some parts of the program were contradicting with each other.

[23]In Aghion, Bolton and Fries (1999), bank managers care their own private benefit.

1.6 Conclusion

This chapter focuses on the interaction between the incentives and the macroeconomic effects of zombie lending in Japan. In the model, the incentives and the macroeconomic effects are interdependent; given the incentive of zombie lending, the low rates of return discourage the agents from saving. However, the prices of loans, deposits and land, which are affected by the saving choices of agents, actually justify the incentive for banks to engage in zombie lending. I interpret this interaction as a result of the lack of coordination to resolve non-performing loan problems. I believe that this mechanism highlights the relationship between the decade-long economic downturn and the financial breakdown in Japan.

The bailout policy, which the Japanese government has taken until the very end of the economic downturn, further strengthened banks' incentives to engage in zombie lending. On the contrary, the strict enforcement of loan-loss provisions, the measure that the Japanese government ultimately took, can restore the efficiency of the economy.

Let me conclude by providing a final remark on the possibilities of the extensions of this chapter. There are many immediate possibilities for extending my model, so long as one can keep track of the incentive condition of banks. One can immediately introduce the labor-leisure choice into my model. It would not be a hard task to introduce the entry-exit decision of managers, thereby endogenizing the dynamics of zombie firms. These possible extensions would provide a richer understanding of bank-induced economic stagnation without having to change the fundamental mechanism of this chapter.

A Appendix A

Derivation of (1.6) and (1.7)

By substituting the price equations (1.3) into (1.1) and the budget constraint of households, it is immediate to obtain the first two lines of the systems (1.6) and (1.7). For the firm owners' consumption, substitute the equilibrium condition $q_t m_t = q_t m_{t+1} = q_t \bar{m}$ into the budget constraint of firm owners. Then I obtain:

$$c_{l,t} = \pi_t. \tag{a.1}$$

Then, substituting the first two lines of (1.3) into (a.1) yields:

$$c_{l,t} = (1 - \alpha_k - \alpha_n) y_t. \tag{a.2}$$

Finally, substitute the definition of the output which is $y_t = A_t d_t^{a_k} \bar{m}^{1-\alpha_k-\alpha_n}$ for the normal regime and $y_t = A_t \left(\frac{d_t}{1+(1-\eta)\nu}\right)^{a_k} \bar{m}^{1-\alpha_k-\alpha_n}$ for the refinancing regime. Then I obtain the equilibrium consumption of firm owners as in (1.6) and (1.7).

Now, the land price formula is written as follows:

$$q_t = \sum_{s=t}^{\infty} \tilde{\beta}^{s-t} \left(\frac{c_{l,s}}{c_{l,t}}\right)^{-1} (1 - \alpha_k - \alpha_n) \frac{y_s}{\bar{m}}. \tag{a.3}$$

By using (a.2), (a.3) can be simplified as:

$$\begin{aligned} q_t &= \frac{c_{l,t}}{\bar{m}} \sum_{s=t}^{\infty} \tilde{\beta}^{s-t} \\ &= \frac{\tilde{\beta}}{1-\tilde{\beta}} (1 - \alpha_k - \alpha_n) \frac{y_t}{\bar{m}}. \end{aligned} \tag{a.4}$$

Finally, by substituting the definition of output in each regime, I obtain the last lines of (1.6) and (1.7).

The steady state

The normal equilibrium

By setting $c_{h,t} = c_{h,t+1}$ and $A_t = A_{t+1} = A$ in the first line of (1.6), I obtain:

$$\begin{aligned} 1 &= \beta \left[A\alpha_k \left(d^N\right)^{\alpha_k - 1} \bar{m}^{1-\alpha_k-\alpha_n} + 1 - \delta \right], \\ &\leftrightarrow \\ d^N &= \left(\frac{\beta A \alpha_k \bar{m}^{1-\alpha_k-\alpha_n}}{1-\beta+\beta\delta} \right)^{\frac{1}{1-\alpha_k}}. \end{aligned} \tag{a.5}$$

Now, set $d_t = d_{t+1} = d^N$. By substituting (a.5) into (1.6), I obtain:

$$\begin{aligned} c_h^N &= A\left(\alpha_k + \alpha_n\right)\left(d^N\right)^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} - \delta d^N, \\ c_l^N &= A(1-\alpha_k-\alpha_n)\left(d^N\right)^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n}. \\ q^N &= \frac{\tilde{\beta}}{1-\tilde{\beta}} A(1-\alpha_k-\alpha_n)\left(d^N\right) \bar{m}^{-\alpha_k-\alpha_n}. \end{aligned} \tag{a.6}$$

The refinancing equilibrium

Again, by setting $c_{h,t} = c_{h,t+1}$ and $A_t = A_{t+1} = A$ in the first line of (1.7), I obtain:

$$\begin{aligned} 1 &= \beta \left\{ \tfrac{1}{1+(1-\eta)\nu} \left[A\alpha_k \left(\tfrac{d^R}{1+(1-\eta)\nu} \right)^{\alpha_k-1} \bar{m}^{1-\alpha_k-\alpha_n} + 1 - \delta \right] \right\}, \\ &\longleftrightarrow \\ d^R &= \left(\frac{\beta A \alpha_k \bar{m}^{1-\alpha_k-\alpha_n}}{1+(1-\eta)\nu-\beta+\beta\delta} \right)^{\frac{1}{1-\alpha_k}}. \end{aligned} \tag{a.7}$$

Now, set $d_t = d_{t+1} = d^R$. By substituting (a.7) into (1.7), I obtain:

$$\begin{aligned} c_h^R &= A\left(\alpha_k + \alpha_n\right)\left(\tfrac{d^R}{1+(1-\eta)\nu} \right)^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} - \tfrac{(1-\eta)\nu+\delta}{1+(1-\eta)\nu} d^R, \\ c_l^R &= A(1-\alpha_k-\alpha_n)\left(\tfrac{d^R}{1+(1-\eta)\nu} \right)^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n}, \\ q^R &= \frac{\tilde{\beta}}{1-\tilde{\beta}} A(1-\alpha_k-\alpha_n)\left(d^R\right) \bar{m}^{-\alpha_k-\alpha_n}. \end{aligned} \tag{a.8}$$

Proof of proposition 1

I first show the local properties around the steady state and then I show the global properties of the dynamics.

The first two equations in (1.6) can be rewritten as a system of non-linear first order equations:

$$\begin{aligned} d_{t+1} &= A_t\left(\alpha_k+\alpha_n\right)d_t^{\alpha_k}\bar{m}^{1-\alpha_k-\alpha_n}+\left(1-\delta\right)d_t-c_{h,t}, &\text{(a.9)}\\ c_{h,t+1} &= \beta c_{h,t}\left\{A_{t+1}\alpha_k\left[A_t\left(\alpha_k+\alpha_n\right)d_t^{\alpha_k}\bar{m}^{1-\alpha_k-\alpha_n}+\left(1-\delta\right)d_t-c_{h,t}\right]^{\alpha_k-1}\bar{m}^{1-\alpha_k-\alpha_n}+1-\delta\right\}. &\text{(a.10)}\end{aligned}$$

Assume $A_t = A_{t+1}$. By taking the first order Taylor approximation around the steady state, I obtain the following linearized system:

$$\begin{pmatrix} d_{t+1} \\ c_{h,t+1} \end{pmatrix} = J\begin{pmatrix} d_t \\ c_{h,t} \end{pmatrix}, \quad J = \begin{pmatrix} \lambda_1 & -1 \\ -\lambda_1\lambda_2 & 1+\lambda_2 \end{pmatrix}.$$

where $\lambda_1 = \left(\alpha_k+\alpha_n\right)\left(\frac{1-\beta+\beta\delta}{\beta}\right)+1-\delta$ and $\lambda_2 = \left(1-\alpha_k\right)\left(1-\beta+\beta\delta\right)\left[\left(\frac{\alpha_k+\alpha_n}{\alpha_k}\right)\left(\frac{1-\beta+\beta\delta}{\beta}\right)-\delta\right]$.

To verify the local saddle-point stability, one of the eigenvalues of matrix J must be inside the unit circle whereas the other must be outside of the unit circle. The eigenvalues are obtained by solving the following quadratic equation:

$$x^2-(\lambda_1+\lambda_2+1)x+\lambda_1 = 0. \tag{a.11}$$

Now, define $f(x) = x^2 - (\lambda_1 + \lambda_2 + 1)x + \lambda_1$. It is immediate to see:

$$\begin{aligned} f(0) &= \lambda_1 > 0, \\ f(1) &= -\lambda_2 < 0, \\ f'(0) &= -(\lambda_1 + \lambda_2 + 1) < 0. \end{aligned} \tag{a.12}$$

By (a.12), we can see that one of the eigenvalues lies inside the unit circle and the other lies outside the unit circle. Thus, the system is locally saddle-point stable. Moreover, the smaller eigenvalue is greater than zero. This implies the dynamics around the steady state is locally monotone[24] in the sense $\{d_{s+1}, c_{h,s}\}_{s=t}^{\infty}$ is an increasing sequence if $d_t < d^N$ and d_t is close enough to d^N.

To verify the global saddle-point stability, it is useful to see the direction of d_{t+1} and $c_{h,t+1}$ given $(d_t, c_{h,t})$. The equation (a.9) and (a.10) can be rearranged as:

$$\Delta d_{t+1} = A(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} - \delta d_t - c_{h,t}, \tag{a.13}$$

$$\Delta c_{h,t+1} = \beta c_{h,t} \left\{ A\alpha_k \left[A(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta) d_t - c_{h,t} \right]^{\alpha_k - 1} \bar{m}^{1-\alpha_k-\alpha_n} + 1 - \delta - \frac{1}{\beta} \right\}. \tag{a.14}$$

By (a.13), $c_{h,t} = A(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} - \delta d_t$ when $\Delta d_{t+1} = 0$. Hence, $\Delta d_{t+1} < 0$ when $c_{h,t} > A(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} - \delta d_t$ i.e. $c_{h,t}$ is above $\Delta d_{t+1} = 0$ locus. On the other hand, $\Delta d_{t+1} > 0$ when $c_{h,t}$ is below $\Delta d_{t+1} = 0$ locus.

By (a.14), $A\alpha_k d_{t+1}^{\alpha_k - 1} \bar{m}^{1-\alpha_k-\alpha_n} - \frac{1-\beta+\beta\delta}{\beta} = 0$ when $\Delta c_{h,t+1} = 0$. By substituting (a.9), this is equivalent to say $c_{h,t} = A(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta) d_t - d^N$ when $\Delta c_{h,t+1} = 0$. Hence $\Delta c_{h,t+1} > 0$ when $c_{h,t}$ is above $\Delta c_{h,t+1} = 0$ locus. On the other hand, $\Delta c_{h,t+1} < 0$ when $c_{h,t}$ is below $\Delta c_{h,t+1} = 0$ locus.

These arguments boil down to the arrows in Figure A-1. By the direction of d_{t+1} and

[24] It can be proved that the smaller eigenvalue of J governs the dynamics along the stable arm of the system.

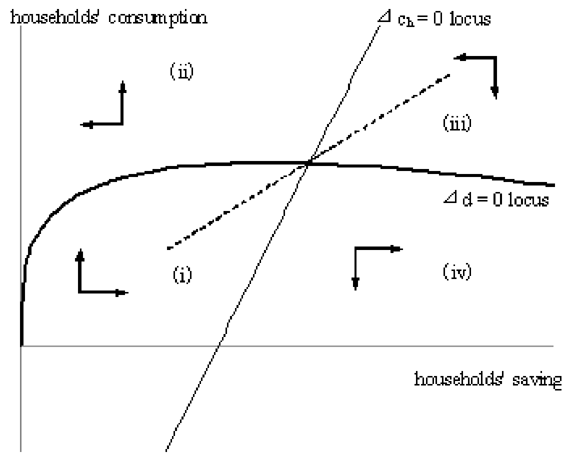

Figure A-1

$c_{h,t+1}$ given $(d_t, c_{h,t})$, one can see that the dynamics is globally saddle-point stable[25].

To show the global monotonicity, it suffices to show that $(d_{t+1}, c_{h,t+1})$ never jumps into region (iii) when $(d_t, c_{h,t})$ is in region (i) and vice versa in Figure A-1. Assume $(d_t, c_{h,t})$ is in region (i). That is $(d_t, c_{h,t})$ satisfies the following conditions:

$$\begin{aligned} d_t &< d^N, \\ c_{h,t} &\leq A(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} - \delta d_t, \\ c_{h,t} &\geq A(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta)d_t - d^N. \end{aligned} \tag{a.15}$$

By (a.9) and the second line of (a.15), the following inequality holds:

$$\begin{aligned} d_{t+1} &= A_t(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta) d_t - c_{h,t} \\ &\leq \begin{matrix} A_t(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta) d_t \\ -A(\alpha_k + \alpha_n) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta)d_t - d^N \end{matrix} \\ &= d^N. \end{aligned} \tag{a.16}$$

[25]The origin in the (d, c_h) space, that is $(d_{s+1}, c_{h,s}) = (0,0)$ for $\forall s \geq t$, is also an equilibrium. However, this equlibrium is uninteresting one. So I ignore this equilibrium.

This implies, at least, $(d_{t+1}, c_{h,t+1})$ never jumps into region (iii) in the figure when $(d_t, c_{h,t})$ is in region (i) since $d_{t+1} \leq d^N$. Now assume $(d_t, c_{h,t})$ is in region (iii). That is $(d_t, c_{h,t})$ satisfies the following conditions:

$$\begin{aligned} d_t &> d^N, \\ c_{h,t} &\geq A\left(\alpha_k + \alpha_n\right) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} - \delta d_t, \\ c_{h,t} &\leq A\left(\alpha_k + \alpha_n\right) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta) d_t - d^N. \end{aligned} \tag{a.17}$$

By (a.9) and the second line of (a.17), the following inequality holds:

$$\begin{aligned} d_{t+1} &= A_t\left(\alpha_k + \alpha_n\right) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta)\, d_t - c_{h,t} \\ &\geq \begin{array}{c} A_t\left(\alpha_k + \alpha_n\right) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta)\, d \\ -A\left(\alpha_k + \alpha_n\right) d_t^{\alpha_k} \bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta) d_t - d^N \end{array} \\ &= d^N. \end{aligned} \tag{a.18}$$

This implies, at least, $(d_{t+1}, c_{h,t+1})$ never jumps into region (i) in the figure when $(d_t, c_{h,t})$ is in region (iii) since $d_{t+1} \geq d^N$, completing the proof.■

I can apply the exactly the same logic for the global saddle-point stability and global monotonicity of the refinancing equilibrium.

Proof of lemma 1

$(\tilde{R}_s - R_s\nu)L_{0,s} < q_s\overline{m}$ must be satisfied if time-s were to be in the normal regime. In the normal regime, $\tilde{R}_s = R_s = \alpha_k A_s d_s^{a_k-1}\bar{m}^{1-\alpha_k-\alpha_n} + (1-\delta)$, $L_{0,s} = d_s$ and $q_s = \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n)A_s d_s^{a_k}\bar{m}^{-\alpha_k-\alpha_n}$. Then we obtain (1.9) by substituting these conditions into the incentive condition in (1.4).

Similarly, $(\tilde{R}_s - R_s\nu)L_{0,s} \geq q_s\overline{m}$ must be satisfied if time-s were to be in the refinancing regime. In the refinancing regime, $\frac{1}{1+(1-\eta)\nu}R_s = \tilde{R}_s = \alpha_k A_s \left(\frac{d_s}{1+(1-\eta)\nu}\right)^{a_k-1}\bar{m}^{1-\alpha_k-\alpha_n} +$

$(1-\delta)$, $L_{0,s} = \frac{d_s}{1+(1-\eta)\nu}$ and $q_s = \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n)A_s\left(\frac{d_s}{1+(1-\eta)\nu}\right)^{a_k}\bar{m}^{-\alpha_k-\alpha_n}$. Then, we obtain (1.10) by substituting these conditions into the incentive condition in (1.5). ■

Proof of proposition 2

The left hand side of the inequality (1.9) is an increasing function of d_s. Then there exists threshold d^{N*} such that for $\forall d_s \geq d^{N*}$, the inequality (1.9) is not satisfied. Then, the normal regime is not supported at time-s if $d_s \geq d^{N*}$.

Similar argument proves that there exists threshold d^{R*} such that for $\forall d_s \geq d^{R*}$, the inequality (1.10) is always satisfied. Then, for $\forall d_s \geq \max\{d^{N*}, d^{R*}\}$, only the the refinancing regime is supported at time-s if an allocation which satisfies definition 3 were to be an equilibrium. Finally, it is immediate to show that $d^* = \max\{d^{N*}, d^{R*}\}$, completing the proof.■

Proof of proposition 3

By substituting the equilibrium condition of normal equilibrium into the equation (1.11), I obtain:

$$(1-\nu+\theta)[\alpha_k + \frac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}}d_t^{1-a_k}] \geq \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n). \tag{a.19}$$

Obviously, this is true for $\forall\theta \in [0,\nu]$ as long as the assumption in the proposition 3 holds.■

Proof of proposition 4

Given the requirement of loan loss provision $\gamma \in [0,\nu]$ and given the level of saving d_s at time-s, notmal regime can be supported at time-s if:

$$(1-\nu-\gamma)[\alpha_k + \frac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}}(d_s)^{1-a_k}] < \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n). \tag{a.20}$$

The left hand side of (a.20) is increasing in d_s and decreasing in γ. Under normal equilibrium, households' saving never exceeds d^N if the initial level of saving, d_t, satisfies $d_t < d^N$. Now assume that normal equilibrium is supported at a certain level of $\gamma \in [0, \nu]$. Then (a.20) must be satisfied on the course of the dynamics of normal equilibrium. Since $\{d_s\}_{s=t}^{\infty}$ is an increasing sequence which converges to d^N, the left hand side of (a.20) increases over time and converges to $(1-\nu-\gamma)[\alpha_k + \frac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}}\left(d^N\right)^{1-a_k}]$.

Now, Let $0 \leq \gamma^* \leq \nu$ satisfy the following equation:

$$(1-\nu-\gamma^*)[\alpha_k + \frac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}}\left(d^N\right)^{1-a_k}] = \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n). \tag{a.21}$$

Then for $\forall\gamma \in (\gamma^*, \nu]$, $(1-\nu-\gamma)[\alpha_k + \frac{1-\delta}{A\bar{m}^{1-\alpha_k-\alpha_n}}\left(d^N\right)^{1-a_k}] < \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\alpha_k-\alpha_n)$. This implies (a.20) is satisfied for $\forall\gamma \in (\gamma^*, \nu]$, $\forall d_s \leq d^N$. Thus normal equilibrium is supported under $\forall\gamma \in (\gamma^*, \nu]$.■

One-sector dynamic general equilibrium model

The model in the main section of this chapter assumes that firms rent capital from households while they own land. And firm owners repay debts to households while they obtain dividend from land. Basically, firms issue debts to households while they issue shares to themselves. This is a non-standard assumption in terms of the asset market structures of standard neo-classical dynamic general equilibrium models. However, the pay off of this assumption is that I can obtain the analytical properties of the equilibria.

In this section, I build a one-sector dynamic general equilibrium model by unifying households and firm owners in the main section. And I illustrate that the properties which I showed in the main section apply to the standard one-sector model. The setup of the unified households in this model is as follows:

$$\max \sum_{s=t}^{\infty} \beta^{s-t} \log c_s,$$
$$s.t.\ c_t + d_{t+1} + q_t m_{t+1} = w_t + R_t d_t + (q_t + \xi_t) m_t.$$

These households can allocate their wealth to bank deposits and land. I separate banks into commercial banks in which households deposit and trust banks in which households entrust land. ξ denotes the rental rate of land which trust banks pay to households. The roles of commercial banks are the same as the banks in the main section. Trust banks manage the land which households entrust. They rent the land to firms with rental rate ξ. One assumption which I make here is a subordinate structure of bank loans. Commercial banks issue priority credit to firms. On the other hand, the lending of trust bank (of the form of land) is subordinate lending. If a firm defaults, the commercial bank has an priority to squeeze the assets of the firm. Hence the commercial bank takes the land and the trust bank is left with nothing. This assumption enables me to incorporate exactly the same financial contract as in section 1.3.1 into this model.

There are a unit mass of firms. Firms are divided into good firms and poor firms. The technologies and the incentives of good firms and poor firms are the same as the benchmark model. I assume that commercial banks and trust banks cannot identify poor firms ex ante. Once firms engage in production, their problem is defined as follows:

$$\begin{aligned}
\max \pi_t &= y_t + (1-\delta)k_t - w_t n_t - \tilde{R}_t L_{0,t} - \xi_t m_t, \\
s.t.\ k_t &= L_{0,t}, \\
y_t &= A_t (n_t)^{\alpha_n} (k_t)^{\alpha_k} (m_t)^{1-\alpha_n-\alpha_k}.
\end{aligned}$$

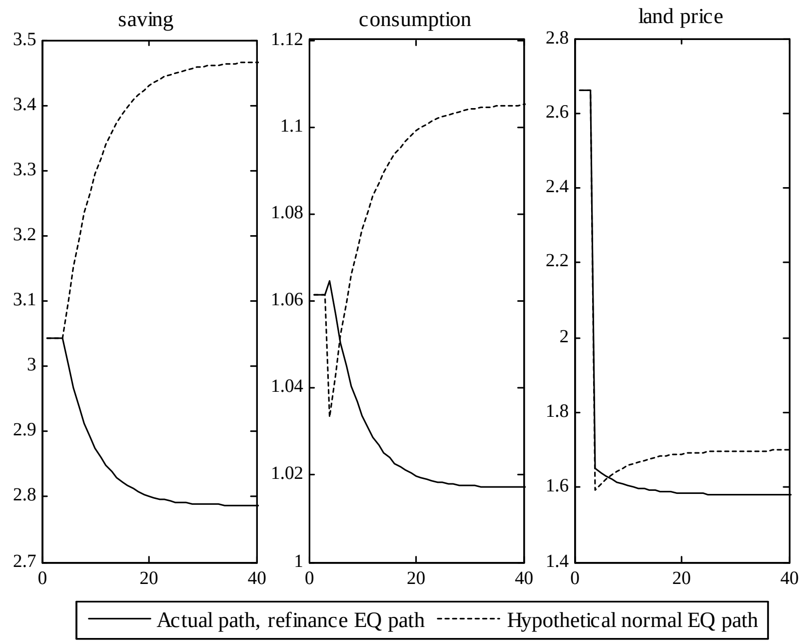

Figure A-2: Impulse responses of one sector DGE model to a permanent capital share shock

By solving problems above, I obtain the following optimality conditions:

$$
\begin{aligned}
\frac{1}{c_t} &= \frac{\beta R_{t+1}}{c_{t+1}}, \\
\frac{q_t}{c_t} &= \frac{\beta}{c_{t+1}}\left(\xi_{t+1} + q_{t+1}\right), \\
w_t &= \alpha_n y_t, \\
\tilde{R}_t &= \alpha_k \frac{y_t}{L_{0,t}} + (1-\delta), \\
\xi_t &= (1-\alpha_n-\alpha_k)\frac{y_t}{\bar{m}} :
\end{aligned}
$$

The definitions of regime and regime path are the same as Definition 1 and Definition 2. The definition of equilibrium is slightly modified but it does not change essentially.

Now I conduct the same simulation as in section 1.4.3. I gave a permanent shock to capital share α_k. The parameter values are the same as section 1.4.3.

Figure A-2 shows the impulse responses of the economy. As in the benchmark model

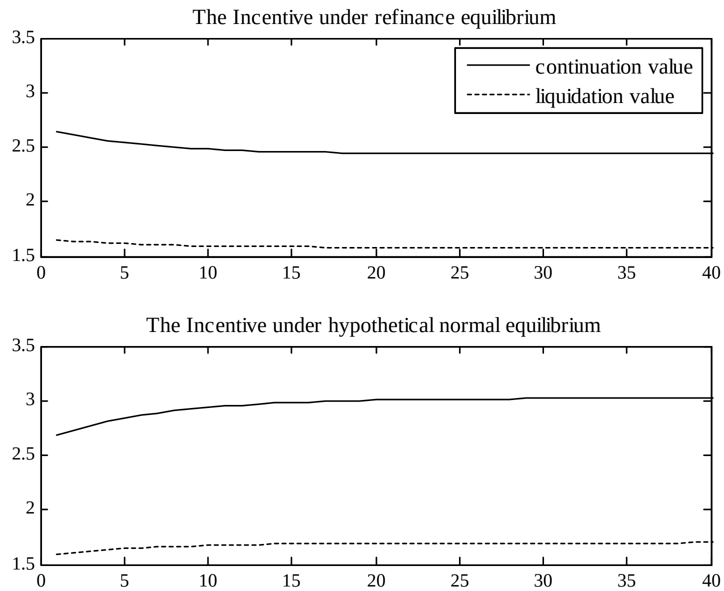

Figure A-3: The incentives of commercial banks

of the main section, households reduce the amount of saving over time. This is because households anticipate that commercial banks will engage in zombie lending. The land price plunges on impact and it continues to decline over time. Households consumption eventually converges to the steady state of the refinancing equilibrium which is lower than that of normal equilibrium.

One of the problem in this version of model is that we are not able to calculate the liquidation value of firms analytically. This is because we cannot obtain the analytical formula of the land price. Instead I compute it numerically and I show that the incentive condition of commercial banks support the refinancing equilibrium while it does not support the normal equilibrium.

Figure A-3 shows that the continuation value is greater than the liquidation value along the path of the refinancing equilibrium: The refinancing equilibirum is supported as an equilibrium. On the other hand, the liquidation value is less than the continuation value along the path of the normal equilibrium: The normal equilibrium is not supported.

Chapter 2

A Model of Securitization-Induced Economic Crises

2.1 Introduction

The subprime crisis and the following economic recession are one of the worst economic events since the great depression in U.S. One important aspect of this crisis is that it started with the collapse of the subprime mortgage market in early 2007.[26] Then the turmoil spread to various financial markets and liquidity dried up in U.S. financial market as a whole. Eventually, the real economy fell into recession and we are yet to see vigorous recovery as of the end of 2009. Although the analysis of the entire mechanism of this crisis is still ongoing research, a deep root of this crisis is likely to lie in the securitization of subprime mortgages because it is the securitization market that collapsed first. Many analysts now view that subprime mortgages were cheaply financed relative to their risks and housing market overheated as a result.

This chapter attempts to construct a model of boom-and-bust cycles induced by securitization. In the model, risky loans, which can be considered as subprime mortgages, are

[26] According to Brunnermeier (2009). It well documents the entire chronology of this crisis.

securitized. Ashcraft and Schuremann (2008) present that there are several stages of information asymmetries in the process of securitization that could be causes for the subprime crisis. I focus on the information asymmetry between issuers and investors on the quality of underlying assets. And I examine the mechanism that securitization makes financing risky loans cheap thereby igniting booms in an aggregate economy. I also examine why and how the rapid expansion of securitization suddenly ends by endogenizing the return of securitized assets along with business cycles.

The following two ingredients are the keys and what distinguish my model from standard housing DSGE models: i) Investors mismeasure the default risks of the risky mortgages, i.e. a deviation from rational expectation. ii) The security design of securitized assets amplifies measurement errors in the default risks of risky mortgages. By embedding these features in a dynamic housing model, the model generates boom-and-bust cycles systematically and gives a comprehensive understanding of crises induced by securitization.

The summary of a boom-and-bust cycle is as follows: the process begins with a deviation from rational expectation by assuming that investors mismeasure the default risks of risky loans. This assumption seems to be an inexpensive way to generate booms based on irrational behavior. However, the deviation from rational expectation alone is not enough to generate boom-and-bust cycles. It is the security design of securitized assets that enables risky loans to be cheaply financed. An important step in securitization is called tranching, which means slicing the payout of the pool of underlying assets: tranching determines the priority of loss absorption among different classes of securities backed by collateral. A specific example of tranching can be seen in Coval et al. (2009), however, the point is that securitization can be a device for informed agents to reallocate returns of collateral in favor of them when there are uninformed agents. By this security design, issuers get overcompensated, risky loans are cheaply financed and booms in a economy are ignited.

The sustainability of a boom depends on the profitability of securitization. In my model, it is endogenized by connecting house price inflation (hereafter HPI) with the recovery rate of defaulted loans, which is one of the components of the default risks of underlying assets; risky loans are collateralized by housing stock. Therefore the recovery rate when loans are in default consists of HPI. In this setting, the dynamics of HPI can cause the sudden end of boom phases. As HPI declines on the course of dynamics, the expected return of the securitized assets declines over time and, at some point, it is no longer high enough for issuers to be willing to engage in securitization business. At this point, the securitization market collapses and borrowers are forced to undergo severe adjustments.

There are good reasons to focus on the specifications above. Some of the information on the quality of underlying assets is likely to be masked to investors due to complicated processes of securitization. Even credit ratings by professional rating agencies turned out to be inaccurate in evaluating risks of securitized assets. This is evident from the fact that many of AAA rated securitized assets defaulted and were devalued significantly in 2007. The information asymmetry between issuers and investors could have significant impacts on the generation of the boom-and-bust cycle. Coval, Jurek and Stafford (2009) and Heitfield (2009) illustrate that default risks of securitized assets are quite sensitive to default risks of underlying assets. Agents who have richer information can take advantage of this fact to obtain high return from securitization. Finally, the sharp decline in HPI right before the subprime crisis is one of the plausible candidates that triggered the crisis. Gerardi, Shapiro and Willen (2007) find that cumulative HPI has statistically and economically significant impacts on the default probability of mortgages. If subprime mortgages take a significant part of the collateral in securitization, then the returns of the securitized assets should be sensitive to the development of HPI.

The remaining parts of this chapter are organized as follows. Section 2.2 describes the model. I first describe the structure of the securitization market focusing on information available to each type of agents. Then I incorporate the securitization market into a dynamic model of housing choices. Section 2.3 parameterizes and simulates the model. Here I check whether a boom-and-bust cycle emerges under a reasonable parameterization. Section 2.4

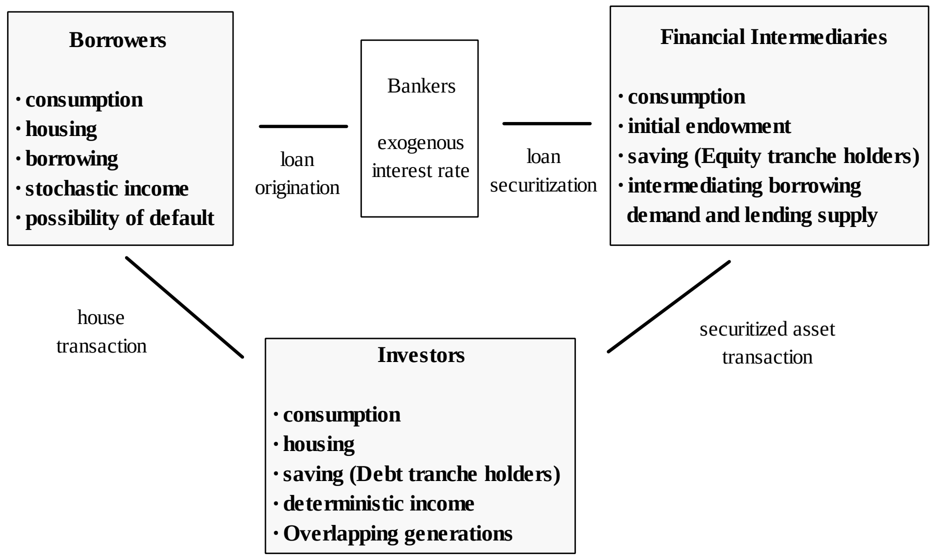

Figure 2-1: Overall structure of the economy

discusses the implication of the model and finally section 2.5 concludes.

2.2 The model

This section presents the structure of the model. Figure 2-1 depicts the structure. The securitization market and the housing market are the centers of this analysis. In the securitization market, financial intermediaries purchase loans, create securitized assets and trade them with investors. The default risks of the loans are the key determinants of the expected return of the securitized assets. Investors have subjective belief on the default risks while financial intermediaries evaluate them accurately. As we will see later, this information asymmetry is the fundamental source to generate boom-and-bust cycles. In the housing market, borrowers and investors trade housing stock whose supply is fixed to $\bar{h}$. Their dynamic choices on housing, consumption and saving (borrowing) determine the housing allocation. Debts of borrowers are collateralized by the housing stock that they own. House value thus plays an important role in determining the recovery rate of defaulted loans. This in turn affects the return of securitized assets.

2.2.1 The securitization market

Individual borrowers

There are measure one borrowers. They are divided into N types evenly.[27] A borrower obtains loan from a bank with the principal amount B_b. The loan is one-period lending with the gross interest rate R_m which is exogenous.[28] Each type of borrowers may default their loans in the next period. The following latent factor X_n governs the default of type n:

$$
\begin{aligned}
&X_n = \sqrt{\rho} Z + \sqrt{1-\rho}\varepsilon_n, \\
&Z \sim N(0,1), \;\; \varepsilon_n \sim N(0,1), \;\; Z \perp \varepsilon_n, \;\; \rho \in [0,1], \\
&X_n \leq \Phi^{-1}(\pi) \Leftrightarrow \text{type } n \text{ defaults}, \\
&Q : \text{the recovery rate when default.}
\end{aligned}
\tag{2.1}
$$

X_n is a linear combination of the aggregate factor Z which is common to all types and the idiosyncratic factor ε_n which is specific to type n. Z and ε_n ($\forall n$) are stochastic, standard normal and independent. ρ is the weight attached to the aggregate factor.[29] Type n borrowers default if $X_n \leq \Phi^{-1}(\pi)$, where $\Phi^{-1}()$ is the inverse distribution function of standard normal distribution. In other words, the marginal probability that type n defaults is π. If $\rho \neq 0$, the latent factors $\{X_1, X_2, \cdots, X_N\}$ are correlated. In this case, defaults among different types are also correlated. Finally, Q denotes the recovery rate of the loans when they are defaulted. I assume that it is common to all types and $Q < R_m$. The parameters (π, ρ, Q) characterize the default risks of loans. We will see that they are critical to the returns of the securitized assets that I introduce in this model. π and ρ are exogenous

[27] As we will see later, each type of borrowers may default their loans. If borrowers are divided unevenly, not only the number of types that default but also which types default matters as each type differs in population. We may be able to assume that every borrowers in $[0,1]$ interval are heterogeneous. However, this complicates measuring of the joint distribution of defaults. Therefore, it is convenient to assume borrowers are divided evenly, which implies N is a positive integer.

[28] R_m is still exogenous in the dynamic economy in the section 2.2.2.

[29] $\left(\sqrt{\rho}\right)^2 + \left(\sqrt{1-\rho}\right)^2 = 1$ implies that X_n is also a standard normal. $\rho \in [0,1]$ implies the bilateral correlation between any pair of latent factors is positive.

parameters throughout this chapter. On the other hand, I will endogenize Q in the dynamic housing choice model in section 2.2.2.

Securitization

Financial intermediaries and investors are involved in the securitization market.[30] The most important assumption in this chapter is that financial intermediaries and investors have different information on the default risks of loans: financial intermediaries know true (π, ρ) while investors have own belief on these parameters. I denote investor's belief as $(\tilde{\pi}, \tilde{\rho})$. I describe how financial intermediaries exploit returns by pooling and tranching that are the essences of securitization.

First, financial intermediaries purchase loans from banks and pool them. They diversify their portfolios so they purchase N types of loans evenly. In the next period, $n \in \{0, 1, 2, \cdots, N\}$ types of loans in the collateral could default. Therefore, the expected rate of return of the collateral, $E(R_c)$, is:

$$E(R_c) = \sum_{n=0}^{N} \left[\left(\frac{N-n}{N} \right) R_m + \frac{n}{N} Q \right] \Pr(n).$$

$\Pr(n)$ is the probability that n types of loans default. Note π and ρ affect $\Pr(n)$.[31]

Next, financial intermediaries issue two types of securities backed by the pool of loans: The payoffs of these two securities are contingent on the payoff of the collateral. I call one type of securities as Debt tranche[32] and the other type as Equity tranche. The following

[30] Banks are also involved in the securitization market. However, banks act mechanically which originate loans to borrowers with an exogenous interest rate R_m and sell them to financial intermediaries. There is no optimization for banks.

[31] It is difficult to obtain an analytical expression of $\Pr(n)$ due to the complexity of solving multiple integrals of correlated multivariate normal distributions. Instead, I solve it numerically using Matlab codes developed by Alan Genz, a professor of Mathematics at Washington State University. These are available at http://www.math.wsu.edu/faculty/genz/homepage.

[32] In the real world, market participants in securitization markets often call each class of securitized asset as a "tranche". Tranche is originally a French word which means "slice" in English.

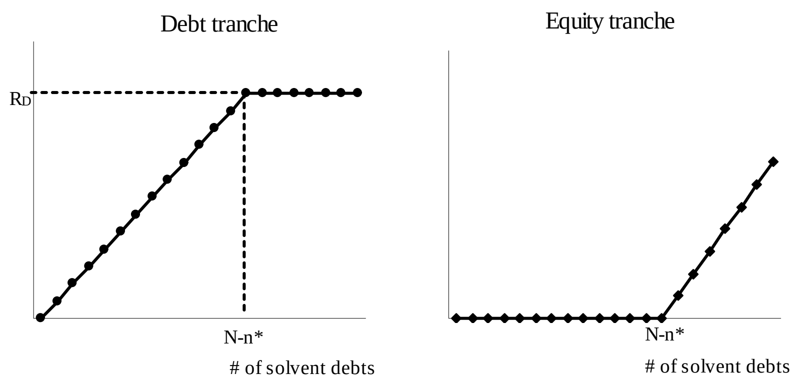

Figure 2-2: payoff schedules of each type of the securitized assets

formulas show the return structures:

$$
\begin{aligned}
R_d &= \begin{cases} D & \text{if } n \leq n^* \\ \frac{N-n}{N} R_m + \frac{n}{N} Q & \text{if } n \geq n^* + 1, \end{cases} \\
R_e &= \begin{cases} \frac{N-n}{N} R_m + \frac{n}{N} Q - D & \text{if } n \leq n^* \\ 0 & \text{if } n \geq n^* + 1, \end{cases} \\
&\frac{N-n^*-1}{N} R_m + \frac{n^*+1}{N} Q < D \leq \frac{N-n^*}{N} R_m + \frac{n^*}{N} Q.
\end{aligned} \tag{2.2}
$$

R_d and R_e are the rates of return of Debt tranche and Equity tranche normalized by the principal of the collateral. D can be interpreted as the notional value of Debt tranche.[33] Figure 2-2 visualizes the return structures. As the payoff structures above show, financial intermediaries issue different classes of securities from a pool of homogenous loans. The returns of the securitized assets are contingent on the total payoff of underlying assets. In this model, Debt tranche has a seniority to be paid out: Equity tranche first cover the losses of underlying assets and the notional value of Debt tranche, D, is secured if $n \leq n^*$. Debt tranche begins to suffer losses only when the number of default gets large.

Finally, financial intermediaries sell Debt tranche to investors while they keep holding Equity tranche. This structure is assumed in this model but it is advantageous to financial

[33] In relation with D, we call n^* the attachment point of Debt tranche.

intermediaries when investors underestimate (π, ρ): Financial intermediaries can lower D as investors tend to overestimate the expected return of Debt tranche. This implies more return is allocated to Equity tranche. When there are uninformed investors in the market, financial intermediaries can be benefited by holding Equity tranche. The securitiy design is crucial to allocate more returns to informed agents.

Now, I introduce the decision mechanisms of investors and financial intermediaries:

$$\frac{\tilde{E}(R_d)}{(1-l)} = D\sum_{n=0}^{n^*}\widetilde{\Pr}(n) + \sum_{n=n^*+1}^{n^*}\left[\frac{N-n}{N}R_m + \frac{n}{N}Q\right]\widetilde{\Pr}(n) \geq R, \tag{2.3}$$

$$\frac{E(R_e)}{l} = \sum_{n=0}^{n^*}\left[\frac{N-n}{N}R_m + \frac{n}{N}Q - D\right]\Pr(n) \geq R_f. \tag{2.4}$$

$l \in [0, 1]$ denotes the ratio of the principal of Equity tranche to the principal of the collateral. I assume that the sum of the principal of Debt tranche and Equity tranche is equal to the principal of the collateral. l is thus the leverage from financial intermediary's point of view. I assume l is exogenous throughout the model. $\tilde{E}(R_d)/(1-l)$ is the expected rate of return of Debt tranche given the subjective belief $(\tilde{\pi}, \tilde{\rho})$. $\widetilde{\Pr}(n)$ is the subjective probability that n types of loans default. I assume investors are risk neutral. Investors thus require $\tilde{E}(R_d)/(1-l)$ to be at least as much as R, the market interest rate.[34] Similarly, financial intermediaries require $E(R_e)/l$, the expected rate of return of Equity tranche, to be at least as much as R_f. I assume $R_f > R$. $R_f - R$ represents the excess return that financial intermediaries require to be willingly to hold Equity tranche in a reduced form way. Remember that Equity tranche is risky in the sense that it pays positive returns only when n is small.

Table 2-1 shows the sensitivity of the expected returns of the securitized assets to investor's belief on the default risks. Column 3 assumes that investors have true information, (π, ρ). Column 4 and 5 show the extent to which financial intermediaries are able to re-

[34]We can easily verify that there exists unique D such that (2.3) holds with equality under the assumption $R_m \geq R(1-l)$.

description	symbol	true information	low $\tilde{\pi}$	low $\tilde{\pi}, \tilde{\rho}$	low $\tilde{\pi}, \tilde{\rho}, Q$
default risk, true	(π, ρ)	$(0.2, 0.8)$	$(0.2, 0.8)$	$(0.2, 0.8)$	$(0.2, 0.8)$
default risk, belief	$(\tilde{\pi}, \tilde{\rho})$	$(0.2, 0.8)$	$(0.1, 0.8)$	$(0.1, 0.2)$	$(0.1, 0.2)$
recovery rate	Q	0.9	0.9	0.9	0.85
attachment point	D	0.920	0.919	0.918	0.918
expected return, Collateral	$E(R_c)$	1.02	1.02	1.02	1.01
expected return, Debt	$E(R_d)/(1-l)$	1.02	1.019	1.018	1.013
expected return, Equity	$E(R_e)/l$	1.02	1.029	1.033	0.984

I set $R = 1.02$, $R_m = 1.05$, $l = 0.1$ and $N = 5$. These parameters are unchanged throughout this experiment.

Table 2-1: Sensitivities of the expected return of securitized assets to default risks

allocate the return of collateral in favor of them when investors underestimate the default risks, i.e. $\tilde{\pi} < \pi$ and $\tilde{\rho} < \rho$.[35] In this case, D is set lower than what it is in the case of no information asymmetry. Consequently, financial intermediaries are able to raise the expected return of Equity tranche. In Column 6, I illustrate that the high expected return of Equity tranche blows out when Q declines. We can see that the decline of Q directly lowers $E(R_e)$ by (2.4). There is also an indirect effect through which a decline in Q raises D though the effect is tiny in this numerical example.

The above exercise illustrates that securitization is a way to reallocate the return of the collateral when investors are uninformed. Note, however, pooling alone doesn't give an opportunity to reallocate returns. If financial intermediaries sold a fractional claim to the collateral to investors, then the expected return of residual claims, which are held by financial intermediaries, would be still the same as that of the collateral. Investors would overestimate the expected return of the collateral, however the mismeasurement would not reflect to the return of securities which financial intermediaries hold. Instead, the key lies in tranching. Financial intermediaries actively reallocate the return of collateral by tailoring security design.

[35] The sum of the expected return of Debt tranche and Equity tranche must coincide with the expected return of the collateral, that is $(1-l)E(R_d) + lE(R_e) = E(R_c)$.

The ability to reallocate the return may be confined by the fluctuation of parameters which are common knowledge to everyone. In the above exercise, Q functions in this way. Later, I relate Q with HPI which is determined endogenously. The endogenous movement of Q is the source of variation in the profitability of securitization in my model.

2.2.2 Dynamic optimizations and the housing market.

Borrowers

Borrowers maximize the discounted sum of the period utility:

$$\max \tilde{E}_t \sum_{s=0}^{\infty} \beta_b^{s-t} U\left(c_{b,s}, h_{b,s}\right), \qquad (2.5)$$
$$U(c_b, h_b) = c_b + \gamma_b \ln h_b.$$

Their period utility function is the sum of the utility derived from consumption, c_b, and housing service that is proportional to the log of housing stock, h_b. γ_b can be interpreted as the weight attached to housing service. β_b is the discount rate of borrowers. I assume $\beta_b < R_m$, where R_m is the loan interest rate. The low discount rate ensures that borrowers accumulate positive debt outstandings in equilibrium. $\tilde{E}_t$ is the expectation operator given borrower's assessment on the default risks. I assume that borrowers have the same belief as investors.[36]

The flow of funds of borrowers is:

$$c_{b,t} + q_t h_{b,t+1} - \psi\left(\frac{h_{b,t+1}}{h_{b,t}} - 1\right)^2 h_{b,t} - B_{b,t+1} = A_t(h_{b,t}, B_{b,t}, n_t). \qquad (2.6)$$

In the RHS, $A_t(h_{b,t}, B_{b,t}, n_t)$ is the resources available to borrowers. It is a function of the existing housing stock, debt outstandings, $B_{b,t}$, and the default rate, n_t/N. A more

[36]Borrowers must have optimistic belief to generate booms. If not, borrowers forsee that the securitization is not profitable for financial intermediaries and they internalize the collapse of the securitization market. Difference in belief between investors and borrowers is non essential. All that matter is their belief deviate from the true stochastic process.

detailed explanation on A_t is described below. The LHS is the net expenditure. Borrowers consume, choose the new level of housing stock and the new level of borrowing. The house price when they acquire new housing stock is denoted by q_t. When they change the level of housing stock, they pay the quadratic adjustment cost, denoted by $\psi\left(\frac{h_{b,t+1}}{h_{b,t}}-1\right)^2 h_{b,t}$. ψ measures the magnitude of this cost. Because borrowers hate the fluctuation of housing service while they do not the fluctuation of consumption, they adjust their housing stock to a desired level very quickly without the adjustment cost. As we will see in the section 2.3.3, the gradual adjustment of housing stock is necessary to have sustained periods of economic booms induced by securitization.[37]

One more constraint that borrowers face is the borrowing limit:

$$B_{b,t+1} \leq q_t h_{b,t+1}. \tag{2.7}$$

As $\beta_b < R_m$, borrowers may borrow infinite amount without the borrowing limit.

Taking into account (2.5), (2.6) and (2.7) together, the Bellman equation of borrower's problem can be written as:

$$\begin{aligned} V_b\left(h_{b,t}, B_{b,t}, n_t\right) &= \max_{h_{b,t+1}, B_{b,t+1}} \left\{ \begin{array}{l} A_t - q_t h_{b,t+1} + \psi\left(\frac{h_{b,t+1}}{h_{b,t}}-1\right)^2 h_{b,t} + B_{b,t+1} \\ +\gamma_b \ln h_{b,t} + \beta_b \tilde{E}_{b,t}\left(V_b\left(h_{b,t+1}, B_{b,t+1}, n_{t+1}\right)\right) \end{array} \right\} \\ s.t \;\; B_{b,t+1} &\leq q_t h_{b,t+1}. \end{aligned} \tag{2.8}$$

$(h_{b,t}, B_{b,t}, n_t)$ summarizes the states of borrowers.[38] The exogenous state in this economy is the default rate, n_t/N. Here I assume that borrowing, B_b is available to borrowers.[39] This

[37] Iacoviello (2009) argues that the quadratic adjustment cost is inappropriate to describe the cost associated with the housing transactions. I simply state that the quadratic adjustment cost is a short-cut to generate sticky aggregate housing investment in the real world.

[38] In equilibrium, $(h_{b,t}, B_{b,t}, n_t)$ also summarizes the states of the aggregate economy. See detailed explanation in Appendix B.

[39] Indeed, this formula supposes that borrowing is available in any subsequent periods. We must assume that $(\tilde{\pi}, \tilde{\rho})$ is consistent with this supposition. That is (2.4) should be satisfied in any reachable states in the future under borrower's belief.

is the case when financial intermediaries intermediate borrowing demand and investment supply. As I describe the financial intermediary's behavior later, they may not intermediate the financial needs. In this case, their problem is simplified as:

$$\hat{V}_b\left(h_{b,t},0,n_t\right)=\max_{h_{b,t+1}}\left\{\begin{array}{c}A_t-q_th_{b,t+1}+\psi\left(\frac{h_{b,t+1}}{h_{b,t}}-1\right)^2h_{b,t}\\+\gamma_b\ln h_{b,t}+\beta_b\tilde{E}_{b,t}\left(\hat{V}_b\left(h_{b,t+1},0,n_{t+1}\right)\right)\end{array}\right\}. \tag{2.9}$$

Because borrowers have no access to borrowing, $B_b=0$, the new value function, $\hat{V}_b$,[40] is distinguished from the former one, V_b.

To complete the problem, I briefly describe the formula of $A_t(h_{b,t},B_{b,t},n_t)$. The resource of borrowers consists of followings: Wage payment to their labor and the value of existing housing stock. The loan repayment is a subtraction to their resource. Among them, the most important assumption is unemployment possibility which has one-to-one relationship with loan defaults.

To be clear, unemployment is solely an exogenous process to borrowers: every borrowers are willing to supply a unit of labor but job opportunities are randomly assigned in each period. A more detailed explanation of the unemployment process follows in section 2.2.3, however, an important consequence is that unemployed borrowers are unable to repay debt therefore they default. In this case, the fire sale of their housing stocks takes place. These are the twists in otherwise a standard dynamic housing choice problem *a la* Iacoviello (2005, 2009).

Given the employment opportunity, borrower's ability to repay loan is:

$$\begin{array}{ll}\min\left\{w+\tilde{q}_th_{b,t},R_mB_{b,t}\right\} & \text{if employed,}\\ \min\left\{\tilde{q}_th_{b,t},R_mB_{b,t}\right\} & \text{if unemployed.}\end{array}$$

[40] There should be a time, say τ, such that borrowers have positive existing debt outstanding $B_{b.\tau}>0$ but new borrowing is not available. We thus need to define one more value function which is just for the transition period. I relegate this to Appendix B.

w is the wage which is assumed to be fixed over time. $\tilde{q}_t$ is the fire sale price of house which conceptually differs from the normal house price, q_t. I assume borrowers first use wage to repay debts. After they exhaust wage, they begin to sell existing housing stock to repay remaining obligations. If wage is sufficiently high and if the fire sale price of house is sufficiently low, employed borrowers repay fully while unemployed borrowers default and sell their entire housing stocks.[41]

Because of the employment status, borrowers become heterogeneous ex post even though they are homogeneous ex ante. To deal with a homogeneous agent problem, I assume that there exists a perfect insurance regarding after-loan-repayment wealth: after each borrower repays whatever amount possible, their wealth is redistributed and equalized. Remember that I assumed that borrowers were homogeneous in terms of the amount of borrowing in section 2.2.1. Without the perfect insurance, borrowers would become heterogeneous in one period and the amount of borrowing would vary.

Ultimately, $A_t(h_{b,t}, B_{b,t}, n_t)$ denotes the equally redistributed wealth. Because wealth is redistributed, the default rate, n_t/N, matters for per capita wealth. I relegate the complete explanation of the wealth redistribution process in Appendix B.

Investors

Investors live two periods. The population of investors relative to borrowers is N_i. I call them "young" when they live in the first period and "old" when they live in the second period. Their objective function is:

$$\max c_{y,t} + \gamma_y \ln h_{y,t} + \beta \tilde{E}_t \left[c_{o,t+1} + \gamma_o \ln h_{o,t+1} \right]. \tag{2.10}$$

[41] I assume that the market for new loans is not open at the time borrowers repay debts. Together with the overlapping structure of investors which I describe below, this assumption lets us avoid complications which arise from heterogeneous expectations on HPI which has one-to-one relationship with the recovery rate of defaulted loans. In principle, I need homogeneous expectation on the recovery rate even agents have different information on the default risks, (π, ρ). A more detailed explanation is found in section 2.2.3.

As borrowers, they derive utility from consumption and housing service. The subscript y stands for the young and the subscript o stands for the old. For example, $c_{o,t+1}$ denotes the consumption of old investors at time $t+1$. γ_y and γ_o are the weights attached to housing service in each period. β is their discount rate. As in borrower's problem, $\tilde{E}_t$ is the expectation operator given investor's subjective belief on the stochastic employment opportunities.

Old investors die after they consume. In each period, a new cohort of investors of the same population is born. That is the population of investors is fixed at every periods. When old investors die, the new born young investors inherit the wealth of old investors, which is of the form of housing stock. Their flow of funds is thus:

$$\begin{aligned} c_{y,t} + B_{d,t+1} + q_t h_{y,t} &= w + q_t h_{o,t}, \\ c_{o,t+1} + \tilde{q}_{t+1} h_{o,t+1} &= R_{t+1} B_{d,t+1} + \tilde{q}_{t+1} h_{y,t}. \end{aligned} \tag{2.11}$$

The first line is the budget constraint of young investors and the second line is that of old investors. When they are young, they obtain wage by supplying a unit of labor, consume, $c_{y,t}$, invest in Debt tranche, $B_{d,t+1}$, and decide the new level of housing stock, $h_{y,t}$. Young investors trade housing stock after the fire sale of housing stock is over.[42] The return of Debt tranche realizes in the next period when young investors become old. The return, R_{t+1}, is contingent on n_t. At the same time, old investors trade housing stock further. They absorb the fire sale supply of housing stock which defaulted borrowers release to the market. The housing stock at the end of life is denoted by $h_{o,t+1}$.

[42] They trade housing stock with borrowers who have repaid their debts and whose wealth is already redistributed.

The FOCs of investors' problem are the followings:

$$
\begin{aligned}
1 &= \beta \tilde{E}_t \left(R_{t+1} \right), \\
q_t &= \frac{\gamma_y}{h_{y,t}} + \beta \tilde{E}_t \left(\tilde{q}_{t+1} \right), \\
\tilde{q}_{t+1} &= \frac{\gamma_o}{h_{o,t+1}}.
\end{aligned}
\tag{2.12}
$$

The first equation is the Euler equation on Debt tranche holdings. Investor's belief matters as the subjective expected return, $\tilde{E}_t \left(R_{t+1} \right)$, enters in the equation. By the first equation, any assets must offer the rate of return equal to $1/\beta$ in investor's expectation. Therefore $1/\beta$ is actually the market interest rate, R, in equilibrium. It is consistent with the risk neutral assumption in the securitization model above.[43]

By now we are able to see more detailed distinction between q and $\tilde{q}$. When investors are young, they trade housing stock with borrowers who already repay debt obligations. At this point, the fire sale of housing stock is over. They thus trade house with the normal house price, q. At the beginning of the next period, old investors, who are used to be young one period ago, further trade housing stock. Here, I assume that old investors absorb the fire sale supply of housing stocks with house price being the fire sale price, $\tilde{q}$.

Financial intermediaries

There are measure one financial intermediaries who are infinitely lived. They obtain endowment w_f in each period. If (2.4) is satisfied, financial intermediaries provide lending to borrowers by issuing the securitized assets. In this case, the fraction l of borrower's loan is ultimately financed by financial intermediary's endowment.[44] Otherwise, they step away from the securitization market and consume all of the endowment. This implies borrowers are not able to borrow. I denote $c_{f,t}$ and $B_{e,t+1}$ as consumption and investment to Equity

[43] This implies their saving demand (lending supply) curve becomes flat at $\tilde{E}_t \left(R_{t+1} \right) = 1/\beta_i$.

[44] I assume e_f is large enough so that financial intermediaries can contribute portion l of the outstandings of the loans.

tranche respectively.

One important assumption is that borrowers and investors learn the true parameters, (π, ρ), immediately at the first time financial intermediaries withdraw from the securitization market. And I assume that financial intermediaries will not return to the securitization market thereafter. This is a strong and crucial assumption to have crises in this model. Although I do not introduce a systematic learning of the parameters, this assumption could be justified in the following way: Borrowers and investors put zero probability on the event that financial intermediaries withdraw from the securitization market under certain optimistic belief $(\tilde{\pi}, \tilde{\rho})$. That is they think (2.4) should be satisfied whatever possible paths the economy undergoes given their information. Then, the fact that financial intermediaries withdraw from the market has borrowers and investors reconsider the profitability of the securitization. Eventually, they revise their belief and I assume that they somehow learn the true parameters (π, ρ) at once. Moreover, it is possible that (2.4) is no longer satisfied once investors learn the true parameters: Financial intermediaries have to raise D under no information asymmetry. This lowers the expected return of Equity tranche. If the decline of the expected return were large enough, financial intermediaries would have no incentive to return to the securitization market.

2.2.3 Incorporating the securitization market

In each period, the securitization in section 2.2.1 is incorporated into the dynamic economy in section 2.2.2. However, we must clear several issues. First of all, we must specify the unemployment process and relate it to defaults of borrowers. Second the recovery rate, Q, should be endogenize so that we fully utilize the dynamic model in this chapter.

For the first issue, I model unemployment as a proxy of default. In section 2.2.1, I define (2.1) as "default process". In the dynamic model in section 2.2.2, I redefine it as the exogenous unemployment process for borrowers: the latent factor X_n governs the job opportunities of type n borrowers. If $X_n \leq \Phi^{-1}(\pi)$, type n borrowers are unemployed.

As assumed in section 2.2.1, investors and borrowers have subjective belief on the parameters (π, ρ). It is denoted as $(\tilde{\pi}, \tilde{\rho})$. Additionally, I assume that $(Z_{t1}, \varepsilon_{1,t1}, \cdots, \varepsilon_{N,t1})$ are independent of $(Z_{t2}, \varepsilon_{1,t2}, \cdots, \varepsilon_{N,t2})$ for $\forall t_1 \neq t_2$. And I parameterize the model such that unemployed borrowers default.[45]

For the second issue, the equivalence between unemployment and default helps to relate HPI with the recovery rate, Q. The maximum amount that unemployed borrowers can repay is the fire sale value of their housing stocks, $\tilde{q}_{t+1} h_{b,t+1}$. Assuming that the borrowing constraint (2.7) binds in equilibrium, we can express borrower's housing stock as $h_{b,t+1} = B_{b,t+1}/q_t$. Combining these two, we can express the recovery rate as $\pi_{q,t+1} \equiv \tilde{q}_{t+1}/q_t$ which is determined endogenously in the equilibrium. As such, the endogenous fluctuation of HPI is the key determinant of the profitability of the securitization in this model (See (2.4) with replacing Q with $\pi_{q,t+1}$).

In (2.12), $\tilde{q}_{t+1}$ depends only on the variables at time $t+1$. By this fact we can avoid complications which arise from heterogeneous HPI expectations. Suppose investors are infinitely lived and suppose new borrowing is allowed when borrowers repay existing debts. In this case, there would be no distinction between q and $\tilde{q}$ as in the standard housing DSGE model. Then future house price, q_{t+1} would depend on $h_{b,t+2}, h_{b,t+3}$ and so on as it would be the discounted sum of the future flow of the marginal utility of housing service. Because investors and financial intermediaries have different information on the unemployment risks, they may have different views on the profitability of the securitization in the future. This might be reflected to different views on future housing allocations because securitization provides borrowers with resources to purchase home. Then there would be a disagreement on the expectation of q_{t+1} which would complicate the solution. By the non-standard overlapping generation structure for investors, we can guarantee that there is

[45] It is not immediate that unemployment is equivalent to default. Even a borrower is unemployed, he could have enough wealth to repay his debt if $\tilde{q}_t h_{b,t} \geq R_m B_{b,t}$. I carefully parameterize the model so that unemployed borrowers default while employed borrowers have sufficient resources to repay debts ($w + \tilde{q}_t h_{b,t} \geq R_m B_{b,t}$) in equilibrium. In what follows, I assume that the parameters of this economy satisfy this criterion.

no disagreement on $\pi_{q,t+1}$.

2.2.4 Equilibrium

In this economy, consumption good is produced by labor in one-to-one relation:

$$y_t = AL_t.$$

Assuming perfect competition, $w = A$. In equilibrium, L_t is the sum of labor supply by investors and borrowers which is determined exogenously. Investors supply a fixed amount of labor while borrowers supply labor stochastically.

The equilibrium is an allocation of $\{h_{b,s+1}, h_{y,s}, h_{o,s+1}, B_{b,s+1}, B_{d,s+1}, B_{e,s+1}\}_{s=t}^{\tau-1}$, $\{h_{b,s+1}, h_{y,s}, h_{o,s+1}\}_{s=\tau}^{\infty}$, the house price $\{q_s, \tilde{q}_s\}_{s=t}^{\infty}$, the realization of the unemployment $\{n_s\}_{s=t}^{\infty}$, and the threshold time τ associated with the sustainability of securitization given the initial state $(h_{b,t}, B_{b,t})$ such that

1. For $s \leq \tau - 1$, $(h_{b,s+1}, B_{b,s+1})$ solve borrower's maximization problem (2.8) given the sequence of house prices that borrowers presume. And $(h_{y,s}, h_{o,s+1}, B_{d,s+1})$ solve investor's maximization problem (2.10). In this phase, financial intermediaries issue the securitized assets and $B_{e,s+1} > 0$. During this phase, borrowers and investors have subjective belief $(\tilde{\pi}, \tilde{\rho})$.

2. At $s = \tau$ (2.4) is violated for the first time. Financial intermediaries withdraw from the market and borrowers and investors learn (π, ρ). A rescue plan is implemented to save borrowers if necessary (See Appendix B to understand why we need the rescue plan and how I implement it). For $s \geq \tau$, financial intermediaries no longer issue the securitized assets.

3. For $s > \tau$, $h_{b,s+1}$ solves borrower's maximization problem (2.9) given the sequence of the house prices. $(h_{y,s}, h_{o,s+1})$ solves investor's maximization problem (2.10) with

restricting $B_{d,s+1} = 0$. In this phase $B_{e,s+1} = 0$.

4. Market clearing conditions are satisfied:

$$
\begin{aligned}
\text{the good market} &: \quad c_{b,s} + c_{y,s} + c_{o,s} + c_{f,s} = \left(\frac{n_s}{N} + N_i\right) y + w_e \\
\text{the housing market} &: \quad h_{b,s+1} + N_i h_{y,s} = \bar{h} \\
\text{the securitization market} &: \quad B_{b,s+1} = N_i B_{d,s+1} + B_{e,s+1}
\end{aligned}
$$

To be precise, there remain exogenous variables which DSGE models usually treat endogenous: borrower's employment is solely governed by the exogenous stochastic process. The loan interest rate, R_m, and the leverage ratio, l, also remain exogenous. The loan interest rate and the leverage affect the return of securitized assets if they are endogenize. Because making them endogenous complicates the solution of the model a lot, I leave them exogenous. The markets clear given these exogenous components. Although the price of loans is exogenous, the assumptions in this model, such as the leverage is exougenous, help us to clear all markets.

The economy is divided into two phases: One is the periods when financial intermediaries engage in securitization and the other is the periods when they keep away from it. I refer the former periods to the securitization regime and the latter to the non-securitization regime. Let τ denote the threshold period that the economy changes from the securitization regime to the non-securitization regime: In periods $s \equiv \{t, t+1, \cdots \tau - 1\}$, financial intermediaries issue the securitized assets. Thereafter financial intermediaries keep away from the market. τ can be ∞ or t, i.e. one of the two regimes continues entire periods. However it is never possible that the securitization regime follows after the non-securitization regime by the assumptions above.

The periods $s \leq \tau - 1$ are said to be a boom phase if $\tau > t$ under the belief $(\tilde{\pi}, \tilde{\rho})$ but $\tau = t$ if borrowers and investors knew true parameters. Intuitively, a boom is a phase when borrowers accumulate debt outstandings and increase housing demand because of the mismeasurements of the default risks. The period τ is said to be a bust period, or equivalently a crisis period, if a boom phase unfolds at time t but the incentive condition (2.4) is violated for the first time at time τ. At time τ, financial intermediaries suddenly withdraw from the securitization market and borrower's borrowing demand is no longer financed. At this period, borrowers are forced to adjust their consumption-housing plan.

2.3 Model implications

The question that I would like to address in this section is whether the optimistic belief, $(\tilde{\pi}, \tilde{\rho})$, induces boom-and-burst cycles in the securitization market and the housing market under reasonable parameterizations. To check this, first I parameterize the model to match some of the moments of actual data. Then I examine the properties of equilibrium to get an intuition about the conditions that crises occur. Finally I simulate the model and check whether a crisis emerges in the model economy.

2.3.1 Parameterizations

The parameters in this model are $(\pi, \rho, \tilde{\pi}, \tilde{\rho}, R_m, l, \beta_b, \beta, w, \gamma_b, \gamma_y, \gamma_o, \bar{h}, N, N_i, R_f, \psi)$. The time frequency of this model is annual. Table 2-2 summarizes the parameterization. I target many of the parameters so that the data generated by the model is consistent with data in U.S. during 2000s when subprime mortgage securitization grew rapidly. I set $\beta = 0.98$ which implies the annual real interest rate is about 2%. This is consistent with the real interest rate imputed by 10-year Treasury inflation-protected securities (TIPS). R_m, the real loan interest rate, is set to 5%. This is consistent with the subprime mortgage interest rates shown in Nedauld and Scherlund (2009); the nominal interest rates of subprime mortgages that were securitized were around $7 \sim 8.5\%$. Assuming that expected inflation is 2~3$\%$, which is consistent with the average U.S. inflation in past 10 years, the real interest rate of subprime mortgages can be approximated by 5%. In this model, we can interpret the loans

parameter	description	value	parameter	description	value
π	unemployment probability (true)	0.125	w	wage	0.75
ρ	unemployment correlation (true)	0.8	γ_b	housing weight, borrowers	0.1
$\tilde{\pi}$	unemployment probability (belief)	0.075	γ_y	housing weight, young investors	0.025
$\tilde{\rho}$	unemployment correlation (belief)	0.2	γ_o	housing weight, old investors	1.0
R_m	real loan interest rate	1.05	$\bar{h}$	housing supply	2.5
l	principal share, Equity tranche	0.19	N_i	population of investors	2
β_b	discount rate, borrowers	0.98	R_f	required return, FIs	1.05
β	discount rate, investors	0.9	ψ	housing adjustment cost	2.5

Table 2-2: Parameterization of the model

as subprime mortgages because they are collateralized by housing stocks and they have high default probability. Together with $\beta_b = 0.9$,[46] the calibration of R_m implies borrowers actually borrow in the equilibrium.

The set of parameters $(w, \gamma_b, \gamma_y, \gamma_o, \bar{h}, N_i)$ targets housing wealth-output ratio. During 2000s, this ratio averaged 1.25 according to the data released by Bureau of Economic Analysis. The set of parameters implies the housing wealth-output ratio in the model reaches to 1.25 when the economy reaches to the steady state of the securitization regime.

The fact that there are many more tranches than two in typical securitization challenges the parameterization of l, the share of Equity tranche's principal. In U.S. rating companies give credit ratings to collateralized debt obligations (CDO). In this chapter, I assume that Debt tranche represents AAA rated tranches in subprime mortgage securitization in U.S. According to Nedauld and Scherlund (2009), the principal share of the AAA rated tranches floated around 0.75~0.85. In this parameterization, I set $1 - l = 0.79$. R_f is set to be greater than the market interest rate. As described in section 2.2.1, the excess return, $R_f - R$, captures the compensation that financial intermediaries require for the riskiness of Equity tranche in a reduced form way. The excess return is 300bps in this parameterization. Ashcraft and Schuremann (2008) calculate that the spread between BBB rated tranches and

[46] This value is roughly consistent with the calibration of impatient households in Iacoviello (2005, 2009).

AAA rated tranches is about 1000bps as of July 24 2007. Though the data was retrieved in the midst of the subprime crisis and I don't have the data before the crisis unfolded, 300bps is not an unacceptable order.

$(\pi, \rho, \tilde{\pi}, \tilde{\rho})$ are key parameters in this model but they are the parameters whose information is hard to obtain. I simply assume that $(\pi, \rho) = (0.125, 0.8)$. The marginal probability that a borrower loses job is much higher than the national unemployment rate in U.S. This captures the notion that borrowers in this model are quite risky. I assume $(\tilde{\pi}, \tilde{\rho}) = (0.075, 0.2)$ which implies borrowers and investors have an optimistic view on the default risks.

2.3.2 The properties of the solution

The policy functions of the problem (2.8) under equilibrium house prices shape the dynamics of the securitization regime. The policy functions are denoted by $h_{b,t+1} = g_h(h_{b,t}, B_{b,t}, n_t)$ and $B_{b,t+1} = g_b(h_{b,t}, B_{b,t}, n_t)$.[47]

To illustrate the solution properties, I show the case of $N = 2$, the simplest case possible in this model. With the parameters above, Equity tranche earns strictly positive return when $n = 0$ or 1. Within these two cases, HPI affects the return of Equity tranche when $n = 1$.

Figure 2-3 shows the housing choice of borrowers, $g_h(h_{b,t}, B_{b,t}, n_t)$. I fix $B_{b,t}$ in this figure and I focus on how borrowers choose the level of housing stock when $h_{b,t}$ and n_t change. When $n_t = 0$, that is when no borrower defaults, the policy function crosses the 45 degree line only once from above. When, $n_t = 2$, that is when every borrowers default, the policy function is on the 45 degree line until the existing housing stock gets sufficiently large.

The policy function above implies that borrowers increase their housing stock when the level of the existing housing stock is small. However, they are allowed to do so only when

[47] The method of computing the equilibrium prices and the policy functions can be seen in the Appendix B.

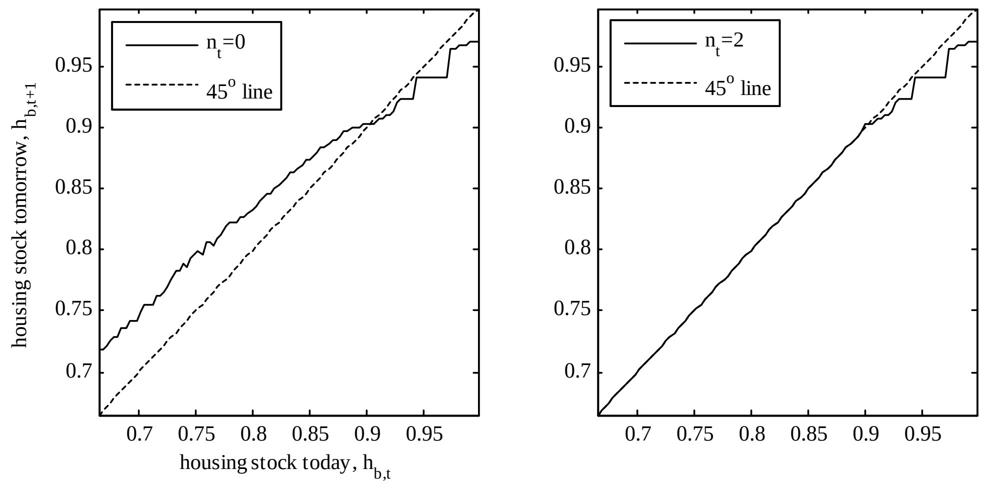

Figure 2-3: The policy function on housing: $g_h(h_{b,t}, B_{b,t}, n_t)$

the default rate is low. When many borrowers default, e.g. $n_t = 2$, per-capita wealth is close to zero and they are not afford to change the level of housing stock by paying the housing adjustment cost. All that they can do is to maintain the existing level of housing stock.[48] Intuitively, they manage to live in the same house by accepting low consumption when the per-capita wealth is small.

Figure 2-4 shows the policy function on borrowing, $g_b(h_{b,t}, B_{b,t}, n_t)$, along with the choice of house value, $g_h(h_{b,t}, B_{b,t}, n_t)$ multiplied by q_t. The horizontal line represents the existing level of housing stock and the vertical line represents the choice of new borrowing as well as new housing value. As the existing level of housing stock increases, borrowers increase the amount of borrowing. Moreover, the borrowing constraint binds upto a certain level of housing stock: the amount of borrowing coincides with the value of housing stock until the economy reaches to the steady state from below.[49] Recall that the borrowing constraint must bind to claim that the HPI $= Q$. This condition is satisfied whenever the economy starts below the steady state of the securitization regime.

[48]By the form of the borrowing constraint, (2.7), it is possible to maintain the same level of housing stock without paying the housing adjustment cost.

[49]The threshold (dashed line in Figure 2-4) is exactly the steady state of the securitization regime.

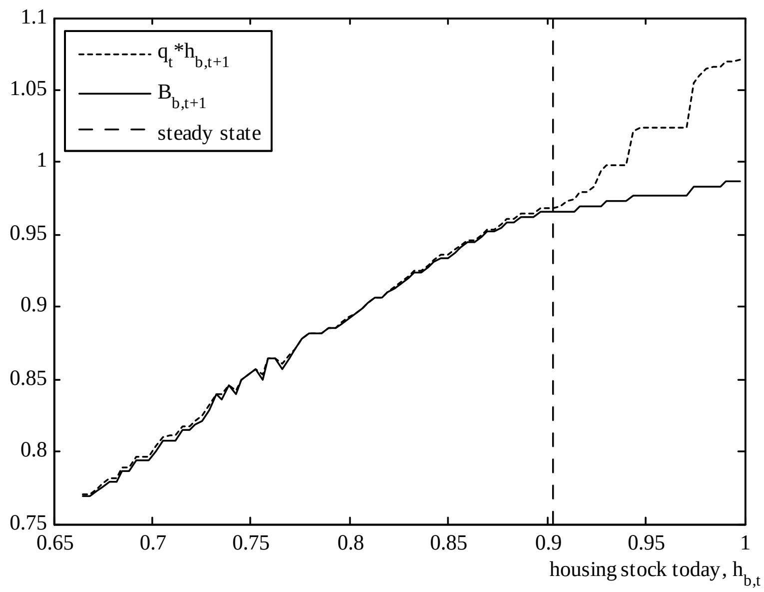

Figure 2-4: The borrowing choice of borrowers ($n_t = 0$).

2.3.3 Simulation

Figure 2-5 shows the simulation result. The lower-middle figure shows the simulated path of the unemployment rate of borrowers (not the aggregate economy).[50] It is based on random draws of the latent factors in (2.1). I assume $N = 6$ in this simulation. The simulation result is not sensitive to the choice of N.[51]

The other figures in Figure 2-5 show the paths of endogenous variables. The solid lines plot the actual paths of the economy while the dotted lines plot the paths of the economy had the securitization regime continued indefinitely. Borrowers increase housing stock for the first several years when they begin with a low level of housing stock. On this course, borrowers accumulate debt outstandings due to the expansion of the borrowing capacity.

Borrower's housing stock suddenly drops after 4 years (see the solid line in the upper-left figure). At this point, the expected return of Equity tranche falls short of R_f, the

[50] With the calibration, the economy wide unemployment rate is at most 33%.

[51] What matters is the expectation of $\pi_{q,t+1}$ in the states for which $\pi_{q,t+1}$ affects the return of Equity tranche; by (2.4), we can see that the time t assessment on one-period ahead recovery rates in the states $n_{t+1} < n^*$ is important for the profitability of securitization. To this end, what matters is the choice of $(\pi, \rho, \tilde{\pi}, \tilde{\rho})$ not N.

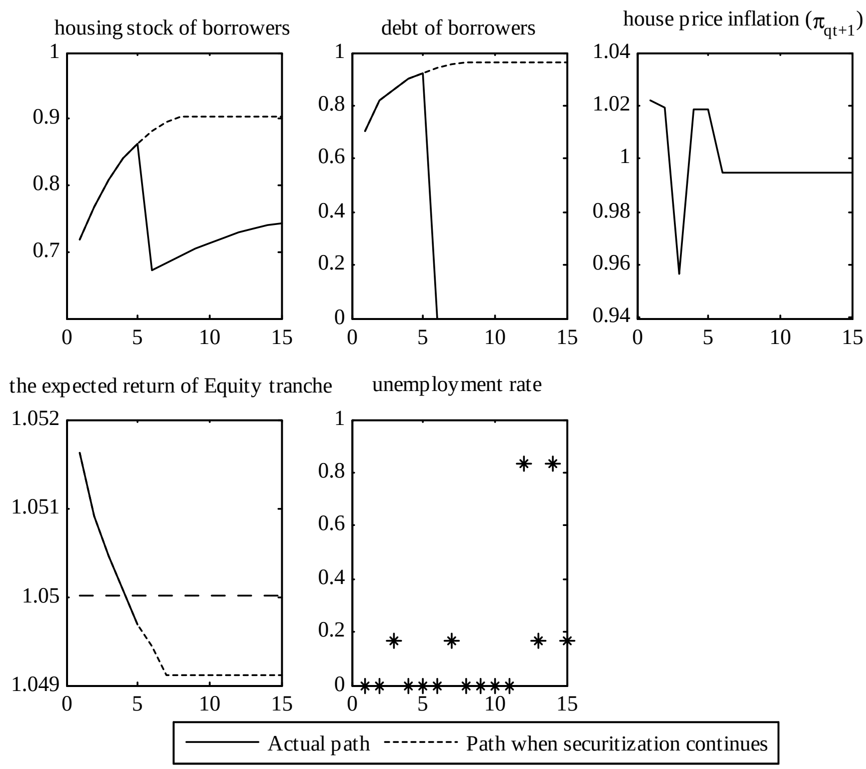

Figure 2-5: A simulation result

required return of financial intermediaries, for the first time (See the lower-left figure in Figure 2-5). Financial intermediaries withdraw from the securitization market and a crisis unfolds at that point. Borrowers and investors revise $(\tilde{\pi}, \tilde{\rho})$. By the assumption, they learn the true parameters immediately. Under new information, the securitization is no longer profitable for financial intermediaries. They thus keep away from the securitization. Borrowers are forced to adjust their housing stock substantially because they are no longer allowed to borrow.[52] Thereafter, borrowers accumulate housing stock gradually, but it never reaches to the level once it was the securitization regime.

HPI is volatile and it is correlated with the unemployment state while the economy is in the securitization regime: When realized n is small, the realized HPI is high. The housing fire sale drives the fluctuation of HPI: by (2.12), the fire sale price $\tilde{q}_{t+1}$ relative to the normal

[52] See the detailed procedure of housing stock adjustment at crises in Appendix B.

Time	t, t+1,⋯,τ-1	τ	τ+1,τ+2,⋯
Regime	Securitization regime	Securitization regime collapses	Non-securitization regime
Investor's perception on the default risks	Own belief	Learn true parameters	True parameters
Borrower's housing stock	Increase over time	Plunge	Increase but never restore previous level
HPI expectation	Decline over time	flat	flat
Return on Equity tranche	Above required return	Below required return	Below required return

Table 2-3: The flow of events and the direction of the dynamics.

price q_t is high when the fire sale supply is small, that is when n is small.[53] This relationship disappears after financial intermediaries withdraw from the securitization market. Because the housing fire sale no longer takes place in this regime, there is no variation in $\tilde{q}_{t+1}$.

Table 2-3 summarizes the entire chronology of the boom-and-bust cycle. The boom emerges when investors have optimistic views on the default risks of loans. In this periods, financial intermediaries can enjoy excessively high return from Equity tranche. However, a magma accumulates at the same time: the expected HPI declines over time as the speed of expansion slows down. Eventually, the expected HPI hits the threshold that the expected return of Equity tranche is break even with R_f. At this point, financial intermediaries suddenly stop issuing the securitized assets and a crisis unfolds.

A negative shock, high unemployment, is not necessary to trigger the crisis. In fact, the crisis unfolds when $n = 0$ in this simulation. All that matter is one-period-ahead expectation of HPI in the states where it affects the return of Equity tranche. Figure 2-6 visualizes this claim. The upper-left figure shows the path of the expected return of Equity tranche. It declines over time. This corresponds to the decline of HPI in states where it matters for the return of Equity tranche (see the two figures in the bottom row of Figure

[53]To have high HPI during securitization regime, as is observed in U.S, we need to have a series of good draws of the latent factors.

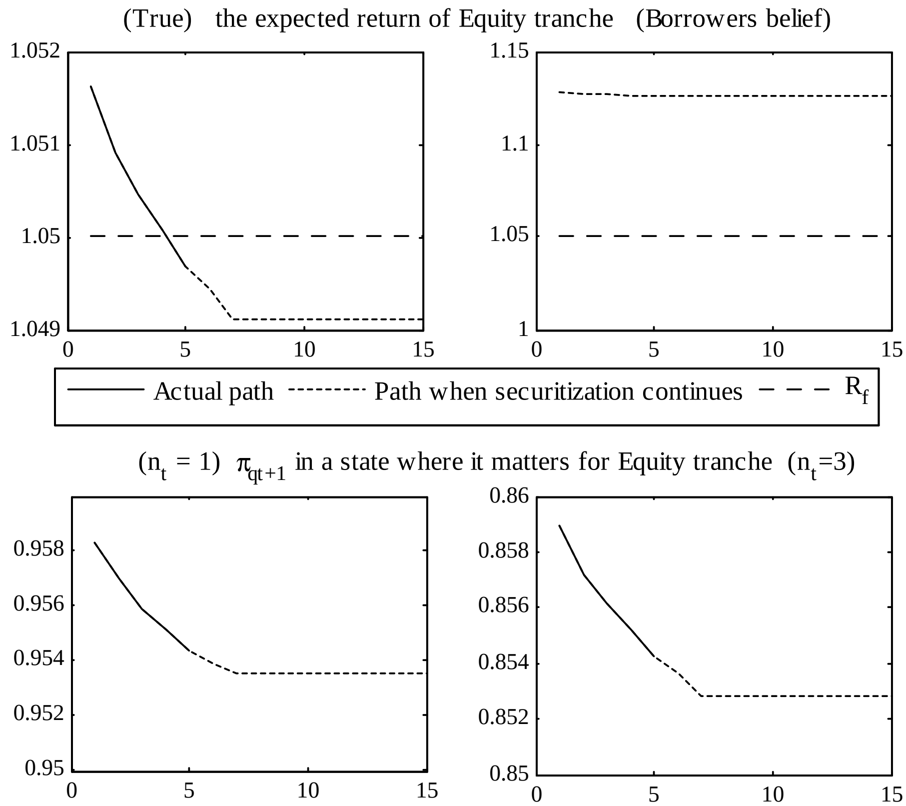

Figure 2-6: House price inflation and the expected return of Equity tranche

2-6. I only show the cases when $n = 1$ and 3 but the same argument holds for the all relevant states). After 4 years, the HPI is no-longer high enough to sustain the expected return of Equity tranche above R_f.

If borrowers knew the financial intermediary's incentive truly, then borrowers might foresee that the securitization regime would collapse at some point in the future.[54] However, borrowers overestimate the expected return of Equity tranche under their belief. The upper-right figure in Figure 2-6 plots borrower's view on the expected return of Equity tranche. Borrowers presume that it is well above R_f. So, borrowers (as well as investors) never anticipate that the securitization regime collapses.[55] Borrowers, who anticipate the

[54]This implies the Bellman equation (2.8) is not appropriate to describe the problem of borrowers; (2.8) assumes the securitization regime continues indefinitely.

[55]To justify this claim, it is not sufficient to check the expected return of Equity tranche in realized states. We must check it in any states that are reachable given the initial state $(h_{b,t}, B_{b,t})$. Here reachable states mean the states that the economy can arrive with non-zero probability.

securitization regime continues, plan to accumulate housing stocks and debt outstandings further as dotted lines in Figure 2-5.

2.3.4 A counter factual exercise

This section demonstrates the importance of the assumptions made in this chapter. I conduct a following experiment: I remove one of the two crucial assumptions, a deviation from rational expectation and the availability of securitization, and check whether the expected return of securities that financial intermediaries would hold exceeds R_f.

In Figure 2-7, at each point of time, I assume that the economy begins with the housing stock and debt outstandings that borrowers choose in the simulation in the section 2.3.3. The line with circle marker shows the expected return of Equity tranche when investors know the true default risks, (π, ρ). The line with square marker shows the expected return of securities when securitization is not available to financial intermediaries. In the latter case, I assume financial intermediaries can pool the loans but cannot tranche the claims. Hence, the expected return shown with the square marker is the expected return of the fractional claim to the pool of the loans. The latter exercise demonstrates the importance of tranching.

It is the combination of the two assumptions that gives financial intermediaries an opportunity to obtain high return. When returning to rational expectation, that is when investors know (π, ρ) truly, financial intermediaries have to give higher return to investors. This lowers the expected return of Equity tranche. When tranching is not available, financial intermediaries have no way to reallocate the return of the collateral. In this case, information structure is irrelevant for the securities that financial intermediaries would hold.

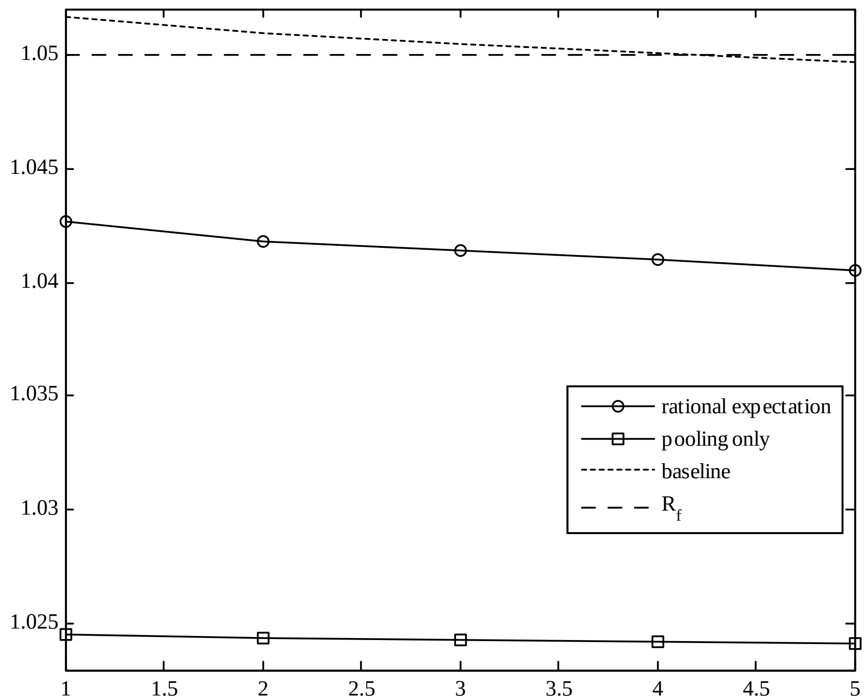

Figure 2-7: The expected return of securities that financial intermediaries hold under alternative assumptions.

2.4 Discussions

2.4.1 The mechanism of Boom and Bust

Coval et al (2009) and Heitfield (2009) suggest that securitization magnifies errors in measuring the default risks of underlying assets: Holders of junior claims are overcompensated if holders of senior claims underestimate the default risks. They argue that securitization of subprime mortgages grew explosively as a result of mispricings of securitized assets.

The main contribution of this chapter is to utilize the above idea in a dynamic macroeconomic model. By deviating from rational expectation in a specific way, I attempt to create boom-and-bust cycles in the aggregate economy. When investors mismeasure the default risks of underlying assets, the security design of securitized assets facilitate financing risky lending that would not be taken place under rational expectation. Then borrowers increase housing demand due to the extended availability of resources. The overpricing of the securitized assets induces speculative expansions of the aggregate economy.

The evolution of HPI plays a key role to generate crises. This is consistent with recent empirical findings on mortgage defaults, such as Gerardi, Shapiro and Willen (2007): They find a statistically significant relationship between the depreciation of house price and the increase of the default probability of mortgages. They also find that the impact is economically significant. When many mortgages default due to house price depreciation, then the return of mortgage backed securitized assets should also decline. I incorporate this finding by connecting HPI to the recovery rate in my model.

Resource availability for borrowers fluctuates during the boom-and-bust cycles in this model. Broadly speaking, this cycle is induced by preference shocks on housing in the word of macroeconomics. Iacoviello and Neri (2009) give one of the interpretations of the housing preference shock as follows:

> *"cyclical variation in the availability of resources needed to purchase housing"*.

In this model, the financial intermediary's incentive condition (2.4) endogenously determines the availability of resources. The "shock" in this model is the mismeasurement of default risks by borrowers and investors. This raises the expected return of Equity tranche above R_f for a while thereby giving additional resources to borrowers. However, the expected return declines over time as the recovery rate gradually declines. Eventually, it falls below R_f and borrowers are no longer able to obtain resources for home purchase. We could generate the same kind of dynamics by fluctuating housing preference parameters in housing DSGE models.

2.4.2 Some missing parts

Although the model in this chapter has salient features to generate securitization-induced economic crises, it is not able to replicate some facts about the subprime crisis in U.S. First of all, this model is not able to generate high HPI observed in the data. In Figure 2-5, (real) HPI during the securitization regime is at most 2%. This is much lower than the peak HPI in U.S during 2000s. Even I parameterize the model to raise HPI in the securitization regime, the HPI in this model falls well short of the data.

Another limitation of the current version of the model is that aggregate output is exogenous so as to incorporate an involved financial market with bounded rationality. A boom in the financial market thus does not lead to a boom in the good market. However a small change can fix this problem. For example, assuming a neo-classical style labor market for investors with a specific preference could generate output increases during the securitization regime.[56] In the neo-classical labor market, agents increase labor supply when the marginal utility of consumption is high. In the securitization regime, consumption of investors tends to decline because investors allocate more resources to investment. Then they would try to supply more labor because of the high marginal utility of consumption. Therefore the boom in the financial market could be transmitted to the good market.

Finally, the model economy generates no slumps after the securitization market collapses. Borrowers undergo only one time adjustment at the time of crisis but economic activities, such as housing of borrowers, immediately recover thereafter. On the contrary, U.S. economy has slumped for sustained periods after the collapse of subprime securitization markets. A part of reason is that this model does not have the financial accelerator effect in the non-securitization regime, which generates persistent and amplified effects on the aggregate output[57]: I assume financial intermediaries keep away from intermediat-

[56] An example of the preferences might be $(c_{y,t})^{v}\left[\psi_l(\bar{l}-l_{y,t})\right]^{1-v}+\gamma_y \ln h_{y,t}$ for young investors. We can show that this preserves the risk neutral assumption with respect to investment while making the labor supply inversely related to their consumption.

[57] Bernanke, Gertler and Gilchrist (1998) is a representative model.

ing financial needs of borrowers. With this setting, there is no way to exert the financial accelerator effect in the non-securitization regime.

2.5 Conclusion

We have seen increasing popularity of securitization to finance mortgages in U.S. The practice started with prime mortgages which the government agencies called Freddie Mac and Fannie May have securitized. Securitization eventually reached its arms to subprime mortgages and this has been said to an ideal way to deliver home to every Americans: Securitization enables investors to hold a class of safe assets which are collateralize by risky mortgages. And this enhanced the home ownership of households who have not been eligible to acquire mortgages. Indeed, there is virtue of securitization as DeMarzo (2005) argues.[58] However, the accuracy of information on underlying assets is much important when evaluating the risks of securitized assets due to tranching. And there is a potential danger that informed agents abuse their superiority. This chapter draws attention to this danger. I illustrate that the information asymmetry regarding with the quality of underlying assets can induce boom-and-bust cycles by securitization.

The theme that underlies in this chapter is that securitization is nothing a miracle to generate safe assets from a pool of risky assets. To be concrete, it is true that a portion of securitized assets can have solid returns with low default probability. However, the riskiness of underlying assets does not change by securitization. Tranching is a specific way to reallocate the risks of the pool of underlying assets. If securitization creates a class of safe assets from a pool of risky assets, the residual claims should be quite risky. But this part is masked when investors mismeasure the risks of underlying assets. In this chapter, I assume investors have subjective belief *a priori* so there is no way to correct the bias. But in reality the bias can be corrected by implementing due diligence to underlying assets in

[58] In DeMarzo (2005), the informed agents reveal true information on the quality of risky assets in equilibrium.

various stages. At least we can take cautious views to securitized assets when we are not confident about the information on underlying assets.

Toward future, the model must be tested to survive when we add more realistic features both observed in the real world and studied by standard macroeconomic literature. One possible critique to this model would be as follows: The market share of subprime mortgage securitization in the vast U.S. financial markets is relatively small and how such a small market can induce large fluctuations? This can be tested by including standard markets in macroeconomic literature like bond and prime mortgage markets to my model and define the market size of subprime mortgages. This may weaken the forces of this model. On the other hand, we may be able to construct a model where resonating effects among various markets actually create more large and lingering fluctuations in the economy. If so, some defects of my model may actually dispel.

B Appendix B

The wealth redistribution process of borrowers

After borrowers repay debts, the perfect insurance plan is executed and their wealth is redistributed equally. In this process, the consumption good and housing stock is equally redistributed.

$$
\begin{aligned}
\tilde{c}_t &= \frac{N-n_t}{N}\max\left\{0, w - R_m B_{b,t}\right\}, \\
\tilde{h}_{b,t} &= h_{b,t} - \frac{N-n_t}{N}\min\left\{h_{b,t}, \frac{\max\left\{R_m B_{b,t}-w, 0\right\}}{\tilde{q}_t}\right\} - \frac{n_t}{N}\min\left\{h_{b,t}, \frac{R_m B_{b,t}}{\tilde{q}_t}\right\}.
\end{aligned}
$$

$\tilde{c}_t$ represents the consumption good that each agent possesses after redistribution. Similarly, $\tilde{h}_{b,t}$ represents the level of housing stock after the redistribution. In $\tilde{c}_t$, the $\max$ part appears because wages of employed borrowers may not be sufficient to repay entire debts. In $\tilde{h}_{b,t}$, the $\min$ part appears because housing value may not be sufficient to repay the remaining obligations.

At the time borrower's wealth is redistributed and borrowers make decisions on the new level of housing stock, the fire sale phase already ended. Therefore, the market value of borrower's wealth, denoted by $A_t(h_{b,t}, B_{b,t}, n_t)$, is:

$$
A_t(h_{b,t}, B_{b,t}, n_t) = \tilde{c}_t + q_t \tilde{h}_{b,t}. \tag{b.1}
$$

The existence and the uniqueness of $\tilde{q}_{t+1}$

We can derive the analytical solution of the fire sale price of housing stock by the overlapping generation structure in investor's problem. By the third equation of (2.12), we can see that $\tilde{q}_{t+1}$ doesn't depend on future housing stocks such as $h_{o,t+2}$. Because old investors die soon after they consume, they do not care the value of house (utility derived from housing) in the future. To obtain the exact formula in terms of the states of this economy, I first introduce the housing market clearing condition at the fire sale phase.

$$\frac{N-n_{t+1}}{N}\min\left\{h_{b,t+1},\max\left\{\frac{R_mB_{b,t+1}-w}{\tilde{q}_{t+1}},0\right\}\right\}+\frac{n_{t+1}}{N}\min\left\{h_{b,t+1},\frac{R_mB_{b,t+1}}{\tilde{q}_{t+1}}\right\}=N_i(h_{o,t+1}-h_{y,t}).$$

(b.1) tells the fire sale supply of housing stocks must be totally absorbed by old investors.

By substituting the market clearing condition into the third equation of (2.12).

$$\tilde{q}_{t+1}=\frac{\begin{array}{c}\gamma_oN_iN-(N-n_{t+1})\min\{\tilde{q}_{t+1}h_{b,t+1},\max\{R_mB_{b,t+1}-w,0\}\}\\-n_{t+1}\min\{\tilde{q}_{t+1}h_{b,t+1},R_mB_{b,t+1}\}\end{array}}{N(\bar{h}-h_{b,t+1})}.$$

This formula still include $\tilde{q}_{t+1}$ in both sides. Now define $f(\tilde{q}_{t+1})$ as follows.

$$f(\tilde{q}_{t+1})=\frac{\begin{array}{c}\gamma_oN_iN-(N-n_{t+1})\min\{\tilde{q}_{t+1}h_{b,t+1},\max\{R_mB_{b,t+1}-w,0\}\}\\-n_{t+1}\min\{\tilde{q}_{t+1}h_{b,t+1},R_mB_{b,t+1}\}\end{array}}{N(\bar{h}-h_{b,t+1})}.$$

We can immediately see following properties of $f(\tilde{q}_{t+1})$: i) $f(0)>0$, ii) $f(\tilde{q}_{t+1})$ is continuous in $\tilde{q}_{t+1}$ and iii) $f(\tilde{q}_{t+1})$ is weakly decreasing in $\tilde{q}_{t+1}$. This implies $f(\tilde{q}_{t+1})$ crosses with 45 degree line exactly only once from above in $(\tilde{q}_{t+1}, f(\tilde{q}_{t+1}))$ space. This proves the existence and the uniqueness of $\tilde{q}_{t+1}$. Also it is immediate that $\tilde{q}_{t+1}>0$.■

The exact solution depends on parameters $(h_{b,t+1}, B_{b,t+1}, n_{t+1}, w, R_m)$ and it can be expressed as follows.

$$\tilde{q}_{t+1}=\begin{cases}\frac{\gamma_oN_iN-NR_mB_{b,t+1}+(N-n_{t+1})w}{N(\bar{h}-h_{b,t+1})} & if\ \tilde{q}_{t+1}h_{b,t+1}>R_mB_{b,t+1}\\ \frac{\gamma_oN_iN-(N-n_{t+1})(R_mB_{b,t+1}-w)}{N\bar{h}-(N-n_{t+1})h_{b,t+1}} & if\ R_mB_{b,t+1}\geq\tilde{q}_{t+1}h_{b,t+1}>R_mB_{b,t+1}-w\\ \frac{\gamma_oN_i}{\bar{h}} & otherwise\end{cases}.\tag{b.2}$$

Note (h_b, B_b, n) describe not only the states of borrowers but also the states of the aggregated economy: Given h_b, we can uncover investors' housing stock by the housing market clearing conditions. By the first equation of (2.12), the supply curve of the securitized assets becomes flat at $\tilde{E}_t(R_{t+1}) = 1/\beta$. This implies the demand side determines the equilibrium borrowing.[59] Finally, n is the only exogenous state in this economy.

By the second equation of (2.12), we can express q_t as a function of $(h_{b,t+1}, B_{b,t+1}, n_{t+1})$ too. From time t perspective, $(h_{b,t+1}, B_{b,t+1})$ must be solved by the optimization of borrowers. If we could know the policy functions associated with the borrower's problem (2.8), then we were able to solve q_t as a function of $(h_{b,t}, B_{b,t}, n_t, n_{t+1})$. In this chapter, I solve the policy functions numerically, therefore the exact formula of q_t is not known.

The problem of borrowers at the crisis

At the end of the securitization regime, that is at the onset of time τ, borrowers have to solve a specific problem to that period. First they owe debts that is supposed to be positive. After the loan repayment, they decide how much to consume and how much housing stock to trade given the wealth. However, they have to do so without borrowing. From time τ onward, their borrowing ability goes down to zero. Therefore, borrower's problem at time τ can be represented as follows:

$$V_{b,\tau}(h_{b,\tau}, B_{b,\tau}, n_\tau) = \max_{h_{b,\tau+1}} \left\{ \begin{array}{c} A_\tau(h_{b,\tau}, B_{b,\tau}, n_\tau) - q_\tau h_{b,\tau+1} + \psi \left(\frac{h_{b,\tau+1}}{h_{b,\tau}} - 1 \right)^2 h_{b,\tau} \\ +\gamma_b \ln h_{b,\tau} + \beta_b E_{b,\tau} \left(\hat{V}_b \left(h_{b,\tau+1}, 0, n_{\tau+1} \right) \right) \end{array} \right\}. \tag{b.3}$$

$V_{b,\tau}(h_{b,\tau}, B_{b,\tau}, n_\tau)$ can be solved readily if we know $\hat{V}_b(h_{b,\tau+1}, 0, n_{\tau+1})$, the solution of the problem (2.9).

A critical problem in (b.3) is that there may be no feasible plan $h_{b,\tau+1} \in [0, \bar{h}]$ such that the non-negativity consumption constraint is satisfied; Increasing the level of housing stock may not be feasible because they cannot borrow. Selling is also problematic because they have to pay the housing adjustment cost. When $A_\tau(h_{b,\tau}, B_{b,\tau}, n_\tau)$ is small, there may

[59] Precisely, we need additional conditions to verify this statement. First, young investor's wealth must be sufficiently large to finance the borrowing demand. i.e. $N_i(w + q_t h_{o,t} - q_t h_{y,t}) \geq B_{b,t+1}$ in equilibrium. Second, financial intermediaries must participate in the securitization market and contribute their wealth to keep Equity tranche. I assume these conditions are met. In section 2-3, I explain what consists of the state when financial intermediaries withdraw from the securitization market.

be no feasible housing plan to manage this tradeoff. To avoid this problem, I assume that there is one time government intervention such that the government redistributes wealth from new born investors to borrowers if:

$$\sup_{h_{b,\tau+1}} \left\{ A_\tau(h_{b,\tau}, B_{b,\tau}, n_\tau) - q_\tau h_{b,\tau+1} + \psi \left(\frac{h_{b,\tau+1}}{h_{b,\tau}} - 1 \right)^2 h_{b,\tau} \right\} < 0.$$

Let G_τ be the amount of wealth redistribution from young investors to borrowers:

$$G_\tau = - \sup_{h_{b,\tau+1}} \left\{ A_\tau(h_{b,\tau}, B_{b,\tau}, n_\tau) - q_\tau h_{b,\tau+1} + \psi \left(\frac{h_{b,\tau+1}}{h_{b,\tau}} - 1 \right)^2 h_{b,\tau} \right\}. \quad \text{(b.4)}$$

In other words, the rescue plan finds the scheme such that the redistribution of wealth from investors to borrowers is minimized. In this plan, it is assumed that borrowers accept zero consumption.

There is one more question in this rescue plan: What q_τ we should apply in (b.4)? I assume the government internalize q_τ. By the house price formulas in (2.12) and by the fact $h_{o,\tau+1} = h_{y,\tau} = \frac{\bar{h} - h_{b,t+1}}{N_i}$ in the non securitization regime, q_τ is expressed as follows:

$$q_\tau = \frac{N_i(\gamma_y + \beta\gamma_o)}{\bar{h} - h_{b,t+1}}. \quad \text{(b.5)}$$

The government finds:

$$h^*_{b,\tau+1} = \arg \max_{h_{b,\tau+1} \in [0,\bar{h}]} \left\{ A_\tau(h_{b,\tau}, B_{b,\tau}, n_\tau) - q_\tau h_{b,\tau+1} + \psi \left(\frac{h_{b,\tau+1}}{h_{b,\tau}} - 1 \right)^2 h_{b,\tau} \right\}.$$

The government internalizes q_τ as in (b.5). If G_τ under $h^*_{b,\tau+1}$ is negative, then it implements the rescue plan. Otherwise, the market equilbrium without intervensions should exist and the government would not step in. In the simulation in section 2.3.3, $A_\tau(h_{b,\tau}, B_{b,\tau}, n_\tau)$ is low enough that the government actually implements the rescue plan.

Numerical computation of the equilibrium

To solve the equilibrium, we have to evaluate the value function (2.8) and (2.9) given the house prices, q and $\tilde{q}$, and we have to check whether the house prices are consistent with the formula (2.12). Conceptually, the house price and the choice of housing and borrowing are determined simultaneously in equilibrium. However, I compute the value function and house price sequentially and iterate this process until the housing market clearing condition is met.

The detailed procedure is as follows:

1. Compute $\Pr(n)$ and $\widetilde{\Pr}(n)$. Given the parameters $(\pi, \rho, \tilde{\pi}, \tilde{\rho}, N)$, they can be expressed as:

2.

$$\begin{aligned}\Pr(n) &= \binom{N}{n} \Pr(X_1 \leq \Phi^{-1}(\pi), \cdots, X_n \leq \Phi^{-1}(\pi), X_{n+1} > \Phi^{-1}(\pi), \cdots, X_N > \Phi^{-1}(\pi)), \\ \widetilde{\Pr}(n) &= \binom{N}{n} \Pr(X_1 \leq \Phi^{-1}(\tilde{\pi}), \cdots, X_n \leq \Phi^{-1}(\tilde{\pi}), X_{n+1} > \Phi^{-1}(\tilde{\pi}), \cdots, X_N > \Phi^{-1}(\tilde{\pi})).\end{aligned}$$

   I evaluate them numerically.

3. Create a discretized state space (h_b, B_b, n) which approximates the true state space.

4. Analytically solve $\tilde{q}$ for each state (h_b, B_b, n) using (b.2).

5. Guess equilibrium normal house price, denoted by q^0, for each state (h_b, B_b, n).

6. Guess initial value function $V_b^0(h_b, B_b, n)$ (for the non-securitization regime guess $\hat{V}_b^0(h_b, 0, n)$).

   (Hereafter, I only describe the procedure in the securitization regime. The essence is the same for the non-securitization regime.)

7. Compute the policy fuctions $g_h^1(h_b, B_b, n)$ and $g_B^1(h_b, B_b, n)$. Also compute the associated new value function $V_b^1(h_b, B_b, n)$ as:

$$\left[g_h^1(h_b, B_b, n), g_B^1(h_b, B_b, n)\right] =$$
$$\arg\max_{h_b', B_b'} \left\{ \begin{array}{c} A^0 - q^0 h_b' + \psi\left(\frac{h_b'}{h_b} - 1\right)^2 h_b + B_b' \\ +\gamma_b \ln h_b + \beta_b E_b\left(V_b^0\left(h_b', B_b', n'\right)\right) \end{array} \right\}.$$

$$V_b^1(h_b, B_b, n) = \begin{array}{c} A^0 - q^0 g_h^1(h_b, B_b, n) + \psi\left(\frac{g_h^1(h_b, B_b, n)}{h_b} - 1\right)^2 h_b + g_B^1(h_b, B_b, n) \\ \gamma_b \ln h_b + \beta_b E_b\left(V_b^0\left(g_h^1(h_b, B_b, n), g_B^1(h_b, B_b, n), n'\right)\right) \end{array}.$$

8. Check whether V_b^1 and V_b^0 are close enough. If not repeat 6 until V_b^m and V_b^{m-1} gets close enough. In this process, the price q^0 is fixed.

9. Compute the housing demand of investors given q^0 by (2.12). Let $g_y(q^0(h_b, B_b, n))$ be the housing demand function of investors. Check whether $g_h^m(h_b, B_b, n) +$ $N_i g_y(q^0(h_b, B_b, n)) = \bar{h}$.

10. If excess demand in housing market, raise the price, i.e $q^1 > q^0$. If excess supply, lower the price, i.e $q^1 < q^0$. Then repeat 6-8 with the new house price. To restart the value function evaluation, it is convenient to start with the converged value function under q^0.

11. Repeat 9 until the housing market is close enough to clear.

12. Finally we obtain equilibrium house prices, the policy functions and the associated value function.

Chapter 3

Credit Availability to Multinational Enterprises and International Business Cycles

3.1 Introduction

One of the characteristics of globalization is growing share of international trade conducted by multinational enterprises. For example, U.S. international trade associated with multinational enterprises accounts over two-third of her total exports and about a half of her total imports.[61] While it is widely perceived that economic activities of multinational enterprises have non-negligible impacts on local economies, the precise mechanisms through which trade associated with multinational enterprises affects local economies are not so clear-cut.

This chapter examines the transmission mechanisms of international business cycles induced by the economic activities of multinational enterprises (hereafter MNEs). The focus of this chapter is the mechanisms through which MNEs' borrowing capacity in local

[61] See Kozlow (2006).

financial markets acts on international business cycle transmissions. I address this issue by constructing two alternative models in which the main difference is the borrowing capacity of MNEs: One model assumes that MNEs do not face borrowing limits while MNEs face borrowing constraints in the other model. And I examine how the two models respond differently to country specific productivity shocks.

This chapter is among the earliest studies that focus on the financial positions of MNEs. I show that the international business cycle transmissions differ dramatically in the specifications of MNEs' borrowing capacity. The unique structure of my models is that MNEs locate their production units in both countries and they raise funds in local financial markets. MNEs locate their production units in both countries to obtain a scarce input, real estate. Real estate is a locally traded asset. In the model with the borrowing constraints, MNEs' borrowing capacities in each country are limited up to the value of real estate that they own. This specification highlights the importance of the local assets for the propagation of international business cycles.

The main findings of this chapter are followings: i) Cyclical properties of real variables heavily depend on the borrowing capacity of MNEs. ii) Real estate prices tend to co-move regardless of the specifications of the borrowing capacity but the mechanisms differ in the specifications. When MNEs' borrowing capacities are unconstrained, the substitution effect and the international risk sharing channel dominate in response to country specific productivity shocks, and the famous international co-movement puzzle remains as in Backus, Kehoe and Kydland (1992). The cross-country real estate price correlation is also positive due to the international risk sharing channel: households, regardless of where they live, increase local real estate demand because the international risk sharing channel provides them with resources to invest in real estate. However, when MNEs' borrowing capacity is limited up to the value of collateral in each country, the international co-movement puzzle is overturned: the cross-country output correlation turns to be positive and the cross-country consumption correlation becomes much lower. With this setup, the wealth effect induced by MNEs investment decisions drives the result. A marginal increase in the value of collateralizable assets has a strong multiplier effect on investment. In my model, MNEs can enjoy this effect in both countries due to the borrowing constraints that they face in the both countries. This incentivizes MNEs to increase investment in both countries regardless of the origin of the shocks. The co-movement of real estate prices is also driven by the wealth effect in this case. As Kehoe and Perri (2002) and Iacoviello and Minetti (2006) illustrate, a key to resolve the co-movement puzzles is to introduce limited enforceability in international financial contracts.

This chapter expands the result of Dietrich (2004) in which he studies a static small open economy with MNEs to a two-country dynamic general equilibrium model. He shows a marginal increase of MNEs' wealth leads to increases in investment in home and foreign country when there are limited enforceability problems in implementing loan contracts. The model with borrowing constraints in this chapter is intended to expand his result to a dynamic large country model and to study international business cycle properties. The mechanism of the wealth effect is similar to Kiyotaki and Moore (1997). The deep root of the wealth effect is the reduction of agency costs under asymmetric information problems that is discussed in Holmström and Tirole (1998) and others. In terms of the form of borrowing constraints, which mirrors the agency cost problem, I follow Kiyotaki and Moore (1997) and Iacoviello (2005).

There already exist papers that examine the roles of MNEs in international business cycle transmissions. For example, Burstein, Kurz and Tesar (2005) consider a production chain in core regions (e.g. the U.S.) and their peripheries (e.g. NAFTA for the U.S.) as a source of positive cross-country output correlations. They assume intermediate inputs produced in the core region and their periphery are more like complements. Under this assumption, a positive productivity shock in one country enhances production in other countries because of the complementarity. This is a novel contribution but this chapter focuses on the characteristics of goods that we observe in trade between developed and developing countries. Instead, I focus on financial positions of MNEs when goods produced in different countries are more like substitutes.[62] This structure is more suitable to resolve the international co-movement puzzle among developed countries where goods produced in these countries are more like substitutes.

Finally, the technical aspects of my models are much owed to Iacoviello and Minetti (2006). They build a international macro model in which local entrepreneurs borrow from home lenders and foreign lenders. An important characteristic of their model is that the marginal propensity of borrowing capacity to the change of collateral value is higher for domestic borrowings than for foreign borrowings. My model retreats from the different marginal propensities of borrowing capacity, but the setup and the solution methods are quite similar to their model.

This chapter is organized as follows. Section 3.2 presents two alternative models in which the main difference is the borrowing capacity of MNEs. Section 3.3 describes the properties of impulse response functions and the results of simulations. Section 3.4 gives concluding remarks.

3.2 The Models

I build two alternative models in which the main difference is the borrowing capacity of MNEs. The common features of the two models are followings: The models consist of two countries, each represented by a unit mass of households with identical preference. In addition, there exists a unit mass of homogeneous multinational entrepreneurs who seek profit opportunities both in home and foreign country. MNEs locate their production units both in home and foreign country and they produce the same single good everywhere. The nationality of MNEs is irrelevant in my models because I assume no barrier in entering good markets and no trade costs. I assume only MNEs have production technology. In each

[62] In my model, the good produced in home country is a perfect substitute of the good produced in foreign country.

country, there is real estate which is in fixed supply, locally traded and held either MNEs or local households. MNEs can enter into local markets to acquire real estate. The real estate held by MNEs is used as an input of production. Real estate held by households cannot be used as an input of production but it yields housing service to households. Finally, agents trade uncontingent bonds which can be traded internationally. Hence the economy has two types of assets, real estate and bonds.

3.2.1 The frictionless model

I start from the frictionless model in which MNEs do not face borrowing limits.[63] MNEs derive utility from consumption, denoted by c^e:

$$\max \sum_{t=0}^{\infty} \gamma^t \ln c_t^e.$$

γ is the subjective discount rate of MNEs. The superscript e denotes variables of the MNEs. They also produce the single good y^e by locating production units in both countries. They hire labor l from households and input their real estate holdings h^e in each country. The production technology is represented by a constant-return-to-scale Cobb-Douglas formulation:

$$\begin{aligned} y_t^e &= y_t + y_t', \\ y_t &= A_t (h_{t-1}^e)^{\alpha} (l_t)^{1-\alpha}. \\ y_t' &= A_t' (h_{t-1}'^e)^{\alpha} (l_t')^{1-\alpha}. \end{aligned}$$

α is the real estate share in the Cobb-Douglas technology. The superscript $'$ denotes variables in foreign country. Thus, y denotes output in home country and y' denotes output in foreign country. I assume that MNEs take the level of technology A and A' as given.

[63] By a transversality condition, MNEs are prohibited to roll over debt infinitely and to play Ponzi-scheme.

Because the goods produced in home country and foreign country are homogeneous, the allocation of resources in equilibrium is such that the prices of the good are the same between home and foreign country.

I assume that the existence of MNEs is given. This may be a strict assumption. Markusen (1998) and Helpman, Melitz and Yeaple (2004) examine conditions that firms implement foreign direct investment (FDI) rather than exporting.[64] Cavallari (2007), using dynamic models, studies how difference in firms' integration strategies affects international business cycle transmissions.[65] I do not consider the integration strategies of firms because my focus is on how MNEs transmit shocks from country to country rather than why they choose to be MNEs. It would be a richer study to incorporate endogeneity of firms' integration strategies, but I leave it a future course of studies.

It is worth noting the properties of real estate. Real estate serves as an input of production as well as a means of savings to MNEs. It is not traded internationally and its price relative to the consumption good is denoted by q. The total supply of real estate is fixed to $\bar{h}$ in each country. MNEs enter into local markets and purchase real estate. A technical assumption is that MNEs cannot rent real estate from households. Only MNEs' real estate holdings matter for production. Given this, the flow of funds of MNEs is:

$$\begin{aligned} c_t^e + q_t h_t^e + q_t' h_t'^e + r_{t-1} b_{t-1}^e + r_{t-1}' b_{t-1}'^e + \tfrac{\xi}{2}(b_t^e)^2 + \tfrac{\xi}{2}(b_t'^e)^2 = \\ y_t^e - w_t l_t - w_t' l_t' + q_t h_{t-1}^e + q_t' h_{t-1}'^e + b_t^e + b_t'^e + \tau_t^e + \tau_t'^e. \end{aligned}$$

The LHS is the expenditures of MNEs. They consume, purchase real estate and repay debts, b and b', with gross interest rates r and r'. The RHS is the resources of MNEs. Their income is output net of labor compensation. The wage rate is w. They have initial real estate wealth and they can borrow from households. A technical point is that the quadratic adjustment costs on debts. ξ controls the degree of the adjustment costs. The adjustment costs ensure

[64] These studies introduce plant specific fixed cost and variable transportation cost. They show that firms choose FDI when the transportation cost incurred by exports overweigh the fixed cost of FDI.

[65] She does not report detailed statistics on business cycle properties generated by the model. Also, she assumes that domestic firms only serve their goods in foreign market.

us that the model has a unique non-stochastic steady state. MNEs pay the adjustment costs to a financial intermediary when they borrow but the financial intermediary pays them back to MNEs in a lump-sum manner, denoted by τ^e.[66]

Under this setting, I obtain the optimality conditions on real estate holdings as follows[67]:

$$\begin{aligned} \frac{q_t}{c_t^e} &= \gamma E_t\left[\frac{1}{c_{t+1}^e}\left(\alpha\frac{y_{t+1}}{h_t^e}\right)\right] + \gamma E_t\left(\frac{q_{t+1}}{c_{t+1}^e}\right), \\ \frac{q_t'}{c_t^e} &= \gamma E_t\left[\frac{1}{c_{t+1}^e}\left(\alpha\frac{y_{t+1}'}{h_t'^e}\right)\right] + \gamma E_t\left(\frac{q_{t+1}'}{c_{t+1}^e}\right). \end{aligned} \tag{3.1}$$

MNEs equate the marginal utility of selling real estate and the marginal benefit of holding real estate. The latter consists of the expected marginal product of real estate and the expected selling price tomorrow in unit of the marginal utility of consumption tomorrow.

The system of equations (3.1) indicates that MNEs' decisions on real estate holdings in each country are tied with their consumption. One source that the prices of local assets are not independent of foreign shocks is the portfolio decisions by MNEs. For example, an increase of MNEs' consumption today motivates them to increase real estate demand in both countries if future is constant.

The problem of domestic households is standard except that owning real estate yields utility to them:

$$\max \sum_{t=0}^{\infty}\beta^t\left[\ln c_t + \psi\ln h_t - \frac{(l_t)^\eta}{\eta}\right].$$

β is the subjective discount rate of households, ψ is the weight attached to the service flow of housing and $\eta - 1$ is the inverse of the elasticity of labor supply.

Their budget constraint is:

$$c_t + q_t h_t + r_{t-1}b_{t-1} + r_{t-1}^* b_{t-1}^* + \tfrac{\xi}{2}(b_t)^2 + \tfrac{\xi}{2}(b_t^*)^2 =$$
$$w_t l_t + q_t h_{t-1} + b_t + b_t^* + \tau_t + \tau_t^*.$$

[66] Schmitt-Grohé and Uribe (2003) discuss this issue in detail .

[67] The full list of the equations can be seen in Appendix C.

b denotes borrowings from MNEs and r denotes their interest rate. Correspondingly, b^* and r^* are those with foreign households. There is a room for home and foreign households to share their income risks. Again I introduce the quadratic adjustment costs on bond holdings, b and b^*, to obtain the unique steady state. The problem of foreign households is symmetric to domestic households.

Together with the standard Euler equation and the labor supply equation, I obtain households' optimality condition on real estate holdings as follows:

$$\frac{q_t}{c_t} = \frac{\psi}{h_t} + \beta E_t \left(\frac{q_{t+1}}{c_{t+1}} \right). \tag{3.2}$$

They equate the marginal utility of selling real estate and the marginal benefit of holding real estate. The logic is the same as (3.1). The latter consists of the marginal utility of housing service today and the expected marginal utility of selling real estate tomorrow.

Finally, I impose $\beta = \gamma$ for the frictionless model to avoid over accumulation of financial assets by the households.

3.2.2 The model with borrowing constraints

For the alternative model, I assume MNEs face following limited enforceability problems on financial contracts: production technology is firm specific in that no one can take over the production processes once MNEs obtain borrowings. And lenders, households in my model, cannot make MNEs precommit to engage in production. Under this assumption, MNEs can take a renegotiation strategy to reduce debt repayments. Assuming that MNEs have full bargaining power, household's capacity to recover loan losses is limited up to the value of collateral they took from MNEs. Under this assumption households could squeeze nothing if there lending had no collateral. Thus only collateralized borrowings are available to MNEs. This logic is the same as that in Kiyotaki and Moore (1997).

The collateral is real estate in my model. MNEs can borrow from households in both countries. But domestic households accept real estate in home country only as collateral and vice versa. As a result, there are two new constraints to the MNEs' problem:

$$\begin{aligned} r_t b_t^e &\le m E_t(q_{t+1} h_t^e), \\ r_t' b_t'^e &\le m E_t(q_{t+1}' h_t'^e). \end{aligned} \tag{3.3}$$

Here, m is the loan-to-value ratio.[68]

Because these constraints pin down the unique steady state of this economy, I dropped the adjustment costs associated with MNEs' debt holdings. The flow of funds of MNEs is thus modified as:

$$\begin{aligned} c_t^e + q_t h_t^e + q_t' h_t'^e + r_{t-1} b_{t-1}^e + r_{t-1}' b_{t-1}'^e = \\ y_t^e - w_t l_t - w_t' l_t' + q_t h_{t-1}^e + q_t' h_{t-1}'^e + b_t^e + b_t'^e. \end{aligned}$$

Finally, to make MNEs always debtors, I impose $\beta > \gamma$ for this model. As MNEs are impatient relative to households, they always have incentives to borrow the maximum amount in the neighborhood of the steady state.[69]

The first-order conditions for real estate holdings are modified as follows:

$$\begin{aligned} \frac{q_t}{c_t^e} &= \gamma E_t \left[\frac{1}{c_{t+1}^e} \left(\alpha \frac{y_{t+1}}{h_t^e} \right) \right] + \gamma E_t \left(\frac{q_{t+1}}{c_{t+1}^e} \right) + m \lambda_t E_t(q_{t+1}), \\ \frac{q_t'}{c_t^e} &= \gamma E_t \left[\frac{1}{c_{t+1}^e} \left(\alpha \frac{y_{t+1}'}{h_t'^e} \right) \right] + \gamma E_t \left(\frac{q_{t+1}'}{c_{t+1}^e} \right) + m \lambda_t' E_t(q_{t+1}'). \end{aligned} \tag{3.4}$$

λ_t and λ_t' are Lagrange multipliers associated with the borrowing constraints (3.3). In the RHS, I have additional terms $m\lambda_t E_t(q_{t+1})$ and $m\lambda_t' E_t(q_{t+1}')$ that represent the marginal benefits of relaxing the borrowing constraints. A relaxation of the borrowing constraints

[68] The fraction $(1 - m)$ can be interpreted as the cost to liquidate MNEs' assets.

[69] Because MNEs are risk averse, MNEs may have incentives to save rather than borrow if today's income is higher than future income. I restrict my attention in the neighborhood of the steady state with a small degree of shocks so that the borrowing constraints always bind. A simulation by Iacoviello (2005) shows that if the variances of shocks are small, the borrowing constraints always bind in the neighborhood of the steady state.

enables MNEs to increase investment in real estate. The increased real estate holdings and the associated house price appreciation relax the borrowing constraints further. An infinite sequence of this logical circulation generates a strong multiplier effect even if the initial increase of collateral value is marginal.[70]

We can think that λ_t and λ'_t capture the marginal benefit of the net worth of MNEs.[71] As Kiyotaki and Moore (1997) and Hormström and Tirole (1998) proved, an increase of net worth reduces agency costs under asymmetric information problems. Not only a pure increase of resource but also the reduction of the agency costs create more than one-to-one increase in investment. I will show that this effect creates completely different output dynamics compared to the frictionless model.

3.2.3 Productivity shocks

The exogenous shocks in my models are country specific productivity shocks. I assume log of productivity follows a VAR(1) process. The two shocks may be correlated.

$$\begin{bmatrix} \log A_t \\ \log A'_t \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A'_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon'_t \end{bmatrix},$$

$$var \begin{bmatrix} \epsilon_t \\ \epsilon'_t \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}.$$

ρ_{11} and ρ_{22} measure the persistencies of the productivity process of each country. ρ_{12} and ρ_{21} measure the spillover effects. $\sigma_{12} \neq 0$ implies that the two shocks are correlated.

[70] Although the borrowing constraints bind, MNEs increase both consumption and investment in response to the marginal increase of their net worth under my specification. The consumption-investment allocation depends on the degree of risk aversion. The smaller the degree of risk aversion, the larger share of incremental resources is allocated to consumption. This weakens the multiplier effect which emerges from the relaxation of the borrowing constraints because MNEs' real estate accumulation is slowed.

[71] The net worth at the beginning of period t is defined as $y_t^e - w_t l_t - w'_t l'_t + q_t h_{t-1}^e + q'_t h'^e_{t-1} - r_{t-1} b_{t-1}^e - r_{r-1} b'^e_{t-1}$.

3.2.4 Equilibrium condition

In equilibrium, real estate demand must be equalized to real estate supply:

$$h_t + h_t^e = \bar{h}, \quad h_t' + h_t'^e = \bar{h}'.$$

Bond markets' clearing conditions are:

$$b_t + b_t^e = 0, \quad b_t' + b_t'^e = 0, \quad b_t^* + b_t'^* = 0.$$

In the model with the borrowing constraints, the constraints bind in both countries:

$$r_t b_t^e = mE_t(q_{t+1} h_t^e), \quad r_t' b_t'^e = mE_t(q_{t+1}' h_t'^e).$$

The good market clearing is:

$$y_t + y_t' = c_t + c_t' + c_t^e.$$

3.3 The Results

3.3.1 The methods

I calibrate the deep parameters of the models, linearize the models in the neighborhood of their non-stochastic steady states and inspect impulse responses to a productivity shock in home country. I also simulate the models by specifying exogenous shock processes and I report cross-country correlations of various endogenous variables generated by the simulation.[72] And I compare them with actual data as well as data simulated by standard international business cycle literature.

[72] I utilize "Toolkit" which is developed by Uhlig (1997) when I solve the models numerically.

Description	Symbol	Parameter Values	
		Frictionless model	Financial friction
Discount rate, households	β	0.99	0.99
Discount rate, entrepreneurs	γ	0.99	0.98
Labor-wage elasticity	$\frac{1}{\eta-1}$	0.05	0.05
Weight on housing service	ψ	0.1	0.1
Real estate share	α	0.1	0.1
Loan-to-value ratio	m	0.9	0.9
Adjustment cost of bond	ξ	0.00001	0.00001
Real estate supply	$\bar{h}, \bar{h}'$	1	1

Table 3-1: The parameter values of the economy

3.3.2 Calibration

Table 3-1 represents the values of the deep parameters. The parameter values are based on Iacoviello and Minetti (2006) and Kollman (1996). Time frequency is quarterly. $\beta = 0.99$ implies the annual interest rate is 4% in the steady states. I set $\gamma = 0.98$ for the model with the borrowing constraints so that the borrowing constraints bind in the neighborhood of the steady state. The loan-to-value ratio m is set to 0.9, which is in line with actual lending practices. The settings of α and ψ imply that MNEs own roughly 50% of real estate in each country. The coefficient on the adjustment costs of bond holdings is set to be tiny. I introduce the adjustment costs only to pin down the unique steady state in each version of the models. Therefore I set ξ so that the adjustment costs themselves do not generate a powerful force to the dynamics of the models. I normalize real estate supply to one in each country.[73]

Table 3-2 represents two specifications on the evolution of productivity. The first column represents the standard setting: the productivity is highly persistent but the shocks have no international spillover effects. Shocks in home and foreign country (ϵ_t and ϵ_t') are correlated and the correlation is set to be 0.2. These values are taken from Kollman (1996).

[73] The results in this paper are not sensitive to the level of real estate supply.

Description	Symbol	Parameter Values	
		Standard	Spillover
Persistency of shocks	ρ_{ii}	0.95	0.95
Spillover effect	ρ_{ij}	0	0.02
Variance of shocks	σ_i^2	0.0073	0.0073
Covariance of shocks	σ_{ij}	0.0015	0.0015

Table 3-2: Parameters on the productivity process

The second column incorporates the spillover effect. Since the spillover effect is one of the main issues regarding with FDI, it would be a recommended experiment to allow the spillover effect.[74]

One may think that the spillover coefficient is small. However, the VAR(1) process of productivity implies that foreign country productivity rises by 0.15% at peak when a home country shock raises home productivity by 1%.

3.3.3 Impulse responses

Frictionless economy

Figure 3-1 shows the impulse responses to a temporary productivity shock in home country. In this setup, I assume there is no persistency in the productivity process (i.e $\rho_{ii} = 0$). Output increases in home country but falls in foreign country. Similarly, labor hours move in opposite directions between the two countries. These responses emerge because of the resource reallocation conducted by MNEs. However, consumption rises in both countries. Though foreign households do not experience a favorable shock, they borrow from other agents in the economy and consume more through an international risk sharing channel.[75] Notice that the agents adjusts everything instantaneously in the frictionless

[74] Estimations based on macroeconomic data, such as Kollman (1996) suggest that the international spillover effects are almost zero. However, recent microeconometric studies, such as Keller and Yeaple (2005), find that there are significant spillover effects from FDI.

[75] Kehoe and Perri (2002) illustrate that the fluctuations of a economy in a single bond world quite resemble to those of a complete market economy, where agents can trade state contingent claims for every possible

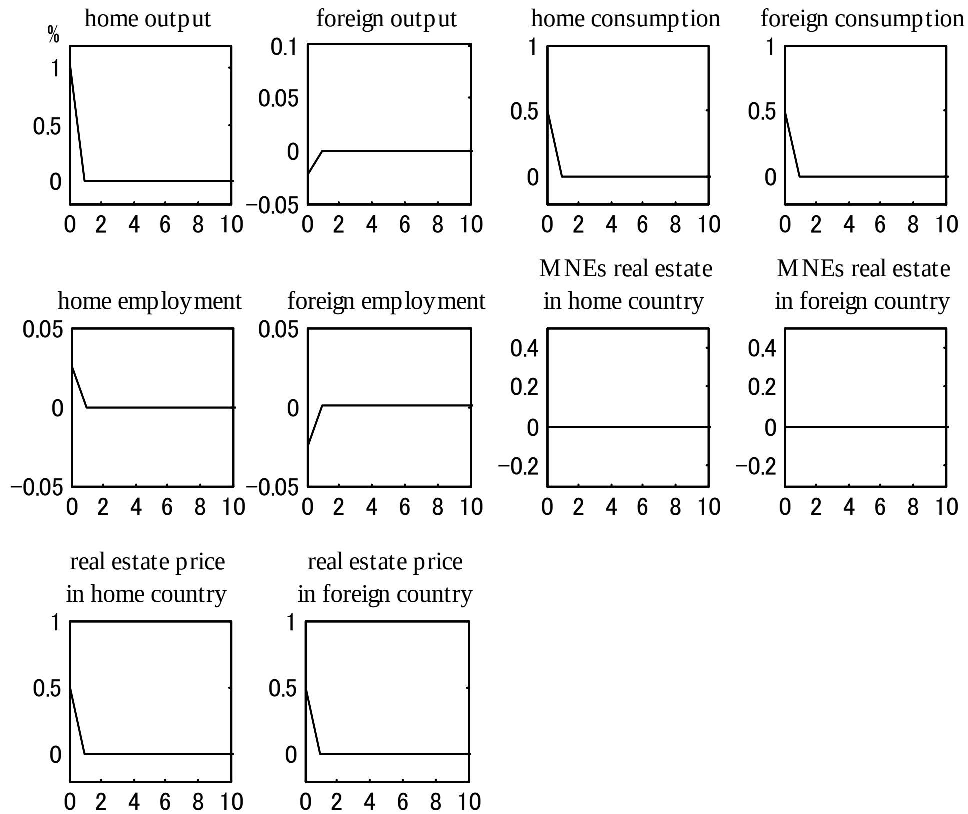

Figure 3-1: Impulse responses to a 1% temporary productivity shock in home country. (The frictionless economy)

economy.

The responses of real estate related variables are somewhat difficult to interpret. The real estate prices rise in both countries but the real estate allocation does not change virtually. Particularly, the real estate price in foreign country rises even though it doesn't experience a positive shock.

To understand the dynamics of real estate markets, it is informative to see the FOCs on real estate holdings. The following equations are the restatement of FOCs on foreign

states in the future.

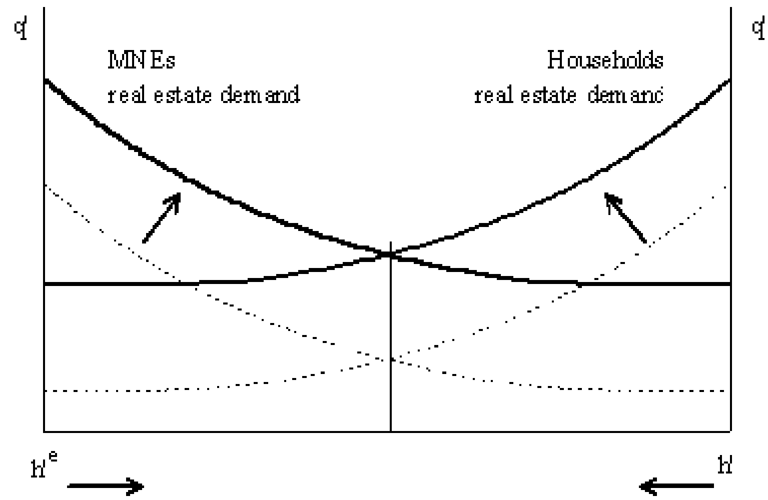

Figure 3-2: The price and the allocation of real estate in equilibrium (foreign country)

country real estate holdings in (3.1) and (3.2):

$$\frac{q'_t}{c^e_t} = \gamma E_t\left[\frac{1}{c^e_{t+1}}\left(\alpha\frac{y'_{t+1}}{h'^e_t}\right)\right] + \gamma E_t\left(\frac{q'_{t+1}}{c^e_{t+1}}\right),$$

$$\frac{q'_t}{c'_t} = \frac{\psi}{h'_t} + \beta E_t\left(\frac{q'_{t+1}}{c'_{t+1}}\right).$$

These equations tell that the foreign real estate price, q'_t, and the foreign real estate holdings of each agent, h'_t and h'^e_t, have a negative relationship holding future variables constant: The demand curves of real estate have standard downward slopes.

Figure 3-2 illustrates what happens in the foreign real estate market. The equilibrium price rises because both agents increase real estate demands. MNEs save a portion of incremental income due to consumption smoothing. Because the temporary shock does not change the profitability of investment in the future, MNEs have incentives to invest not only in home country but also in foreign country. For foreign households, income transfers through the international risk sharing channel induces them to increase real estate demand.

With the parameter values of Table 3-1, the equilibrium allocation of real estate does not change virtually because the increase of real estate demand of one agent just offsets that of the other. The offsetting force comes from the fact that real estate supply is fixed

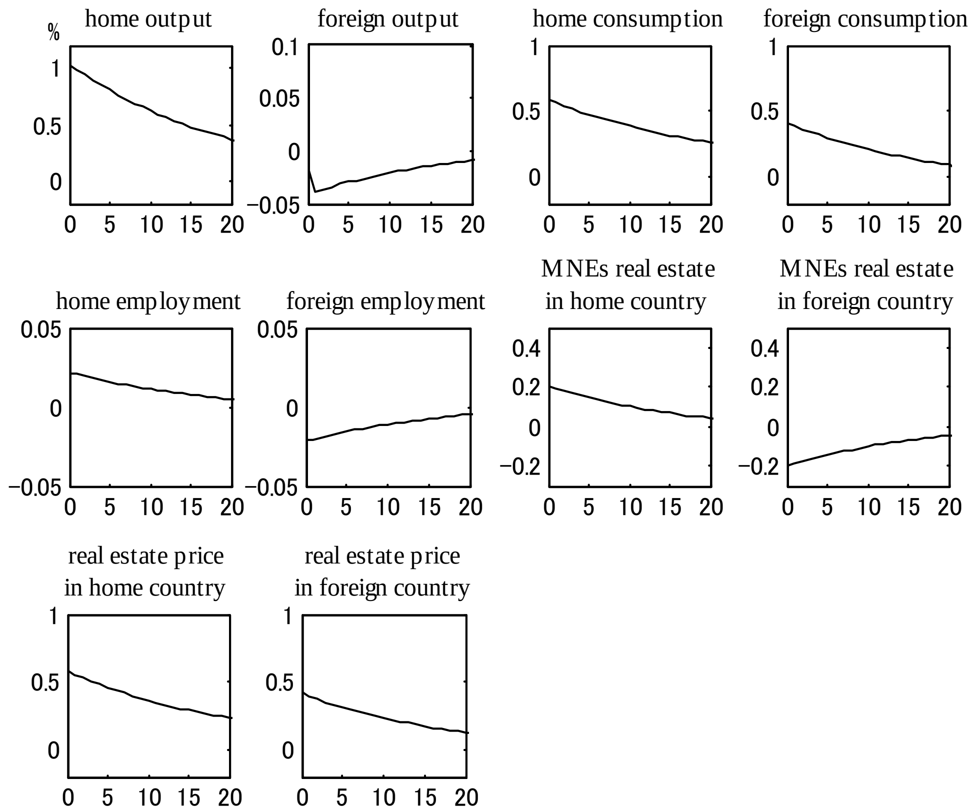

Figure 3-3: Impulse responses to a 1% persistent home productivity shock ($\rho_{11} = 0.95$, The frictionless model)

and the economy has no way to increase aggregate savings.

The real estate price co-movement arises because the phenomenon illustrated in Figure 3-2 occurs in both countries. From the perspective of MNEs' optimization, MNEs increase real estate demand in both countries. Recall the system of equations (3.1). An increase of consumption induces MNEs to valuate saving more. Since future productivity is the same across countries, MNEs increase real estate demand at the same rate in both countries.

The invariance of real estate allocation is not a general result. A change in parameters, preferences and other things create different dynamics in real estate allocation.[76]

[76]For instance, when the degree of risk aversion differs between MNEs and households, the dynamics of real estate allocation change. I confirm that MNEs sells real estate and increase consumption further in response to the productivity shock if MNEs are more risk loving than households.

Figure 3-3 shows responses to a positive home productivity shock when the productivity process is persistent (i.e $\rho_{ii} = 0.95$). In this scenario, MNEs purchase real estate in home country but sell in foreign country. MNEs, knowing future productivity will rise in home country, have strong incentives to shift production resources to home country. This desire is strong enough that the actual real estate holdings increase in home country even the price rises in the equilibrium. Notice the divergence of real estate holdings directly leads to the divergence of output.

Asset price co-movement is preserved in this scenario. Although the real estate demand of MNEs in foreign country is dampened by the substitution effect, this restores the marginal product of real estate (Intuitively, this is a movement along the demand curve of MNEs in Figure 3-2). And the international risk sharing channel increases the real estate demand of foreign households (This is an upward shift of the demand curve of foreign households). These forces push up the price of foreign real estate. However, this general equilibrium effect is at odds with phenomena in the real world. For example, the international risk sharing implies that MNEs lend to foreign households. Intuitively, this rarely happens in the real world. The asset price co-movement is a sort of by-product in this frictionless model.

The economy with the borrowing constraints - the wealth effect

An important feature of the model with the borrowing constraints is that marginal increases of the net worth of constrained agents have powerful multiplier effects on investment. Kiyotaki and Moore (1997) show that, in a closed economy, positive shocks to the net worth increase real estate holdings of credit-constrained firms and they show that aggregate output increases by more than the magnitude of initial shocks. Dietrich (2004) studies an static small open economy model with financially constrained MNEs. In his model, MNEs have two types of investment opportunities: Domestic investment and FDI. He shows that an increase of MNEs' net worth induces them to increase FDI as well as domestic investment. Under the financial constraint, which is similar to the one developed in this chapter, underinvestment occurs both in domestic investment and FDI. This implies MNEs would have incentives to increase investment in both countries if they obtained an incremental resource. The extent to which MNEs increase each type of investment is determined by its profitability. That is, MNEs increase investment in both countries so that marginal returns of domestic investment and FDI become the same.

The basic idea of Dietrich (2004) is carried over to my model as the borrowing constraints in my model cause underinvestment in both countries.[77] The marginal increase of the net worth of MNEs would push up investment in both countries. To confirm this intuition, I conduct a following experiment. After all agents have completed their optimization at time $t-1$, I reallocate real estate in home country from home households to MNEs so that the real estate holdings of MNEs increase by 1% initially. And I investigate how this reallocation affects the optimization of MNEs and the fluctuation of the economy from time t onward. This experiment looks into how the economy responds to a marginal increase of the net worth of MNEs.

Figure 3-4 shows the impulse responses. The impulse responses tell three things: i) MNEs increase real estate holdings in both countries. ii) The magnitude of the increase of real estate holdings is more than the magnitude of the initial shock. iii) Output increases in both countries.

The first observation confirms my intuition above. Because underinvestment occurs in both countries, an increase of the net worth induces MNEs to invest in both countries. And the magnitude of the increase is almost the same across countries.[78] A desire to equate the marginal benefit of real estate holdings leads MNEs to invest in both countries almost equally.

[77] The proof is in Appendix C.

[78] The general equilibrium effect causes tiny differences in the magnitude of the response of real estate investment. Especially, home households face a negative shock but foreign households don't. This creates a difference in consumption between the two types of households which in turn creates a slight difference in the equilibrium allocation of real estate in the two countries.

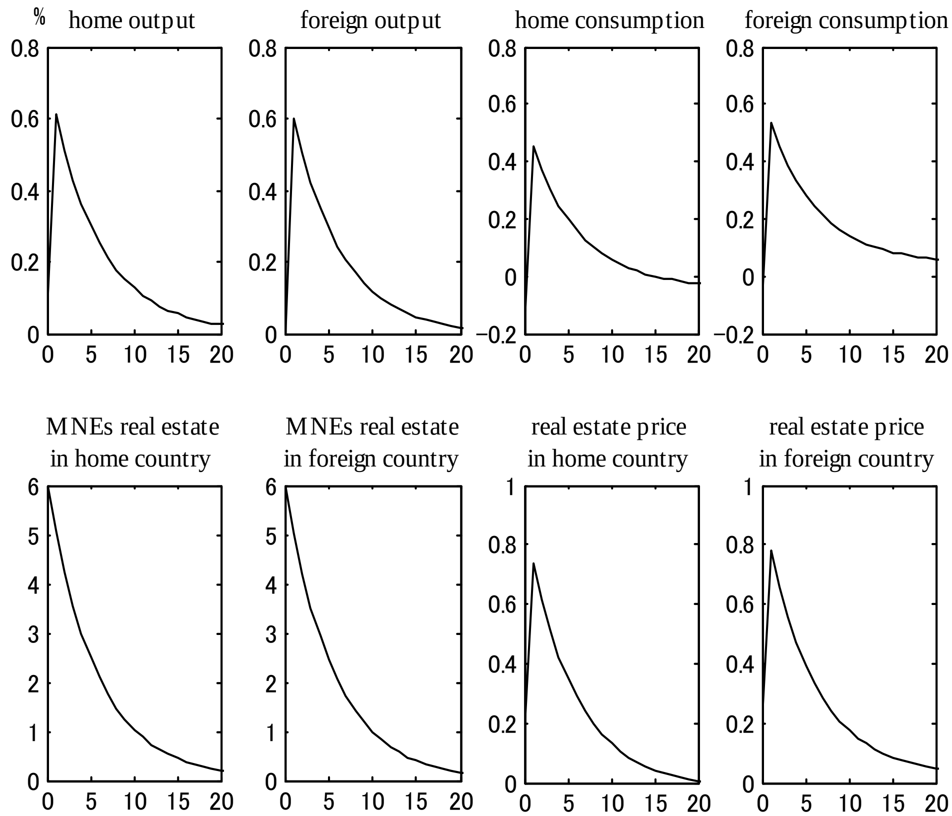

Figure 3-4: Impulse responses to a 1% land reallocation shock in home country. (The model with the borrowing constraints)

The large responses of real estate holdings come from the multiplier effect of relaxing the borrowing constraints. Investment in real estate increases assets of MNEs. MNEs can utilize the incremental assets as collateral for borrowing. This in turn increases the resources for investment further. This sequence creates more than one-to-one effect on MNEs' real estate holdings. This force is captured by $m\lambda_t q_{t+1}$ in equation (3.4). I call it the wealth effect of MNEs' net worth.

In my model, the above force works in both countries. MNEs can obtain the strong multiplier effect not only investing in home country but also investing in foreign country despite the fact that the initial shock hits home country. In all, MNEs take advantage of the multiplier effects in both countries and invest until the marginal benefits of real estate holdings become the same across the two countries.

With this mechanism in our mind, let us see the impulse responses when a positive productivity shock hits home country. Here I assume that the productivity process is persistent (i.e $\rho_{ii} = 0.95$). Figure 3-5 shows the impulse responses. MNEs purchase real estate in both countries in the equilibrium. There are two forces exerting in this economy. Because the shock is in favor of home country production, there would be a desire to reallocate resources from foreign country to home country. Namely, this is the substitution effect.[79] However, the positive productivity shock increases the net worth of MNEs. Thus the wealth effect, which induces MNEs to increase investment in both countries, arises. Under the parameter values in Table 3-1, the wealth effect well dominates and MNEs increase real estate holdings in both countries. As a consequence, output rises in both countries.

Asset price co-movement appears again in this experiment. However, the mechanism is different from that of the frictionless model. MNEs increases real estate demand in both countries as opposed to the case of the frictionless model (See Figure 3-2). The relaxation of the borrowing constraints shifts up the demand curve of MNEs in both countries. This puts pressure on real estate prices. Foreign households increase real estate demand too, but the main force is increase of labor income rather than the international risk sharing channel.[80] The increase of output in both countries increases the labor income of households in both countries. Notice MNEs borrow more from foreign households in the equilibrium. This is a sharp contrast to the impulse response of the frictionless model. The portfolio decisions of MNEs under the presence of the wealth effect are the main driver of the asset price co-movement in the model with the borrowing constraints.

[79] In the labor market, hours worked increase in home country while they decrease in foreign country. This captures the substitution effect exerting in this model.

[80] There is still a channel of international risk sharing because I allow bond trading between home and foreign households. However, the asset price co-movement arises even if I shut down the international risk sharing channel.

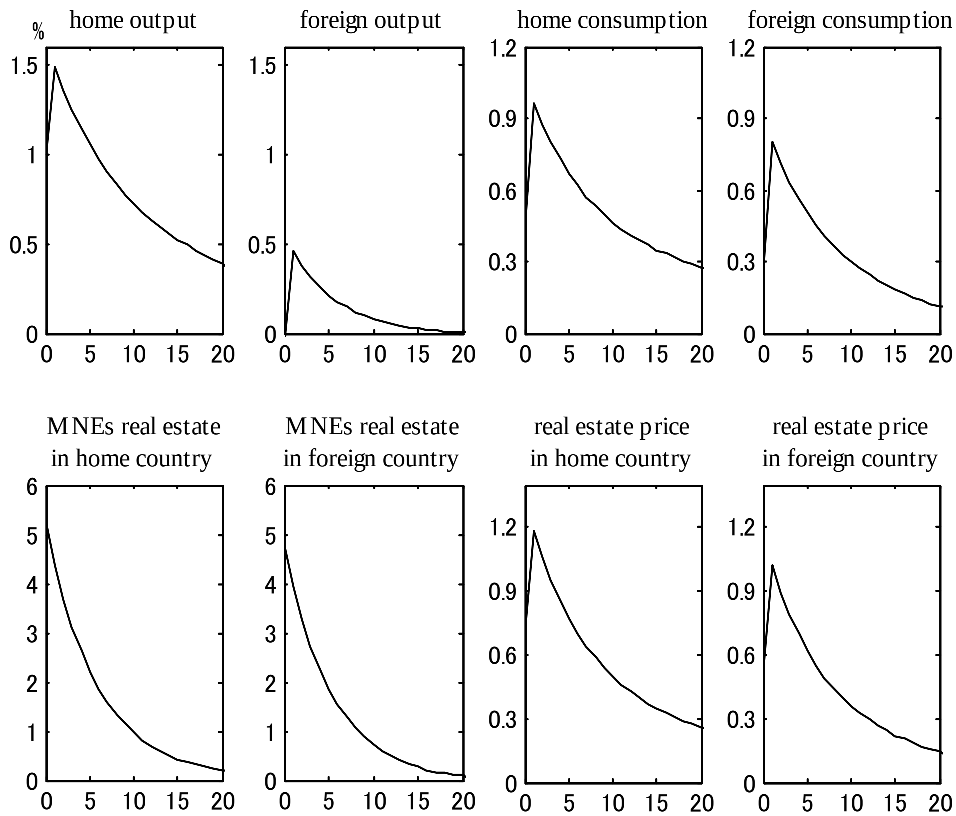

Figure 3-5: Impulse responses to a 1% persistent productivity shock in home country. ($\rho_{11} = 0.95$. The model with the borrowing constraints)

3.3.4 Simulation

Before simulating the model, I modified my models to include variable capital. I assume that the variable capital enters into production function and it is owned by MNEs. Hence, the production function is modified as:

$$\begin{aligned} k_t^e &= k_t + k_t', \\ y_t^e &= y_t + y_t' = A_t(h_{t-1}^e)^{\alpha}(k_{t-1})^{\nu}(l_t)^{1-\alpha-\nu} + A_t'(h_{t-1}'^e)^{\alpha}(k_{t-1}')^{\nu}(l_t')^{1-\alpha-\nu}. \end{aligned}$$

ν is the capital share. I set $\nu = 0.23$. I fix α as in Table 3-1. This implies the labor share $1 - \alpha - v = 0.67$. Capital can be freely relocated between home and foreign country and it depreciates at the rate $\delta = 0.025$. The budget constraint of MNEs now includes the evolution of capital.[81] Households' budget constraints do not change.

As for the model with the borrowing constraints, the variable capital also serves as collateral for borrowing. I assume that domestic households take only capital in home country as collateral and vice versa. The borrowing constraints of MNEs are thus modified as:

$$\begin{aligned} r_t b_t^e &\leq m\left[E_t(q_{t+1} h_t^e) + k_t\right], \\ r_t' b_t'^e &\leq m\left[E_t(q_{t+1}' h_t'^e) + k_t'\right]. \end{aligned}$$

These modifications create a channel for the economy to increase aggregate savings. And they make my models more comparable to international business cycle models as many of them have variable capital.

In addition, I exclude bond transactions between home and foreign households in the model with the borrowing constraints. In the real world, opportunities for international risk sharing among working households may be restricted considerably. In my models, this can be achieved by excluding opportunities for bond trading between two types of households.

I focus on the cross-country correlations of various endogenous variables. Table 3-3 shows the simulation results. Column 1 represents the result when MNEs do not face the borrowing constraints. Column 2 shows the result with the borrowing constraints but without the spillover effect of productivity shocks. Column 3 shows the result when we also have the spillover effect. I compare my results with a standard international RBC model (Column 4, based on Backus Kehoe and Kydland (1992)) and data of developed countries (Column 5, based on Kehoe and Perri (2002) and Iacoviello and Minetti (2006)).

The frictionless economy (Column 1) predicts negative cross-country correlations in output, capital investment and employment. It also predicts unreasonably high cross-

[81] See Appendix C for the exact form of the budget constraint.

Description	No friction	Financial friction		BKK	data
		No spillover	With spillover		
International correlation					
Output y, y'	-0.19	0.19	0.21	-0.18	0.51
Consumption c, c'	0.94	0.27	0.47	0.88	0.32
Employment l, l'	-0.96	0.20	0.12	$n.a$	0.43
Capital investment i, i'	-0.82	-0.24	-0.08	$n.a$	0.29
Real estate prices q, q'	0.94	0.37	0.56	$n.a$	0.50

Notes: BKK represents baseline result of Backus Kehoe and Kydland (1992) in which they assume no friction in the model. For, column 4, the first four values are correlations between U.S. variables and 15 EU-country variables which are based on Kehoe and Perri (2002). The correlation of real estate prices represents that between U.S. and Japan based on Iacoviello and Minetti (2006). All series are HP filtered with smoothing parameter λ=1600.

Table 3-3: Business cycle properties

country correlation in consumption. Basically, these are what standard international RBC models predict. Without any frictions in financial markets, whether the firms are MNEs or exporting firms is irrelevant for the cross-country correlations.

The models with the borrowing constraints (Column 2 & 3) yield positive cross-country output correlations. The dominance of the wealth effect generates positive co-movement in output. The lower cross-country correlations in consumption and real estate prices are derived from the exclusion of the international risk sharing channel. These are the wedges between my models (Column 2 & 3) and standard international RBC models. Eliminating opportunities for risk sharing reduces the cross-country consumption correlation dramatically. Allowing the spillover effect widens the gap between consumption and output. This is also observed in Kehoe and Perri (2002) and Heathcore and Perri (2002).

Cross-country correlations in factor inputs become closer to the actual data though there are still discrepancies. As for the labor market, labor hours in each country turn to be positively correlated. From the perspective of foreign households, the removal of bond trading with home households contains the increase of consumption when a positive productivity shock hits home country. This in turn contains the leftward shift of the labor supply curve.[82] On the other hand, an increase of the marginal product of labor in foreign country induces MNEs to increase labor demand given real wage.[83] These forces result in more employment in foreign country and create positive co-movement of labor hours.

Although discrepancies between simulated data and actual data still remain, the model with the borrowing constraints fits better to actual data. Financially constrained MNEs are a persuasive factor of explaining the fluctuation of actual economies.

3.4 Conclusion

This chapter examines the transmission mechanisms of international business cycles when there are internationally operated entrepreneurs. The focus of this chapter is how business cycles are transmitted from one country to another when the borrowing capacity of MNEs is limited. I find that cross-country output correlation depends on MNEs' ability to raise funds in financial markets. Without restriction in the borrowing capacity, the substitution effect dominates in response to productivity shocks and output tends to be negatively correlated. When MNEs' borrowing capacity is limited, the wealth effect dominates and cross-country output correlation becomes positive. The local asset prices co-move positively in response to country specific productivity shocks. This is robust to various specifications of the models but underlying mechanisms differ in the specifications of borrowing capacity. For the frictionless model, the high degree of international risk sharing is the driver of co-movement but it is at odds with the phenomenon in the real world because firms lend to households. On the other hand, if MNEs are constrained in borrowing capacity, the wealth effect and their portfolio decisions have first order impacts on the asset price co-movement. As recent international business cycle literature reveals, my findings suggest that limited enforceability on financial contracts is a key factor to explain

[82] The labor supply equation is $l_t^{\eta-1} = w_t/c_t$. An increase of household's consumption shifts the labor supply curve leftward in a partial equilibrium context.

[83] In a partial equilibrium context, the marginal product of labor rises when MNEs increase real estate holdings because of the Cobb-Douglas technology.

international business cycle properties.

C Appendix C

In this appendix, I describe the details of the models in chapter 3. I present the optimality condition of the agents. Then I describe the steady states and the equations that characterize the dynamics around the steady states.

The Frictionless Economy Model

FOCs and Market Clearing Conditions

By solving home households' problem, I obtain:

$$
\begin{aligned}
&(l_t)^{\eta-1} = \frac{w_t}{c_t}, \\
&\frac{1}{c_t}(1-\xi b_t) = \beta r_t E_t\left(\frac{1}{c_{t+1}}\right), \\
&\frac{1}{c_t}(1-\xi b_t^*) = \beta r_t^* E_t\left(\frac{1}{c_{t+1}}\right), \\
&\frac{q_t}{c_t} = \frac{\psi}{h_t} + \beta E_t\left(\frac{q_{t+1}}{c_{t+1}}\right).
\end{aligned}
$$

These are labor supply equation, Euler equations on debt choices and the optimality condition regarding real estate holdings respectively. Foreign households' optimality conditions are symmetric to those of home households.

By solving MNEs' problem, I obtain:

$$
\begin{aligned}
&w_t = (1-\alpha)\frac{y_t}{l_t}, \\
&w'_t = (1-\alpha)\frac{y'_t}{l'_t}, \\
&\frac{1}{c^e_t}(1-\xi b^e_t) = \gamma r_t E_t\left(\frac{1}{c^e_{t+1}}\right), \\
&\frac{1}{c^e_t}(1-\xi b'^e_t) = \gamma r'_t E_t\left(\frac{1}{c^e_{t+1}}\right), \\
&\frac{q_t}{c^e_t} = \gamma E_t\left[\frac{1}{c^e_{t+1}}\left(\alpha\frac{y_{t+1}}{h^e_t}\right)\right] + \gamma E_t\left(\frac{q_{t+1}}{c^e_{t+1}}\right), \\
&\frac{q'_t}{c^e_t} = \gamma E_t\left[\frac{1}{c^e_{t+1}}\left(\alpha\frac{y'_{t+1}}{h'^e_t}\right)\right] + \gamma E_t\left(\frac{q'_{t+1}}{c^e_{t+1}}\right).
\end{aligned}
$$

These are, in order, labor demand equations in each country, Euler equations on debt choices and the optimality conditions regarding real estate holdings of MNEs.

The unknowns are $\{c, c', c^e, y, y', y^e, h, h', h^e, h'^e, l, l', b, b', b^e, b'^e, b^*, q, q', w, w', r, r', r^*\}$.[84]

Home households' optimality conditions, corresponding foreign households' optimality conditions, MNEs' optimality conditions, the market clearing conditions, two households' budget constraints and three production equations (presented in section 3.2.1) form 24 equations for 24 unknowns.[85]

[84] The total factor productivities, A and A', are also unknown parameters. These are determined by a exogenous process.

[85] One of the budget constraints of the three agents is redundant.

The Steady State

The steady state can be summarized as follows:

$$
\begin{aligned}
r &= r' = r^* = \beta^{-1}, \\
b &= b' = b^e = b'^e = b^* = 0, \\
\frac{q}{c} &= \frac{(1-\beta)h}{\psi}, \quad \frac{q'}{c'} = \frac{(1-\beta)h'}{\psi}, \\
\frac{qh^e}{y} &= \frac{q'h'^e}{y'} = \frac{\alpha\beta}{1-\beta}, \\
\frac{c}{y} &= \frac{c'}{y'} = (1-\alpha), \\
\frac{c}{y} &= (1-\alpha)l^{-\eta}, \quad \frac{c'}{y'} = (1-\alpha)l'^{-\eta}, \\
\frac{h^e}{h} &= \frac{h'^e}{h'} = \frac{\alpha\beta}{\psi(1-\alpha)}, \\
\frac{y'}{y} &= \left(\frac{\bar{h}'}{\bar{h}}\right)^{\alpha}, \\
\frac{c^e}{y} &= \alpha\left[1 + \left(\frac{\bar{h}'}{\bar{h}}\right)^{\alpha}\right], \\
A &= A' = 1.
\end{aligned}
$$

The steady state level of debts are zero because I assume that there are quadratic adjustment costs on debt holdings around $b = 0$. The total value of real estate is the real estate share, α, times the discounted sum of the value of output (The forth row). household's consumption-output ratio is obtained by evaluating the households' budget constraints at the steady state. the relative output is proportional to the relative real estate supply (the eighth row). By the good market clearing condition the ratio of MNEs' consumption to home output is given by the ninth row. Steady state productivity level is normalized to one.

Log Linearization

Followings are the system of log-linearized equations.

$$\widehat{c_t} = \widehat{y_t} - \eta\widehat{l_t},$$

$$\widehat{c'_t} = \widehat{y'_t} - \eta\widehat{l'_t},$$

$$\widehat{c_t} = E_t\widehat{c_{t+1}} - \widehat{r_t} + \xi db^e_t,$$

$$\widehat{c_t} = E_t\widehat{c_{t+1}} - \widehat{r^*_t} - \xi db^*_t,$$

$$\widehat{c'_t} = E_t\widehat{c'_{t+1}} - \widehat{r'_t} + \xi db'^e_t,$$

$$\widehat{c'_t} = E_t\widehat{c'_{t+1}} - \widehat{r^*_t} + \xi db^*_t,$$

$$\widehat{q_t} - \widehat{c_t} = \frac{(1-\beta)h^e}{h}\widehat{h^e_t} + \beta E_t\left(\widehat{q_{t+1}} - \widehat{c_{t+1}}\right),$$

$$\widehat{q'_t} - \widehat{c'_t} = \frac{(1-\beta)h'^e}{h'}\widehat{h'^e_t} + \beta E_t\left(\widehat{q'_{t+1}} - \widehat{c'_{t+1}}\right),$$

$$\widehat{c^e_t} = E_t\widehat{c^e_{t+1}} - \widehat{r_t} - \xi db^e_t.$$

$$\widehat{c^e_t} = E_t\widehat{c^e_{t+1}} - \widehat{r'_t} - \xi db'^e_t.$$

$$\widehat{q_t} - \widehat{c^e_t} = (1-\beta)\left[E_t(\widehat{y_{t+1}}) - \widehat{h^e_t}\right] + \beta E_t(\widehat{q_{t+1}}) - E_t(\widehat{c^e_{t+1}}),$$

$$\widehat{q'_t} - \widehat{c^e_t} = (1-\beta)\left[E_t(\widehat{y'_{t+1}}) - \widehat{h'^e_t}\right] + \beta E_t(\widehat{q'_{t+1}}) - E_t(\widehat{c^e_{t+1}}),$$

$$\frac{c}{y}\widehat{c_t} - \frac{qh^e}{y}\widehat{h^e_t} - \frac{1}{\beta y}db^e_{t-1} + \frac{1}{\beta y}db^*_{t-1} = (1-\alpha)\widehat{y_t} - \frac{qh^e}{y}\widehat{h^e_{t-1}} - \frac{1}{y}db^e_t + \frac{1}{y}db^*_t,$$

$$\frac{c'}{y'}\widehat{c'_t} - \frac{q'h'^e}{y'}\widehat{h'^e_t} - \frac{1}{\beta y'}db'^e_{t-1} - \frac{1}{\beta y'}db^*_{t-1} = (1-\alpha)\widehat{y'_t} - \frac{q'h'^e}{y'}\widehat{h'^e_{t-1}} - \frac{1}{y'}db'^e_t - \frac{1}{y'}db^*_t,$$

$$\widehat{y_t} + \frac{y'}{y}\widehat{y'_t} = \frac{c}{y}\widehat{c_t} + \frac{c'}{y'} \times \frac{y'}{y}\widehat{c'_t} + \frac{c^e}{y}\widehat{c^e_t},$$

$$\widehat{y_t} = \widehat{A_t} + \alpha\widehat{h^e_{t-1}} + (1-\alpha)\widehat{l_t},$$

$$\widehat{y'_t} = \widehat{A'_t} + \alpha\widehat{h'^e_{t-1}} + (1-\alpha)\widehat{l'_t}.$$

A technical complication is how to log-linearize equations containing the debts b^e, b'^e, and b^* because these variables are zero in the steady state. I proceed with the following ways: For all variables except debts, I define $\hat{x}$ as log deviation from the steady state. For bonds, I define db as deviations in level from the steady state.

Together with the evolution of productivity in section 3.2.3, the equations above form 19 equations for 19 unknowns, $\{\widehat{c}$, $\widehat{c'}$, $\widehat{c^e}$, $\widehat{y}$, $\widehat{y'}$, $\widehat{y^e}$, $\widehat{h}^e$, $\widehat{h}'^e$, $\widehat{l}$, $\widehat{l'}$, db^e, db'^e, db^*, $\widehat{q}$, $\widehat{q'}$, $\widehat{r}$, $\widehat{r'}$, $\widehat{r^*}\}$.

The model with the borrowing constraints

The Model

Multinational Entrepreneurs (MNEs)

The amount that MNEs can borrow from households is now limited up to a fraction of the expected value of collateral. The collateral is real estate in this model. Home households only accept real estate in home country as collateral and vice versa.

$$\max \quad \sum_{t=0}^{\infty} \gamma^t \ln c_t^e,$$

$$s.t. \quad \begin{gathered} c_t^e + q_t h_t^e + q_t' h_t'^e + r_{t-1} b_{t-1}^e + r_{t-1}' b_{t-1}'^e = \\ y_t^e - w_t l_t - w_t' l_t' + q_t h_{t-1}^e + q_t' h_{t-1}'^e + b_t^e + b_t'^e, \\ r_t b_t^e \leq m E_t(q_{t+1} h_t^e), \\ r_t' b_t'^e \leq m E_t(q_{t+1}' h_t'^e), \\ y_t^e = y_t + y_t', \\ y_t = A_t (h_{t-1}^e)^\alpha (l_t)^{1-\alpha}, \quad y_t' = A_t' (h_{t-1}'^e)^\alpha (l_t')^{1-\alpha}. \end{gathered}$$

I assume $\beta > \gamma$ in this model so that the borrowing constraints bind in the neighborhood of the steady state. Since the borrowing constraints pin down the unique steady state of the debts, b^e and b'^e, I drop the quadratic adjustment costs regarding with these debts.

Households

Because households no longer pay for the adjustment costs on debts b and b', their budget constraints are modified as follows:

$$\begin{aligned} & c_t + q_t h_t + r_{t-1} b_{t-1} + r^*_{t-1} b^*_{t-1} + \tfrac{\xi}{2}(b^*_t)^2 \\ & = w_t l_t + q_t h_{t-1} + b_t + b^*_t + \tau^*_t. \end{aligned}$$

Notice the adjustment cost on b^*, which represents the financial transaction between home and foreign households, remains because nothing other than the adjustment cost pins down the unique steady state of b^*.

FOCs and Market Clearing Conditions

MNEs optimality conditions are given as follows:

$$\begin{aligned} & w_t = (1-\alpha)\frac{y_t}{l_t}, \\ & w'_t = (1-\alpha)\frac{y'_t}{l'_t}, \\ & \frac{1}{c^e_t} = \gamma r_t E_t\left(\frac{1}{c^e_{t+1}}\right) + r_t \lambda^e_t, \\ & \frac{1}{c^e_t} = \gamma r'_t E_t\left(\frac{1}{c^e_{t+1}}\right) + r_t \lambda'^e_t, \\ & \frac{q_t}{c^e_t} = \gamma E_t\left[\frac{1}{c^e_{t+1}}\left(\alpha\frac{y_{t+1}}{h^e_t}\right)\right] + \gamma E_t\left(\frac{q_{t+1}}{c^e_{t+1}}\right) + m\lambda^e_t E_t(q_{t+1}), \\ & \frac{q'_t}{c^e_t} = \gamma E_t\left[\frac{1}{c^e_{t+1}}\left(\alpha\frac{y'_{t+1}}{h'^e_t}\right)\right] + \gamma E_t\left(\frac{q'_{t+1}}{c^e_{t+1}}\right) + m\lambda'^e_t E_t(q'_{t+1}). \end{aligned}$$

λ^e and λ'^e represent Lagrange multipliers associated with the borrowing constraints. Euler equations on debt choices and real estate holdings now include the marginal benefit of relaxing the borrowing constraints. By eliminating the Lagrange multipliers, we get:

$$\frac{q_t}{c_t^e} = \gamma E_t\left[\frac{1}{c_{t+1}^e}\left(\alpha\frac{y_{t+1}}{h_t^e}\right)\right] + E_t\left\{q_{t+1}\left[\frac{m}{r_t c_t^e} + \frac{(1-m)\gamma}{c_{t+1}^e}\right]\right\},$$
$$\frac{q_t'}{c_t^e} = \gamma E_t\left[\frac{1}{c_{t+1}^e}\left(\alpha\frac{y_{t+1}'}{h_t'^e}\right)\right] + E_t\left\{q_{t+1}'\left[\frac{m}{r_t' c_t^e} + \frac{(1-m)\gamma}{c_{t+1}'^e}\right]\right\}.$$

For home households' optimality conditions, there is a slight change in the optimality condition on debt transaction with MNEs (the second row):

$$(l_t)^{\eta-1} = \frac{w_t}{c_t},$$
$$\frac{1}{c_t} = \beta r_t E_t\left(\frac{1}{c_{t+1}}\right),$$
$$\frac{1}{c_t}(1-\xi b_t^*) = \beta r_t^* E_t\left(\frac{1}{c_{t+1}}\right),$$
$$\frac{q_t}{c_t} = \frac{\psi}{h_t} + \beta E_t\left(\frac{q_{t+1}}{c_{t+1}}\right).$$

Foreign households' optimality conditions are also modified accordingly.

The Steady State

The steady state is summarized as follows:

$$
\begin{aligned}
& r = r' = r^* = \beta^{-1}, \\
& b^* = 0, \\
& \frac{q}{c} = \frac{(1-\beta)h}{\psi}, \qquad \frac{q'}{c'} = \frac{(1-\beta)h'}{\psi}, \\
& \frac{qh^e}{y} = \frac{q'h'^e}{y'} = \frac{\alpha\gamma}{1-\phi} \qquad where \;\; \phi = m\beta + (1-m)\gamma, \\
& \frac{qh^e}{b} = \frac{q'h'^e}{b'} = \frac{1}{m\beta}, \\
& \frac{b}{y} = \frac{b'}{y'} = \frac{m\alpha\beta\gamma}{1-\phi}, \\
& \frac{c}{y} = \frac{c'}{y'} = (1-\alpha) + \frac{m\alpha\beta(\beta^{-1}-1)\gamma}{1-\phi}, \\
& \frac{h^e}{h} = \frac{h'^e}{h'} = \frac{\alpha(1-\beta)\gamma}{\psi(1-\phi)} \times \frac{y}{c}, \\
& l = l' = \left[(1-\alpha)\frac{y}{c}\right]^{\frac{1}{\eta}}, \\
& \frac{y'}{y} = \left(\frac{\bar{h}'}{\bar{h}}\right)^{\alpha}, \\
& \frac{c^e}{y} = \left(1 - \frac{c}{y}\right)\left(1 + \frac{y'}{y}\right), \\
& A = A' = 1.
\end{aligned}
$$

In this model, the debts of MNEs are positive (the forth and the fifth row). They are obtained by evaluating the borrowing constraints at the steady state.

Log Linearization

The system of log linearized equations is as follows:

$$\widehat{c_t} = \widehat{y_t} - \eta\widehat{l_t},$$

$$\widehat{c'_t} = \widehat{y'_t} - \eta\widehat{l'_t},$$

$$\widehat{c_t} = E_t\widehat{c_{t+1}} - \widehat{r_t},$$

$$\widehat{c'_t} = E_t\widehat{c'_{t+1}} - \widehat{r'_t},$$

$$\widehat{r_t} = \widehat{r^*_t} + \xi db^*_t,$$

$$\widehat{r'_t} = \widehat{r^*_t} - \xi db^*_t,$$

$$\widehat{q_t} - \widehat{c_t} = \frac{(1-\beta)h^e}{h}\widehat{h^e_t} + \beta E_t\left(\widehat{q_{t+1}} - \widehat{c_{t+1}}\right),$$

$$\widehat{q'_t} - \widehat{c'_t} = \frac{(1-\beta)h'^e}{h'}\widehat{h'^e_t} + \beta E_t\left(\widehat{q'_{t+1}} - \widehat{c'_{t+1}}\right),$$

$$\widehat{q_t} - (1-m\beta)\widehat{c^e_t}$$
$$= (1-\phi)\left[E_t(\widehat{y_{t+1}}) - \widehat{h^e_t}\right] + \phi E_t(\widehat{q_{t+1}}) - (1-m\beta)E_t(\widehat{c^e_{t+1}}) - m\beta r_t,$$

$$\widehat{q'_t} - (1-m\beta)\widehat{c^e_t}$$
$$= (1-\phi)\left[E_t(\widehat{y'_{t+1}}) - \widehat{h'^e_t}\right] + \phi E_t(\widehat{q'_{t+1}}) - (1-m\beta)E_t(\widehat{c^e_{t+1}}) - m\beta r'_t,$$

$$\frac{c}{y}\widehat{c_t} - \frac{qh^e}{y}\widehat{h^e_t} - \frac{b}{\beta y}(\widehat{b^e_{t-1}} + \widehat{r_{t-1}}) + \frac{1}{\beta y}db^*_{t-1} = (1-\alpha)\widehat{y_t} - \frac{qh^e}{y}\widehat{h^e_{t-1}} - \frac{b}{y}\widehat{b^e_t} + \frac{1}{y}db^*_t,$$

$$\tfrac{c'}{y'}\widehat{c'_t} - \tfrac{q'h'^e}{y'}\widehat{h'^e_t} - \tfrac{b}{\beta y}(\widehat{b'^e_{t-1}} + \widehat{r'_{t-1}}) - \tfrac{1}{\beta y'}db^*_{t-1}$$
$$= (1-\alpha)\widehat{y'_t} - \tfrac{q'h'^e}{y'}\widehat{h'^e_{t-1}} - \tfrac{b}{y}\widehat{b'^e_t} - \tfrac{1}{y'}db^*_t,$$

$$\widehat{y_t} + \frac{y'}{y}\widehat{y'_t} = \frac{c}{y}\widehat{c_t} + \frac{c'}{y'} \times \frac{y'}{y}\widehat{c'_t} + \frac{c^e}{y}\widehat{c^e_t},$$

$$\widehat{b^e_t} + \widehat{r_t} = E_t(\widehat{q_{t+1}}) + \widehat{h^e_t},$$

$$\widehat{b'^e_t} + \widehat{r'_t} = E_t(\widehat{q'_{t+1}}) + \widehat{h'^e_t},$$

$$\widehat{y_t} = \widehat{A_t} + \alpha\widehat{h^e_{t-1}} + (1-\alpha)\widehat{l_t},$$

$$\widehat{y'_t} = \widehat{A'_t} + \alpha\widehat{h'^e_{t-1}} + (1-\alpha)\widehat{l'_t}.$$

Together with the productivity specification this system forms 19 equations for 19 unknowns, $\{\widehat{c}, \widehat{c}', \widehat{c}^e, \widehat{y}, \widehat{y}', \widehat{y}^e, \widehat{h}^e, \widehat{h}'^e, \widehat{l}, \widehat{l}', \widehat{b}^e, \widehat{b}'^e, db^*, \widehat{q}, \widehat{q}', \widehat{r}, \widehat{r}', \widehat{r}^*\}$.

The Model with Variable Capital

In the models with variable capital, production functions and the law of the motion of capital are:

$$
\begin{aligned}
k_t^e &= k_t + k_t', \\
y_t &= A_t(h_{t-1}^e)^\alpha (k_{t-1})^\nu (l_t)^{1-\alpha-\nu}, \\
y_t' &= A_t(h_{t-1}'^e)^\alpha (k_{t-1}')^\nu (l_t')^{1-\alpha-\nu}, \\
i_t &= k_t - (1-\delta)k_{t-1}, \\
i_t' &= k_t' - (1-\delta)k_{t-1}'.
\end{aligned}
$$

i and i' are capital investment of MNEs in each country. δ is the depreciation rate of capital and v is the capital share in production function.

When MNEs do not face the borrowing constraints, the flow of funds of MNEs is modified as:

$$
\begin{gathered}
c_t^e + i_t + i_t' + q_t h_t^e + q_t' h_t'^e + r_{t-1} b_{t-1}^e + r_{t-1}' b_{t-1}'^e + \tfrac{\xi}{2}(b_t^e)^2 + \tfrac{\xi}{2}(b_t'^e)^2 = \\
y_t^e - w_t l_t - w_t' l_t' + q_t h_{t-1}^e + q_t' h_{t-1}'^e + b_t^e + b_t'^e + \tau_t^e + \tau_t'^e,
\end{gathered}
$$

When MNEs face the borrowing constraints, I assume that capital can be used as collateral for borrowing:

$$
\begin{gathered}
r_t b_t^e \leq m\left[E_t(q_{t+1} h_t^e) + k_t\right], \\
r_t' b_t'^e \leq m\left[E_t(q_{t+1} h_t'^e) + k_t'\right], \\
c_t^e + i_t + i_t' + q_t h_t^e + q_t' h_t'^e + r_{t-1} b_{t-1}^e + r_{t-1}' b_{t-1}'^e = \\
y_t^e - w_t l_t - w_t' l_t' + q_t h_{t-1}^e + q_t' h_{t-1}'^e + b_t^e + b_t'^e.
\end{gathered}
$$

The objective function of MNEs and the problems of households are not changed.

For the frictionless model, the optimality conditions for capital holdings are given as:

$$\frac{1}{c_t^e} = \gamma E_t \left[\frac{1}{c_{t+1}^e} \left(\nu \frac{y_{t+1}}{k_t^e} + 1 - \delta \right) \right],$$
$$\frac{1}{c_t'^e} = \gamma E_t \left[\frac{1}{c_{t+1}'^e} \left(\nu \frac{y'_{t+1}}{k_t'^e} + 1 - \delta \right) \right].$$

For the model with the borrowing constraints, the optimality conditions for capital holdings are:

$$\frac{1}{c_t^e} = \gamma E_t \left[\frac{1}{c_{t+1}^e} \left(\nu \frac{y_{t+1}}{k_t^e} + 1 - \delta \right) \right] + m\lambda_t^e,$$
$$\frac{1}{c_t'^e} = \gamma E_t \left[\frac{1}{c_{t+1}'^e} \left(\nu \frac{y'_{t+1}}{k_t'^e} + 1 - \delta \right) \right] + m\lambda_t'^e.$$

In the chapter, I exclude the bond transaction between home households and foreign households in the model with the borrowing constraints. Thus, home households' budget constraint is modified as follows:

$$c_t + q_t h_t + r_{t-1} b_{t-1} = w_t l_t + q_t h_{t-1} + b_t.$$

Proof of inefficiency in the financial friction model

In this section, I show that the steady state allocation of real estate in the model with the borrowing constraints (but without capital) is inefficient. I show this by showing that there is a Pareto improvement allocation.

To prove the claim, we need to derive steady state levels of output, real estate allocation and consumption. First, I retrieve the following steady state equations from above:

$$\frac{c}{y} = (1-\alpha) + \frac{m\alpha\beta(\beta^{-1}-1)\gamma}{1-\phi} \equiv \theta_1,$$
$$\frac{h^e}{h} = \frac{\alpha(1-\beta)\gamma}{\psi(1-\phi)} \times \frac{y}{c}.$$

By these conditions, we can obtain the level of real estate holdings as:

$$\begin{aligned} \frac{h^e}{h} &= \frac{\alpha(1-\beta)\gamma}{\psi(1-\phi)} \times \frac{1}{\theta_1} \equiv \theta_2, \\ &\Rightarrow (1+\theta_2)h = \bar{h}, \\ &\Rightarrow h = \frac{1}{1+\theta_2}\bar{h}, \qquad h^e = \frac{\theta_2}{1+\theta_2}\bar{h}. \end{aligned} \tag{c.1}$$

The steady state labor hours (see the ninth row of the steady state equations in the model with the borrowing constraints) are given by:

$$\begin{aligned} l &= \left[(1-\alpha)\frac{y}{c}\right]^{\frac{1}{\eta}}, \\ &\Longrightarrow \left[\frac{(1-\alpha)}{\theta_1}\right]^{\frac{1}{\eta}}. \end{aligned} \tag{c.2}$$

Combining (c.1) and (c.2) yields:

$$\begin{aligned} y &= A\left(\frac{\theta_2}{1+\theta_2}\bar{h}\right)^{\alpha}\left[\frac{(1-\alpha)}{\theta_1}\right]^{\frac{1-\alpha}{\eta}}, \\ c &= \theta_1 y, \\ &= A\theta_1\left(\frac{\theta_2}{1+\theta_2}\bar{h}\right)^{\alpha}\left[\frac{(1-\alpha)}{\theta_1}\right]^{\frac{1-\alpha}{\eta}}. \end{aligned}$$

Suppose a marginal unit of real estate is reallocated from home households to MNEs in the steady state.

$$h^e_{new} = h^e + dh, \quad h_{new} = h - dh. \tag{c.3}$$

This reallocation changes two things. i) Output in home country increases by the marginal product of real estate. ii) Home households' utility decreases by the marginal utility of housing service.[86] Notice i) and ii) are measured by:

$$\begin{aligned}\frac{\partial y}{\partial h^e} &= \frac{\alpha y}{h^e} = \alpha A \left[\left(\frac{\theta_2 \bar{h}}{1+\theta_2} \right) \left(\frac{\theta_1}{1-\alpha} \right)^{1/\eta} \right]^{\alpha-1}, \\ \frac{\partial u}{\partial h} &= \frac{\psi}{h} = \frac{\psi(1+\theta_2)}{\bar{h}}.\end{aligned}$$

Suppose all the incremental income is distributed to home households. And suppose home households consume all the incremental income and they do not change labor supply. These imply neither the debt position of each agent nor the leisure of home households changes. If, for home households, the gain from incremental consumption is higher than the loss from giving up housing service, then the new allocation is a Pareto improvement: all the agents are at least as better off as in the original steady state and home households are strictly better off than in the original steady state.

The marginal utility of consumption of home households at the steady state is:

$$\frac{\partial u}{\partial c} = \frac{1}{c} = \frac{1}{A\theta_1 \left[\theta_2/(1+\theta_2) \times \bar{h}^{\alpha}\right] \left[(1-\alpha)/\theta_1\right]^{\frac{1-\alpha}{\eta}}}.$$

Hence, the gain from incremental consumption is:

$$\begin{aligned}\frac{\partial u}{\partial c} \times \frac{\partial y}{\partial h^e} \times dh &= \frac{\alpha A \left[\theta_2/(1+\theta_2) \times \bar{h}\right]^{\alpha-1} \left[(1-\alpha)/\theta_1\right]^{(1-\alpha)/\eta}}{A\theta_1 \left[\theta_2/(1+\theta_2) \times \bar{h}\right]^{\alpha} \left[(1-\alpha)/\theta_1\right]^{(1-\alpha)/\eta}} \times dh \\ &= \frac{\alpha(1+\theta_2)}{\theta_1\theta_2\bar{h}} \times dh.\end{aligned}$$

[86] I assume MNEs do not utilize incremental real estate as collateral of borrowing. This is not an optimal response of MNEs but it satisfies the borrowing constraints. Hence there is no problem in showing inefficiency.

The loss from giving up the marginal unit of housing service is:

$$-\frac{\partial u}{\partial h} \times dh = -\frac{\psi(1+\theta_2)}{\bar{h}} \times dh.$$

The net change of home households' utility is:

$$\begin{aligned}\left(\frac{\partial u}{\partial c}\frac{\partial y}{\partial h^e} - \frac{\partial u}{\partial h}\right) dh &= \left[\frac{\alpha(1+\theta_2)}{\theta_1\theta_2\bar{h}} - \frac{\psi(1+\theta_2)}{\bar{h}}\right] dh \\ &= \left[\frac{\alpha}{\theta_1\theta_2} - \psi\right]\frac{(1+\theta_2)}{\bar{h}} dh. \qquad \text{(c.4)}\end{aligned}$$

Recall

$$\theta_2 = \frac{\alpha(1-\beta)\gamma}{\psi(1-\phi)} \times \frac{1}{\theta_1}.$$

Then (c.4) is rewritten as:

$$\begin{aligned}\left(\frac{\partial u}{\partial c}\frac{\partial y}{\partial h^e} - \frac{\partial u}{\partial h}\right) dh &= \left[\frac{\alpha\psi(1-\phi)}{\alpha(1-\beta)\gamma} - \psi\right]\frac{(1+\theta_2)}{\bar{h}} dh \\ &= \left[\frac{(1-\phi)}{(1-\beta)\gamma} - 1\right]\frac{\psi(1+\theta_2)}{\bar{h}} dh.\end{aligned}$$

Notice $(1-\phi) > (1-\beta)$ and $\gamma < 1$. Then,

$$\begin{aligned}\left[\frac{(1-\phi)}{(1-\beta)\gamma} - 1\right] &> 0 \quad \Rightarrow \\ \left(\frac{\partial u}{\partial c}\frac{\partial y}{\partial h^e} - \frac{\partial u}{\partial h}\right) dh &= \left[\frac{(1-\phi)}{(1-\beta)\gamma} - 1\right]\frac{\psi(1+\theta_2)}{\bar{h}} dh > 0.\end{aligned}$$

In all, the allocation (c.3) and $c_{new} = c + (\partial y/\partial h^e) \times dh$ is feasible. And the allocation makes home households strictly better off while MNEs and foreign households are as better off as in the original steady state. Thus, reallocating real estate from home households to MNEs attains a Pareto improvement which implies the original steady state is inefficient.[87]

[87] Notice allocating all the incremental output, $(\partial y/\partial h^e) \times dh$, to the households is not necessary. A sufficient condition that home households are strictly better off is $\frac{(1-\phi)\kappa}{(1-\beta)\gamma} > 1$, where κ is the share of incremental output which is reallocated to home households.

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