

The Early Modern Debate on the Problem of Matter's Divisibility: A Neo-Aristotelian Solution

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THE EARLY MODERN DEBATE ON THE PROBLEM OF MATTER'S DIVISIBILITY:

A NEO-ARISTOTELIAN SOLUTION

A dissertation

by

COLIN EDWARD CONNORS

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Abstract

My dissertation focuses on the problem of matter's divisibility in the 17th-18th centuries. The problem of material divisibility is a focal point at which the metaphysical principle of simplicity and the mathematical principle of space's infinite divisibility conflict. The principle of simplicity is the metaphysical requirement that there must be a simple or indivisible being that is the constitutive foundation of all composite things in nature. Without simple beings, there cannot be composite beings. The mathematical principle of space's infinite divisibility is a staple of Euclidean geometry: space must be divisible into infinitely smaller parts because indivisibles or points cannot compose extension. Without reconciling these metaphysical and mathematical principles, one can call into question the integrity of mathematics and metaphysics.

Metaphysical contradiction results from the application of metaphysical simplicity to the composition of material bodies that occupy infinitely divisible space. How can a simple being constitute a material object while occupying a space that lacks a smallest part? Should we assume that a composite material object (such as the paper in front of the reader) *exists in* an infinitely divisible space, then the simple beings must occupy a space that consists of ever smaller spaces. The simple being thereby appears to consist of parts *simpler* than itself—a metaphysical contradiction.

Philosophers resolve this contradiction by either modifying the metaphysical principle of simplicity to allow for the occupation of infinitely divisible space, or have simply dismissed one principle for the sake of preserving the other principle. The rejection of one principle for preserving the other principle is an undesirable path. Philosophers would either forfeit any

attempt to account for the composition of material reality by rejecting simplicity or deny understanding of geometry heretofore via the rejection of space's infinite divisibility.

My objective in this dissertation is two-fold: 1.) to provide an historical analysis of various philosophers' attempts to reconcile simplicity and infinite divisibility or to argue for the exclusive nature of the said principles; 2.) to articulate a reconciliation between simplicity and infinite divisibility. Underlying both objectives is my attempt to draw a connection between the metaphysical principle of simplicity and the metaphysical principle of sufficient reason. Having shown in the historical section that each philosopher implicitly references a modified version of the principle of sufficient reason when articulating their theories of metaphysical simplicity, I will use this common principle to develop a Neo-Aristotelian solution to the problem of material divisibility. This Neo-Aristotelian solution differs from other accounts in the historical section by including a potential parts theory of material divisibility while modifying the principle of simplicity: simple beings are no longer conceived as constitutive parts of a material thing, but as the sources of unity for a natural composite being.

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Dissertation Introduction and Synopsis

During the 17th and 18th centuries, philosophers witness a conflict between proponents of the truths of mathematics and proponents of the truths of metaphysics. This conflict is not only metaphysical in nature, but also epistemic in character. In this debate, one must either deem that the conflict leaves either set of principles irreconcilable, eschewing a truth of metaphysics in order to preserve a truth of mathematics or vice-versa; or one must find some way to reconcile the seemingly conflicting principles; or one might believe that the conflict is unsolvable, casting both sets of principles into the region of knowledge that is unattainable by feeble human reason. The clash centers on the question of whether material bodies that occupy infinitely divisible space must be infinitely divisible.

The principles at stake in this dispute are the metaphysical truth of simplicity and the mathematical truth of space's infinite divisibility. The metaphysical truth of simplicity states that all beings must either be simple beings or be composed of simple beings. It is a principle traceable to the Neo-Platonic philosopher named Proclus. Proclus argues in the *Elements of Theology* that there must be what he calls "The One" or pure unity. Without something that is one or is considered one being, no composition or multitude could exist. Proclus writes: "For if it in no respects participates of *The One*, neither will the whole be one whole, nor each of the many of which the multitude consists; but there will also be a certain multitude arising from each of these, and this will be the case to infinity."¹ Unity or simplicity enables the something to be called *a thing*; otherwise, nothing at all would exist. The principle of simplicity thereby is a fundamental principle of metaphysics that must be true in order for there to exist anything at all since composition presupposes simplicity.

¹ Proclus. *Proclus: Elements of Theology*. Thomas Taylor, ed. Frome, Somerset, UK: The Prometheus Trust, 1998. Proposition I.

The mathematical truth of space's infinite divisibility states that space can be divided without limit since there is no smallest and indivisible part of space. It is a truth propounded in Euclid's *Elements* from Euclid's notion of a line as "...a breadthless length."² Since Euclid conceives a point as "...that which has no part", it could not possibly be the sole constituent of a line or any geometrical objects because partless things cannot be aligned next to one another as we will explore later in this dissertation.³ It would be absurd to consider the Euclidean principle of the infinite divisibility false since a number of mathematical theorems are based on such a principle, including the Pythagorean theorem. Without infinitely divisible space, it would be very difficult for a mathematician to prove the Pythagorean theorem because no irrational numbers would be possible. Thus, a rejection of space's infinite divisibility would result in a repudiation of commonly accepted mathematical theorems.

The affirmation of both of these principles is integral to the preservation of the coherence of human knowledge and the value placed on the disciplines of metaphysics and mathematics. Should one deny one principle in favor of the other principle, then the result is an exclusive field of human knowledge: metaphysical knowledge and mathematical knowledge collide. The affirmation of both principles is thereby necessary for the preservation of the human intellect. Without compatibility of knowledge systems, human knowledge rests in subjectivity rather than objectivity. Knowledge is not about the world which we inhabit, but is only about that which we reason whose truth and terms are determined solely by the theorizer.

² Euclid. *Euclid's Elements*. Dana Denmore and Heath, eds. Ann Arbor, Michigan: Sheridan Books, Inc., 2007. Book One, Definition Two.

³ This point is similar to the annihilation argument, which is against the possibility of unextended minima composing material objects or space. See Euclid's *Elements*, Book One, Definition One.

The apparent irreconcilability of simplicity and infinite divisibility results so long as simple beings can act as limits placed on material bodies which are given as being divisible *ad infinitum*. Such bodies must be infinitely divisible because they exist in infinitely divisible space. An infinitely divisible space is a space whose parts can always be divided into smaller parts: a given space can be divided into halves, fourths, eights, sixteenths, etc...⁴ Yet the complication arises as soon as one also admits that such infinitely divisible material bodies must also consist of simple (indivisible) parts. The supposedly simple beings must forfeit their simplicity by virtue of occupying infinitely divisible space: the indivisible being exists in a space that is divisible beyond the indivisibility of said simple being. The simple and indivisible is now composite and divisible. We are left with the following contradictory metaphysical dilemma: the simple being must not be simple or the infinitely divisible material object cannot be infinitely divisible. But neither alternative is tenable for the status of human knowledge.

We will find later in the historical section of this dissertation that there are two divergent theories of the relation between bodies and space: one set of theories in which matter exists *in* space and *alongside* other bodies, and another theory in which matter (and space) are ultimately phenomenal or perceptions within each simple being (all composition reducible to perception). The former theory, which I will call the matter in space theorem (MIST) is found in the thought of David Hume, George Berkeley, as well as the pre-critical and critical theories of Immanuel Kant. It corresponds to what might also be called a “common sense” view of matter and space:

⁴ Lady Anne Conway, a thinker in the Early Modern period, describes the concept of the infinitely small in connection to God’s infinite powers well: “Since it cannot be denied that God can place one creature inside another, he could therefore place two as easily as one, or four as easily as two, or eight as easily as four, so that he would thereby multiply them endlessly by always placing smaller creatures inside larger ones.” The aforementioned illustration should help the reader grasp the nature of infinite divisibility. Infinite divisibility does not require the thinker to grasp the entire series of possible divisions to be made in a given space; rather, the thinker should grasp only that one *could make* such divisions. See Conway, Anne. *The Principles of the Most Ancient and Modern Philosophy*. Allison P. Coudert and Taylor Corse, eds. Cambridge: Cambridge University Press, 1996. 17.

our perceptions of material objects existing to the left/right of or above/below other bodies that are *in* space in the same way that we would consider food to be *in* a Tupperware container are (mostly) accurate depictions of the reality of matter and space.⁵

The latter theory, which I call space in matter theorem (SIMT), is found in Leibnizian metaphysics. Although we perceive the world according to the common sense view, the deeper metaphysical understanding of the world shows that it is not a really accurate depiction of matter and space, which are only perceptions in each simple being and so called composite things are not really things, but mere aggregates at best. My contention is that if one assumes MIST, the problem of reconciling matter and space is especially apparent. The reader will see how this assumption works out in conjunction with a couple other key assumptions belonging particularly to actual parts theory, and then I will provide an overview of the dissertation and the key assumptions I will make pertinent to the problem of matter's divisibility.

The simultaneous affirmation of metaphysical simplicity and space's infinite divisibility results in the contradiction either that simple beings must possess parts or that infinitely divisible space must possess an ultimate part and thereby an ultimate division. This contradiction results particularly if we assume that a simple being exists *within* a container-like space and *alongside* other simple beings. The simple being must possess simpler parts because it fills every infinitely smaller part of space. The simple being is really non-simple. It does not help to argue that space contains no smallest part; otherwise, space could not be infinitely divisible.

While SIMT sets the stage for the incompatibility between simplicity and infinite divisibility, it is not a sufficient condition for the said incompatibility. Inherent is the reasoning

⁵ The reader should be aware that there are variations of MIST when examining the three philosophers mentioned. I will elaborate these variations further later in the introduction.

that results in the aforementioned metaphysical dilemma is the widely held actual parts theory. Actual parts theory states that the parts of a divisible whole must be already actually existent opposed to being merely potentially existent until they are actually divided. While I will explain this theory more explicitly in the opinion chapter in which I weigh in on the debate between actual parts and potential parts theorists, the actual parts theory coupled with MIST constrains these metaphysicians to the perspective that simple beings can only be constitutive parts of an actual composite whole.

If we are to accept metaphysical simplicity and MIST, then the simple beings must be constitutive actual parts of the actual composite whole. Given MIST, simplicity is either a composite whole or an ultimate part. For a simple being, whether MIST or SIMT is assumed, must be most fundamental—a being that cannot consist or be grounded completely in the existence of other beings of which it is the whole. By this very notion of simplicity, we have excluded the composite whole from being considered the simple being because composition is dependent on its parts which are also regarded to be actual beings given actual parts theory. If we had assumed potential parts theory and wish to preserve metaphysical simplicity in some sense, the opposite conclusion would be reached—the simple being can only function as the actual composite whole because the parts are potential rather than actual. It should thereby be clear that the assumption of actual parts theory coupled with MIST necessitates the conclusion that simple beings must be constitutive parts of the actual composite whole.

I also note that one actual parts proponent studied in this dissertation, Leibniz, maintains SIMT and reaches a similar conclusion as his MIST counterparts that simple beings are in some sense “constitutive” parts of composite material objects. However, there is a key difference

between the simple beings found in Leibnizian thought versus the thought of his counterparts: composite material objects are *metaphysically derived* from their correspondent simple beings. This metaphysical grounding is phenomenal or based on the perceptions inherent within each simple being. The simple beings in Leibniz's rendition of SIMT are therefore not so much constitutive parts of their composite wholes as we would say that the bricks are constitutive parts of a wall, but are the grounding beings for their composite beings that exist as the perceptions of simple beings. My proposed solution falls under a category distinct from the philosophers investigated: my Neo-Aristotelian solution subscribes to potential parts theory and to MIST.

My broader goals in this dissertation are to investigate a series of responses to the problem of matter's divisibility in the Early Modern period of philosophy and then to propose my own solution to the said conflict. I intend to show in the historical analysis that the metaphysical principle of simplicity is closely connected to what I will call the principle of (material) sufficient reason. The principle of (material) sufficient reason is a modification of the principle of sufficient reason as found in Leibnizian thought. Nothing happens without a cause or reason for its existence or as Leibniz states in the *Monadology*:

“And the principle of sufficient reason, on the strength of which we hold that no fact can ever be true or existent, no statement correct, unless there is a sufficient reason why things are as they are and not otherwise—even if in most cases we can't know what the reason is.”⁶

Applied to the problem of material divisibility and a philosopher's belief in metaphysical simplicity, there must be a cause or reason for a particular composite thing existing rather than some other composite thing. The cause or reason is the simple being whose conjunction with

⁶ Leibniz “Monadology”, Paragraph 32.

other simple beings provides a cause for this composite material object rather than another material object.

Having shown that all the philosophers in the historical chapters uphold the connection between metaphysical simplicity and the principle of material sufficient reason, I intend to take the conclusion reached in this historical analysis and incorporate it into my solution to the problem of material divisibility. The particular conclusion allows me to formulate my Neo-Aristotelian solution to the problem of matter's divisibility with the confidence that my solution, although it based on potential parts theory, shares with all the other theories explored in this dissertation the belief that metaphysical simplicity and sufficient reason are intricately connected. This common ground decouples the perspective that the simple being *must be* a constitutive part of a material whole in the sense that simplicity always implies that the simple being is "smaller" than the original whole envisioned. According to this perspective, the simple being is "smaller" than its composite in the same way that we would say the brick is "smaller" than the wall. But the historical analysis reveals that the simple being *need only* be either *the* metaphysical ground for or *one of the* metaphysical grounds for a composite. I intend to exploit this decoupling of simplicity with being the constitutive part of a composite thing by showing that the actual composite whole can also be the sufficient reason for its parts assuming that there is a principle of organization or unity which I borrow from Aristotelian metaphysics, the substantial form.

Having described the trajectory of this dissertation, I now provide a synopsis of the historical section while also clarifying a few key terms. The historical section is divided

according to how each philosopher attempts to either reconcile or divide metaphysics and mathematics through a common theory of the relation between matter and space.

The first major chapter is an analysis the thought of philosophers who espouse MIST. Philosophers in this chapter include David Hume, George Berkeley, and the pre-critical Immanuel Kant. A sub-chapter is devoted to the theory of each philosopher. David Hume and George Berkeley are organized back to back respectively as proponents of the metaphysical principle of simplicity to the exclusion of the mathematical principle of space's infinite divisibility. The pre-critical Kant sub-chapter follows the treatment of Berkeley since the pre-critical Kant can be viewed as a continuation of the perspective that the only way to reconcile simplicity and infinite divisibility is to affirm the seemingly contradictory position that simple beings possess distinguishable parts: a theory of simple beings to which Berkeley subscribes, but also rejects any attempt to bridge his theory of simplicity with space's infinite divisibility. My objective in the sub-chapters on Hume and Berkeley is to determine why Hume supports a theory of simplicity in which the simple beings, which both he and Berkeley call "minima", cannot consist of extended parts, while Berkeley allows for such simple beings to consist of extended parts. The resulting goal of the sub-chapter on the pre-critical Immanuel Kant is to explore why Kant believes that simple beings can consist of distinguishable parts and exist in infinitely divisible space without forfeiting their simplicity.

For terminological purposes, I call the distinguishable parts that exist in Berkelean and pre-critical Kantian minima "intellectually divisible or distinguishable parts". In order to grasp properly intellectual distinction or divisibility, I will first define "divisibility" broadly speaking

and then contrast the three forms of divisibility discussed in this dissertation including intellectual divisibility.

Broadly speaking, “divisibility” refers to something’s capacity to be distinguished into at least two parts. The parts that would result from an actual division would either be non-beings of their own accord, distinct beings of their own accord, or distinct beings of their own accord that are also capable of occupying different parts of space: intellectual, metaphysical, and physical respectively.⁷ The problem of matter’s infinite divisibility concerns metaphysical and intellectual division of matter opposed to physical division. An example of physical division would begin to help the reader understand the contrast between these forms of distinction/divisibility. To divide a sheet of paper physically is to divide or distinguish the top half from the bottom half such that top half and bottom half of the sheet of paper exist separately from one another and *could* occupy different locations in space. This form of division is not the key focus of this dissertation since we are not questioning whether matter consists of atoms or physically indivisible parts. Rather, we are investigating whether matter consists of metaphysically indivisible or simple parts, dubbed minima.

Metaphysical divisibility is a thing’s capacity to be distinguished into at least two separately existing parts without necessarily being physically divisible. For example, atoms exist in various sizes according to Epicurus; however, what exists in various sizes also possesses distinguishable parts. An Epicurean atom, although physically indivisible, is metaphysically divisible by virtue of possessing a left/right and top/bottom sides. Such sides are distinct but

⁷ I have borrowed these concepts somewhat from Holden, Thomas. *The Architecture of Matter: Galileo to Kant*. Oxford: Oxford University Press, 2004. 11-16. I have collapsed formal divisibility to intellectual divisibility for the purposes of this dissertation.

could never be physically separated out naturally. It is also conceivable that the different parts of the atoms could be metaphysically distinct beings since the atom is only said to be uncuttable by natural means. Thus, an Epicurean atom is not identical to the minima or metaphysically simple beings into which I inquire. The metaphysical principle of simplicity states that there are metaphysically indivisible things—the so-called simple beings. Thus, metaphysical simplicity and metaphysical indivisibility are identical.

Intellectual divisibility is a thing's capacity to be distinguished into parts that cannot exist separately. Such divisibility is derived from intellectual distinctions, which include different kinds of relations like spatial orientation, such as "to the right of" and "to the left of", and familial relations, like father-son. In the case of the father-son relation, such a relation itself is not a being on its own since the relation depends on something that is the father which also requires something that is the son. Intellectual divisibility is equivalent to what I refer to as the "distinguishable parts" of minima and is especially evident in Berkelean and pre-critical Kantian minima.⁸

Berkelean and pre-critical Kantian extended minima are metaphysically indivisible units that possess size or extension. Since all extended things are divisible, they are metaphysically indivisible and intellectually divisible. If the parts of extended minima could exist separately, then the problems associated with the infinite divisibility of matter would repeat. By contrast, unextended minima, like those found in Hume's *Treatise of Human Nature* and in Leibnizian metaphysics, are metaphysically indivisible and intellectually indivisible. These two forms of

⁸ We shall find that Leibniz's metaphysics includes two forms of intellectual divisibility. There is the standard quantitative form of intellectual divisibility and then an alternative qualitative form of intellectual divisibility. Quantitative intellectual divisibility makes minima extended, while qualitative intellectual divisibility can be found in certain forms of unextended minima that possess a number of qualitative properties. The latter can be found in unextended minima as is evident in Leibniz's *Monadology*.

minima are therefore differentiated according to whether such minima are intellectually divisible. Now that I have clarified somewhat the forms of divisibility pertinent to the our discussion of matter's divisibility, I return now to my interpretation of each MIST philosopher's account of simplicity starting with David Hume and George Berkeley together, and then culminating with the pre-critical Kant's account of simplicity and its compatibility with infinite divisibility.

Hume's and Berkeley's accounts of simplicity are based on whether color is dependent on extension. The addition of color to minima, particularly in the case of David Hume's unextended minima, is necessary to preserve the existence of minima. Although dealing with different metaphysical issues, Hume's move to preserve his unextended minima through the addition of color is reminiscent of Leibniz's addition of quality to distinguish between his simple beings, monads. Without color in minima, or qualities in Leibnizian monads, simple beings could not exist given the constraints of these philosophers' theories. For Hume, color is independent of extension because color is distinct from and therefore fully separable from extension, while color, although distinct from extension, is inseparable from extension according to George Berkeley. The Hume and Berkeley sub-chapters are devoted particularly to articulating each philosopher's "separability criterion" of ideas that guides their thought process on color and extension for their minima.

The pre-critical Immanuel Kant's "force-shell" theory is Kant's attempt to bridge simplicity and infinite divisibility while subscribing to MIST. By attempting to bridge simplicity and infinite divisibility, the pre-critical Kant must directly struggle with the problem of how a simple being can occupy infinitely divisible space without forfeiting its simplicity. Hume and Berkeley conveniently avoid this problem by denying the space is infinitely divisible and can

more readily account for simple beings in their metaphysics. Kant's minima, the force-shells, are puncta surrounded by a wall of attractive and repulsive forces. It is in this chapter that I explicitly identify the principle of (material) sufficient reason as the real grounding of the metaphysical principle of simplicity. I then connect the pre-critical Kant's statements regarding simplicity and sufficient reason to Hume's and Berkeley's accounts of minima.

At the end of the pre-critical Kant chapter, I detail an objection from Thomas Holden against force-shell theory. This objection is called the "hollow worlds argument". In this objection, Holden argues that force-shells cannot be beings at all because a punctum is not a being. The result of this lack of being or substance in the world is the false conclusion that the world is a mere system of relations. But relations cannot exist without *beings* to be related. Whether or not Holden's objection is sound is irrelevant to my purpose in the pre-critical Kant chapter, but I do highlight the objection as a real *potential* issue for a force-shell theorist to deal with in metaphysical inquiries. The "hollow worlds argument" sets up for the philosopher dealt with in the next chapter, Leibniz.

The Leibniz chapter follows the one on the pre-critical Kant in part as a solution to the "hollow worlds argument" that *potentially* plagues force-shell theory via a reconceptualization of the relation between matter and space: SIMT. I show how Leibnizian SIMT solves not only the problem of substantiality in force-shell theory, but also successfully dodges the problem of accounting for the simplicity of simple beings that exist in infinitely divisible space as is evident in MIST solutions. I will also articulate how this SIMT based solution to the problem of matter's divisibility connects the metaphysical principle of simplicity to the principle of (material) sufficient reason.

While Leibniz's SIMT provides a potential solution to the problem of matter's divisibility, it invites a host of other problems including how the deeper metaphysical reality of monads is connected to the reality of material bodies and space itself. This problem allows this author to segue into the solution that the Critical Immanuel Kant proposes to the problem of matter's divisibility in the second antinomy of the *Critique of Pure Reason*.

Kant deems that all previous attempts to account for matter's divisibility are problematic because such philosophers have concluded a truth regarding the *reality* of the things rather than only pertaining to the *appearances* of things. Reason cannot penetrate truths of things as they are in themselves, but only the truths of how things *appear to us*. It actively structures our experiences according to the forms of intuition of space and time, and the various categories of the understanding, resulting only in the knowledge of phenomena rather than noumena. For Kant, I will show how the metaphysical principle of simplicity and, in turn, its correspondent principle of (material) sufficient reason must be relegated to the noumenal realm. The result is that the problem matter's divisibility cannot be resolved noumenally, but philosophers can affirm that matter is (potentially) infinitely divisible from the phenomenal standpoint inasmuch as the form of our external intuitions, space, is originally intuited as infinitely divisible.

My opinion chapter is therefore set up in order to reaffirm the connection between metaphysical simplicity and the principle of sufficient reason while attempting to breakdown certain key assumptions of Kant's second antinomy. It is not part of my goal to object to Kant's transcendental idealism itself. Thus, I am not claiming that the antinomies are void. Rather, I claim that the reasoning Kant directly provides for his thesis that simple beings exist and the anti-thesis that no simple beings exist relies on an unfair rendition of the respective theories.

Kant does not acknowledge the possibility that metaphysical simplicity and sufficient reason are interconnected.

Kant's second antinomy falls apart if a simple being need not be a constitutive part a composite material object. My Neo-Aristotelian theory of potentially infinitely divisible reaffirms the interconnection between simplicity and sufficient reason, allowing for simple beings to ground composite things not as constitutive parts but as unitary wholes via substantial form. Thus, the second Kantian antinomy works only if actual parts theory is true. But, I will argue that actual parts theory is false and potential parts theory is true. Thus, we cannot remain mired in the Kantian antinomy.

Before delving into the first chapter of the dissertation, I would like to clarify a few terms and concepts, as well as explain the background of my Neo-Aristotelian solution to the problem of matter's divisibility. The clarification of these terms and concepts will help the reader understand more clearly the nature of the problem of matter's divisibility. I will begin with definitions of matter and space and then discuss the Aristotelian concept of place as the background of my theory of space in my Neo-Aristotelian potential parts doctrine

Matter is conceived as being either active or passive. If matter is active, then it typically would be conceived to exhibit a field of attractive and repulsive fields of force. These fields of forces would either emanate from some center punctum/particle as shown in Kantian force-shell theory or some sort of active power as explained in Leibnizian conatus theory. If matter is passive, then it is either purely extension as is true in Cartesian theory or is simply an extended object endowed with impenetrability as is true in Newtonian theory. When investigating the philosophers detailed in this dissertation, I am merely looking at their theories for what is

typically regarded as a “natural composite being” or a being as distinguished from space that could move in space.⁹

In my argument for an Aristotelian theory of material divisibility, I will define matter in the active sense with a modification: material bodies are extended beings that possess the capacity to repel or attract other extended beings by virtue of their possession of a substantial form. The substantial form is from Aristotelian natural philosophy in which matter distinctly considered is passivity but retains its activity by virtue of an organizing principle—the substantial form.¹⁰ This concept of matter will play a significant role in the reformulation of metaphysical simplicity, resulting in the near identity of the metaphysical principle of simplicity and the principle of sufficient reason.

Space is bare extension or dimension in which solid and impenetrable bodies move. The precise nature of space is a central point of dispute between natural philosophers during the Early Modern period, notably between Sir Isaac Newton and Gottfried Leibniz. I will briefly summarize each of the two positions.

Sir Isaac Newton has advocated a theory in which space can exist independently of material bodies. He writes:

⁹ My investigative definition of matter as “the movable in space” comes from Aristotle’s understanding of material bodies in his *Physics*. This investigative definition of matter is apt for most of the philosophers studied in this dissertation well except for David Hume and George Berkeley who do not clearly distinguish matter from space. Although both philosophers consider space and matter as composites of colored minima, it still seems possible to consider the material objects in their theories as the sets of minima which we consider under the abstract idea of “substance” that appear to move in relation to other sets of minima considered under an abstract idea of a different “substance”.

¹⁰ This definition shows that I am more in agreement with Leibniz than Descartes on the nature of matter. For Descartes, matter is an extended substance, while Leibniz argued that force rather than mere extension defines matter. See Joseph, H.W.B. *Lectures on the Philosophy of Leibniz*. Oxford: Clarendon Press, 1949, 24-30; and Strickland, Lloyd. *The Shorter Leibniz Texts: A Collection of New Translations*. New York, NY: Continuum, 2006. 122-130.

“Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is commonly taken for immovable space; such is the dimension of a subterraneous, an aerial, or celestial space, determined by its position in respect of the earth.”¹¹

Absolute space is the space that we come to know through the intellect, while relative space is the space known through sensation. Relative space cannot be properly understood save as a part of that absolute space which is immoveable and permanent. Newton supposed that space must possess such a nature in part in order to make his system of physics operate correctly.¹²

Leibniz disagreed with Newton, arguing that space is no more than the coexistences of material bodies on account of the law of contradiction, his principles of the identity of indiscernibles, and that of sufficient reason.¹³ He specifically dispenses with the absolute character of space, leaving relative space—an *order* of coexistences; or more precisely, Leibniz deems the absolute character of space as a mere *ens rationis* rather than as an *ens reale*. A mathematical necessity does not require there being a real being. Leibniz writes:

“For space indicates. . . an order of things existing at the same time, considered just as existing together, without bringing in any details about what they are like. When we see a number of things together, one becomes aware of this order among them.”¹⁴

The *order* of coexistences can only be acknowledged through reason but does not exist independently of either reason or material objects. Sensation merely reveals to us a particular material object and its coexistent material object, but reason “abstracts” from such experiences to

¹¹ Newton, Isaac. *Scholium to the Definitions in ‘Philosophiae Naturalis Principia Mathematica’*, Bk. 1. (1689); translation by Andrew Motte, 1729, rev. Florian Cajori. Berkeley: University of California Press, 1934. pp. 6-12. 6.

¹² Al-Azm, Sadik. *The Origins of Kant’s Arguments in the Antinomies*. Oxford: Clarendon Press, 1972. 75-79

¹³ Leibniz, Gottfried. *The Leibniz-Clarke Correspondence: Together with Extracts from Newton’s ‘Principia’ and ‘Optics’*. H.D. Alexander, ed. New York: Manchester University Press, 1998.32; 34-36. (hereon abbreviated as H.D. Alexander, page number)

¹⁴ H.D. Alexander, 30.

realize the *order* of coexistences—space. This theory of space becomes an integral component of the treatment of material divisibility in the Leibniz chapter—matter and space are really well-founded phenomenal existences rather than real beings.

I will neither undertake a thorough analysis of these doctrines of space nor declare my support of one or the other since such a study does not genuinely impact the precise debate on matter's divisibility. I now provide an account of Aristotelian place since I borrow extensively from Aristotelian thought to formulate my Neo-Aristotelian solution to the problem of matter's divisibility

Aristotle details his account of place in Book IV of the *Physics*. He defines the place of a *thing* as "...the innermost motionless boundary of what contains it."¹⁵ Two points are immediately worthy to observe based on this definition. Note first that Aristotle grasps "place" not as an abstract or absolute being or property that exists apart from particular material bodies, but as an aspect of *something*. There are no places without things, as there are no material things without places. For one tends to think of an existent something as being somewhere rather than nowhere.¹⁶ The second crucial point in this definition is that place requires the existence of at least two finite material bodies. One of the two finite material objects contains the other material body. The finite material body that is contained is also said to exist in a place. Thus, the outer material body, unless it is also contained by some other material, is not located in a place.

Place or *topos* is dependent on an order of extended bodies. The place of one body in isolation cannot be identified per Aristotle's definition. Per Aristotle's definition also, bodies can

¹⁵ Aristotle, "Physics" in *The Complete Works of Aristotle*. By Jonathan Barnes, ed. Volume One. Princeton, Princeton University Press, 1984. 212a20.

¹⁶ *Physics*, 208a30.

only be finite rather than infinite. For an infinitely extended body cannot contain another finite extended body because it lacks any limitation or boundary whatsoever. In order to understand Aristotle's concept of the place of a thing better, let us grasp his arguments for his definition.

Aristotle's arguments for his definition of the place of a thing can be understood in two phases: the first phase is his attempt to distinguish place from body, while the second phase is to show why place is neither form, nor matter, nor extension. It is by negating the previously mentioned three possibilities that Aristotle formulates his definition of place. The argument in the first phase is based on the observation of the mutual replacement of material objects. Aristotle illustrates mutual replacement: "Where water now is, there in turn, when the water has gone out as from a vessel, air is present; and at another time another body occupies the same place."¹⁷ A body, such as water, cannot be identical to place unless one admits either that the water remains in the same place despite the observation that it moves, which is a contradiction, or the impossibility of two bodies existing in the same place, such as the water and air occupying the same location. The body is what is moveable, while place is immoveable.

Aristotle then questions whether place is something beyond the existence of bodies or whether place can only exist through bodies. Since place is something immoveable and bodies can pass in and out of the same place, it appears that place can exist without bodies. Alternatively, one could also argue that place is dependent on bodies since one cannot properly think of a place without thinking of something occupying that place relative to another material body. The next phase of Aristotle's argumentation establishes the manner by which place is said to exist. The existence of place presupposes the existence of bodies.

¹⁷ *Physics*, 208b5.

Aristotle establishes that place is a limit between bodies by excluding the possibility that form, matter, or extension is identical to place. First, I will explain briefly the nature of form and matter according to Aristotle in order to grasp more fully his arguments that place depends on a material body. Aristotle subscribed to a metaphysical view of hylomorphism, according to which material or natural bodies exist as a union of matter and form. Barring discussion of prime matter, matter is that *out of which* something exists or is made.¹⁸ For example a human being is made out of or composed of flesh and bone. Flesh and bone are the matter of a human being. The form is the structure or organization of the human being. It gives shape to the matter or makes the matter a certain *kind* of thing.¹⁹ In a human being, the rational soul is the form because it animates the flesh and bone of the body, enabling the corporeal human body to move. It makes the human being a human being rather than a dog. Form and matter in natural beings are interdependent. It is impossible for a form to exist without matter. A form, as structure, is the structure of *something*. Alternatively, matter cannot exist without form. For every matter from which something is made possesses its own form. For instance, flesh and bone are made out of further material, carbon and water. Even the matter of a human being possesses its own matter which has been structure according to the forms of “fleshness” and “boneness”. Having described hylomorphism, we can know understand better Aristotle’s argument against form or matter being identical to place.

Place cannot be form because form pertains to the being itself, while place pertains to body.²⁰ Aristotle notes that place might be identical to form since it is difficult to distinguish between the container and what is contained. Imagine a glass vase. The vase exists because a

¹⁸ *Physics*, 194b25.

¹⁹ *Physics*, 194b25.

²⁰ *Physics*, 211b10.

certain form or shape known as “vaseness” has shaped or actualized the matter, the glass. It appears that the form, “vaseness”, exists wherever this glass exists. The place of the glass vase and the form are coincidental. For the vase’s location is simply the entirety of the vase, while the form shapes the entirety of the matter. Given the coincidence of the form and the place, place and form appear to be identical. However, despite the truth that the form and the place are coincident boundaries or limits of this particular material object, Aristotle keenly points out that the form is the boundary of a *thing* and the place is the boundary of a *body*. The form makes a thing this sort of thing, but the place delimits the matter which exists through the form. Therefore, place and form are not identical. Let us see why matter is distinguishable from place.

Matter cannot be place because matter is not separable from a thing, but place is separable from a thing.²¹ Let us first understand why matter and place might be identical. Matter and place might appear to be identical since matter and place are in continuity. This continuity is best expressed when Aristotle provides a comparison between mutual replacement and qualitative change:

“For just as in change of quality there is something which was formerly black and is now white, or formerly soft and now hard—this is why we say that the matter exists—so place, because it presents a similar phenomenon, is thought to exist—*only in the one case we say so because what was air is now water, in the other because where air formerly was there is now water.*”²² (my italics added)

Matter and place appear to be identical by continuity because each underlies its respective objects. Matter underlies qualitative changes, as the matter of wax underlies

²¹ *Physics*, 211b30.

²² *Physics*, 212a1.

the change from the wax being soft to the wax being hard. Place underlies a substance's or thing's change in location. *Where* there is now water, there is now air. However, place and matter are not identical despite this similarity by continuity. While matter and place underlie certain kinds of changes, place is separable from a thing and matter is inseparable from a being.

Recall Aristotle's theory that natural things are hylomorphic—form and matter are interdependent and treated as *one* thing. Wherever there is form there is also matter and vice-versa. Yet we observe that form and matter, one thing, can change place or undergo mutual replacement with another natural body. One natural body moves into the place which the other natural body has previously occupied. The place remains the same, but the matter of the natural bodies has moved with that natural body to another place. Thus, the matter of air has moved to a different place, while the matter of water has moved to the place which the air has previously occupied given the aforementioned example of mutual replacement. The place of a thing is separable from a particular natural body, while the matter is inseparable from the thing. Therefore, place and matter are not identical. Let us now see why extension and place cannot be identical.

Extension cannot be place because then extension would be something that exists independently of the body. Aristotle writes of the supposed identification between place and extension: "The extension between the extremities is thought to be something, because what is contained and separate may often be changed while the container remains the same...--the assumption being that the extension is something over and above the body displaced."²³ The

²³ *Physics*, 211b15.

observation in the case of mutual replacement is that place or a certain extension remains while the body moves to a different place. For example, a body of water measuring two feet in diameter moves entirely to a different place. It appears that a two foot diameter absence of the water, now two feet in diameter of air, currently occupies the same place. That measure or boundary of bodies, the two feet of extension, seems to be identical to the place.

Yet Aristotle argues that treating extension as place makes extension into what it is not, something that exists independently of bodies. If place were identical to extension, extension being something distinct from the body, an infinity of places would result.²⁴ For extension is divisible *ad infinitum* by its nature. Thus, if water were to change place, it would change place as a whole in addition to its parts, being an extended natural body. For something to change place completely, all of its extended parts would have to change place as well. Thus, there would have to be a place of a place of a place without limit. This result strikes the human mind as absurd because we conceive a body to be in one place, as a whole, rather than in an infinity of places. Therefore, place cannot be extension and extension cannot be conceived as underlying all location changes (independent of a body) unlike place. Let us now determine the nature of place according to Aristotle.

Aristotle's arguments against the identification of place with form, matter, and extension show three distinct features of place. First, place is a *container and boundary* of bodies. Aristotle's argument against the identity of place and form shows that place is dependent on bodies. Should no bodies exist, then place cannot exist. Second, place is a *separable* container of bodies. The argument against the identification of matter and place shows that place underlies

²⁴ *Physics*, 211b20.

all locomotion. By underlying all locomotion, it exists independently of *each* body in existence; however, place cannot exist if no bodies exist. Thus, in the case of the mutual replacement of air and water, the place of a body is independent of *this* air and *this* water. Yet place would not exist if nothing, including *this* air and *this* water, exists. Hence, we have an understanding of why Aristotle refers to “place” not simply as “place itself”, but the “place *of a body*”. Finally, place is an accident distinct from the accident of extension or the size of a body. We derive this feature of place from Aristotle’s argument against the identification of extension and place; otherwise, an infinity of places would result. Place is not to be found in the accidental category of quantity, unlike extension. Thus, it would be improper to ascribe an individual number to an extension of a place itself unless one is referring to the extension of a natural body occupying a place. In and of itself, place is not divisible. Yet it attains a certain kind of division through the existence of a natural body’s extension. For space is a boundary or container, and boundaries or containers take on features according to that of which they are boundaries or containers.

We can now summarize why Aristotle thinks that place and space depend on natural bodies. As accidents, or certain characteristics that exist through something else, place and extension or space requires something that exists by itself. Natural bodies are substances on which accidents such as extension and place exist.

I will take this Aristotelian notion that place and space are interdependent as true in my opinion chapter. This theory fits particularly well with a theory of matter and space which incorporates substantial form as part of the account of material simplicity. I will also hold as true two other claims for the purposes of my dissertation argumentation. First, per the dissertation topic question, I will not dispute whether space is infinitely divisible. Second, I will assume that

the distinction between intellectual divisibility, metaphysical divisibility, and physical divisibility are intellectually sound. Having outlined the structure of the dissertation and summarized my argument, let us now begin the historical part, starting with the theories that matter consists of simple beings.

Chapter One:
**The Priority of the Metaphysical Principle of Simplicity in
Hume, Berkeley, and the Pre-Critical Kant**

Chapter One Introduction

This chapter focuses on the proponents of the theory that material bodies exist *in* space and *alongside* other material bodies, and how these proponents either support the reconciliation between metaphysical simplicity and space's infinite divisibility or attempt to prioritize one principle to the exclusion of the other. I call this theory "Matter-in-Space-Theorem" (MIST). There are three philosophers investigated in this chapter. David Hume and George Berkeley argue for the priority of metaphysical simplicity over space's infinite divisibility, while the pre-critical Immanuel Kant attempts to reconcile simplicity and infinite divisibility. My broader argument in this chapter is that these philosopher's notions of metaphysical simplicity in their diverse forms are intricately connected to the metaphysical principle of sufficient reason, particularly evident in the pre-critical Kant's argument for force-shells. My specific goals for each chapter are to account for why Hume and Berkeley differ with respect to their particular notions of simplicity despite their similar epistemic backgrounds and how the pre-critical Kant can allow for a simple being to exist in infinitely divisible space without forfeiting its simplicity.

I shall demonstrate the differences between Hume and Berkeley on the nature of simplicity based on the separability criterion of ideas. The separability criterion of ideas is the condition required in order to claim that at least one idea is distinct and separable from another non-contradictory idea, and that these two ideas can be readily combined to form one new idea or be separated from one another into two completely distinct ideas. Despite their similar arguments, each of which are founded on the views that *esse est percipi*, Berkeley and Hume differ regarding the nature of minima because the former believes that color depends on extension and the latter maintains that color can exist without extension. Thus, the minimum for

Berkeley must be extended since minima must be colored in order to distinguish one minimum from another minimum and color can only exist through extension; a minimum for Hume can exist unextended since color can exist without extension and minima are distinguished from one another according to their distinct colors. Hume thereby maintains a stricter or more extreme form of the separability of ideas than George Berkeley. This stricter form allows Hume to argue that color can exist in something unextended since color and the minimum are two distinct and non-contradictory ideas and two distinct non-contradictory ideas can be combined in any order or fashion—a conclusion which Berkeley cannot reach.

After detailing Hume's and Berkeley's theories of the nature of minima, we will focus on Kant's account of extended minima, dubbed the "force-shell theory". Unlike Hume's and Berkeley's accounts of minima and space, we shall find that Kant distinguishes between the material body and the space or place in which the material body exists. For Hume and Berkeley, since *esse est percipi* and our perceptions of space consist of colored minima or material bodies, there is no true distinction between space and material bodies. For Berkeley and Hume, a material body exists wherever space is said to exist since space is simply a complex impression of colored minima or objects. Every idea possesses content and content represents a material body. However, Immanuel Kant distinguishes sharply between space and material bodies via a distinction between external determination and internal determination respectively.

We shall consult Kant's pre-critical work called the *Physical Monadology* in order to grasp his notion of material divisibility. The *Physical Monadology* will provide us with Kant's arguments for the infinite divisibility of space as a series of external determinations and his arguments for "force-shells" or extended minima that compose material bodies via internal

determination. I will argue that internal determinations are based on the principle of sufficient reason. Now that I have described the three positions on the nature of minima in material divisibility, I will examine a particular problem in the thought of David Hume and George Berkeley: their ideas of infinity. By fleshing out their ideas of infinity, the reader should come to a better understanding of why Berkeley and Hume think that space and material bodies are finite.

In order to understand how one's idea of infinity influences one's position on infinite divisibility, we will examine the ideas of Locke, Berkeley, and Hume. Note that John Locke maintains that the idea of space is infinitely divisible, upholding the principles of Euclidean geometry; however, George Berkeley and David Hume argue that space is only finitely divisible and consists of perceptual minima.

All three Empiricists acknowledge the reality of the positive idea of infinity, that one must perceive immediately an infinity of parts; however, Locke, Berkeley, and Hume also deny that the finite human mind can possess the positive idea of infinity since it cannot complete an infinite process. What requires investigation is why Berkeley and Hume deny the negative ideas of infinity and infinite divisibility and why Locke acknowledges the negative ideas of infinity and infinite divisibility.

Hume and Berkeley must deny the existence of the negative ideas of infinity and infinite divisibility if they deny the existence of negative ideas. It is only by determining why Berkeley and Hume deny the existence of negative ideas that one can understand why they deny the negative ideas of infinity and infinite divisibility. Since John Locke admits the existence of negative ideas, it is crucial to discover why negative ideas are present in Locke's epistemology. Once the reason or set of reasons for negative ideas being present in Locke's epistemology have

been found, it is then necessary to find the absence of these reasons or even their contrary principles in Berkeley's and Hume's epistemologies. It is found that negative ideas are possible in Locke's epistemology because of his reference to external objects or substances as "that which I know not what", a blueprint or pattern for negative ideas lying at the heart of Lockean epistemology; however, Berkeley and Hume deny the existence of external objects or omit reference to them respectively.

This chapter on the Empiricists consists of four sections, while an additional section is devoted to Immanuel Kant's force-shell theory. The first section is an account of Locke's theory of negative ideas and theory of external objects or substances. The second section focuses on George Berkeley's *esse est percipi* principle and its connection to his argument against infinite divisibility and theory of minimum perceptions. The third section is an investigation in Hume's theory of impressions and ideas and how Hume argues against space's infinite divisibility, outlining a theory of colored minimum sensibles. The fourth section is an attempt to discover why Hume and Berkeley reach different conclusions concerning the nature of minima through differing separability criteria of ideas. I will briefly show how this separability criterion of ideas is connected to the principle of material sufficient reason. Finally, in the fifth section, I will analyze Kant's arguments for his "force-shell theory", paying close attention to his distinction between the infinite divisibility of space and the finite divisibility of matter via the problem of the minimum's simplicity. It is in this Kant sub-chapter that I will connect the principle of sufficient reason to metaphysical simplicity explicitly.

Prior to beginning the Empiricist accounts of material bodies consisting of minima, we will turn our attention to the first two "horns" of Pierre Bayle's Trilemma from his entry on Zeno in

the *Historical and Critical Dictionary*. I would like to show why the assertion of the existence of minima is reasonable when contemplating the issues concerning infinite divisibility, but problematic when one attempts to construct space (and matter) from minima, whether these minima are mathematical or physical points. Bayle's goal in writing the Trilemma is not only to show skepticism concerning the reality of extension, but to humble human reason.²⁵ Bayle humbles human reason via a *reductio ad absurdum* of three "horns" or possibilities of the constitution of extension: mathematical points, physical points, and infinite divisibility. Mathematical points and physical points represent unextended minima and extended minima respectively, and will be the focus of this summary of Bayle's Trilemma. Some attention will be paid to Bayle's arguments against infinite divisibility in order to show why the assertion of the existence of minima appears reasonable. In the fourth chapter of this section, I will show how Hume's and Berkeley's accounts of *colored* minima provide a fourth alternative to the three horns of Bayle's Trilemma. The reality of minima, whether unextended or extended, appears to be absurd chiefly because of a conflict between two requirements: being the ultimate constituents of space or bodies (metaphysical indivisibility), and being entities. Let's begin with the problem of infinite divisibility in order to demonstrate why it might seem reasonable to assert the existence of minima and then investigate Bayle's accounts of mathematical points and physical points.

Prior to the analysis of Bayle's Trilemma, an understanding of the connection between Zeno's paradoxes of motion and the Trilemma is necessary since Bayle's Trilemma is framed vis-à-vis Zeno's paradoxes. Zeno's four paradoxes of motion detail two possible constitution of

²⁵ Cummins, Phillip. "Bayle, Hume, and Reid on Extension, Composites and Simples". *History of Philosophy Quarterly*. Vol. 7 No. 3 (July, 1990) pp. 299-314. 302.

space: infinitely divisible or finitely divisible and consisting of mathematical points.²⁶ The dichotomy paradox is an example of the former and the stadium paradox is an example of the latter.²⁷ If one can show that any possible constitution of an entity is incoherent, then the entity in question cannot exist. Thus, if space is either infinitely divisible, composed of physical atoms or mathematical points, and all three of those possibilities are incoherent, then space cannot exist. Motion cannot exist if space cannot exist. Thus, if Bayle thinks that if Zeno's goal is to show that motion does not exist, he can accomplish the same end by proving the non-existence of space. The proof of the non-existence of space is also a means of showing the non-existence of pluralities. Zeno also puts forth arguments to show that a plurality of things does not exist. Without space, pluralities in space are impossible. Therefore, Bayle's Trilemma is connected not only to Zeno's paradoxes of motion, but also to his paradoxes of pluralities.

Infinitely divisible space is found problematic because of how the parts of infinitely divisible space stand in relation to one another. Bayle introduces a variety of arguments against infinitely divisible space in his *Dictionary*, but only the contiguity argument is analyzed in this paper. The contiguity argument shows that objects in infinitely divisible space cannot touch one another because each part of an infinitely divisible object is separate from its other parts to infinity. These two claims are incompatible: space is infinitely divisible and every part of an extended body must immediately touch an adjacent part.

Pierre Bayle argues that three Aristotelian principles that there is no vacuum, and that space is infinitely divisible results in the claim that no part of extension completely touches another

²⁶ Bayle, Pierre. Richard Popkin, ed. *Historical and Critical Dictionary: Selections*. Indianapolis: Hackett Publishing, Inc., 1991. 353-358.

²⁷ For a summary of the stadium paradox, see either Bayle, 357-358 or Russell, Bertrand. "The Problem of Infinity Considered Historically". Salmon, Wesley C., (ed.). *Zeno's Paradoxes*. Indianapolis and Cambridge: Hackett Publishing Company, Inc., 1970. 51-54.

part.²⁸ Bayle argues that it is problematic for an extended object not to have contiguous parts since the existence of an extended object requires the immediate contact of its parts. He states his case by means of the following principle: “Now when a thing cannot have all its existence necessarily requires, it is certain that its existence is impossible.”²⁹ Imagine two supposedly adjacent segments of a pen which is an infinitely divisible extended object. For contiguity to exist, the segments of the pen must touch each other. Yet should space be infinitely divisible, there is always an ever-diminishing part of pen to be found in either segment of the pen. Thus, there is no “ultimate” part of each segment of the pen to touch any other part of the pen. Since there is no vacuum and extension requires contact between parts (for extension consists solely of extension), it seems that the Aristotelian notion of infinitely divisible space is incompatible with the very nature of extension itself.

The contiguity argument shows that the existence of minima appears to be necessary since minima can “fill in the gap” between bodies. Should no smallest part of space exist, then it is impossible for two material objects to touch one another. Thus, in order for at least two bodies to touch, smallest parts of space must exist. Minima are the smallest possible parts of space or material bodies because they are metaphysically indivisible. Thus, minima make it possible for objects in space to touch or be in direct contact with one another. Whether the minima be unextended or extended, a smallest possible part of space exists. Thus, minima solve the problem of the contiguity argument, allowing for a basic observation of experience to occur: that objects can be in direct contact.

²⁸ Bayle, 363.

²⁹ Bayle, 363.

There are two qualities of minima: the first being their indivisibility, as the smallest parts of space, and their requirement of being something rather than nothing—that they enter into the composition of material objects or of diverse spaces. We will find that these two qualities of minima conflict with one another through Bayle’s account of mathematical points and physical points: mathematical points cannot be entities due to their indivisibility, while physical points cannot be indivisible while being entities that can compose space and material objects.

Bayle argues against the possibility of mathematical points composing space because points are non-entities.³⁰ We shall find that mathematical points are non-entities due to their indivisibility. Without elucidating his claim that such points are non-entities, Bayle refers the reader to the Scholastic and Ancient disputes about this matter. He cites one such argument in particular, credited to Aristotle, dubbed the annihilation argument. The annihilation argument is found in Book VI of the *Physics* wherein Aristotle demonstrates that continuums cannot be composed of smallest parts.³¹ This argument is also detailed in David Hume’s *Treatise of Human Nature* before Hume discusses his theory of tangible or colored minimum sensibles.³² It is based on the principles that mathematical points are partless or metaphysically indivisible, and are the sole constituents of space. The indivisibility of such a point is incompatible with this point touching other similar points in order to compose space. The result is the “annihilation of” the indivisible points such that one point overlaps the other minima. The argument runs as follows:

³⁰ Bayle, 360. Bayle references this principle to Antoine Arnauld’s and Pierre Nicole’s *Port Royal Logic*.

³¹ McKeon, Richard. *The Basic Works of Aristotle*. “Physics” New York: The Modern Library, 2001. 231b1-5.

³² Hume, David. T.I.II.IV.4. See also Jacquette, 22-24.

1. If mathematical points comprise space, they must touch one another either by part or by whole.
2. They cannot touch one another by part because these points are partless.
3. They cannot touch one another by whole since they would be entirely “in” one another.

-If they are “in” one another entirely, then it is impossible to distinguish between them.

Therefore, mathematical points cannot comprise space.

The upshot of this annihilation argument is that it is impossible to distinguish between mathematical points due to their complete indivisibility or “partlessness”. If one cannot distinguish between mathematical points, then a multiplicity of points cannot comprise space. Yet if it is impossible to distinguish between mathematical points and nothing lacks distinction, then mathematical points are “nothings” or non-entities. Based on the argument, one can see that mathematical points cannot compose space due to their complete indivisibility or partlessness. Granted, their complete indivisibility is compatible with the fundamental notion of minima being the smallest possible parts of space or material bodies; however, this complete indivisibility makes such mathematical points into nothings. One can easily compare mathematical points to unextended minima: like mathematical points, unextended minima are completely indivisible. In fact, mathematical points and unextended minima are indistinguishable due to their utter indivisibility. Thus, a thinker who subscribes to the theory of unextended minima constituting bodies or space, must deal with the same issue that plagues mathematical points: the incompatibility between being indivisible and being an entity, something that can be distinguished from other similar minima and compose space.

Bayle claims that physical points cannot be the ultimate constituents of space because they are extended and have parts. We shall find that physical points face the opposite incompatibility that mathematical points face: while physical points are entities because they can compose space due to the existence of their parts, they fail to be *minima* because of their divisibility. Here is Bayle's argument against physical points:³³

1. Extended things have right, left, top, and bottom sides.
2. Physical points are extended things.
3. Thus, physical points have right, left, top, and bottom sides.
4. If physical points have distinguishable sides, then physical points or atoms are conceptually divisible.
5. What is conceptually divisible cannot be a true point or simple things since it lacks conceptual indivisibility.

Therefore, physical points are not truly simple constituents of material bodies.³⁴

Physical points circumvent the problem that mathematical points face as given through the annihilation argument; however, by possessing parts, they cannot qualify as minima, being divisible. By possessing parts, physical points can compose space and be entities. Thus, physical points are certainly something rather than nothing; however, they cannot be the simple parts of bodies. Yet by solving the problem that plagues mathematical points, proponents of physical points create a new issue for their theory of minima: physical points cannot be

³³ Bayle, 360.

³⁴ *Ibid.*, 360.

completely indivisible. By their conceptual divisibility, parts smaller than these supposedly ultimate constituents of space and bodies exist. The smallest parts of physical points would be better representative of true minima given that these smaller parts would be conceptually indivisible; however, the same problem regarding mathematical point would result: such smaller, ultimate parts would be non-entities. Now let's compare physical points to theories of extended minima by George Berkeley and Immanuel Kant.

Physical points are similar to extended minima in that they are extended, being intellectually divisible and supposedly metaphysically indivisible. Bayle credits physical points to Epicurus, an Ancient philosopher who once believed that reality ultimately consists of atoms or "uncuttables". Such Epicurean atoms are the ultimate constituents of natural bodies because one cannot separate the purported parts of an atom by physical force into distinct locations.

"Uncuttable" expresses a lower limit to the possible physical division one could make to a physical point.³⁵ Extended minima are not only physically indivisible, but also metaphysically indivisible, particularly in George Berkeley's and Immanuel Kant's theories. Thus, although extension implies division and smaller parts, the smaller parts of extended minima cannot count as beings that could exist separately from the entirety of the extended minimum itself.

Hypothetically speaking, if one were to separate the parts of an extended minimum from the extended minimum, such parts would cease to exist. Yet one can still distinguish according to intellectual observation that extended minima possess parts—extended minima must be intellectually divisible. In Berkeley's theory of extended minima, the combination of metaphysical indivisibility and intellectual divisibility of these minima is due to the

³⁵ Konstan, David. "Atomism and its Heritage: Minimal Parts". *Ancient Philosophy*. Vol. 2, No. 2. Fall 1982. 60-75. 61.

interdependence between color, extension, and the necessity of finite perceptions; in Kant's theory of extended minima, the "force-shells" are metaphysically indivisible and intellectually divisible on account of a distinction between the external relations of space and the internal relations minima in material bodies. We shall explore the two aforementioned points in greater detail in the Berkeley and Kant chapters of this section.

We can infer that the problem of physical points is similar to the problem of extended minima: both physical points and extended minima are intellectually divisible, but they forfeit their simplicity by virtue of their intellectual divisibility. Now that the nature of and problems associated with minima are clarified, let's explore the background of Berkeley's and Hume's theories of infinity via John Locke's account of negative ideas.

Chapter One-Section A: John Locke and Negative Ideas

John Locke, the first of the British Empiricists, maintains that space is infinitely divisible through a negative idea of infinite divisibility. Rather than imagining all possible divisions to a given space, one need only contemplate the possibility of making further divisions. In order to determine why two later British Empiricists, George Berkeley and David Hume, deny the possibility of a negative idea of infinitely divisible space and all negative ideas, it is necessary to study why Locke accepts a negative idea of infinite divisibility. Only by understanding another philosopher who shares a similar epistemological theory yet affirms the existence of negative ideas can one determine why Berkeley and Hume remove negative ideas from their epistemologies. A reference to substance or external objects founds Locke's theory of negative ideas. Substance as "that which I know not what" is the blueprint for all negative ideas, lying at

the core of Lockean epistemology. Locke's reference to external objects enables the formulation of a negative idea of infinity and infinite divisibility³⁶

Before proving the thesis of this section, some clarification of important terms is necessary: infinite divisibility, the positive idea of infinity, negative ideas, and the negative idea of infinity. Bear in mind that only an empiricist understanding of these terms is being implemented. First term, the idea of infinitely divisible space is a given space, although finite in its totality, is divisible without limit. One can divide space *ad infinitum* without arriving at a smallest space. For example, let us assume that a given space measures eight feet long and it is to be divided by halves. The first division splits the space into two halves, each measuring four feet. A four foot space can also be divided into two foot halves. This process can continue without end if space is infinitely divisible. However, the question of whether a finite space can consist of an infinity of parts is under dispute between philosophers like Locke who maintain that space is infinitely divisible as previously described and philosophers like Hume who would find the repeated process of division incomprehensible. This difference in comprehensibility is related to different notions of infinity.

Second term, the positive idea of infinity is the mental picturing or imagining of an infinity of parts. To imagine or mentally picture an infinity of parts is to view in one's mind an unlimited number of individual, visible images. The visible images might be colored points, colored shapes, or some other visual image that is imaginable. They are parts which form an infinite totality of visible images. Thus, something is infinite if and only if it has an infinity of

³⁶ It is interesting to note that because Hume does not acknowledge the existence of external objects, he claims that none of the materialist claims are knowable. See Smith, Kemp. *The Philosophy of David Hume*. London: Macmillan and Co., Ltd, 1941. 321-322.

parts. Locke, Berkeley, and Hume maintain that the finite human mind cannot attain the positive idea of infinity since the mind can only complete a finite process.³⁷ Nothing that is capable of only finite processes can complete something infinite in either addition or division through successive synthesis. Every product of a finite mental process, or the product and possession of an idea, is finite. Thus, process and product are linked together as finite if the mind is finite. If the positive idea of infinity requires the mental picturing of an infinity of parts, which is the product of an infinite process, and an infinite process is impossible if the mind is finite, then the mind cannot attain a positive idea of infinity. Furthermore, if the positive idea of infinity is impossible, then the positive idea of infinite divisibility is also impossible since the positive idea of infinity entails that the idea of infinite divisibility requires the mental picturing of an infinity of parts. Notice that the positive idea of infinity entails that no finite quantity or space be infinitely divisible. If something is infinite if and only if it has an infinity of parts, then something infinitely divisible cannot be finite because infinite divisibility requires that there be an infinity of parts; however, an infinity of parts means that the quantity or space is infinite! Yet the finite is not the infinite! So much for the infinite divisibility of finite space given the positive idea of infinity. The only other possibility for infinite divisibility is through the negative idea of infinity.

Third term, negative ideas signify the absence of positive ideas.³⁸ This form of absence is most like the Aristotelian notion of privation. A privation signifies the absence of a quality or power in something that could possess the quality or power. For example, blindness is the

³⁷ Locke, 152; Hume, 24-25; Smith, 291-293; Flew, Antony. "Infinite Divisibility in Hume's *Treatise*," in D.W. Livingston and J.T. King (eds.) *Hume: A Re-Evaluation*. New York: Fordham, University Press, 1976. 259; Jacquette, Dale. *David Hume's Critique of Infinity*. Boston: Brill, 2000. 6-7.

³⁸ Locke, 330; Jacquette, 68; 72-73.

privation of sight in the eye. It is an eye's nature to see. Thus, blindness is the absence of what the eye is naturally capable. Ignorance is an idea which signifies the absence of knowledge, something positive. When ignorance is present, one does not really possess something, but *could* possess something, namely knowledge.³⁹ Knowledge is the product of some mental process, the possession of a positive idea. Thus, if one has the negative idea of ignorance, there is a mental process or form of thought which is attendant with the negative idea; however, that of which one is ignorant, a specific form of knowledge, is not present in the mind as the perception of mental processes' product is not perceived. The mental process which produces and constitutes the negative idea of ignorance is the thought or realization that one's knowledge about something is lacking but also that that missing knowledge could be attained. Thus, negative ideas do not really lack content *per se* since no distinction would be possible for different negative ideas.

Rather, they possess the content of a mental process which *refers* to the *possibility* of the possession of positive content, the perception of the mental processes' product. They are referential by referring to the content of positive ideas. Every idea, therefore, is constituted by some mental process that is the bare content of an idea. Whether or not the product (positive content) or object of the mental process is present determines a positive idea from a negative idea. If the idea is identical to its object, perception of the mental processes' product is necessary and the idea is positive. Positive content and the perception of the product of a mental

³⁹ The sense of "could" here requires clarification. In some cases of negative ideas like ignorance, the possession of its object, knowledge, is possible despite the finitude of the human mind. "Could" can mean the possibility of attainment: the product of the mental process is perceivable by the mind. In other cases of negative ideas like infinity, "could" cannot mean the possibility of attainment. Instead it would mean that the product would be attained if the mind could implement a greater mental process than is possible with the finitude of the mind. As will be explained, the mind can see that there is no contradiction in continuing the addition of quantities without limit despite the fact that the human mind cannot complete the process. If one wishes to build in the negative idea of actual infinity instead of leaving negative ideas at the level of potential infinity, one would surmise that the negative idea of actual infinity is attainable if the mental process of addition moves beyond mere successive synthesis. Thus, "could" has two different senses pertaining to negative ideas. The difference depends on the nature of the product of the mental process and the sophistication of the mental process itself.

process are equivalent concepts. Knowledge, given through some mental process, requires information about something (its object). However, if the idea is not its object, then it could be a negative idea. Ignorance, also given through some mental process, lacks its object called knowledge. Thus negative ideas are dependent on positive ideas, but the human mind need not possess or have possessed the positive idea in order to have its corresponding negative idea. This understanding of negative ideas is necessary for grasping the negative idea of infinity and infinite divisibility.

Fourth term, the negative idea of infinity is given through the idea of “unboundedness”. “Unboundedness” is viewing a quantity, although finite, in terms of the possibility of adding another quantity to it or dividing the given quantity without limit.⁴⁰ One need not perceive the product of the addition or division, but merely entertain the possibility of having the product of the addition or division. The mental process is possible without having to entertain the reality of the product. Process and product are not necessarily linked together. Unlike the positive idea of infinity, knowledge of the process is not necessarily linked to the perception of the product. One can see from the negative idea of infinity as “unboundedness” that something finite can consist of an infinity of parts—is infinitely divisible. The division of a quantity without limit shows that the finite quantity can have an infinity of parts, being “without limit”, but the parts cannot exceed the whole quantity. The parts cannot be identical in size. Rather, they must be of ever diminishing size. The ever diminishing quantities need not be perceived. The positive idea of infinite divisibility presupposes that the parts of an infinite quantity are the same size. Thus, an infinity of parts that are the same size entail an infinite quantity, but an infinity of quantities of an ever diminishing size add up to a finite quantity.

⁴⁰ Locke, 153-155.

Locke, Berkeley, and Hume agree that a positive idea of infinity or infinite divisibility is unattainable for the human mind. Locke, through negative ideas, can claim a weaker association between the knowledge of a process and the perception of the product; however, Berkeley and Hume, since only positive ideas are possible, must maintain that the knowledge of a process requires the perception of the product. The culmination of these distinctions between negative and positive ideas is that the idea of infinity must be an infinite idea for Berkeley and Hume, while the idea of infinity is not itself an infinite idea for Locke. For Locke, the idea *of* infinity refers to the process which is deemed possible without having the idea or the product itself be infinite. Ideas refer to something other than themselves: there is the idea, which is the mental process, and then there is the object to which the idea refers or represents, the product of the mental process. The product of the mental process need not be perceived or given by the thinking of the mental process. However, for Berkeley and Hume, ideas are identical to what they would signify: what one would call the idea of infinity is itself an idea that is infinite or an infinite idea

Chapter One-Section A.I: Locke on the Nature of Ideas and Infinity

Locke's definition of ideas of sensation incorporates the idea of an external object: an object that is not itself the mind's idea. This inclusion of external objects shows that ideas refer to or represent something other than themselves: the idea of an object and the object itself are not identical. One's ideas refer to the external object by being caused by them. Thus, the person need not perceive all the positive content of something in mind in order to have an idea of the object, making negative ideas possible. The original negative idea in Locke's epistemology is that of external objects or substance as "that which I know not what". Substance is unknown

since all that one perceives are the qualities, none of which are the nature of the substance. From this possibility of negative ideas given external objects, Locke can formulate a negative idea of infinity and infinite divisibility as unboundedness. Locke's affirmation of the existence of external objects in his definition of ideas of sensation makes the idea of infinitely divisible space possible because one could perceive the process of division continuing *ad infinitum* without perceiving an infinity of parts in space. I will investigate Locke's definition of an idea of sensation, showing how the incorporation of external objects into the aforesaid definition makes negative ideas possible through the representational theory of ideas. Through Locke's negative ideas, the negative ideas of infinity and infinitely divisible space are possible.

No formal defense of negative ideas is present in Locke's chief work, *An Essay Concerning Human Understanding*. One can assume that Locke thinks the existence of negative ideas is self-evident to anyone who examines the contents of one's mind, or negative ideas are built in to Locke's epistemological system. The only alternative is that Locke's epistemology makes negative ideas possible, particularly the definition of the idea of sensation.

Locke's definition of the idea of sensation includes the concept of an external object. The idea of sensation refers to or represents external objects. By referring to something that is not in the mind of the thinker, the possibility of a negative idea is clear. Since my mind only perceives the qualities of the external object rather than the external object itself, one possesses a negative idea of the external object by virtue of the fact that the external object is other than what is perceived in the mind. From the theory of negative ideas, the idea of infinity as "unboundedness" arises: one need only see that quantities can be added to an existent quantity *ad infinitum*. An infinity of parts need not be perceived in order to have the idea of infinity.

Similarly, the negative idea of infinitely divisible space entails only that any number of divisions can be made to a given space *ad infinitum*. Not all parts of the infinitely divisible space need to be perceived. Locke's idea of sensation guarantees the possibility of negative ideas, which make ideas of infinity and infinitely divisible space possible. It is now necessary to link Locke's idea of sensation to his theory of negative ideas.

After arguing against innate ideas, John Locke discerns the origin of ideas with which experience furnishes human beings. The idea of sensation is the first idea which Locke describes. It includes all qualities derived from the senses: color, sound, taste, etc...⁴¹ Locke defines the idea of sensation as the idea which "...when I say the senses convey into the mind, I mean, they from external objects convey into the mind what produces there those perceptions."⁴² External objects presumably exist outside or independent of the mind. Ideas are the immediate objects of thought, unlike external objects.⁴³ Being the immediate objects of thought, ideas are dependent on thought.⁴⁴ Yet thinking is always directed toward its object, an idea. External objects are the sources of the ideas of sensation. They cause the existence of the ideas of sensation. Ideas of sensation represent external objects partly because the external objects cause them.⁴⁵ By representing external objects or qualities of external objects, ideas not only refer to external objects, but also take on certain qualities of what they represent: a picture theory of ideas. A picture of a real landscape is like a representative idea because this picture is formed with certain qualities also found in the landscape. This picture of a real landscape is possible only if the landscape either exists or has existed. Like a picture of a real landscape, it is not

⁴¹ Locke, 56.

⁴² *Ibid.*, 56.

⁴³ Chappell, Vere. "Locke's Theory of Ideas". *Cambridge Companion to Locke*. Cambridge: Cambridge University Press, 2006. pp. 26-55. 30-31.

⁴⁴ Chappell, 28-29.

⁴⁵ This is true of our ideas of primary qualities rather than secondary qualities.

simply causation that ideas of sensation refer to external objects, however. Thomas Lennon maintains that causation is neither sufficient nor necessary for references.⁴⁶ Causation is not always sufficient for representation since something can cause an effect without the effect representing the cause. Lennon uses the example of heat causing water to boil.⁴⁷ Boiling water does not represent heat since the heat is in the water. Something represented is not in the thing doing the representing; otherwise, what is doing the representing would simply be what is represented when representation indicates that what is represented is apart from what is representing. Lennon argues that causation is not necessary for representation since a cause can be absent despite something being represented.⁴⁸ One can represent a dead person through another object without having the dead person present in order to be represented. How can an idea represent an external object if not by causation requires elucidation.

Lennon maintains that the mind gives the idea a reference to the essence of the external object. Essence is what a thing is. The mind itself, rather than the idea, does the referring. In this manner, simple ideas of sensation intimate external objects. They intimate the external objects by helping the human mind distinguish things.⁴⁹ Presumably, the idea contains information about the external object to which it refers. Yet how does the idea contain information about the said external object?

Locke uses the idea called white to illustrate the connection between the causation of an idea and how the mind refers to the external object. He states: “Whilst I write this, I have, by the Paper affecting my Eyes, that *Idea* produced in my Mind, which whatever *Object* causes, I call

⁴⁶ Lennon, Thomas M. “Locke on Ideas and Representation”. *Cambridge Companion to Locke’s Essay*. Cambridge: Cambridge University Press, 2007. pp. 231-257. 249-250.

⁴⁷ Lennon, 250.

⁴⁸ *Ibid.*, 250.

⁴⁹ Lennon, 254.

White.”⁵⁰ Something about the external object, the paper in this case, tends to cause a particular idea called white in the mind. Causation is necessary for the mind to refer the idea to the external object. It is not always necessary for representation because the mind can recall the idea without the external object’s presence. It is not sufficient for representation because the mind does the referring. Nevertheless, causation makes representation possible. For the external object must first bring about the simple idea of sensation because ideas are derived from experience. Therefore ideas represent external objects by being caused by the external objects. However, it is the mind that refers the ideas to external object which tends to cause the said idea.

Ideas describe external objects in Locke’s epistemology. Thus, ideas are not identical to the external objects. One describes external objects through the manner by which external objects have produced ideas. Locke’s distinction between the ideas of primary and secondary qualities shows that humans do not fully grasp external objects; instead, humans describe external objects as they appear rather than how they are in themselves.⁵¹ Secondary qualities are “...powers to produce various sensations in us by their primary qualities.”⁵² They do not really exist in external objects. Examples include sensible qualities like colors, sounds, tastes, etc... Since secondary qualities do not really exist in external bodies, ideas of secondary qualities are not really resemblances of external objects. Ideas of sensible qualities do not capture the quality present in an external object. They are like pictures that are garbled and do not accurately depict that of which they are pictures. However, if they are like pictures, then they refer to something else. Although our idea of red does not resemble any redness in the external object, the idea of red shows that the external object tends to produce a red sensation in us.

⁵⁰ Lennon, 253; Locke, 537.

⁵¹ Lennon, 255-257.

⁵² Locke, 84.

Primary qualities really exist in external objects because such qualities cannot be annihilated in thought without annihilation of our idea of body.⁵³ They explain the secondary qualities of which humans have ideas.⁵⁴ Examples of primary qualities include figure, motion and rest, bulk, etc...⁵⁵ Since primary qualities really exist in external objects, ideas of primary qualities resemble external objects.⁵⁶ Ideas, especially of primary qualities, represent external objects in the same way that a lawyer represents a court case. The lawyer presents the court case by conveying the same information found in the court case brief.⁵⁷ Yet the lawyer is not identical to the court case brief itself in the same way that the idea is not identical to the external object. Since an idea is dependent on thought, no idea fully represents an external object. Locke can maintain that external objects exist but cannot know what they are. An external object, as a substance which underlies certain qualities, is “that which I know not what.”⁵⁸ For one can only perceive the substance’s qualities rather than the substance itself, as one can only perceive one’s ideas of external objects rather than the external objects themselves. A negative idea of substance exists at the core of Locke’s theory of ideas.

Now that the possibility of negative ideas has been established, Locke’s negative idea of infinity is understandable. The origin of the idea of infinite space arises from the expansion of the ideas of particular spaces.⁵⁹ Each idea of particular spaces is finite; however, the realization

⁵³ *Ibid.*, 84

⁵⁴ Jacovides, Michael. “Locke’s Distinction Between Primary and Secondary Qualities”. *Cambridge Companion to Lockes’ Essay*. Cambridge: Cambridge University Press, 2007. pp. 101-129. 104-106.

⁵⁵ Locke, 84. See Locke’s argument that certain qualities really exist in bodies.

⁵⁶ Jacovides, 106-108.

⁵⁷ Thomas Lennon distinguishes between two theses of representation: surrogate and presentation. He explains these theses in terms of a lawyer representing a client in a court case and a lawyer representing a case in court. The surrogate thesis leads to the conclusion that ideas hide their objects as a lawyer takes the place of a client and never reveals the client himself. The presentation thesis is that ideas bear the qualities of their objects as a lawyer presents his case in court. For a more detailed discussion of this difference, see Lennon, 232-234.

⁵⁸ Locke, 232-233.

⁵⁹ *Ibid.*, 153.

that one can add any number of particular spaces to a given space constitutes the negative idea of infinite space. Similarly, every idea of a quantity is itself finite. Yet one can add an indefinite number of quantities to a given finite quantity, constituting the negative idea of infinity. As infinity is a boundless quantity, infinite space is a boundless space. Locke maintains that a negative idea of infinity never represents the process of addition as completed, a positive idea of infinity.⁶⁰ Otherwise, the mind has already passed over all the endless additions of quantities. How can a finite mind complete what is endless through successive addition? How is Locke's negative idea of infinity or infinite space applicable to the negative idea of infinite divisibility?

The endless process of addition constitutes the negative idea of infinity. The negative idea of infinity does not require a "picturing" of the result of additions. Instead, it requires that one see that the operation of addition can be carried out indefinitely. If the idea of endless addition constitutes the idea of infinity, endless division is the essence of infinite divisibility. One need not perceive an infinity of parts in something that is infinitely divisible. Instead, it is necessary that the division be carried out with reference to a designated whole. In this respect, infinite divisibility exists by thinking of the process of division: $\frac{1}{2}$, $\frac{1}{4}$, etc.... The sixteenth part of an infinitely divisible object need not be perceived just as the entirety of a quantity, having added the trillionth quantity, need not be pictured as an idea in one's mind. A limit exists pertaining to the mind's capacity to imagine certain finite quantities whether through addition or through division. If these finite quantities, although they cannot be pictured in the mind, are possible, why need one picture an infinity of parts for the idea of infinity or infinite divisibility? One need only see that the process of addition or division can continue *ad infinitum*. The

⁶⁰ *Ibid.*, 155.

negative idea of infinity or infinite divisibility is not an infinite process, unlike a positive idea of infinity or infinite divisibility.

To summarize the argument in this section, Locke's negative ideas allow him to entertain the possibility of negative ideas of infinity and infinite divisibility—infinity as unboundedness in terms of division or addition. Since Locke's definition of an idea of sensation incorporates a reference to an external object, the representational theory of ideas, and the idea is not the external object itself, the negative idea of external objects or substances arises: "that which I know not what." Knowledge does not necessarily require perception of the product of a mental process since one can have negative knowledge of something. Consequently, only the knowledge that a process of addition or division can be thought of to continue *ad infinitum* without having to perceive every particular addition or division of parts is itself a negative idea of infinity or infinite divisibility respectively. The idea of infinity is not an infinite idea for Locke.

Chapter One-Section A Conclusion

Locke concludes that space is infinitely divisible through his negative idea of infinity as "unboundedness". Negative ideas are possible through a reference to external objects in the definition of ideas of sensation. One's ideas represent external objects, but are not themselves the external object. Ideas are distinct from the objects they represent. Thus, Locke's first negative idea is of the ultimate nature of external objects. Given negative ideas, a mental process is not necessarily linked to the perception of its product, content of an idea. With the negative idea of infinity, the perception of an infinity of parts is unnecessary. Only the realization that the process of addition or division can continue without limit is necessary. Pertaining to infinite

divisibility, not all of the parts of this space need to be perceived. One must acknowledge that it is possible to divide a quantity in a ratio so that a series of ever-diminishing parts exists (proportional parts). The possibility of the mental process via negative ideas does not depend on the perception of the product of the mental process. Given the mental process of division by ratios, space need not consist of smallest parts. Therefore, negative ideas make the idea of infinitely divisible space possible.

Paradoxically, a difference in the origin of ideas makes certain ideas possible and other ideas impossible. If ideas represent external objects, then one's (negative) idea of infinity is an idea of a mental process without the perception of its product: an infinity of parts. Ideas are not always identified with their objects. If no external objects are referenced, then one's (positive) idea of infinity is an idea which is itself infinite and consists of an infinity of parts. The idea of a mental process is necessarily linked with the perception of its product: the idea is its object (or product). If ideas are their objects, then knowledge and perception are strongly linked: one perceives the mental process only if its product is perceived. If ideas are not their objects but represent their objects, then knowledge and perception are less strongly linked: one perceives the possibility of a process but need not perceive the product of the process. Infinity is unknowable and contradictory if ideas are their objects, and is knowable and coherent if ideas are not their objects. The reason the findings of this thesis are paradoxical is because ideas are supposed to "take on" certain content; however, the content itself that is usually supposed to make an idea possible or impossible. Yet if there are different natures of ideas themselves, ignoring their content, one would not suspect that such different natures constrain what content is acceptable or unacceptable.

One can tell already that the discussion on infinite divisibility could move from a more mathematical and metaphysical dispute to a more epistemological discussion concerning the nature of ideas. Nevertheless, such a debate might be question-begging between Locke, Berkeley, and Hume because their theory of ideas is the foundation of their respective systems. One would simply have to show that there is knowledge which is necessarily evident in itself and cannot be accounted for under any of the British Empiricists' epistemologies. This endeavor, however, is more appropriate for a different thesis.

Chapter One Section B: Berkeley, Negative Ideas, and Perceptual Minima

Unlike John Locke, the negative idea of infinity is conspicuously absent from Berkeley's epistemology. Although Berkeley never explicitly argues against the existence of negative ideas, one can formulate a possible argument that Berkeley would use through his refutation of the existence of external objects. He refutes the existence of external objects on the ground that, because we only perceive the qualities of the external object without perceiving the external object itself, the external object is unperceived. If ideas must be perceived through the *esse est percipi* principle, one cannot have an idea (perception) of something unperceivable, the external object.⁶¹ Negative ideas are partly unperceived: one never perceives the result of the mental process. Since the result is unperceivable, the mind cannot entertain a negative idea according to Berkeley. Without reference to something other than immediate perceptions, impressions and ideas, it is impossible to form a negative idea: ideas must have positive content. Ideas can only have positive content if only immediate perceptions are possible. Negative content is the absence of perceiving the result of a mental process. Since all that one has in mind is an

⁶¹ *Esse est percipi* is really a move toward the concrete versus the abstract. See Wild, John. *George Berkeley: A Study of His Life and Philosophy*. New York: Russell & Russell, Inc., 1962. 87-89; 125-128.

immediate perception, any further perception or object cannot be discussed. To perceive a mental process without the result would be to perceive something non-immediate. Without the possibility of non-immediate perception, the negative ideas of infinity and infinite divisibility are impossible and the existence of the minimum sensible is necessary.

First, I will show how Berkeley's *esse est percipi* principle necessitates the existence of only positive ideas. Second, I will analyze Berkeley's chief argument against infinite divisibility, the perception argument, and link *esse est percipi* to the perception argument. We shall find that *esse est percipi* functions as a metaphysical and epistemological principle for Berkeley.

Integral to understanding Berkeley's rejection of negative ideas is the distinction between immediate versus mediate perception. According to Berkeley, what one perceives is really something immediate rather than mediate. What is mediately perceived is perceived through something else. What is immediately perceived is not perceived through something else. For example, in his *Dialogues*, George Berkeley discusses the scientific account of sound and claims that what scientists propose as the nature of sound is something we mediately perceive: a sound wave. When one has heard a sound, it is unclear to the percipient that sound waves cause vibrations in the ear drum. What one immediately perceives is not a series of sound waves or the vibration of the ear drum, but what one indescribably experiences as sound. Yet what one mediately perceives is not actually what one perceives since one does not really perceive the sound waves which cause the vibrations in one's ear drum. Thus, mediate "perception" is really a lack of perception: what is mediately perceived is really unperceived, while what is immediately perceived is really perceived.

George Berkeley's metaphysics and epistemology begin with the principle *esse est percipi*. In the *Three Dialogues*, Hylas and Philonus debate whether *to exist* and *to be perceived* are independent principles. Hylas, a proponent of scientific materialism, maintains *esse* and *percipi* are non-identical and independent principles, while Philonus, representing Berkeley, believes that *esse* is dependent on *percipi*.⁶² Hylas and Philonus appeal to common sense to support their respective positions. Common people are inclined to maintain that something can exist without being perceived by something according to Hylas; however, Philonus notes that common sense shows that the reason a person claims that something exists is because it is or has been perceived: "Ask the gardener, why he thinks yonder cherry-tree exists in the garden, and he shall tell you, because he sees and feels it; in a word, because he perceives it by his senses. Ask him why he thinks an orange tree not to be there, and he shall tell you, because he does not perceive it."⁶³ The common person claims that something exists because he has perceived it, while something does not exist because it is not perceived. He would also hold that that what is perceived must be perceivable. Consequently, the existence of something is dependent on whether it can be an object of perception. Berkeley excludes contradictions like square-circles and other inconsistencies from existing because such objects are imperceivable.⁶⁴ How many sides does a square-circle have if it is unthinkable for it to have one and four sides? Thus, if something is imperceivable, it has no *esse*.

If something is unperceived, then it has no *esse*. A common person would agree that something could exist without it being perceived in the present. Hylas would believe that home

⁶² Berkeley, George. *The Works of George Berkeley: In Two Volumes-Volume I*. London: Adamant Media Corporation, 2005. 204.

⁶³ Berkeley, 204.

⁶⁴ *Ibid.*, 203-204.

still exists while being at work. Berkeley's point, however, does not seem to be that since *I* do not perceive an orange tree, it does not exist. Rather, the common person finds it necessary to posit the existence of something if it is perceived since the basis of believing that something exists is founded on it being perceived. To preserve this common opinion, Berkeley posits the existence of God who perceives all ideas that different souls could perceive.⁶⁵ Although I do not presently perceive the orange tree, it must be that case that God perceives it. Granted, if there is a contrary situation, such as perceiving an apple tree instead of an orange tree, one could safely say that the orange tree does not exist. For it is impossible that an orange tree be an apple tree when an apple tree is present at a certain location instead of an orange tree. This is an instance of showing that what has *esse* must be perceivable—being consistent or non-contradictory since it is impossible for an apple tree to be an orange tree—and must be perceived as present at a certain location in order to have *esse*—that the orange tree's absence excludes the apple tree from existing and the apple tree's presence means that it exists. *Non-percipi* does not guarantee that something has no *esse* although something must be perceived by God in order to have *esse*. Actual *percipi* guarantees the existence of something's *esse* and something must be able to be perceived in order to have *esse*. It is consistent to maintain that perception and *esse* are inextricably related, while maintaining the common sense idea that something can continue to exist without it being perceived.

It is important to realize that *esse est percipi* does not rule out the existence of the real world. Rather, the real world is a conglomeration of ideas in the mind of God per Berkeley's idealism.⁶⁶ The dispute centers on what counts as real. Idealism is the view that only ideas or sensations

⁶⁵ *Ibid.*, 205.

⁶⁶ Kingston, F.T. *The Metaphysics of George Berkeley*. Lewiston: The Edwin Melen Press, 1992. 32; 50.

exist. Idealism is in counter-distinction to materialism, the view that reality is matter or corporeal. Reality, as described later, is ideal rather than material for Berkeley. Hylas later objects to Philonus' claim that *esse est percipi* on the ground that the beauty and reality of the world is lost. Philonus responds that if *esse est percipi* is true, the world still exists and appears in the same way as if it were material.⁶⁷ *Esse est percipi* preserves the order and beauty of the natural world: the rivers flow as they would if they were matter, and the valleys are beautiful as they would be if they were material. Berkeley does not abolish the real world, rather he clarifies what counts as real per *esse est percipi*.

Based on *esse est percipi*, Berkeley's reason for rejecting the claim that humans have a positive idea of infinity is clear. Since one claims that something exists because it is perceived and that something does not exist because it is not perceived, if one perceives something contrary to a purported fact or observation, such a thing does not exist. Since the human mind is finite, it can only grasp ideas which are finite: "...it being of the nature of infinite not to be comprehended by that which is finite."⁶⁸ If the positive idea of infinity entails the conceptualization of an infinite quantity of ideas or parts, the finite mind neither does nor can perceive the infinity of parts. Every idea of a quantity the mind forms is finite because it cannot complete a successive infinite process of addition. One perceives only the contrary of the infinite, namely the finite—a finite quantity. Since one only perceives finite ideas of quantity, and the finite mind cannot complete an infinite process, the human mind cannot attain the positive idea of infinity. What is not perceived has no *esse*: the positive idea of infinity. If an object is not perceived and something else to the contrary is perceived, the object has no *esse*.

⁶⁷ Berkeley, 184.

⁶⁸ Berkeley, 73.

One must perceive a smallest part of space if *esse est percipi* is true: infinitely divisible space has no *esse*. If the positive idea of infinity entails the perception of an infinity of parts, then the positive idea of infinite divisibility also guarantees the perception of an infinity of infinitely small parts. However, how could something have an infinity of infinitely small parts and remain finite? If something consists of an infinity of parts, then it is infinite by definition according to Berkeley. If one admits that the human mind is finite, then it is also true that the human's power of perception is finite. Whatever is of finite power can only effect or receive some product or object that is also finite. Thus, it is impossible for the finite human mind to divide a given finite space *ad infinitum*. The finite human mind must reach an end to its perceptions of space, reaching what is perceivable as the smallest part of space: the minimum perception. Alternately, Berkeley argues from a conceptual incompatibility between the positive idea of infinity and finitude, assuming the negative idea of infinity is impossible. A finite space cannot contain an infinity of parts; moreover, if an infinity of parts cannot constitute finite space or quantity, it must be true that a smallest part is perceivable. It also follows that the perception of a smallest part precludes infinite divisibility since there cannot be a smallest part if something is infinitely divisible.⁶⁹ Thus, what one actually perceives is contrary to space being infinitely divisible since space cannot be both infinitely divisible and finitely divisible.

To show why Berkeley excludes negative ideas, *esse est percipi* must be linked to the distinction between what is immediately versus what is mediately perceived and the denial of material or external objects. If what is immediately perceived alone ultimately has *esse* and what has *esse* must be perceived, an idea cannot have negative content because negative ideas have or

⁶⁹ This reasoning, dubbed the perception argument, is detailed later in this paper.

indicate something which cannot be perceived or pictured immediately—the result of a mental process.

Key to understanding Berkeley's rejection of negative ideas is to grasp the distinction between what is immediately versus mediately perceived. The distinction between immediate and mediate perception arises when discussing sensible qualities and their relation to material substances.⁷⁰ For Berkeley, what is immediately perceived is an idea or a sensation. A sensation, like blue or round, is ultimately an idea or perception in the mind. Ideas cannot exist outside the mind. One need only inspect the contents of one's mind to conclude that what is immediately perceived is an idea. Unlike Locke, ideas are sensible things instead of representations of sensible things.⁷¹ Hylas argues that there is something other than what is immediately perceived, namely a material object which is mediately perceived.⁷² Material objects, unlike ideas, can exist without the mind. Ideas are like pictures or representations of material objects in the same way that a statue of Julius Caesar bears certain qualities that belong to the real Julius Caesar according to Hylas.⁷³ However, Berkeley notes that if one perceives the statue or picture of Julius Casesar, what one perceives is really a configuration of colors and shapes according to some symmetry.⁷⁴ One does not perceive the real Julius Caesar through the picture or statue of Julius Caesar. The one who thinks of the real Julius Caesar through the statue of Julius Caesar does not think of the real Julius Caesar via the ideas or sensations constituting the statue; otherwise, the person who does not know the real Julius Caesar would recognize the picture or statue as being of Julius Caesar. One associates the statue or picture of

⁷⁰ Berkeley, 178-179.

⁷¹ Bolton, Martha Brandt. "Berkeley's Objection to Abstract ideas and Unconceived Objects". Doney Willis, ed. *Berkeley on Abstraction and Abstract Ideas*. New York and London: Garland Publishing, Inc., 1989. 99-120.

⁷² *Ibid.*, 178.

⁷³ Berkeley, 178.

⁷⁴ *Ibid.*, 179.

Julius Caesar with the real Julius Caesar via memory. Therefore, what one immediately perceives is no more than what is given as an idea or sensation. One cannot claim that something other than what is immediately perceived is associated with or exists in addition to what is immediately perceived simply based on what is given in immediate perception. Thus far, to be is to be immediately perceived.

Now we must settle how *esse* should be conceived in relation to the being of ideas. Konrad Wogan distinguishes between *esse* as *essentia* and *esse* as *existentia*, maintaining that the latter is the proper understanding of *esse* despite the fact that Berkeley waffles between the two understandings of *esse*.⁷⁵ If ideas must be perceived in order to exist, then *esse* as *essentia* means that being perceived is essential to ideas. Ideas, being what they are, cannot be unperceived in the same way that a horse cannot be a non-animal. *Esse* as *existentia* means that being perceived is a property of ideas. Wogan refers to a property of ideas as something that might not belong to the said idea: the idea can be without that property belonging to it.⁷⁶ An idea need not be perceived in order to be an idea. Wogan maintains that “*esse* interpreted as essential” is an essential and analytic proposition, while “*esse* interpreted as existential” is a non-essential and non-analytic proposition.⁷⁷ Since *esse est percipi* is a demonstrable principle, it must be true that *esse* is understood as *existentia* rather than as *essentia* according to Wogan.⁷⁸ Wogan formulates a demonstration of *esse est percipi*, showing that *esse* should be understood as *existentia*:⁷⁹

⁷⁵ Wogan, Konrad Marc. “Berkeley’s Sensationalism and the *Esse est Percipi* Principle”. Armstrong, D.M. and Martin, C.B. eds. *Berkeley: A Collection of Critical Essays*. New York: Garland Publishing, Inc. 1998. 321-323.

⁷⁶ Wogan, 321.

⁷⁷ *Ibid.*, 321-322.

⁷⁸ *Ibid.*, 322.

⁷⁹ *Ibid.*, 332.

1. An idea, X, exists unperceived.
2. I perceive that X exists unperceived.
3. Therefore, X is perceived and unperceived.

If one attempts to perceive an unperceived idea, the contradiction that the idea is perceived and unperceived results. The contradiction is avoidable if one affirms the tautology that it is possible to perceive only perceivable ideas or that only ideas existing as perceived can be perceived. The percipient adds being perceived to the idea; however, it is true that the percipient can only perceive perceivable ideas. Consequently, there may be unperceived ideas but such ideas cannot be perceived without contradiction. To be, *esse*, as being perceived must be a non-essential and non-analytic property of an idea: *esse* as *existentia*.

However, *esse* seems to be more like the idea's *essentia* rather than its *existentia*. To show that *esse* is the idea's *essentia*, a return to Berkeley's introduction of the principle *esse est percipi* from his *Dialogues* is necessary. Philonus states:

“Ask the gardener, why he thinks yonder cherry-tree exists in the garden, and he shall tell you, because he sees and feels it; in a word, because he perceives it by his senses. Ask him why he thinks an orange tree not to be there, and he shall tell you, because he does not perceive it.”⁸⁰

It is true to say that there is an “I” which perceives an idea as Konrad sets up his argument; however, if *esse* were *existentia*, the gardener would not believe that something exists because he perceives it. Although the gardener does not perceive an idea, the fact that something must perceive an idea in order for the idea to be (whether it is only God who perceives the idea or someone else) shows that *esse* and perception are inextricably linked. In order for the idea to be, it must be perceived. If Berkeley holds that all reality is a set of ideas (save notions) or that

⁸⁰ Berkeley, 204.

objects are the ideas, and without being perceived, ideas cannot exist, *esse* must be essential to ideas. Without being perceived, no idea exists. Being perceived cannot be added to an existent idea, *contra esse as existentia*; otherwise, there could be some idea which no percipient is perceiving. This claim runs contrary to Berkeley positing God's existence to account for the belief that ideas or things continue to be despite the fact that "I" do not perceive them.

Therefore, *esse* must be interpreted as *essentia*.

Berkeley does allow for things to be mediately perceived by the mind, linking immediate perceptions of different senses. His example is that of a sound coming from a coach.⁸¹ It is only by experiencing a certain sound and also perceiving the coach that one could infer the existence of the coach from the specific sound. Experience links two immediate perceptions. However, if the coach were something other than a conglomeration of ideas or sensations, no experience of the coach would be possible. Thus, although the coach is not immediately perceived upon hearing the sound, the coach is still a separate immediate perception. However, Berkeley admits that it is only sound that is truly heard rather than the coach. Mediately perceived things exist save only by the association of two or more immediate perceptions. Rational inferences between perceptions are possible, except that one must have perceived immediately either the coach or the sound in order to infer that the sound is from the coach.⁸² Mediate perception is founded on immediate perception. What is immediately perceived, therefore, is the foundation of what exists for George Berkeley: that which is neither an immediate perception nor founded upon an immediate perception has no *esse*.

⁸¹ Berkeley, 179.

⁸² Kingston, 39-40.

Now that the distinction between immediate and mediate perception is clear, the reasons Berkeley rejects material or external objects are understandable. If *esse est percipi* is true, and only ideas or sensations are immediately perceived, and material objects are imperceptible by themselves, then no material or external objects exist. This argument which Berkeley employs is called the master argument, which is found in the *Dialogues between Hylas and Philonus* and *A Treatise Concerning the Principles of Human Knowledge*. This chapter contains a synopsis of the master argument from the two aforementioned texts: it is that it is logically impossible to perceive what is imperceptible in itself.⁸³ Analysis of the master argument reveals that Berkeley collapses conceiving and possibility into perceiving, showing that what is immediately perceived cannot be a negative idea.

Berkeley's rejection of external objects or substances stems from Locke's account of substances and distinction between primary and secondary qualities. Locke maintains that a substance is "that which I know not what". He formulates this definition based on his distinction between primary and secondary qualities coupled with the claim that the primary and secondary qualities, while inhering in the substance, are not the substance. What one perceives of material objects is their qualities: extension, motion, color, etc... Substance is unperceived save as a substratum for its qualities. Berkeley, through *esse est percipi*, shows that it is impossible to form an idea of substances or external objects because they are unperceived.

If substances are unperceived, then they cannot exist if nothing unperceivable exists. If all that exists is something immediately perceived, an idea in the mind, and substances are unperceived by definition, it is incomprehensible to perceive a substance. If one could perceive

⁸³ Bolton, 112; Fogelin, Robert. "Hume and Berkeley on the Proofs of Infinite Divisibility". *Philosophical Review*. 97, 47-69. Jan. 1998. 55; 65-67.

a substance, then there would be something being perceived that is actually unperceived.⁸⁴

However, it is contradictory to be perceived and unperceived. Therefore, since it is impossible for a substance or external object to be perceived, it cannot exist. Moreover, the substance cannot be an idea of any sort because only ideas exist and are immediately perceived and substance is not an idea. Consequently, it is impossible to formulate a positive or negative idea of a substance. Therefore, Berkeley has no recourse to negative ideas through the presupposed existence of external objects.

If some aspect of a negative idea must be unperceived in order to be a negative idea, Berkeley must reject the existence of negative ideas due to *esse est percipi*. Negative ideas include negative content or content that one cannot perceive. Negative content is the absence of the result of some process of thinking. As is true of the negative idea of infinity, one can recognize the possibility of adding an indefinite number of parts to a given quantity without perceiving the infinity of parts as a result of one's addition. Silence, a negative idea, indicates the absence of perceiving a sound which could exist. Since every negative idea signifies the absence of some positive content, and one cannot really perceive the absence for absences are of things that are unperceived, negative ideas cannot exist if *esse est percipi*. Unlike Locke, Berkeley must reject negative ideas altogether, precluding the possibility of the negative ideas of infinity and infinite divisibility.

Perception is of one's immediate experience. If something is not immediately perceived or cannot be immediately perceived by anything, it cannot exist. Immediate perception, as has been shown, precludes the existence of negative content. Negative content cannot be perceived

⁸⁴ Berkeley (Principles), 95;

in any sense because it is the absence of positive content. Perception is always of something positive. Thus, if something exists only if it is perceived, then immediate perception is only of positive content. Something perceived is always *positive*. Being positive, Konrad describes perception as "...an apprehension which neither contains nor presupposes any inference or transition from one mental act to another."⁸⁵ If perception is always positive, there is no lack in what one perceives—content is always present. If I perceive a square, all four sides must be evident in the perception. One cannot merely take three lines without the fourth yet entertain that there is some hidden fourth line that would form a square. One cannot claim from this example that what one perceives is a negative idea of a square because what one is not immediately aware of, or especially what *no one* is immediately aware of, exists. Therefore, if what is immediately perceived in experience must have positive content, and *esse est percipi* necessitates there being only immediate perception, then *esse est percipi* precludes the possibility of negative ideas.

The aforementioned account of *esse est percipi*, immediate perception, and rejection of negative ideas indicates that Berkeley collapses conceiving into perceiving, and perceiving into possibility. One can study the meaning of conceivability via the meaning of inconceivability. Inconceivability is a failure of thought due to some incompatibility of the concepts involved or the incapacity of a faculty of thought to grasp a given object.⁸⁶ Two incompatible concepts are not only inconceivable, but impossible or cannot be. An example of incompatible concepts are square and circle. A square-circle could never exist because one figure has four sides and the other figure has only one side: a square-circle would have four sides and only one side

⁸⁵ Wogan, 323.

⁸⁶ Doney, Willis. "Berkeley's Argument Against Abstract Ideas". Creery, Walter E., ed. *George Berkeley: Critical Assessments*. Vol. II. London and New York: Routledge, 1991. 138.

simultaneously, which is contradictory. An object may be inconceivable not because of a contradiction, but because of the failure of the faculty of thought to entertain a given concept. An example of such a concept is the positive idea of infinity. Unless one reasons otherwise, the finite human mind cannot complete an infinite process and conceive an infinity of parts. However, an infinite mind could grasp an infinity of parts. The positive idea of infinity is inconceivable according to the finite human mind, but is conceivable according to the infinite mind of God. This form of inconceivability is not impossibility because something is impossible if it cannot exist, as two incompatible concepts are impossible because what they signify cannot be. Thus, conceivability presupposes that the given object of thought is possible (it can exist because there is no incompatibility of concepts involved), and that the faculty of thought can entertain the specified object. Possibility and conceivability are not identical; however, conceivability presupposes possibility.

Perception is observation or immediate experience.⁸⁷ To say that something is not perceived is to say “I do not observe X”. On this understanding of perception, to argue that something does not exist because it is unobservable is to say that “If I do not observe X, it does not exist”.⁸⁸ Thus, observability and impossibility are inextricably linked. While Berkeley’s method of argumentation against negative ideas and external objects seems similar to the subjective observability criterion of perception previously mentioned, Berkeley’s theory of perception and existence is not really from the standpoint of the subject, but from the object. If Berkeley’s theory of perception and existence is really from the standpoint of the object instead of the subject, then conceivability is identical to perception, and perception is identical to

⁸⁷ Doney, 139.

⁸⁸ This argument is from Willis Doney’s rendition of Berkeley’s purported argument from unobservability against abstract ideas. See Doney, 137-138.

possibility. Notice in the argument against substances or material objects that Berkeley does not simply say that “*I* cannot perceive what is meant by substance”, but that it is something which *no* percipient can perceive by definition. Substances are imperceivable by definition, not by the incapacity of an individual’s faculty of perception to perceive the substance.⁸⁹ Since substances are imperceivable by nature, not even the infinite mind of God could perceive substances. If *esse est percipi* is true, then if something is imperceivable by definition, it cannot exist. Perception and possibility are identical for Berkeley.

Conceivability and perception are also identical for Berkeley because conceivability does not imply anything beyond what one immediately perceives. John Locke would argue that negative ideas are conceivable even though one could never directly perceive or be immediately aware of what content is absent from the negative idea—that is to say, the positive content which the negative idea signifies is absent. Conceivability is not limited to perception according to Locke. However, if Berkeley rejects negative ideas since they are partly imperceivable, what is conceivable can only be what one perceives or is immediately aware of: something must have positive content if it is conceivable. Ideas must be fully accessible to the mind that possesses them.⁹⁰ Thus, Berkeley preserves conceivability’s presupposition of possibility and the claim that some objects might not be perceivable for certain beings’ faculties, but limits the scope of what things can fall under the faculty of conceivability: conceiving is ultimately perception or direct awareness. Thus, Berkeley identifies conceiving with perceiving, and perceiving with possibility, ultimately limiting the scope of beings to perceivable beings.

⁸⁹ George Pitcher renders Berkeley’s argument against abstract ideas in a similar manner: if an object or state of affairs is impossible, then an idea of an object or state of affairs is also impossible. Granted, Berkeley argues that objects are ideas. So, the antecedent is really identical to the consequent. See Doney, 142-143, and Pitcher, George. *Berkeley*. London: Routledge and Kegan Paul, 1977.

⁹⁰ Bolton, 107.

Before beginning the comparison of Berkeley's rejection of negative ideas to Hume's, the question of whether Berkeley is an imagist arises from his *esse est percipi* principle. The answer to this question depends on what one means by "imagist". If an imagist is one who believes that ideas are only particular, imaginable, and sensible qualities, then Berkeley is an imagist. However, if an imagist is one who holds that ideas cannot function as signs for something else, Berkeley is not an imagist. A mid-way position between these two theories is reachable, calling Berkeley an imagist on account of the claim that ideas can be signs for other ideas. This synthesis is similar to the claim that mediate perception (ideas as signs) is based on immediate perception (ideas as immediately perceived). Therefore, an imagist believes that all ideas, whether they are signs or not, are ultimately either things which are immediately perceived (are the ideas themselves) or refer to things that are immediately perceived. An imagist would simply agree with Berkeley's account of mediate perception.

To arrive at this conclusion, two contrasting positions on whether Berkeley is an imagist are provided: Martha Brandt Bolton, who believes that Berkeley is an imagist, and E.J. Furlong, who holds that Berkeley is not an imagist. Martha Brandt Bolton argues that Berkeley is an imagist because he tends to speak of ideas in terms of what is sensed in experience such as colors and sound to refute the claim that what is imperceptible can exist.⁹¹ If ideas are immediately perceived and only what is immediately perceived exists, Berkeley cannot help but be an imagist since an imagist believes that ideas are particular, imaginable sense qualities on Bolton's understanding. However, E.J. Furlong believes that Berkeley is not an imagist because an

⁹¹ Bolton, 105.

imagist would not allow for thinking *with* the aid of images (ideas as particular sense qualities).⁹² Rather, an imagist would only allow for thinking the images themselves. If one thinks *with* the aid of images, one uses the image (whether an idea or sensible quality) to think of things that are not found in the image itself. In Berkeley's theory of mathematics, Furlong writes that Berkeley might take a particular triangle, an image, to stand for all triangles in order to demonstrate a truth about all triangles.⁹³ The particular triangle, which is an image, points to something other than itself. The positions of Furlong and Bolton are not mutually exclusive, however.

If one notices that Berkeley distinguishes between what is mediately versus immediately perceived, and that Berkeley's images stand for things that are ultimately of particular sensible qualities, Berkeley can still be an imagist and maintain that an idea can stand for something other than itself provided that this something else is also an idea contra Furlong and Bolton. What is immediately perceived is an idea for Berkeley. An idea is a particular, sensible quality immediately evident to the percipient. Mediate perception occurs whenever the percipient is led to think of another idea based on perceiving the association of a given idea with another idea: as one associates a particular sound with the wheels of a coach. According to Bolton and Furlong, an imagist would only believe that immediate perception is possible. Mediate perception is impossible for Berkeley according to Bolton, and the fact that Berkeley admits the use of ideas as signs (mediate perception) demonstrates that Berkeley is no imagist according to Furlong. If Bolton and Furlong follow through with their claims, Furlong would convince Bolton that Berkeley is not an imagist as is evident in Berkeley's philosophy of mathematics; however, Bolton is right that Berkeley is an imagist when it comes to ideas that do not function as signs.

⁹² Furlong, E.J. "Symposium: Abstract Ideas and Images". *Berkeley on Abstraction and Abstract Ideas*. Willis Doney (ed.). New York and London: Garland Publishing, Inc., 1989. 169-184. 179-180.

⁹³ Furlong, 182-183.

Despite their differences, Bolton and Furlong must acknowledge that Berkeley's incorporation and understanding of mediate perception in his metaphysics and epistemology can show that Berkeley is an imagist if an imagist holds that all ideas are or are based on particular sensible qualities. Berkeley's theory of mediate perception is founded on immediate perception: things mediately perceived must have once been immediately perceived. Furlong's example of the idea of a particular triangle standing for all triangles incorporates Berkeley's theory of mediate perception. The particular triangle, which is the sign, refers to other triangles (ideas) which could be found or based on immediate experience. Only on the basis of having immediately perceived other triangles could one begin to perceive the possibility of a particular triangle standing for other triangles. Ideas as resemblances of other ideas are founded on immediate perception: they begin as something immediately perceived and end with things that were or could be immediately perceived (the other resembling ideas). Bolton provides the groundwork for an imagist, immediate perception, while Furlong unwittingly adds a further dimension to imagist thinking in accord with Berkeley, mediate perception. Therefore, Berkeley is an imagist, but not strictly in Bolton's or Furlong's sense; however, Bolton's and Furlong's ideas and observations concerning whether Berkeley is an imagist are incorporated into this new synthetic understanding of Berkeley as an imagist.

Berkeley's minimum perception argument is the result of affirming the finitude of the human mind and denying the positive idea of infinity: the human mind, being finite, must reach a minimum in its perceptions. Berkeley begins his argument with the following claim: since finite space is an idea in the human mind, and *esse est percipi*, each part of the finite space must be

perceivable.⁹⁴ Otherwise, there would be some part that is not perceived and would not exist.

Jose A. Robles writes that Berkeley maintains two important propositions through the minimum perception argument: "...the finiteness of the finite and the perceptibility of the existent."⁹⁵

Consequently, Berkeley's argument runs as follows:

1. If innumerable parts cannot be perceived in finite space, then finite space cannot be infinitely divisible.

2. Innumerable parts cannot be perceived in finite space.

Therefore, finite space cannot be infinitely divisible.

Infinitely divisible space must contain an infinity of parts whether the positive or negative ideas of infinity are granted. The positive idea of infinity, that one pictures an infinity of parts of an infinite quantity, clearly contains an infinity of parts that may or may not be identical in size per definition. If one applies the positive idea of infinity to infinite divisibility, it is clear that an infinity of parts must be present in the perception of infinitely divisible space. Thus, if an infinity of parts is imperceivable, then one cannot claim that the space is infinitely divisible, which is precisely what Berkeley claims in the minimum perception argument. The negative idea of infinity does not require the picturing of an infinity of parts. One need only perceive that a mental process of addition or division can continue indefinitely without perceiving all results of the mental process of addition or division. If one applies the negative idea of infinity to infinite divisibility, there would be an infinity of ever diminishing quantities in the given quantity; however, the perception of all the ever diminishing quantities and especially

⁹⁴ Berkeley, 135.

⁹⁵ Robles, Jose A. "Berkeley: Scepticism, Matter, and Infinite Divisibility." Popkin, Richard H., (Ed.). *Scepticism in the History of Philosophy*. Boston: Kluwer Academic Publishers, 1996. 87-97. 95.

the imperceptibly minute quantities of a given space, is unnecessary. One need only perceive that the process of dividing a quantity according to a certain ratio is necessary to entertain a negative idea of infinite divisibility. Since Berkeley rejects negative ideas, he must reject the negative idea of infinite divisibility. Since only the positive idea of infinite divisibility or the negative idea of infinite divisibility is possible, and there are no negative ideas, Berkeley entertains only the positive idea of infinite divisibility. Since *esse est percipi* is true and only positive ideas are possible for Berkeley, all parts of space must be perceptible.⁹⁶ If the positive idea of infinite divisibility is entertained, then some parts of infinitely divisible space would be imperceptible because the mind is finite. However, what is imperceptible does not exist. Therefore, since there aren't innumerable parts perceptible in a given finite space, space cannot be infinitely divisible. There must be some sort of minimum perception belonging to the idea of space, the minimum sensible. Thus, if one shows that there are perceptual minima or minimum sensibles, infinitely divisible space is impossible. Thus far, we have seen how *esse est percipi* and the rejection of negative ideas have led Berkeley to deny the infinite divisibility of space and affirm the existence of some perceptual minima or minimum sensible.⁹⁷ Consequently, it is absurd for a finite space to be infinitely divisible given the positive idea of infinity because a finite space would contain an infinity of parts, making it infinite instead of finite.

We have seen that Berkeley's *esse est percipi* principle necessitates the claim that what exists either is immediately perceived or is reducible to what is immediately perceived. What is immediately perceived is an idea with positive rather than negative content. No negative ideas

⁹⁶ Berkeley affirms what is often called perceptual space and rejects physical space (space full of real objects). See Turbayne, Colin Murray. "Berkeley and Russell on Space". Creery, Walter E (ed.). *George Berkeley: Critical Assessments*. Vol. I. London: Routledge, 1991. 140-150. 145-148.

⁹⁷ Gray, David. "Berkeley's Theory of Space". *History of Philosophy*, 16, 415-434. 20p. 415-434. 416-417.

are possible if immediate perception is the primary form of perception via *esse est percipi*: no negative ideas exist because they are not immediately perceivable. Thus, if one cannot perceive immediately an infinite division to one's idea of space because one perceives a smallest part, then space cannot be infinitely divisible. Otherwise, a smallest part would be imperceivable. Perceiving infinite division via negative ideas is also impossible since no negative ideas exist. Infinite divisibility is impossible whether expressed as a positive or a negative idea because such infinite division is imperceivable. Not only does *esse est percipi* govern what exists, but also what can be known. It is a metaphysical and epistemological principle. We will now see the similarities between Hume and Berkeley concerning the denial of infinite divisibility and negative ideas.

Chapter One-Section C: Hume, Negative Ideas, and Minimum Sensibles

In his *Treatise of Human Nature*, Hume entertains various arguments against the infinite divisibility of space, arguing that the idea of space is composed of minimum sensibles. Minimum sensibles are like colored points that compose space as pixels on a computer screen compose the image that is on the computer screen. Like Berkeley, Hume rejects negative ideas on the basis of his theory of impressions and ideas: if all that is known are the contents of one's mind (impressions and ideas) and force and vivacity (clarity) characterize impressions and ideas, then no negative ideas can exist: only positive ideas or ideas with only positive content can exist. Like Berkeley, no reference to external objects is evident in Hume's thought. Hume also follows Berkeley's distinction between immediate and mediate perception, claiming that mediate perception is reducible to immediate perception. Thus, one can say that Hume and Berkeley subscribe to the *esse est percipi* principle, which makes negative ideas impossible; however,

Berkeley's *esse est percipi* principle is metaphysical and epistemological in nature, while Hume's *esse est percipi* principle is only epistemological—Berkeley asserts a truth about reality and what can be known, while Hume admits a truth of only what can be known. Hume's distinction between impressions and ideas via vivacity and his lack of reference to external objects makes the idea of infinitely divisible space impossible because an infinity of parts in finite space is neither perceivable nor possible.

In order to demonstrate this thesis, I shall first assess the reasons Dale Jacquette cites for Hume's rejection of negative ideas and critique Jacquette's reasons. Next, I will provide an alternate account of Hume's rejection of negative ideas beginning with his distinction between impressions and ideas through force and vivacity. Third, I will analyze two of Hume's arguments against infinite divisibility, showing their connection to Hume's rejection of negative ideas. Fourth, I shall link Hume's rejection of negative ideas to Berkeley's rejection of negative ideas and his *esse est percipi* principle. Finally, I will summarize and formulate Hume's argument against negative ideas in the context of his denial of infinitely divisible space.

Chapter One-Section C.I: Jacquette's Account of Hume and Negative Ideas

Dale Jacquette observes that David Hume only entertains only the possibility of a negative idea of a vacuum.⁹⁸ Hume argues against the possibility of a negative idea of absolute darkness in the same manner by which he argues against the negative idea of the vacuum: there would be no content producing impression.⁹⁹ The argument against the negative idea of the

⁹⁸ Jacquette, 69-74.

⁹⁹ Jacquette maintains that when Hume speaks of "darkness" he really means "absolute darkness". He notes that Hume's argument is weakened in the case of a man who can see perceive twilight: darkness is partial is a partial negation of light. Since the twilight light has content, one could frame a negative idea of partial darkness rather than

vacuum is based on Hume's copy theory of impressions and ideas, and also his idea of space consisting of minimum sensibles. The copy theory of impressions and ideas is that every simple idea is derived from a corresponding simple impression. A simple idea is an idea that cannot be distinguished into other ideas; a complex idea is distinguishable into simple ideas.¹⁰⁰ The color blue is a simple idea because this idea cannot be distinguished into ideas of another kind. Since simple ideas lack the capability of being further distinguished into something else, they are indivisible. Golden mountain is a complex idea distinguishable into the ideas of gold and mountain. Sometimes, ideas constituting complex ideas can be distinguished further into other ideas: take the case of mountain in golden mountain. Per the copy theory of impressions and ideas, the simple idea of blue is derived from the simple impression of blue. Ideas are fainter and less lively perceptions than their impression counterparts. Since space is a collection of perceptual minima, the impressions always have content. Thus, the ideas which impressions produce can only be positive.¹⁰¹ If vacuum is the absence of minima, then we cannot have an idea of a vacuum since ideas are copies of impressions and there is no corresponding impression of a vacuum possible.¹⁰²

Presumably, one can formulate an argument against all negative ideas, including a negative idea of infinitely divisible space, from Hume's argument against the negative idea of a vacuum. Jacquette observes that since all simple impressions are also minimum sensibles, which have content, no negative idea can result from the simple impressions.¹⁰³ Otherwise, minimum sensibles are without content and fail to be minimum sensibles. They are distinguishable from

absolute darkness. See Jacquette, Dale. "Hume on Infinite Divisibility and the Negative Idea of a Vacuum". *British Journal for the History of Philosophy*. 10(3), August, 2002. 72-75.

¹⁰⁰ Hume, 5.

¹⁰¹ Jacquette, 72.

¹⁰² Jacquette, "Hume...", 413-435. 425; 430-432.

¹⁰³ Jacquette "Hume...", 422.

one another according to their content: one minimum can be green and form part of a table, while another minimum is blue and forms part of the sky.

Why must impressions have content? Jacquette correctly follows Hume's argument against the negative idea of a vacuum; however, it is a *petitio principio* to argue against all negative ideas by assuming that simple impressions are minimum sensibles. Note also that Hume argues against the negative idea of a vacuum after arguing for his theory of space composed of minimum sensibles. This order of arguments shows that, if Hume believes that negative ideas are impossible, the reason for his belief must precede the arguments against infinitely divisible space and against the negative idea of a vacuum. One must search for the basis of his denial of negative ideas in his theory of impressions and ideas.

Chapter One-Section C.II: An Alternate Account of Hume's Colored Minimum Sensibles and Denial of Negative Ideas

Hume's distinction between impressions and ideas through force and vivacity and an absence of a reference to external objects show that only positive ideas are possible. Impressions and ideas are perceptions, or contents of thought.¹⁰⁴ Impressions are those perceptions which "...enter with most force and violence."¹⁰⁵ Examples of impressions include sensations (soft, cold, blue, etc...) and emotions (joy, anger, fear, etc...). Ideas are those perceptions which "...are the faint images of these (impressions)..."¹⁰⁶ (my emphasis added). Implicit in this distinction is the copy theory of impressions and ideas. The copy theory states that every simple idea is a copy of its corresponding simple impression. There cannot be a simple idea without

¹⁰⁴ Jacquette, 6-7.

¹⁰⁵ Hume, 5.

¹⁰⁶ *Ibid.*, 5.

having had the corresponding simple impression. It would be impossible to frame the simple idea called blue if there is not simple impression called blue. Note that, unlike Locke, Hume never incorporates external objects into his definitions of impressions and ideas. One might suspect that impressions are or replace external objects in Hume's copy theory; however, external objects are not perceptions. Unlike Locke, Hume has no recourse to a derivation of negative ideas via the idea of external objects.

Despite the absence of external objects, why ideas are only positive is unclear. If ideas are copies of impressions, the reason ideas can only be positive must rest in a truth about impressions. If impressions can only be positive, then no negative ideas are possible. Why might Hume believe that impressions can only have positive content?

Hume never clearly states in *A Treatise of Human Nature* that impressions can only have positive content. Hume's formulation of impressions and ideas provides the reason for impressions necessarily having positive content. Impressions and ideas are distinguishable according to their force and vivacity. Force and vivacity are based on how one feels about the perceptions. Trudy Govier maintains that the force of a perception is its staying power, while the vivacity of the perception is its clarity.¹⁰⁷ Impressions have more force or staying power than ideas because they influence the development of ideas, while ideas have less force or staying power than impressions because they cannot influence the development of impressions. Ideas are copies of impressions and impressions are not copies of ideas. Moreover, ideas have less force than impressions because the mind cannot maintain the perceptions known as ideas for as

¹⁰⁷ Govier, Trudy. "Variations on Force and Vivacity in Hume". *The Philosophical Quarterly*. Vol. 22, No. 86. Jan., 1972. pp. 44-52. 45-46; 48-49.

long a time as those perceptions which are called impressions.¹⁰⁸ Thus, impressions enter the mind with greater force than ideas presumably because the mind has less control over such perceptions than with ideas. Impressions are more vivacious than their idea counterparts because more detail is present in impressions than in ideas. If one examines the impression of a colored paper and attempts to remember that paper in all its details, it is usually the case that the idea of the details in the colored paper are less distinct. The precise degree of color in the paper may be less clear. Despite exceptions, such as the vivacity and force of a nightmare, Hume finds the difference between impressions and ideas as clear as the difference between feeling and thinking.¹⁰⁹ Therefore, force concerns how the perceptions affect us, while vivacity indicates the clarity of the details in the perception. It is the vivacity of impressions that makes ideas have only positive content possible.

It is incoherent for vivacity to characterize negative content. Negative content is the absence of positive content. How can an absence of something be clearer than any other absence? Mental processes characterize and produce ideas. Negative ideas lack the completion or perception of their object, the product of the mental process. The product of the mental process and the mental process characterize positive ideas. Thus, positive content requires perception of the mental processes' product. Only the perception of a product can more or less clear. For how can a mental process be less clear than another mental process? Otherwise, one would not be able to distinguish the mental process itself: can one find one mental process of addition less clear than another process of addition? Thus, it is incoherent for whatever has only negative content to be clearer than something else which has negative content. Impressions are

¹⁰⁸ Hume, 10-11.

¹⁰⁹ *Ibid.*, 5.

clearer perceptions than their idea counterparts. This statement means that even impressions are characterized according to mental processes and the product of those processes. Mental processes and the perception of the mental processes' product must also characterize impressions. For if impressions exist only in the mind as perceptions, then they must exist through mental operations or processes. If impressions could have negative content, then there would be an instance when an absence would have clearer details than its idea counterpart. Yet an absence of something cannot be copied so that the copy would be less clear than the original.¹¹⁰ Therefore, if impressions are more vivacious than their idea counterparts, they must have only positive content. If they can only have positive content, and simple ideas are always copies of simple impressions, then no idea can be copies of the absence of positive content. Therefore, no negative ideas are possible.

Without negative ideas, then one must perceive an infinity of parts to have the idea of infinity or infinite divisibility, which is impossible if the mind is finite. Hume's notion of infinity is implicit in his notion of infinite divisibility: "'Tis also obvious, that whatever is capable of being divided *in infinitum*, must consist of an infinite number of parts, and that 'tis impossible to set any bounds to the number of parts, and that 'tis impossible to set any bounds to the number of parts, without setting bounds at the same time to the division."¹¹¹ Notice the correspondence between the number of parts to the number of divisions. Something with a finite number of parts is finitely divisible, while something with an infinity of parts is infinitely

¹¹⁰ Dale Jacquette argues a Neo-Humean account of negative ideas based on impressions that have only partially positive. A negative idea of partial darkness is based on impressions of diminishing light as would exist during twilight hours. This negative idea is not a pure absence of positive content because positive content is required to frame such a negative idea. The negative idea is not an absence of positive content, but refers to the diminution of positive content. See Jacquette, "Hume...", 426-429.

¹¹¹ Hume, 24; Flew, 260. Flew claims that the assumption that only something infinite must consist of an infinity of parts generates the conclusion that finite space cannot be infinitely divisible. Yet it is only true that whatever is infinitely divisible can be divided without limit.

divisible. Thus, the number of parts dictates whether something is finite or infinite. Finite quantities always have a finite number of parts. Infinite quantities have an infinity of parts. Since a finite quantity has a finitude of parts, it cannot be infinitely divisible. Otherwise, a finite quantity would have an infinity of parts. Since infinity dictates that an infinite number of parts be present in a quantity, only an infinite quantity is infinitely divisible. However, why does Hume associate the number of divisions with the number of parts?

Hume's rejection of negative ideas entails that the result of a process must be perceivable. If one adds a quantity to another quantity, then the result of the process must be perceivable. Given the finitude of the human mind, which could only perceive the result of a finite process, and the rejection of negative ideas, to entertain the idea of continuing an additive or divisional process *ad infinitum* is impossible. Even if one could not perceive the billionth addition to a quantity, a finite mind could complete the finite process given a certain amount of time. However, a finite mind could never complete an infinite process. If the completion of a process necessarily requires the perception of the product of the process, then since an infinite process cannot be completed, an infinite product cannot be perceived. The idea of infinity requires an infinite process of addition by a finite mind. Since the result of the process must be perceivable, the idea of infinity would be an infinite idea for Hume. To have a process which cannot be completed is to have an absence of positive content, a negative idea of a certain process. Therefore, by necessarily linking process and product together, and by denying negative ideas, knowledge must be perceivable for ideas must have positive content. However, why must an infinity of parts necessarily equal an infinite quantity when those who maintain that space is infinitely divisible hold that space is divisible into an infinity of parts and still be finite?

Hume's principle that knowledge requires perception coupled with simple/complex dichotomy of ideas necessitate the claim that an infinity of parts always equals an infinite quantity. Proponents of infinite divisibility distinguish between aliquot and proportional parts.¹¹² Thijssen claims that Hume's distinction between aliquot and proportional parts comes from Pierre Bayle's *Systeme de Philosophie, Oeuvres Diverses*, IV.¹¹³ Proponents of infinite divisibility believe that an infinity of aliquot parts necessarily equals an infinite quantity, while an infinity of proportional parts can equal a finite quantity. Aliquot parts are quantities that are equal to one another. Fifteen aliquot parts are equal to fifteen units, a finite quantity. Thus, an infinity of aliquot parts equals infinity. Aliquot parts match the kinds of parts of which Hume is thinking when rejecting the idea of infinity. Aliquot parts correspond to Hume's positive idea of infinity and infinity divisibility: a finitude of aliquot parts is a finite quantity, while an infinitude of aliquot parts is an infinite quantity. Proportional parts are parts that exist in a ratio to one another and to the whole. To divide a space by continually halving it is to have a space which could be divisible into an infinity of parts, but the sum of the parts is finite: just as $1/2$, $1/4$, $1/8$... converge on 1, a finite quantity. Each proportional part is proportionally smaller than the previous part according to a ratio. A finite quantity can consist of an infinitude of ever diminishing quantities, proportional parts. Why Hume rejects proportional parts requires determination.

Hume rejects proportional parts because ideas are simple or complex and knowledge of a mental process requires perception of the product of the mental process. Negative ideas, as mere

¹¹² Jacquette, 141-142; Fogelin, Robert. "Hume and Berkeley on the Proofs of Infinite Divisibility." *The Philosophical Review*. Vol. 97, No. 1. Jan., 1988. pp. 47-69. 53-55

¹¹³ Thijssen, J.M.M.H. "David Hume and John Keill and the Structure of Continua". *Journal of the History of Ideas*. Vol. 53, No.2. April-Jun., 1992, pp. 271-286. 280; Jacquette, 142.

absences of positive ideas, are neither simple nor complex ideas. Simple ideas have positive content, or entail the perception of the product of a mental process, that is not distinguishable into other positive content. A complex idea consists of simple ideas. Since no negative idea has positive content, and complex ideas require positive content by being a complex of simple ideas, no negative idea is a complex idea. Yet if positive content of an idea is either homogeneous or multifarious, then our ideas of quantity are either simple and indivisible or complex and divisible. Thus, whether quantity is simple or complex, indivisible quantities must exist. With proportional parts, however, one can never end the process of division. There is no ultimate or indivisible quantity at which the process of division ends. Quantities that are proportional parts, therefore, cannot exist for Hume. Aliquot parts neatly fit into Hume's simple/complex idea dichotomy, however. Aliquot parts are indivisible like simple ideas and can be compounded to form a complex idea of a quantity. But only an infinite quantity can consist of an infinity of parts, while a finite quantity can only consist of a finitude of parts.

Marina Frasca-Spada maintains that Hume rejects proportional parts because their distinction as aliquot parts is unfounded.¹¹⁴ Proportional parts are ultimately indistinguishable from aliquot parts because one cannot divide proportional parts without limit. The result of dividing purported proportional parts is the perception of an indivisible perception, an aliquot part. Presumably, if the mind is finite, there is an end to the number of possible divisions to the idea of space.¹¹⁵ Aliquot parts provide imaginal content to proportional parts. Yet the purported proportional parts are not real proportional parts because there is some common unit by which

¹¹⁴ Frasca-Spada, Marina. *Space and the Self in Hume's Treatise*. New York : Cambridge University Press, 1998. 42-43.

¹¹⁵ This experiment is known as the grain of sand argument, which shall be investigated later. One's idea of space is not infinitely divisible.

every proportional part is divisible: the minimum aliquot part. Proportional parts cannot have a common unit because they are infinitely divisible. Thus, proportional parts are reducible to aliquot parts.

However, Frasca-Spada does not fully account for why proportional parts are reducible to aliquot parts: even if the mind is finite, proponents of proportional parts maintain that proportional parts are possible. Hume would commit a *petitio principio* against the proponents of proportional parts by claiming that a finite mind cannot conceive of proportional parts. Rather than arguing that the mind is finite and cannot conceive of proportional parts, the rejection of proportional parts is based on Hume's rejection of negative ideas.

While there is still a *petitio principio* on Hume's part since proponents of proportional parts would argue that negative ideas are possible, clarification of the crucial issue advances the debate. If negative ideas are impossible, then Hume's claim that the finitude of the human mind precludes the possibility of proportional parts is valid. If negative ideas are possible, then proportional parts are distinguishable from aliquot parts and Hume's argument from the finitude of the human mind is invalid. Frasca-Spada's claim that Hume rejects the distinction between proportional and aliquot parts due to the finitude of the human mind does not highlight the crucial issue between Hume and proponents of proportional parts: whether negative ideas are possible.

However, Hume acknowledges perceptions without reference to an external object. Since ideas are copies of impressions and vivacity characterizes impressions and ideas, no negative ideas exist because clarity or vivacity can only distinguish positive content, the perception of the product of a mental process—absence cannot be more or less clear in one

instance rather than in another instance. Thus, there are no negative ideas if knowledge of a mental process requires perception of the product of the mental process. Without negative ideas, the idea of infinity and infinite divisibility entail an infinite number of parts. If the human mind is finite, it cannot complete an infinite process and cannot perceive an infinity of parts. Thus, the mind does not really have an idea of infinite divisibility or infinity. Infinite divisibility is an incoherent concept since only a finite quantity is finitely divisible for Hume. We shall now examine some of Hume's arguments against the infinite divisibility of space in order to show the aforementioned principles at work in the arguments.

**Chapter One-Section C.III: Hume's Grain of Sand Argument, Ink Spot Experiment, and
*Esse Est Percipi***

Only two of Hume's arguments are analyzed in this thesis: the grain of sand argument and the ink spot experiment. The grain of sand argument and the ink spot experiment show that perceptions are not infinitely divisible. In the grain of sand argument, space is only finitely divisible because one cannot produce a distinct image that is smaller than the grain of sand, purportedly the smallest perceivable part of the idea of space. The ink spot experiment shows that impressions of space are only finitely divisible because there is a point at which a spot of ink on a paper vanishes if one moves far enough away from it. The goal of this section is to show how the principles uncovered in the last section underlie Hume's grain of sand argument and ink spot experiment. The principles that knowledge of a mental process requires the perception of the product of the mental process, that no negative ideas exist, and the simple/complex dichotomy of perceptions are the basis for the grain of sand argument and ink spot experiment.

The grain of sand argument shows that the idea of space is only finitely divisible.

Assuming the finitude of the human mind, one reaches a minimum perception because only something infinite has an infinity of parts:

“When you tell me of the thousandth and ten thousandth part of a grain of sand, I have a distinct idea of these numbers and their different proportions; but the images, which I form in my mind to represent the things themselves, are nothing different from each other, nor inferior to that image, by which I represent the grain of sand itself, which is so vastly to exceed them.”¹¹⁶

Frasca-Spada notes Hume’s denial of proportional parts in this argument.¹¹⁷ The denial of proportional parts is based on the denial of negative ideas as discussed earlier. The finitude of the human mind is limited with respect to the number of possible divisions it can make. A boundary is reached when one is unable to distinguish clearly the products of division.¹¹⁸ Thus, there must be a smallest possible division in one’s idea of space or of a body since a division smaller than the smallest possible division is imperceptible.

If one attempts to divide space and the product of the division is no longer perceived, then the idea of space cannot be further divided. There is an end to the perception of the products of division. What one perceives of the idea of space is knowledge of the idea of space. Since there are no negative ideas, if space were infinitely divisible, then one could perceive a series of ever diminishing divisions to the idea of space. If there is a smallest possible division, then any part of space that is purportedly smaller than the smallest part is indistinguishable from the smallest possible division. If something is indistinguishable from something else, then the two or more things are identical. Consequently, the idea of the grain of sand, later called the minimum sensible, is the ultimate constituent of the idea of space. Grains of sand comprise the idea of

¹¹⁶ Hume, 25.

¹¹⁷ Frasca-Spada, 41-43.

¹¹⁸ Jacquette, Dale. “Hume on Infinite Divisibility and Sensible Extensionless Indivisibles”. *Journal of the History of Philosophy*. 34. 61-78. Jan. 1996. 65.

space. If space is infinitely divisible, then it would be possible to perceive a division smaller than the idea of a grain of sand. Thus, the denial of negative ideas and the affirmation that knowledge of a mental process requires perception of the mental processes' product underlie the grain of sand argument.

There is no distinction between subjective states of affairs and objective knowledge for Hume. If no negative ideas exist, then one cannot claim that there are parts of space smaller than the perceptible parts. Ideas must be dependent on perception if there are no negative ideas. Thus, if something is not perceived in an idea, one does not have an idea of what is unperceived. Moreover, if something is not perceived in an idea, then the idea cannot contain what is unperceived in it. Thus, one's idea of space cannot include imperceptible parts if there are no negative ideas and ideas depend on perception. If one's idea of space must contain minima, then it is reasonable to infer that one's impression of space must also contain minima.

Hume could argue that one's impression of space contains minima from the claim that one's idea of space contains minima from his copy theory of ideas and simple/complex perception dichotomy. Since there are simple, indivisible ideas that comprise the idea of space and all simple ideas are copies of simple impressions, there must be perceptual minima of the impressions of space. The aforementioned argument is valid given Hume's theory of the origin of ideas; however, it would not fully convince someone who maintains the possibility that there is a different impression of space, one that is not accounted for under the simple/complex perception dichotomy. If one permits, as Locke, the possibility of a negative idea of external objects, then one's senses or impressions would indicate that there is something not directly perceived. What is not directly perceived but is somehow indicated through the impressions

would be negative content. One's perception would only indicate parts or aspects of the external object without revealing the entirety or all aspects of the external object. Negative content is not susceptible to the simple/complex dichotomy of perceptions. To remove this possible objection, the ink spot experiment shows that there is no impression of infinitely divisible space because there is some point at which an object, an ink spot in this case, becomes imperceptible—perceptual minima exist. The manner of experiment is not different from the grain of sand argument; however, the two arguments differ with respect to what is proven: minimum sensibilia for impressions from the ink spot experiment and minimum sensibilia for ideas from the grain of sand argument. If there is no impression of infinitely divisible space, then there cannot be an idea of infinitely divisible space since all ideas of space are formed from the minimum impressions of space.

Hume's ink spot experiment shows that there are perceptual minima in impressions because there is always a point at which an image vanishes from sight.¹¹⁹ The experiment requires that one imagine an ink spot on a piece of paper. The ink spot is visible; however, as one retires in distance from the ink spot, the spot becomes less visible. Consequently, there is a distance at which the ink spot is no longer visible. The ink spot is indivisible just prior to its disappearance from one's vision.¹²⁰ Hume's minimum impressions are extensionless or indivisible, yet visible if one adds color to them.¹²¹ Since one's impressions of space ultimately resolve into perceptual minima or minimum sensibles, the impression of space consists of minimum sensibles. If one's impression of space consists of minimum sensibles, and simple

¹¹⁹ Jacquette, 114-115.

¹²⁰ Hume, 25.

¹²¹ One can wonder how color is applicable to something indivisible if color requires extension in order to be perceptible. Hume provides no immediate answer to this question; however, if minima are indivisible and extensionless just prior to their annihilation, then they must already be visible or discernable. Otherwise, how could one detect that a length has vanished immediately after it became indivisible?

ideas are ultimately copies of simple impressions, one's ideas of space must consist of minimum sensibles. The ideas of minimum sensibles are copies of the impressions of minimum sensibles. However, the principles of Hume's ink spot experiment are not evident: why must impressions of space be resolvable into minimum sensibles? An inductive experiment concerning the impression of space cannot show absolutely that no impression of space could be infinitely divisible.

Hume must maintain that any impression of space must be finitely divisible. If impressions must have positive content, then there must be simple impressions of minimum sensibles. Positive content is either homogeneous or multifarious: of one kind or of a combination of kinds. Note that positive content being of one kind or a combination of kinds corresponds to Hume simple/complex dichotomy of perceptions. If space is a complex impression, then it must consist of simple impressions, minimum sensibles.¹²² The complex impression of space must be finitely divisible and consist of minimum sensibles.¹²³

Antony Flew raises an objection to Hume's ink spot experiment: how can one compare the relative sizes of ideas or impressions to real objects given the fact that one can examine an object in a microscope that is perceived to be of a different size than if examined with the naked eye? Two contrary measures are applicable to the same thing. Hume might respond that humans know nothing about the physical objects, because of his distinction between impressions and ideas: impressions cannot be derived from physical objects because we are only acquainted with our perceptions. Consequently, there aren't contrary measures of the same thing (the physical objects).

¹²² Jacquette, "Hume on Infinite Divisibility and the Negative Idea of the Vacuum". 416-417; 420.

¹²³ Flew, Antony. "Infinite Divisibility in Hume's *Treatise*". 261-262.

This objection would certainly work against Locke who refers to external objects in his definition of the ideas of sensation; however, Locke's space is infinitely divisible. Pertaining to Hume, one might argue that Hume's theory is problematic without physical or external objects because cannot measure accurately one idea if one takes into account different visual levels: the microscope versus the naked eye. Despite every idea consisting of minima, the naked eye does not reveal the same idea to be of identical size. The naked eye reveals the impression of a creature to be ten minimum sensibles, while the microscope shows the impression of the creature to be one hundred minimum sensibles. What is the real size of the impression or idea of the creature? Either the impressions or ideas are not of the same creature, or minimum sensibles are merely limitations on what is perceivable at a time without precluding the possibility of sizes smaller than minimum sensibles. Thus, Hume's principles of measurement are problematic if there are no ideas of substances or external objects distinct from one's perceptions.

The principle that knowledge of a mental process requires the perception of the mental processes' product is evident in the grain of sand argument and the ink spot experiment. This principle is evident in the former because no negative ideas exist, and is evident in the latter because all impressions must have positive content. In the grain of sand argument, the fact that no negative ideas exist requires that all ideas have positive content. Yet positive ideas must be perceived since ideas are perceptions, and perceptions must be perceived in order to exist; otherwise, the positive idea would be viewed to lack positive content. If an idea lacks positive content, it is negative. Yet positive ideas cannot have an absence of positive content; otherwise, they would not be positive ideas. Consequently, one's positive idea of space, since finite minds cannot perceive an infinity of parts, must consist of indivisible minimum sensibles (grains of sand): minimum sensibles must be perceivable in any idea of space. In the ink spot experiment,

the fact that impressions are more vivacious perceptions than their idea counterparts guarantees that minima must be perceived in any complex impression of space. Vivacious perceptions or impressions must have positive content. Positive content, however, is perceivable as either simple or complex. Being a complex impression, space must be perceived as consisting of simple impressions, minimum sensibles. Thus, the grain of sand argument and ink spot experiment show that, if any knowledge of space exists, one's knowledge of space must correspond to the manner by which one perceives space. Whether it be of an idea or an impression, one must perceive every part of space in order to have knowledge of space. Since negative ideas do not exist and impressions must have content, space must consist of simple impressions or ideas, minimum sensibles.

Now that the reasons Berkeley rejects negative ideas and the negative ideas of infinity are clear, it is easier to draw comparisons to Hume's implicit rejection of the negative ideas. Hume nowhere rejects negative ideas overtly. Instead, he rejects only the negative idea of a vacuum.¹²⁴ Dale Jacquette believes that one can formulate an argument against negative ideas from Hume's rejection of the negative idea of a vacuum.¹²⁵ While Jacquette's argumentation is apt, Hume's rejection of negative ideas is better linked to his distinction between impressions and ideas according to vivacity or clarity. Jacquette's rendition of Hume's argument becomes circular because impressions of minimum sensibles must be presupposed in order to show that there can be no negative ideas; however, the rejection of negative ideas must be demonstrated before arguing that minimum sensibles exist. Minimum sensibles are proven because there is no negative idea of infinity or infinite divisibility. Yet the rejection of negative ideas entails the

¹²⁴ Jacquette, Dale. *David Hume's Critique of Infinity*. Boston: Brill, 2000. 69-74.

¹²⁵ Jacquette, Dale. "Hume on Infinite Divisibility and the Negative Idea of a Vacuum". *British Journal for the History of Philosophy*. 10(3), 413-435. August, 2002. 72-75.

rejection of the negative ideas of infinity and infinite divisibility—circularity! Hume’s rejection of negative ideas according to his distinction between impressions and ideas according to vivacity or clarity avoids this circularity because the origin of Hume’s rejection of negative ideas lies at the heart of his epistemology rather than due to some later conclusion. It also provides a basis for linking Hume to Berkeley’s rejection of negative ideas, particularly the negative ideas of infinity and infinite divisibility. Hume and Berkeley preclude the possibility of negative ideas because of their rejection of external or material objects, coupled with the fact that only positive ideas are possible: every aspect of an idea must be perceivable by the mind. Berkeley and Hume agree that *esse est percipi*, but Berkeley maintains that *esse* is of existence, while Hume believes that *esse* is that of which we can have knowledge.

Unlike Locke but like Berkeley, Hume never builds external objects into his epistemology; unlike Berkeley, Hume never argues against the existence of external objects—instead he simply finds the distinction between existence and external existence unfounded.¹²⁶ Since he has no recourse to external objects in his theory of impressions and ideas, Hume cannot base a theory of negative ideas on the negative idea of external objects. The source of the distinction between impressions and ideas is not from external objects, but the perceptions’ vivacity or clarity. Impressions are clearer or more vivacious perceptions than their idea counterparts. If vivacity can only be ascribed to something that has positive content, then if ideas are copies and less vivacious perceptions of impressions, no negative ideas can exist.

¹²⁶ Hume, David. *A Treatise of Human Nature*. New York: Barnes and Noble Publishing, 2005. 54-55.

Trudy Govier claims that vivacity in Hume's distinction between impressions and ideas is identical to the meaning of clarity.¹²⁷ To say that impressions are more vivacious perceptions than their idea counterparts is to maintain that impressions are clearer to the percipient than their idea counterparts. Hume characterizes ideas as being more dull copies of their impression counterparts. The difference between impressions and ideas is like the experience of feeling versus thinking.¹²⁸ To feel something impacts the percipient more than to think something. One need only inspect his perceptions to discern the difference between impressions and ideas according to Hume.¹²⁹ The impression of blue, which enters the mind through the senses, is clearer than one's idea of blue in the sense that the impression of blue appears more vibrant than the idea of blue. One perceives the blueness in the impression of blue more so than in the idea of blue. The ability to describe the notion of clarity or vivacity becomes more difficult because clarity is something experienced rather than thought of: words can only go so far in describing clarity.

However, clarity only describes positive content since clarity never characterizes negative content. Negative content is the absence of positive content. One might ask how one form of absence can be any clearer than any other form of absence. If impressions are clearer perceptions than their idea counterparts, and absence characterizes negative content, how can one form of perception be clearer given negative content than any other form of content? One form of negative content is distinct from another form of negative content not because of an absence *per se* but because of a reference to a distinct form of positive content. Yet the positive content

¹²⁷ Govier, Trudy. "Variations on Force and Vivacity in Hume". *The Philosophical Quarterly*. Vol. 22, No. 86. Jan., 1972. pp. 44-52. 45-46; 48-49.

¹²⁸ Hume, 10-11.

¹²⁹ Hume, 5-7.

is not found in the negative content itself. Absence *qua* absence is not distinct from another absence for there is no distinguishable content within the absence. Without content, there is no way to say “this absence is clearer than that absence”. One cannot say that the impression of the absence of blue is any clearer than the idea of the absence of blue. Therefore, vivacity or clarity requires positive content if impressions are understood as more vivacious perceptions than their idea counterparts.

Since impressions can only have positive content, and ideas are copies of impressions, no negative ideas are possible. Hume maintains that all simple ideas are copies of their simple impression counterparts.¹³⁰ The simple idea of green contains the same content as the simple impression of green; however, the two perceptions are distinct according to vivacity. If ideas are ultimately copies of impressions, then whatever is true of impressions must also be true of ideas. Since impressions only have positive content, ideas must have positive content. Negative ideas require negative content. If only positive content is available, then there cannot be negative ideas formed from impressions. All impressions, therefore, are positive idea producing perceptions.¹³¹

Hume scholars, such as D.G.C. McNabb and John Laird, largely concur that Hume never acknowledges the existence of external objects; however, some Hume scholars, like Robert Anderson and John Passmore, maintain that Hume incorporates external objects into his epistemology.¹³² D.G.C. McNabb and John Laird cite passages from the *Treatise* referenced

¹³⁰ Hume, 6-7. Jacquette, 44.

¹³¹ Jacquette, “Hume on...”, 425.

¹³² Anderson, Robert F. “The Location, Extension, Shape, and Size of Hume’s Perceptions”. *Hume: A Re-Evaluation*. Livingston, Donald W. and King, James T. (eds.) New York: Fordham University Press, 1976. 153-171. 157-159.

earlier that indicate impressions and ideas refer to nothing beyond themselves.¹³³ Thus, perceptions are complete because they are single, never revealing their sources or whether they have a source; perceptions are also all that the human mind has to work with: if humans can only form either an impression or idea, nothing beyond impressions and ideas can count as knowledge. John Passmore and Robert Anderson emphasize Hume's statements in the *Enquiry* over those in the *Treatise* concerning the existence of external objects.¹³⁴ Passmore quotes a number of passages from the *Enquiry* in which he alludes to the dictates of human reason, as many philosophical and non-philosophical people hold, incorporating the belief in external objects.¹³⁵ According to Passmore and Anderson, since Hume has never stated that he disbelieves in this particular dictate of reason, and claims that philosophical reflection leads to belief in external objects, Hume accepts the existence of external objects in the *Enquiry*.¹³⁶ Anderson connects the *Enquiry* passages quoted by Passmore to several *Treatise* passages purportedly indicating Hume's belief in external objects.¹³⁷ It is worth noting, however, that Hume refers to the belief in external objects in those passages as belonging to the vulgar or uninformed opinion. Yet Hume is not in agreement with the vulgar opinion! That is to say, if one acknowledges the existence of impressions and ideas, and one is to have knowledge of external objects, then it is only through impressions and ideas that one *could* have knowledge of external objects according to Hume's true opinion. Thus, impressions and ideas do not grant us any knowledge of external objects. If Hume really believed that impressions and ideas give us knowledge of external objects, then why did he never refer to external objects when defining

¹³³ Laird, John. *Hume's Philosophy of Human Nature*. (London, 1932). 29-30.; D.G.C. MacNabb. "David Hume". *The Encyclopedia of Philosophy*. Paul Edwards, ed. 8 vols. (New York, 1967), IV. 78.

¹³⁴ Anderson, 158-159.

¹³⁵ Passmore, John. *Hume's Intentions*. Rev. ed. (New York, 1968). 90-91.

¹³⁶ Anderson, 158-159.

¹³⁷ Anderson, 159-160.

impressions, which would presumably be the resultant perception of external objects. Thus, whenever Hume refers to “object” in the *Treatise*, one should interpret this term as impression. For ideas are copies of impressions, not of objects because impressions are never stated to be copies of anything. Thus, MacNabb’s and Laird’s claims that Hume rejects external objects seem more fitting, being in agreement with the position of this thesis.

In addition to the *Enquiry* and *Treatise* passages that Anderson cites, Anderson argues that Hume’s rejection of the *Treatise*, coupled with the aforementioned references to the *Enquiry*, should fortify the position that Hume believes in the existence of external objects.¹³⁸ The beginning of the *Enquiry* includes a brief account of Hume finding that the *Treatise* was written in the “wild passions” of his youth, and should be regarded as an immature work. However, even though Hume rejects the *Treatise*, this claim would not prevent the thrust of this thesis being true: for what is being handled is the problem of infinite divisibility as it appears in the *Treatise* only. Furthermore, even if Anderson is right to claim that Hume never objects to the principle of human reason positing the existence of external objects, one cannot assume that the absence of objection is evidence of acceptance. An alternative would be that Hume is agnostic with respect to the existence of external objects in the *Enquiry*. Thus, Anderson’s claims are inconclusive. Taking what has been stated into account, it appears the absence of external objects in Hume’s epistemology is true and leads to his rejection of negative ideas.

Thus far, it is clear that Hume and Berkeley exclude the possibility of negative ideas on account of the absence of external objects and because perceptions must have positive content whether due to some notion of *esse* as is true with Berkeley, or due to some distinction in

¹³⁸ Anderson, 161-162.

epistemology, vivacity, as is the case with Hume. One might also tie the two philosophers' rejection of negative ideas to the priority of immediate perception in their respective philosophies. Immediate perception is more fundamental in Hume's and Berkeley's philosophy due to their being an awareness only of ideas or impressions/sensations. If immediate perception is prior to mediate perception, and one limits knowledge or being to what is perceived, *esse est percipi* is present in Hume's and Berkeley's philosophy. Hume's version of *esse est percipi* describes esse in terms of what is knowable, while Berkeley's version of *esse est percipi* describes esse in terms of what exists. *Esse est percipi*, in whatever form, precludes the existence of negative ideas.

While Berkeley's claim that immediate perception is prior to mediate perception is already clear from his argument against external objects and the association between a sound and a coach, Hume's belief that immediate perception is prior to mediate perception is evident after examining the statement of his project in the *Treatise*. Hume states that the goal and methodology of his *Treatise* is to frame an experimental epistemology from the inspection of the contents of his mind.¹³⁹ One cannot infer anything beyond the contents of one's mind. What is present in one's mind is a perception, whether an impression or an idea. Perceptions are perceived immediately by themselves. If mediate perception is possible according to Hume, it must be through the association of certain ideas. Yet the ideas themselves were immediately perceived. Thus, Hume allows for the association between a sound and a coach (mediate perception) as does Berkeley provided that what is mediately perceived was once immediately perceived. Therefore, in Hume's and Berkeley's philosophy, immediate perception plays a primary role in their theories of knowledge.

¹³⁹ Hume, XXIII-XXVII; Jacquette, 3-4.

Berkeley's notion of *esse* in *esse est percipi* is metaphysical as is clear from his argument against external objects, while Hume's notion of *esse* in *esse est percipi* indicates what is knowable because he never admits the reality of external objects in his epistemology. Since Berkeley argues against material or external objects on the basis of *esse est percipi*, *esse* must mean existence in a metaphysical sense. To say that something is imperceivable means that it cannot be. Since material or external objects are imperceivable, they cannot be. Hume never discusses the possibility of external objects even though he refers to external existence, which is found to be incoherent according to his epistemology. If one can only claim to know about something on the basis of whether it is or has been content in one's mind, if material objects are not any of the contents of one's mind, then one cannot claim to know whether or not they exist. Although Hume maintains that what is conceivable is really possible and what is inconceivable is really impossible, although one cannot conceive of a material object, its existence is not precluded. For material objects cannot be found among the mind's content. What is conceivable or inconceivable is limited according to what is present as mental content or perceptions. Something is conceivable if there is a corresponding perception. Something is inconceivable if there is a contradiction in the respective perception. What is other than a perception cannot fall under Hume's theory of conceivability and inconceivability. Since Hume admits that what is present to the mind is its perceptions, and the mind can only claim knowledge to what is present as perception, *esse* means knowledge for Hume.

We have found that Hume and Berkeley reject the negative idea of infinity and infinite divisibility as "unboundedness" on account of their rejection of negative ideas. Negative ideas, signifying the absence of positive ideas or positive content, are impossible on account of the rejection of material or external objects, and that whatever is immediately perceived must have

positive content. Hume and Berkeley maintain that we are only acquainted with what we immediately perceive, the ideas or impressions in our minds. What we immediately perceive is not an external object as Hume and Berkeley note; however, Berkeley argues against external objects on account of *esse est percipi*, while Hume maintains a skeptical position concerning the existence of material objects. The result of Berkeley's denial of negative ideas due to *esse est percipi* is that no absence of positive content is perceivable: what is immediately perceived must be positive content or a positive idea since immediate awareness is of something rather than the absence of something; otherwise, something imperceivable would exist, namely an absence. The consequence of Hume's skepticism concerning external objects is that perceptions are distinguished according to their vivacity or clarity: impressions are more vivacious perceptions than their idea counterparts. Clarity always describes positive content. Impressions, being more vivacious than ideas, must have only positive content *a fortiori*. If ideas are copies of impressions, the positive content of impressions is also found in ideas. Thus, only positive ideas are possible because only positive content is possible. The culmination of the aforementioned conclusions is that Berkeley and Hume admit that *esse est percipi*; however, Berkeley clearly interprets *esse* in terms of existence, while Hume defines *esse* in terms of what can be known. Beginning with what is immediately perceived, which is the basis of *esse est percipi*, both philosophers conclude that no negative ideas exist. Without negative ideas, no negative idea of infinity or infinite divisibility is possible.

Hume not only incorporates the same notion of infinity and infinite divisibility as Berkeley, but also Berkeley's minimum perception argument. The finitude of the mind leads Hume to conclude that there must also be some sort of minimum in one's perceptions: "In rejecting the infinite capacity of the mind, we suppose it may arrive at an end in the division of

its ideas...¹⁴⁰ The minimum in one's perceptions is a simple, indivisible idea. These minimum perceptions are the result of the last possible division to one's idea of space. Thus, space is ultimately a compound idea of minimum perceptions or minimum sensibles according to Berkeley and Hume. Thus, Hume's affirmation of the finitude of the mind, the rejection of negative ideas, coupled with the simple/complex perception dichotomy neatly fit together in his epistemology. Following Berkeley, since only the positive idea of infinity is possible, a finite space cannot contain an infinity of parts and cannot be infinitely divisible according to Hume. All three of these affirmations or negations concerning the nature of human knowledge and ideas lead to the denial of infinitely divisible space and the conclusion that there must be some minimum perception constituting the idea of space.

The grain of sand argument supports the contention that there is a minimum perception or sensible constituting space and Hume begins by examining this minimum idea or perception that results especially from the finitude of the human mind. Hume calls this minimum idea or perception a "grain of sand". He argues that if there is a minimum idea or perception, then it would be impossible to divide it further because it is a simple idea having no distinguishable parts: "'Tis therefore certain, that the imagination reaches a *minimum*, and may raise up to itself an idea, of which it cannot conceive any sub-division, and which cannot be diminished without a total annihilation."¹⁴¹ Latent in this argument is Hume principle that whatever is distinguishable is separable. This principle is the result of Hume's simple/complex idea dichotomy. If something is a simple idea, then it can exist without other simple ideas because one simple idea in no way implies another simple idea. For example, the simple idea of blue does not indicate

¹⁴⁰ Hume, 24-25; Flew, 259.

¹⁴¹ Hume, 25.

the existence of the simple idea of yellow since blue can exist without yellow and vice-versa.

The idea of blue must be simple because there is no other distinguishable idea that constitutes the idea of blue: the simple idea of blue is only blue. Thus, no simple idea has distinguishable parts or consists of other ideas and no simple idea indicates the existence of another simple idea.

Taking these two aforementioned facets of simple ideas, it is possible to understand how the principle of whatever is distinguishable is separable operates in the grain of sand argument. If the grain of sand is the minimum idea or perception, then it must be a simple idea. If it is a simple idea, then it would be absurd to distinguish a part of it. Hence, there cannot be a thousandth or one-hundredth part. Such parts are not distinguishable in the grain of sand; yet what cannot be distinguished within something cannot really constitute the idea in question. If space were infinitely divisible, then it would have been possible to distinguish clearly the parts of a grain of sand. Yet such a distinction is impossible given the finitude of the human mind, the denial of negative ideas, and the affirmation of the simple/complex idea dichotomy.

Hume's neglect of negative ideas prevents him from having an idea of infinity or infinite divisibility. Hume rejects negative ideas because impressions can only produce positive ideas since they are clearer perceptions than their idea counterparts. Clarity always implies positive content. Thus, Hume's argument against infinite divisibility is as follows:

1. If there is an idea of infinite divisibility, it is either positive or negative.
2. A positive idea of infinite divisibility cannot be attained given the finitude of the human mind.
3. There is no negative idea of infinite divisibility because there are no negative ideas.

Therefore, there is no idea of infinite divisibility or infinite divisibility is an incoherent idea.

Chapter One-Section C: Conclusion

It is clear from Berkeley's and Hume's denial of negative ideas that the rejection of the negative idea of infinite divisibility stems from the lack of reference to external or material objects in their epistemologies and metaphysics. Although their precise reasons for lacking reference to external objects are distinct, their affirmation of the finitude of the human mind and the *esse est percipi* principle force both philosophers to deny external objects and negative ideas. Without external objects and negative ideas, reality and knowledge are limited to whatever is apparent or could be apparent to the mind. Thus, there is no distinction between the real or physical world and the mental world in Berkeley's and Hume's metaphysics. Only whatever one immediately perceives can be knowledge.

Whether all metaphysics and epistemologies that lack reference to external objects necessarily lead to the denial of negative ideas is undetermined. What is required for the existence of negative ideas is a reference to something which cannot be immediately perceived. What cannot be immediately perceived is something that is possibly radically other than one's ideas. If one accepts the possibility of an innate idea of God, like Descartes, an idea which cannot be accounted for solely through human reason, then one has a referent which is not immediately perceived. However, Berkeley and Hume reject the existence of innate ideas. Unlike Hume, Berkeley refers to notions which have active beings like God and souls as their

objects. Why Berkeley could not build a system of negative ideas from notions or God's perception of ideas which one could not see is unknown. One could surmise that Berkeley maintains that notions and ideas are inherently unrelated on account of their being active and passive respectively. Whatever the answer is to the question of the necessary relation between negative ideas and the denial of external objects is, it is clear why Berkeley and Hume reject the existence of negative ideas and the negative idea of infinite divisibility through their denial of external objects.

Now that I have clarified the similarities between Hume and Berkeley on their negative ideas of infinity and infinite divisibility, let's focus our attention on their differences regarding the nature of minima. By determining why Hume thinks that minima are unextended and Berkeley believes that minima are extended, I will link their respective theories to the main issue of concern in this dissertation: whether the minimum's location is identical to the minimum itself.

Chapter One-Section D: Berkeley versus Hume on the Nature of Minima

Let us suppose that perceptual minima compose our idea of space like pixels compose an image on a computer screen. Since perceptual minima compose the idea of space, space cannot be infinitely divisible because the existence of perceptual minima limits the number of possible divisions one could make to space. Thus, the size of a given finite space is measured according to the number of minima that compose the finite space. Even if it is true that perceptual minima compose the idea of space, what is the nature of the minima? Are perceptual minima extended or unextended? One might argue that perceptual minima must be extensionless since an extended minimum is divisible into smaller parts, there existing something smaller than a

minimum. But if minima are extensionless, how is it possible to perceive something with perceptual qualities like color if perceptual qualities require extension? Can one perceive color without that color being extended? Moreover, how can one align an extensionless part next to another extensionless part if the extensionless parts lack distinguishable sides? Extended and extensionless minima are problematic since either alternative leads to contradiction. If either alternative leads to contradiction, then one could claim that the idea of finitely divisible space is also contradictory. We must determine why a thinker would argue for perceptual minima being either extended or extensionless.

The philosophers, George Berkeley and David Hume, agree that perceptual minima or colored points comprise our ideas of space. Yet Berkeley believes that minima are extended, while Hume argues that minima are unextended. What makes this divergence in their theories of minima baffling is the observation that their arguments for space consisting of perceptual minima are similar. If their arguments are similar, then one would expect that Berkeley and Hume agree that minima are extended or unextended. One suspects that their concepts of minima and perception differ. We shall find that Berkeley and Hume differ with respect to their theories of minima because of different separability criteria for ideas. Berkeley's weaker separability criterion stems from his *esse est percipi* principle and claim that color requires perception in order to be perceived, while Hume's stronger separability criterion comes from a stronger emphasis on the nature of simple or undivided ideas—if an idea has distinguishable content from another idea, then the ideas can exist apart from one another. Thus, Berkeley must argue that minima are extended since unextended minima cannot be colored and it is only through color that minima are distinguishable from one another; however, Hume must maintain that minima are unextended because extended minima have distinguishable parts, which are

separable from the entirety of the minimum. Consequently, Berkeley's minima instances of a whole with parts possessing parts that are not more ultimate than the whole itself; Hume's minima are examples of a whole without parts conditioning the possibility of a greater whole. Not only does the dispute concerning the nature of minima impact the divisibility of space, but shows a difference in the nature of ideas and perception.

I shall detail Berkeley's and Hume's accounts of colored minimum perceptions as a response to Pierre Bayle's Trilemma, providing an account of Humean space composed of minima. I will also provide evidence for Berkeley's and Hume's conclusions that minima are extended and unextended respectively through their minimum perception and annihilation arguments respectively. I shall then determine why Berkeley and Hume differ on the nature of minima, concentrating on the reasons Berkeley's minima are extended. Berkeley reasons that minima must be extended due to the interdependence between color, extension, and the necessity of finite perceptions, while Hume maintains that minima must be unextended because of the independence of color and extension, and the necessity of finite perceptions.

Chapter One-Section D.I: Berkeley, Hume, and Pierre Bayle on Minima

The absence of negative ideas due to Berkeley's *esse est percipi* and Hume's distinction between impressions and ideas has shown why Berkeley and Hume reject the infinite divisibility of space, concluding that space consists of minima. Yet how does space consist of minimum sensibles? Does it consist of minimum sensibles in the manner by which a house, as space, is merely the sum of the volumes of a collection of bricks: the composition thesis? Are the minimum sensibles related to one another as particular whole numbers are related to one another

along a number line, each being defined as non-overlapping parts: the dispositional thesis?¹⁴²

These two theses highlight an important point regarding the notion of simplicity afforded through MIST and the belief in the finite capacity of the human mind: the simple being is a constitutive unit not only of matter but also of space. In this manner, the features of matter must conform to the characteristics of space.

I will show which theory of finitely divisible space Berkeley's and Hume's denial of negative ideas lead, focusing especially on Hume's account of minima from his *Treatise*. In order to understand the two philosophers' theories of space, it is necessary to trace the idea of the minimum sensible to Pierre Bayle. Pierre Bayle, in his *Historical and Critical Dictionary*, has written an article on Zeno of Elea in which he details the problem of infinite divisibility. After detailing Zeno's four paradoxes of motion, Bayle provides an interpretation of the import of Zeno's paradoxes of motion: Zeno's paradoxes of motion could show that space does not exist.¹⁴³ Bayle's Trilemma leads to a skeptical conclusion concerning the reality of space. David Hume and George Berkeley acknowledge Bayle's Trilemma; however, Hume formulates a fourth alternative that evades Bayle's skeptical conclusion that space or extension does not exist, namely that of tangible or colored minimum sensibles, while Berkeley disagrees with Bayle that extended physical points cannot compose extension, founding his theory of extended minima.¹⁴⁴ An investigation into Hume's response to the annihilation argument, which shows that mathematical points cannot comprise space, reveals Hume's theory of colored unextended minimum sensibles. By arguing for tangible or colored minimum sensibles, Hume argues for a

¹⁴² Falkenstein, Lorne. "Space and Time". *The Blackwell Guide to Hume's Treatise*. Malden, MA: Blackwell Publishing, 2006. 59-76. 63.

¹⁴³ Bayle, Pierre. *Historical and Critical Dictionary : Selections*. translated, with an introduction and notes, by Richard H. Popkin, with assistance of Craig Bush. Indianapolis: Hackett Publishing, Co, 1991. .351-359.

¹⁴⁴ Jacquette, 22.

dispositional theory of space since the color of the minimum sensibles and the claim that space is originally a compound impression, allow the minimum sensibles to be particular locations in space rather than having the minimum sensibles occupy particular locations of space: the addition of color makes each minimum distinct, preventing annihilation, and by being found originally in a compound impression, each minimum is defined as being non-overlapping with other minima. Hume's space is dispositional rather than compositional. The principle that knowledge of a mental process requires perception of the mental processes' product makes Humean space dispositional instead of compositional. Now let's see how Hume and Berkeley respond to Bayle's Trilemma, beginning with Hume's and Berkeley's thoughts on mathematical points.

Hume finds Bayle's rejection of mathematical points helpful because minimum sensibles must be extensionless like the mathematical points. Minimum sensibles must be extensionless because it would be contradictory to claim that minima are extended. Whatever is extended has parts. Such extended minima are distinguishable into smaller parts. What consists of smaller parts cannot be a minimum. Donald Baxter can help our understanding of Hume's claim that minima cannot consist of smaller parts.¹⁴⁵ Anything that has spatial parts is extended. What is extended must have at least two distinct termini. By being distinguishable from one another, two termini are separable. For no termini necessitates the existence of another termini. What is separable is divisible. Therefore, what has spatial parts is divisible. If minima were divisible or extended, they would have parts smaller than themselves. This claim is contradictory because minima must be the smallest parts of bodies or extension. Therefore, minima must be

¹⁴⁵ Baxter, Donald. "Hume on Infinite Divisibility". *History of Philosophical Quarterly*. Vol. 5, No. 2, April 1988. 133-140. 134; 138-139.

extensionless. However, their extensionlessness does not solve the annihilation argument. Therefore, minima must be extensionless and have some other quality or set of qualities that precludes their annihilation. The solution is purportedly found in Bayle's account of physical points. Now let's see why Berkeley would also object to mathematical points comprising space and bodies.

Berkeley, if he rejects the possibility of color without extension, finds extensionless minima impossible via the annihilation argument; however, Hume, since he entertains the possibility of color without extension, accepts minima being extensionless. For Berkeley, minima cannot be extensionless and constitute space and bodies since the annihilation argument is sound. Not only does Berkeley believe that minima cannot be extensionless due to the validity of the annihilation argument, but he thinks that minima cannot be unextended since, unlike Hume, color and solidity belong only to what is extended. Berkeley's thoughts concerning the dependence of color on extension stem from an argument against the distinction between primary and secondary qualities. The upshot of Berkeley's argument is that secondary qualities like color, sound, etc... depend on primary qualities like extension, motion, etc... and vice-versa instead of solely having secondary properties dependent on primary qualities. Hence, we will have understood Berkeley's weaker separability criterion of ideas. Unlike Berkeley, Hume possesses a stronger separability criterion of ideas on account of his grain of sand argument and complete separability of simple ideas. Color and the minimum are perceived as two distinct ideas and must be separable. Thus, there is no contradiction in combining color and something extensionless like minima on account of there being two distinct simple ideas. Thus, color can exist without extension and Hume can provide his alternative to Bayle's Trilemma, unextended

colored minimum sensibles. Now let's determine what Hume and Berkeley think about physical points in relation to their theories of colored minima.

Hume *agrees* with Bayle that physical points cannot be the ultimate constituents of space and bodies since physical points possess parts, but he *disagrees* with Bayle that physicality can only be applied to a minimum that is extended; however, Berkeley *disagrees* with Bayle's conclusion that minima cannot be extended, but *agrees* with Bayle's contention that physicality or color can only belong to extended minima. Once again, Hume's and Berkeley's difference on the separability criterion of ideas distinguishes their responses to Bayle's Trilemma. Let's begin with Hume's account of physical points and then discuss Berkeley's interpretation of physical points. Hume incorporates the physicality of Epicurean points in his theory of tangible or colored minimum sensibles. Hume and Bayle, therefore argue that there are no extended indivisible points because any extension is divisible.¹⁴⁶ Physicality solves the problem that plagued the possibility of mathematical points comprising space: annihilation. Recall that mathematical points cannot be aligned by wholes because they would be in one another. The addition of physicality, solidity or tangibility, to these points would exclude the possibility of their annihilation according to Hume.¹⁴⁷ The solid or tangible minima would repel one another, preventing their annihilation. Epicurean atoms or physical points are indivisible physically and are divisible conceptually, but they can be aligned. Their physicality enables their alignment. Yet they cannot be the ultimate constituents of space because they are divisible conceptually.¹⁴⁸ Mathematical points are conceptually indivisible, but they cannot be aligned with one another

¹⁴⁶ Creery, Walter E., ed. *George Berkeley: Critical Assessments Volume II*. David Raynor, "Minima Sensibilia in Berkeley and Hume". London: Routledge, 1991 372-373.

¹⁴⁷ *Treatise* I.II.IV.6.

¹⁴⁸ Raynor, David. "Minima Sensibilia in Berkeley and Hume". Creery, Walter E., ed. *George Berkeley: Critical Assessments Volume II*. London: Routledge, 1991 372-373.

without their annihilation. Yet the addition of physicality to mathematical points prevents their annihilation. Seeing the positives and negatives of the theories of mathematical points and physical/Epicurean atoms, Hume incorporates the conceptual indivisibility of mathematical points and the physicality of Epicurean atoms to his theory of tangible minima.¹⁴⁹

Berkeley disagrees with Bayle that the smallest constituents of space cannot be extended, siding with a theory of minima similar to physical points. Berkeley and Hume agree that the addition of solidity or tangibility to these points would exclude the possibility of their annihilation because then one point is distinguishable from another point.¹⁵⁰ The solidity or tangibility of the minima would repel one another, preventing their annihilation. Moreover, by virtue of being extended, Berkeley's minima cannot face the same issue that unextended minima would face: annihilation. The crucial issue with which Berkeley and Hume deal in the problem of minima is the extent to which something extended can be metaphysically indivisible. Berkeley clearly believes that extended minima are metaphysically indivisible, but Hume would claim that extended minima is contradictory since whatever is extended is also divisible metaphysically.

Dale Jacquette believes that Hume incorporates the property of extensionlessness into his theory of minima from Bayle's refutation of physical points, reinforcing Hume's assertion that whatever is extended is also metaphysically divisible.¹⁵¹ Thus, Berkeley must disagree with the claim that whatever is extended is always metaphysically divisible. This disagreement is perplexing given that for most extended things, Berkeley would be able to think that such things

¹⁴⁹ Jacquette, 25-27.

¹⁵⁰ Hume, 35.

¹⁵¹ Jacquette, 26-27.

are metaphysically divisible. If Berkeley perceives a wooden table which is clearly divisible into two, independently existing halves as two blocks of wood, this table's extension grants it metaphysical divisibility; however, an extended minimum cannot be divided into two independently existing halves without annihilation.

Surely, Berkeley must think that the extended minimum is at least intellectually divisible; otherwise, Berkeley cannot share a similar idea of space or extension as other philosophers. Why then does Berkeley allow this exception concerning metaphysical divisibility and minima? The answer, as I shall show, is a matter of a weaker separability criterion of ideas than Hume, forcing Berkeley to conclude that color or tangibility can only be perceived through extension. Thus, Berkeley's weaker separability criterion of ideas is linked to his *esse est percipi* principle, which requires that extension is necessary for the perception of color, unlike Hume who argues that color and the minimum are distinct ideas via his stronger separability criterion of ideas. After I demonstrate this difference between Berkeley and Hume on perception and color, I will show how Berkeley believes that the minimum's location must be distinct from the minimum itself through his weaker separability criterion of ideas and why Hume thinks that the minimum's location must be identical to the minimum itself.

We shall find that the separability of ideas for each Empiricist philosopher is a matter of perception. Part of Berkeley's issue concerning unextended minima is the difficulty of perceiving these minute parts of bodies or space. Thus, Berkeley finds the existence of unextended minima doubtful due to his *esse est percipi* principle. Interestingly, Hume also finds the perception of these minute parts of space difficult. Hume writes in his *Treatise of Human Nature*:

“For as the points, which enter into the composition of any line or surface, whether perceived by the sight or touch, are so minute and so confounded with each other, that ‘tis utterly impossible for the mind to compute their number, such a computation will never afford us a standard, by which we may judge of propositions.”¹⁵²

When attempting to determine whether certain lines are longer, shorter, or equal to one another, one cannot inspect the entire length of the line by inspecting the points themselves. What this observation shows is that perceptual minima, while forming a line or extension, are indistinct and imperceivable in themselves. Berkeley could find common ground on this matter with Hume and persuade him that unextended minima cannot exist if human knowledge relies on immediate perception. He thinks that extension is at least intellectually divisible, but ultimately composes of metaphysically indivisible extensions. The parts of the extended minimum exist and are perceivable only by being part of the minimum according to Berkeley. Therefore, Berkeley’s minima are most like Epicurean atoms or physical points by being extended, while Hume’s minima are most like mathematical points and also combined with the physicality of Epicurean atoms or physical points to prevent their annihilation.¹⁵³ What is left undetermined is precisely how Berkeley thinks that color requires extension. We shall determine this point after seeing how Hume thinks that his colored unextended minimum is an alternative to Bayle’s Trilemma.

Taking Bayle’s Trilemma into account, Hume’s fourth alternative in response to the annihilation argument can be formulated. The refutation of space being infinitely divisible necessitates space being composed of mathematical points, physical points, or something else. Mathematical points cannot comprise space because such points, simply being extensionless, would be annihilated upon contact. Physical points cannot be the ultimate constituents of space

¹⁵² Hume, 38.

¹⁵³ Jacquette, 25-27.

because they have parts. Yet they can compose space because of their physicality. Thus, space must be finitely divisible and composed of something that is extensionless and physical.

The addition of color or solidity to the minimum sensibles circumvents the annihilation problem and necessitates a relational or dispositional view of space. Hume claims that the addition of color to minimum sensibles makes distinction between minima possible: "...I ask any one, if he sees a necessity, that a colored or tangible point should be annihilated upon the approach of another coloured or tangible point?"¹⁵⁴ What has plagued the alignment of extensionless mathematical points is the fact that the distinction between mathematical points in the same place is impossible. Mathematical points in the same place are indistinguishable by their extensionless natures. If the minima are distinguishable from one another, then the annihilation problem is circumvented. The annihilation problem is circumvented because the addition of color to minima distinguishes the whole of one minimum from the whole of another minimum. With a distinction between different wholes in place, one minimum cannot annihilate another minimum upon contact. Thus, since distinction enables separation per Hume's concept of simple ideas and color is a simple idea, the whole of the minima, cannot merge upon contact by being distinct through color. The result of aligning colored or tangible minimum sensibles is an idea or impression which is compound or divisible, space. Space is originally a complex impression of colored minimum sensibles. How then do these unextended colored minima "compose" a space: do they comprise the complex impression of space in the same way that a multitude of bricks would compose a wall, dubbed the "compositional" thesis, or by some other means, which I will call the "dispositional" thesis? I shall answer that Hume's unextended colored minima comprise space "dispositionally" rather than "compositionally" given that Hume

¹⁵⁴ Hume, 35.

perceives the minima through the complex impression of space rather than perceiving the minima prior to the complex impression of space itself. I shall thus link the two aforementioned ways of viewing the minimum in relation to the complex of space to the “compositional” and “dispositional” theories of finitely divisible space.

Prior to delving in to an argument for Hume’s space being discrete and dispositional rather than discrete and compositional, a brief examination of infinitely divisible, discrete, compositional, and dispositional space is necessary. Discrete space is only finitely divisible with every point on a line having an immediate successor as all whole numbers on the number line—2, 3, 4...—have an immediate successor.¹⁵⁵ Unlike discrete space, points have no immediate successor in infinitely divisible space. Whether discrete space is compositional or dispositional, objects or the minimum sensibles determine spatial relations instead of having spatial relations pre-exist and determine the objects’ relations toward one another as would be the case if space were infinitely divisible.¹⁵⁶ For the whole of infinitely divisible space determines its parts, while the parts of discrete space determine the whole of discrete space. In dispositional discrete space, the two minima pick out the two immediately adjacent locations instead of one colored minimum being put next to another colored minimum and having the new space equal the sum of the quantities of the two colored minima.¹⁵⁷ Thus, two colored minima pick out two locations next to one another in dispositional space, while the resultant impression of two colored minima in compositional space add their volumes together. The minima in discrete dispositional space form space by being defined as non-overlapping parts. By being non-overlapping parts, they

¹⁵⁵ Falkenstein, 62-63.

¹⁵⁶ Slouk, Howard. “Hume and the Perception of Spatial Magnitude”. *Canadian Journal of Philosophy*. 34(3) 355-374. Sept. 2004. 359.

¹⁵⁷ Falkenstein, 62

form two distinct spatial locations. The minima in discrete compositional space are not automatically defined as non-overlapping but are non-overlapping by being aligned next to one another. Thus, one can see that space is defined by the number of locations that colored minima “pick out” in discrete dispositional space, while space is the summation of volumes of aligned colored minima in discrete compositional space.

Being a complex impression of colored minimum sensibles, Hume’s space is discrete and dispositional rather than compositional. Yet one could offer an objection to the dispositional thesis on account of Hume’s response to the annihilation argument. Hume speaks of the blue and the red colored minimum sensibles being *aligned* next to one another to form a compound and divisible impression of colored space.¹⁵⁸ If space were dispositional, the minima would not have to go through a process of being alignment to form space.¹⁵⁹ However, space is dispositional since the idea of space is copied from a compound or complex impression of space.¹⁶⁰ It is not copied individually from the simple impressions, colored minima, but from the entirety of the simple impressions forming a compound impression. The colored minima, by being part of a compound impression, are disposed in a certain manner by being perceived as non-overlapping. Despite being inseparable from the colored minima, the idea of the manner of disposition still allows for the separability of the colored minima per Hume’s separability criterion of distinct ideas: if an idea is distinct from another idea, then one idea can exist or be separate from the other idea.¹⁶¹ According to Lorne Falkenstein, if one interprets the colored minima as distinct perceptions from one another and also from the compound impression of

¹⁵⁸ Hume, 35.

¹⁵⁹ Slouk, 360-361.

¹⁶⁰ Falkenstein, 68.

¹⁶¹ T 1.1.7.18; SBN, 25)

colored minima, forming at least three perceptions, then one can still conceive the colored minima individually, as Hume does in response to the annihilation argument, and also the colored minima being part of the compound impression of space.¹⁶² Imagine that there are two colored minima, a blue and a red one, coupled with the idea of a blue minimum that is conceived to the right of a red minimum. One can distinguish between the separate blue and red minimums. Yet the conception of a blue minimum alone can be conceived without also conceiving of a blue minimum next to a red minimum and vice-versa. However, the relation between a blue minimum and a red minimum cannot be conceived apart from the minima: the relation presupposes the colored minima. Thus, the relational discrete concept of space is compatible with Hume's separability criterion of ideas provided that one can distinguish between the minima as related to (disposed toward) one another and the minima themselves. Hume, therefore, advances that space is discrete or finitely divisible, consisting of colored, extensionless minimum sensibles according to a manner of disposition, being non-overlapping.

Pierre Bayle's Trilemma is important for understanding Berkeley's and Hume's account of minima since the absence of an account of minima renders space an impossibility. The two possible constituents of space important for this section are mathematical points and physical or Epicurean points. Mathematical points are extensionless, while physical points are extended and solid. Berkeley and Hume affirm the existence of space consisting of colored minimum sensibles.¹⁶³ Berkeley clearly takes issue with mathematical points (or indivisibles constituting space) being extensionless. He incorporates the extension of physical points into his theory of colored and extended minimum sensibles; Hume, however, recognizes that the ultimate

¹⁶² Falkenstein, 68-69.

¹⁶³ Falkenstein, Lorne. "Space and Time". *The Blackwell Guide to Hume's Treatise*. Malden, MA: Blackwell Publishing, 2006. 59-76. 60-64; Jacquette, 7-8; Robles, 95; Gray, 416.

constituent of space is extensionless and includes color or solidity from physical points to formulate his theory of unextended, colored minimum sensibles.

Berkeley's solution to Bayle's Trilemma is similar to Hume's save that the minima are extended rather than unextended. Berkeley takes issue with Bayle's criticism of physical points. He acknowledges the possibility that minima can be extended without the possibility of being separable. Why he admits this possibility requires further exploration. Now let's see how Berkeley's weaker separability criterion of ideas is shown through his account of color and extension.

Chapter One-Section D.II: Berkeley versus Hume on the Nature of Extended Minima

It is questionable whether Berkeley believes that his minima are extended. Most of the arguments in favor of Berkeley's minima being unextended come from various passages in the *Alciphron* and the *New Theory of Vision*. David Raynor argues that Berkeley's minima must be unextended because it is contradictory to say that something is indivisible and has parts.¹⁶⁴ He also cites a passage from *Alciphron* in which minima are the same no matter what sort of thing the percipient is: the appearance of long and short distances from a certain perspective is identical in magnitude.¹⁶⁵ Yet if long and short are of identical magnitude, the greater is equal to the less, they lack magnitude. David Gray cites a passage from Berkeley's *New Theory of Vision* to argue that minima cannot consist of parts; otherwise the minima of a mite would be different from that of a human being.¹⁶⁶ If the mite's minima were smaller than that of the human's, then the mite's minima could be multiplied to equal the relative size of the human's minima. The

¹⁶⁴ Raynor, 370-371.

¹⁶⁵ *Ibid.*, 371.

¹⁶⁶ Gray, 417.

human's minima would no longer be indivisible—no longer a *minimum*. Thus, minima, being indivisible, must be identical and unextended. For, if they weren't indivisible as is the case the mite's minimum versus that of the human, then one minimum would be larger than another minimum; however, it is absurd to have two different minimums since being a minimum precludes there being something smaller than what is called a minimum. One could argue that if a minimum is indivisible and extension is divisible, then no minimum could be extended.¹⁶⁷ Daniel Flage argues similarly that Berkeley's minima must be unextended because they are indivisible and indivisibles must be unextended according to current geometrical assumptions.¹⁶⁸ Given these reasonable assumptions and principles, it is difficult to see why Berkeley's minima could be extended.

Commentators, such as Douglas Jesseph and H.M. Bracken, argue that Berkeley's minima are extended. Bracken argues that Berkeley's denial of abstraction or abstract ideas necessitates the combination of color and extension: if color requires extension, then if minima are colored (as Hume later claims), minima must be extended.¹⁶⁹ In the *Dialogues*, Berkeley argues that color and extension (space) are inseparable through his denial of the distinction between primary and secondary qualities. The refutation of primary and secondary qualities is due to the denial of abstract ideas. The denial of abstract ideas is traceable to *esse est percipi*: if an idea can only be perceived in terms of possessing particular qualities, then abstract ideas cannot exist.¹⁷⁰ Contra Daniel Flage, Douglas Jesseph maintains that Berkeley's mathematics

¹⁶⁷ Raynor, 372; Gray, 417-418.

¹⁶⁸ Flage, Daniel. *Berkeley's Doctrine of Notions: A Reconstruction Based on his Theory of Meaning*. New York: St. Martin's Press, 1987. 50.

¹⁶⁹ Bracken, H.M. *Bayle, Berkeley, and Hume: Eighteenth Century Studies II*. New York and London: St. Martin's Press, 1977-1978. 227-245; Jesseph, 20-33.

¹⁷⁰ Many types of abstract ideas are argued against in Berkeley's *Principles* titled by various commentators as types A, B, and C. See Beardslay, Monroe C. "Berkeley on 'Abstract Ideas'". *Berkeley: A Collection of Critical Essays*.

and geometry run counter to the current mathematical and geometrical assumptions of his time.¹⁷¹ Like most of his philosophy, Berkeley emphasizes the concrete over the abstract through *esse est percipi*, unlike current mathematicians and geometricians.¹⁷² Jesseph also argues, referring to the annihilation argument, that it is impossible for a collection of unextended things to compose space or extension, or that something visible could be composed of invisible parts.¹⁷³ It seems, therefore, that there are good reasons to suppose that Berkeley's minimum sensibles are extended rather than unextended.

If one examines Berkeley's refutation of the distinction between primary and secondary qualities due to *esse est percipi*, it is clear that Berkeley's minima must be extended since color cannot exist without being extended and it is impossible for unextended things to comprise extension. In the *Dialogues*, Berkeley claims that primary and secondary qualities are inseparable. He first shows that primary qualities, such as motion, cannot be perceived without secondary qualities:

“But how doth it follow that because I can pronounce the word *motion* by itself, I can form the idea of it in my mind exclusive of body? Or because theorems may be made of extension without any mention of *great* or *small*, or any other sensible mode or quality...”¹⁷⁴

Although someone can speak of primary qualities without mentioning a secondary quality, it is evident that one cannot perceive a particular primary quality without also perceiving particular secondary qualities. Certain secondary qualities belong to primary qualities. If there is a motion, it is either fast or slow. If there is extension, it is either large or small. If primary

Armstrong and St. Martin, C.B., eds. New York and London: Garland Publishing, Inc., 1988. 409-426. 410-416; Pappas, George S. “Abstract Ideas and the ‘esse est percipi’”. New York: Garland Publishing, Inc., 1989. 185-211. 191-195; 207-209; Craig, E.J. “Berkeley's Attack on Abstract Ideas”. *Berkeley: A Collection of Critical Essays*. Armstrong and St. Martin, C.B., eds. New York and London: Garland Publishing, Inc., 1988. 121-133. 121-126.

¹⁷¹ Jesseph, 57.

¹⁷² Wild, 176-178.

¹⁷³ Jesseph, 57.

¹⁷⁴ Berkeley, 170.

qualities exist, then there are necessarily secondary qualities attendant with them. Primary qualities, now being inseparable from secondary qualities, lose their independent status which had distinguished them from secondary qualities. The converse holds also for Berkeley, making secondary qualities dependent on primary qualities: “But do not colours appear to the eye as coexisting in the same place with extension and figures?”¹⁷⁵ Since the perception of colors is indistinguishable from extension or figure, color and extension are interdependent. One cannot perceive color without perceiving extension and vice versa. Berkeley acknowledges that unextended things, like mathematical points, cannot comprise something extended.¹⁷⁶ One cannot use color to aid in distinguishing between mathematical points or minimum sensibles since color can only exist as extended. Berkeley’s minima must be extended.

Taking a passage from the *New Theory of Vision* in which Berkeley asks the reader to “conceive colour without visible extension”, David Raynor argues that Berkeley’s focus is not really proving that color must exist with extension, but that extension cannot exist without color.¹⁷⁷ If the visual field is always extended, we never perceive minima alone. If minima are colored, we would never perceive color without extension. Thus, minima could still be colored while having them always be parts of extension. Hence, color is ascribable to unextended things, while always being perceived with extension.

Raynor’s observation from the *New Theory of Vision* is apt; however, Berkeley has never mentioned whether the minima are colored by themselves or by being constituents of extension. If the latter is true, then color really belongs to what is extended rather than what is unextended;

¹⁷⁵ Berkeley, 178.

¹⁷⁶ Raynor, 371.

¹⁷⁷ Raynor, 371.

if the former is true, then Raynor is correct. What is clear from the *New Theory of Vision* passage is that color is always perceived to be attendant with extension. Simply because one's visual field is never solely of one minimum, it is not the case that whatever is true of the visual field as a whole is also true of its ultimate parts, the minima. Raynor must begin with the assumption that minima are colored in order to reconcile the claims that we can never perceive color without extension and that color can be unextended. For Berkeley nowhere mentions that something extensionless is colored rather than something extended being colored. Raynor's *New Theory of Vision* quotation is indifferent to either claim of the relation between color and extension. Moreover, if Berkeley finds that minima are difficult to distinguish from one another, how can we claim that color has been added to them? Extensionless things are indistinct or invisible.¹⁷⁸ It is absurd to combine something visible like color to something invisible like a minimum or point. If the passages from the *Dialogues* have been interpreted correctly, the inseparability of color and extension shows that color is extended given *esse est percipi*.

Since color is imperceivable apart from extension, Berkeley's theory of perception indicates a weaker form of the separability criterion of ideas: if two or more ideas are distinct from one another and are extended, then one idea can exist without the other idea. If Berkeley maintained a stronger form of the separability criterion, as Hume does, one idea would be separable from another idea if the ideas have distinguishable content. Thus, Berkeley's weaker form of separability criterion shows us a different meaning of the minimum as indivisible: a minimum is indivisible if it is perceivable according to the smallest possible part of extension. Although this minimum has parts because it is extended, one must hold that this minimum is the most ultimate part of extension. The most ultimate part of extension must be perceivable. If

¹⁷⁸ Raynor, 371.

minima alone are imperceptible, then they would not exist. Consequently, the parts of the ultimate extended parts of extension cannot exist independently of the extended minimum. To counter David Gray's and David Raynor's contentions that an indivisible must be unextended because extension is divisible, only extended minima are perceptible and exist. "Minimum" refers to whatever is perceptible according to the smallest perceptible extension. If extension is only finitely divisible, then extension comes in "chunks" of smallest possible extensions. It is these "chunks" of smallest possible extensions that we call minimum sensibles in Berkeley's philosophy of geometry.

We have seen that Berkeley's *esse est percipi* principle and refutation of the distinction between primary and secondary qualities have led to the conclusion that minima are extended because color cannot exist without extension due to a weaker separability criterion of ideas. Hume, however, maintains a stronger separability criterion of ideas given his claim that color can be unextended.¹⁷⁹ Minimum sensibles must be colored in order to avoid the annihilation problem according to Hume. Noticeably absent from the *Treatise* is a discussion of primary and secondary qualities. The aforementioned point is likely due to Hume's agnosticism with respect to external objects, while Berkeley sought to refute the existence of external objects. For distinction between primary and secondary qualities exists only if there are external objects. We are left to examine Hume's arguments for minimum sensibles, such as the grain of sand argument to show why minima can be unextended and colored. What is shown is that Hume's grain of sand argument necessitates the perception of something absolutely indivisible, while Berkeley's minimum perception argument never specifies whether minima have parts.

¹⁷⁹ Smith, Norman Kemp. *The Philosophy of David Hume: A Critical Study of its Origins and Central Doctrines*. London: Macmillan and Co. Limited, 1941. 277.

Hume's grain of sand argument shows that if minima have parts, then these parts would be clearly distinguishable from the minimum itself. Hume takes the idea of the minimum sensible or grain of sand, asking whether it is possible to distinguish the minimum into parts.¹⁸⁰ However, no one can distinguish any lesser part than what is already the minimum. Presumably, if one part were distinguishable from another part, then the parts must have their own content.¹⁸¹ Since extension is measurable, if a minimum were three inches, the measure of the minimum shows that the minimum has parts. The content of the parts is a smaller quantity or size. Smaller quantities or sizes are distinguishable and separable from larger quantities or sizes; otherwise, smaller quantities or sizes must be identical to larger quantities or sizes. If minima had parts, then there would be something more minimal than what is already the minimum, which is contradictory.¹⁸² Since nothing distinguishable from the minimum constitutes the minimum, the minimum sensible must be extensionless. Consequently, the minimum sensible is a simple idea that constitutes the idea of space.

Since the minimum sensible and color are simple ideas, they are independent of one another and combinable to form an extensionless colored minimum sensible. The color green is distinguishable from the minimum sensible because a minimum sensible is not necessarily green.¹⁸³ Although one might not necessarily perceive the color green without being combined with a minimum sensible, the content that constitutes the color green is distinguishable from the

¹⁸⁰ Hume, 25.

¹⁸¹ This claim is derived from Hume's principle that all distinct ideas are separable. See Hume, 22.

¹⁸² Raynor, 372.

¹⁸³ Given Hume's account of the abstract ideas, color and the figure colored (or extension) are distinctions of reason. Color and the figure colored are indistinguishable according to perception, but are distinguishable according to different resemblances. Gerald Vision claims that if simple ideas are abstract particulars for Hume, then color and extension are separable because they are their own distinct ideas. Thus, while sense perception never reveals color without extension, if color and extension are abstract particulars, the color can exist without extension. Consequently, color and extensionless minimum sensibles are combinable. See Hume, 23 and Vision, Gerald. "Hume's Attack on Abstract Ideas: Real and Imagined". *Dialogue: Canadian Philosophical Review*. 18, 528-537, 10 p. December, 1979. 194-195.

content that constitutes the minimum sensible. For green does not measure extension, but the minimum sensibles that comprise extension measure extension. Whenever one idea is separable from another idea, the two ideas are combinable provided that they are not contraries. Thus, green and the minimum sensible (extensionless) are combinable by being two simple ideas, the minimum sensible is extensionless via the grain of sand argument. Unlike Berkeley, color and extensionlessness are compatible for Hume due to a stronger separability principle: whatever (simple) ideas are distinguishable according to distinct content, must be separable.

Unlike Hume's grain of sand argument, Berkeley's minimum perception argument only shows that there must be a minimum perception, omitting whether such minimum is extended or extensionless. We have seen that the minimum perception argument shows that space is not infinitely divisible and must consist of minima. Hume uses this argument to show that the minimum must be extensionless as is the case in the grain of sand argument. Berkeley only establishes in the minimum perception argument that innumerable parts do not constitute finite space.¹⁸⁴ Nothing is stated about the parts that comprise finite space. Whether minima are extended or unextended is inconclusive given the absence of information.

Berkeley has an argument akin to Hume's grain of sand argument; however, unlike Hume's grain of sand argument, whether minima are extended or unextended remains ambiguous. Berkeley states in the *Principles*: "There is no such thing as the ten-thousandth part of an *inch*."¹⁸⁵ Instead of beginning from the minimum sensible like Hume, Berkeley starts with a divisible measure, an inch. We know that an inch cannot be infinitely divisible. Thus, there is a finitude of minima constituting the inch. By beginning reflection with a divisible quantity,

¹⁸⁴ Berkeley, 135.

¹⁸⁵ Berkeley, 136.

Berkeley leaves open the possibility that minima are extended. It is conceivable that a finite quantity of extended colored minimum sensibles could compose extension as a series of extensionless colored minimum sensibles could comprise space. However, since Hume begins with the minimum sensible itself and observes the impossibility of dividing something that lacks parts, minima must be extensionless. Only if one takes into account Berkeley's refutation of the distinction between primary and secondary qualities must minima be extended because color is not combinable with what is extensionless. Since Hume has no recourse to primary and secondary qualities and provides the grain of sand argument, minimum sensibles must be extensionless.

Chapter One-Section D.III: Berkeley, Hume, and Metaphysical Simplicity

Now that we have seen why Berkeley and Hume differ regarding their separability criterion of ideas on extension and color, let's show how these principles inform their theories of metaphysical simplicity and its compatibility with intellectually distinguishable parts. We have seen how Berkeley's principle of *esse est percipi* led to his denial of the distinction between primary and secondary properties, resulting in the interdependence of color and extension via a weaker separability criterion of ideas. Hume's distinction between color and the minimum via his response to the annihilation argument enables him to formulate a stronger separability criterion of ideas, paving the way for the possibility of colored unextended minima. I also intend to draw out these differences concerning the separability criterion of ideas more explicitly.

Metaphysical divisibility and intellectual divisibility relate to the parts or aspects of thing which are distinguishable from one another. Something that is metaphysically divisible consists of two parts or aspects which can exist apart from one another and count as their own distinct

beings. To divide a block of wood into two equal halves is an instance of metaphysical division because each half of the block of wood continues to exist separately. Note that the aforementioned example of halving a block of wood also counts as physical division.

Physical division is a kind of metaphysical division that pertains to the use of natural or physical forces to divide at least two parts of a thing into distinct spatial locations. Something can be physically *indivisible* while being metaphysically divisible. One cannot divide physically an Epicurean atom, which possesses extension, but one can also perceive the parts of an extended atom which could be divided into supposedly smaller atoms. It is only by having something be metaphysically *indivisible* that the physical *indivisibility* of a being is guaranteed. For there are no parts to divide physically into two distinct locations in space if there is no possibility of distinguishing at least two separately existing parts of a thing. Thus, being the opposite of metaphysical divisibility, metaphysical indivisibility is the *incapacity* of a thing to be divided into at least two parts or aspects that could exist separately from one another.

This incapacity to be divided into at least two separately existing parts of a thing results either from a lack of intellectually divisible parts or from a necessary interdependence between the parts or aspects of a thing. Intellectually divisible parts are the non-identical parts of a thing that may or may not possess the ability to exist separately from one another. For example, should one believe that color depends on extension for its existence, as Berkeley believes in his theory of minima, then color is only intellectually divisible or distinguishable from extension. Color cannot exist apart from extension, but is not identical to extension given that an extension can be colored differently. Intellectual indivisibility, therefore, indicates that something does not possess any distinguishable parts. A mathematical point is intellectually indivisible since it is

defined as “that which has no part” in Euclid’s *Elements*. Something that is intellectually indivisible is also metaphysically indivisible since there aren’t any non-identical parts of an intellectually indivisible thing to exist separately from one another. Thus, it is possible for something to be intellectually divisible and metaphysically indivisible; however, something cannot be metaphysically divisible without being intellectually divisible. Berkeley’s colored extended minima are intellectually divisible and metaphysically indivisible, while Hume’s colored unextended minima are metaphysically indivisible and intellectually indivisible.

I will demonstrate the metaphysical indivisibility of Hume’s colored unextended minima results from a lack of intellectually divisible parts, while the metaphysical simplicity of Berkeley’s colored extended minima is due to an interdependence of color and extension. For Berkeley, the minimum’s simplicity is best understood as being identical to the limits of the minimum’s intellectually distinguishable parts.

Berkeley’s refutation of the distinction between primary and secondary properties forces his simple beings to be extended and colored. Without their distinction, primary and secondary qualities are interdependent for their existence. Color, a purported secondary quality, can only exist through being extended, a supposed primary quality. Now if what is colored is extended and what is extended is colored, and also space and material bodies are complexes of minima, minima must be colored *and* extended.

Despite the conclusion that simple beings (minima) must be extended while retaining their simplicity, it is still baffling to claim that whatever is extended is divisible while also claiming that an *extended* minimum is indivisible. We run into the same problem Hume and Bayle have had with physical points and is a problem which the pre-critical Kant will have to combat given

his force-shell theory. Whatever is extended, like a fourteen inch block of wood, is metaphysically divisible into parts since such parts say, of the wooden block, could exist separately. How could Berkeley argue that an extended thing can be metaphysically simple or indivisible? To clarify Berkeley's argument, I will formulate a *reductio ad absurdum* argument that Berkeley's extended minima are metaphysically indivisible through his reasonings concerning color and extension:

1. Assume that colored extended minima are metaphysically divisible, that one is to divide metaphysically such a colored extended minimum.
2. If one were to divide metaphysically this colored extended minimum in half, two unextended indivisibles would result.
3. The unextended indivisibles cannot be colored per Berkeley's account of color and extension.
4. Two unextended, colorless indivisibles are indistinguishable from one another
 -Hume and Berkeley admit this point as well as proponents of material infinite divisibility. We have a principle common to all parties involved in the problem of matter's divisibility.
5. Should two unextended indivisibles be indistinguishable from one another, then they would cease to exist.
6. Thus, if one were to divide metaphysically the colored extended minimum in half, two colorless unextended indivisibles would result.

7. Therefore, by dividing metaphysically the colored extended minimum, the two resultant parts of the colored extended minimum would cease to exist. But this is absurd by definition of metaphysical divisibility.

Thus, it is impossible to assert the metaphysical divisibility of Berkeley's colored extended minima. While Berkeley's minima are metaphysically indivisible, they are also intellectually divisible by virtue of their being extended. Yet the extension of Berkeley's minima is not a collection of at least two smaller parts that can exist separately from each other and the extended minimum. By virtue of being extended, Berkeley's minima are intellectually divisible; by virtue of color's dependence on extension and the need for minima or indivisible parts of space, Berkeley's minima are metaphysically indivisible. Berkeley's minima are metaphysically indivisible through the existence of at least two intellectually divisible and interdependent parts. The simplicity of Berkelean minima is therefore identical to whole of the intellectually distinguishable parts on account of the interconnection between color and extension.

Hume believes that the minimum's simplicity rests in its intellectual indivisibility since Hume distinguishes sharply between minima and color via his response to the annihilation argument. The annihilation argument is directed against the possibility of mathematical points comprising space and matter. Hume must counter this annihilation argument since his minima are unextended. Being unextended, it is difficult to distinguish one minimum from the other. It is difficult to align one minimum next to another minimum without one minimum penetrating another minimum. Hume's solution is the addition of color or solidity to these minima in order to distinguish one minimum from another minimum. Thus, Hume thinks that color can exist

without being attributed to something that is extended. Color and extensionlessness preserves the metaphysical indivisibility of Hume's colored unextended minima.

Hume rejects the notion that a minimum could be extended since extension is divisible and whatever is divisible is separable. Thus, Hume precludes the possibility of his minima possessing parts. Hume's minima are therefore intellectually indivisible. Through their intellectual indivisibility, colored unextended minima must also be metaphysically indivisible because such minima possess no parts whatsoever that could exist separately.

What we can find from these investigations is that the notion of a simple idea is different for each philosopher: simplicity always entails intellectual indivisibility for Hume, but simplicity only entails metaphysical indivisibility for Berkeley. Berkeley's rejection of the distinction between primary and secondary qualities necessitated color's dependence on extension, requiring that minima be extended. By being extended, Berkeley's minima are metaphysically indivisible and intellectually divisible. Since color depends on extension, Berkeley proposes a weaker separability criterion of ideas. One might suspect that a weaker separability criterion of ideas also necessitates the existence of extended minima, and the combination of metaphysical indivisibility and intellectual divisibility in turn pertaining to the nature of minima. Hume distinguishes sharply between the color and the unextended minimum, allowing for the combination via a stronger separability criterion of ideas as shown earlier in this section. It seems that Hume's stronger separability criterion of ideas makes possible the combination of metaphysical indivisibility and intellectual indivisibility as well. I shall now summarize my conclusions regarding negative ideas and infinity according to Locke, Berkeley, and Hume.

Chapter One-Sections A-D Conclusion

We have seen why John Locke accepts the possibility of a negative idea of infinite divisibility through his affirmation of substance as “that which I know not what”. Built into Locke’s concept of substance or external object is the blueprint of negative ideas, constituting in part the foundation of his theory of ideas. Through negative ideas, an idea can refer to something that is not imagined in the mind—one need not perceive all the content to which an idea may refer. However, Berkeley and Hume lack reference to external objects in their epistemologies. They lack recourse to external objects to found a theory of negative ideas. Berkeley’s emphasis on immediate perception through *esse est percipi* and Hume’s distinction between impressions and ideas according to force and vivacity necessitate positive content, precluding the existence of negative ideas. Without negative ideas, Berkeley and Hume cannot entertain the negative idea of infinitely divisible space. Since the human mind is finite and only positive ideas are possible, entailing that only what the mind immediately perceives counts as reality and knowledge, the human mind cannot complete an infinite process of division and perceive a minimum composing its idea of space, the minimum sensible.

Although Berkeley and Hume share remarkably similar principles and arguments for perceptual minima, the nature of the minima is distinct for each philosopher. The dispute centers on whether color and extension are separable, and the extent to which ideas are separable. For Berkeley, color and extension are inseparable due to his refutation of the distinction between primary and secondary qualities. Unextended minima cannot be colored, not preventing their annihilation. If minima are colored, then they must be extended. However, Hume thinks that color and extension are separable as indicated in his grain of sand argument and response to the

annihilation argument. Color is applicable to unextended minima, forming unextended colored minima. Therefore, through their distinct separability criterion of ideas yet similar theory of ideas through the denial of negative ideas, Berkeley and Hume agree that the idea of space consists of minimum sensibles, but they differ on whether the minima are extended or unextended.

Indivisibility differs radically for Berkeley and Hume. Indivisibility, defined generally, means that something cannot be divided further without the possible remainder ceasing to exist. For Berkeley, indivisibility is relative to the smallest extension. The parts of this smallest extension are perceivable only by being parts of the minimum itself. If one were to divide the minimum, the parts would no longer be perceivable and would cease to exist. For Hume, indivisibility requires that there be nothing further distinguishable that would compose the given minimum. No further parts are perceivable in any manner. Indivisibility for Hume is extensionlessness, while indivisibility for Berkeley is the smallest extension with respect to minimum sensibles.

I would like to point out a latent claim of the principle of (material) sufficient reason in Berkelean and Humean minima. Berkeley argues that color's necessary connection to extension requires that minima be extended. One might take this necessary connection between extension and color as the *sufficient ground* for all composition—precisely in accordance with the principle of material sufficient reason. This sufficient ground thereby divorces Berkeley from the perspective that simplicity *must* function in terms of lacking distinguishable parts. Instead, the extended minimum is a unit not because it is absolutely indivisible but because it is the *sufficient reason* as a unit for their being composition in the first place.

One might thereby compare this notion of a *sufficient ground* to Hume's unextended minima. Hume's minima are surely absolutely indivisible, but they possess their own determination, namely color, without which the minima would be nothing at all. Without minima, no composition would be possible. Thus, colored unextended minima function similarly as the sufficient ground of composition like Berkelean colored extended minima both from the standpoint of requiring color for their continued existence. I will argue in the pre-critical Kant sub-chapter that force-shells are metaphysically simple because they possess their own internal determination or sufficient ground for material composition similar to the manner by which Berkelean and Humean minima possess color as the ground for their metaphysical simplicity and for other composition. Unlike Berkeley and Hume, the pre-critical Kant totally rejects the belief that matter must conform to the features of the space which it occupies through the possession of a distinct internal determination or *sufficient ground* of its simplicity.

Chapter One-Section E: Immanuel Kant and Force-Shells

In the *Physical Monadology*, the pre-critical Immanuel Kant proposes a theory that material bodies occupying infinitely divisible space consist of smallest parts. The smallest parts of these material bodies are called force-shells. Force-shells are indivisible puncta surrounded by a force that either attracts or repels other force-shells according to their distance from one another. They are metaphysically simple or indivisible while being intellectually divisible without forfeiting their simplicity.

Upon hearing Kant's proposal that material bodies can be finitely divisible and occupy infinitely divisible space, this assertion strikes a proponent of material infinite divisibility, as decidedly counter-intuitive and repugnant to reason. The source of this repugnance is the

reasonable belief that the features of a material body must conform to the characteristics of the space which it occupies. Should a space be divisible ad infinitum, there is no smallest part of space to occupy. Thus, a supposedly smallest part of a material body, being only finitely divisible, must be further divisible. Kant must either re-describe the notion of simplicity for his theory of material divisibility or reject the reasonable belief that matter must conform to the features of the space which it occupies, or a combination of both alternatives. He must choose at least one of the aforementioned options to resolve the tension between the metaphysical principle of simplicity and the mathematical truth of space's infinite divisibility.

Kant resolves the metaphysical and mathematical tension by a reformulation of the metaphysical principle of simplicity in terms of the principle of sufficient reason, rejecting the belief that matter must conform to the space which it occupies. The principle of sufficient reason states that a reason must be given for something existing or being of a certain determination rather than otherwise. Should a being provide the basis for its own determinations or existence, it thereby is the sufficient reason for itself. Such a being is metaphysically simple in the sense that it would be contradictory to provide another being or more fundamental aspect of the said being to account for its determinations or indivisibility.

In his force-shell theory Kant lays the framework for the reconceptualization of the metaphysical principle of simplicity via the distinction between internal and external determinations. External determinations constitute the existence of space or the divisibility of a material object into parts. For an object in infinitely divisible space consists of spatially distinct parts, such as left/right and top/bottom. Internal determinations consist of attributes that a being possesses of its own accord. The simplicity considered through the principle of sufficient reason

would be an example of an internal determination. Considered according to external determinations alone, force-shells are divisible; however, force-shells are indivisible when considered according to internal determination. Thus, Kant's distinction between internal and external determinations underlies his distinction between matter and space, indivisibility and divisibility, and enables the reconciliation between the metaphysical principle simplicity and the geometric truth of space's infinite divisibility.

I will link Kant's distinction between internal and external determinations and his force-shell theory to the crucial issue of this dissertation: whether the minimum's simplicity is compatible with intellectually distinguishable parts. The force-shell's internal determination guarantees its metaphysical indivisibility, while the force-shell's external relations to either other parts of space or other force-shells show its intellectual divisibility. Kant's force-shells are therefore comparable to Berkeley's colored extended minima in regards to their metaphysical indivisibility and intellectual divisibility.

To demonstrate my thesis, I will first compare Kant's force-shell theory to Roger Boscovich's account of force-shells as extended minima. A Jesuit thinker, Roger Boscovich has argued a very similar theory of minima by addressing the annihilation problem of unextended minima. He claims that a gap separates each punctum of a force-shell via attractive and repulsive forces that emanate from each punctum. Note that it is unknown whether Kant actually read Boscovich's writings.¹⁸⁶ The value of such a comparison between the two theories is that the reader gains a greater understanding of the philosophical import of force-shell theory in general: by virtue of their extension, force-shells effectively counter the annihilation argument

¹⁸⁶ Holden, 244-245.

which plagues Hume's theory of unextended colored minima while providing a means by which to reconcile the metaphysical principle of simplicity and the mathematical truth of space's infinite divisibility. The reader should note that, unlike Kant, Boscovich believes that space is finitely divisible.¹⁸⁷ This difference between Boscovich and Kant places Boscovich's force-shell theory much closer to the Berkeley's theory of extended minima than Kant.

I will then study Kant's pre-critical work, the *Physical Monadology* to analyze Kant's arguments for the infinite divisibility of space and for the existence of force-shells based on his distinction between internal and external determinations. Having analyzed Kant's arguments for the infinite divisibility of space and for the existence of force-shells, I will show how Kant's distinction between internal and external determinations results in the compatibility between the minimum's simplicity and its intellectually distinguishable parts.

Chapter One-Section E.I:

Force-Shells as Extended Minima: Tracing the Force-Shell Theory to Roger Boscovich

Roger Boscovich provides a detailed account of his force-shell theory in his *Theory of Natural Philosophy*. He argues for force-shells as a variant of extended minima in order to resolve the problems associated with the theory that unextended minima comprise either matter or space. A proponent of unextended minima, like David Hume, would face the problem of composition posed via the annihilation argument. Something unextended cannot be aligned next another unextended thing because all unextended things lack distinguishable sides. Without

¹⁸⁷ Brittan, Gordon G. "The Role of the Law of Continuity in Boscovich's Theory of Matter." *R.J. Boscovich: Vita E Attiva Scientifica-His Life and Scientific Work*. Rome: Treccani, 1993. 222.

distinguishable sides, either a gap or unextended things would have to overlap one another and become indistinguishable.

Philosophers like Hume attempt to solve this issue by ascribing color to his unextended minima. Whether or not Hume responds successfully to the annihilation argument, force-shell theorists like Boscovich seek to avoid this compositional dilemma altogether by supposing that the minima be extended instead. The question for Boscovich is how his minima can be extended when minima are typically interpreted as being indivisible by virtue of their non-extension. Boscovich considers the simplicity of a minimum as a focal point of inherently active matter. Let's first contextualize Boscovich's force-shell theory in the context of the sciences of the time.

Roger Boscovich formulates his force-shell theory of matter in his work, *A Theory of Natural Philosophy*, as a response to the Newtonian account of matter through mass or solidity, as well as to the rather occult quality of Newtonian gravitation.¹⁸⁸ Newton regards mass, solidity, extension, etc... as qualities that inhere in matter. These qualities contrast with other qualities that exist only in the mind of the perceiver—like color and sound. Newtonian matter functions as a substratum, underlying these sensible properties. Unlike Newton, Boscovich proposes that mass, solidity, etc... are not really qualities that inhere in matter, but are the result of the arrangement of force-shells. For example, a particular body would be dense rather than light since the force-shells of this body are arranged in a more compact manner. The arrangement of force-shells in each body is due to varying forces of each shell. Thus, Boscovich regards material bodies as aggregates of a buzz of activity, unlike the Newtonian theory of matter

¹⁸⁸ D'Agostino, Salvo. "Boscovich's Physical Theory of Matter and Space." *R.J. Boscovich: Vita E Attiva Scientifica-His Life and Scientific Work*. Rome: Treccani, 1993. 42-43.

in which the matter is conceived of consisting of physically indivisible parts that comprise material objects.

Boscovich thereby advocates a more dynamic theory of matter opposed to the Newtonian atomism in which matter is considered passive rather than active through properties like mass and solidity. It is in part Boscovich's rejections of atomism through his theory of matter as a buzz of activity that paves the way for alternative understandings of matter, particularly for an alternative account of natural phenomena founded on ascribing to matter a more mental rather than physical existence. William Hamilton and Joseph Priestly are two scientific thinkers who borrowed from Boscovich's dynamical/kinematic theory of matter, each thinker formulating either an idealistic theory material objects or a theory that matter is essentially a "buzz of activity" respectively.¹⁸⁹ Kinematic theory is appealing to other scientists largely because of the unicity or simplification of all natural forces into the attraction and repulsion that one force-shell bears to all other force-shells. Having summarized Boscovich's perspective on the Newtonian account of matter as substratum, let's now examine Boscovich's arguments for force-shells.

¹⁸⁹ William Hamilton is a 19th century scientific thinker who espoused a more mental rather than physical theory of the objects of science. He found Boscovich's dynamic or kinematic theory of matter more appealing than the Newtonian concept of matter as substratum due to Hamilton's own metaphysical idealism, the belief that only our immaterial perceptions truly exist. Thus, Boscovich and other scientists view matter via the force-shell theory as a "buzz of activity". Physicists and chemists later adopt this "buzz of activity" as a model for atomic theory, notably the 18th century chemist Joseph Priestly in his *History of the Present State of Discoveries Relating to Vision, Light, and Colour*. Boscovich's force-shells act as an intermediary between mind and bodies, the former existing through thought and non-extension, while the latter existing through being non-thinking and extension. See Kargon, Robert. "William Rowan Hamilton and Boscovichean Atomism." *Journal of the History of Ideas*. Vol. 26, No. 1 (Jan. – Mar., 1965), pp. 137-140. 138-139; Hamilton, letter to Sydney Hamilton, June 30, 1834, in Robert Graves, *Life of Sir William Rowan Hamilton* (Dublin, 1882-89), II, 93-95; Sutton, M.A. "J.F. Daniell and the Boscovichean Atom". *Studies in the History and Philosophy of Science* I. No. 4 1971. 277-292. 277-279; see also Kargon, 139; and see also Priestley, Joseph. *History of the Present State of Discoveries Relating to Vision, Light, and Colour*. (London, 1772). 383-389; Sytnik-Czetwertynski, Janusz. "The Philosophical Foundations of the Kinematic Atomism of Ruder Josip Boscovich". *Forum Philosophicum: Internation Journal of Philosophy*. 2007. 139-155. 140.

Boscovich's argument for the existence of material minima is based on a demonstration of the impossibility of immediate contact between bodies. Unlike other proponents of the existence of minima, Boscovich argues for minima without an overt appeal to the metaphysical or epistemological need for the priority of simplicity over complexity, but from the theory the matter exhibits attractive and repulsive forces. The attractive and repulsive forces prevent the contiguity of matter, precluding immediate contact between bodies. Boscovich writes: "Now, because the repulsive force is indefinitely increased when the distances are indefinitely diminished, it is quite easy to see clearly that no parts of matter can be contiguous to any other part; for the repulsive forces would at once separate one from the other."¹⁹⁰ I will argue partly on the basis of the aforementioned quote that Boscovich's argument for force-shells contains a latent appeal to the principle of simplicity.

The principle of simplicity is found in Boscovich's usage of the repulsive forces of material bodies: a simple being is required to account for the final repulsive force between at least two bodies. Keep in mind that Boscovich conceives bodies as centers of forces whose attractive and repulsive capacities vary depending on the distance from the punctum. Imagine two blocks of wood. If Boscovich's claim concerning the nature of bodies and forces is correct, these two blocks of wood cannot immediately touch one another. For the repulsive forces that each block of wood causes are indefinitely increased the closer something else is to the said block of wood. Thus, some distance will always remain between the two blocks of wood. Since no two blocks of wood are in immediate contact, one can speculate that *something* that is simple prevents another thing that is also supposedly simple from being in immediate contact with it.

¹⁹⁰ Boscovich, Roger S.J. *A Theory of Natural Philosophy*. Cambridge: The MIT Press, 1966. 43.

Let's trace this line of thought further by considering the possible divisibility of the space between the two parts of matter that are not in immediate contact.

The principle of simplicity emerges in Boscovich's thought when one inquires into the divisibility of the space between each part of matter that is not in immediate contact. We must ask whether the distances between parts of matter continue *ad infinitum* or must there be some part of matter that is simple. The distances between the parts of matter cannot be divisible *ad infinitum* since it is not through the distance or empty space itself that the attractive and repulsive forces exist but through matter itself. Boscovich is asserting here that, in order for there to be forces, some source for the forces must exist. Infinitely divisible matter could not have a source for its forces because there is no real starting point from which the forces begin to emanate. Boscovich reasons that if space is not infinitely divisible, there must be some indivisible or simple part of matter that acts as the source of the forces. These minima cannot be composites or physically divisible parts because it is impossible to join two minima together by immediate contact due to the indefinitely increasing repulsive forces that each minimum exhibits.

The upshot of Boscovich's argument against the immediate contact of bodies is space and matter are distinguishable, unlike the case with Berkeley and Hume. Matter is a series of unextended points which repel and attract one another, preventing immediate contact. Without immediate contact, an empty space or vacuum must exist. In the cases of theories in which minima exist, should immediate contact between material bodies be possible, then one must distinguish between matter and space, as is true in the Boscovich's force-shell theory; however, should immediate contact between material bodies be impossible, one need not distinguish between matter and space, as is the case with Berkeley's and Hume's accounts of minima. Let's

now determine how Boscovich's account of minima circumvents an issue called the problem of contact or annihilation that alternate theories of minima face. It is through Boscovich's response to the problem of contact that we can determine that Boscovich's force-shells are really extended minima, being metaphysically indivisible and intellectually divisible.

Boscovich's distinction between physical contact and mathematical contact is the basis for his response to the problem of contact. As we know, the problem of contact, or the annihilation argument, is directed toward the thesis that space and matter consist of unextended minima. Since unextended minima lack parts, they cannot touch one another without the whole of one minimum being wholly in the entirety of another minimum. The two unextended minima are annihilated due to their indistinguishability upon contact.¹⁹¹ The contact as described in the annihilation argument is what Boscovich calls *mathematical contact*. Mathematical contact is the point at which the distance between at least two things is zero.¹⁹² The annihilation of unextended minima occurs due their lack of parts and the minima being situated in relation to one another without any distance. Denying the mathematical contact between force-shells, Boscovich describes an alternate form of contact applicable to his force-shells: *physical contact*. Physical contact is the point at which the distance between two bodies is *imperceptibly* small.¹⁹³ Boscovich's distinction between between physical contact and mathematical contact corresponds to the 20th century scientific distinction between language of perception and language of theory.¹⁹⁴ In Boscovich's force-shell theory, physical contact is achieved between two force-shells because the repulsive forces prevent any further closing of the distance between

¹⁹¹ Boscovich, 59.

¹⁹² Boscovich, 57.

¹⁹³ Boscovich, 57.

¹⁹⁴ D'Agostino, Salvo, 222.

the two force-shells. It is this distinction between mathematical contact and physical contact that allows Boscovich to improve upon the theories of his predecessors by avoiding the annihilation of unextended minima.

I will now argue that, despite contrary references in Boscovich's text, that Boscovich's force-shells are most properly classed as extended minima given their metaphysical indivisibility and formal divisibility as shown through the absence of immediate contact between force-shells. I shall first explain briefly the relation between extended minima and metaphysical/intellectual divisibility, and the relation between unextended minima and metaphysical/intellectual divisibility. Finally, I shall demonstrate that Boscovich's force-shells are properly considered extended minima rather than unextended minima on account of their intellectual divisibility and metaphysical indivisibility.

The determinations of extended minima and unextended minima are founded on the distinctions between intellectual divisibility and metaphysical divisibility. Being the ultimate constituents of matter or space, minima must be metaphysically indivisible since it is impossible to divide the minimum into two parts and have each part exist independently from the other part. Extended minima are metaphysically indivisible and intellectually divisible. They possess parts or aspects that are distinguishable from one another on account of their extension. Yet they are metaphysically indivisible since the intellectually divisible parts cannot exist independently from the same supposed minimum. Unlike extended minima, unextended minima are intellectually indivisible, lacking distinguishable parts whatsoever. Without distinguishable parts, such minima must be metaphysically indivisible because there cannot be any parts that could exist independently from one another. Thus, we shall now see how metaphysical indivisibility and

intellectual divisibility applies to Boscovich's force-shells, showing that force-shells are extended minima based on my criteria for distinguishing between unextended minima and extended minima. Before proving that Boscovich's force-shells are actually extended minima, I would like to turn to the text to see why the force-shells may be considered unextended minima.

Force-shells might be considered unextended minima because Boscovich refers to his force-shells as an improvement over competing theories of unextended minima, emphasizing the metaphysical center of the attractive and repulsive forces of the force-shell: the force-shell *is* principally a punctum or unextended minimum from which the attractive and repulsive forces emanate. Boscovich's repeated textual references to his theory of force-shells as an improvement over other theories of unextended minima seem to show that the force-shells are unextended minima. Boscovich writes as the conclusion in his argument for force-shells: "Thus, the primary elements of matter cannot be composed of contiguous parts, but must be perfectly indivisible and simple..."; and elsewhere on the same page he states: "Besides, I was not the first to introduce the notion of simple non-extended points into physics."¹⁹⁵ Taking the first statement into account, it appears that Boscovich could not intend that his force-shells be extended minima since force-shells are truly indivisible and simple, lacking contiguous parts. Being intellectually divisible, extended minima possess parts which are contiguous to one another. Thus, they cannot be perfectly indivisible and simple as would be true in the case of unextended minima which lack parts whatsoever. Force-shells also cannot be extended minima since Boscovich remarks that other proponents of unextended minima erroneously add sides to the unextended minima: "...because they have mixed up with the simplicity and non-extension that they attribute to the elements that imperfect idea of a sort of round globule having two

¹⁹⁵ Boscovich, 59.

surfaces distinct from one another, an idea they have acquired through the senses...”¹⁹⁶

Boscovich believes that the true idea of unextended minima does not come from the senses, but through reason. He could be referring to Hume’s colored unextended minima, noting that, although Hume thinks that color is compatible with being unextended, the only way to see visually such colored minima is if they have parts. For how could something that is purely unextended be aligned next to something else that is also unextended if each unextended minimum lacks a left side and a right side? Thus, one must conceive these supposedly unextended minima as round globes so that one minimum can be in immediate contact with another minimum. By excluding the round globule interpretation of minima and emphasizing the pure indivisibility and simplicity of minima, Boscovich appears to think that his minima cannot be extended since a round globule is representative of a model for extended minima. Thus, it appears that Boscovich’s force-shells are best categorized as unextended minima; however, this interpretation faces difficulty when considering Boscovich’s claim that these puncta cannot be in immediate contact with one another.

Without immediate contact, unextended minima or puncta require attractive and repulsive forces in order to constitute matter. Boscovich writes: “I, since I do not admit the contiguity of the points themselves, but suppose that any two points of matter are separated from one another, avoid a mighty rock, upon which both of these others come to grief, whilst they build up the continuum from indivisible and non-extended things of this sort.”¹⁹⁷ Thus, it is only by means of the attractive and repulsive forces emanating from the punctum that matter can be composed of force-shells. Matter is not composed purely from what is unextended, but from the forces that

¹⁹⁶ Boscovich, 59.

¹⁹⁷ Boscovich, 59.

emanate from what is unextended. The forces which surround the force-shells's punctum consist of distinguishable sides, preventing the annihilation of the puncta. Thus, it appears that Boscovich's concepts of simplicity and indivisibility are compatible with a minimum being extended as force-shells. The indivisible parts of matter are really extended on account of the attractive and repulsive forces. Despite this response for the textual evidence supporting Boscovich's force-shells as ultimately unextended minima, it is questionable whether metaphysical priority rests in the puncta of the force-shells in isolation, or if metaphysical priority should belong to the entirety of the force-shell. For the source of the forces is the punctum. Should metaphysical priority be given the punctum itself, force-shells are unextended minima; however, if metaphysical priority is given the punctum and the forces, force-shells are extended minima. I will argue that metaphysical priority should be given to the entirety of the force-shell since the punctum and its forces are metaphysically interdependent. Let's first see why Boscovich might give metaphysical priority to the punctum itself, supporting the claim that force-shells are unextended minima.

Boscovich repeatedly refers to his minima being puncta rather than the entirety of the shell surrounding the minima. Behind the preference for the punctum over the entirety of the force-shell is the principle that the ultimate part of matter or space must be the source of matter's entire constitution. It stands to reason that whatever is the source or cause of a thing's characteristic possesses metaphysical priority of the said characteristics. The punctum of the force-shell fulfills this role in part because it is the source of the attractive and repulsive forces that form larger material bodies. It therefore appears that Boscovich's force-shells are unextended minima.

However, the assertion that the punctum could be the ultimately real and constitutive part of matter is incorrect since it is dependent on the attractive and repulsive forces which emanate from itself in order to compose material bodies. Without the attractive and repulsive forces, each punctum would cease to exist via the problems highlighted in the annihilation argument. The attractive and repulsive forces would not exist without the punctum, their center from which they emanate and are organized. Thus, the force-shell is intellectually divisible due to the distinction between the attractive and repulsive forces, and the minimum itself. Being intellectually divisible, the force-shell cannot be an unextended minimum. They are also metaphysically indivisible due to the metaphysical interdependence between the punctum and its attractive and repulsive forces. Thus, force-shells are extended minima because they are metaphysically indivisible and intellectually divisible.

Having shown that Boscovich's force-shells are extended minima, I will now investigate Kant's theory of force-shells as given through his distinction between internal and external determinations. Kant's distinction between internal and external determinations paves the way for the metaphysical indivisibility and intellectual divisibility of force-shells while also allowing for the distinction between matter and space itself respectively. This distinction between matter, which consists of ultimate parts, and space, which is infinitely divisible, marks Immanuel Kant's attempt to reconcile the metaphysical principle of simplicity with the mathematical truth that space is infinitely divisible. I will therefore show that Boscovich's force-shells are remarkably similar to Kant's force-shells on account of their metaphysical indivisibility and intellectual divisibility; however, Kant's force-shells are metaphysically indivisible and intellectually

divisible via his distinction between internal and external relations without presupposing Boscovich's argument against immediate contact.¹⁹⁸

Chapter One-Section E.II:

Kant's Arguments for Space's Infinite Divisibility and the Existence of Force-Shells in

The Physical Monadology

Immanuel Kant compiled his *Physical Monadology* in 1756, outlining a theory in Natural philosophy for the fundamental elements of the material world, the force-shells. Force-shells are indivisible puncta surrounded by a shell of forces according to attraction and repulsion. As one force-shell approaches the punctum of another force-shell, both shells of the distinct puncta generate greater repulsive forces. Thus, no two puncta are in immediate physical contact with one another. Simultaneously, as one force-shell moves away from another force-shell, the two force-shells exhibit greater attractive forces toward one another. These force-shells settle into an ever greater network of force-shells simultaneously exhibiting attractive and repulsive forces toward one another, composing the desks, rocks, and human beings that we experience daily. The force-shell theory is significant since it is a clear attempt to reconcile the metaphysical principle of simplicity with the geometrical truth of space's infinite divisibility as Kant states: "*Metaphysicae cum geometria junctae usus in philosophia naturali....*"¹⁹⁹ Following the rationalism of Christian Wolff, Kant lays out his *Physical Monadology* in proposition format

¹⁹⁸ Noteworthy here is that Kant and Boscovich agree that immediate contact is impossible. Yet the claim that immediate contact is impossible is Boscovich's argument for force-shells, while the claim that immediate contact is impossible is a conclusion of Kant's argument for force-shells.

¹⁹⁹ Kant, Immanuel. "The Physical Monadology". David Walford and Ralf Meerbote, eds. *Immanuel Kant: Theoretical Philosophy, 1755-1770*. Cambridge: Cambridge University Press, 1992. 49; 51.

reminiscent of Euclid's *Elements*. Beginning from the definition of simple substance, Kant outlines his theory of force-shells.

First, I will investigate Kant's argument for the infinite divisibility of space, tying his theory of a system of external relations of absolute space to space's infinite divisibility. Second, I will examine Kant's argument for minima, showing how internal determinations necessitate the existence of minima. I will then compare Kant's theory of force-shells to Boscovich's force-shell theory. Finally, I will show how Kant's force-shells are metaphysically indivisible through internal relations and the principle of sufficient reason, and are intellectually divisible through external determinations. While Kant begins the *Physical Monadology* with his arguments for the existence of minima, we shall start with Kant's account of space's infinite divisibility.

Kant's argument for space's infinite divisibility is strictly geometrical and positive rather than negative. Typical arguments for the infinite divisibility of space prove infinite divisibility are negative, showing that the alternative to infinitely divisible space, the existence of simple parts or minima, is contradictory. In this geometrical argument, Kant demonstrates space's infinite divisibility directly or positively without referring to the possible existence of minima. He cannot attempt to disprove the existence of minima or show that minima are contradictory since he has already argued for minima.²⁰⁰

Kant argues through the possibility of cutting a finite line an indefinite number of times.²⁰¹ Kant begins with a given finite line, XZ, that can be indefinitely extended. Now let another line representing an extended object, AB, be drawn at a right angle to XZ. It is this line, AB, which Kant intends to show must be infinitely divisible given the indefinite extension of

²⁰⁰ Kant "Physical Monadology", 52-53.

²⁰¹ Kant "Physical Monadology", 54-55.

XZ. Next, let another line be drawn parallel to AB, CD, and intersecting XZ at D. Kant now begins the constructions that show the infinite divisibility of the extended objects, line AB, by marking a series of random points on line BZ and connecting each of these random points on BZ to C. Each line thus drawn cuts line AB at some point. Since line XZ is indefinitely extended, BZ is also indefinitely extended toward Z. Thus, an indefinite number of points can be marked on BZ and connected to C and line AB, the extended object can be divided an indefinite (infinite) number of times. The infinite divisibility of line AB is potentially infinitely divisible rather than actually infinitely divisible since one has not drawn an infinite number of lines through it; rather, what has been shown is that AB *can* be divided without limit. One must acknowledge the possibility of knowledge without perceiving every part or aspect of one's idea as one must acknowledge that extended objects are *potentially* infinitely divisible without requiring the perception of every possible division of the extended object.

Kant founds his proof on the claim that knowledge of infinitely divisible lines is independent of the perception of each and every part of a given line. This denial contrasts sharply with Hume's and Berkeley's requirement that knowledge is founded on immediate perception. For then a line is infinitely divisible if and only if the human mind actually perceives the infinity of parts constituting the purportedly infinitely divisible line. Since the human mind is finite, it must perceive an ultimate, indivisible part. By contrast, Kant's proof shows not only that a finite line is infinitely divisible, but also that it is impossible to perceive the ultimate part of a line.

Kant's proof tells us that the human mind can know something which it *could* perceive in terms of recognizing a thought process which would make the idea perceptible regardless of

whether the mind possesses the capacity to complete such a process. Inasmuch as one perceives the possibility of indefinitely cutting a given finite line, it becomes apparent that one can know that the said line is infinitely divisible. What the percipient perceives is a given finite line accompanied with a geometric construction and a process of indefinite division. The basis for the perception of the indefinite process of division is the indefinite addition to a given line. Inasmuch as one does not perceive a limit to the indefinite addition to a line, a limit cannot be granted to the division of a finite line as Kant's construction intends to show.

Before beginning the analysis of Kant's argument for extended minima or force-shells in the *Physical Monadology*, I will provide an overview of the propositions and steps necessary for understanding Kant's account of force-shells. Propositions I, II, V, and VI will serve best this purpose. Propositions I and II provide Kant's reasons for believing that matter consists of metaphysically indivisible parts or minima; however, it is not yet completely clear whether such minima are unextended like Hume's colored unextended minima or extended like Boscovich's force-shells and Berkeley's colored extended minima. Proposition V is an argument for the reconciliation between the metaphysical truth of simplicity and the geometrical truth of space's infinite divisibility on account of the distinction between the principle of sufficient reason and external relations respectively. It also shows that Kant's minima must be extended being both metaphysically indivisible via the principle of sufficient reason, and intellectually divisible through external relations. Finally proposition VI clarifies the nature of Kant's extended minima as force-shells, the entirety of the shell being metaphysically indivisible and intellectually divisible, surrounding a punctum or unextended minimum that is intellectually indivisible and metaphysically indivisible. I will also provide an argument against the possibility of force-shells by Thomas Holden dubbed the "hollow worlds argument" in which the substantiality of force-

shells is called into question. Without substantiality, force-shells are non-entities that could not compose a world which is commonly considered to be filled with entities. Kant's force-shells prevent the double attribution of divisibility and indivisibility to the same thing in the same respect via the distinction between internal and external relations. Also, Kant's force-shells are a unique form of extended minima and also unextended minima, since it is a combination of a punctum (an unextended minimum) and the entirety of a force-shell (an extended minimum).

It is worthy to note that there is a debate amongst Kant scholars as to the precise features of matter. The common interpretation is that Kant conceptualizes matter as an aggregate of puncta surrounded by shells of attractive and repulsive forces. Sheldon Smith has recently combated this common interpretation, claiming instead that Kant actually espouses a theory in which force-shells constitute a mid-way position between the physical continuum theories of matter and the aggregated puncta theories of matter.²⁰² Smith refers to this theory as the "deformable continuum" theory of matter: matter consists of physical spheres of forces that overlap via compression.²⁰³ A "deformable continuum" theory of matter is like the typical continuum theories in the sense that the spheres continuously overlap one another, but such continuum is "deformable" because physical spheres constitute the said continuum, similar to the aggregated puncta theories of matter. While I will not explore Sheldon's argument fully in this dissertation, he appears correct in his view that Kant espouses a "deformable continuum" theory of matter. Sheldon seems correct when considering that such force-shells exist in infinitely divisible space. As such, the shells themselves must be continuous, unlike the shells found in Boscovich's theory in which such shells exist in finitely divisible space. This support of a

²⁰² Smith, Sheldon. "Kant's Picture of Monads in the *Physical Monadology*". *Studies in the History and Philosophy of Science*. 44, 2013. 102-111. 102-105.

²⁰³ Smith, 109-111.

“deformable continuum” supports Kant’s later change to a full physical continuum theory in his Critical work, the *Metaphysical Foundations of Natural Science*. Let’s return to Kant’s argumentation in the *Physical Monadology*.

Propositions I and II taken in conjunction are affirmations of the metaphysical principle of simplicity. Let’s determine how Kant argues for the existence of material simple parts. Proposition I is a definition of simple beings, while proposition II is an application of the priority of simple beings over composite beings. Kant defines a simple being or substance as something that lacks parts. He writes: “A simple substance, which is also called a monad, is one which does not consist of a plurality of parts, any one of which could exist separately from the others.”²⁰⁴ Based on this definition of simple substances or minima, it is unclear whether these minima are extended or unextended. It might appear to the reader initially that these minima must be unextended because they do not consist of a plurality of parts. Yet should one read the next phrase in this definition, namely that any one of these parts could exist separately from the others, only metaphysical indivisibility rather than intellectual indivisibility is evident. Recall that metaphysical indivisibility entails that no parts exist that could exist separately from what is metaphysically indivisible. For something can be metaphysically indivisible and intellectually divisible, as one being while possessing interdependent parts. For this reason, we do not yet know whether these simple beings are more like extended minima or more like unextended minima. All that is evident is that simple substances or beings must be metaphysically indivisible. This notion of simplicity is integral to Kant’s argument that material bodies consist of simple beings via the principle of sufficient reason.

²⁰⁴ Kant “Physical Monadology”, 53.

Kant's argument for simple beings in proposition II is founded on the notion that composition is a mere accidental relation for material bodies. "Accidental relation" applies to the composition of material bodies in a universal rather than particular or even sortal sense. Kant does not mean that composition is accidental to being a particular form of material body such as a table or a rock. For rock or a table are compositional by nature. Rather, he means that composition is accidental to *being a material body*. Composition in material beings is ultimately an accidental relation between parts, each of which could exist separately. Although *being a material table or rock* requires composition as an essential feature, the compositional parts of the said rock and table can be separated, forming either new compositional material bodies or even non-composite beings. Suffice to say that if something is a material body and is compositional then the material body's composition is accidental in nature. Left unanswered is why all composition is accidental rather than essential. Kant does not appear to provide the reader with an argument for this central principle.

In order to grasp the accidental nature of composition, we first have to grasp what Kant means by "composition" and "accidental" in light of his force-shell theory itself. My interpretation of both terms is founded on the essential interconnection between the metaphysical principle of simplicity and the principle of sufficient reason. Force-shells are the simple beings that compose material bodies. In this way, force-shells are the sufficient reason for the composition of material bodies. Presumably, these material bodies can be arranged in any manner of a multitude of force-shells. Taking this groundwork of the principles of sufficient reason and of simplicity, I will put forth an argument that all composition is accidental in light of the metaphysical principle of simplicity. Kant uses the backdrop of these two aforementioned

metaphysical principles and an observation regarding the separability of parts of material bodies to infer that the composition of material bodies is accidental.

The reader should note first that Kant does not equate composition with extension. Force-shells are extended but are not regarded as compositional material bodies unless Kant wishes to enter into an infinite regress of reasons for the said force-shell's composition. Simplicity or non-composition can be preserved despite the simple being possessing distinguishable, but non-separable parts.

The reason force-shells possess distinguishable parts comes from Kant's attempt to reconcile the principles of simplicity and of infinite divisibility. Force-shells must be simple while occupying infinitely divisible space. By occupying infinitely divisible space, they take on the extension of the space which they occupy. Thus, the reason for the composition of material bodies being accidental cannot be found in the nature of extension for Kant, which partly leads me to suspect that Kant's underlying principles must be found elsewhere in his thought, namely through the metaphysical principles of simplicity and of sufficient reason as well as his observation of the separability of parts in the concept of material objects. I will begin from the observation that material parts are enduring existences, revealing the principles of simplicity and principles of sufficient reason at work in his argument to show why Kant thinks that the composition of material bodies is accidental.

Kant's claim that bodies consist of separable parts sets up the reader to inquire into the nature of the parts in question and the reason or cause for the parts of a body existing as they do. Kant seems to link separability to material composition: "Bodies consist of parts, each of which

separately has an enduring existence.”²⁰⁵ One tends to think of material bodies in terms of possessing parts which could exist separately. This thought seems applicable to sensory experience in which we see material bodies come to be through assemblage and cease to be through separation of its parts. The parts of bodies are connected accidentally in which their connection might have been otherwise, whether the alternative is a dissolution of the parts or the assembly of different parts. For instance, the blocks of wood might be assembled into a chair or otherwise as a table; or the said blocks of wood might remain as they are without assembly into some other material object. Left unanswered thus far is why such parts of bodies are separable.

The separability of the ultimate parts of material bodies is based on the notion that the ultimate parts are substances in the sense that their existence endures. Being substances, such parts can stand on their own existence. In this sense, their existence endures despite separation from other parts. With this claim that the parts of material bodies are substances themselves, the reader must conclude that the composition of such substances is accidental. Here is Kant’s argument that the composition of material parts is accidental:

1. The parts of material bodies are substances (by virtue of their recombination and dissolution).
2. If the parts of material bodies are substances, then the composition of such parts to form a material body can vary (be accidental).

Therefore, the composition of such parts to form a material body is accidental.

²⁰⁵ Kant “Physical Monadology”, 53.

We can then infer Kant's definition of composition from the aforementioned argument and see why he believes that the composition of material bodies is accidental: composition is the conjunction of two or more parts, each of which possesses separable enduring existence. Since each part can endure on its own, the conjunction or combination of such parts must be accidental *in relation to the enduring parts themselves*. This point allows for the possibility that a material object's composition itself may be essential *in relation to the composite body*. Both truths are compatible since the attribution of essential and non-essential exist in different respects. Kant must thereby mean that bodies possess accidental composition in relation to the enduring parts themselves. From here, I will argue that the principle of sufficient reason is connected closely to the metaphysical principle of simplicity via Kant's belief that the parts of material bodies possess enduring existence.

Kant's argument for simple substances is meant to address a problem that arises when considering the metaphysical status of the enduring and separable parts of material bodies. The problem Kant highlights is whether matter can consist solely by accidental composition or whether there is some part that is simple, the ultimately enduring part of matter. Should a material body consist solely by composition, lacking a simple part, then it appears that accidental relations do not need to be explicable. There would be no cause or reason for the compositional material bodies to exist the way they do.

The reader should keep in mind that I am not ignoring the role of efficient causes in this analysis. Efficient causes are required to make the composite material body actual, having been assembled from enduring parts. The point I make is that the simple, enduring parts are the sufficient reason for being *this* composite material body rather than *that* composite material

body. In this manner, I am alluding to a sense of material causality as found in Aristotelian Physics. For example, the material reason for this water molecule being different from that water molecule is because each water molecule possesses different hydrogen and oxygen atoms. Furthermore, I think that Kant is ultimately arguing via this principle of sufficient reason that there must be simple, enduring parts for any such composite material bodies since one must account for the material composition of any composite material body (viz the water molecule) through its enduring simple parts. To say that the parts of the water molecule also are accidentally composed without limit is to provide no true account for the water molecule itself. I will term this usage of the principle of sufficient reason as the *material sufficient reason*. The material sufficient reason is the principle underlying Kant's objection to an infinite regress of accidental composition forming matter and the claim that matter consists only of accidentally combined parts without an existent and enduring simple part.

Kant argues that material bodies cannot be solely compositional since accidental relations require an account of their existence and an infinity of such relations merely begs the same questioning into the cause for composition. There must be a sufficient reason for the composition of bodies and this sufficient reason is a simple being.

Here is Kant's argument in premise-conclusion format:²⁰⁶

1. Should bodies exist as composites, consisting of accidental relations between separable parts, then it is possible to annihilate all composition from them.
2. Bodies exist as composites.

²⁰⁶ Kant "Physical Monadology", 53.

3. Thus, it is possible to annihilate all composition from them.
4. If it is possible to annihilate all composition from them, then the ultimately simple parts called monads or minima must remain in existence.

Therefore, the ultimately simple parts called monads or minima must remain in existence.

The first three premises follow from Kant's reasoning concerning the nature of composition, while the thought process behind the fourth premise requires clarification. The reason Kant infers that it is possible to annihilate all composition from bodies given can be articulated by responding to an objection from Thomas Holden. Holden writes that Kant cannot properly infer that all composition can be annihilated from the claim that some composition can be annihilated without presupposing a simple part.²⁰⁷ What is to prevent the possibility of an infinite regress of accidentally composed material bodies? Although some of the parts of a material body are separable, not all material composition can be annihilated, particularly if matter is actually infinitely divisible. Composition is accidental when looking at each instance of compositional material bodies, but is essential when taking into the whole composition of infinitely divisible material bodies. The only way to circumvent the possibility of infinitely divisible compositional material bodies is to assume before being proven the existence of a simple part—there is no infinite regress of accidental composition in infinitely divisible bodies.

The *petitio* argument Holden puts forth against Kant's argument for simple parts or substances appears sound at first blush; however, Kant's notion that composition exists in terms of enduring parts preserves his claim that it is possible to annihilate all composition in material bodies. The key to Kant's argument is the assumption of the validity of the actual parts doctrine:

²⁰⁷ Holden, 175.

Kant's argument would collapse assuming the truth of the potential parts doctrine. Since each part of a material body endures, its arrangement or composition with another enduring material part must be a relation. Kant thereby conceives composition in terms of relation: "...the composition of such parts is a relation, and hence a determination which is in itself contingent, and which can be denied without abrogating the existence of the things having this relation..."²⁰⁸

The dissolution of a composite material body results in the continued existence of its parts. In this manner, the enduring parts of the material body pre-exist the actual composition of the material body. Since each enduring part of the material body pre-exists the material body, it is conceptually possible to annihilate all composition because all composition is founded on the pre-existent enduring parts of material bodies. Should the potential parts doctrine be true, the Kant could not have asserted that the dissolution of composition results in the continued existence of enduring parts since the parts could not pre-exist the whole. Kant is not presuming the existence of a simple part save only by presuming the validity of the actual parts doctrine. A continuist could only object to Kant's argument by objecting to the actual parts theory. Thus, composition is dependent on some pre-existent enduring part based on his notion of material bodies and the actual parts doctrine.

Given this objection, it is my opinion that Kant's reasoning concerning the accidental nature of composition of material bodies resolves the possibility of nothing existing through the separation of infinitely divisible or even finitely divisible matter. I do not claim the following reasoning is found explicitly in Kant's writing, but could be found by inference regarding statements on the nature of composition and material bodies. Kant believes that the parts of composite material bodies remain despite annihilation and that the leftover portion of matter

²⁰⁸ Kant "Physical Monadology", 53.

must be simple.²⁰⁹ However, the objector supposes that such bodies are infinitely divisible. In such a case, we have the exception of there being anything leftover at all upon annihilation of all composition, much less something being simple. In this case, composition would be essential to the nature of material bodies when examining all compositional parts of matter. Kant resolves this issue by implicitly appealing to the principle of sufficient reason: compositional material bodies cannot solely consist of composite parts *ad infinitum* since composition requires explanation through the existence of enduring parts; they cannot consist of nothing either because then no composition would exist. Consequently, the “leftover” from the annihilation of all composition must be simple.

I will now propose an outline of the argument and then provide justification for the exclusion of there being compositional parts or nothing at all leftover from the removal of all composition. Here is an argument to support Kant’s inference that simples exist from removing all composition in matter:

1. If all composition is annihilated in material bodies, then either a further compositional part remains, nothing at all remains, or simple parts remain.
2. Assume that all composition is annihilated in material bodies.
3. It is not the case that compositional parts remain.
4. It is not the case that nothing at all remains.

Therefore, simple parts remain.

²⁰⁹ Kant “Physical Monadology”, 53.

A compositional part cannot remain since each compositional part exists only through an enduring part. Without parts capable of existing independently without composition, there could be no constitutive cause for the composition of a material body. To claim that a compositional part constitutes a material body, which also constitutes another larger body without limit is to remove a basis for a particular constitution being the way it is: an application of the material sufficient reason. Thus, the material body could not exist in that particular arrangement of constitutive parts.

Kant's argument appeals to proponents of finitely divisible matter in which matter must consist of simple parts. Composition ultimately requires simple parts which constitute material bodies. This process of composition cannot continue indefinitely in the case of finitely divisible matter. The result of complete separation of material bodies is a metaphysically indivisible part.

This argument might be directed against proponents of the potential infinite divisibility of matter because such proponents would believe that each division of matter results in a compositional part. Kant rejects such a theory in part because there is no ultimate account of the said composition, covertly invoking the principle of sufficient reason. Moreover, the composition of matter is dependent on the existence of enduring parts. If each enduring part is compositional, then there is no genuinely enduring part. Every part becomes a mere relation and relations depend on enduring constituents according to Kant's reasoning.²¹⁰ Therefore, it appears that matter cannot be potentially infinitely divisible.

²¹⁰ Thomas Holden stresses that Kant is not simply referring to the composition of material bodies as accidental, but all relations as accidental. The claim that all relations are accidental fuels Kant's argument that a simple part must remain; otherwise, no real account of relations is possible. See Holden, 184-186.

Keeping Kant's objective of reconciling the metaphysical principle of simplicity and the mathematical truth of space's infinite divisibility in mind, it appears that there is some question as to whether the simple parts of matter can be simple if they are to occupy infinitely divisible space. The simple part must be divisible because there is no smallest part of space which the simple part is to occupy. Kant recognizes this conflict of principles explicitly: "...it follows that each of these elements will occupy a part of space which admits of yet further division; that is to say, a body will fill some specifiable space."²¹¹ The metaphysical principle of simplicity appears to be forfeited without an apparent solution. The solution Kant proposes is to distinguish between division according to substance versus a distinction of a thing according to space. The result is that simple parts are extended and divisible not according to a division of substance but of space. The simple parts or force-shells are metaphysically indivisible via the principle of sufficient reason and are intellectually divisible by virtue of occupying infinitely divisible space.

Division of something into spatial parts does not necessarily result in the division of the said thing into two other beings or substances since space is based on a system of external relations. Kant writes: "The division of space, however, is not the separation of things, of which one is set apart from another and has a self-sufficient existence of its own. It rather displays a certain plurality of quantity in an external relation."²¹² The simple being does not forfeit its simplicity by virtue of possessing a top/bottom, right/left set of parts since such divisions of space do not actually relate two things or beings. For, *ex hypothesi*, if a division should occur, these theoretical parts could not exist on their own since the being is already simple or

²¹¹ Kant "Physical Monadology", 56.

²¹² Kant "Physical Monadology", 56.

metaphysically indivisible. It remains to be seen on what basis Kant grounds his distinction between division according to things and division according to space.

The basis of Kant's distinction between division of things and division of space is founded on his concept of the enduring part of a material body and their accidental composition, while space is not a substance. The division of a material body is not merely the division of something into distinct spatial locations but is also the division of a thing itself. The division of a thing itself does not result from the division merely of an extended thing, but from the division of something composite or consisting of two presumably separable and enduring things. Since material bodies consist of enduring parts, the division of the matter's results in a division of a thing. The material body becomes at least two distinct, enduring things by division since it already consists of two self-sufficient parts that are accidentally composed. On the other hand, space is not a substance. It is what Kant calls "...an appearance of the external relation of substances..."²¹³. The division of external relations is not a division of substance. Thus, we come to a fuller understanding of Kant's distinction between a division of things and division of space via his concept of the material body's accidental composition. These two divisions thereby enable Kant to claim that his simple beings retain their simplicity while being extended by virtue of occupying infinitely divisible space. We thus come to Proposition VI in which Kant details these simple and ultimately enduring parts of matter as force-shells.

In the sixth proposition, Immanuel Kant details the extension of these simple parts as a punctum surrounded by a sphere of activity. The sphere of activity is the simple being's or monad's act of filling a space. By virtue of filling space, the monad bears external relations to

²¹³ Kant "Physical Monadology", 57.

other minima or simple substances and seeks to prevent other monads from occupying its own sphere of activity.²¹⁴ No immediate contact between monads is thereby possible. The cause of the sphere of activity is a punctum, the center or focal point of the forces.²¹⁵ It is the punctum in conjunction with the sphere of activity that constitutes the metaphysical simplicity of the simple being while also allowing for the extension of the simple being. Kant thereby reconciles the metaphysical principle of simplicity and the mathematical truth of space's infinite divisibility via the principle of sufficient reason and his notion of the accidental composition of material bodies. Yet we will find that Kant's force-shell theory is not without its own difficulties.

Thomas Holden challenges the coherence of Kant's force-shell theory in two stages—the first stage points to the problem of the punctum's independent existence from the force-shell, while the second stage focuses on the emptiness revealed through the lack of an independently existing punctum—collectively dubbed the Hollow World Argument. The upshot of the Hollow World Argument is that Kant's force-shells are shown to be empty, substance-less entities which comprise the world. Presumably, since Kant applies the metaphysical principle of simplicity and demonstrates the existence of substances, force-shells could not be substanceless entities. Let's begin with the first stage of the argument.

Holden argues that a punctum lacks explanatory value since it would never be posited unless it also possesses a force-shell.²¹⁶ Recall that Boscovich posits the force-shell to prevent the immediate physical contact between bodies. Similarly, Kant posits a force-shell surrounding the punctum to reconcile the metaphysical principle of simplicity and the infinite divisibility of

²¹⁴ Kant "Physical Monadology", 57.

²¹⁵ Holden, 259.

²¹⁶ Holden, 264-265.

space. The force-shell must be intellectually divisible to preserve its metaphysical simplicity. Should Kant have posited only a punctum that occupies infinitely divisible space, the punctum would have to be divisible. Holden asks how the punctum can be distinct from the force-shell since the punctum requires the force-shell for its own existence as the force-shell requires the punctum for its own existence.²¹⁷ The punctum therefore is not an independently existing thing; rather, it is merely a modification of the force-shell which already exists.

We might argue that Holden misunderstands the metaphysical connection between the punctum and its force-shell on the ground that the punctum is the cause of the properties of force-shell. Depending on the features of the punctum, a particular variation a of force-shell results. One force-shell would have a greater area of influence than another force-shell due to the differing properties of each punctum. Based on this understanding of the punctum and the force-shell, it appears that the punctum is the cause of the force-shell and this metaphysical dependence is asymmetrical: the force-shell depends on the punctum.

Yet Holden seems to grasp this point already when considering the metaphysical relation between the punctum and its force-shell. He writes of Kant's and Boscovich's force-shell theories: "If this reconstruction is fair, then it seems that Kant and Boscovich are both conceptualizing the *puncta* as substances while treating the force-shells as properties of these point-substances."²¹⁸ Properties exist and are understood through the substance to which they belong. Thus, Holden acknowledges the asymmetrical metaphysical dependence that the shell possesses in relation to the punctum by virtue of being a property of the punctum.

²¹⁷ Holden, 265.

²¹⁸ Holden, 252.

The problem that Holden seems to have is found in Kant's and Boscovich's claims that matter fills space through "relational determinations"—the shell itself.²¹⁹ The punctum does not actually fill the space since it lacks intellectually distinguishable parts. Thus, it is really the shell itself that enables the punctum to interact with other parts of matter and maintain its existence in infinitely divisible space. Matter thereby seems to consist primarily of relational determinations rather than substances.

Without the punctum or the force-shell itself constituting substance, Holden argues that a "hollow world" results. Without some independent existent, the world is no more than a system of dispositions or powers.²²⁰ A world consisting solely of powers is a vacuous world with no source of these powers, a problem that plagues most systems of dynamism.²²¹ Thus, the world is no more than a system of relations. It is a series of relations that are relation between non-things, other relations. For example, there is no desk or table at which the reader is sitting now, but merely a series of repulsive and attractive forces that are not from ultimate parts of matter, but only from other attractive and repulsive forces. Thus, "thinghood" vanishes given Kant's and Boscovich's force-shell theories.

I believe that Holden's two part "hollow worlds" argument raises an important question concerning the substantiality of the punctum and its force-shell. Although the punctum causes the shell's existence, there is also a metaphysical need for there to be a force-shell to preserve the punctum's existence. Without a shell, the punctum's existence as the ultimate constituent of matter is problematic, especially given Kant's need to reconcile metaphysical simplicity and

²¹⁹ Holden, 252.

²²⁰ Holden, 266-267.

²²¹ Holden, 266.

space's infinite divisibility. It might turn out that force-shells do exist, but it appears that they could not be the ultimate constituents of matter.

Taking Holden's "hollow worlds" argument into account, it seems that a potential problem for theories responding to the divisibility of matter by preserving the simplicity of matter and the infinite divisibility of space: the problem of matter existing *in* space.²²² The hollowness problem shows that the substantiality or metaphysical indivisibility of a minimum is lost because the minimum is dependent on its own extension as external relations. Kant's and Boscovich's force-shells are extended through their own shell of activity. The punctum which lies at the center of this force-shell depends on the shell of activity for its existence lest annihilation upon contact with other minima or puncta result. By its dependence on the shell of activity, the punctum's substantiality is lost and a mere series of external relations without substances or internal relations/differences results—the "hollowness of the world". Hollowness results and substantiality collapses into the series of external relations by virtue of the attempt to reconcile the metaphysical principle of simplicity with the geometrical truth of space's infinite divisibility.

The same hollowness argument does not affect Hume's and Berkeley's theories of matter and space. Hume and Berkeley assert the metaphysical principle of simplicity, or perhaps better titled the "epistemic principle of simplicity" given their Empiricism, without holding the infinite divisibility of space as a geometrical truth. Should one wish to reconcile the principles of simplicity and of the infinite divisibility of space, it seems necessary to reconceptualize the relation between matter and space.

²²² Holden, 266-267.

The claim that space exists *in* matter could avoid the substantialiy problem that Kant faces in his force-shell theory. We will find that Leibniz takes a similar trajectory in his attempt to reconcile simplicity and infinite divisibility. *In* is not to be taken in the spatial container sense as we could say that the gift is found “inside” the gift box. Rather, *in* is to be interpreted as saying that space is a property of or arises from the ultimate constituents of matter. Leibniz ultimately argues that space is ideal and dependent on the existence of material bodies to exist. Space exists as series of perceptions of coexistent beings in monads, the ultimate constituents of reality. By virtue of not being extended, these monads cannot consist of a series of external relations in the same manner that extended minima exist partly as a series of external relations.

Chapter One: Conclusion

It becomes clear through Thomas Holden’s Hollow Worlds Objection that Kant’s force-shells, while classified as extended minima, combine elements of unextended minima and extended minima. The punctum is most easily identifiable as the incorporation of a modified theory of unextended minima. As unextended minima are partless like mathematical points, being intellectually indivisible, the puncta are also intellectually indivisible. The independent reality of such puncta is questionable since their existence would cease without the force-shells surrounding them, as Thomas Holden also highlights.²²³ Thus, the puncta of force-shells cannot really be considered metaphysically indivisible beings because they cannot exist independently of the shell.

We have here an exception to the rule that whatever is intellectually indivisible is also metaphysically indivisible in Kantian force-shell theory; however, the puncta could be

²²³ Holden, 264-265.

considered metaphysically indivisible but only as an intellectually distinguishable part of the force-shell. Hence, puncta are not unextended minima in their own right, but are similar to unextended minima inasmuch as they are intellectually indivisible. Thus, the force-shell itself must be an instance of an extended minima, being metaphysically indivisible as a whole, the force and the punctum included, but intellectually divisible by its distinguishable parts, the forces themselves and the punctum itself. By virtue of the punctum at the center of the force-shell, the force-shells themselves are not straightforward extended minima like Berkeley's colored extended minima. There is no center from which forces emanate in the case of Berkeley's extended minima. Berkeley's extended minima are simply metaphysically indivisible parts of matter or space due to the inseparability of color and extension—the parts of such minima are not readily distinguishable. The parts of Kant's and Boscovich's force-shells or extended minima are readily distinguishable from one another, supported through the intellectually indivisible puncta. Assuming one accepts that Berkeley's colored extended minima is the traditional variant of extended minima, it is evident that Kant's and Boscovich's force-shells cannot be classed as the same variant of extended minima as Berkeley's minima. It is by incorporating the puncta into the force-shells, as something that is metaphysically indivisible and intellectually divisible, that Kant's and Boscovich's force-shells incorporate an element of unextended minima into their theories of extended minima. By asserting the simplicity of matter and the infinite divisibility of space, Kant reconciles the geometrical truth of space's infinite divisibility and the metaphysical truth of the priority of simple beings over composite beings.

Chapter Two: Leibniz and Phenomenal Matter and Space

Chapter Two Introduction

The solutions we have studied thus far in the problem of matter's divisibility have required that material objects exist *in* space and *alongside* one another. The "matter in space theorem" (MIST) is a reasonable assumption to make in line with common sense regarding objects and space. A person need only consult his own perceptions of the world he inhabits to see the applicability of MIST. For example, the sheet of paper in front of the reader, a material object, appears to exist alongside other pieces of paper which bear relations to other objects in one's visual field. In this chapter, I will show how Leibniz rejects MIST in order to reconcile the metaphysical principle of simplicity with the mathematical principle of space's infinite divisibility.

Leibniz's theorizes that space and matter are perceptions within simple substances called monads.²²⁴ I will call this Leibnizian theory the "space in matter theorem" (SIMT). Note that "in" does not indicate a spatial container-like relation between matter and space as indicated through MIST; rather, the term "in" denotes a metaphysical dependency between space, spatial objects, and their metaphysical ground, the monad.

SIMT avoids the contradiction inherent in the simultaneous assertion that point-like simple substances exist in infinitely divisible space while also reconciling simplicity and space's infinite divisibility. For in the case of SIMT, it is impossible to ask whether a given monad be divided further into divisible parts by the occupation of infinitely divisible space. The monad is not in space and does not form a constitutive part of space or matter; rather, space and matter

²²⁴ Note that the metaphysical status of matter and space is much disputed in current literature. I am assuming in this chapter that Leibniz is a phenomenalist regarding the existence of matter and space. See the appendix for the Leibniz chapter for an overview of the different interpretations of Leibniz's theory of matter and space.

alone are infinitely divisible while the monad itself is indivisible as the simple being. As a simple being, the each monad exists independently of all other composite things and simple things, save the dominant monad, God. Each monad is a “windowless” soul in this manner, providing the sufficient reason for the existence of composites.²²⁵ While Leibniz’s solution to the problem of simplicity and infinite divisibility is ingenious, it is certainly not without its own issues.

Leibniz’s Idealist metaphysics consists of *at least* two levels of reality: the metaphysical reality of monads and the phenomenal world of matter and space. It is precisely by his formulation of SIMT and rejection of MIST that Leibniz is essentially an Idealist. What allows Leibniz to reject MIST and support SIMT is his objection to quantitative intellectual divisibility as a characteristic of the relation between monads and their correspondent material bodies that are perceived to exist in space. I will ultimately argue that Leibniz’s objection to quantitative intellectual divisibility is based on his conceptualization of the metaphysical principle of simplicity in terms of the principle of (material) sufficient reason. That is to say that a being is simple not because it occupies a smallest part and possesses absolutely no quantitatively distinguishable parts, but because it is a unity and is the sufficient reason for all other non-simple beings. More complex beings are resolvable not by composition “part *alongside* part”, but by “part *grounding* the composition of the whole according to its own sufficient unity”.

Yet it is precisely by the reconceptualization of simplicity in terms of the principle of material sufficient reason and his move to SIMT that Leibniz’s attempted solution runs into real problems concerning the metaphysical relation between part and whole. Leibniz articulates a

²²⁵ Leibniz, Gottfried. “The Monadology, or, the Principles of Philosophy.” Roger Ariew and Daniel Garber, eds. *G.W. Leibniz: Philosophical Essays*. Indianapolis and Cambridge: Hackett Publishing, INC., 1989. Paragraph 7.

theory in which space is potentially infinitely divisible, while matter is actually infinitely divisible. It is with matter's actual infinite divisibility and its correlation to the reality of monads that the issue of the relation between part and whole emerges. Although it is inaccurate to say that monads constitute the composition of material bodies as ultimate parts that coexist *alongside* one another—as Leonhard Euler has argued is characteristic of Leibnizian monadic theory—there is considerable question concerning how Leibniz argues for the actual infinite division of matter through the possession of ideal rather than real relations between parts.²²⁶ Ideal relations are merely mental or appetitive connections shared between monads to form the composite rather than real relations in which the parts of an object exert a force against one another. What is curious about Leibniz's solution is that it seems that if the progression from a whole material object to its infinite of divisible parts is a series of real relations, how is it possible to move from the ideal relation grounded in each monad to the infinity of parts of a material object? This mixup between the real and the ideal concerning the relation between whole and part in Leibnizian SIMT leads a philosopher like Immanuel Kant to declare the problem of reconciling simplicity and infinite divisibility an antinomy.

In order to grasp more clearly why Leibnizian SIMT is significant, let's examine how MIST is at work in the theories of Hume, Berkeley, and the Pre-Critical Kant. Hume, Berkeley, and Pre-Critical Kant, have covertly asserted that material bodies must be understood through their quantitative intellectual divisibility. The simple/composite dichotomy of material bodies results so that the parts of each material body are countable and coexistent irrespective of whether such simple parts are extended or unextended. The countability and coexistence of

²²⁶ Arthur, Richard T. "Russell's Conundrum: On the Relation of Leibniz's Monads to the Continuum". *An Intimate Relation: Studies in the History and Philosophy of Science Presented to Robert E. Butts on his 60th Birthday*. James Robert Brown and Jurgen Mittelstrass, eds. Volume 116. 1989. pp.171-201. 173.

parts results in finitely divisible material bodies. Thus, the simple parts, minima, or force-shells, are the units by which a subject grasps the composition of material bodies (e.g. a material object might consist of 357 simple parts, while another one might consist of 1348 simple parts). Although we do not necessarily say that all forms of simple beings that are countable and coexistent are quantitatively intellectually divisible themselves, as is the case with Hume's unextended colored minima, all of these minima align next to one another to form coexistent and countably distinct bodies.

In order to show how Leibniz reconceptualizes the simple/complex dichotomy by rejecting quantitative intellectual divisibility, I will rely primarily on passages from the *Monadology* and from the *Principles of Nature and Grace*. My interpretation of the relation between monads, matter, and space arises principally from the incompatibilist interpretation of Brandon Look.²²⁷ Look conceives Leibnizian metaphysics as inherently two level: the metaphysical reality of the monads and the phenomenal level of matter and space. While I agree with the phenomenalist “two level” camp of interpreters for the purpose of this chapter, I point out to the reader that there is considerable interpretive controversy regarding the metaphysical status of matter and space in current Leibniz scholarship, which can be divided ultimately into compatibilist and incompatibilist lines of thought.²²⁸ The problem arises when a scholar attempts to square Leibniz's articulation of the reality of matter (composite substances) in what is often termed the “Middle Years” of Leibniz's career with his “Later Years” in which Leibniz apparently suspends his theory of composite substances altogether in total support of the reality

²²⁷ Look, Brandon. “Leibniz's Metaphysics and Metametaphysics: Idealism, Realism, and the Nature of Substance.” *Philosophy Compass*. 11:5, 2010. 871-879.

²²⁸ See Hartz, Glenn. “Why Corporeal Substances Keep Popping Up in Leibniz's Later Philosophy”. *British Journal for the History of Philosophy*. 6:2, 193-207 202-207; Stumpf, Andrew. “Harmonizing Leibniz's Ontology”. *Dialogue*. Volume 51, Issue 3, September 2012, pp. 467-483.

of simple substances, monads. A compatibilist believes that Leibniz continues to support the position that there are composite substances in the “Later Years”, while an incompatibilist denies Leibniz’s commitment to composite substances in the “Later Years” on the pains of contradiction. Attendant with Look’s interpretation in this chapter, I will incorporate an opposing compatibilist position to draw out the Leibniz’s problem of reconciling part and whole given his assertion of the actual infinite divisibility of matter, notably the “three level” interpretive camp including the compatibilist thought of J.E. McGuire and that of Glenn Hartz and J.A. Cover. I will also rely on two of Leibniz’s essays treating the topics of the potential infinite divisibility of space and the actual infinite divisibility of matter, “Created Things Are Actually Infinite” and “On Minimum and Maximum” respectively. Irrespective of whether one supports a compatibilist or incompatibilist position regarding Leibniz’s stance on matter as composite substance, it is clear Leibniz’s attempt to intricately connect unity with actuality fuels the interpretive controversies.²²⁹

Chapter Two: Section A:

Leibniz’s Argument for Simple Beings in the *Monadology* and Pseudo Qualitative Intellectual Divisibility

Leibniz’s argument for and characterization of monads in the *Monadology* demonstrates his rejection of quantitative intellectual divisibility. The word “monad” comes from the Greek word *monas* meaning unity or what is one thing.²³⁰ Leibniz calls these monads “simple substances” since they are indivisible and exist independently of other beings.

²²⁹Hartz, Glenn A. and Cover, J.A. “Space and Time in Leibniz’s Metaphysics”. *Nous*. Vol. 22, No. 4. December 1988. 493-519. 497-502

²³⁰Levey, Samuel. “On Unity and Simple Substance in Leibniz”. *The Leibniz Review*. December 2007. 61-106. 61-62.

The use of the term, “monad”, is indicative of Leibniz’s struggle with the articulation of the metaphysical principle of unity and its application to substances. It becomes clear that Leibniz distinguishes himself from a fellow philosopher like Descartes who argues that substances can be extended and thereby divisible: Leibniz stresses the principle of unity to such an extent that the ultimately real substances are really the simple substances, monads, which lack divisibility. This divisibility, when understood in contradistinction to Descartes’s extended substances, is a form of quantitative intellectual divisibility because the divisible parts of extended substances are spatial. I will argue that Leibniz stresses a strong connection between unity and actuality because of the latent application of the principle of sufficient reason in his argument for monads: monads are ultimately real and one because their cannot be any further natural reason or cause for their existence, providing the sufficient reason for the existence of all other composite things.

In order to grasp more clearly Leibniz’s use of the principle of material sufficient reason and his rejection of quantitative intellectual divisibility, I would like to examine Leibniz’s description of monads possessing qualities. At first glance, Leibniz appears to contradict his definition of monads as “simple substances” when he asserts that monads must possess qualities.²³¹ For if a simple being is without parts, how could something that is simple possess qualitative distinctions of its own being?²³² Wouldn’t the same being be composite rather than simple? For Leibniz, it is clear that to be without parts does not mean that the being is completely undifferentiated like a point which lacks qualities and extensive parts. Leibniz is not adverse to the claim that monads possess qualities without forfeiting their simplicity, as

²³¹ Leibniz “The Monadology, or, the Principles of Philosophy”, Paragraph 8.

²³² Leibniz. “Monadology”, Paragraphs 8-9.

commentator Phillip Cummins argues: “Leibniz was not illogical in endorsing qualitatively complex simple substances. There is no logical connection between having a plurality of qualities and comprising a plurality of parts.”²³³ A changeable being requires the possibility of distinctions or modifications for the being’s metaphysical identity.²³⁴ There cannot be a multiplicity of beings without something to distinguish each member of the multitude. Thus, Leibniz’s principle of unity does not exclude all forms of multiplicity. The question remains how these qualitative parts of monads are said to exist as modifications of monads without forfeiting monadic simplicity.

Kant’s distinction between the external and internal forms of intuition, space and time respectively, provides a foundation for which one can understand quantitative and qualitative intellectual divisibilities or distinctions. Kant describes space as the external form of intuition that allows things to appear as objects outside us.²³⁵ The objects outside us are coexistent. Time is the form of intuition that allows us to contemplate the inner states of the mind.²³⁶ The inner states of the mind are successive rather than coexistent. Unlike the representation of space, the representation of time is one dimensional or linear. It consists of events perceived before or after other events rather than things that are placed “next to” or “alongside” one another. Inasmuch as time is represented as a line, time can be said to possess a certain form of distinction or divisibility. This form of divisibility or distinction, I argue, approximates the qualitative distinctions in monads. Let’s apply Kant’s distinction between space and time as forms of intuition to quantitative intellectual divisibility and qualitative intellectual distinctions.

²³³ Cummins, Phillip. “Bayle, Leibniz, Hume and Reid on Extension, Composites and Simples”. *History of Philosophy Quarterly*. July 1990. 299-314. 304.

²³⁴ Leibniz, “Monadology”, Paragraph 8.

²³⁵ Kant *Critique of Pure Reason*, 3.

²³⁶ Kant’s *Critique of Pure Reason*, 3

The succession or division of time characterizes the qualitative rather than quantitative distinctions found in Leibniz's monads. Leibniz describes the successive character of monadic qualities: "But, besides the principle of the change, there must be a particular series of changes, which constitutes, so to speak, the specific nature and variety of the simple substances."²³⁷ The series of changes is a multiplicity within the monad (unit). Each change is not coexistent with the other changes in the monad as each object is coexistent with other objects in space. Since monads lack this coextensive multiplicity that characterizes space, they also lack quantitative intellectual divisibility. Quantitative intellectual divisibility identifies the coexistent, divisible parts of an extended being. These coexistent parts may or may not be capable of separable existence from each other or from the whole in which they cohere.

Given the truth of SIMT, Leibnizian monads can possess qualities without being considered composite beings. Leibnizian monadic qualities provide the reader with a newfound "pseudo" form of intellectual divisibility based on qualities if interpreted through the Kantian representation of time as a line. Monads are "divisible" through their succession of monadic qualities. Succession through their monadic qualities enables us to map their successive qualities, one after the other, as Kant argues that we can represent time through a line: "And precisely because this internal intuition presents to us no shape or form, we endeavor to supply this want by analogies, and represent the course of time by a line progressing to infinity, the content of which constitutes a series which is only of one dimension."²³⁸ This representation of time through a line does not make time spatial or that monads possess coexistent parts by nature. Rather, it helps us to conceive of the succession of events and, I argue, the succession of qualities

²³⁷ Leibniz "Monadology", Paragraph 13.

²³⁸ Kant, *Critique of Pure Reason*, 11.

in monads which constitutes a particular monad's identity. Inasmuch as we look at monads as possessing this form of "pseudo" intellectual divisibility according to time or succession, Leibniz can simultaneously maintain that monads possess qualities without forfeiting monadic simplicity.

Chapter Two-Section B:

The Principle of Simplicity and the Principle of Material Sufficient Reason

Leibniz's argument for the existence of monads in the *Monadology* provides the reader one of the clearest examples of the application of the metaphysical principle of simplicity. It is also construed as an almost definitional account of the simple/complex dichotomy.²³⁹ This monadological argument provides the reader with the definitional nature of simple/composite being dichotomies. By merely defining monads into existence via definitions of simple or composite beings, the conclusion that monads exist appears to be tautological and question begging. Commentators like Samuel Levey and Brandon Look surmise that there is a missing premise that enables the inference that monads exist from the existence of composites in experience.²⁴⁰

My interpretation of the monadological argument is that it is neither tautological nor question begging; rather, it is evidence of the *aggregate thesis*, a thesis that Daniel Garber has argued is Leibniz's proof for the existence of true unities called monads: "...the reality that an aggregate of individuals has derives from the reality of its parts."²⁴¹ Samuel Levey comments on Garber's rendition of the aggregate thesis: "The metaphor of "deriving" seems to suggest more

²³⁹ Look, 872-874.

²⁴⁰ Levey, Samuel. "On Unity and Simple Substance in Leibniz". *The Leibniz Review*. December 2007. 61-106; Look, 872.

²⁴¹ Garber, Daniel. *Leibniz: Body, Substance, and Monad*. New York: Oxford University Press, 2009. 74.

than Garber's gloss that the aggregate will be real only if the parts are real. At the least it gives ontological priority to the parts. (One could allow that an aggregate is real only if its parts are real without thereby assigning priority to the parts.)"²⁴² If the aggregate is actual, then the reality of the aggregate itself is derived from the actuality of its parts.

It is from this notion of "deriving" that I will argue for the latent principle of material sufficient reason at work in Leibniz's argument for monads. Given the aggregate thesis, I will argue that what makes Leibniz's argument work is a modification of the principle of sufficient reason which I call the principle of material sufficient reason. I say "material" because the principle deals with the compositional relation between parts and wholes. The wholes are metaphysically dependent on their (ultimately) simple parts because the composite wholes are contingent realities consisting of simple parts that are self-subsistent, unitary beings. The Leibnizian principle of simplicity is intricately linked to the principle of material sufficient reason such that Leibnizian monads are simple not merely on account of a lack of quantitatively divisible parts, but by their being the material sufficient reasons of their correspondent composite wholes of which they are the "parts".²⁴³

To demonstrate the connection between Leibniz's principle of simplicity and the principle of material sufficient reason, I will first examine the argument for monads in the *Monadology*. Having shown that the argument in the *Monadology* appears to lack a premise, I will turn to the *Principles of Nature and Grace Based on Reason* for Leibniz's treatment of unity

²⁴² Levey, Samuel. "On Two Theories of Substance in Leibniz: Critical Notice on Daniel Garber, *Leibniz: Body, Substance, Monad*. *Philosophical Review*, Vol. 120, No. 2, 2011. 285-320. 307.

²⁴³ I say "parts" because Leibnizian metaphysics faces the problem of accounting for exactly how monads "enter into" the composition of complex things and whether complex things can be considered substances. See Glenn Hartz's "Why Corporeal Substances Keep Popping Up in Leibniz's Later Philosophy". *British Journal for the History of Philosophy*, 6:2, 193-207. 2008 for a recent critical summary of the aforementioned problems.

and multitude in the parallel monadological argument to clarify Leibniz's inference that simple beings exist. It will be clear from Leibniz's treatment of unity and multitude in the *Principles* that Leibniz implicitly employs the material principle of sufficient reason and that he conceives simple beings primarily in terms of being the sufficient reasons for all other non-simple beings.

Leibniz's argument for the existence of monads in the *Monadology* begins with a definition of the monad attendant with a hidden metaphysical premise that composite beings presuppose the existence of simple beings. Monads are simple. They are the building "blocks" of all existent things: "The monad, which we shall discuss here, is nothing but a simple substance that enters into composites—simple, that is, without parts."²⁴⁴ The monad is metaphysically indivisible. It is self-subsistent, a "windowless soul" as Leibniz describes.²⁴⁵ Inasmuch as monads are self-subsistent beings, there is no further reason for their existence, save the dominant monad, God.²⁴⁶ The following is an analysis of the argument in premise-conclusion format provided through Brandon Look.²⁴⁷

1. If there are composites, then there are simples.
2. There are composites.

Therefore, there are simples.

The argument is *modus ponens* in form. Its second premise is derived from experience, while the first premise founded on the metaphysical claim that composites are collections of simples. The argument's sheer simplicity might convince the reader of the veracity of its

²⁴⁴ Leibniz "Monadology", Paragraph 2.

²⁴⁵ Carr, Herbert Wildon. *The Monadology of Leibniz*. Kensington, London: The Faval Press, 1930. 27-28.

²⁴⁶ Leibniz "Monadology, Paragraphs 38-40.

²⁴⁷ Look, 872.

premises; however, an opponent could take issue with both premises based on the notion of “composite” that Leibniz employs in the argument. If composites are collections of simple beings, then Leibniz has already admitted the existence of simple beings in the second premise—a clear indication of begging the question; however, one might also surmise that Leibniz is attempting to formulate an a priori analytic metaphysical system just as he formulates an a priori analytic system of arithmetic.²⁴⁸ One can clearly perceive that two added to two makes four by analysis of the bare concepts of four and two just as it is possible to conclude that simple beings exist because they are already given through the essence of a composite thing. Metaphysics becomes an analysis of definitions and axioms in the same manner that arithmetic is an analysis of numerical concepts with basic operational axioms.

Yet such a redescription of metaphysics in terms of mathematics does not reveal *why* Leibniz thinks that a compositional whole presupposes the existence of its simple parts. A theory that composites are mere aggregates of simple beings would beg the question for a proponent of the potential parts doctrine. A potential parts proponent would claim the opposite of Leibniz: the actual whole is metaphysically prior to its potentially existing parts. Given the actual existence of a material object, the entirety of the material object must be said to exist actually, while the parts potentially exist because they *could be* divided from the actual whole. It is possible to consider the existence of the parts from the existence of the actual whole. Aristotle and Rene Descartes would be two good proponents of this theory.²⁴⁹ A composite could consist of composites seemingly without recourse to a simple being. Leibniz denies this possibility

²⁴⁸ Jesseph, Douglas M. “Leibniz on the Foundations of the Calculus: The Question of the Reality of Infinitesimal Magnitudes”. *Perspectives on Science*. 6.1&2, 1998. pp. 6-40. 6-9.

²⁴⁹ Arthur “Russell’s Conundrum: On the Relation of Leibniz’s Monads to the Continuum”, 180-181.

through his definition of composite things.²⁵⁰ As a potential parts opponent, he *assumes* the existence of simple beings without explicitly *showing* that composites metaphysically require simple beings for their existence. A parallel argument in the *Principles of Nature and Grace Based on Reason* helps answer why Leibniz believes that composite things are collections of simples.

The parallel argument in the *Principles of Nature and Grace Based on Reason* is distinctive because of the comparison between simple beings and composite beings with unity and multitude respectively. Leibniz writes: “There must be simple substances everywhere, because without simples there would be no composites—without *ones* there could not be *manies*.”²⁵¹ Leibniz’s emphasis on *ones* and *manies* is striking since it advances his strong and universal claim that “there must be simple substances everywhere”—an instance of his belief that existence is a plenum, a world that is completely filled with living souls.²⁵² He did not argue that simple substances must be everywhere in the *Monadology* within the parallel argument for monads.

Piecing together his use of *one* and *many* with his conclusion that simple substances must be everywhere, Leibniz asserts that whatever exists is either one thing or many things. One and many are fundamental metaphysical categories which include an implicit metaphysical dependency relation: many is neither self-sufficient nor truly real because whatever is many depends on its parts; whatever is one is self-sufficient and real. Many things are not a real being, but a *collection* of real beings. For we distinguish one “many” from another “many” because of

²⁵⁰ Leibniz “Monadology”, Paragraph 2.

²⁵¹ Leibniz “Principles of Nature and Grace” Roger Ariew and Daniel Garber, eds. *G.W. Leibniz: Philosophical Essays*. Indianapolis and Cambridge: Hackett Publishing, INC., 1989. Paragraph 1.

²⁵² Leibniz “Monadology”, Paragraphs 8; 69.

its parts. If its parts are themselves “many”, then we can ask the same question of those parts *ad infinitum* without ever grounding the reality of the original many. Consequently, Leibniz can infer that simple beings exist everywhere (ones) assuming the actual parts doctrine and a plenum.

Although the *aggregate thesis* is found in the *Monadology* and the *Principles of Nature and Grace Based on Reason* it is best articulated in the argument for monads in the *Principles* because of Leibniz’s use of *ones* and *manies*. Critiquing Bertrand Russell’s analysis of Leibniz’s theory of perception and aggregates, Richard T.W. Arthur says that the *aggregate thesis* follows from Leibniz’s nominalism regarding the “being” of a collection of unitary things.²⁵³ Arthur writes of Leibniz’s nominalistic account of collections or *manies*:

“On his view, if you have two diamonds, you do not *also* have a set of two diamonds, as an entity distinct from its members, except insofar as it is perceived. Of course, if the two diamonds are together, you can perceive them as a unity, as a pair of diamonds. But this unity, Leibniz claims, is simply a perceived unity.”²⁵⁴

Perception supplies the unity of aggregates or collection as a whole, but the real unity lies in each member or unit of the collection. One and being are thereby convertible for Leibniz: whatever is one is one being, whatever is being is one (thing). Thus, the being of a collection requires grounding and this grounding must be a simple being—the monad.

From his treatment of simple beings and composites according to the distinction between one and many, the principle of material sufficient reason is evident. Leibniz defines the principle of sufficient reason in the *Monadology*:

²⁵³ Arthur, Richard T.W. “Presupposition, Aggregation, and Leibniz’s Argument for a Plurality of Substances”. *The Leibniz Review*, Vol. 21, 2011. 91-115. 95-97.

²⁵⁴ Arthur, 98.

“And the principle of sufficient reason, on the strength of which we hold that no fact can ever be true or existent, no statement correct, unless there is a sufficient reason why things are as they are and not otherwise—even if in most cases we can’t know what the reason is.”²⁵⁵

He applies sufficient reason not only to contingent truths, truths that might be otherwise, but also to necessary truths such as those of mathematics.²⁵⁶ A sufficient reason provides a final account of a necessary or a contingent truth as Leibniz describes of mathematics:

“Eventually their analysis comes to an end, because there are *simple ideas* that can’t be given a definition; and their demonstrations also come to an end, because there are axioms and postulates—in a word, basic principles—which can’t be proved and don’t need to be proved; these are identical propositions, the opposites of which contain explicit contradictions.”²⁵⁷ (My italics added)

Note Leibniz’s use of the term *simple ideas* in the context of the aforementioned quote. In order to provide a sufficient reason for mathematical solutions, recourse to simple ideas is needed—building blocks from which one derives the answer to mathematical problems. Mathematical problems are complex, requiring analysis into their constitutive parts. Leibniz extends this resolution of problems into simple parts or sufficient reason(s) to contingent truths, such as the movement of his pen across the paper on which one might write.²⁵⁸ Thus, the principle of sufficient reason is integral not only to the explanation of natural phenomena and mathematical ideas, but also to the very manner by which human beings acquire knowledge—always in terms of the resolution of something complex into one or a few simple and final ideas.

²⁵⁵ Leibniz “Monadology”, Paragraph 32.

²⁵⁶ Leibniz “Monadology”, Paragraphs 33-35.

²⁵⁷ Leibniz “Monadology”, Paragraph 35.

²⁵⁸ Leibniz “Monadology”, Paragraph 36.

It is from these passages that I develop the principle of material sufficient reason—something simple accounts for whatever is composite or a collection. What makes Leibniz’s argument for monads work is not merely the inference that composites are really collections of simple beings or even the distinction between unity and multiplicity, but the principle of sufficient reason. Just as one accounts for a mathematical problem or theorem through the analysis of simple and final mathematical concepts or principles, the contingent reality of composites can only be understood through the simple beings or monads that form the composites. For a composite, the opposite of the nature of a simple thing, can come to be and cease to be, but simple beings neither come to be nor cease to be by natural means: “Composite things, in contrast with that, can begin or end gradually, through the assembling or scattering of their parts.”²⁵⁹ The sufficient reason for this collection of parts existing rather than some other collection of parts must be founded on the simple beings themselves. Since composites are multitudes and every multitude can come to be and cease to be by the assembling and disassembling of its parts, it possesses a contingent reality which requires a (material) sufficient reason or account for its existence.

I reformulate Leibniz’s argument for monads through the principle of material sufficient reason, merging the form of the argument in the *Monadology* with the broad reaching conclusion in the *Principles of Nature and Grace Based on Reason*:

1. A monad is a simple being.
2. Everything that exists is either a simple being (one thing) or a composite being (many things).

²⁵⁹ Leibniz “Monadology”, Paragraph 6.

3. If a composite being exists, then it requires a material sufficient reason for its existence.
4. This sufficient reason is either another composite being or a simple being.
5. This sufficient reason cannot be another composite being for then an infinite regress of only composite beings would ensue—which is impossible.
6. Thus, the sufficient reason of a composite being must be a collection of simple beings.
7. Composite beings exist everywhere (the plenum principle).

Therefore, simple beings or monads exist everywhere.

The reformulated argument is clearly compatible with the *modus ponens* form as found in the *Monadology*. It also incorporates the principles of unity and multiplicity in the second premise as found in the *Principles*, while including the plenum principle in both texts to reach the strong and universal conclusion that the world is full of monads. It also avoids the objection that Leibniz's argument is a mere tautology or question begging at the outset. The question begging and tautological argumentation is avoided not only through the dilemma in premise two borrowed from the distinction between unity and multitude in the *Principles* but also chiefly through the incorporation of the principle of material sufficient reason.

Is the argument thereby free of potential objections? Not at all! A proponent of the potential parts doctrine could object to the implicit assumption running throughout the whole argument that a composite being requires an account through all of its parts. The assumption that composites require an account through their parts assumes that the parts precede the composite whole—a key principle of an actual parts theorist. It is precisely this actual parts

theorist claim that I will dispute in a later chapter. My solution to the problem of material divisibility is not a removal of all simple beings from nature, but a re-description of them through the commonly shared principle of sufficient reason. The reformulated argument shows *why* Leibniz thinks that composite things presuppose simple beings for their existence; however, it does not answer the question-begging dispute that potential parts proponents might wage.

Having examined the relevant passages in the *Monadology* and the *Principles*, the connection between Leibniz's principles of simplicity and material sufficient reason should be clear. Simple beings function as final explanations for composite things in mathematics and nature. Without the need for final explanations, the positing of simple ideas or beings would be fruitless. Simple beings should not be thought as quantitatively indivisible parts of composite things as one might conceive the quantitative indivisibility of a point, but as sufficient reasons for the existence of other things. Evident in Leibniz's description of monads or simple beings is that such beings are self-sufficient, being the final explanations of all natural events or material objects. Simplicity is really a function of being the material sufficient reason for the simple being itself and other non-simple beings. Without the need for quantitative intellectual divisibility to describe monads, Leibniz's concept of simplicity eschews quantitative intellectual divisibility for a more fundamental metaphysical principle, material sufficient reason. Now that I have argued that the simplicity of monads is based on the principle of material sufficient, which results in the rejection of quantitative intellectual divisibility, I will now focus on Leibniz's response to the problem of matter's divisibility.

Chapter Two-Section C:

Leibniz's Theory of the Divisibility of Matter and Space and his Affirmation of SIMT

Leibniz's affirmation of SIMT through his rejection of quantitative intellectual divisibility enables him to reconcile the metaphysical principle of simplicity with the mathematical principle of space's infinite divisibility. His reconciliation between the two principles is ingenious by preventing the potential objection that a simple being must be further divisible if it occupies an infinitely divisible space. Leibniz dodges this objection by asserting SIMT. For monads are not *in* or *occupying* space in the way the way that we would assert that a simple being that exists according to MIST would be said to be *in* and *occupy* space. Leibniz thereby seems to sufficiently reconcile simplicity with infinite divisibility.

Yet Leibniz asserts that matter and space are actually infinitely divisible and potentially infinitely divisible respectively. This assertion is curious not only because there is an apparent distinction between real being for matter and ideal being for space but because Leibniz is asserting that a material body is resolvable into an actual infinity of extensionless and non-spatial substances. But if matter, as phenomena, is said to be perceived to exist in Euclidean space, then how could one talk about a material object being resolvable into an infinity of point-like substances when a geometrician like Euclid, would never affirm such a possibility for a line to be resolvable into points? Leibniz would thereby have the problem of reconciling the relation between the so-called ultimate "parts" of matter, monads, with the composition of the whole of the material object. Such a problem, I believe, is part of the reason Immanuel Kant in the *Critique of Pure Reason* argues that the problem of determining whether the existence of simple beings is reconcilable with the infinite divisibility of space is an antinomy.

In order to ascertain the problem that Leibniz faces when affirming the actual infinite divisibility of matter and the potential infinite divisibility of space, let's first grasp the Euclidean

nature of space and then grasp why he believes that material objects are actually infinitely divisible. I will use Leibniz's argument for the potential infinite divisibility of space from his essay, "On Minimum and Maximum", and then examine his essay, "Created Things Are Actually Infinite", for his argument that matter is actually infinitely divisible.

Leibniz articulates the Euclidean notion that space cannot consist of mathematical points. By claiming that a point is "that which has no part" and a line is a "breadthless length", Euclid excludes the possibility that points comprise the length of a line since each point lacks sides.²⁶⁰ Leibniz follows suit, arguing that minima cannot comprise a line, nor could a line be resolvable into an infinity of points assuming that the line is infinitely divisible. Should an infinitely divisible line consist of minima, the shorter length of a given line would equal the greater length of the same line.²⁶¹ Consequently, it would seem absurd for Leibniz to posit the actual infinite divisibility of matter. For then a material body would be resolvable into minima. Yet this is precisely what Leibniz asserts regarding the actually infinite divisibility of matter.

Leibniz's incorporation of the distinction between "real" and "ideal" relations enables him to formulate a metaphysical theory that matter can be actually infinitely divisible, while space retains its potential infinite divisibility.²⁶² Space is ideal because it represents merely the possible coexistences of material objects, but the parts of matter bear real relations between each other as Leibniz describes in his essay, "Created Things Are Actually Infinite": "For any body

²⁶⁰ Euclid, *Elements*. Bk 1, Def. 1 + Def. 2.

²⁶¹ Leibniz G.W. "On Minimum and Maximum" in *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686*. Richard T.W. Arthur, ed. New Haven and London: Yale University Press, 2001. 97; See also Leibniz's essay, "Pacidius to Philalethes: A First Philosophy of Motion", in the same text and edition as well as: Levey, Samuel. "The Interval of Motion in Leibniz's *Pacidius Philalethi*". *NOUS* 37:3 (2003). 371-416.

²⁶² McGuire, J.E. "Labyrinthus Continui". Peter K. Muhamer and Robert G. Timball, eds. *Motion and Time, Space and Motion: Interrelations in the History of Philosophy of Science*. Ohio: Ohio State University Press, 1976. 289; Cummins, 303.

whatever is actually divided into several parts, since any body whatever is acted upon by other bodies.”²⁶³ As a plenum, the natural world is inherently active as substances are considered to be naturally active. This belief partly comes from Leibniz’s definition of substance in his *Principles of Nature and Grace Based on Reason*: “Substance is a being that is capable of action.”²⁶⁴ It also comes partly from his notion of a plenum in his *Monadology*, “Thus there is nothing fallow, nothing sterile, nothing dead in the universe, no chaos, no confusion save in appearance, somewhat as it might appear to be in a pond at a distance, in which one would see a confused movement and, as it were, a swarming of fish in the pond, without separately distinguishing the fish themselves.”²⁶⁵ The natural material world is full of activity in contradistinction to Rene Descartes’s conception of matter as passivity, being mere extended substances.²⁶⁶ But the natural world is perceived to exist in infinitely divisible space. Each body is thereby divisible *ad infinitum*. But if each body is divisible *ad infinitum*, then it must be actually so. For each part of an infinitely divisible body must be acting on the part of every other material body.

The result is that space is infinitely divisible in a continuous sense, while matter is infinitely divisible and is *resolvable* ultimately into an actual infinity of monads. There are no ultimate parts of space because space is ideal, while there are ultimate parts of matter since matter is real or active. This distinction between the divisibilities of matter and space directly correlate to the original scholarship of J.E. McGuire and the more recent scholarship of Glenn

²⁶³ Hartz and Cover, 515-516.

²⁶⁴ Leibniz, G.W. “Created Things Are Actually Infinite”. *The Labyrinth of the Continuum: Writings on the Continuum Problem: 1672-1686*. Richard T.W. Arthur, ed. New Haven and London: Yale University Press, 2001. 1393.

²⁶⁵ Leibniz “Principles”, Paragraph 1; Leibniz “Monadology”, Paragraph 69.

²⁶⁶ Levey, Samuel. “Leibniz on Mathematics and the Actual Infinite Division of Matter”. *The Philosophical Review*, Vol. 107, No. 1 (Jan., 1998), pp. 49-96. 51-53.

Hartz and J.A. Cover concerning the metaphysical status of matter and space in Leibnizian thought. McGuire, Hartz, and Cover argue for a “tri-part” metaphysics in which monads occupy one level of reality, matter another (derivative) level of reality, and space an ideal level of reality.²⁶⁷ They argue for the distinction between matter and space by citing Leibniz’s statements that shapes or figures do not lie in material objects themselves, but in the minds of the percipient.²⁶⁸ Shape is added to matter as a perception in the mind. Shape and space in turn are ideal rather than real, meaning that space is a representation of the possible coexistential relations of matter.²⁶⁹

Hartz and Cover also note that material level of reality is distinct from the reality of monads because monads are not foundation of space: space is an ideal relation based on the existence and arrangement of material bodies.²⁷⁰ Thus, according to the “tri-part” interpretation of Leibnizian metaphysics, monadic and material realities are real, while space is an ideal form of reality.²⁷¹ I argue that such an interpretation of matter as actually infinitely divisible, while space being potentially infinitely divisible invites a host of incoherences regarding the monadic relation of material parts and wholes.

The root of Leibniz’s issue concerns the relation between the monadic realm and the so-called material realm evident in both the tri-part levels of reality interpretation by Hartz, Cover, and McGuire as well as the two part reality in Look’s interpretation. If we assume that matter is

²⁶⁷ Hartz, Glenn A. and Cover, J.A. “Space and Time in Leibniz’s Metaphysics”. *Nous*. Vol. 22, No. 4. December 1988. 493-519. 497-508; McGuire, 307-310.

²⁶⁸ Hartz and Cover, 494-496.

²⁶⁹ Hartz and Cover, 515-516.

²⁷⁰ Hartz and Cover, 502-504.

²⁷¹ Note that this interpretation of matter and monad as real could also be valid in the “two-level” interpretation of Brandon Look inasmuch as we buy the claim that matter is distinguishable from space as a real relation is distinguishable from an ideal relation.

actually divisible *ad infinitum*, we can surely conceive that a given whole of a material body is resolvable into a reality of infinitely divided parts reaching the monadic or simple metaphysical level. This inference is warranted when Leibniz claims that each body consists of actual smaller bodies without limit on account of each part of a material body acting on another material part. Keep in mind that monads do not *compose* material bodies since Leibniz does not endorse MIST despite that material bodies consist of discrete rather than continuous parts.²⁷² Due to a completed process of division, an infinity of monads are the foundation of, in some sense, the material bodies.

The problem arises when attempting to account for the resolution of a material body to its actual infinity of monads and then revert from the monadic level to the composite level of the material body. Despite the assertion that minima cannot comprise a line or other quantum because true minima lack parts as is the case in Leibniz's Euclidean theory of space, how can these indivisible "windowless" beings ever be compounded in any respect so as to form a composite material body given SIMT? It is precisely this "windowlessness" of monads that causes the problem. For Leibniz, the relations between monads are ideal rather than real with the infinitely populated world of monads synched through pre-established harmony.²⁷³ Leibniz writes of the relation between each monad and its corresponding body:

"Thus, although each created monad *represents* the whole universe, it *represents more distinctly* the body that is exclusively assigned to it, and of which it forms the entelechy [see note in 18]. And just as that body expresses the whole universe through the interconnection of all

²⁷² Arthur, Richard T. "Russell's Conundrum: On the Relation of Leibniz's Monads to the Continuum." 196-197.

²⁷³ See Woolhouse, Roger. "Pre-Established Harmony Returned: Ishiguro versus the Tradition". *Studia Leibnitiana*, Bd. 17, H. 2. 1985. 204-219 for a complete account of recent literature on Pre-Established Harmony in Leibniz.

matter in the plenum, the soul also represents the entire universe by representing its particular body.”²⁷⁴ (my italics added)

Note that Leibniz *does not say* that each monad *interacts with* or *acts upon* its corresponding material body and correspondent monads. Rather, each monad *represents* its correspondent body in particular and the universe of bodies (and monads) in general. This representation of its particular body and the universe is according to perception rather than by actually interacting with its body and the remaining universe. The monad is its own center of activity, but that does not mean its activity is the interaction with other monads via composition to form bodies.

That the relation between each monad is ultimately ideal rather than real is a criticism which Johann Gottfried Herder raised against Leibniz’s metaphysics in the 18th century. Courtesy of Nigel DeSouza’s analysis, Herder buys Leibniz’s argument for monads but objects to the “windowlessness” of the monads, which results in the world ultimately consisting of ideal rather than real relations.²⁷⁵ Herder thinks it is absurd for something that possesses its own activity and reflects other beings in its essence to bear only ideal relations toward the other things.²⁷⁶ By raising this criticism of Leibniz’s philosophy through Herder, the reader can grasp a fuller critical context of the reception of Leibniz’s metaphysics. This context shows the reader that Leibniz conceived the relation between monads and other monads to be ideal rather than real on account of their “windowlessness”.

²⁷⁴ Leibniz “Monadology”, Paragraph 62.

²⁷⁵ DeSouza, Nigel. “Leibniz in the Eighteenth Century: Herder’s Critical Reflections on the Principles of Nature and Grace”. *British Journal for the History of Philosophy* 20:4, 773-795. 2012. 776-777.

²⁷⁶ DeSouza, 776-777.

We now arrive at the essential contradiction given Leibniz's assertion of the actual infinite divisibility of matter and the composition of material bodies through monads: it is absurd for a whole to bear real relations to its parts, while the ultimate parts bear only ideal relations toward the whole and to each other. An actual whole requires the actuality of its parts. The actual wholes' parts *really* compose the whole of which they are parts and consequently bear *real* relations toward one another. Real relations require activity between the parts. But the ultimate parts, Leibnizian monads, do not act on each other at all. Leibniz cannot assert that matter is actually infinitely divisible on the ground that each part of the body acts on other parts while being infinitely divisible, but then assert that the ultimate metaphysical reality of composite things is the monad, which then bears only ideal relations to the other monads which comprise composite things since monads are not beholden to quantitative intellectual divisibility. Leibniz cannot have his cake and eat it too; he cannot claim that the whole bears actual relations to its parts, while articulating a theory in which the ultimate parts of the whole ideally relate to one another and to the whole.

Chapter Two Conclusion

Although Leibniz's attempt to reconcile simplicity and infinite divisibility is unique versus the other philosophers studied thus far, it appears that it is not without great difficulty for Leibniz to resolve the ideal versus real part/whole distinction. When examining Kant's resolution to the primary problem of the dissertation in his *Critique of Pure Reason*, it is clear that Leibniz's mistake when formulating his resolution to the conflict between simplicity and infinite divisibility is that he assumes to possess knowledge of ultimate reality. When consulting phenomena or experience itself, Kant would argue that Leibniz could claim the infinite

divisibility of matter and of space, but with the provision that matter and space are potentially infinitely divisible without recourse to a simple being.

Yet Leibniz successfully provides a potential resolution to the hollow worlds problem that apparently plagues the Pre-Critical Kant's force-shell theory. By rejecting quantitative intellectual divisibility, Leibniz's world is full of beings/souls and their activities rather than a world that is simply full of activity. Leibniz also avoids the apparent pitfall of Kant's force-shell theory by virtue of his inversion of qualitative intellectual divisibility. Kant's force-shells are quantitatively divisible since they exert a shell of attractive and repulsive forces. But this shell of forces surrounds an indivisible point that, if Holden is correct, depends on its shell for its existence. The indivisible punctum thereby loses its substantiality, unlike Leibnizian monads whose substantiality is preserved through their self-sufficiency. It is likely that Leibniz rejects action at a distance on the ground that action at a distance requires quantitative intellectual divisibility, a form of divisibility that Leibniz precludes in his monadological theory.

As we close this Leibniz chapter, I would like to point out that it is precisely Leibniz's appeal to the actual parts theory that I will take issue with in the opinion chapter. Without actual parts theory, it is clear that Leibniz's monadological theory would collapse, lacking an account for the actuality and priority of the parts over the whole. It is part of my objective to show that it is possible to reconcile simplicity with infinite divisibility without actual parts theory. Let's investigate the Critical Kant's response to the problematic relation between the metaphysical principle of simplicity and the mathematical principle of space's infinite divisibility.

Chapter Three:
Kant and the Antinomy of Matter's Divisibility

Chapter Three Introduction

The philosophers we have studied thus far tend to provide a definite response to this question of matter's infinite divisibility. We have seen David Hume and George Berkeley affirm the view that matter consists of minima, whether unextended or extended. We have also seen philosophers like Leibniz and the pre-critical Immanuel Kant attempt to reconcile infinite divisibility and minima. It would be noteworthy should a philosopher deem that this question is unanswerable. This is what Immanuel Kant proposes due to the limitations of human reason.

In the *Critique of Pure Reason* Kant provides four antinomies of pure reason. An antinomy results when human reason inquires into what transcends all possible experience. Prior efforts to resolve the conflict between the principles of simplicity and infinite divisibility have failed because philosophers have sought to account for the divisibility of 'noumenal' matter. Kant's critical response to the continuum problem includes two results based on his distinction between *noumena* and *phenomena*: noumenally speaking, the divisibility of matter is unsolvable; phenomenally speaking, matter is infinitely divisible because Euclidean space is an a priori formal intuition and the basis for our experiences of phenomena. Kant presents his solution in the second antinomy of the *Critique*. He finds the arguments supporting metaphysical truth of simplicity in the thesis or the mathematical truth of infinite divisibility in the antithesis equally valid; however, the inferences proponents of the thesis and antithesis make extend beyond possible experience. For the sake of preserving science and the unity of knowledge of possible experiences, Kant relegates the principle of simplicity to noumena and confines the extent of science's conclusions regarding matter to phenomena. The conflict between the metaphysical

truth of simplicity and the mathematical truth of space's infinite divisibility is too great a problem for human reason to bear when studying the nature of reality apart from experience.

The result of Kant's second antinomy is the dissolution of the distinction between the simplicity of the minimum and the minimum's distinguishable parts. The concepts of intellectual and metaphysical divisibility also become null and void. Kant's relegation of the metaphysical notion of simplicity to what is beyond human experience sets up his account for the divisibility of matter as a phenomenon. Since the foundations of experience do not afford the human being any evidence of an indivisible or simple being, material objects must be infinitely divisible as the space which they occupy is infinitely divisible. I will show how Kant's treatment of space as an a priori formal intuition results in his distinction between phenomena and noumena, preventing him from entertaining the possibility of a distinction between the simplicity of a minimum versus the minimum's possible distinguishable parts.

Kant's critical response to the infinite divisibility of matter as phenomena is incomplete without grasping his argument for matter's infinite divisibility in his *Metaphysical Foundations of Natural Science*, a work written during Kant's critical period.²⁷⁷ This critical position contrasts remarkably with his pre-critical force-shell theory. My goal in this section is not only to determine why Kant qualifies his response to the question of material divisibility, but also to discover why he deems that his force-shells are impossible given his critical theory.

The basis for Kant's rejection of force-shells comes from his theory that space is an a priori formal intuition. Since space is infinitely divisible and is the form of all our intuitions, making experience possible, every phenomenon that is presented in human experience must

²⁷⁷ Friedman, Michael. "Metaphysical Foundations of Newtonian Science". Robert E. Butts, ed. *Kant's Philosophy of Physical Science*. Boston: D. Reidel Publishing Company, 1986. pp. 25-60 25.

conform to the conditions of space. Matter must take on the features of the space which it occupies; otherwise, it would lie completely outside the realm of human experience.

I will first study Kant's argument for the infinite divisibility of space in the *Metaphysical Foundations of Natural Science*. Kant demonstrates his departure from his force-shell theory on the ground that matter is a plenum—a principle found in Leibniz's metaphysics but also an application of his claim that space is an a priori formal intuition. Second, I will account for Kant's two arguments that space is an a priori formal intuition in the "Transcendental Aesthetic" of his *Critique of Pure Reason*. Formal intuition of space makes experiences possible, removing the possibility that a simple being is a possible object of experience and guaranteeing that matter is a plenum. Third, I will focus on the thesis and anti-thesis in the Second Antinomy, providing Kant's solution according to the distinction between phenomena and noumena as presented in the "Transcendental Aesthetic". Kant therefore affirms the principle that matter must conform to the space which it occupies, unlike his earlier force-shell theory in which matter need not conform to the features of the space which it occupies.

Chapter Three-Section A:

The Metaphysical Foundations of Natural Science-

The Plenum Principle and Matter's Infinite Divisibility

Kant's *Metaphysical Foundations of Natural Science* is a critical work written between the first and second editions of the *Critique of Pure Reason*. This text is not only the promised application of his principles to natural science as expounded in the *Critique*, it is also his attempt to provide a metaphysical account of Newtonian science, particularly of the various physical

Laws described in Newton's *Principia*.²⁷⁸ A significant aspect of Kant's metaphysics of nature is his account of matter in the section of the *Metaphysical Foundations* entitled "Metaphysical Foundations of Dynamics". Kant argues for the infinite divisibility of matter, rejecting his previous force-shell theory. Key to his argument regarding the divisibility of matter is the Leibnizian principle of the plenum: each part of matter is matter. I intend to show that Kant's reference to the Leibnizian plenum principle is an application of his theory that space is an a priori formal intuition, removing the possibility that matter consists of simple parts or monads.

Prior to beginning a formal account of Kant's argument for material infinite divisibility, I will provide preliminary summaries and responses to questions concerning the need for Kant to provide a metaphysical account of science, as well as a question concerning a "gap" in the structure of Kant's Critical theory of matter. First, I will provide a sketch of the meaning of metaphysical accounts in Kantian theory. Second, I will elucidate a gap in Kant's explanatory theory of science relating to causation. These responses will provide the reader greater awareness of the crucial issues of matter's divisibility and how it fits into his Critical elucidation of Newtonian science.

Natural science, the study of the bodies of external sense, presupposes a philosophy of nature.²⁷⁹ In the *Metaphysical Foundations*, a philosophy of nature is a special "metaphysical account" and critique of Newtonian science.²⁸⁰ Kant claims that Newtonian science requires a

²⁷⁸ Butts, Robert E. "Introduction: Kant's Quest for a Method of Metaphysics". Robert E. Butts, ed. *Kant's Philosophy of Physical Science*. Boston: D. Reidel Publishing Company, 1986. pp. 1-24. 15-18.

²⁷⁹ Kant, Immanuel. "Metaphysical Foundations of Natural Science" in *Kant's Philosophy of Material Nature*. James W. Ellington, ed. Hackett Publishing Company: Indianapolis, Indiana, 1985. 467. (abbreviation: Kant, MFNS)

²⁸⁰ Friedman, 25-26.

metaphysical account because it rests on empirical concepts.²⁸¹ Empirical concepts require grounding through a priori principles. The *Metaphysical Foundations* is Kant's attempt to ground Newtonian science through a priori principles, but with a special empirical character via the notion of matter as the movable in space.²⁸² This metaphysics of nature thereby focuses on the form of natural inquiry rather than the matter or the objects of study in a particular science.²⁸³ For example, chemistry studies the chemical interactions of atoms and molecules, while biology studies living things. A metaphysics of nature studies neither atoms and molecules nor living things strictly speaking; rather, a metaphysician of nature would examine what principles and objects both particular sciences share in common: matter as the movable in space.

It is precisely matter as the movable in space, the form of all empirical concepts, that Kant studies in his philosophy of nature. This way, Kant hopes to ground the Newtonian mechanistic model of matter with a dynamical model. Dynamical forces in Kantian natural philosophy replace the lifeless atoms in Democritean and Newtonian physics. Kant, however, focuses the bulk of his criticism toward Newtonian physics.²⁸⁴

My focus in the *Metaphysical Foundations* is on the chapters entitled the “Metaphysical Foundations of Phoronomy” and the “Metaphysical Foundations of Dynamics”. The examination begins with Kant's definition of matter in the “Metaphysical Foundations of Phoronomy” as the “movable in space”. It continues with an account of Kant's argument in support of the infinite divisibility of matter in the chapter on dynamics. By comparing the notion

²⁸¹ Kant, MFNS, 469.

²⁸² Kant, MFNS, 480; Westphal, Kenneth. “Does Kant's ‘Metaphysical Foundations of Natural Science’ Fill a Gap in the ‘Critique of Pure Reason’?”. *Synthese*, Vol. 103, No. 1 (Apr., 1995), pp. 43-86. 51.

²⁸³ Watkins, Eric. “The Argumentative Structure of Kant's Metaphysical Foundations of Natural Science”. *Journal of the History of Philosophy*. 36:4 October, 1998. Pp. 567-593. 569-573.

²⁸⁴ Michael Friedman argues that it is not so much a mechanical model of matter that is Kant's chief target in the *Metaphysical Foundations*, but Newtonian absolute space. See Friedman, 30.

of empirical space and its essential connection to the existence of matter to space as a universal form of external intuitions, it will be evident that matter must fill each part of empirical space.

It is important to stress that Kant is critiquing the mathematical-mechanical model in which impenetrability is the primary property that defines matter, because impenetrability requires explanation via dynamical forces.²⁸⁵ He claims that impenetrability cannot be discovered in experience; rather, things are relatively impenetrable by virtue of a balance in attractive and repulsive forces between two parts of matter.²⁸⁶ Impenetrability is a relative rather than absolute feature of matter. Should one hold that matter's impenetrability is absolute is tantamount to agreeing with the circular explanation that matter is impenetrable because it is impenetrable.²⁸⁷ Kant thereby accepts that impenetrability is the result of the dynamical forces of matter.

Kant thinks that a special argument for matter's infinite divisibility is needed beyond the affirmation of space's infinite divisibility.²⁸⁸ We will witness Kant's application of the a priori form of the unity of space's features to that of matter: since matter is perceived through the a priori formal intuition of space which is given as infinitely divisible, matter must conform to space as material content is structured according to its form. Prior to delving into this argument, I would like to detail briefly Kant's definitions of matter and space.

Kant divides the chapters in his *Metaphysical Foundations of Natural Science* according to a particular treatment of motion. It is in the first chapter, entitled "Metaphysical Foundations

²⁸⁵ Brittan, Gordon G. Jr. "Kant's Two Grand Hypotheses". R. E. Butts, ed. *Kant's Philosophy of Physical Science*. Boston: D. Reidel Publishing Company, 1986. 61-94. 78.

²⁸⁶ Brittan, 79-80.

²⁸⁷ Brittain, 80.

²⁸⁸ Kant, MFNS, 504-505.

of Phoronomy”, in which he details motion according to pure quantity without regard to any quality matter possesses.²⁸⁹ In order to grasp this notion of motion, Kant defines matter as “the movable in space” and space as that in which all motion takes place.²⁹⁰ His definition of matter and space relies heavily on the notion that motion cannot exist unless there is something that is moving (matter) and something pure or absolute in which the motion takes place (space).²⁹¹

Kant conceives motion in terms of a subject, matter, and its form, space.

The space under investigation in the *Metaphysical Foundations* is empirical space, which Kant defines as the totality of all objects of experience. It is thereby dependent on matter for this existence. This dependency on matter is the framework from which I will argue that matter is a plenum for Kant. Since it is the totality of all objects, empirical space is capable of motion in the sense that it moves according to one’s acquaintance with the totality of experienced objects in space. For example, let’s say that a person is looking at a table with a laptop and a coffee mug placed to the laptop’s left from the perspective of the percipient. Now, that same person moves to his right, shifting his perspective of the table and the two objects on the table. The objects and the table itself take on a new set of relations. Empirical space is thereby subjective in nature, but dependent on space as a form of the external intuition as described in the *Critique*, as I will describe later. Neither empirical space nor space as a form of our external intuitions exists apart from human consciousness. With the notions of matter and empirical space in mind, let’s examine Kant’s argument for the potential infinite divisibility of matter in his chapter entitled the “Metaphysical Foundations of Dynamics”.

²⁸⁹ Kant, MFNS, 477.

²⁹⁰ Kant, MFNS, 480.

²⁹¹ Kant, MFNS, 481.

The “Metaphysical Foundations of Dynamics” focuses on motion conceived in matter as what Kant calls “an original moving force”.²⁹² By conceptualizing motion as a quality of matter, an original moving force, Kant reiterates his pre-critical theory of dynamical matter—matter is inherently active rather than inert. In the first explication and the first proposition, Kant argues that matter fills space by its own moving force rather than by its own extension. This theory contrasts with Cartesian theories of matter in which matter fills space through its extension.²⁹³ Since some forms of matter are more or less solid than others, like the difference between air and water, there must be some cause to the varying solidity of matter. The attractive and repulsive forces cause the varying degrees of matter’s solidity.²⁹⁴ Thus, Kant shows his support for dynamical matter similar to Leibnizian theory, while differing from the mathematical-mechanical models of Descartes and Newton.

Kant shows the validity of the plenum principle in his argument for the infinite divisibility of space. Acknowledging that a proof of matter’s infinite divisibility on the basis of space’s infinite divisibility is inadequate, Kant argues for a material plenum on the basis of the existence of an indefinite quantity of resistive focal centers between any two given parts of matter. I hope to show through the rest of this section that the plenum principle is connected to Kant’s claim of the uniformity of space as shown in the “Transcendental Aesthetic” of the *Critique of Pure Reason*. Finally, I will delve into Kant’s argument for the principle of the plenum and analyze it as an application of the uniformity of space from the standpoint that space is an a priori formal intuition.

²⁹² Kant, MFNS, 477.

²⁹³ Kant, MFNS, 496-497.

²⁹⁴ Kant, MFNS, 498.

Kant's argumentation relies heavily on the claim that matter is only relatively impenetrable rather than absolutely impenetrable. Kant no longer conceptualizes matter as consisting of puncta from which attractive and resistant forces emanate. Since Kant describes impenetrability and other qualities of matter in terms of attractive and repulsive forces, it is the attractive and repulsive forces that constitute matter's essence rather than the common qualities of matter like solidity or impenetrability.

The relative impenetrability of matter is founded on the theory that matter is dynamical. Matter cannot be absolutely penetrated; otherwise, an infinite compressive force would have to be possible.²⁹⁵ Kant describes the relative impenetrability of matter:

“But this force has a degree which can be overcome, and hence matter's space of extension can be diminished, i.e., its space can be penetrated in a certain measure by a given compressive force...”²⁹⁶

The upshot of relative impenetrability is that a material object can reduce another material object's extension via sufficient compressive forces; however, the finite compressive forces cannot completely overcome the ever increasing resistive forces of the other material object since no infinite compressive force is possible. Matter is relatively impenetrable given that one particular material object's compressive forces do not overcome another object's ever-increasing resistive forces.²⁹⁷ For Kant, one can empirically verify relative impenetrability but cannot do the same for absolute impenetrability because an infinite compressive force cannot exist. With this notion of relative impenetrability in mind, we are ready to examine the crux of Kant's argument for infinitely divisible matter.

²⁹⁵ Kant, MFNS, 501.

²⁹⁶ Kant, MFNS, 502.

²⁹⁷ Brittan, 82.

Kant requires the plenum principle since space's infinite divisibility alone does not prove matter's infinite divisibility. He thereby adds a contentious claim that each part of matter is itself matter: "But in a space filled with matter every part of the space contains repulsive force to counteract on all sides all remaining parts...Hence every part of space filled by matter is of itself movable..."²⁹⁸ Kant acknowledge this claim begs the question against the monadists.²⁹⁹ Such a claim would also beg the question against a Newtonian, who supports the existence of matter. Let's see how this plenum principle applies to the argument:

1. Matter fills infinitely divisible space by its own force of extension.
2. If matter fills infinitely divisible space by its own force of extension, then matter must also be infinitely divisible assuming that space is filled with matter.
3. (The plenum principle) If matter fills infinitely divisible space, then space is filled with matter.
4. Space is filled with matter.

Therefore, matter must also be infinitely divisible.³⁰⁰

A lucid understanding of Kant's first premise is key for the argument since it purportedly justifies the inference that matter is also infinitely divisible and that space is filled with matter (the plenum principle). A problem arises when considering Kant's pre-critical and critical thoughts on the nature of matter's activity of filling space by the force of its extension: why can Kant admit in the *Metaphysical Foundations* that matter fills space by the force of its extension

²⁹⁸ Kant, MFNS, 503-504.

²⁹⁹ Kant, MFNS, 504.

³⁰⁰ Kant, MFNS, 502-504.

without admitting the existence of a force-shell as he does in the *Physical Monadology*? Kant spends considerable effort to show that the monadist cannot argue for the existence of a force-shell or simple part of matter given the activity of matter filling space by its force of extension. The crux of Kant's argument is that the puncta of force-shells cannot be substances because they are generated from the interaction between other resistant forces.

In the *Physical Monadology* Kant approaches the question of the infinite divisibility of matter from the standpoint of substance, while he considers matter in the "Metaphysical Foundations of Dynamics" from the notion of the motion by which matter fills space. In the *Physical Monadology*, Kant argues initially that matter must consist of simple parts, substances, because the accidental relations of the composite material objects would be ungrounded; however, he argues against the claim that that the puncta in force-shells could serve as the foundation of substances.

In the *Metaphysical Foundations* the reasoning behind the plenum principle is that the force-shell would collapse if no centers of resistive forces between any two points of the shell exist. Keep in mind that Kant is working with the definition of matter as the movable in space. Here is the plenum argument:³⁰¹

1. Let's assume that a physical monad (force-shell) exists.
2. If such a physical monad exists, then it is possible to designate a point inside the diameter of the shell given the infinite divisibility of space.

³⁰¹ Kant, MFNS, 504-505.

3. Yet this point inside the force-shell (existing between the center of the shell and a designated exterior part of the shell) must resist the shell's center and the corresponding exterior point along the shell's circumference.
4. If this point inside the force-shell resists the center and the exterior point, then this point is movable and is therefore matter too.
5. If this point is matter too, then the force-shell does not exist, and matter is infinitely divisible.

Therefore, matter is infinitely divisible.

Matter exists as a physical continuum, a series of overlapping series of repulsive and attractive forces. The puncta from the *Physical Monadology* are thereby gone in Kant's version of matter in the *Metaphysical Foundations*. Thus, the repulsive and attractive forces are matter itself in physical continuum theory since each part of this physical being is movable in space.³⁰² Yet the plenum principle argument for the physical continuum is questionable: why must the force-shell collapse without an indefinite number of repulsive force centers?

Justification for this claim is difficult given only the repulsive forces of matter that Kant has already established in the *Metaphysical Foundations*. Since the repulsive forces gradually diminish the further a point is from the shell, there is only a force that moves other things away from the shell's center. The intermediate point would then be only a certain degree of repulsive force in the entire shell. Since each point in the shell exhibits only repulsive forces, it would be impossible for the center and the circumference point to collapse into one another. For there is

³⁰² Kant, MFNS, 505.

only a force in one direction resulting in the two points remaining distant from one another. We therefore would either conclude that Kant begs the question in the sense that he presupposes a physical continuum in order to prove a physical continuum, or we might engage in an alternative argumentative interpretation based on the *Critique of Pure Reason* and his treatment of space in the “Transcendental Aesthetic”.

Kant’s *Metaphysical Foundations of Natural Science* is an application of his conclusions reached for scientific inquiry in the *Critique of Pure Reason*. It is an application of the *Critique* particularly in the sense that the universality of scientific truths is founded on the Categories and the forms of intuition, time and space. Since I am dealing with the nature of matter which occupies space, I will analyze Kant’s arguments that our experiences cannot exclude space in the chapter of the *Critique* entitled, the “Transcendental Aesthetic”. My decision to incorporate an analysis of Kant’s argument for the universality of space is to provide the framework for my eventual claim that Kant incorporates the plenum principle as a universal characteristic of matter as space is a universal form of all our outward experiences.

In the “Transcendental Aesthetic”, Kant provides the intellectual background and argumentation for the groundwork of a theory of appearances—the principles of sensibility.³⁰³ Space, is not a mere conception derived from experience; rather, it is the form of all outer intuitions.³⁰⁴ What Kant means here is that space makes possible all experiences of objects that we intuit as belonging outside ourselves. By contrast, other thinkers have argued for the reality of an independently existing and eternal space, like Samuel Clarke, or have proposed that space

³⁰³ Ewing, A.C. *A Short Commentary on Kant’s ‘Critique of Pure Reason’*. Chicago, Illinois: The University of Chicago Press, 1950. 28-29; *Kant Critique*, 1-3.

³⁰⁴ *Kant Critique of Pure Reason*. Norman Kemp Smith, ed. Translated by Andrew Fiala (Introduction), J.M.D. Micklejohn (Translation). Barnes and Noble Publishing, 2004. 3.

is merely a confused representation of the coexistences of material objects, such as Leibniz.³⁰⁵ It is precisely by setting up space as a transcendental condition for experience, as a form of intuition, that Kant eventually relegates the metaphysical principle of simplicity to the realm of noumena, things that are beyond possible existence.

My goal with this treatment of Kant's theory of space in the "Transcendental Aesthetic" is to argue by analogy that Kant's statement's regarding the universality of space lays the framework for grasping matter as a plenum in the *Metaphysical Foundations*. Just as infinitely divisible space permeates all our intuitions of objects outside us in empirical space, and empirical space is founded on the existence of matter, matter too must exist wherever empirical space exists. This comparison I am making to the universality of space in the *Critique* is a difficult path to take, but is ultimately justifiable in my opinion because of the connections between the *Critique* and its application to physical science, the *Metaphysical Foundations*.

To accomplish this task, I will initially define important terms on which Kant founds his argumentation regarding space, namely intuition, sensibility, matter, and form. Next, I will briefly analyze Kant's two arguments in the "Transcendental Aesthetic", and note relevant conclusions from the argument regarding the universality of space. Finally, I will link space as a form of our intuition to the framework for empirical space, connecting the universality of space in all experiences with matter as a plenum in empirical space.

Intuition and sensibility are two key concepts on which Kant bases his argumentation in the "Transcendental Aesthetic". Kant defines intuition as the manner by which our knowledge of

³⁰⁵ Caird, Edward. *A Critical Account of the Philosophy of Kant: With an Historical Introduction*. Volume I. Bristol, England: Thoemmes Press, 1999. 244-246.

objects relates to objects.³⁰⁶ He considers sensible intuition as the only kind of intuition because we can only receive representations through the senses.³⁰⁷ Space and time are two forms of sensible intuition. Sensibility is the capacity for receiving representations in some mode by which we can be affected by objects.³⁰⁸ No sensation or intuition can exist unless an object is given. Norman Kemp Smith notes that Kant implicitly critiques the Leibniz-Wolff distinction between sensibility (appearance) and thought.³⁰⁹ The Leibniz-Wolff distinction between thought and appearance is a matter of clarity rather than content; however, Kant claims that it is impossible to consider the things which appear to us as the same knowledge as things as they are in themselves—irrespective of the clarity of our perceptions. We thereby have a division between *phenomena*, things as they appear to us, and *noumena*, things as they are in themselves. It is with *phenomena* that Kant is chiefly concerned since *noumena* are unknowable, being outside sensible intuitions.³¹⁰

The matter and form of phenomena are two principles governing the objects of human experience. When an object is given by sensuous intuition, the intuition is considered empirical, the object of the intuition being the *phenomenon*.³¹¹ Matter is the sensation or content of the phenomenon, while form is the arrangement of the matter of sensation. Matter is given to us after experience, while form is given to us before experience. For instance, Kant would argue

³⁰⁶ Kant, *Critique*, 1.

³⁰⁷ Smith, Norman Kemp. *A Commentary to Kant's 'Critique of Pure Reason'*. New York, NY: Noble Offset Printers, INC., 1962. 80-81.

³⁰⁸ Kant, *Critique*, 1.

³⁰⁹ Smith *Commentaries*, 143-144.

³¹⁰ There is debate amongst certain Kant scholars like James Van Cleve and Michael Devitt as to whether the phenomena/noumena distinction results in a one world or two world theory: are phenomena and noumena two aspects of the same thing or are they two distinct things? See Van Cleve, James. *Problems from Kant*. Oxford: Oxford University Press, 1999. 143-146; Devitt, Michael. *Realism and Truth*. Princeton, N.J.: Princeton University Press, 1984. 59; Bird, Graham. *Kant's Theory of Knowledge*. London: Routledge and Kegan Paul, 1962. 37; Dreyer, D.P. *Kant's Solution for Verification in Metaphysics*. London: Allen and Unwin, 1966. 513

³¹¹ Kant, *Critique*, 2.

that the red ball a person perceives is a phenomenon in which the color yellow is the matter, while the form of the red ball's extension is provided through human consciousness. Without space as a form of the sensible intuition, no representations would exist and thereby no experiences. The mind therefore, via the forms of intuitions, makes experiences possible. Kant's Copernican revolution in philosophy is thereby evident: the mind actively constructs human experiences and the knowledge gained from the said experiences instead of the mind passively accepting knowledge from human experiences.³¹² It is from the standpoint of the mind structuring experiences in terms of space as a form of sensible intuition that the plenum principle is supplied as a condition for the experience of empirical space. The plenum principle is founded on the notion that empirical space which exists only through synthesis of the parts depends on a space filled with matter for its synthesis.

My claims concerning empirical space are based on a reading of a conflict between two interpretations of space in the *Aesthetic* and *Analytic* portions of the *Critique*, as well as certain statements regarding empirical space found in the section on Phoronomy from the *Metaphysical Foundations of Natural Science*. I will begin with the treatment of space in the *Aesthetic* and the *Analytic*, briefly providing an overview of the interpretive conflict and then analyzing the arguments put forth for space as an a priori form of intuition in the *Aesthetic*. Finally, I will connect Kant's statements regarding empirical space as a synthesis of a sensible manifold to his statements regarding empirical space founded on matter as the "movable in space".

The interpretive difficulties regarding the nature of space are based on the question of whether space is given by intuition as an a priori whole or whether space is a synthesis of some

³¹² Ewing, 15-17.

empirical manifold. Kant argues in the *Aesthetic* that space is given as an a priori whole by intuition, but later argues in the *Analytic* that space is constructed via synthesis.³¹³ In the former case space is a whole that precedes and conditions its parts, while the parts precede and condition the whole of space in the latter case. In the *Aesthetic*, space is given as a whole from which parts are gleaned by some form of limitation, while the whole space is assembled by the synthesis of manifold parts in the *Analytic*.³¹⁴ Space in the *Aesthetic* is what Kant calls a *totum analyticum*, while it is a *totum syntheticum* in the *Analytic*.

Norman Kemp Smith notes that Kant does not appear to acknowledge this conflict between the *Aesthetic* and the *Analytic*; nevertheless, Smith proposes a solution to this conflict on the ground that the two notions of space are given through a distinction between what he terms “...the *a priori* manifolds of space and time and the empirical manifold which is apprehended in and through them.”³¹⁵ According to this solution, the empirical manifold can only be synthesized via the intuition of a universal space that is given beforehand. Space in the *Analytic* refers to this empirical manifold that is synthesized only on the presupposition of the space elaborated in the *Aesthetic*. Space in the *Aesthetic* is thereby known according to Smith as the “principle of synthesis”, while space in the *Analytic* is known as a “synthesized totality”.³¹⁶ Kant seems to regard space as an intuition that is *totum analyticum* rather than *totum syntheticum*.

The concern with these two interpretations of space is avoided if we perceive Kant’s descriptions as a preparation for his remarks concerning empirical space and the infinite

³¹³ Smith *Commentaries*, 94-96.

³¹⁴ Smith *Commentaries*, 94-99.

³¹⁵ Smith *Commentaries*, 95.

³¹⁶ Smith *Commentaries*, 97-98.

divisibility of matter in his *Metaphysical Foundations of Natural Science*. Inasmuch as empirical space is theorized as a *totum syntheticum* and this notion of space continues in the *Metaphysical Foundations*, Kant has built the plenum principle in to his natural philosophy. In order to understand Kant's remarks concerning empirical space and the potential connection to the plenum principle, I will briefly analyze Kant's arguments in the *Aesthetic* that establish the a priority of space.

The first argument establishes the non-empirical origin of space, while the second argument shows that space is a necessary representation of all outward experiences.³¹⁷ These arguments ultimately establish that space is transcendently ideal while sustaining the principle that space is empirically real.³¹⁸ Kant concludes that space cannot be a concept that has been derived from particular experiences since particular experiences of spatial relations presuppose a representation of space. He argues as follows:

1. If space is a concept that has been derived from outward experiences, then particular spatial experiences can be understood apart from an already given representation of space.
2. But particular spatial experiences cannot be understood apart from an already given representation of space.

Therefore, space is not a concept that has been derived from outward experiences.

The argumentative form appears valid as *modus tollens*. The complication arises when considering the justification for the claim that particular experiences cannot be understood from

³¹⁷ Kant *Critique*, 4; Caird, 242-244.

³¹⁸ Van Cleve notes how these arguments are similar to the ones found in Kant's *Prolegomena to Any Future Metaphysics*. See Van Cleve, 44-51.

an already given representation of space. While oversimplified in scope, it appears that there are two camps for interpreting the second premise of the first argument: 1.) A.C. Ewing, Cohen, Riehl, Caird, and Watson argue that Kant is identifying a logical rather than psychological priority of spatial relations vis-à-vis the representation of space itself. 2.) Norman Kemp Smith regards the first argument on psychological rather logical grounds vis-à-vis the claim that sensations differ qualitatively and are non-spatial.³¹⁹ Let's now examine both positions.

According to the logical interpretation, particular spatial relations like left/right, up/down logically presuppose space (as a Cartesian grid if you will) in order to grasp the aforementioned distinctions. Otherwise, a vicious circle would result since spatial relations cannot be perceived save in space as space could not be perceived without spatial relations.³²⁰ The human mind already possesses a Cartesian coordinate grid into which particular spatial relations are plotted and understood.

The psychological interpretation is founded on the claim that qualities from sensations are non-spatial and presuppose a representation of space in order for the sensitive qualities to be presented to human consciousness. This interpretation imports more than is explicitly stated as having been proven: not only is space of a non-empirical origin, it is imposed on experiences via the mind.³²¹ This psychological interpretation thereby gives a positive explanation of the origin of the representation of space rather than providing merely an argument for a negative conclusion. If sensations themselves are non-spatial, the representation space must be non-empirical since empirical content is provided through the sensations (the negative conclusion).

³¹⁹ Ewing, 33-35; Smith *Commentaries*, 102; Watson, John. *The Philosophy of Kant Explained*. New York: Garland Publishing, 1976. 83.

³²⁰ Ewing, 34.

³²¹ Smith *Commentaries*, 101.

Yet the reader can infer according to Smith that the representation of space must be supplied (by the mind). Despite this additional positive conclusion provided through the psychological interpretation, commentators disagree with the psychological line of interpretation provided by Norman Kemp Smith. This disagreement is founded primarily on Kant's language in the *Aesthetic*.³²²

I believe that the logical interpretation is a more natural reading of the first argument, there is some validity to Smith's claim regarding the complexity of tendencies and issues in the *Critique of Pure Reason*. This complexity of issues arises from the very intent of the *Critique* as a critique and synthesis of the epistemologies of his day: British Empiricism and Continental Rationalism. The psychological aspect of Kant's argumentation arises from his critique of British Empiricism, particularly that of David Hume who awoke Kant from his "dogmatic slumber". Hume's account of human nature and knowledge is inherently psychological. We might thereby expect a critique of such a theory to be somewhat psychological as well. The logical and psychological interpretations of Kant's first argument are therefore not necessarily mutually exclusive. Let's proceed to the second argument.

Kant concludes that space is a necessary representation a priori of all external intuitions since phenomenal objects presuppose space but not vice-versa. Here is an analysis:³²³

1. If we cannot imagine the non-existence of space, but can imagine a space without external objects, then space must be an independent representation a priori of all external phenomena.

³²² Ewing, 34; Smith *Commentaries*, 102-103

³²³ Kant *Critique*, 4.

2. We cannot imagine the non-existence of space, but can imagine a space without external objects.

Therefore, space must be an independent representation a priori of all external phenomena.

There is considerable controversy involved in the interpretation of this argument in light of the ambiguity of two words Kant uses in the German language: *vorstellung* (represent) and *denken* (think). Both “represent” and “think” are generally translated as “imagine”. Based on the German, Kant states in support of the second premise that we cannot represent (*vorstellung*) the non-existence of space though we may think (*denken*) of a space without external objects. A.C. Ewing summarizes the ambiguity well: “If “represent” means “think” we can represent both; if it means “make a mental image of” we can represent neither.”³²⁴ In the former case in which representation is identical to thinking in terms of considering the logical possibility of something, the ability to represent the absence of space and the ability to consider space without objects are clearly feasible. For there is no contradiction in affirming the logical possibility of a non-spatial world as there would be no contradiction in considering a space without objects as we would consider abstractly an empty Cartesian coordinate grid. In the latter case in which representation means “make a mental image of”, the absence of space cannot be imagined since imagination requires space while the space without objects cannot be made into mental imagery presumably because a mental image is of some object. We are thus left with an embarrassing interpretive dilemma which leaves the very meaning of Kant’s second argument opaque.

³²⁴ Ewing, 35.

Commentators like A.C. Ewing and Norman Kemp Smith have offered solutions to this difficulty by referring to some other sense of awareness beyond logical conceptualizations. Ewing writes that Kant may refer to a vague sense in which a human being is aware of an indefinitely extending space in our experience.³²⁵ In this way, space can be thought of without objects since our awareness is of indefinitely extending space and we cannot eliminate this awareness since the awareness is essential to our experiences of definite material objects. Norman Kemp Smith provides a similar form of interpreting the argument on the ground that Kant is referring to the psychological rather than logical impossibility of representing the absence of space: “For the criterion employed is not the impossibility of thinking otherwise, but our incapacity to represent the specific element as absent.”³²⁶ Smith’s interpretation of the second argument is thereby consistent with his interpretation of the first argument inasmuch as Kant is highlighting a psychological rather than logical point in each argument. It might be true that Kant is not ultimately referring to thinking and representation in terms of mental imagery or logical conceptualization, but in terms of some sense of awareness attendant with experience. Irrespective of the true interpretation and soundness of the arguments, the upshot of the arguments is that space is a form of outer intuition that is a universal or uniform condition of all human experiences.

Since space is a universal or uniform condition of all human experiences, it is given as what Kant calls an empirical reality and transcendental ideality. Within human experience, space is empirically real—a brute fact of human experience. Space is also transcendently ideal

³²⁵ Ewing, 35.

³²⁶ Smith *Commentaries*, 103-104.

since because it conditions the possibility of outward human experiences.³²⁷ I will now connect Kant's statements regarding the universality and uniformity of space in the *Critique* is connected to the uniformity of matter—the plenum principle—in the notion of empirical space in the *Metaphysical Foundations of Natural Science*.

The plenum principle is latent in Kant's statements regarding the nature of empirical space and matter in the “Metaphysical Foundations of Phoronomy”. Space in the *Analytic* sets the stage for a notion of empirical space to be applied in natural sciences via the notion that space is an a priori form of external intuition. Space in the *Analytic* is a synthesis from parts of space. The synthesis of space in the *Analytic* is remarkably similar to the synthesis of the parts of empirical space in the chapter on Phoronomy: “...and this space as the sum total of all objects of experience and itself an object of experience is called empirical space. Now such space insofar as it is material is itself movable.”³²⁸ Kant clearly states that empirical space is itself matter, being reducible to the totality of all (material) objects of experience. If we believe that empirical space is itself matter, then we must affirm the plenum principle inherent in Kant's treatment of empirical space: (empirical) space is a material plenum. We thereby have a Leibnizian account of matter inasmuch as space is filled with matter and each part of matter is matter.

The plenum principle allows Kant to claim that each part of the force-shell possesses its own repulsive force. By virtue of Kant's definition of matter as what fills space by its repulsive forces, each part of the shell is matter. Physical monad or force-shell theory collapses into physical continuum theory because empirical space depends on matter filling it. Thus, there is

³²⁷ Kant *Critique*, 7-8.

³²⁸ Kant MFNS, 481.

no empty space in empirical space given the statements in the chapter on Phoronomy. One could thereby compare the uniformity and universality of matter in empirical space to the universality and uniformity of space as the form of external experiences: the plenum principle is the condition of the possibility of empirical space as space is the condition of the possibility of all external phenomena.

Chapter Three-Section B:

The Second Antinomy and the Principle of Simplicity

It has become clear that Kant's notion of empirical space and of the matter as the phenomenon which constitutes it is a physical continuum on account of the plenum principle; however, Kant regards the inquiry into the divisibility of noumenal matter as an antinomy in the *Critique of Pure Reason*. In the *Critique*, Kant limits human knowledge to the realm of possible or sensible experiences—phenomena. Given this limitation of human knowledge, Kant hopes to show that a metaphysician's attempt to apply categories like simplicity and infinite divisibility to objects beyond possible experience results in contradiction. My goal in this section is to investigate Kant's notion of metaphysical simplicity and why he argues that such a notion should be relegated only to the noumenal realm in the second antinomy. Kant's notion of substance used in the second antinomy informs his concept of simplicity: by omitting the principle of sufficient reason from his account of substance and instead basing the notion of substance on the possibility of intellectually distinguishable parts. Kant thereby abandons the distinction between the simplicity of a minimum and the minimum's distinguishable parts.

I will first analyze the thesis and the anti-thesis of the second antinomy, particularly the interpretive difficulties that concern the notion of substance and help determine Kant's audience for the second antinomy. I will then show that Kant omits the principle of sufficient reason in his account of simplicity and substance and incorporates the notion that a simple substance is a being that lacks intellectually distinguishable parts.

While I will analyze the thesis and antithesis arguments in greater depth later in this chapter, a brief overview of the argumentation is helpful for grasping the various interpretations. The thesis and antithesis arguments are representative of the conflict between the metaphysical truth of simplicity and the mathematical truth of space's infinite divisibility. The thesis argument shows that simple substances exist because composition presupposes simple beings as constituents. The antithesis argument demonstrates that no simple substances exist since space is infinitely divisible and nothing simple can occupy infinitely divisible space without forfeiting its simplicity. Interpretive difficulties stem from consideration of what Kant means in these arguments by "substance", "simple", and "composite". I will argue not only the common view that "substance" refers to matter, but that Kant's antinomy is invalid inasmuch as he misrepresents his target audience for the thesis (Leibnizians and Wolfians) by omitting the principle of sufficient reason from his notions of "substance" and "simple".

There is considerable dispute amongst commentators not only as to whether the arguments in the second antinomy are sound, but also as to the actual content and focus of the antinomy itself. The real thorn concerning the actual content and focus of the second antinomy lies in interpreting the proponents of the anti-thesis, whom Kant calls the "Pure Empiricists".³²⁹

³²⁹ Kant *Critique*, 262-266.

While I do not claim to know who the real proponents of the anti-thesis are, I will provide relevant content needed to understand the dispute. In order to tackle the problematic interpretation of the proponents of the anti-thesis, an understanding of the general content and scope of the second antinomy is necessary.

The common interpretation of the second antinomy, espoused by T.D. Weldon, Van Cleve, and C.D. Broad, is that Kant focuses on the nature of the structure of matter itself—whether material objects consist of indivisibles (atoms) or are infinitely divisible.³³⁰

Rationalists, arguing on the basis of the metaphysical principle of simplicity, maintain that simple parts of matter must exist, while “Pure” Empiricists like David Hume argue that no simple parts exist. Another commentator, Oscar Schmirge, argues that the second antinomy is about the nature of the internal ground of all spatio-temporal objects of experience.³³¹

Rationalists claim that the internal ground of all spatio-temporal objects must be simple substances, while the “pure” empiricists claim that space, rather than substance, is absolutely primary. My opinion is that the common interpretation is correct inasmuch as the other views collapse into a discussion of material structure.

A basis for the structure of matter interpretation of the second antinomy rests on Kant’s reference to substances “occupying space”. Van Cleve comments on Kant’s phrase of “occupying space” in the first premise of the Antithesis: “By ‘occupying’ a space Kant means

³³⁰ Weldon, T.D. *Kant’s ‘Critique of Pure Reason’*. 2nd Edition. Oxford: Clarendon Press, 1958. 204-210; 293-294; Van Cleve, James. “Reflections on Kant’s Second Antinomy”. *Synthese* Vol, 47 No. 3. Part II (June 1981), pp. 481-494. 481-483; Radner, Michael. “Unlocking the Second Antinomy: Kant and Wolffe”. *Journal of the History of Philosophy*. 36:3 July, 1998. Pp. 413-441. 413-414; Broad, C.D.”Kant’s Mathematical Antinomies: The Presidential Address”. *Proceedings of the Aristotelian Society*. New Series. Vol. 55 (1954-1955). pp.1-22. 1-2.

³³¹ Schmirge, Oscar. “What is Kant’s Second Antinomy About?” *Kant-Studien*. 97. pp. 272-300. 272-274.

filling it as opposed to merely falling within it.”³³² Assuming that Kant’s usage of substance is consistent in both the thesis and anti-thesis, “substance” must refer to a material object. For if Kant is using substance in different senses upon comparing the thesis and anti-thesis, an antinomy could not result.

The proponents for the thesis are the Leibnizians and Wolfians on the basis of Kant’s statements on the nature of substance. Michael Radner traces Kant’s use of the term “substance” to Leibnizian-Wolffian substance metaphysics.³³³ C.D. Broad agrees with Radner, citing the Leibnizian truth in the thesis that substances are non-composite.³³⁴ The proponents for the thesis are therefore clear.

The exact proponents for the anti-thesis are unclear particularly because the usual suspects for the “pure empiricists” would not support space’s infinite divisibility. Schmierge notes that the usual suspects for empiricism, like David Hume and George Berkeley, cannot be the “pure empiricists” to which Kant alludes since neither philosopher argues for infinitely divisible space.³³⁵ Another possibility for the “pure empiricist” is Euler given his anti-Leibnizian/Wolffian argument that matter is infinitely divisible since space is infinitely divisible.³³⁶ The Aristotelian and Cartesian schools would also be possible anti-thesis proponents since each school supports space’s infinite divisibility contra Leibniz. Given the anti-Leibnizian-Wolffian bent of the antithesis, it seems best to characterize the second antinomy as a critique of

³³² Van Cleve “Reflections...”, 482-483.

³³³ Radner, 416-417.

³³⁴ Broad, 16-17.

³³⁵ Schmierge, 291.

³³⁶ Euler, L. *Gedanken von den Elementen der Corper, in welchen das Lehr-Gebäude von den einfachen Dingen und Monaden geprüfet, und das wahre Wesen der Corper entdeckt wird*. Berlin: Royal Academy of Sciences. Published in OO.III, 2, 349-366. Chapter II, Paragraph 12.
OO III, 2, 349-366.

the application of Leibnizian-Wolffian substance metaphysics to the structure of matter: human knowledge of composite substances (material objects) and their simple substances (monads) incorporates phenomenal and noumenal realms.

The statements of the thesis and the anti-thesis are invalid statements due to the limitations of human reason. Kant argues that the human mind cannot know these noumenal beings (simple substances) as part of our experiences. Simple beings cannot be part of a possible experience since such possible experiences occur only through infinitely divisible space as a form of our external intuition. Yet it is beyond the capability of human reason to claim that no such simple substances exist since then all composition would be nullified according to Leibnizian-Wolffian principles.

Having been preceded by a number of commentators who have suggested an alternative understanding of the second antinomy—like Heinz Heimsoeth and Sadik Al-Azm—Michelle Grier argues that that second antinomy is a question of the divisibility of substance rather than the divisibility of matter.³³⁷ A problem arises when considering the possibility that Kant effectively refers to two different kinds of matter in the thesis and anti-thesis: metaphysical matter for the thesis and ordinary physical matter for the anti-thesis. Kant's inclusion of mental substance in the discussion of his second antinomy makes evident an alternative form of matter beyond physical matter.³³⁸ Grier argues that that proponents of the thesis put substance prior to

³³⁷ Heimsoeth, Heinz. *Atom, Seele, Monad*. Weisbaden: Steiner, 1960. 277-279; Al-Azm, Sadik. *The Origin of Kant's Arguments in the Antinomies*. New York: Oxford University Press, 1972. 46; Grier, Michelle. *Kant's Doctrine of the Transcendental Illusion*. Cambridge: Cambridge University Press, 2001. 194-197.

³³⁸ Kant *Critique*, 264-266.; Schmiege, 273-274.

space, metaphysical matter, while proponents of the antithesis place space prior to substance, which is ordinary matter³³⁹

Schmierge partly disagrees with Grier, maintaining instead that proponents of the thesis and the antithesis believe that matter precedes space.³⁴⁰ He keenly points out that proponents of the antithesis cannot hold that space is prior to matter because such a view is Kant's own transcendental idealist view, a *synthesis* of the thesis and antithesis views.³⁴¹ Kant synthesizes the two views by supposing the matter as phenomenon is infinitely divisible, while matter as noumenon undetermined to infinite divisibility, open to the possibility that noumenal matter consists of simple parts. Proponents of the thesis and antithesis hope to reach the unconditioned from the conditioned.³⁴² In order to reach the unconditioned, it is necessary to consider material objects apart from the experiences of said objects. This act involves removing the empirical element of impenetrability from matter, leaving behind the conditioned element in this antinomy, matter's occupation of space—the “whole given in its bounds”.³⁴³ Yet this activity of seeking the unconditioned whole (the simple substances or substance as infinitely divisible) is problematic because we seek what is ultimately beyond a possible experience. For proponents of the thesis, metaphysical matter as simple substances is the internal ground of all spatio-temporal objects, while metaphysical matter given as a unitary whole is the internal ground of all spatio-temporal objects for proponents of the antithesis.³⁴⁴ Matter is given only in experience and when we attempt to isolate what is not given in experience save as a mere concept, an antinomy results.

³³⁹ Grier, 200-208.

³⁴⁰ Schmierge, 277-278.

³⁴¹ Schmierge, 277.

³⁴² Schmierge, 278-283.

³⁴³ Schmierge, 281-282.

³⁴⁴ Schmierge, 290-291; 292-293.

My opinion is that the common and alternative interpretations are not all that distinct from one another. Schmiege's interpretation is good inasmuch as it highlights the metaphysical need for an internal ground of spatio-temporal phenomena which is noumenal in origin; however, regardless of how one interprets this internal ground, such discussion of a ground inevitably incorporates a discussion of the structure of matter as a discussion of material structure must involve the ground of spatio-temporal phenomena. To regard the simple substance as the ground of material objects is to say that they consist of simples in some way; similarly, to consider matter as a unitary being is to claim that material structure is also infinitely divisible. Even Grier and Schmiege acknowledge that such discussions bear an intimate relation to one another.³⁴⁵

While interpreters like Michelle Grier and Oscar Schmiege are right to point out that Kant's second antinomy also involves discussions of mental substance, mental substances are not said to be *in space* in the same way that extended corporeal beings are *in space*.³⁴⁶ Also, Kant's statement regarding mental substances could simply be a reference to the Leibnizian concepts of substances as windowless souls. That is to say, since only something mental could qualify as a simple substance and form the foundation of material phenomena in Leibnizian substance theory, the second antinomy could still simply focus on the nature of the structure of corporeal matter—a consideration given in Leibniz's continuum problem. Thus, it seems to me that the material and internal grounding interpretations are ultimately identical in substance and nominally distinct.

³⁴⁵ Grier, 194-199; Schmiege, 278-279.

³⁴⁶ Kant *Critique*, 234-235.

Now that we have highlighted the two forms of interpreting Kant's second antinomy, let's turn our attention to interpretive objections to the validity of the antinomy itself. Brigitte Falkenberg, Bertrand Russell, and Dale Jacquette highlight different Rationalist notions of simplicity and alternative theories of the relations between matter and space. The result of these alternative interpretations is that Kant's second antinomy is invalid because he assumes either a notion of relation between space and matter that a proponent of the thesis would not follow or an idea of simplicity which a proponent of the thesis would not regard as true. Evident in these interpretations is the suggestion that part of Kant's problem in the second antinomy is his attempt to simplify all positions into one categorical argument based on the assumption that Euclidean space is the only space in which objects of experience exist.

An evident case in which the problem of Euclidean space arises is in Russell's and Falkenberg's keen observations that the proof of the thesis does not require the notion that such simple substances occupy Euclidean space.³⁴⁷ In Leibnizian theory matter is not in space. Rather, space is in the simple substances (monads) as a perception of the order of other simple substances. Concepts of simple and composite require only the principle of sufficient reason to account for the "compoundedness" of existent composites.

Russell and Falkenberg conclude that proponents of the thesis are considering what they call the "logical individual" instead of the "mereological atom": the latter being what Kant considers as integral to the thesis.³⁴⁸ The "logical individual" or simple substance is an existent that is independent of external relations—a notion which we will find does not enter into Kant's

³⁴⁷ Falkenberg, Brigitte. *Kants Kosmologie : die wissenschaftliche Revolution der Naturphilosophie im 18. Jahrhundert*. Frankfurt am Main : V. Klostermann, 2000. 231-234.

³⁴⁸ Falkenberg. 234.

rendition of the thesis argument. Kant therefore presupposes that the simple substance *enters into* the constitution of the composite substance in the same way that a brick *enters into* the composition of a wall. Matter is *in* space—a paradigm which Leibniz has rejected as a proponent of simple substances. Let's now examine Oscar Schmierge's objection to Falkenberg's line of thought.

Oscar Schmierge rejects Falkenberg's claim that composite substances are not in space; otherwise, the antinomy could not be valid.³⁴⁹ Furthermore, Kant's *Critique* requires that the so-called composite substances and simple substances appear to the human percipient in space because space is the form of all outer intuitions. As shown in the argument for the antithesis, the error on the part of the thesis proponents is that they assume an intelligible connection to simple, non-spatial substances: what Kant considers to be noumena. Without this essential ordering of phenomena and noumena as described in the *Aesthetic*, the second antinomy is invalid. I will answer whether or not Schmierge has responded successfully to Falkenberg later.

Another alternative theory of space and matter comes from David Hume, a philosopher who is neither a Rationalist supporting the thesis nor an Empiricist found in the camp of antithesis proponents. Since space itself is filled with perceptual content per Hume's theory of impressions and ideas, no vacuum or empty space can exist. Dale Jacquette argues against Kant's second antinomy, referencing a purported bias in Kant's thought toward Euclidean geometry and removing Hume's theory from possible acknowledgment.³⁵⁰ Thus, Kant's may already beg the question with his second antinomy by excluding the possibility that space is

³⁴⁹ Schmierge, 290-291.

³⁵⁰ Jacquette, Dale. "Kant's Second Antinomy and Hume's Theory of Extensionless Indivisibles." *Kant-Studien*. Vol. 84(1) 1993. 50-51.

finitely divisible. Kant would therefore have to either reject the mathematical truth of Euclidean space's infinite divisibility or deem the metaphysical principle of simplicity invalid given the setup of this antinomy.

We are thus left with the problem that Kant categorizes the thesis and antithesis too generally in such a way that alternative theories for simple substances are either ignored or misrepresented a purported opponent's argument in support of the existence of simple substances. Kant's mistake is that he omits the connection between the principles of simplicity and of sufficient reason from the discussion of the thesis and antithesis. This omission is problematic because the principle of sufficient reason provides the basis for Leibniz's inference that simple beings exist from the existence of aggregates. The principle of sufficient reason characterizes Leibnizian simplicity in such a way that simple beings are not merely understood as necessary parts of aggregates, but that they are the foundation for determining the existence of one aggregate versus another aggregate. Kant omits this foundational principle when he writes that simple beings exist only because composites exist—Kant repeats the apparent tautology of which Leibniz is originally accused. Without the principle of sufficient reason, Kant must relegate knowledge of simple beings and the principle of simplicity to the noumenal realm.

The second antinomy is valid if the thesis and antithesis prove opposite conclusions and refer to the same concepts. The problem is whether Kant consistently applies the concepts of simple substance and composite substance in the thesis and antithesis and particularly whether he has neglected an aspect of simplicity that proponents of the thesis might use to support their position. Should Kant fail to use the concept of substance consistently or should he employ the concept of substance in his rendition of the thesis argument differently than a proponent of the

thesis would argue, it would appear that Kant has presupposed his Transcendental Idealism in order to prove his theory as P.F. Strawson argues.³⁵¹

Following the analyses of Norman Kemp Smith and C.D. Broad, I will argue that Kant fails to employ the principle of sufficient reason in his argumentation expounded in the thesis position, effectively reducing the notion of simplicity in Leibizian thought to “non-extended” and composite to “extended”. Even charitably granting that Kant employs a latent notion of sufficient reason in his thesis argument, it is clear that the antithesis argument contains only notions of extension and non-extension. The omission of the principle of sufficient reason in discussions of simple substances sets up Kant for presupposing his Transcendental Idealist position regarding matter’s divisibility. Kant cannot consider the question whether the simplicity of the minimum could include intellectually distinguishable parts.

In order to demonstrate my opinion for this concluding chapter of the Critical Kant section, I will first analyze the arguments for the thesis and the antithesis. Then by paying close attention to the analysis of Norman Kemp Smith and C.D. Broad, I will note first how Kant omits the principle of sufficient reason from his account of simple substances in the thesis and antithesis, resulting in the identification of “simple” and “composite” with “non-extended” and “extended”. Without the principle of sufficient reason, Kant must consider the question of whether a minimum’s simplicity is compatible with its possession of intellectually distinguishable parts unknowable.

Kant’s rendition of the thesis argument relies on the notion that substance is what is capable of independent existence.³⁵² The format of the argument is *reductio ad absurdum*. The

³⁵¹ Strawson, P.F. *The Bounds of Sense*. London: Methuen, 1966. 175.

absurdity the thesis intends to show is that it is absurd to assume that simple parts or substances do not exist. By virtue of being the sufficient reason for aggregates and not possessing a further material sufficient reason for their own existence, simple beings must be independent existents. We will eventually see that Kant does not make full use of this principle of sufficient reason after considering also the antithesis argument. If Kant has argued in the *Physical Monadology* for the compatibility between simplicity and the occupation of infinitely divisible space on the basis of the principle of sufficient reason, we might ask whether Kant has fully acknowledged the connection between the principle of sufficient reason and the principle of simplicity in the thesis argument of the *Critique*. Here is an analysis of the thesis argument:³⁵³

1. Assume that composite substances do not consist of simple substances or parts.
2. If composite substances do not consist of simple substances or parts, then nothing would exist assuming that all composition is annihilated in thought.
3. (assumed premise) But if nothing would exist assuming that all composition is annihilated in thought, either it is impossible to annihilate all composition in thought or something simple must remain.
4. If it is impossible to annihilate all composition in thought, then there must be some simple part or substance.

Therefore, simple parts or substances must exist.

³⁵² Broad, 16-17.

³⁵³ Kant *Critique*, 259-260.

Kant's thesis argument for simple parts in the *Critique* does not completely match his argument for simple beings in the *Physical Monadology* because the critical Kant does not attempt to show the existence of simples from matter as accidental composition. In the *Critique*, Kant merely begins from the assumption that simples do not exist and proves a contradiction in the notion of composite being assuming that all composite beings were annihilated in thought. Thus, the *Critique* argument is *reductio ad absurdum*. In the *Physical Monadology*, Kant begins from the accidental composition of matter, noting that simple beings are necessary for preserving the particular form of accidentally composed matter, but also the very existence of matter itself. Thus, the *Physical Monadology* argument is ultimately *modus ponens*. This difference thereby fuels my conclusion that Kant misrepresents the Leibnizian-Wolfian argumentation for simple beings by eschewing the principle of sufficient reason from the argument.

C.D. Broad agrees with me that Kant's rendition of the argument does not fully embrace the Leibnizian argumentation. Integral to his argument for simple substances or parts, Leibniz attempts to demonstrate an inherent absurdity in the notion of dividing a given composite *ad infinitum* without reaching some existent simple being. Broad comments on the intent of Leibniz's argumentation: "Leibniz tried to make us see that the notion of a whole that is composed of parts, which are themselves composed of parts...and so on without end, involves a regress which is vicious. Kant merely shows that a 'compound substance' would not answer to his definition of a 'substance'."³⁵⁴ Leibniz argues for simple substances understanding simplicity as a necessary and sufficient reason or explanatory account for the composite things we experience as material objects. By contrast, Kant merely argues for simple substances on the

³⁵⁴ Broad, 17.

basis that an aggregate cannot be an independent existent.³⁵⁵ A “composite or aggregated substance” is contradictory since then the said substance would be conceived through “simpler” substances out of which it is composed.

The twist to Leibniz’s argument arises when considering how a sufficient reason is necessary for composite things; otherwise, an infinite regress of composites cannot exist. By referring to a need for “simples” because there are “composites” in his *Monadology* Leibniz is not simply arguing why there cannot be any composite substances. Rather, he is arguing from the need for accounting for the existence of anything composite through the existence of simples.³⁵⁶ Substituting another composite being as an explanation for another composite being is non-explanatory. Composites cannot consist solely of composites. Thus, Kant sidesteps the issue of providing an explanatory account of composites through simple substances. He elects instead to elaborate on the notion of substances as independent existences to show why it is contradictory to suppose that composite substances exist. Kant’s argument in the antithesis is especially revealing insofar as he omits the principle of sufficient reason from his account of simple beings.

Kant’s antithesis argument relies on substituting the concepts of unextended and extended for the terms “simple” and “composite” via the imposition of the infinite divisibility of space on the purported simple substances. Like the thesis argument, the logical form of the antithesis is *reductio ad absurdum*. Here is an analysis of the antithesis argument:³⁵⁷

³⁵⁵ Kemp Smith, 489; Broad, 16-18.

³⁵⁶ Leibniz *Monadology*, Proposition I

³⁵⁷ Kant *Critique*, 259-260.

1. Assume that composite substances consist of simple substances and such composite substances occupy infinitely divisible space.
2. If composite substances consist of simple substances and occupy infinitely divisible space, then each part of the composite substance (simple substances) must occupy infinitely divisible space.
3. But if each simple substance occupies infinitely divisible space, then the simple substance must be further divisible and thereby not be simple.

Therefore, composite substances do not consist of simple substances.

The second part of the antithesis, which shows that nothing simple exists anywhere in the world is ultimately an assertion that not simple being can be demonstrated to exist in our inner or outer experiences.³⁵⁸ Since time and space are infinitely divisible and do not possess a smallest part, these cannot contain simple substances. Let's return to the main argument of the antithesis.

Kant alters the notions of simple and composite to exclude the principle of sufficient reason by substituting an empirical element to the metaphysical concepts: unextended and extended. It is clear that the antithesis argument contains empirical elements when it is argued that nothing simple can be found anywhere in experience.³⁵⁹ What forms these experiences of outer intuitions is space, which is understood as extension. By maintaining that every part of space consists of spaces, Kant identifies composition with extension or possessing spatial parts.

³⁵⁸ Kant *Critique*, 260-261.

³⁵⁹ Kant *Critique*, 260-264.

To say that something is simple is tantamount to affirming that something is unextended or non-spatial.

In Leibnizian thought simple and composite are not identical in meaning to unextended and extended as Norman Kemp Smith notes well.³⁶⁰ Kant also has to assume that simples occupy or fill space rather than merely “falling within” space as C.D. Broad observes.³⁶¹ Should simple substances merely “fall within” space as a point is said to fall within a line as its limit, Kant could not argue that such simple substances be extended.³⁶² By considering space first when determining matter’s divisibility, Kant eschews the Leibnizian principle of sufficient reason and cannot perceive how simple substances exist.³⁶³

Kant’s removal of the principle of sufficient reason is due largely to his replacement of the principle of sufficient reason with empirical causality via the limitations of human reason. In this way, Kant constrains phenomena to empirical causality which is exclusively tied to one event following another event due to a rule. The principle of sufficient reason is connected to the causality between beings rather than events, including the relation between simple beings and their respective composites. Since the mind cannot know noumena, the ultimately real beings, it can only understand phenomena through empirical causality. Without the principle of sufficient reason, Kant cannot entertain the possibility that simple beings exist in the phenomenal world.

³⁶⁰ Kemp Smith, 489; Radner, 423-425.

³⁶¹ Broad, 20.

³⁶² This point is particularly true considering the earlier investigation of space and monads in which space exists in monads as a web of perceptions rather than the converse. In which case a monad or simple substance would not have to occupy or fall within any part of space.

³⁶³ Norman Kemp Smith keenly observes an incoherence when comparing the antithesis argument with Kant’s argument against force-shells in his *Metaphysical Foundations of Natural Science*. He cannot maintain that matter’s infinite divisibility is proven solely through space’s infinite divisibility in the *Critique* while also denying that such a proof is possible in the *Metaphysical Foundations*. See Kemp Smith, 491-492.

Chapter Three Conclusion

Kant's solution is that we cannot determine the features of matter from a noumenal standpoint; however, it is true that matter is potentially infinitely divisible from a phenomenal view as shown in the *Metaphysical Foundations*. The thesis and antithesis positions are flawed and dogmatic; yet the antithesis position is deemed of greater value by Kant because at least the empiricist proponents rest their claims mostly on knowledge established through the senses rather than mere concepts.³⁶⁴ Nevertheless, anti-thesis proponents are wrong because they claim that space exists over and above matter as noumena.³⁶⁵ Contradiction arises when we consider space and matter are concrete and complete realities rather than abstract and incomplete existences.³⁶⁶

By differentiating between matter considered as phenomena versus matter conceived as noumena, Kant's Critical solution avoids the hollow worlds problem that poses a problem for his pre-critical force-shell theory. Recall that the hollow worlds problem arises because the force-shell itself is a non-substantial punctum surrounded by a field of energy. The aggregation of non-substantial puncta result in a "hollow world": a system of external force relations without any real being to which these external forces relate. By deeming phenomenal matter infinitely divisible as shown particularly in the *Metaphysical Foundations of Natural Science*, Kant avoids the use of non-substantial puncta in his account of matter. Van Cleve explains Kant's solution well:

³⁶⁴ Weldon, 205-206.

³⁶⁵ Schmirge, 292.

³⁶⁶ Caird, Edward. *A Critical Account of the Philosophy of Kant: With an Historical Introduction*. Volume I. Bristol, England: Thoemmes Press, 1999. 600-602.

“We cannot say that an exploring series is infinite, for no matter how long we have embarked upon it, it will not be true that infinitely many perceptions have occurred. And we cannot say that an exploring series is finite, for in that case it would terminate in a “limiting” member, but no such member is possible.”³⁶⁷

The result is that matter is potentially infinitely divisible: there cannot be any ultimate member given (contra the thesis), but the entire series of conditioned or divisible parts of matter cannot be given in human experience (contra the antithesis). Whether non-substantial puncta or simple beings exist as noumena is unknowable according to Kant’s Transcendental Idealism.

Kant’s division between noumenal and phenomenal accounts of matter results in the relegation of the principle of simplicity to noumena. He thereby eschews a metaphysical principle in order to preserve a mathematical principle of space’s infinite divisibility; however, he does not claim that matter’s infinite divisibility extends to matter conceived as noumena. By relegating simplicity to the noumena through the second antinomy, Kant has removed the question of whether a minimum’s simplicity can include intellectually distinguishable parts. By doing so, the difference between metaphysical and intellectual divisibility is null. Kant’s second antinomy is called into question by the omission of the principle of sufficient reason and its neglect of alternative theories of the material composition. It is partly through the aforementioned omissions that I will argue in the next section for a Neo-Aristotelian solution to the problem of matter’s divisibility.

³⁶⁷ Van Cleve, James. “Reflections on Kant’s Second Antinomy”. *Synthese*. Vol. 47 No. 3. Kant’s ‘Critique of Pure Reason’. Part II June 1981. Pp. 481-494. 487.

Chapter Four:
A Defense of a Neo-Aristotelian Potential Parts Doctrine

Chapter Four Introduction

The Early Modern period is complete with a variety of responses to the question of whether metaphysical simplicity is compatible with space's infinite divisibility. Some philosophers like David Hume and George Berkeley prioritize metaphysical simplicity over space's infinite divisibility. Both philosophers advocate a theory in which a finite quantity of perceptual minima comprises perceptions of matter/space, but differ with respect to whether the perceptual minima are extended or unextended. Other philosophers like the critical Immanuel Kant argue for the priority of space's infinite divisibility over metaphysical simplicity, claiming that all knowledge is based on experience and a simple part or being cannot be found in experience. But philosophers like Leibniz and the pre-critical Immanuel Kant argue for a reconciliation between metaphysical simplicity and space's infinite divisibility. Despite their differences, the majority of these philosophers save the critical Immanuel Kant support the actual parts doctrine, a theory in which the parts of a divisible actual whole must also be actual or actually existent prior to their division from the whole.³⁶⁸ Should the actual parts doctrine be false, the particular responses these philosophers develop to resolve the problem of matter's divisibility must collapse.

I will disagree with the actual parts doctrine as stated, arguing to the contrary that an undivided composite whole need only consist of potentially existent parts. This argument is a revival of the Aristotelian potential parts theory attendant with an attempt to bridge the principle of metaphysical simplicity with an Aristotelian theory of substantial forms via the introduction of a theoretical form of matter called "neutral matter". The inversion of the means by which simple

³⁶⁸ Holden, 80.

beings are conceptualized within the material object and the introduction of “neutral matter” are my key contributions to the debate. Simple beings are no longer constitutive parts of the composite material body, but are identified with the substantial form or the very principle of unity of material things. Although I disagree with proponents of the actual parts doctrine, my Neo-Aristotelian response shares with actual proponents the close association between metaphysical simplicity and the principle of material sufficient reason as uncovered in the historical section of this dissertation.

This Neo-Aristotelian theory requires an inversion of the typical means by which metaphysical simplicity is conceptualized. Rather than having the simple being exist as an actual part of an undivided or divided composite whole, the simple being is identified as the unitary being itself whether or not that single being consists of parts. Since the substantial form is the principle of unity for a being, it partly makes all the potentially divisible parts possible in conjunction with matter, the Aristotelian principle of potency. Simplicity exists in the composite or simple whole itself, making the potentially divisible parts possible.

Thomas Holden’s *The Architecture of Matter: Galileo to Kant* will serve as a framework for addressing the conflict between actual parts theorists and potential parts theorists. Thomas Holden argues that the actual and potential parts doctrines’ proponents reach a stalemate because each side assumes the counter principle that their opposition presupposes. I will review one form of argumentation, namely the arguments from divisibility, and articulate the question-begging nature of each argument.

I intend to break this stalemate by arguing for the metaphysical identification of unity and being actual: wherever there’s *one thing*, there is something that is actual; wherever there is

something actual, there is *one thing*. I will call this the Actuality-Unity Principle or AUP. Incorporating a form of matter which is *indifferent* to being either finitely or infinitely divisible, I will show that the actual parts theorists inherently beg the question when supposing that an actual being exists. Actual parts theorists arbitrarily limit the assumed actual being existing in this neutral matter to being the simple being itself or being both one thing and many things simultaneously should an assumed actual being consist of “simpler” actual beings. It is only through the potential parts doctrine that we can have compositional variations in the first place. I will then conclude with a brief overview of Aristotelian substantial forms and show how these forms might serve as the simple beings of such a potential parts theory. If I am successful, I will have reconciled metaphysical simplicity with space’s infinite divisibility via a revival of an Aristotelian solution to an Early Modern problem.

Before beginning an analysis of Holden’s work, I would like to point out my argument assumes the infinite divisibility of space. This assumption apparently rules out Hume and Berkeley from being my opponents; however, the inference that Hume and Berkeley are no longer my opponents in this debate is not warranted because of my assumption that it is possible to talk about a matter or space that is *indifferent* to being either finitely or infinitely divisible. I call this matter “neutral matter”, much akin to Aristotelian prime matter. It stands to reason that if an actual parts theorist must assume that the actual being is the simple being or that the actual being is contradictorily composed of simple actual parts, such an argument applies to actual parts theorists whether or not they affirm or deny space’s infinite divisibility.

Chapter Four-Section A:

The Problem of Divisibility in the Actual and Potential Parts Doctrines:

The Arguments from Divisibility

The actual parts and potential parts doctrines result in an impasse partly because each camp believes in two contrary notions of divisibility. Holden notes that actual parts proponents believe that division *can* occur only if there already exists at least two actual parts in the undivided composite whole.³⁶⁹ The actual parts pre-exist their division. Potential parts theorists deny the pre-existence of actual parts, claiming that it is only through the act of division that the parts become actual. Division *creates* the actual parts. It is because the actual parts theorist presupposes the actuality of parts prior to their division from the whole, while a potential parts theorist holds that actual parts are posterior to their division that both sides reach an impasse.

Despite their differences, the two camps presuppose a key metaphysical principle which I will call the metaphysical priority of actuality over potentiality. Actuality over potentiality is rooted in Aristotelian metaphysics: something possesses the capacity to act (potency) insofar as it is already (actual). To show this principle latent in Aristotelian metaphysics, I will reference a passage in which Aristotle describes the priority of actuality over potentiality with respect to formula. Aristotle writes of the priority of actuality over potentiality in Book IX of his

Metaphysics:

“...for that which is in the primary sense potential is potential because it is possible for it to become actual, e.g. I mean by ‘capable of building’ that which can build, and by ‘capable of seeing’ that which can see, and by ‘visible’ that which can be seen.”³⁷⁰

³⁶⁹ Holden, 78.

³⁷⁰ Aristotle. “Metaphysics” in *The Complete Works of Aristotle: The Revised Oxford Edition*. Jonathan Barnes, ed. Volume 2. Princeton: Bollingen Series LXXI, 1995. 1049b15.

From here, it should be clear to the reader that just as the “visible” is what can be seen, so too must the “divisible” be that which can be divided. That which can be divided is an actual composite whole, while its “divisible” character is the actual being’s potentiality of being divided. Something can only be divided (potentiality) in so far as it is an existent divisible thing (actuality). It is precisely a question of whether an undivided whole’s parts are actual or potential that distinguishes Actual Parts Theory from Potential Parts Theory.

In Actual Parts Theory, the capacity of division (potentiality) presupposes the actuality of the parts to be divided (actuality). The parts precede or condition the possibility of the whole. Consequently, actual parts theorists must attribute metaphysical simplicity to an ultimate part of a composite whole.

Actuality over potentiality is latent also in Potential Parts Theory. The undivided whole can only be divided into actual parts (potency) insofar as there is an undivided whole that is divisible (actuality). Thus, the whole precedes or conditions the possibility of the parts. While the metaphysical principle of simplicity appears abandoned in the case of Potential Parts Theory, I will later attempt to reconcile metaphysical simplicity with Potential Parts Theory by showing how metaphysical simplicity can be attributed to the undivided whole inasmuch as the whole is the material sufficient reason for its parts.

It is from this inversion of the priority of whole and part that I will bridge the connection between metaphysical simplicity and the unitary being or undivided whole. With actuality over potentiality in mind, let’s examine Holden’s analysis of the arguments from divisibility from the Actual Parts Theory and Potential Parts Theory.

Thomas Holden traces the actual parts divisibility argument from the thought of Charleton, an atomist, to David Hume.³⁷¹ I would like to note that Charleton defends atomism strictly speaking. Since all atomists are actual parts theorists, Charleton's defense of atomism can also be taken as an indirect defense of actual parts theory. With this important point in mind, Holden argues that what unites these arguments for actual parts theory is the claim that division merely *reveals* the parts.³⁷² This mode of revealing is shown to be linked to their deduceability from their existence in the whole. Charleton comments on this deduceability:³⁷³

“As those parts, which are deduced from a Continuum, must be praesistent therein before deduction (else whence are they deduceable?) so also must those which yet remain deduceable, be actually existent therein, otherwise they are not deduceable from it.”

The parts must be actual already in the whole in order to be divisible—it is the actuality of the parts which grants their potentiality of being divided. Imagine that one is to divide a cake into two halves. According to actual parts theorists, the only way for the cake to be divided is if the portion of the cake to be divided already exists. For if these portions of the cake do not exist, then there would be nothing to divide. Moreover, the entire cake would not exist. Thus, without actual parts, there aren't any parts to be divided.

An actual parts theorist links the number of actual parts to the number of potential divisions. David Hume is a good representative of this belief when he states:³⁷⁴

“Tis also obvious that whatever is capable of being divided *in infinitum*, must consist of an infinite number of parts, and that 'tis impossible to set any bounds to the number of parts without setting bounds at the same time to the division.”

³⁷¹ Holden, 106-108.

³⁷² Holden, 107.

³⁷³ Charleton, Walter. *Physiologia Epicuro-gassendo-Charletomana*. New York: Johnson Reprint Co., 1966. 93.

³⁷⁴ Hume *Treatise*, 1.2.1.2.

If something consists of X number of parts, then I can only divide this thing X number of times. If matter is infinitely divisible, then it must be actually infinitely divisible since the parts of matter must already be actual. Note that Hume argues for the finite divisibility of matter and space. Thus, Actual Parts Theory does not necessarily lend itself to the actual infinite divisibility of matter.

The impasse is already evident between proponents of the actual and potential parts doctrines: actual parts theorists will not accept the claim by potential parts proponents that the divisible parts can be given through the undivided whole. To say that the parts exist potentially is nonsense for actual parts theorists.

The divisibility argument in favor of Potential Parts Theory is the claim that Actual Parts Theory rests on a conceptual confusion of unity (undividedness) and separateness. Holden uses the arguments from Digby and Collins to detail the conceptual framework for potential parts. Digby, a proponent of the Aristotelian potential parts doctrine, links the natures of quantity and divisibility: “*Quantity or Biggnesse*, is nothing else but divisibility...Quantity is but one *whole* that may indeed be cut into so many severall partes: but those partes are not really there, till by division they are parceled out...”³⁷⁵ Digby alludes to a central claim of the potential parts doctrine which relies on abstract ideas or universals: an undetermined whole quantity is divisible by its nature. This whole is undetermined because it is *any* multiplicity of things rather than *this* multiplicity, and quantity is a multiplicity. Important to notice here is the inherent claim that quantity is infinitely divisible. Digby claims only that the parts of a quantity do not exist unless

³⁷⁵ Digby, Kenelm. *Two Treatises, in the one of which the Nature of Bodies is expounded; in the other, the Nature of Man's Soule; is looked into*. Stuttgart: Friedrich Fromman Verlag, 1970. 99; Holden 118-119.

they are “parceled out”. Let’s understand better why Digby claims the potential existence of quantitative parts.

Integral to Digby’s argument for potential parts is the observation that the division of a quantity into parts is to divide a given whole into at least two distinct wholes. Digby writes of the cessation of the preceding whole through division: “...and then, the whole (out of which they are made) ceaseth to be any longer; and the parts succeeded in lieu of it; and are, every one of them, a new *whole*.”³⁷⁶ The actuality of something according to Potential Parts Theory is that it forms a whole being—actuality is of the whole rather than the part. I will now formulate the argument for potential parts:

1. Something is actual if and only if it is a whole.
2. A part of an undivided whole material object is not a whole.

Therefore, a part of an undivided whole material object is not something actual.

It is evident from this formulation that something is actually divided if and only if it should consist of several wholes. A proponent of Potential Parts Theory, Collins claims that the actual parts doctrine is erroneous because the actuality of the parts of matter is identical to affirming that matter is already actually divided: “...he makes it consist of *actually separate and distinct Parts* (though I wonder at the same time he should make it divisible, when by its consisting of separate and distinct Parts it is actually divided.”³⁷⁷ It would be pointless to

³⁷⁶ Digby, 10.

³⁷⁷ Collins, Anthony. *A Letter to the learned Mr. Henry Dodwell, containing some remarks on a (pretended) Demonstration of the Natural Immortality of the Soul* (1709), in Samuel Clarke, *The Works of Samuel Clarke*, 4 vols. New York: Garland, 1978. iii. 749-753. 751.

attempt dividing matter if is already divided via its actual parts. Potential Parts Theory therefore appears to be in sync with the manner by which we conceive division.

The first premise is the focal point of the dispute. Actual parts theorists would deny that being whole is a necessary and sufficient condition for being actual. They would agree with potential parts theorists that being actual is being whole since the complete material object is actually composed of actual material parts. But I will counter the Actual parts theorist's contention that all parts are actual, arguing that being an actual part is parasitic on being a whole or unity. In order to set the stage for the argument let's investigate the supposed impasse more closely.

Holden notes that by assuming one of two theories of division, a thinker is lead to one particular theory of part and whole, resulting in upholding either Potential Parts Theory or Actual Parts Theory.³⁷⁸ Despite this remark, Holden acknowledges the possibility of a reformulated metaphysical argument for establishing the truth of either Potential Parts Theory or Actual Parts Theory.³⁷⁹ I will take up Holden on this challenge, providing a reformulated metaphysical argument in favor of Potential Parts Theory by means of Actuality-Unity Principle. I will now argue for the Actuality-Unity Principle, paying close attention to its grounding in Actuality over Potentiality.

Prior to my argument for Actuality-Unity Principle, I would like to describe Aristotle's theory of the potential infinite divisibility of space considering that Actuality-Unity Principle is ultimately from Aristotelian metaphysics. Aristotle is considered to be the earliest proponent of potential infinite divisibility. His arguments for the infinite divisibility of motion, time, and

³⁷⁸ Holden, 129.

³⁷⁹ Holden, 130-131.

space are found in Book VI of the *Physics*.³⁸⁰ Note that Aristotle never argues explicitly for the potential infinite divisibility of matter. It is possible that Aristotle simply assumed that the reader would suppose the infinite divisibility of matter given the truth he argues for the potential infinite divisibility of extension, time, and motion since matter occupies space.

The best evidence that Aristotle believes that matter is potentially infinitely divisible is found in his arguments against Democritean atomism. Democritean atomism is the theory that the world consists of void and physically indivisible parts of matter.³⁸¹ While the argument against Democritus removes physically indivisible parts from matter, it should be noted that Aristotle does not argue explicitly against simple beings. What he argues against is that matter consists of uncuttable parts of various shapes and sizes. It is possible to construe this argument as an explicit attack on the constitutive parts rendition of the actual parts doctrine. Matter is not built from the ground up by actually indivisible or simple things, rather matter must be given as a being in potentiality. Since matter is a principle of potency and exists in infinitely divisible space, it is potentially infinitely divisible.

With an understanding of Aristotle's theory of infinitely divisible space, let's grasp more clearly Aristotle's theory of actuality and potentiality. By grasping actuality and potentiality more clearly, the reader can know better my Neo-Aristotelian solution to the problem of matter's divisibility.

Actuality is best understood by contrast to something being in potency. Aristotle in his *Metaphysics* illustrates this contrast through the example of a block of wood that could be made into a statue of Hermes. He writes: "...we say that potentially, for instance, a statue of Hermes is

³⁸⁰ Aristotle. "Physics". See Book VI chapters 1-6.

³⁸¹ Aristotle. "Physics". Book IV chapter 6.

in the block of wood and the half-line is in the whole, because it *might be separated out*, and even the man who is not studying we call a man of science, if he is capable of studying. Otherwise, actually.”³⁸² (my italics added) The key phrase is that a being in potency *might be separated out* from an actually existent thing. To separate out one thing from another is to make one thing numerically or countably distinct from the original actual being. When Aristotle says that the statue of Hermes is in an unformed block of wood, it is not true that the statue of Hermes is definitely countable in the said block of wood. One does not say that there is a block of wood—*one thing*—and then the statue of Hermes—another *one thing*—within the same *one thing*, the block of wood. Otherwise, there would be two things existing as the same one being.

Being in potency is indeterminate and inclusive, while being in actuality is determinate and exclusive. We cannot say that the actual statue of Hermes is in the *unformed* block of wood; otherwise, the block of wood could not be *unformed*. The unformed nature of the block of wood allows the same block of wood to be potentially a statue of Hermes or some other form of statue. Actuality thereby adds form to the block of wood to determine the block of wood as a statue (of Hermes) rather than as a statue (of Zeus). Rather than claiming that a block of wood formed into a statue of Hermes is the block of wood (one thing) *and* the form or statue of Hermes (another thing), one need only say that there is a *formed* block of wood or the statue of Hermes. There are not two numerically distinct things but only *one thing* because the block of wood is *actually formed* into the statue of Hermes.

Aristotle’s example of the half-line potentially existing in the whole line is particularly appropriate when conceiving how the parts of an extended body are potential parts rather than

³⁸² Aristotle. “Metaphysics”. 1048a30.

actual parts. For if the half-line were actual, then it would be numerically distinct from the whole line based on the Actuality-Unity Principle, but the numerical unity of the actual parts must be somehow bound with the numerical unity of the whole line. Yet when we consider a whole line, we do not consider all of the potential divisions we could make to the said line: halves, fourths, eights, etc... For the line is *undivided* as a block of wood is *unformed* relative to being either a statue of Hermes or of Zeus. We do not think of a whole as being divided into halves, fourths—only that it could be divided depending on what sort of actual whole there is.

This description of the supposed potential parts of an undivided whole still begs the question for an actual parts theorist. The actual parts theorist can merely suggest that the actual parts must precede their division in order to conceive an undivided whole that is divisible into any number of parts.

A potential parts theorist like Aristotle could respond by noting that there are different forms of division. In the case of a whole line, let us assume that the parts are actual and that there are actual half-lines and third-lines. For if the half-line is actual, how can we also understand the whole to be actually divided into halves while also being divided into thirds? Such actual measurements are incommensurate. We would have to say that the third-line is an actual part of the half-line, but we do not grasp the kind of division of parts in terms of the third line when examining the potential division of a half-line. Thus, we can only affirm that the *undivided* whole line only consists of potential parts on the pain of being determinate or actualized in terms of actually consisting of the half-line or the third-line.

We have thus far studied being in potency in terms of a capacity of *being separated out*, which I have understood as a capacity for attaining numerical unity or being a *single thing*. In

order to grasp Actuality-Unity Principle more clearly, we will focus on the nature of actuality and understand according to what I will call “indexability”.

The ability to index or “point to” a being is a criterion that determines what is actual rather than what is potential.³⁸³ Beings in potency cannot be indexed or pointed to because such beings cannot be readily identifiable as part of or from a given actual being. This process of indexing would appeal particularly to those philosophers like Berkeley and Hume who subscribe to a theory of knowledge and metaphysics based on immediate perception. It is concrete in its application: for it is not possible to immediately perceive the statue of Hermes in the *unformed* block of wood or the half-line in the *undivided* line without also giving due notice to the statue of Zeus in the *unformed* block of wood or the third-line in the *undivided* line. What we do immediately perceive or is indexible in such instances is really the *unformed* block of wood which *could become* a statue of Hermes or of Zeus and the *undivided* line which *could be* divided into halves and thirds.

When the artisan makes the statue of Hermes from the *unformed* block of wood, or someone actually divides a given *undivided* line, the statue of Hermes, or the *formed* block of wood, and the half-line from the *divided* line, are indexible beings. They are numerically one, having been *separated out* from some other actual being that possessed the potency to be either the statue of Hermes or the half-line. When such beings are indexible or made actual/numerically one thing, there is an exclusion of other beings that are in potency. Aristotle describes this indexability well in a passage from Book III of his *Physics*: “Take for instance the buildable: the actuality of the buildable as buildable is the process of building. For the actuality

³⁸³ The reader should keep in mind that I am only discussing natural material beings rather than beings that could be considered immaterial such as objects of religious consideration.

must be either this or the house. But when there is a house, the buildable is no longer there.”³⁸⁴

(my italics added) The buildable *could be* any number of different things, including a house.

But when the form of a house (the actuality) is imprinted in the matter, the potentiality cannot be there because what is *there* (what is indexable) is present. In the same manner, the block of wood *being* the statue of Hermes means that it *is not* a statue of Zeus; the line *being* divided into halves means that it *is not* divided into thirds.

The act of division merely “reveals” the actual parts of an undivided line as Hume and Walter Charleton have argued.³⁸⁵ The act of division can only add actuality to what were previously potential parts by making them indexable. The parts’ “deduceability” in Charleton’s language is such from the *undivided* line itself. For it is conceptually possible to halve any line whatsoever based on the Euclidean definition of a line as a “breadthless” length.³⁸⁶ The reader should now be clear as to the distinction between potential parts and actual parts according to what I have termed “indexability” or “numerical distinction” which has its foundation in the Aristotelian distinction between actuality and potentiality.

This description of actuality and potentiality via an Aristotelian potential parts doctrine does not really address one of the central tenets of the actual parts doctrine: material objects, properly conceived, consist of actual parts that constitute the given whole. The upshot is that a philosopher like Leibniz would dispute the claim that all actual things are numerically one thing assuming that the phenomenalist interpretation is true. For the actuality of a composite or aggregate thing is founded on the actuality of the monads which are simple beings and

³⁸⁴ Aristotle *Physics*, 201b10.

³⁸⁵ Charleton, 93; Hume *Treatise* 1.2.1.2

³⁸⁶ Euclid. *Elements*. Book I, Definition 2.

aggregates/composites are not numerically one thing. Part of what fuels this objection is the actual parts theorist's principle that the part conditions the whole, while a potential parts theorist maintains that the whole conditions the part. I would like to develop this difference a bit further in order to contextualize my argument for potential parts better.

Despite this difference, I note that Leibniz and any other actual parts theorist subscribe to Actuality-Unity Principle inasmuch as it describes the simple being that composites and aggregates require for their existence. The Actuality-Unity Principle is latent in these philosophers' arguments for simple beings because simple beings are the first actualities and these first actualities are numerically single things. Even the actualities of composites and aggregates are parasitic on the actuality and numerical unity of the simples.

Despite this parasitic relation between composites/aggregates and their constitutive simple parts, it is precisely on this point that actual parts theorists differ from potential parts theorists concerning Actuality-Unity Principle. Actual parts theorists uphold a narrow application of Actuality-Unity Principle to only the simple beings. This narrow application of Actuality-Unity Principle partly follows from their belief that parts are metaphysically prior to their whole. Potential Parts Theory includes a broad application of Actuality-Unity Principle in the sense that whatever is actually must be numerically one thing, whether that being is simple or composite. Since the whole precedes the parts, should a composite thing be actual, then the composite is the numerically one thing in existence, while its parts are merely potentially existent.

Remarkably, the simple/composite metaphysical relation is rewritten in Potential Parts Theory. What actual parts theorists might consider a "composite" being to be really numerically *one thing* in Potential Parts Theory. But to be numerically *one thing* in Actual Parts Theory is

virtually identical to saying that something is a simple being. Thus, the actual parts theorist would critique a potential parts theorist on precisely this conceptual ground, failing to realize that Potential Parts Theory requires such an inversion of part/whole and simple/complex relations.

A tree would be considered to be numerically one thing in the case of Potential Parts Theory despite consisting of a parts which *could be separated* out from the whole. It is a composite only in the sense that it possesses potential parts, but is considered a simple being by virtue of the fact that it is considered to be a *single thing*. The problem we will study later is what makes it possible for a chair, or any other material thing to be considered *a being* in light of Potential Parts Theory. It is precisely because I turn to Aristotelian substantial form to account for things being *one thing* rather than some *other one thing* that my proposed theory is *Aristotelian*. The substantial form, or the whole unified being is the principle of material sufficient reason for its potential parts. We will see how Actual Parts Theory inverts this basis for the principle of material sufficient reason to the actual parts of the composite whole rather than to the very whole itself.

The same tree according to Actual Parts Theory could only be considered an aggregate at best. It is a composite whose parts are actual and ultimately simple. These actual simple parts condition the actuality of this aggregative chair. The chair is not a simple being in the sense of being a *single* thing, but is a *multitude of single things*. As an aggregate, this material object's identity is supplied through the mind as Leibniz has argued (reference). The material objects actual simple parts, as I have argued previously throughout this dissertation, is a principle of material sufficient reason for the composite whole—the exact opposite emphasis versus Potential Parts Theory. My argument is an attempt to show that only the whole can condition the part;

otherwise, an actual parts theorist would have to suppose that a composite whole is numerically one thing and not numerically one thing simultaneously given AUP and what I will describe as “neutral matter”.

Chapter Four-Section B: Argument against the Actual Parts Theory

My interpretation of Holden’s analysis of the impasse between the actual parts and potential parts doctrines is that neither side attempts to prove their principles of divisibility from a neutral standpoint. My contribution to ending this stalemate is what I will call “neutral matter”. “Neutral matter” is indeterminate quantity—an abstraction from all material objects. Its indeterminacy is relative to its possession of actual parts or potential parts. All that can be said about it is its potency to possessing either ultimate simple part or an infinity of potential parts.

Before delving into the argument for PPT, I would like to describe a little more fully “neutral matter” by comparing it to the image of a blank canvas. A blank canvas is *unformed* until the artist applies paint to the board, forming an image. The blankness of the canvass is an openness or receptivity to images drawn in different ways. These different ways of drawing might include straight, unbroken lines or straight, broken lines.³⁸⁷ These different ways of drawing lines are analogous to the formation of material bodies by either actual, simple parts or potential, continuous parts. The artisan need not draw any particular form of a line in such a case; however, the artisan may be restricted to drawing a specific type of line assuming certain rules. Analogously, there is no limitation given the actuality of a material object in neutral matter;

³⁸⁷ There is no need to add quantitative form to the actual being given AUP. The positing of actuality, whether through substantial form or some other means, also makes that being metaphysically *one thing*.

however, certain metaphysical principles may restrict the material object to consist of either potential parts or actual parts.

The reader may think that the blank canvas analogy makes neutral matter appear to be identical to space. Like space, the blank canvas has its own dimensions *onto which* the artisan may draw figures. The images are arranged according to different kinds of spatial orientations relative to one another in the same way that material objects exist *alongside* one another in space. For space is more like a blank canvas onto which one may paint images of objects just as material objects are said to exist in space. However, neutral matter is not space because it concerns the form of actual and the moveable rather than pure potential and immoveable.

The objector is right to point out that neutral matter and space are very similar in terms of being pure potencies, but neutral matter is not “spread out” like space is. “Neutral matter”, whether one takes a Newtonian or Leibnizian theory of space, is a mere abstraction of particular material objects. These material objects are actual and moveable rather than merely possible and immoveable like space. Aristotle provides us with a pertinent contrast between place and body in the *Physics*: “We assume first that place contains that of which it is the place, and is no part of a thing...again, that place can be left behind by the thing and is separable...”³⁸⁸. Unlike place (space) and material bodies, “neutral matter” does not exist as its own entity in the sense that bodies are said to *move into* different places: nothing “moves into” neutral matter” and “neutral matter” does not move. It is a mere thought experiment, a pre-condition of neutrality regarding the metaphysical structure of matter regardless of the metaphysical status of space.

³⁸⁸ Aristotle “Physics”, 211a1.

Strictly formulated, neutral matter is the *supposition* of any actual material object prior to its determination as consisting of either actual parts or potential parts. It is an abstraction or *ens rationis* rather than an individual thing or *ens reale*. It has the same metaphysical status as the abstraction of humanity from individual human beings, such as Peter or Paul: neutral matter encompasses both actual parts and potential parts but is neither one of them in the same way that humanity includes Peter and Paul but is neither Peter nor Paul. “Neutral matter” is thereby indifferent to the diverse theories of space evident in the Early Modern period, whether that Space is Newtonian, Leibnizian, or Kantian. For all Early Modern theories of space are compatible with the existence of matter, whether the matter consists of actual parts or potential parts. Thus, the result of neutral matter in conjunction with a substantial form is an actual, moveable object while space is either the mere coexistence of such objects (Leibniz), the a priori form of our external intuition (critical Kant), or the container in which material objects exist (Newton).

I will show that Potential Parts Theory is true given neutral matter under the constraint of Actuality-Unity Principle. Should we assume Actual Parts Theory is true, then the same material object must be numerically one and not numerically one simultaneously. I will also assume the following: 1.) Neutral matter is conceptually feasible. 2.) Actuality-Unity Principle is accepted by both potential parts and actual parts proponents. 3.) The metaphysical identity of the whole is based on the metaphysical identity of its parts if the parts are actual. The upshot is that the potential parts doctrine best preserves Actuality-Unity Principle and the principles governing whole/part relations. Here is the argument for the potential parts doctrine:

1. Something is actual if and only if it is numerically one thing.

2. Assume that there exists actual matter.
3. Thus, the existent matter is numerically one thing.
4. Suppose on principle that this numerically one actual matter consists of only actual parts.

-Given neutral matter, we cannot assume the given actual matter to be a minimum, nor that the matter is infinitely divisible and consists of actual parts or potential parts.

5. Thus, these actual parts are also numerically one (and separate) things.
6. But if the identity of the whole is based on its parts, and the parts are numerically distinct things both from each other and from the whole, then the whole is numerically one thing and not one thing simultaneously.
7. The identity of the whole is based on its parts.
8. Thus, the whole is one thing and not one thing simultaneously.

Therefore, actual matter cannot consist of only actual parts.

Actual parts theorists can either deny Actuality-Unity Principle and/or modify how the identity of the whole is based on the identity of the parts. Denying Actuality-Unity Principle is not a good option because the assertion that minima, of any form, exists is founded on the Actuality-Unity Principle. Hume's and Berkeley's simple/complex idea dichotomy presupposes Actuality-Unity Principle since their simple ideas, of which the complex ideas are based, are actual by virtue of being numerically one idea. Complex ideas are not really identical to being one thing, but are collections or aggregates of simple ideas. Leibniz and the pre-Critical Immanuel Kant must ultimately accept Actuality-Unity Principle since they regard the existence of their monads and force-shells respectively as numerically singular identities. What is

regarded as ultimately real or actual in their respective theories are the monads for Leibniz and force-shells for Kant: the ultimately real or actual things are numerically one identities.

Modifying the identity of the whole based on the identity of the parts is more feasible than denying Actuality-Unity Principle outright; however, actual parts theorists cannot do so without presupposing actual parts theory itself. Actual parts proponents are thereby disputing the first premise, namely that all actualities are numerical unities.

Collections of numerically one and actual beings apparently cohere well with their statements on the application of Actuality-Unity Principle to constitutive parts. The collection or whole does not exist over and above the parts of the collection. For what we know is what is actual of its own accord and is numerically one thing in turn. A brick wall does not possess a distinct existence or actuality apart from the actual existence of its bricks. In this manner, the whole or brick wall is considered “actual”, but there is no confusion between the numerical identities of its parts and its own numerically one identity for it is not actual or numerically one of its own accord. Actual parts proponents could thereby avoid the resultant contradiction of being numerically one and not being numerically by virtue of denying that it possesses an actuality separate from its parts.

But this response is to no avail because it requires the actual parts theorists’ assumptions regarding the constitutive parts application of Actuality-Unity Principle. Consider for a moment how actual parts proponents would attempt to alter the application of Actuality-Unity Principle to the actuality of matter starting from neutral matter itself. The argument begins from the supposition that there is an actual material object existent in neutral matter.

I would also like to note to the reader that neutral matter is only required inasmuch as we can suppose that proponents of both sides begin from a neutral ground. However, the following argument does not absolutely require neutral matter for its validity. It is a full *reductio ad absurdum* of Actual Parts Theory. The upshot is that an actual parts proponent must assume that the given actual material object is either a minimum or is composed of minima if he denies the connection between actuality and numerical unity.

My defense of the first premise thereby runs as follows:

1. The actual material object in neutral matter must be either numerically one thing or not numerically one thing.
2. If the actual material object is numerically one thing, then the actual parts theorist has already assumed what we intended to show.
3. If the actual material object is not numerically one thing, then it either consists of minimal actual parts (simple beings that composes other composite or aggregative beings), or it consists of parts that are not (immediately) numerically singular things.
4. If the actual material object consists of minimal actual parts, then we have what we sought to prove.
5. If the actual material object does not (immediately) consist of minimal actual parts, then the parts either do not consist of actual minima or they do ultimately consist of actual minima.
6. If these parts do not consist of actual minima, then Actual Parts Theory is false since there is no truly actual part that grounds all the composites (no simple

being or we admit an infinite regress of infinitely smaller parts that cannot be actual because there is no constitutive ground for them—Potential Parts Theory).

7. If these parts consist of ultimately minimal actual parts, then we have what we sought to prove

Therefore, the material object in neutral matter must be numerically one thing with its parts open to determination (parts in potency).

The upshot of my defense of Actuality-Unity Principle is that Actual Parts Theory proponents bear the burden of proof assuming that neutral matter is valid. To arrive at Actual Parts Theory, it is necessary to presuppose that either the original actual material object is a minimum or that it must consist of actual minimal or simple parts. But such a constraint is nothing more than Actual Parts Theory! Thus, neutral matter naturally lends itself to Potential Parts Theory given the lack of determination of parts in such a theory. Actual Parts Theory requires the assumption of principles beyond the common principles shared between the two camps. But the foundation of Potential Parts Theory is based precisely on the two theories' points of common ground.

Chapter Four-Section C:

Substantial Form and Neo-Aristotelian Potential Parts

Having demonstrated the validity of Potential Parts Theory given neutral matter and Actuality-Unity Principle, an actual parts proponent could still inquire into the grounding or reason for the actual whole possessing its potential parts. In the historical chapters of this

dissertation I have argued that the metaphysical principle of simplicity is grounded ultimately in the principle of sufficient reason (PSR). The actual parts theorist would question the grounding principle behind the whole conditioning its potential parts? How can such parts be considered *potential* or as possessing the capacity to be separated from the whole? My response is based on the metaphysics of Aristotle: a principle of unity, the substantial form, is the sufficient reason for the potential parts as part of an actual, numerically one being.

Actual parts theorists are right to inquire into the sufficient reason for the existence of potential parts in the undivided actual whole, especially considering that Potential Parts Theory and Actual Parts Theory presuppose the principle of sufficient reason. The origin of such presupposition is latent in the two camps' subscription to Actuality over Potentiality. Whenever something is in potency or liable to change or vary, reason demands a cause or reason to explain this potency—particularly in cases whereby a potency is actualized. In order to grasp this objection to Potential Parts Theory and understand the inversion between part/whole and simplicity versus composition that culminates in the application of substantial form as the sufficient reason for the potential parts of matter, I would like to draw out more explicitly how Actual Parts Theory and Potential Parts Theory apply PSR, drawing from Actuality over Potentiality. We will see that the key difference between proponents of Actual Parts Theory and Potential Parts Theory on the metaphysical status of the parts of a composite whole is based on whether said composition is an accidental or an essential relation between parts.

Actual parts theorists' application of PSR via Actuality over Potentiality should be particularly apparent considering the actual parts theorists studied in the historical section of this dissertation. All of these theorists' arguments for simple parts as the (material) sufficient reason

for composition stems from the observation that composition is accidental. Composition is accidental because composite things come to be and cease to be, changing their qualities. Assuming Actuality over Potentiality, there must be an actuality or reason to account for this potency.

The result of this observation that composition is accidental and subscription to Actuality over Potentiality is the following metaphysical dilemma: either everything is composition (and thereby accidentally composed through and through), or there is a simple thing which accounts for the composition of other beings. The only conclusion possible given Actuality over Potentiality, which prioritizes actuality over potency, is that composition ultimately presupposes a simple being for its own account of material composition. This aforementioned argument for actual simple parts of composite wholes is latent in the theories of Hume, Berkeley, pre-critical Kant, and Leibniz.³⁸⁹ The assumption that composition is accidental constrains Actual Parts Theory proponents to the position that the simple being must be a compositional part and sufficient reason, in some sense, of the compositional whole.

This constraint that the simple thing must be a constituent part of an actual whole is not present in Potential Parts Theory since composition is essential to a whole thing. The only proponent of Potential Parts Theory reviewed in the historical section is the critical Immanuel Kant who argues in the *Metaphysical Foundations of Natural Science* that simple beings (force-shells) cannot exist because matter fills all of extension through its own activity or force.³⁹⁰ Since matter is really a field of activity without focal points of said activity, it is not actually infinitely divided to infinity and lacks actual, simple parts.

³⁸⁹ Hume *Treatise*, 1.2.1.2; Berkeley, 179; Kant *Physical Monadology*, 53; Leibniz “Monadology”, Paragraph 6.

³⁹⁰ Kant *MFNS* 504-505.

A Potential Parts Theory proponent interprets composition initially from the whole. For someone like the critical Kant, a Potential Parts Theory proponent does not begin from the claim that composition is accidental *and then* seeks an account for the said composition. Rather, he begins with the assertion that matter is *already* spread space through and through. Matter's actuality is already given as a whole or as *a thing*. It is not conceptually viewed from parts to whole but from whole to parts. Since matter is also given as divisible, the metaphysical status of its parts is given as *undetermined*. For what is necessarily given is the whole, but the parts are said to be only because matter is also divisible. Since matter is divisible, parts are also essential to composition despite these parts being designated as potential parts rather than actual parts.

The reader should clearly note that, like Actual Parts Theory, Potential Parts Theory borrows PSR from Actuality over Potentiality, but applies PSR differently on account of considering composition an essential rather than accidental characteristic of matter. Potential Parts Theory clearly requires Actuality over Potentiality since there is no question of the potentiality of parts without the actuality of a whole. PSR comes into play on this ground since the whole material object, which is given as a *divisible* thing, grounds the claim that there are potential parts. *How* such parts can be separated from the whole is based on what sort of whole is present. The capacity for the division of a diamond is quite different from that of a block of wood. The account cannot be grounded simply on the parts of said object since the parts are in potency. There must be something in the whole that accounts for this potential division.

My response for this account, the sufficient reason for potential division of the parts, is the Aristotelian substantial form. The sufficient reason is not the whole material object itself, but its ground of unity or of being considered an actual thing. In this way, the substantial form is the

simple being that replaces the monads of Leibniz, the force-shells of the pre-critical Kant, the perceptual minima of Berkeley and Hume. In Potential Parts Theory, the simple being is not a constituent of the composite whole, but penetrates the whole as its principle of actuality and unity.

Chapter Four-Section D:

Aristotelian Substantial Form and PPT- The Reconciliation between

Simplicity and Infinite Divisibility

Substantial form is a good candidate for being the PSR of and simple being on which undivided composite wholes rely for their actuality and unity. It serves the role as PSR for composite material objects because it is the source of actuality of a thing, making something the being that it is. Substantial form also penetrates the entirety of the composite whole and cannot itself be divided metaphysically into any other “simpler” beings. This incorporation of substantial form into composite material objects is essentially Aristotelian by virtue of presupposing a hylomorphic understanding of nature, but is new because it revitalizes the Aristotelian understanding of nature in terms of simplicity/composition, and of actual versus potential parts in the Early Modern debate on matter’s divisibility.

Aristotle’s theory of substantial form is integral to my attempt to reconcile metaphysical simplicity and space’s infinite divisibility because substantial form is a source of unity or “simplicity” for beings and is compatible with matter’s potential infinite divisibility. Considering how I have already described hylomorphism in the general introduction, I will spare the reader a redescription of such theory and summarize instead Aristotle’s distinction between

substantial form and accidental form and connect substantial form to my solution to matter's divisibility.

Aristotle distinguishes forms which make a being a substance or thing which underlies qualities versus forms that make something a certain quality or characteristic. The substantial form of humanity would make a being a human, while the accidental form of blueness colors something (a substance) blue. The substantial form is primary because there could not be any being in existence without it; there would be nothing that the accidental form could cause for something to possess some characteristic or accident.

It is important in the thought experiment involving neutral matter that it is the substantial form which makes the material object *a being*; otherwise, the material object must be a characteristic and not a *thing*. Per Actuality-Unity Principle, the material object that is assumed actually existent in neutral matter is one being. As substantial form makes a being what it is (one thing as a composite of form and matter) in Aristotelian metaphysics, so too is there a principle of unity in the case of the material objects in the thought experiment of neutral matter. There must be *at least* one thing *if* we assume that matter is actualized.

This principle of unity, the substantial form, works particularly well with matter's potential infinite divisibility. The application of substantial form makes that material object one thing, but allows for a being to possess separable parts. The material object is thereby a whole whose parts *could be separated out*. A potential part is not separated from the whole unless it "receives" its own substantial form. A substantial form is thereby a principle of limitation for matter. It enables a *flexibility* attendant with the potential infinite divisibility of material parts while also being a principle of limitation for actual material parts. Substantial form begins with

the whole irrespective of the innumerable potential parts in the same manner that one begins with the whole of potentially infinitely divisible quantities before examining the innumerable parts.

My Neo-Aristotelian theory of the potential parts doctrine not only reconciles simplicity and infinite divisibility, but also shares with all other actual parts theories investigated in this dissertation the close association between the principle of actuality, simplicity, and sufficient reason. The simple being in this Neo-Aristotelian solution is the whole or totality of a material object itself considered as *one being* given the substantial form. No matter which actual parts theory one subscribes to, the simple being is necessarily regarded as *an actual being* irrespective of whether it possesses intellectually distinguishable parts (Berkeley and pre-critical Kant) or is intellectually indivisible quantitatively (Hume and Leibniz). What each actual parts theorist and I hold in common also is that the simple being is identified with the principle of material sufficient reason; however, the principle of material sufficient reason in my Neo-Aristotelian solution is *understood* differently from the manner it is applied in the other actual parts theories.

This difference in the application of material sufficient reason is evident from the truth that the sufficient reason or simple being in actual parts theories functions as the *constitutive element* of the composite whole, while the simple being is the *ground* for the possibility of composite things without itself being a *constitutive element* of the composite whole. Actual parts theories require that simple beings are unit quantities, while my Neo-Aristotelian theory specifies only that the whole on account of its substantial form that is *one being* contains a potential infinity of parts as the ground. The substantial form is not a part of the whole, but completely penetrates the entirety of the potential parts. It cannot be aligned “next to” another substantial form save incidentally through the quantitative accidents of space/place as simple beings are

aligned “next to” one another to form composite material objects in the thoughts of Hume, Berkeley, and pre-critical Kant. Nor does material reality exist at a different phenomenal level than the ultimate metaphysical reality of substantial forms as is the case with the metaphysical relation between monads and matter/space in Leibnizian thought. Without the truth of actual parts doctrine, Hume’s, Berkeley’s, the pre-critical Kant’s, Leibniz’s theories of matter’s divisibility must be false.

This Neo-Aristotelian theory of matter’s divisibility also circumvents the Kant’s second antinomy in the *Critique of Pure Reason*. Although we have inferred matter’s infinite divisibility from space’s infinite divisibility, it is impossible to infer also that the simple being must be divisible and thereby forfeit its simplicity. The substantial form need not be divided for we can only inquire into the particular divisions of material objects through the said substantial form. The substantial form is a *metaphysical unit* that makes a being what it is as *a thing*. It entirely penetrates a given being without being in one part any more or less than another part of the same being. This new doctrine avoids the conclusion of the second antinomy while also preserving the common sense intuition that matter is *in* space. There is only one reality, but because the ultimate grounding principles of beings is based on integral wholes via substantial forms, simple beings exist and are necessary for potentially infinitely divisible matter.

Chapter Four Conclusion

The argument provided for the Aristotelian potential parts doctrine not only breaks the stalemate to which Holden alludes in his work on the structure of matter in the Early Modern period, but also reconciles simplicity and infinite divisibility in a remarkable way. It breaks the stalemate by providing a metaphysically neutral form of matter which is simply known as a

composite whole. This whole is composite by virtue of space's infinite divisibility. One cannot suppose the actual parts doctrine and implement an indivisible and actual part of matter. Yet this composite whole does not forfeit simplicity by virtue of the substantial form as the sufficient reason for the organization of the composite parts.

The result is that the nature of simplicity is re-envisioned to account for the potential parts doctrine. The simple being is no longer a minimum or constitutive unit of a composite material body, but the sufficient reason for the organization of the composite whole—the substantial form. The metaphysical principle that simplicity is prior to composition is preserved despite the potentially infinite division of matter.

This formulation of a Neo-Aristotelian potential parts doctrine would not have been possible without finding the common principles and a form of matter indifferent to possessing either potential parts or actual parts. By applying Actuality-Unity Principle to neutral matter, it becomes clear that the potential parts doctrine is more metaphysically sound than the actual parts doctrine. Despite disagreements with actual parts proponents, this Aristotelian potential parts theory is not entirely devoid of contributions given by actual parts proponents, particularly considering the relations between the principles of sufficient reason and simplicity. The result is a new theory of simplicity and the reconciliation between the principles of simplicity and space's infinite divisibility.

Appendix:

Interpretations of Leibnizian Space and Matter

An authentic interpretation of Leibniz's metaphysical theory of matter and space has proven to be a problem more labyrinthian than the issue it was meant resolve: the composition of the continuum. What complicates an accurate interpretation of his metaphysical theory is that Leibniz changes his mind repeatedly throughout his career, fleshing out his apparently different theories via short essays or letters to his contemporary philosophers and scientists.

There is no *magnum opus* to which a Leibniz scholar can point in order to help provide a means by which to connect the different short works and formulate a coherent theory. Kant scholars have the *Critique of Pure Reason*; Hume scholars can refer to the *Treatise of Human Nature* or the *Enquiry Concerning Human Understanding*; while Spinoza scholars can organize a theory centered on the *Ethics*. While a *magnum opus* itself cannot resolve all interpretive difficulties, it can serve as a starting point for a genuine interpretive theory.

For Leibniz scholars, interpretive difficulties arise from the fact that it is possible to select any particular essay or letter as a focal point for an interpretation. Perhaps Leibniz specifically intends these interpretive difficulties as a reflection of one of his central metaphysical tenets in his later metaphysics: just as there exists a plenum of monads or individual souls with each possessing its own perspective of the world, there are any number of individual texts to select as a focal point for interpretation of all the Leibnizian short works. The task for the Leibniz scholar is to figure out how to piece together all of the individual texts to help convey a latent, presumably coherent message just as all of the monadic perceptions are linked together according to a pre-established harmony.

My task in this appendix is to provide a broad overview of the interpretive positions regarding Leibniz's stance on the metaphysical status of matter and space in his "later" years. Thus, I will not provide a "pre-established harmony" for interpreting Leibnizian metaphysics. The reader should already be somewhat acquainted with a few interpretations having read the Leibniz chapter, including that of Brandon Look, J.E. McGuire, and that of Hartz and Cover. Glenn Hartz's "Why Corporeal Substances Keep Popping Up in Leibniz's Later Philosophy" (2008) will be my particular guide when deciphering the general interpretations regarding Leibniz's theory of matter. Although I argue my claims in the Leibniz chapter based on the interpretation of the incompatibilist or Extreme Idealist position of Brandon Look, I will actually side with a rather ingenious interpretation of the difficulties provided by Andrew Stump. Stump is a compatibilist or Moderate Idealist who argues that matter can be called a composite substance provided that "substance" is applied to things by analogical predication instead of univocal predication.

In order to grasp the controversial interpretive positions more clearly, I will clarify what is meant by "Idealism" broadly speaking, as well as the distinction between Extreme Idealism and Moderate Idealism. These terms should be understood in the context of Leibniz's "later" metaphysics. I will use the phrases or terms "composite substance", "corporeal substance", "matter", and "body" interchangeably unless otherwise noted.

Having surveyed the vast amount of scholarship on Leibniz's scholarship, it is clear that the majority of scholars agree that Leibniz is an Idealist, save Phemister who argues that corporeal substances are the only true substances in Leibniz's metaphysics—monads are really

the animal souls combined with organic bodies.³⁹¹ Idealism is the metaphysical theory that mind or soul is the foundation of reality and everything else is mind-dependent. Robert Adams defines Leibnizian Idealism the best: “The most fundamental principle of Leibniz’s metaphysics is that “there is nothing in things except simple substances, and in them perception and appetite.”³⁹² Thus, Leibnizian Idealism not merely a claim that everything is composed of simple substances, but that such simple substances possess perceptions and appetites. Paul Lodge comments: “A construction of the whole of reality out of perceiving substances and their perceptions and appetites exemplifies a broadly idealist approach to metaphysics.”³⁹³ The reader should note that Leibniz has not always been an idealist. Leibniz does not qualify as an Idealist in his “early” years; but he may be an Idealist in his “middle” years, and is definitely an Idealist of some form in his “late” years.

The question pertinent in this appendix is whether or not Leibniz’s commitment to monadic reality in his “later” years is compatible or incompatible with his purported belief in the reality of composite substances—a remnant theory from his “middle” years. The proposition that his commitment to monads is incompatible with the existence of composite substances is Extreme Idealism, while the assertion that monads can coexist with the requirements of there being composite substances is Moderate Idealism.³⁹⁴ Given the range of potential positions regarding Leibniz’s continued commitment to composite substances, the criterion for composite substances needs to be discussed.

³⁹¹ Phemister, P. *Leibniz and the Natural World*. Dordrecht: Springer, 2005. 31.

³⁹² Adams, Robert. *Leibniz: Determinist, Theist, Idealist*. Oxford: Oxford University Press, 1994. 214.

³⁹³ Lodge, Paul. “Garber’s Interpretations of Leibniz on Corporeal Substances in the Middle Years”. Volume 15 2005. pp. 1-26. 25.

³⁹⁴ I borrow these terms from Glenn Hartz’s essay which I will use to great extent in this appendix, “Why Corporeal Substances Keep Popping up in Leibniz’s Later Years.” *British Journal for the History of Philosophy*, 6:2. 1998. 193-207.

I will supply a material biconditional statement as the definition of composite substance: composite substance exists if and only if the said being possess a unity of its own distinct from monadic unity with accompanying active and passive forces. It is not enough to say that a composite substance exists if and only if a certain aggregate of monads also exist. For Leibniz distinguishes sharply between an aggregate and a composite amounting to what McDonough terms as an *inorganic* versus an *organic* unity: “The central thrust of the argument is simply that extended bodies can be divided into two classes: those having genuine, organic, *per se* unity, like an extended living person, and those having merely accidental, non-organic, derivative unity, like a stack of logs or a pile of bricks.”³⁹⁵ Thus, it is not enough for a compatibilist to claim that Leibniz believes in composite substances by virtue of their being a certain aggregate of monads whose perceptions are harmonious so as to “form” a table or a chair. A composite substance is not a plurality of unities (an aggregation), but a unity of plurality. For a plurality of unities is nothing more than the unities themselves. The mind supplies that unity which allows a person to call an aggregate of bricks a “wall” rather than merely referring to each brick that constitutes a said “wall”. What unifies a plurality must be something more than what is being unified while also being the plurality itself. The Aristotelian substantial form plays this role of unification by virtue of providing a structure to the so-called ‘matter’ or parts of a being to call it *one thing* rather than *many things*—precisely what McDonough means by *organic unity*.³⁹⁶ Although a composite substance presupposes monads for its existence, the unity of the composite substance cannot be the unity of the monads.

³⁹⁵ McDonough, Jeffrey. “Leibniz’s Conciliatory Account of Substance.” *Philosophers’ Imprint*. Volume 13, No.6. April, 2013. 1-23. 5.

³⁹⁶ McDonough, 5-7.

With the distinction between organic and inorganic unity in mind, let's turn to a more in depth examination of the development of Leibniz's metaphysics. Leibniz's career is distinguished into his "early", "middle", and "late" periods on the basis of his changing theories regarding the metaphysical status of matter and space. The "early" period marks the time in which Leibniz espoused a form of atomism, regarding mathematics as the only valid description of nature. Matter and space are thereby the true realities of natural phenomena. The "middle" period marks the time in which Leibniz shifted from an atomistic account of nature to a combination of modern mathematical physics with an Aristotelian physics of substantial forms or "natures". By advocating a theory of substantial forms, Leibniz espoused a theory of composite substances—a theory that material objects consist of an infinity of parts unified through a pseudo Aristotelian substantial form. The "later" period demarcates a supposed transition period in which Leibniz formulates officially his monadological theory of simple substances. The point of contention between commentators is precisely whether Leibniz continues to accept his belief in composite substances in his "later" years. Leibniz's struggle to articulate the relation between actuality (being) and unity underlies the conflicting compatibilist and incompatibilist interpretations.

I will run initially through Leibniz's appeal to atomism in his early career, briefly detailing his objection to atomism. Having briefly detailed Leibniz's objection to atomism, I will summarize his attempt to bridge Aristotelian substantial form with the mathematical science, setting the stage for his articulation of the ultimate expression of being and unity in his later career concerning the existence of monads. It is at this point that I will use Hartz's "Why Corporeal Substances Keep Popping Up in Leibniz's Later Philosophy" as a guide to analyze the variety of compatibilist and incompatibilist position, arguing for Andrew Stumpf's compatibilist

interpretation. I point out that the list of positions summarized in this appendix is neither exhaustive nor complete. Certain compatibilist positions, if true, would also remove the distinction between Leibniz's "Middle" and "Later" years. Jefferey McDonough argues this claim contra Dan Garber's central thesis that the "Middle" years constitute a distinct time period in Leibniz's thought.³⁹⁷ My hope is that Andrew Stumpf's compatibilist interpretation could be used to harmonize the compatibilist positions and demonstrate to incompatibilists a method for interpreting Leibnizian unity without sacrificing the fundamental metaphysical unity that characterizes monads in the "Later" years.

Leibniz's earlier writings point to an atomistic theory of space and matter similar to the theories proposed by Democritus and Leucippus. Richard Arthur writes that Leibniz's earliest memories of embracing atomism date back to a 1661 letter written to Nicole Remond in which Leibniz describes his opinions shifting from those of the Schoolmen to the "New Science".³⁹⁸ The materialism of the New Science is attractive to Leibniz for its appeal to mathematical, mechanistic, and imaginative descriptions for physical events rather than the abstract theories of substantial form distinctive of Scholasticism.³⁹⁹ Such atoms are not completely indivisible per Democritean theory, possessing different sizes and shapes. The essence of such atoms is principally extension. These atoms are thereby indistinguishable from one another save by their differing motions and positions in space relative to one another—matter is distinguishable only

³⁹⁷ McDonough, Jefferey. "Comments on Dan Garber's *Leibniz: Body, Substance, and Monad*". Presented at the Eastern Division Meeting of the American Philosophical Association, December 2011. 8-9.

³⁹⁸ Arthur, Richard. "The Enigma of Leibniz's Atomism". *Oxford Studies in Early Modern Philosophy*. Volume 1. 2003. 183-227. 183-184.

³⁹⁹ Leibniz, Gottfried. *Die philosophischen schriften von Gottfried Wilhelm Leibniz*, vol. IIIC. I. Gerhardt (ed.) pp. 618-624. GIII p.620.

through its extrinsic relations to other material objects.⁴⁰⁰ There is no need for Aristotelian substantial form or intrinsic natures in the New Science as there is for the Schoolmen.

Leibniz later finds the imaginative appeal of atomism lackluster as he considers the absence of a genuine metaphysical account of the New Science's mathematics. He writes: "...when I looked for the ultimate reasons for mechanism, and even for the laws of motion, I was greatly surprised to see that they could not be found in mathematics but that I should have to return to metaphysics."⁴⁰¹ His return to metaphysics lies in the absence of any genuine method for individuating atoms from one another if their essence is bare extension, possessing merely extrinsic relations to other atoms whose individual difference also remain mysterious.⁴⁰² This critique of atomism is especially evident in his 1690 essay, "A Demonstration against Atoms Taken from the Contact of Atoms". Leibniz shows that the atoms of Galileo and Gassendi cannot be "uncuttable" while possessing different shapes and sizes.⁴⁰³

Underlying these arguments against Galileo and Gassendi is Leibniz's belief that an explanation of natural phenomena through external relations without internal natures results in contradiction when attempting to describe the nature of atoms. Mass, motion, shape, size, and impenetrability do not properly describe physical events, let alone the reality of atoms.⁴⁰⁴ Leibniz thereby argues that *force* rather than *extension* defines material objects. Yet these forces also require explanation through some sort of principle of unity, namely a substantial form. By invoking substantial form, Leibniz provides physics with a means by which to sort things that are

⁴⁰⁰ Hartz, Glenn. "Materialist Ontology: The Leibnizian Way". *History of Philosophy Quarterly*. Volume 1, No. 3. July 1984. Pp. 315-332. 316-317.

⁴⁰¹ Leibniz, Gottfried W. "Discourse on Metaphysics". X.

⁴⁰² Hartz "Materialist Ontology...", 315.

⁴⁰³ Leibniz, Gottfried. "A Demonstration Against Atoms Taken From the Contact of Atoms (23-24 October 1690)". Lloyd Strickland, ed. *Leibniz: The Shorter Leibniz Texts*. London: Continuum, 2006. 119-121.

⁴⁰⁴ Hartz "Materialist Ontology", 316-321.

mere aggregates (lacking a principle of unity) versus things that are physically *one* thing via substantial form.⁴⁰⁵

Leibniz proposes his theory of substantial form as a bridge between the Scholastic theory of substantial forms in nature and the scientific mechanics of his age, marking the so-called “Middle Years” of Leibnizian metaphysics.⁴⁰⁶ Having derived the theory from Aristotle’s account of hylomorphism, Scholastics explained a natural being’s activities in terms of its substantial form, which makes a being what it is. For example, the human soul makes a given body a human body. By contrast, Enlightenment era scientists applied mathematics to their understanding of nature; however, in order to make their mathematics work, such scientists presupposed the existence of atoms or physical points. Thus, Enlightenment era scientific thinkers would explain the burning sensation one would experience upon placing a hand on a flame through the motion of particles that constitute the flame itself and describe natural events through the mathematical properties of particles—size, shape, and motion; a Scholastic thinker would account for the burning sensation through the *tendency* in the flame to produce such sensations in us as like produces like—fire burns the hand because fire possesses “burnness”.⁴⁰⁷ Leibniz incorporates both theoretical avenues of thought by using substantial forms as the principle that unifies the complexity of material bodies, which are described in terms of mathematics. The metaphysical substantial forms organize the bodies that are mathematically described in terms of shape and number.

⁴⁰⁵ Leve, Samuel. “On Two Theories of Substance: Critical Notice of Daniel Garber, ‘Leibniz: Body, Substance, and Monad’”. *Philosophical Review*. Volume 120, No. 2 2011. 285-320. 291.

⁴⁰⁶ Leibniz, G.W. “Discourse on Metaphysics”. In *G.W. Leibniz: Philosophical Essays*. Translated by Roger Ariew and Daniel Garber. Indianapolis, IN: Hackett Publishing Company, 1989. 22 pp. 54-55.

⁴⁰⁷ Garber, Daniel and Rauzy, Jean Baptiste. “Leibniz on Body, Matter, and Extension”. *Aristotelian Society: Supplementary Volume*, Supp. (78), 23-40. 18 p. 2004. . 23-25.

The result of this conjunction between substantial form and mathematics is the composite substance—a being that possesses a “composite unity” of form and matter. The composite substance is akin to Aristotelian natural substances, what Garber has called “quasi-Aristotelian substances.”⁴⁰⁸ This “composite unity” constitutes the being of material beings, particularly that of living things. Composite substances are often distinguished from “aggregates”, which are mere collections of substances. Leibniz’s continued support for composite substances becomes an issue during his “Later Years” in which he apparently replaces all mention of matter or composite substances with the use of the term “substance” to refer primarily to monads rather than composite material things. Contrary to other scholars, Garber ultimately thinks that the problem of Idealism never cropped up for Leibniz in his “Middle Years”; Leibniz only wished to contrast himself for the Cartesian theory that substances can be extended.⁴⁰⁹ Thus, by arguing explicitly for the existence and metaphysical priority of monads or simple substances over matter, Leibniz finds himself struggling in his “Later Years” to reconcile an apparently extreme idealist theory with the existence of real corporeal substance leftover from his “Middle Years”.

The question that has arisen in contemporary Leibniz scholarship is whether Leibniz remains committed to corporeal/composite substances in the “Later Years”. Commentators typically take on either a compatibilist or an incompatibilist position regarding Leibniz’s commitment to composite substances. It is Leibniz’s struggle to connect the metaphysical principles of being (actuality) and unity that fuels the controversy. In order to navigate the variety of compatibilist and incompatibilist positions, I will look to Glenn Hartz for guidance,

⁴⁰⁸ Garber, Daniel. “Leibniz and the Foundations of Physics: The Middle Years”, in *The Natural Philosophy of Leibniz*, eds. K. Okruhlik and J. R. Brown. Dordrecht, D. Reidel, 1985. 27-130.

⁴⁰⁹ Garber “Leibniz on Body, Matter, and Extension”, 30-40.

but then propose Andrew Stumpf's attempted analogical reasoning concerning Leibniz's use of the term "substance" as a solution.

Arguing a rather nuanced agnostic position regarding the existence of composite substance, Hartz provides the reader a critique of the different interpretations of Leibnizian composite substance. This critique results in Hartz's view that the Exclusive Idealist doctrine and the more moderate Idealist-Corporeal Substance doctrine are flawed. Hartz writes of his precise position in the very concluding sentence in the article: "One of those motifs was Exclusive Idealism, another Idealism Plus Corporeal Substances. Leibniz never managed to choose between them, and neither will any successful interpretation."⁴¹⁰ This quotation shows that Hartz's position represents a form of what I would like to call Leibnizian "Meta-Incompatibilism". Leibnizian "Meta-Incompatibilism" states that the *resolution* of all forms of compatibilism and specific forms of incompatibilism (e.g. phenomenalism) is impossible. In other words, all forms of Extreme Idealism and Moderate Idealism are really false. In this vein, Hartz's "meta-incompatibilism" is akin to the Kantian antinomy: just as reason itself cannot resolve the conflict between contrary metaphysical positions that concern the knowledge of noumena, it is also impossible to assert that Exclusive Idealism or Idealism-Corporeal Substances could be true.

Hartz critiques four arguments in favor of some form of Extreme Idealism. These arguments fail because they force the reader to misinterpret the text by construing composite substances as lacking a unity distinct from the unity of the monads.⁴¹¹ Assuming that Extreme Idealists ignore critical aspects of Leibniz's theories, and Moderate Idealists cannot correctly

⁴¹⁰ Hartz "Why Corporeal Substances", 207.

⁴¹¹ Hartz "Why Corporeal Substances", 194.

point out a suitable interpretation for the unity of composite substances distinct from aggregative unity and monadic unity, Leibnizian “Meta-Incompatibilism” ensues.

The first and most widely supported theory of Extreme Idealism, Phenomenalism, finds its place in the thought of scholars like C.D. Broad and Catherine Wilson. It is the theory everything other than monads must be some form of phenomena—a perception or appearance that exists within the said windowless souls. Phenomenalist commentators often cite Leibniz’s letters to De Volder as evidence of his commitment to the claim that matter is a “well-founded phenomenon”.⁴¹² This phrase, “well-founded phenomenon”, means only that matter is a derivative being distinguishable from bare extension by virtue of possessing its own force.⁴¹³ On the other hand, space is a mere phenomenon, being based solely on extension. Matter, although ultimately a perception within each monad, possesses a level of reality distinct from matter by virtue of its own activity or force.

It is Montgomery Furth who originally cites the De Volder letters as evidence for Leibniz’s phenomenistic tendencies in his “Later Years”.⁴¹⁴ Leibniz disputes De Volder’s claim that substances can be extended. De Volder, a Cartesian metaphysician, argues for extended substances on the basis that whatever is conceived *per se* is a substance and that extension is conceived *per se*.⁴¹⁵ Lodge notes that De Volder does not seem to back up his claims in the argument save to describe the nature of substance as whatever is conceived as one

⁴¹² Hartz “Why Corporeal Substances...”, 194.

⁴¹³ Shneider, Christina. “Leibniz’s Theory of Space-Time: An Approach from his Metaphysics”. *Monist: An International Quarterly Journal of General Philosophical Inquiry*. 81(4), 612-632, 21 p. October 1998. 623-624.

⁴¹⁴ Furth, Montgomery. “Monadology” in the *Leibniz Review*, 76 (1967), pp. 169-200. 186-188. Rutherford, Donald P. “Phenomenalism and the Reality of Body in Leibniz’s Later Philosophy”. *Studia Leibnitia*. Band XII/1 (1990). pp. 11-28. 13.

⁴¹⁵ Lodge, Paul. “The Debate Over Extended Substance in Leibniz’s Correspondence with De Volder”. *International Studies in the Philosophy of Science*. Vol. 15, No. 2, 2001. 155-165. 155-156.

whole thing from which nothing can be subtracted without forfeiting the being itself, a concept which Lodge calls the “conceptual simplicity definition”.⁴¹⁶ Leibniz takes issue with De Volder’s claim that extension would qualify as something which is conceptually simple for extension is a plurality of coexistences.⁴¹⁷ Since matter is a plurality, it cannot be a substance. Thus, matter can only be phenomenal, possessing no capacity to exist on its own. A material object is an aspect of the perception in each monad.

Relying on the work of Robert Adams, Hartz notes that Leibniz never refers to composite substances as phenomena and sharply distinguishes between them from aggregates.⁴¹⁸ He writes: “Moreover, Leibniz clearly distinguished the aggregate-part of a corporeal substance (the ‘organic body’) from the larger corporeal substance (the organic body + dominant monad).”⁴¹⁹ The dominant monad is the soul of the larger corporeal substance, while the organic body is its matter—a clear application of Aristotelian hylomorphism to understanding nature.

Another scholar, Brandon Look, echoes Hartz’s criticism of the phenomenalistic interpretation of composite substances or matter; however, Look argues ultimately in favor of Leibnizian Extreme Idealism.⁴²⁰ One of the passages Look cites to support his claim that composite substances are genuine substances is the third paragraph in the *Principles of Nature and Grace* to support Leibniz’s commitment to composite substances. In the third paragraph, which I have cited to support the phenomenalistic interpretation, Leibniz states: “Every simple substance (or individual monad) is the centre and source of unity of a composite substance such

⁴¹⁶ Lodge, 156.

⁴¹⁷ Lodge, 157-158.

⁴¹⁸ Hartz “Why Corporeal Substances...”, 195; Adams, 267.

⁴¹⁹ Hartz “Why Corporeal Substances...”, 195.

⁴²⁰ Look, Brandon. “Leibniz’s Metaphysics and Metametaphysics: Idealism, Realism, and the Nature of Substance.” *Philosophy Compass*. Volume 5, Issue 11, 2010. 871-879. 871; 879.

as an animal; the central monad is surrounded by a mass made up of an infinity of other monads which constitute its body.”⁴²¹ Look writes of this citation: “According to Leibniz, then, the monad itself is a compound of soul and matter; or, more precisely, the *completed substance* of the monad is made up of entelechy and primary matter, that is, primitive active and passive powers.”⁴²² Look invokes the notion that monads are *completed substances* from the standpoint that the soul/dominant monad of an animal, plant, or human being requires its organic body. Thus, it appears that Leibniz sticks to a theory in which composite substances are real or are substances in their own right.

However, Look and I note that the reality of such composite substances are considered substances or unified beings only from the standpoint of their dominant monad and from the aggregates of monads that compose the organic body.⁴²³ Even if we accept that composite substances are contrasted from aggregates as Hartz claims, there is still the problem of how one should account for a *per se* unity of a composite substance given the dominant monad’s relation to its organic body.⁴²⁴ We will return to this problem later when examining the problems concerning unity and corporeal substances later.

The second proposal for Extreme Idealism is the aggregative theory. The aggregative theory apparently grants more reality to matter by designating certain aggregations of monads as the material thing. Matter is no longer strictly “phenomenal” or the resultant perception from each monad. Citing Leibniz’s apparent failure to provide an adequate account of the union of soul and body, Rutherford holds that unity and reality belong to monads while matter is merely

⁴²¹ Leibniz “Principles...” Paragraph 3.

⁴²² Look “Leibniz’s Metaphysics and Metametaphysics...”, 874.

⁴²³ Look “Leibniz’s Metaphysics and Metametaphysics”, 875.

⁴²⁴ Look, “Leibniz’s Metaphysics and Metametaphysics”, 875.

aggregative. He writes: “Leibniz's deep metaphysics . . . is the metaphysics of monads, in which all other beings, including living creatures, are no more than "phenomena" and "results"”⁴²⁵ But this aggregate theory is not without its issues.

Yet the aggregative theory does not really grant matter any ultimately greater reality than the phenomenal view because matter does not possess any greater unity or reality than the collection of each of the monads themselves. Moreover, the unity of an aggregate does not come from the aggregate itself, but from the subject perceiving the aggregate.⁴²⁶ Thus, the aggregative theory cannot satisfy the requirement for the composite substance to which Leibniz seems to allude according to Hartz.⁴²⁷ The upshot is that the aggregative theory lapses into Extreme Idealism, a theory which does not account for Leibniz’s repeated references to corporeal substances.

The third theory in support of Extreme Idealism is the *Reductio ad Vinculum Argument*. The *ad Vinculum* theory is found in several of Leibniz’s letters to Arnauld and Des Bosses, cited originally by J.E. Erdmann. Erdmann writes: “. . .that there is here an additional element that transforms them into something real, a *realizans*, which, in the letters to Des Bosses, is called a *vinculum substantiate*; that on this account, while every mere body is *substantiae*, or a *substantiatum*, a living body is also a *substantia (composite)*.”⁴²⁸ The composite or *substantia* to which Erdmann refers is ultimately a substantially bonded being; however, this substantially

⁴²⁵ Rutherford, 282.

⁴²⁶ Arthur, Richard T.W. “Presupposition, Aggregation, and Leibniz’s Argument for a Plurality of Substances”. *The Leibniz Review*, Vol. 21, 2011. 91-115. 98.

⁴²⁷ Hartz, 196.

⁴²⁸ J. E. Erdmann, *History of Philosophy*, trans. W. S. Hough, 2nd ed., 3 vols. London: Sonnenschein, 1891. 188-189.

bonded thing is incompatible with the notion that composite substance possess a unity *per se*.⁴²⁹

Thus, according to Erdmann, commentators should ignore Leibniz's repeated references to corporeal substances as things which are supposedly *unum per se* and advocate a reduction in which all references to matter or corporeal substances are really substantially bonded things.⁴³⁰

Hartz argues that Leibniz's references to substantial bonds as found in the Des Bosses correspondences are made in error.⁴³¹ He argues that Leibniz supplies the substantial bond doctrine which constitutes a source of unity for the composite substances despite Leibniz's own admission that the source of unity for composite substances is unknown in his later metaphysics.⁴³² While commentators argue that Leibniz's substantial bond doctrine is an implicit admission that the doctrine of corporeal substances is false, Hartz thinks it merely shows that only the substantial bond doctrine is false, while the corporeal substance doctrine could be true. He writes: "*They* take this as a reason to deny canonical corporeal substances; I take it merely as a reason to deny substantial bonds..."⁴³³ Should the reader accept Hartz's claim that the substantial bonds doctrine is really a mere experiment that Leibniz entertains rather than a genuine theory to account for the unity of composite substances, the truth of doctrine of corporeal substances is independent of the truth of bonded corporeal substances.

The fourth argument in favor of Extreme Idealism that Hartz critiques is the *Philosophical Argument*. The *Philosophical Argument* is straightforward in its appeal to simplicity. Hartz writes: "How pristine, neat, clean, simple, powerful - yes, beautiful - is this

⁴²⁹ Hartz, 196-197.

⁴³⁰ Erdmann, 187-189.

⁴³¹ Hartz, 197-198.

⁴³² The source of unity in composite or corporeal substances in his middle metaphysics is the substantial form, which the monads have replaced in the later metaphysics.

⁴³³ Hartz, 198.

monadology, which in a single stroke transforms the vulgar empiricist's rag-bag world of space, matter, and phenomena into pure intelligibility! If you start with things that *can't* have parts, the problem of unity is instantly solved - no need to search for 'principles of unity' or worry about borderline cases.” Should Leibnizian metaphysics begin from the existence of monads, which are simple beings, then there is no need to provide an account of corporeal bodies. Such bodies are reducible to monads as forms of intelligibility. Hartz notes that there is no single author of such an argument except that the *Philosophical Argument* is akin to a process of thought that underlies arguments for Extreme Idealism, notably that of Montgomery Furth.⁴³⁴ Furth claims that all mention of bodies or the perception of bodies is merely a lapse in Leibniz's terminology.⁴³⁵ Despite the great explanatory power of monadic reality, Leibniz uses terms like “body” to connect the monadological theory with a reader's apparent experiences. “Body”, “corporeal substance”, and “composite substance” are mere words in Leibnizian Extreme Idealism according to Furth.

Hartz responds to the *Philosophical Argument* by alleging that its proponents construct their interpretations for their own philosophical purpose rather than to discover what the philosopher's actual theory might actually be.⁴³⁶ He writes: “But I insist on commentator honesty: if the philosophy is what's driving one's interpretation, just say that plainly - that the ideas being dug out are being dug out for a philosophical purpose that may run counter to the historical figure's intentions.”⁴³⁷ By doing so, proponents of Extreme Idealism construe terms like “body”, “corporeal substance”, and “composite substance” to mean something other than

⁴³⁴ Hartz, 198.

⁴³⁵ Furth, Montgomery. “Monadology” *Leibniz: A Collection of Critical Essays*, ed. Harry G. Frankfurt Notre Dame, Ind.: University of Notre Dame Press, 1976. pp. 99-135. 120-121.

⁴³⁶ Hartz, 198-200.

⁴³⁷ Hartz, 199.

what Leibniz originally intends. Hartz proceeds to investigate the thought of Donald Rutherford to demonstrate how a proponent of the *Philosophical Argument* has misconstrued Leibniz's true intent of the aforementioned terms.⁴³⁸ Rutherford is guilty of prioritizing certain sections of Leibnizian texts over others that support Extreme Idealism to the exclusion of genuine composite substances.⁴³⁹ Hartz writes: "...overlooks the fact that the text *says* (and thus certainly 'implies!') that there is 'more than an accidental unity'."⁴⁴⁰ Proponents of the *Philosophical Argument* thereby read the Leibnizian text primarily for supporting their overarching conclusions, ignoring the other textual parts which would seem to indicate a conclusion contrary to their own.

Having critiqued four direct arguments against there being genuine corporeal substances in later Leibnizian metaphysics, Hartz provides an overview of two arguments favoring tension between Leibniz's assertion that monads and composite substances exist. He notes that these two arguments actually demonstrate Leibniz's implicit belief in composite substances despite lacking a means by which to articulate their substantial unity successfully.⁴⁴¹ The two arguments, the No Principle of Unity Argument and the Argument from Inconsistent Ontological Commitments, underlie the thought of most proponents of Extreme Idealism.

Hartz identifies Donald Rutherford as a major proponent of the No Principle of Unity Argument, which Rutherford uses to defend Extreme Idealism. Leibniz strongly adheres to the Principle of Unity which is the principle that all individual things or substances possess *per se* unity.⁴⁴² When articulating a theory of composite substances, particularly the union of soul and

⁴³⁸ Hartz, 199-201.

⁴³⁹ Rutherford, Donald. *Leibniz and the Rational Order of Nature*. Cambridge: Cambridge University Press, 1995. 292-293.

⁴⁴⁰ Hartz, 200.

⁴⁴¹ Hartz, 202-205.

⁴⁴² Hartz, 202.

body, Rutherford notes that Leibniz is at a loss to supply the reader with a principle of unity beyond a pre-established harmony.⁴⁴³ Without a principle of unity, commentators like Rutherford argue for a Leibnizian Extreme Idealism.

Yet Hartz notes that the No Principle of Unity Argument could actually demonstrate a latent tension between Extreme Idealism and Moderate Idealism in later Leibnizian metaphysics. Hartz writes: "...I think we're stuck with candidly admitting that there is a gaping hole at the center of Leibniz's late-mature corporeal substance doctrine - and that this is a serious shortcoming. He bought, but could not pay for (in his own *coin*), *per se* unity for corporeal substances."⁴⁴⁴ Assuming that Hartz's objections to the arguments for Extreme Idealism are true, Leibniz is definitely committed to corporeal substances but is unable to provide an account of their *per se* unity without contradiction. The conclusion that should be reached is that corporeal substances are present in Leibniz's later metaphysics, but their existence is fundamentally incompatible with monadic unity. We thereby have a fundamental or "meta-incompatibility" between the assertion of both monadically substantial and corporeally substantial realities: we cannot affirm Extreme Idealism alone without neglecting a number of Leibnizian texts, but Moderate Idealism is rife with its own obscurities and an objective Leibnizian scholar cannot state that Leibniz has successfully dealt with the obscurities.

The next argument ultimately in support of meta-incompatibilism is the argument from Inconsistent Ontological Commitments. Originally cast as an argument in support of Extreme Idealism, the argument from Inconsistent Ontological Commitments forces the reader to pick from either Extreme Idealism or Moderate Idealism since it is impossible to maintain both forms

⁴⁴³ Rutherford, 292-296.

⁴⁴⁴ Hartz, 202.

of interpretation simultaneously.⁴⁴⁵ Since Moderate Idealism is rife with its own obscurities, Extreme Idealism must be the best selection.

Catherine Wilson, Hartz writes, faces this same crossroads when reading Leibniz's later metaphysics while also observing that Leibniz himself is unaware of the metaphysical-logical inconsistency. Hartz writes: "He seems almost unaware of their logical incompatibility—probably because, as Catherine Wilson has argued persuasively, he was in love with two radically divergent pictures of the ultimate ground floor metaphysic."⁴⁴⁶ While tempted to subscribe to Extreme Idealism, Wilson decides to acknowledge the unsettling truth that Leibniz remains committed to two incompatible metaphysical systems: the simultaneous assertion that monads and corporeal substances exist.⁴⁴⁷ Hartz argues that this fundamental incompatibility should be interpreted as a sign that Leibniz himself is neither an Extreme Idealist nor a Moderate Idealist. Leibniz is fundamentally a philosopher who enjoys experimenting with a variety of theories without committing to any single theory.⁴⁴⁸ Leibniz *wants* to support a theory of composite substances, but cannot *provide* the reader with a sound theory of corporeal substances' unity without contradicting monadic unity or merely claiming that composite things are merely aggregates of monadic unity.

Hartz's rendition of Leibnizian "meta-incompatibilism" leaves the reader no choice to reach the painful conclusion that there is no definitive interpretation of Leibniz's later metaphysics regarding the existence of composite substances. Yet if the reader accepts the assumption that Leibniz's later metaphysics remains consistent with his middle metaphysics,

⁴⁴⁵ Hartz, 203.

⁴⁴⁶ Hartz, 204; Wilson, Catherine. "Reply to Cover's 1993 *Review of Leibniz's Metaphysics*". *Leibniz Society Review* 4. 1994. 5-8

⁴⁴⁷ Wilson, 7-8.

⁴⁴⁸ Hartz, 206.

then it would appear that Extreme Idealism is the only valid option in order to remain true to metaphysical and logical consistency.

The reader can understand this need for logical consistency upon reflection of a useful rule when determining which conclusion or interpretation is sound: a sound interpretation of any theory requires that the reader select the interpretation or conclusion that proves the least problematic regarding the assumptions of their author. Although Extreme Idealism necessitates the exclusion of certain textual information, it is the least problematic theory regarding explanatory power. Moderate Idealism is too inconsistent regarding the account of corporeal substances' unity, the result being that there is only monadic unity or being by aggregation.

Nevertheless, a Moderate Idealist or even a "Meta-Incompatibilist" like Hartz could object to such a position, claiming that an Extreme Idealist ignores all textual references to corporeal substances. Extreme Idealists are thereby "cherry-picking" Leibniz's texts for their own philosophical purposes.

In response, Extreme Idealists could simply say that that they acknowledge Leibniz's textual references to composite substances, but that such references are without merit. If Moderate Idealists and "Meta-Incompatibilists" acknowledge that Leibniz has unsuccessfully explained the unity of composite substances in his later metaphysics, that the very term "composite substance" contains no genuine metaphysical reference. Such non-genuine metaphysical references could merely be means by which Leibniz wishes to assuage a potential fear on the reader's part that the monadological world is radically inconsistent with the world of natural physics. Thus, I believe that Hartz has not fully argued for his "Meta-Incompatibilism" effectively due to this potential ground for the description of composite substances. Yet, even

this argument for Extreme Idealism falls short if there is a means by which to account for the unity of composite substances implicit in Leibniz's metaphysics.

I'd like to draw the reader's attention to a potential account of the unity of corporeal substances to which Hartz alludes and Andrew Stumpf articulates: analogical predication of the term "unity" in Leibniz's metaphysics. Hartz acknowledges this account of analogical predication of *per se* unity as a potential solution to the problem that Moderate Idealists face, but puts this possible solution down: "But all indications are that he used this phrase to mark out an absolute feature—one that was "all or nothing".⁴⁴⁹ Hartz does not cite directly any supporting texts for his claim that *per se* is univocally said of substance, relying instead on the *absence of evidence* for an analogical use of unity or substance; however, his claims here make some sense by virtue of the manner by which Leibniz argues for monads on the basis that such beings must be *purely* simple or else they could not be true (simple) substances. There is no alternative to the true form of the substances of nature, the monads, by virtue of their *per se* or monadic unity. Of course, an *absence of evidence* is not *evidence of absence*. Moreover, if Hartz truly believes what he says regarding *per se* unity, then how could he really buy into Leibniz's references to corporeal substances as genuine substances throughout the text? Despite the textual issues, if Hartz asserts a univocal form of *per se* unity of substances, then he cannot really regard corporeal substances as "substances". In which case, Hartz should really be an Extreme Idealist and merely "explain away" the references to corporeal substances. Andrew Stumpf critiques Hartz's views, taking issue with Hartz's "Meta-Incompatibilism".

⁴⁴⁹ Hartz, 204-205.

Andrew Stumpf argues that Hartz's theory of what I call "Meta-Incompatibilism" stems from a passage in the "Metaphysical Consequences of the Principle of Reason", in which Leibniz appears to contradict his statements regarding the existence of composite substances.⁴⁵⁰ Leibniz initially acknowledges simple and composite substances: "A substance is either simple, such as a soul, which has no parts, or it is composite, such as an animal, which consists of a soul and an organic body."⁴⁵¹ But he seems to contradict the possibility of composite substances: "And because an organic body, or any body whatsoever, can again be resolved into substances endowed with organic bodies, it is evident that *in the end there are simple substances alone*, and that in them are the sources of all things and of the modifications that come to things."⁴⁵² Coupled with a critique of Leibniz's theory of Pre-Established Harmony as a sufficient principle of unity for composite substances, and a lack of evidence for an analogical use of "substance" and "unity", Hartz can only conclude that Leibniz is fundamentally confused regarding the existence of composite substances in his later metaphysics.

Stumpf initially takes issue with Hartz's claim that the Principle of Pre-Established Harmony (PEH) cannot serve as a sufficient ground for the unity of composite substances. Monads independently exist from one another. Should there be a unity of monads, it could only be unity of aggregates, which cannot be *per se* unity.⁴⁵³ Stumpf objects to Hartz's contention that monads cannot be unified to form a *per se* composite unity: "The *PEH* is not an arbitrary effort on Leibniz's part to unify entities that are highly resistant to such unification."⁴⁵⁴ God

⁴⁵⁰ Stumpf, Andrew. "Harmonizing Leibniz's Ontology". *Dialogue*, Volume 51, Issue 03. September 2012. pp 467 – 483. 471

⁴⁵¹ Leibniz, Gottfried. "Metaphysical Consequences of the Principle of Reason." *Leibniz: Philosophical Writings*, ed. G. H. R. Parkinson, trans. M. Morris and G. H. R. Parkinson. Totowa: Rowman and Littlefield, 1975. 174-175.

⁴⁵² Leibniz, "Metaphysical...", 174-175.

⁴⁵³ Hartz, 203.

⁴⁵⁴ Stumpf, 471.

becomes the mediator in the unification of monads, which are ordered according to greater or lesser degrees of perfection.⁴⁵⁵ God recognizes this ordering of perfection according to the intrinsic nature of monads, in such a way to unify subordinate (less perfect) monads to their dominant (more perfect) monads. Thus, monads are unified *metaphysically* rather than *physically*. *Metaphysical unification* does not entail that monads interact causally with each other, but are linked through God according to their intrinsic degrees of perfection. Thus, Leibniz can retain PEH as a sufficient *per se* principle of unity for composite substances without forfeiting the independence of monads.

Assuming the validity of the claim that PEH is a sufficient *per se* principle of unity for composite substances, Stumpf articulates an analogical form of predication for “substance” and “unity” in Leibnizian thought. He acknowledges Hartz’s observation that Leibniz does not explicitly draw any formulate different senses of substance or unity: “I contend that even if Leibniz does not *explicitly* draw the distinction, the fact that he has it in mind is evident from its effects on his different metaphysical projects.”⁴⁵⁶ Stumpf’s method of interpretation is thereby based on inferring causes from certain effects. Since Leibniz applies the same term “substance” to corporeal things and to monads, it appears that “substance” cannot mean precisely the same thing but must be relatedly used in each instance. Granted, this inference is valid only if Leibniz is attempting to formulate a coherent metaphysics.

Stumpf begins his argument for the analogical predication of “substance” and “unity” with an overview of a similar interpretive issue that has cropped up in Aristotle’s usage of “substance” in the *Categories* and in the *Metaphysics*. Aristotle considers individual things to be

⁴⁵⁵ Stumpf, 471.

⁴⁵⁶ Stumpf, 474.

“primary substances” in the *Categories*, while the forms are the “primary substances” in the *Metaphysics* Book Z. This application of “primary substance” in the *Categories* and in the *Metaphysics* seems incoherent since Aristotle appears to exclude such references to forms in the *Categories* and to individuals in the *Metaphysics*.⁴⁵⁷ To resolve this difficulty, scholars like Wedin refer to a passage from *Metaphysics* D in which Leibniz refers to two senses of “substance”:

1. The ultimate subject, which is not further predicated of anything else.
2. Whatever, being a this, is also separable—such is each thing’s shape and form.⁴⁵⁸

The first sense describes “substance” as applied in the *Categories*, while the second sense describes “substance” as applied in the *Metaphysics* Book Z. Wedin calls the first sense *ontological* and the second sense *structural*, arguing that the second sense is more fundamental than the first sense since the form is the cause of the structure or unity of the ultimate subject.⁴⁵⁹ Thus, the use of the term “substance” indicates that something is *one thing*, but can be applied according to different senses of being *one thing*. An individual is a substance in a derivative sense because we cannot say that a subject is really more than *one thing*; a form is a substance in a primary sense because it makes an individual, derivative substance *one thing* without being another subject itself. Thus, Aristotle’s usage of “substance” appears incoherent at first glance; however, by noting that by tracing the usage of one sense of “substance” to the usage of another sense of “substance”.

⁴⁵⁷ Stumpf, 474

⁴⁵⁸ Aristotle *Metaphysics*, 1041a6

⁴⁵⁹ Wedin, M. *Aristotle’s Theory of Substance*. New York: Oxford University Press, 2000. 449

Stumpf compares the dominant monad to the form in Aristotle's *Metaphysics*, and composite substance to the individual in Aristotle's *Categories*.⁴⁶⁰ He begins the comparison by noting Leibniz's references to wanting to make his thoughts consistent with those of Aristotle:

“At first, when I had freed myself from the yoke of Aristotle, I was in favour of [material] atoms and the void, because this view best satisfies the imagination. But thinking again about this, after much meditation I saw that it is impossible to find the principles of a real unity in matter alone, or in what is only passive, since this is nothing but a collection or aggregation of parts ad infinitum... So, in order to get to these real unities I had to have recourse to a formal atom, since a material thing cannot simultaneously be material and perfectly indivisible, or possessed of a genuine unity. So it was necessary to recall and, as it were, to rehabilitate substantial forms, which are so much decried these days... I found, then, that the nature of substantial forms consists in force, and that from this there follows something analogous to feeling and desire; and that they must therefore be understood along the lines of our notion of souls.”

Assuming this desire to be consistent with Aristotle, it is possible to re-examine the passages Hartz cites as proof of his “Meta-Incompatibilism” and read the passages through the new lens of analogical predication. Stumpf cites this particular passage from Leibniz for reinterpretation:

“For even though every simple substance has an organic body which corresponds to it—otherwise it would not have any kind of orderly relation to other things in the universe, nor would it act or be acted upon in an orderly way—yet by itself it is without parts.”⁴⁶¹

Here we find that simple substances provide the organized relations needed for each composite body to interact with one another in the world of physics. The comparison Stumpf wants to draw here is that the simple substances (dominant monads) organize their composite bodies in the same way that forms organize the individuals as composite substances. Leibniz's statement that there are *in the end*, only simple substances is an affirmation that simple substances are the ultimate sources of the unity of composite substances.⁴⁶² He is not denying the existence of composite substances. Following the thought of Pratt, composite substances

⁴⁶⁰ Stumpf, 475

⁴⁶¹ Leibniz “Metaphysical Consequences...”, 174-175.

⁴⁶² Stumpf, 476

possess *integrity* rather than pure unity.⁴⁶³ *Integrity* is a form of unity that distinguishes composite substances from being mere aggregates: break apart an aggregate of units and the units will remain in existence; break apart a composite substances and the parts of the composite substance cannot remain in existence.⁴⁶⁴ Thus, we have Stumpf's analogical rendition of Leibnizian "unity" and "substance" borrowed from Aristotelian metaphysics.

Hartz might object that analogical predication cannot be what Leibniz has in mind for the *integrity* of composite substances is because there would be no reason for Leibniz to switch from the use of substantial forms in his middle years to monads in his later years. The term *monad*, borrowed from Henry More's *Enchiridion Metaphysicam*, conveys a sense that monads are separate from the existence and unity of other monads, being windowless souls.⁴⁶⁵ Analogical predication of "substance" and "unity" cannot be applied to monads and composite substances since the original or primary referent in the analogy, the monad, is radically independent of anything else. "Substance" and "unity" can only be applied univocally to monads and composite substances.

While Hartz would be right to point out the radical independence of monads, he cannot deny that there is a distinction and inherent order existent in each monad. Each monad is organized into dominant and subordinate monads—a matter of degrees. While the substantiality of composite substances cannot be accounted for by looking at each monad versus other monads simply as independent existences, it is possible to look at the inherent ordering of the perceptions of each monad to note the order and unity that each one expresses via PEH.

⁴⁶³ Stumpf, 476; Pratt, V. "One for Leibniz." *Sorites* 4: 10-20. 15-20

⁴⁶⁴ Stumpf, 477

⁴⁶⁵ Wilson, Catherine. *Leibniz's Metaphysics: A Historical and Comparative Study*. Princeton, NJ: Princeton University Press, 1989. 181.

In PEH the dominant monad organizes the unity of the subordinate monads that form the composite body. Such organization distinguishes the dominant and subordinate monads from being a mere aggregate whose unity is supplied through the mind in part because monads are non-spatial.⁴⁶⁶ Something that is spatial and distinct in its substance could only possess an aggregative unity in the same way that a person would note the unity of an aggregate of bricks, a “wall”. Since monads that count as a composite substance are substantially rather than accidentally unified, they must be distinguished from aggregates and be substances in a derivative sense.

Assuming that the dominant monad possesses unity *simpliciter*, while composite substances possess *integrity*, Hartz has not proven that Leibniz’s theory of substance in his later metaphysics succumbs to a deep incompatibilism. When articulating such an incompatibilist position, it pays to keep in mind the influence of past philosophers on a given philosopher. This note is especially valid given Leibniz’s attempt to bridge the new science with that of Aristotelain/Scholastic philosophy.

⁴⁶⁶ Cover, J.A. and Hartz, Glenn. “Are Leibnizian Monads Spatial?” *History of Philosophy Quarterly*. Volume 11. Volume 3. July 1994. 295-316. 296-297

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