Essays on International Asset Portfolios and Commodities Trade

Author: Marketa Halova

Boston College Electronic Thesis or Dissertation, 2012

Boston College

The Graduate School of Arts and Sciences

Department of Economics

ESSAYS ON INTERNATIONAL ASSET PORTFOLIOS AND COMMODITIES TRADE

a dissertation

by

MARKETA W. HALOVA

submitted in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

August 2012

ESSAYS ON INTERNATIONAL ASSET PORTFOLIOS AND COMMODITIES TRADE

MARKETA W. HALOVA

Dissertation Advisors: Christopher Baum, Fabio Ghironi and Georg Strasser

Do events in the natural gas market cause repercussions in the crude oil market? In light of the enormous impact that price movements in the two largest U.S. energy markets have on the economy, it is important to understand not just the individual markets but also how they relate to one another. On this front, the literature presents a puzzle: while economic theory suggests that the oil and gas markets are interlinked through a bi-directional causal relationship, empirical research has concluded that the oil market affects the gas market but not vice versa. The first chapter of this dissertation improves on the previous studies in two ways: by using high-frequency, intraday oil and gas futures prices and by analyzing the effect of specific news announcements from the weekly oil and gas inventory reports. The results dispel the notion of one-way causality and provide support for the theory. The reaction of the futures volatility and returns is asymmetric, although this asymmetry does not follow the "good news" vs. "bad news" pattern from stock and bond markets; the response depends on whether the shock is driven by oil or gas inventory gluts or shortages. The two-way causality holds not only for the nearby futures contract but also for contracts of longer maturities. These findings underscore the importance of analyzing financial markets in a multi-market context.

The second chapter of this dissertation asks whether volatility and trading volume evolve in a unidirectional or bidirectional, contemporaneous or lagged relationship in the crude oil

and natural gas futures markets. This question is important because it affects trading and government regulation but previous studies have come to conflicting conclusions. Their main shortcoming is the low frequency of data used in the analysis. This chapter improves on the previous studies in three ways: by using high-frequency, intraday oil and gas futures prices and volume, by including trading not only during the day but also during the night, and by analyzing not only the nearby futures contract but also contracts with longer maturities. For the nearby contract, Granger-causality tests show that past values of volume help explain volatility which agrees with the Sequential Information Arrival Hypothesis. Past values of volatility have explanatory power for volume only when absolute return is used as the volatility measure; when the conditional variance from GARCH models is used as the volatility measure, the causality in this direction disappears. These results change when low-frequency daily data is applied. It is also shown that the volatility-volume relationship differs for contracts with longer maturities. These findings are relevant for regulations, such as trader position limits recently adopted by the U.S. Commodity Futures Trade Commission.

The third chapter of this dissertation investigates whether the production structure of firms affects international optimal portfolios, risk-sharing, and response of terms of trade (TOT) to shocks. The answer to this question would enhance our understanding of the home equity bias, yet it has not been addressed in the theoretical literature. This chapter studies the question in a two-country dynamic stochastic general equilibrium model with endogenous portfolio allocation. It shows that the optimal portfolio includes more home equity as the production structure changes from exporter-only, i.e., firms operating in their home countries and serving foreign markets by exports, to multi-national-company-extreme (MNC), i.e., firms hiring labor in both countries and producing locally in both countries.

This shift occurs because changing the firms' production structure eliminates exposure to technology differences and allows the home household to accomplish the same diversification with less foreign equity. The production structure also has implications for the effect of technology shocks on the TOT. Under the exporter-only setup, a shock to technology causes a standard TOT deterioration, whereas under the MNC-extreme setup, a shock to technology leads to a TOT improvement. By producing testable predictions, this chapter underscores the need to take firms' production structure into account when analyzing international optimal portfolios, risk sharing, and response of the TOT to technology shocks. This is especially important since empirical research has generated conflicting results.

Acknowledgements:

The author thanks advisers Christopher Baum, Fabio Ghironi and Georg Strasser for their guidance, Donald Cox and Richard Tresch for their very helpful comments, and the Boston College Library, especially Barbara Mento, for assistance in obtaining the data.

`

Table of Contents

CHAPTER 1

GAS DOES AFFECT OIL: EVIDENCE FROM INTRADAY PRICES AND INVENTORY ANNOUNCEMENTS

1 Introduction

This paper studies the relationship between crude oil (referred to as "oil") and natural gas (referred to as "gas"). It investigates whether there is two-way causality as suggested by economic theory or one-way causality as evidenced by previous empirical studies.

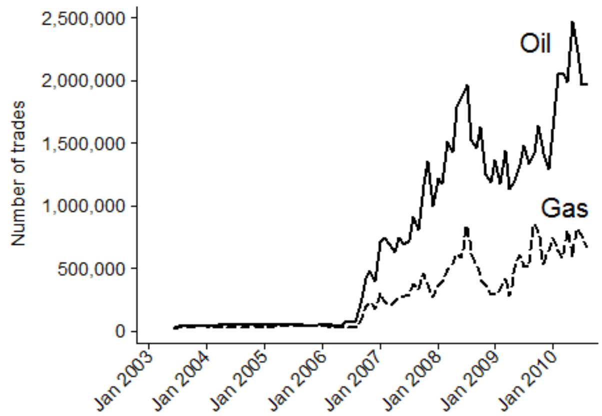
Price movements in markets for these two largest U.S. energy sources exert an enormous influence on the economy.[1] For example, Elder and Serletis (2010) show that oil price volatility has a negative effect on investment, durables consumption and aggregate output, and exacerbates the response of the economy to a negative oil price shock while dampening the response to a positive oil price shock. In addition, trading in the oil and gas futures markets has increased as shown by Figure 1, ranking the oil and gas futures as the first and the second largest energy futures, and the first and the ninth largest commodity futures by volume in 2008, respectively.[2] In light of this enormous impact of oil and gas on the macroeconomy as well as the financial markets, it is important to understand not just the markets for the individual commodities but also how they relate to one another.

On this front, the literature presents a puzzle. Economic theory suggests the existence

[1] In 2009, oil and gas accounted for 37% and 25% of the energy use in the U.S. See the EIA Annual Energy Review 2009, Table 1.3.

[2] Futures Industry Magazine Annual Volume Survey: 2008 A Wild Ride.

Figure 1: Number of futures trades by month



Notes: This figure shows the number of nearby futures trades executed on NYMEX by month.

Source: Tick Data, Inc.

of two-way causality between oil and gas markets for several reasons.[3] From the demand perspective, oil and gas are substitutes because a portion of both the power generation and industrial sectors has the ability to switch between gas and products refined from crude oil as the production input. Therefore, an increasing oil price resulting from rising demand will increase the price of gas as some firms in these sectors switch from oil to gas. The relative price of oil and gas also influences decisions to build new facilities designed to consume a certain type of fuel. From the supply perspective, an increasing oil price resulting from increasing demand may lead to increased drilling and production of gas to substitute away from the more expensive oil, causing an increase in gas supply with a subsequent decrease in gas price. The gas price may also fall because gas is frequently found in the same underground reservoirs as oil, so an increasing oil demand satisfied by a higher production of oil may result in a higher production of gas, reducing the gas price. At the same time, however, the increasing oil price may intensify competition for resources, such as drilling rigs,

[3] Villar and Joutz (2006) provide a summary of the economic theory.

production facilities, and engineering and operations staff used in exploration and production of both oil and gas, causing an increase in the gas price. In addition, the linkages between oil and gas prices should have strengthened because of the development of liquified natural gas technology (LNG), which allows the transportation of gas throughout the world and creates a world market for gas, a process that has accelerated since the 1990s.

However, the empirical research reviewed in Section 2 has concluded that the causality runs only in one direction: the oil market affects the gas market but not vice versa. Two explanations are usually stated. First, the oil market is larger than the gas market. Second, the oil market is global with prices determined by world supply and demand whereas the gas market is local with prices determined by local supply and demand. The first explanation does not seem satisfying since gas is almost as important an energy source as oil for the U.S. The second explanation also does not seem to hold because oil and gas are partial substitutes in the power generation and industrial sectors and the LNG technology has connected the local gas markets.

Since the conflict between theoretical and empirical research represents a puzzle with unsatisfying explanations, this paper investigates the contradiction. It improves on the previous studies in two ways. First, it uses high-frequency, *intraday* New York Mercantile Exchange (NYMEX) oil and gas futures prices, in contrast to observations taken at a lower frequency, such as the daily, weekly or monthly data used by the previous papers, that do not capture price interdependencies accurately. Second, it analyzes the effect of *specific* news announcements from the weekly oil and gas inventory reports published by the Energy Information Administration (EIA) of the U.S. Department of Energy instead of conducting a general analysis of the relationship between the oil and gas prices using, for example, Granger causality tests as the previous studies. Even though these inventory reports are the most watched weekly indicators in the oil and gas markets, their effects have not been fully analyzed.[4]

Three results follow from this analysis. First, the notion of one-way causality is dispelled, providing support for the bi-directional causality indicated by the theory for the first time. Gas inventory announcements affect oil futures volatility and return, and oil inventory announcements affect gas futures volatility and return. These cross-commodity effects turn out to be substantial. The effect of *gas* inventory announcements on *oil* price volatility is more than twice as strong as the effect of *oil* inventory announcements on *gas* price volatility, highlighting the importance of spillovers from the gas market to the oil market.

Second, the model allows for different reactions of price volatility and return to inventory gluts and shortages.[5] However, in contrast to the "good news" vs. "bad news" pattern found in stock and bond markets, the response differs by commodity and type of the shock. A glut has a larger impact than a shortage for the effect of *oil* inventory announcements on *oil* price volatility. In contrast, a glut has a slightly smaller impact than a shortage for the effect of *gas* inventory announcements on *oil* price volatility, which is also the case for the effect of *gas* inventory announcements on *gas* price volatility. Finally, a glut has the same effect as a shortage for the effect of *oil* inventory announcements on *gas* price volatility. This highlights the importance of analyzing the data in detail since a general analysis is unable

[4]For example, the Financial Times noted as of September 21, 2011: "The U.S. is the world's largest oil consumer and its weekly petroleum status reports are closely watched."

[5]Except for Gregoire and Boucher (2008), who analyze the effect of *gas* inventory announcements on *gas* price using daily data, previous papers have not analyzed the asymmetry between inventory gluts and shortages.

to uncover these differences.

Third, in contrast to previous studies that analyzed only the nearby futures contract for the following month, this paper includes contracts with all maturities to investigate how current news announcements affect financial instruments at various points in the future. The two-way causality indicated by the cross-commodity effect holds not only for the nearby contract but also for the following seven months' contracts. The asymmetry effect also persists across these maturities. The results described above are, therefore, robust to the maturity structure. In general, the coefficients decrease as the length of the contract increases, supporting the Samuelson (1965) theorem that futures contracts become more volatile as their expiration date approaches.

In addition to our validating the economic theories on the two largest U.S. energy markets, identifying precise linkages between these markets is important for several other reasons. Numerous studies have highlighted the importance of volatility spillovers between the oil market and financial markets, e.g., stock markets and exchange rate markets, a topic of interest to regulators and policy-makers.[6] To fully understand these spillovers, it is important to realize that the oil volatility partially originates in another energy market, i.e., the gas market.

Also, as documented by Buyuksahin, Haigh, Harris, Overdahl and Robe (2008) and Basu and Gavin (2011), the recent dramatic rise in oil and gas futures trading evidenced by Figure 1 is mainly due to an increasing number of financial institutions participating in commodity derivatives such as oil and gas futures. According to Buyuksahin et al (2008),

[6]See, for example, Kilian and Park (2009), Cifarelli and Paladino (2010), and Hammoudeh, Yuan and McAleer (2010).

the share of traditional commercial traders, such as oil and gas exploration, production, and refining companies who use the futures for commercial hedging decreased from 83% in 2000 to 55% in 2008. At the same time, according to Basu and Gavin (2011), non-commercial financial traders, such as investment banks, mutual funds, pension funds, university endowment funds and hedge funds, increased their investments in commodity derivatives, in an attempt to diversify their stock market risk and chase higher yield, higher risk investments. Since financial institutions played a key role in the recent economic crisis by investing in complex financial products such as derivatives, and had to be bailed out by the taxpayers, it is imperative to understand the markets in which they have an increasing exposure.[7]

Finally, this paper contributes to the discussion of whether oil prices are exogenous or endogenous to the macroeconomy. While earlier papers, e.g., Hamilton (1985), attribute oil price changes to exogenous events, such as political instability producing disruptions in oil supply, more recent studies, e.g., Barsky and Kilian (2002, 2004) and Kilian (2009), have challenged this view by showing that oil price changes can endogenously follow from demand changes.[8] This paper provides another piece of evidence that oil prices evolve in a bi-directional causal relationship instead of being exogenous. This improved understanding of the linkages between the oil and gas markets may provide insight into modeling dependencies of the macroeconomy on the energy markets. Krichene (2007) develops a model for analyzing

[7]Trading in oil and gas options based on oil and gas futures has also been on the rise, with financial institutions again becoming more dominant market players. Since the correct pricing of options relies on the ability to reliably estimate volatility of the underlying asset, i.e., that of the futures contract, it is again critical to understand how the volatility is partially driven by spillovers from another market.

[8]Hamilton (2008) and Kilian (2008) provide literature reviews of the relationship between oil prices and the macroeconomy.

the oil and gas markets with a role for monetary policy. However, in his simultaneous equations, the gas demand and supply are affected by oil but the oil demand and supply are not affected by gas. Such a misspecification may lead to misguided policies. These considerations underscore the importance of analyzing energy (and financial) markets in a multi-market context.

2 Literature Review

Although theory suggests a two-way relationship between the oil and gas markets, the empirical research by Pindyck (2004), Villar and Joutz (2006), Asche, Osmundsen and Sandsmark (2007), Brown and Yucel (2007) building on an earlier paper by Brown (2005), and Hartley, Medlock and Rosthal (2008) concluded that the causality runs only in one direction: the oil market affects the gas market but not vice versa.[9] Onour (2009) also noted that the gas price is affected by the oil price, especially at low oil prices (below $40 per barrel), but did not investigate the sensitivity of oil to gas. See Table 9 in Appendix A for an overview of the data used in these papers.[10]

[9]Pindyck (2004) conducted Granger causality tests using the conditional volatility from GARCH models. When estimating the GARCH model with daily data, he concluded that the oil market affected the gas market but not the other way around which he said was the expected result. When estimating the GARCH model with weekly data, he obtained the opposite result but disregarded it, arguing that correlations between the oil and gas volatilities are low in the weekly model.

[10]Several other papers have studied the relationship between oil and gas markets without analyzing causality. Yucel and Guo (1994) showed that oil, gas and coal prices were cointegrated from 1974 through 1990 although only oil and coal prices were cointegrated from 1947 through 1974. Panagiotidis and Rutledge (2007) also found a cointegrating relationship between oil and gas prices, while Serletis and Rangel-Ruiz

While the above papers claim that the gas market does not affect the oil market, this study uncovers evidence of such a relationship. It differs from the previous empirical papers in two respects. First, it takes advantage of NYMEX *intraday* oil and gas futures prices, i.e., prices at a higher frequency than the daily, weekly or monthly prices used by the previous papers. Second, in contrast to the previous papers that conduct a general analysis of the relationship between the oil and gas prices using, for example, Granger causality tests, this paper analyzes the effect of specific news announcements: the weekly oil and gas inventory reports published by the EIA.

Four papers have analyzed the effect of inventory announcements on intraday energy prices. Linn and Zhu (2004) studied the effect of *gas* inventory announcements on intraday *gas* futures price return and volatility from 1999 through 2002 and concluded that volatility increased after inventory announcements. However, they included only the actual inventory level. This is an issue because the efficient market theory states that only the unanticipated component of news announcements matters because the anticipated component has been built into price forecasts by rational economic agents. Therefore, the correct methodology

---

(2004) reported that the oil and gas relationship had weakened and Bachmeier and Griffin (2006) concluded that the oil, gas and coal markets were only very weakly integrated. Wang, Wu and Yang (2008) showed that the correlation between oil and gas prices did not exhibit long-memory, implying a weak relationship in the long run. Marzo and Zagaglia (2008) noted that the correlation between the oil and gas prices has been rising since 2000 although this correlation was low on average over two thirds of the sample period. See Table 10 in Appendix A for an overview of the data used in these papers. Also, related papers studied relationships between other commodities. Serletis and Herbert (1999) focused on daily gas, fuel oil, and power prices from 1996 through 1997, while Ohana (2010) analyzed daily spot gas and heating oil futures prices from 2000 through 2009.

for analyzing the impact of news announcements on prices requires that market expectations be subtracted from the actual value to arrive at the unanticipated component.

Gay, Simkins and Turac (2009) investigated whether investors learned about analyst accuracy using *gas* inventory announcements and *gas* futures prices from 1997 through 2005. Focusing only on futures return and disregarding price volatility, they concluded that the oil futures return reacted to analyst forecasts.

Bjursell, Gentle and Wang (2009) considered, among other things, the effect of *oil* inventory announcements on *oil* intraday volatility and the effect of *gas* inventory announcements on *gas* intraday volatility. Instead of subtracting the market expectations from the actual values to arrive at the unexpected component, they estimated the "surprise" based on historical inventory levels. They concluded that this "surprise" increased volatility in gas and heating oil prices.

Chang, Daouk and Wang (2009) investigated whether investors learn about analyst accuracy using *oil* inventory announcements and *oil* futures prices from 2003 through 2005. Focusing only on price return and disregarding price volatility, they concluded that the oil price return rises (falls) when analysts forecast a decrease (increase) in supplies.[11]

None of these four papers on the effect of inventory announcements focused on the cross-commodity effect, i.e., the effect of *gas* inventory announcements on the *oil* price and the effect of *oil* inventory announcements on the *gas* price. Also, these papers did not distinguish

[11] Three of the above papers on the causal relationships between the oil and gas markets allowed inventory to play a limited role. Villar and Joutz (2006), Brown and Yucel (2007), and Hartley, Medlock and Rosthal (2008) included the gas inventory but ignored the oil inventory. More importantly, they included only the actual inventory, i.e., they failed to subtract market expectations of the inventory.

between positive and negative surprises, which prevented them from discussing asymmetries between effects of inventory shortages and gluts. Finally, they focused only on the nearby futures contract, ignoring the effect on contracts with longer maturities.[12]

3 Data

3.1 Oil and Gas Futures Prices

Oil and gas futures contracts are traded on two main exchanges: the NYMEX in Chicago and the InterContinental Exchange (ICE) in London. This paper focuses on the NYMEX futures data because NYMEX is more liquid than ICE.[13] This proprietary data is provided by Tick Data, Inc., a company that specializes in intraday time series data for equities, futures and options.[14] The data are transaction data, i.e., not bid-ask quotes.

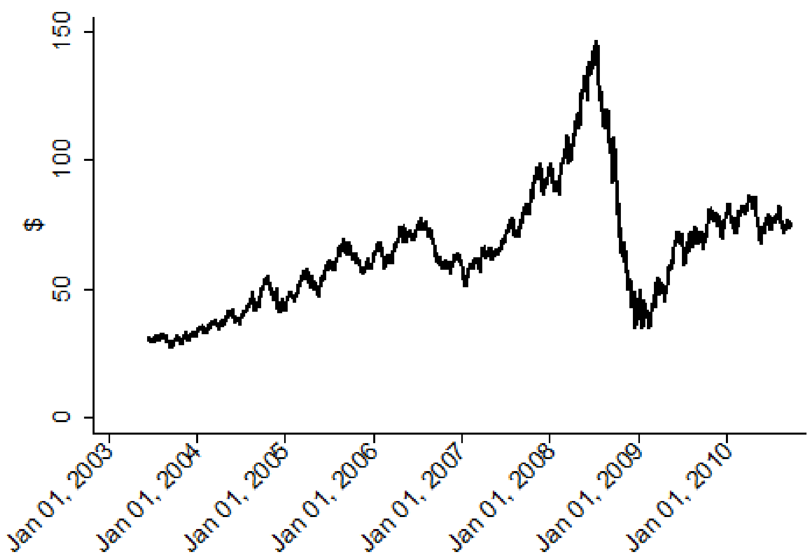
[12]Seven papers have analyzed the effect of other news announcements on oil prices. Ghura (1990) focused on the impact of five U.S. macroeconomic announcements, exchange rates and interest rates on price returns of oil and nineteen other commodities using daily prices from 1985 through 1989. Kilian and Vega (2008) conducted a more comprehensive study by analyzing the impact of thirty U.S. macroeconomic announcements on price returns of oil and gasoline using daily data from 1983 through 2008. Roache and Rossi (2009) analyzed the impact of thirteen U.S. and European announcements on price returns and volatilities of oil and eleven other commodities using daily data from 1997 through 2009. Guidi, Russell and Tarbert (2006), Schmidbauer and Rosch (2009), and Lin and Tamvakis (2010) studied the effect of OPEC decisions while Leinert (2010) considered the effect of unanticipated discoveries of giant oil fields. Three papers analyzed the effect of other announcements on gas prices: Ates and Wang (2007), Mu (2007), and Gregoire and Boucher (2008).

[13]See Downey (2009) for description of the oil market.

[14]See www.tickdata.com.

Light Sweet Crude Oil futures started trading on NYMEX under the symbol CL on March 30, 1987. Currently, the trading day takes place on Mondays through Fridays from 9:00 a.m. till 2:30 p.m. Eastern Time.[15] The standardized contract is for 1,000 barrels with the price quoted in U.S. dollars per barrel. Futures contracts with different maturities are available. The shortest maturity, called the nearby contract, is for the following month. The longest maturity is nine years. Trading ceases on the third business day prior to the twenty-fifth calendar day of the month preceding the delivery. At expiration, oil has to be physically delivered to Cushing, OK.[16] Figure 2 presents the oil nearby contract price from 2003 to 2010.

Figure 2: Crude oil futures price



Notes: This figure shows the price of the NYMEX crude oil nearby futures contract per barrel in U.S. dollars.

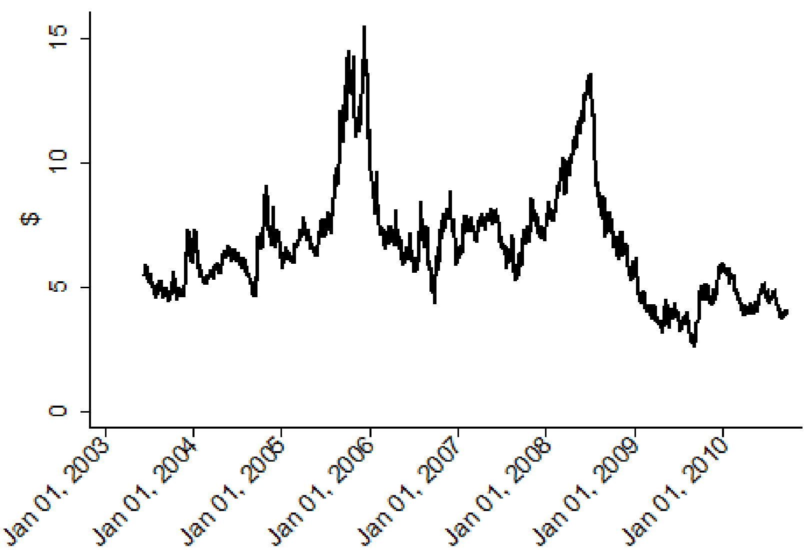
Source: Tick Data, Inc.

[15]Until January 31, 2007, the trading day started at 10:00 a.m. Night trading is not analyzed in this paper since the day and night trading sessions may differ from the information arrival standpoint.

[16]See website of CME Group that owns NYMEX: www.cmegroup.com.

Natural gas futures started trading on NYMEX under the symbol NG on January 4, 1993. The trading hours are identical to those for oil. The standardized contract is for 10,000 million British thermal units (mmBTU) with the price quoted in U.S. dollars per mmBTU. Maturities from one month to twelve years are available. Trading ceases three business days prior to the first day of the delivery month. At expiration, gas has to be physically delivered to Henry Hub, LA. Figure 3 presents the gas nearby contract price from 2003 to 2010.

Figure 3: Natural gas futures price



Notes: This figure shows the price of the NYMEX natural gas nearby futures contract per million British thermal units in U.S. dollars.

Source: Tick Data, Inc.

Very few market participants choose to make physical delivery at contract expiration. Instead, they roll over their positions into a new contract. Two methodologies are used in this paper to create a continuous record of the futures contract prices. In the first methodology, the current contract is used until the expiration date. However, as market participants begin rolling over their positions on the preceding days, trading may be thin during the last days before the contract expiration date, causing unusual price movements. Therefore, the second methodology switches to the next contract as soon as its daily contract volume exceeds the current contract volume. The results do not materially differ between the two methods. Therefore, only results using the expiration date method are reported.[17]

3.2 Weekly Petroleum Status Report

The data on the U.S. oil inventory come from the Weekly Petroleum Status Report published by the EIA. This report is prepared by the EIA based on companies submitting weekly forms stating their current oil inventory, as mandated by law.[18] The first report was published on August 20, 1982. The report is released weekly on Wednesday at 10:30 am for the week ending on the previous Friday. If Monday, Tuesday or Wednesday fall on a public holiday, the report is released on the following Thursday at 11:00 am.[19] See Appendix B for a discussion of the adjustment to holidays. Only commercial inventory is considered in this paper, i.e., the Strategic Petroleum Reserves held by the U.S. government are excluded. The data are in thousands of barrels.

[17]Another method chooses a particular date, e.g., the 15th calendar day of each month to switch from the nearby contract to the second nearby contract in an attempt to model the market participants' behavior. This approach is not used in this paper because it is arbitrary.

[18]See www.eia.doe.gov.

[19]These holiday weeks are skewing the intraday pattern graphs in Sections 5.1.1 and 5.2.1. The regressions are correct because the data show the inventory reports on the days and intervals when they are actually released. As a robustness check, however, these weeks are eliminated from the data. The results do not change materially. The results reported in this paper include the holiday weeks.

3.3 Weekly Natural Gas Storage Report

The data on the gas inventory in the the U.S. Lower 48 states come from the Weekly Natural Gas Report published by the EIA based on companies submitting weekly forms stating their current gas inventory, as mandated by law.[20] The first report was published on December 31, 1993. The report is released weekly on Thursday at 10:30 am for the week ending on the previous Friday unless Thursday falls on a public holiday. There are twelve such holiday weeks in the sample period. It is assumed that the report was released at 10:30 am on the following Friday during these holiday weeks.[21] See Appendix B for a discussion of the adjustment to holidays. The data are in billions of cubic feet.

3.4 Expectations of the Weekly Petroleum Status Report

Bloomberg conducts a weekly survey of approximately twenty industry experts asking them what they expect the oil inventory excluding the Strategic Petroleum Reserves to be once released by the EIA, and publishes the minimum, maximum, mean and median values of the expert forecasts (in thousands of barrels). This paper uses the median forecast.[22] The

---

[20]See www.eia.doe.gov.

[21]The American Gas Association (AGA) published this report prior to the end of April of 2002. The AGA released the report on Wednesdays after NYMEX market closing time from December 31, 1993 until February 25, 2000 and between 2:00 pm and 2:15 pm from March 3, 2000 until April 26, 2002. The EIA took over on May 3, 2002. However, as this study is limited to the period from June 13, 2003 until September 24, 2010, it is not affected by the change of the release day from Wednesday to Thursday.

[22]The data can be found on the Bloomberg terminal under the code ECO with "North America" under Region, "U.S." under Country, and "10) Energy/Commodities" under Type. The list of economists surveyed, including the date of the survey, can be obtained by typing in "12".

survey is published on Monday or Tuesday, i.e., prior to the actual values being released by the EIA. The first survey was conducted on June 13, 2003.[23] As explained in Section 4, the "surprise" is calculated as the difference between the EIA inventory announcement and the Bloomberg survey forecast.

3.5 Expectations of the Weekly Natural Gas Storage Report

Bloomberg conducts a weekly survey of approximately twenty-five industry experts asking them what they expect the gas inventory to be once released by the EIA (in billions of cubic feet), using the same methodology as for oil. The first survey was conducted on May 3, 2002.

4 Methodology

Deciding on the length of the intraday time interval involves choosing between noise due to the data microstructure and loss of information. Using the volatility signature plot that shows the scaled realized volatility, i.e., daily average of squared returns, against time intervals in multiples of one minute as outlined by Andersen, Bollerslev, Diebold and Labys (2000), the 10-minute interval is chosen as the appropriate length. See Appendix C for volatility signature plots for oil and gas prices.[24] For example, the first 10-minute interval

[23] Note that Bloomberg asks the experts by how much they expect the inventory to change compared with the previous week. Therefore, to arrive at the expected inventory, it is necessary to add the previous week's actual inventory and the experts' expectation of the change in inventory.

[24] See Dacorogna, Gencay, Muller, Olsen and Pictet (2001) for a discussion of scaling factors. Also, note that the realized volatility is used only to choose the appropriate interval. It is not used in the regressions. The dependent variable in the volatility regressions is defined as the absolute return.

contains trades from 9:00:00 till 9:09:59.[25] There are 27 and 33 intervals within a trading day depending on whether the market opens at 10 a.m. or 9 a.m. The days when the NYMEX market closes earlier, usually due to an upcoming holiday, are eliminated to prevent skewing intraday patterns.[26] Missing prices are set equal to the previous prices.[27] The resulting sample period contains 54,884 10-minute intervals on 1,826 days from June 13, 2003 until September 24, 2010, a period of 380 weeks.

Let P_j be the price at the end of 10-minute interval *j*. Since both oil and gas price series are non-stationary, the return, i.e., the difference between the log price at the end of interval *j* and the log price at the end of interval *j-1*, $R_j \equiv \ln(P_j) - \ln(P_{j-1})$, is used. The resulting return series is stationary as gauged by an augmented Dickey-Fuller test.

This paper analyzes the effect of announcements on price returns as well as volatility, defined as the absolute value of the returns, $|R_j|$, following, for example, Ding, Granger and Engle (1993), Ederington and Lee (1993), Gwilym, McMillan and Speight (1999), McKenzie (1999), Bollerslev, Cai and Song (2000), and Ederington and Guan (2005). See Figures 4 and 5 for oil and gas absolute price returns, respectively.

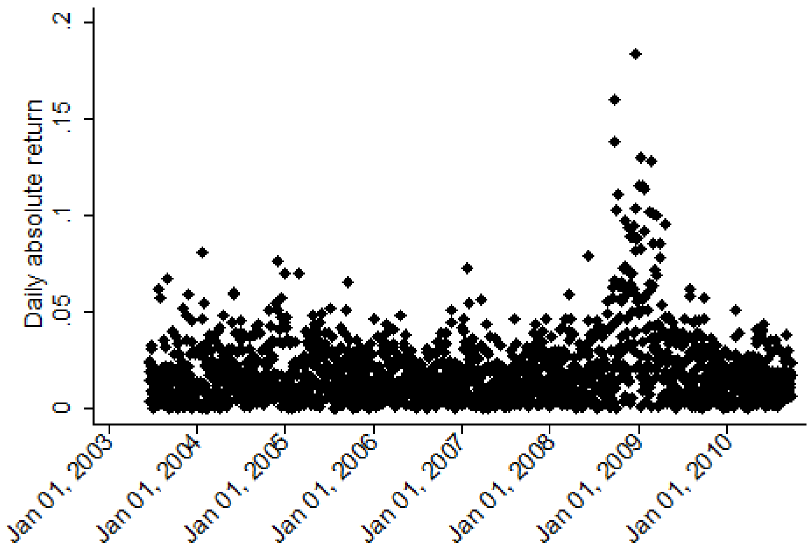
The methodology for analyzing the effect of news announcements on prices assumes efficient markets, implying that only the unanticipated component of news announcements matters because the anticipated component has been built into market participants' price forecasts. The unexpected component is the difference between the actual value, A_{kj}, and

[25] As a robustness check, the analysis is repeated using 15-minute and 30-minute intervals. The results do not change materially.

[26] There are only 20 such days within the sample period.

[27] There are very few such missing observations (0.20% and 0.45% of all observations for the oil and gas nearby contracts, respectively).

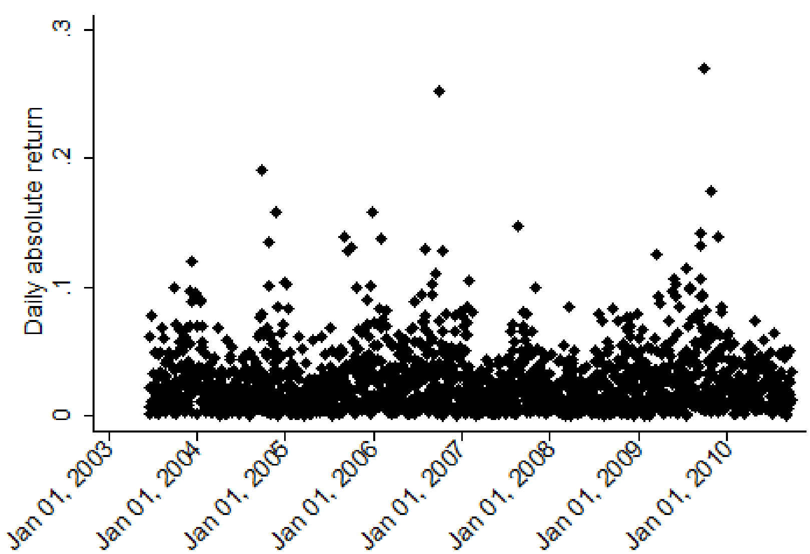
Figure 4: Crude oil futures price volatility



Notes: This figure shows the price volatility of the NYMEX crude oil nearby futures contract defined as absolute return.

Source: Tick Data, Inc.

Figure 5: Natural gas futures price volatility



Notes: This figure shows the price volatility of the NYMEX natural gas nearby futures contract defined as absolute return.

Source: Tick Data, Inc.

the expected value, E_{kj}, where $k \in \{O, G\}$ stands for oil and gas announcements. Since this difference is in thousands of barrels and billions of cubic feet for oil and gas, respectively, it is divided by the actual value and then multiplied by 100. The resulting "surprise", $S_{kj} \equiv \frac{A_{kj}-E_{kj}}{A_{kj}} \times 100$, can then be interpreted as a percentage deviation of the expectations from the actual values.[28] Statistics for the oil inventory surprise, S_O, and the gas inventory surprise, S_G, are summarized in Table 1. Since the mean values of these variables are close to zero, the Bloomberg survey can be considered unbiased, i.e., the analysts do not systematically overforecast or underforecast.

Table 1: Summary statistics for oil and gas inventory surprise variables

	Min	Max	Mean	Standard Deviation
Oil inventory surprise, S_O	-2.85%	2.82%	-.02%	.94%
Gas inventory surprise, S_G	-1.55%	2.93%	.05%	.46%

A positive surprise, $S_{kj} > 0$, i.e., $A_{kj} > E_{kj}$, means that the actual inventory, when announced by the EIA, is higher than what the analysts expected. In other words, the analysts underforecast the actual inventory resulting in an inventory glut. A negative surprise, $S_{kj} < 0$, i.e., $A_{kj} < E_{kj}$, means that the actual inventory, when announced by the EIA, is

[28] Balduzzi, Elton and Green (2001) implement another methodology for standardizing announcement units. They divide the difference between the actual and expected values by its sample standard deviation σ_k and interpret the coefficient as the change in oil price return for one standard deviation change in the surprise. In this paper, dividing by the actual value is preferred to allow for interpreting the surprise as a percentage deviation of the expectation from the actual value.

lower than what the analysts expected. In other words, the analysts overforecast the actual inventory, resulting in an inventory shortage.

To allow for the possibility of asymmetric reaction of the price return and volatility to shortages and gluts, indicators, $I(S_{kj} > 0)$, are created for oil and gas that take on value of 1 if $S_{kj} > 0$ and 0 otherwise. These indicators are then multiplied by the surprise, i.e., $S_{kj} \times I(S_{kj} > 0)$. This allows interpreting the coefficient on the surprise, S_{kj}, as the effect of a shortage. The sum of the coefficients on S_{kj} and $S_{kj} \times I(S_{kj} > 0)$ can then be interpreted as the effect of a glut.

The effect of the announcements is analyzed using ordinary least squares (OLS) regression. Several control variables are included in the regressions in addition to the above surprise variables. As suggested by Andersen, Bollerslev, Diebold and Vega (2003), lags of surprise and dependent variable are added to allow for autocorrelation.[29]

A beginning-of-day dummy is included to account for unusual price movements at the beginning of the day and to account for the change in the beginning of the trading day from 10:00 to 9:00 on February 1, 2007. This dummy takes on the value of 1 during the first interval of the day and 0 in all other intervals. An end-of-day dummy is included in the same way to account for unusual price movements at the end of the day. These time-of-the-day effects have been identified in many financial markets, for example by Becker, Finnerty and Kopecky (1993), Gwilym, McMillan and Speight (1999), Bollerslev, Cai and Song (2000), and Linn and Zhu (2004).[30] A first-trading-day dummy is included that takes on the value

[29] 33 lags are used since a trading day includes 33 10-minute intervals. In an alternative specification, the model is modified to include lagged cross-commodity terms, i.e., the *oil* price volatility (return) equations include *gas* price volatility (return) lags. The results do not change.

[30] Alternative specifications are run where the beginning-of-day (end-of-day) dummy took on the value of

of 1 in all intervals on the day after a non-trading day, i.e., after a weekend or a holiday, to allow for effects due to the market being closed for an extended period of time. A trader composition variable, defined as the ratio of non-commercial financial traders volume to the traditional commercial traders volume, is added to account for a change in the composition of firms trading oil futures, as documented by Buyuksahin et al (2008). These authors show that the proportion of non-commercial financial traders has been on the rise and the proportion of traditional commercial traders has declined. The three-month Treasury bill rate is included because Pindyck (2004) shows that this interest rate helps explain the oil return because it affects the cost of holding inventory.

Trading volume is added to account for various unobservable sources of volatility. One unit of volume represents 1,000 executed contracts. Disagreement exists on whether contemporaneous or lagged volume should be used. Foster (1995) concluded that oil volatility and volume are contemporaneously correlated and driven by the same factors, assumed to be information. He showed that lagged volume explains current volatility which means that lagged volume rather than contemporaneous volume should be used as a control variable to avoid endogeneity issues. Kocagil and Shachmurove (1998) corroborated this by providing evidence of two-way Granger causality between oil volatility and volume. Fujihara and Mougoue (1997) argued that volume is a significant explanatory variable for oil volatility even though factors other than volume affect the persistence of volatility. In contrast, Girard, Sinha and Biswas (2008) concluded that volume not only leads oil and gas volatility but is exogenous to it, which implies that it can be used for forecasting volatility and as a proxy for the flow of information, suggesting contemporaneous volume *can* be included in

1 for the first (last) two and three intervals. The results do not change.

the regressions. For gas, Herbert (1995) concluded that lagged volume influences current price volatility but lagged price volatility has much less of an influence on current volume. Since the previous papers disagree on whether contemporaneous or lagged volume should be used as a control variable in the volatility regression, both specifications are implemented. The results are very similar. The lagged volume specification is reported in this paper.[31]

Control variables for gasoline inventory, distillate fuel oil (referred to as "distillate") inventory and refinery utilization are added because these data are included in the Weekly Petroleum Storage Report.[32] Since these data are released at the same time as the oil inventory data, their announcements could possibly skew the results. There are no other weekly announcements made by any U.S. government agency which coincide with the oil and gas inventory announcements.[33]

For the futures *return*, the regression is:

$$R_j = \alpha + \Sigma_{i=1}^{I}\beta_i R_{j-i} + \Sigma_{k=1}^{K}\Sigma_{l=0}^{L}\gamma_{kl}S_{k,j-l} + \Sigma_{k=1}^{K}\Sigma_{l=0}^{L}\delta_{kl}S_{k,j-l} \times I(S_{k,j-l} > 0) + \Sigma_{m=1}^{M}\theta_m\{Z_m\} + \varepsilon_j, \quad (1)$$

where $\{k\}$ includes oil, gas, gasoline, distillate and refinery utilization, i and l stand for

[31] In an alternative method of accounting for various unobservable sources of volatility, the Chicago Board Options Exchange Volatility Index (VIX) is included as a measure of the stock market volatility and investor sentiment in general. The results do not change.

[32] Distillate includes diesel fuels used in on-highway engines, e.g., automobiles and trucks, and off-highway engines, e.g., railroad locomotives and agricultural machinery, as well as fuel oils used for heating and power generation. Refinery utilization is defined as the ratio of gross inputs used in the atmospheric oil distillation units to the operable capacity of the units. For more detail, see Definitions of Petroleum Products and Other Terms on www.eia.doe.gov.

[33] The Weekly Petroleum Storage Report includes other data, e.g., propane inventory and jet fuel inventory. However, Bloomberg does not conduct surveys for these commodities. Therefore, they cannot be included in the regression. It is unlikely they would have a major effect on the results of this paper.

lags of returns and surprises, respectively, and $\{Z_m\}$ includes dummies for the beginning-of-day, end-of-day and the first-trading-day, and controls for trader composition, three-month Treasury bill rate, and lagged volume.

Similarly, for the futures *volatility*, the regression is:

$$|R_j| = \alpha + \Sigma_{i=1}^{I}\beta_i|R_{j-i}| + \Sigma_{k=1}^{K}\Sigma_{l=0}^{L}\gamma_{kl}S_{k,j-l} + \Sigma_{k=1}^{K}\Sigma_{l=0}^{L}\delta_{kl}S_{k,j-l} \times I(S_{k,j-l} > 0) + \Sigma_{m=1}^{M}\theta_m\{Z_m\} + \varepsilon_j. \quad (2)$$

Newey-West standard errors are used to account for heteroskedasticity and autocorrelation in the error term ε_j.[34]

Since the oil and gas prices exhibit time-varying volatility depicted in Figures 4 and 5, generalized autoregressive conditional heteroskedasticity (GARCH) models lend themselves as tools for analyzing the data in addition to the OLS. Four specifications are presented. First, the GARCH(1,1) model that includes one ARCH term and one GARCH term with the Gaussian distribution of errors is used as a benchmark:

$$R_j = \alpha + \Sigma_{i=1}^{I}\beta_i R_{j-i} + \Sigma_{k=1}^{K}\Sigma_{l=0}^{L}\gamma_{kl}S_{k,j-l} + \Sigma_{k=1}^{K}\Sigma_{l=0}^{L}\delta_{kl}S_{k,j-l} \times I(S_{k,j-l} > 0) + \Sigma_{m=1}^{M}\theta_m\{Z_m\} + \varepsilon_j, \quad (3)$$

$$h_j^2 = \nu + \phi h_{j-1}^2 + \zeta\varepsilon_{j-1}, \quad (4)$$

where equation (3) is the mean equation, equation (4) is the conditional variance equation, and the distribution of the error conditional on an information set at time j, Ψ_j, is assumed to be $\varepsilon_j|\Psi_j \sim N[0, h_j^2]$.

Second, a GARCH(1,1) model with the t distribution is implemented to account for the distribution's heavy tails. Third, a GARCH-in-mean(1,1) model with the Gaussian distribution that lets the expected value of the return depend on its conditional variance is

[34] Note that the above regressions do not suffer from endogeneity due to a two-way relationship between oil prices and oil inventory because the inventory variable on the right-hand side is not the actual inventory; it is the *unanticipated* component of the inventory changes. As such, it is not a decision variable of the market participants.

used. The mean equation then becomes:

$$R_j = \alpha + \Sigma_{i=1}^{I} \beta_i R_{j-i} + \Sigma_{k=1}^{K} \Sigma_{l=0}^{L} \gamma_{kl} S_{k,j-l} + \Sigma_{k=1}^{K} \Sigma_{l=0}^{L} \delta_{kl} S_{k,j-l} \times I(S_{k,j-l} > 0) + \Sigma_{m=1}^{M} \theta_m \{Z_m\} + \psi h_j^2 + \varepsilon_j. \quad (5)$$

Finally, an EGARCH(1,1) model with the Gaussian distribution that allows for an asymmetric reaction to positive and negative innovations is implemented with the conditional variance equation:

$$\log(h_j^2) = \nu + \phi \log(h_{j-1}^2) + \omega \left| \frac{\varepsilon_{j-1}}{\sqrt{h_{j-1}}} \right| + \rho \frac{\varepsilon_{j-1}}{\sqrt{h_{j-1}}}, \quad (6)$$

where the term $\rho \frac{\varepsilon_{j-1}}{\sqrt{h_{j-1}}}$ captures the asymmetry because positive innovations $\varepsilon_j > 0$ are allowed to have different effects on the conditional variance than negative innovations $\varepsilon_j < 0$.

The above regressions are run for the futures nearby contract as well as the contracts of longer maturities to investigate how current news announcements affect financial instruments at various points in the future.

5 Results

5.1 Oil Price Volatility

5.1.1 Intraday Pattern Graphs

Figure 6 shows the intraday pattern of oil price volatility.[35] In panel a), absolute returns for each 10-minute interval are averaged across all days in the data sample. In addition to a U-

[35]The intraday pattern graphs do not display the first interval to avoid skewing the graphs by the overnight gap as the first interval is affected by not only by the first ten minutes of the trading day but also the period since the market closed on the previous day.

shaped pattern due to the market opening and closing, one feature stands out.[36] The 10:40 a.m. interval shows a spike in volatility. Panels b), c) and d) display the intraday pattern of volatility by day. In panel b), the absolute returns for each interval are averaged only for Mondays. While the U-shaped pattern is still visible, the 10:40 a.m. spike disappears. The graphs for Tuesdays and Fridays, not shown in this paper, look similar. Panel c) shows the intraday volatility pattern for Wednesdays, i.e., the day when the Weekly Petroleum Storage Report is released at 10:30 a.m. As documented by Bjursell, Gentle and Wang (2009), the 10:40 a.m. spike is due to this oil inventory news announcement. However, panel d) for Thursdays shows a similar pattern with a smaller magnitude. This paper hypothesizes that the 10:40 a.m. spike is due to the release of the Weekly Natural Gas Storage Report at 10:30 a.m. This cross-commodity effect, suggesting that the gas market influences the oil market, has not been observed in previous research.[37]

5.1.2 Cross-Commodity Effect

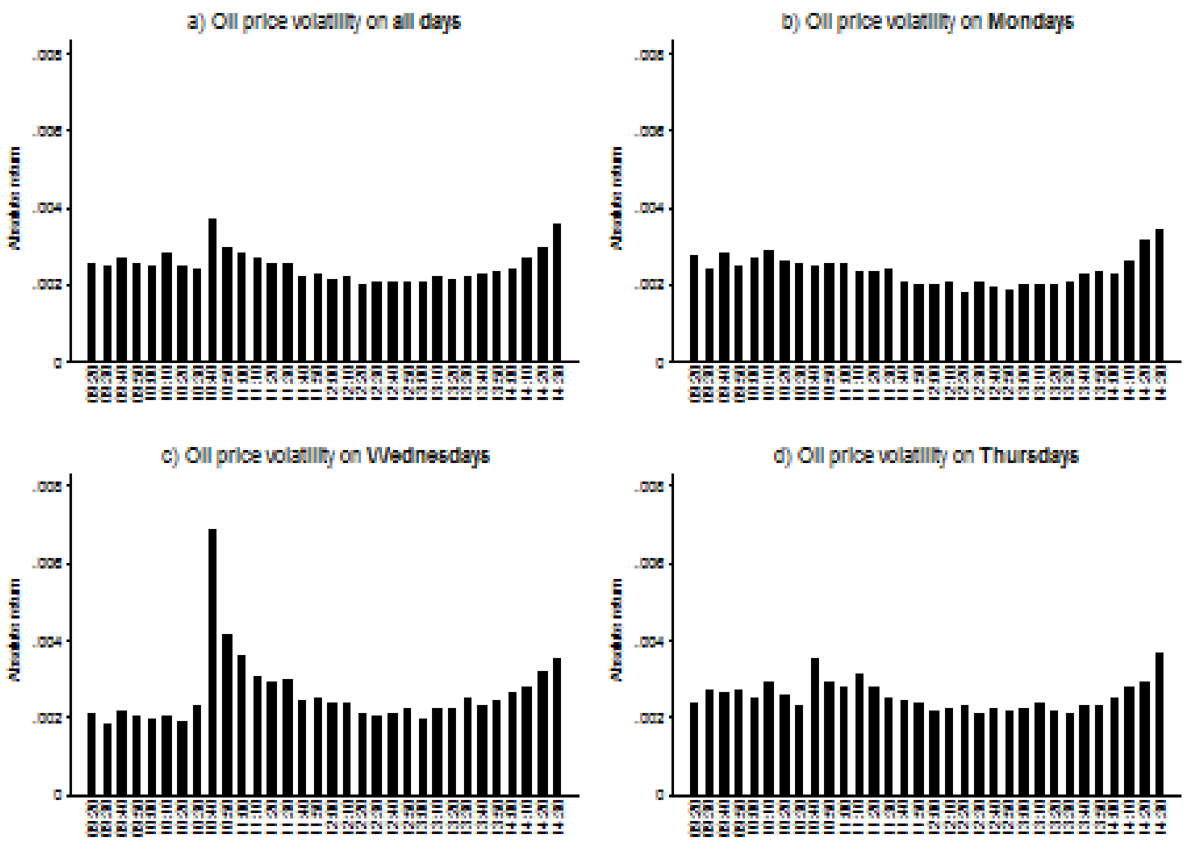
Table 2 shows regression results for the price volatility of the oil nearby futures contract. Specification (1) includes the oil inventory variables as well as control variables that have been used in other papers to explain the oil price volatility, as described in Section 4.[38]

[36] The fact that the data sample includes the period from June 13, 2003 till January 31, 2007 when the market opened at 10 a.m. as well as the period from February 1, 2007 till September 24, 2010 when the market opened at 9 a.m. creates a "double-U" because the volatility is higher after 9 a.m. and after 10 a.m.

[37] The smaller spike on Thursdays in the 11:10 a.m. interval is due to the Weekly Petroleum Storage Report being released on Thursdays at 11:00 a.m. if Monday, Tuesday or Wednesday fall on a holiday, as discussed in Section 3.2.

[38] The first-trading-day dummy, refinery utilization variables, and lags of inventory surprises are not significant, so the specification reported in this paper excludes them.

Figure 6: Intraday pattern of oil price volatility



Notes: This figure shows the intraday pattern of the NYMEX crude oil nearby contract futures price volatility. The volatility is defined as the absolute return. The first interval is not displayed to avoid skewing the graphs by the overnight gap. Source: Tick Data, Inc.

The oil glut variable stands for the situation in which $S_{kj} > 0$, i.e., $A_{kj} > E_{kj}$, and the oil shortage variable stands for the situation in which $S_{kj} < 0$, i.e., $A_{kj} < E_{kj}$. The coefficient of -.00425 on the oil shortage means that the analysts overforecasting the actual inventory by 1% leads to an .00425 increase in volatility.[39] The coefficient of +.00626 on the oil glut means that the analysts underforecasting the actual inventory by 1% leads to an .00626 increase in volatility. These coefficients are substantial given the intraday absolute returns

[39]The change is an increase even though the coefficient has a negative sign because shortage stands for $S_{kj} < 0$, so the coefficient is multiplied by a negative number.

shown in Figure 6.

Specification (2) adds the gas inventory variables. The oil inventory variables are unaffected which means that the gas inventory variables provide additional explanatory power. The coefficients of -.00205 and +.00191 on the gas shortage and the gas glut are about one-half and one-third the size of their oil counterparts, respectively, indicating the gas inventory announcements have a sizeable effect on the oil price volatility. This influence of the gas market on the oil market has not been documented before.

Specification (3) adds control variables for the gasoline and distillate inventory. Even though adding these variables decreases the oil shortage and oil glut coefficients, the gas shortage and gas glut coefficients are unaffected, confirming the cross-commodity effect. In fact, the gas inventory variables become more important relative to the oil inventory variables. The gas shortage coefficient becomes almost as large as the oil shortage coefficient and the gas glut coefficient becomes about half the size of the oil glut coefficient. This cross-commodity effect provides support for the theories on spillovers between the oil and gas markets described in the Introduction.

5.1.3 Joint Model of Oil and Gas Price Volatility and the Asymmetry Effect

To put the effect of *gas* inventory announcements on the *oil* price volatility in perspective, the effect of *oil* inventory announcements on the *gas* price volatility is analyzed for comparison. Since errors may be correlated across the oil and gas regressions, a seemingly unrelated regression (SUR) is estimated.[40] Table 3 displays the results for the oil and gas inventory variables. Again, the coefficients are sizeable given the intraday absolute returns shown in

[40]The SUR results do not materially differ from running regressions separately for oil and gas.

Table 2: Price volatility regressions for oil nearby contract

	(1)	(2)	(3)
Oil shortage	***-.00425	***-.00426	***-.00231
$S < 0$	(.00051)	(.00051)	(.00066)
Oil glut	***.00626	***.00627	***.00433
$S > 0$	(.00074)	(.00074)	(.00077)
Gas shortage		**-.00205	***-.00209
$S < 0$		(.00075)	(.00075)
Gas glut		***.00191	***.00194
$S > 0$		(.00052)	(.00053)
Gasoline shortage			**-.00107
$S < 0$			(.00048)
Gasoline glut			***.00212
$S > 0$			(.00068)
Distillate shortage			**-.00139
$S < 0$			(.00062)
Distillate glut			*.00093
$S > 0$			(.00051)
Beg-of-day dummy	***.00795	***.00796	***.00797
	(.00026)	(.00026)	(.00026)
End-of-day dummy	***.00098	***.00099	***.00100
	(.00008)	(.00008)	(.00008)
Trader composition	***-.00010	***-.00010	***-.00010
	(.00003)	(.00003)	(.00003)
T-bill rate	***-.00008	***-.00008	***-0.00008
	(.00001)	(.00001)	(0.00001)
Volume 1st lag	***.00006	***.00006	***.00006
	(.00001)	(.00001)	.00001
$\|R_j\|$ 1st lag	***.07220	***.07248	***0.07239
	(.01127)	(.01129)	(.01130)
R^2	0.27	0.28	0.28
$RMSE$	0.00318	0.00317	0.00317

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. The number of observations is 54,850 in all specifications. The control variables that are not significant are excluded from the specification reported in this paper. Only the first lags of volume and absolute return are reported to save space.*

Figure 6.

The effect of both *gas gluts and gas shortages* on the *oil* price volatility is more than twice as strong as the effect of *oil gluts and oil shortages* on the *gas* price volatility. This underscores the cross-commodity effect and highlights the importance of the gas market spillovers for the oil market.

In addition to the cross-commodity effect, the joint model results in Table 3 offer an

Table 3: SUR model for oil and gas price volatility

	Oil price volatility	Gas price volatility
Oil shortage	***-.00232	**-.00089
$S < 0$	(.00030)	(.00042)
Oil glut	***.00434	**.00089
$S > 0$	(.00030)	(.00043)
Gas shortage	***-.00217	***-.02432
$S < 0$	(.00061)	(.00087)
Gas glut	***.00200	***.02110
$S > 0$	(.00044)	(.00064)

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. The number of observations is 54,850. Only the oil and gas inventory variables are reported to save space.*

interesting picture of asymmetries. Numerous studies have shown that in stock and bond markets, an unexpected price decrease, i.e., "bad news", is associated with a higher volatility than an unexpected price increase, i.e., "good news" (e.g., Glosten, Jagannathan and Runkle (1993)). In the oil and gas markets, the evidence is mixed. The finding from the stock and bond markets was corroborated by Susmel and Thompson (1997) using monthly gas data from 1975 through 1994 and Wang, Wu and Yang (2008) using intraday gas data from 1995 through 1999. However, Wang, Wu and Yang (2008) did not find any asymmetries in oil. Switzer and El-Khoury (2007) came to the opposite conclusion using daily oil data from 1986 through 2005, showing that positive price shocks lead to higher volatility than negative price shocks. This was also the case in Gregoire and Boucher (2008), who used daily gas data from 2005 through 2007, and Kuper and van Soest (2006), who used monthly oil data from 1970 through 2002.[41]

In our findings, price decreases are associated with inventory gluts and price increases are associated with inventory shortages in accordance with laws of demand and supply as shown in Table 6. The relative response of volatility to the price decreases, i.e., inventory gluts, and price increases, i.e., inventory shortages, differs by commodity. A glut has a larger impact than a shortage for the effect of *oil* inventory announcements on *oil* price volatility. In contrast, a glut has a slightly smaller impact than a shortage for the effect of *gas* inventory announcements on *oil* price volatility, which is also the case for the effect of *gas* inventory announcements on *gas* price volatility. Finally, a glut has the same effect as a shortage for the effect of *oil* inventory announcements on *gas* price volatility. This shows the asymmetry depends on the source of the shock and highlights the importance of analyzing the data in detail since general statements may miss the fine differences.

5.1.4 Effect across Futures Contract Maturities

To investigate how current news announcements affect financial instruments at various points in the future, the specification (3) is run for the nearby contract, denoted as Contract 1 and the following seven months' contracts for oil and gas. Since errors may be correlated across the regressions for the different maturities, SUR is applied.[42]

[41]Note that with the exception of Gregoire and Boucher (2008), these papers on asymmetries did not consider inventories.

[42]This leads to the slight differences between results for the nearby contract reported in Table 2 that uses OLS and Table 4 that uses SUR. Also, since contracts with longer maturities are traded less frequently, these contracts have more missing observations than the nearby contract. Oil contracts 2 through 8 have 0.42%, 6.79%, 25.69%, 46.43%, 60.76%, 71.30% and 77.54% missing observations, respectively. Gas contracts 2

Table 4 displays the results for the oil and gas inventory variables. See Appendix D for the other regressors. The two-way causality indicated by the cross-commodity effect holds across the maturity structure as the oil and gas inventory variables remain significant for the longer maturities. The asymmetry effect also persists across the maturities. The results described above are, therefore, robust to the maturity structure. In general, the coefficients decrease as the length of the contract increases, supporting the Samuelson (1965) theorem, which states that futures contracts become more volatile as their expiration date approaches.

5.1.5 Pre-Announcement and Post-Announcement Effects

The oil and gas inventory announcements affect the oil price volatility even prior to the announcement.[43] The volatility is lower than usual for approximately 70 minutes before the oil announcements and 30 minutes before the gas announcements. After the announcement, the volatility remains higher than usual for approximately 60 minutes following the oil announcements and 20 minutes following the gas announcements. This suggests that the market participants decrease their trading activity while waiting for the inventory report announcements and increase their trading activity once the reports are released.

The oil futures market appears to be approximately as efficient as the gas futures market

through 8 have 1.19%, 5.83%, 11.83%, 17.97%, 24.81%, 33.24% and 40.07% missing observations, respectively. As in the nearby contract, the missing observations are set equal to the previous observation. However, since the number of missing observations varies across the contracts, an unbalanced SUR is applied as a robustness check. The results do not change. Contracts with maturities longer than eight months do not have significant coefficients on the inventory variables, so they are not reported.

[43]The pre-announcement and post-announcement analysis is performed using dummies for the intervals before and after the announcements.

Table 4: Price volatility of contracts with longer maturities

Panel a): Oil contracts

	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6	Contract 7	Contract 8
Oil shortage	***-.00223	***-.00197	***-.00185	**-.00067	***-.00124	*-.00057	-.00005	-.00048
$S < 0$	(.00030)	(.00026)	(.00026)	(.00028)	(.00030)	(.00031)	(.00031)	(.00031)
Oil glut	***.00428	***.00352	***.00334	***.00208	***.00224	***.00083	***.00116	.00043
$S > 0$	(.00031)	(.00027)	(.00027)	(.00029)	(.00031)	(.00031)	(.00032)	(.00032)
Gas shortage	***-.00233	***-.00243	**-.00136	*-.00095	***-.00178	*-.00105	-4.35e-06	-.00021
$S < 0$	(.00062)	(.00054)	(.00054)	(.00058)	(.00063)	(.00063)	(.00065)	(.00064)
Gas glut	***.00241	***.00223	***.00206	***.00135	***.00247	***.00139	***.00167	.00011
$S > 0$	(.00045)	(.00040)	(.00039)	(.00042)	(.00046)	(.00046)	(.00047)	(.00047)

Panel b): Gas contracts

	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6	Contract 7	Contract 8
Oil shortage	**-.00096	**-.00080	***-.00116	***-.00105	**-.00070	-.00053	-.00030	-.00001
$S < 0$	(.00043)	(.00041)	(.00038)	(.00035)	(.00035)	(.00033)	(.00034)	(.00032)
Oil glut	**.00100	.00063	**.00086	.00035	.00043	*.00061	.00038	***.00104
$S > 0$	(.00044)	(.00041)	(.00038)	(.00036)	(.00036)	(.00034)	(.00035)	(.00033)
Gas shortage	***-.02486	***-.02315	***-.02062	***-.01168	***-.01141	***-.01015	***-.00752	***-.00724
$S < 0$	(.00088)	(.00083)	(.00077)	(.00072)	(.00073)	(.00069)	(.00070)	(.00067)
Gas glut	***.02120	***.02059	***.01782	***.01515	***.01220	***.01194	***.00827	***.00766
$S > 0$	(.00064)	(.00061)	(.00056)	(.00052)	(.00053)	(.00050)	(.00051)	(.00049)

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. The number of observations is 54,598. Only the oil and gas inventory variables are reported to save space. See Appendix D for the other regressors.*

in absorbing the inventory news announcements since the gas futures volatility remains higher than usual for approximately 40 minutes after the oil announcements and 30 minutes after the gas announcements.

5.2 Oil Price Return

This section presents the effect of inventory announcements on the oil futures return. While the effect on the price volatility is significant both statistically and economically, the economic significance of the effect on the oil price return is small as evidenced by the low R^2 in Table 5, although the statistical significance remains.

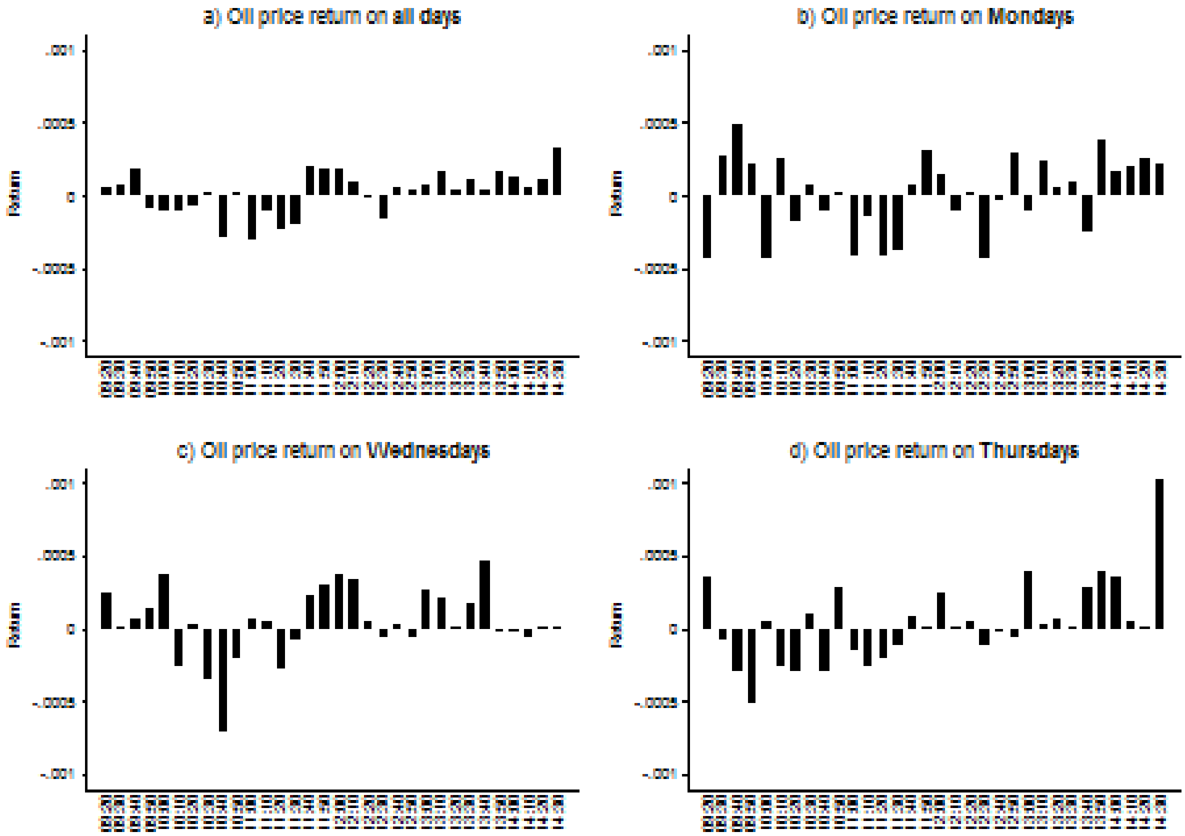
5.2.1 Intraday Pattern Graphs

Figure 7 displays the intraday pattern for the oil futures return. Except for a slight drop in the oil price return during the 10:40 a.m. interval on Wednesday, no clear pattern can be discerned around the oil and gas inventory announcement times. In contrast to the oil price volatility that is increased by both shortages and gluts, the oil price return rises as a result of shortages while dropping as a result of gluts. Since the intraday patterns combine both these increases and decreases, no obvious pattern can be seen. The regression results, however, show that both oil and gas inventory announcements do affect the oil futures return.

5.2.2 Cross-Commodity Effect

Table 5 shows the same three specifications as Table 2. An oil glut decreases the oil price return while the oil shortage increases it, in accordance with the laws of demand and supply. A gas glut decreases the oil price return, suggesting the two commodities are substitutes. However, a gas shortage has no effect on the oil price return, indicating that the substitutability does not hold for gas shortages.

Figure 7: Intraday pattern of oil price return



Notes: This figure shows the intraday pattern of the NYMEX crude oil nearby contract futures price return. The return is defined as the difference between log prices of two subsequent intervals. The first interval is not displayed to avoid skewing the graphs by the overnight gap. Source: Tick Data, Inc.

5.2.3 Joint Model of Oil and Gas Price Return and the Asymmetry Effect

Again, to put the effect of *gas* inventory announcements on the *oil* price return in perspective, the effect of *oil* inventory announcements on the *gas* price return is analyzed, using SUR.

As Table 6 shows, the effect of a *gas* glut on the *oil* price return is as strong as the effect of an *oil* glut on the *gas* price return, highlighting the importance of the gas market for the oil market and providing support for the theories on linkages between the oil and gas markets described in the Introduction. However, while the effect of an oil shortage on

Table 5: Price return regressions for oil nearby contract

	(1)	(2)	(3)
Oil shortage	***-.00302	***-.00301	***-.00409
$S < 0$	(.00069)	(.00069)	(.00070)
Oil glut	***-.00589	***-.00589	***-.00553
$S > 0$	(.00085)	(.00085)	(.00079)
Gas shortage		-.00013	-.00013
$S < 0$		(.00117)	(.00117)
Gas glut		***-.00306	***-.00306
$S > 0$		(.00074)	.00074
Gasoline shortage			***-.00170
$S < 0$			(.00059)
Gasoline glut			***-.00419
$S > 0$			(.00075)
Distillate shortage			***-.00351
$S < 0$			(.00063)
Distillate glut			***-.00208
$S > 0$			(.00060)
Ref util shortage			***.00049
$S < 0$			(.00016)
Ref util glut			.00048
$S > 0$			(.00050)
End-of-day dummy	**.00031	**.00031	**.00031
	(.00013)	(.00013)	(.00013)
R_j 1st lag	*-.01484	*-.01481	*-.01500
	(.00846)	(.00846)	(.00846)
R^2	0.006	0.007	0.014
$RMSE$	0.00462	0.00462	0.00461

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. The number of observations is 54,850 in all specifications. The control variables that are not significant are excluded from the specification reported in this paper. Only the first lag of return is reported to save space.*

the gas price return is significant, the effect of a gas shortage on the oil price return is not, indicating a lack of two-way causality in the gas shortage situations.[44]

Table 6 also reveals that gluts (i.e., an unexpected excess supply) have a larger impact than shortages (i.e., an unexpected excess demand) on oil and gas price returns.

[44]When the regression is run without distinguishing between gas gluts and shortages, the gas inventory surprise variable has a significant coefficient of -.00195, confounding the asymmetric effect. This shows the importance of distinguishing between the inventory gluts and shortages.

Table 6: SUR model for oil and gas price returns

	Oil price return	Gas price return
Oil shortage	***-.00407	**-.00117
$S < 0$	(.00043)	(.00060)
Oil glut	***-.00554	***-.00325
$S > 0$	(.00044)	(.00061)
Gas shortage	-.00013	***-.01887
$S < 0$	(.00088)	(.00122)
Gas glut	***-.00306	***-.02224
$S > 0$	(.00064)	(.00089)

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. The number of observations is 54,850. Only the oil and gas inventory variables are reported to save space.*

5.2.4 Effect across Futures Contracts Maturities

Table 7 displays specification (3) for the oil and gas nearby contracts as well as the following seven months of contracts. Only the oil and gas inventory variables are reported to save space. See Appendix E for the other regressors.

The cross-commodity effect as well as the asymmetry effect hold across the maturity structure since the oil glut, oil surplus and gas glut variables remain significant for the longer maturities. In general, the coefficients decrease as the length of the contract increases, again providing support for the Samuelson (1965) theorem.

Table 7: Price return of contracts with longer maturities

Panel a): Oil contracts

	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6	Contract 7	Contract 8
Oil shortage	***-.00425	***-.00396	***-.00388	***-.00250	***-.00245	***-.00240	***-.00132	***-.00108
$S < 0$	(.00043)	(.00039)	(.00037)	(.00037)	(.00036)	(.00034)	(.00034)	(.00033)
Oil glut	***-.00564	***-.00508	***-.00435	***-.00363	***-.00286	***-.00177	***-.00138	***-.00092
$S > 0$	(.00044)	(.00040)	(.00039)	(.00038)	(.00037)	(.00035)	(.00035)	(.00034)
Gas shortage	.00034	.00085	-.00023	.00037	.00108	-.00029	-.00018	.00103
$S < 0$	(.00088)	(.00080)	(.00077)	(.00075)	(.00073)	(.00070)	(.00069)	(.00067)
Gas glut	***-.00299	***-.00286	***-.00291	***-.00167	***-.00237	**-.00103	***-.00249	-.00047
$S < 0$	(.00064)	(.00058)	(.00056)	(.00055)	(.00053)	(.00051)	(.00050)	(.00049)

Panel b): Gas contracts

	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6	Contract 7	Contract 8
Oil shortage	**-.00122	***-.00162	***-.00153	***-.00168	***-.00160	***-.00126	-.00063	-.00061
$S < 0$	(.00060)	(.00056)	(.00051)	(.00047)	(.00045)	(.00041)	(.00040)	(.00037)
Oil glut	***-.00342	***-.00260	***-.00240	***-.00171	***-.00143	***-.00164	***-.00132	***-.00154
$S > 0$	(.00062)	(.00058)	(.00053)	(.00048)	(.00046)	(.00042)	(.00041)	(.00039)
Gas shortage	***-.01853	***-.01702	***-.01637	***-.00977	***-.00899	***-.00997	***-.00732	***-.00718
$S < 0$	(.00122)	(.00115)	(.00105)	(.00096)	(.00092)	(.00084)	(.00082)	(.00077)
Gas glut	***-.02269	***-.02210	***-.01878	***-.00159	***-.01290	***-.01322	***-.00930	***-.00819
$S < 0$	(.00089)	(.00084)	(.00077)	(.00070)	(.00067)	(.00062)	(.00060)	(.00056)

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. The number of observations is 54,682. Only the oil and gas inventory variables are reported to save space. See Appendix E for the other regressors.*

5.2.5 GARCH Models

In addition to OLS regression, the four GARCH models described in Section 4 are implemented: GARCH(1,1) with a Gaussian distribution of errors, GARCH(1,1) with a t distribution, GARCH-in-mean(1,1) with a Gaussian distribution and EGARCH(1,1) with a Gaussian distribution. Conclusions regarding the cross-commodity, asymmetry and maturity structure effects do not change, although some control variables that are not significant in the OLS regression are significant in the GARCH models. See Table 8 for results.

Table 8: GARCH models

	(1)OLS	(2)GARCH(1,1)	(3)GARCH(1,1)t	(4)GARCH(1,1)-M	(5)EGARCH
Oil shortage	***-.00408	***-.00366	***-.00341	***-.00362	***-.00329
$S < 0$	(.00070)	(.00020)	(.00024)	(.00020)	(.00020)
Oil glut	***-.00555	***-.00573	***-.00421	***-.00574	***-.00468
$S > 0$	(.00079)	(.00017)	(.00024)	(.00017)	(.00022)
Gas shortage	-.00015	-.00036	*-.00082	-.00036	.00080
$S < 0$	(.00117)	(.00055)	(.00049)	(.00055)	(.00053)
Gas glut	***-.00307	***-.00213	***-.00207	***-.00213	***-.00234
$S > 0$	.00074	(-.00031)	(.00039)	(.00031)	(.00044)
Gasoline shortage	***-.00170	***-.00218	***-.00192	***-.00214	***-.00205
$S < 0$	(.00059)	(.00016)	(.00023)	(.00016)	(.00021)
Gasoline glut	***-.00419	***-.00440	***-.00329	***-.00440	***-.00355
$S > 0$	(.00075)	(.00017)	(.00025)	(.00017)	(.00023)
Distillate shortage	***-.00350	***-.00340	***-.00346	***-.00342	***-.00289
$S < 0$	(.00063)	(.00013)	(.00022)	(.00014)	(.00018)
Distillate glut	***-.00208	***-.00167	***-.00214	***-.00167	***-.00160
$S > 0$	(.00060)	(.00014)	(.00022)	(.00014)	(.00020)
Ref.utiliz.shortage	***.00049	***.00038	***.00045	***.00038	***.00027
$S < 0$	(.00016)	(.00008)	(.00012)	(.00009)	(.00011)
Ref.utiliz.glut	.00048	***.00036	*.00033	***.00037	***.00028
$S > 0$	(.00050)	(.00011)	(.00017)	(.00011)	(.00013)
Beg-of-day dummy	-.00009	***-.00037	**.00015	***-.00039	***-.00021
	(.00039)	(.00004)	(.00006)	(.00004)	(.00003)
End-of-day dummy	**.00031	***-.00475	***.00037	***-.00475	***.00052
	(.00013)	(.00002)	(.00006)	(.00002)	(.00007)
1st trading day dummy	-.00002	*-.00004	.00003	*-.00004	
	(.00005)	(.00002)	(.00003)	(.00002)	
Trader composition	-.00001	-.00004	*-.00007	-.00002	
	(.00006)	(.00003)	(.00004)	(.00003)	
T-bill rate	2.60e-06	***.00002	**-.00002	***.00002	***-.00004
	(.00001)	(-7.06e-06)	(7.57e-06)	(7.14e-06)	(2.12e-06)
Volume 1st lag	-4.46e-06	*-5.99e-06	4.29e-07	**-7.75e-06	
	(.00001)	(3.56e-06)	(4.09e-06)	(3.58e-06)	
R_j 1st lag	*-.01499	***-.04904	***-.04800	***-.04999	
	(.00851)	(.00308)	(.00309)	(.00311)	
arch		***.56878	***.00415	***.56878	***-.01060
		(.00382)	(.00026)	(.00391)	(.00018)
garch		***.42869	***.99499	***.42822	***.99976
		(.00226)	(.00030)	(.00230)	(.00003)
archm σ^2				***1.37579	
				(.24542)	
earch-a					***.01807
					(.00016)

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. Columns 1 through 6 report OLS, GARCH(1,1) with Gaussian distribution of errors, GARCH(1,1) with t-distribution, GARCH-in-mean(1,1) with Gaussian distribution and EGARCH(1,1) with Gaussian distribution. Lagged inventory surprise terms are not significant in any specification, so they are not reported. Only the first lags of volume and absolute return are reported to save space. The t-distribution shows 3.2 degrees of freedom. Specification (5) is run without the first trading day, trader composition, lagged volume and lagged return to ensure convergence. The OLS results differ slightly from those reported in Table 5 because the specification in Table 10 includes additional control variables to allow comparison to the GARCH models.*

5.3 Robustness Checks

Robustness checks are performed to ensure that the above results are maintained conditional on the time period, stages of the business cycle, oil and gas price returns, oil and gas price volatility, and oil, gas, gasoline and distillate inventory. Details of these robustness checks are available upon request.

5.3.1 Structural Breaks

Since the oil and gas markets were subject to numerous shocks and developments during the sample period, such as the increase in futures trading, the introduction of LNG technology, the development of the shale gas fields, and the explosion of the Deepwater Horizon drilling rig in the Gulf of Mexico followed by a moratorium on U.S. off-shore drilling, the analysis is performed with dummies for individual years to ensure stability across time. The analysis is repeated with dummies for individual months. In addition, a structural break test is performed following Hansen (2001). The results are unaffected, which means that the conclusions of this paper hold despite these considerable shocks and developments in the energy markets.

5.3.2 Business Cycle

The sample period includes the most severe recession since the Great Depression. Therefore, the sample is split into booms and recession using the National Bureau of Economic Research dating that identified the recession as beginning in December 2007 and ending in June 2009. The results are unaffected, which means that the results exhibit stability across the business cycle. Since the world economy may be entering a second dip, the sample was split into periods before and after December 2007. Again, the results were unaffected. This stands in contrast to Hess, Huang and Niessen (2008), who found that the effect of macroeconomic news announcements on the commodity index futures was dependent on recessions and booms.

5.3.3 Oil and Gas Price Returns

To check that the results are maintained conditional on the oil and gas price returns, dummies taking on the value of 1 for oil and gas prices below their mean and 0 otherwise are created. The oil dummy is significant for the *oil price volatility* with the coefficient of 0.00027 but it does not affect the coefficients on the other variables. The gas dummy is not significant for the *oil price volatility.* Neither dummy is significant for the *oil price return.*

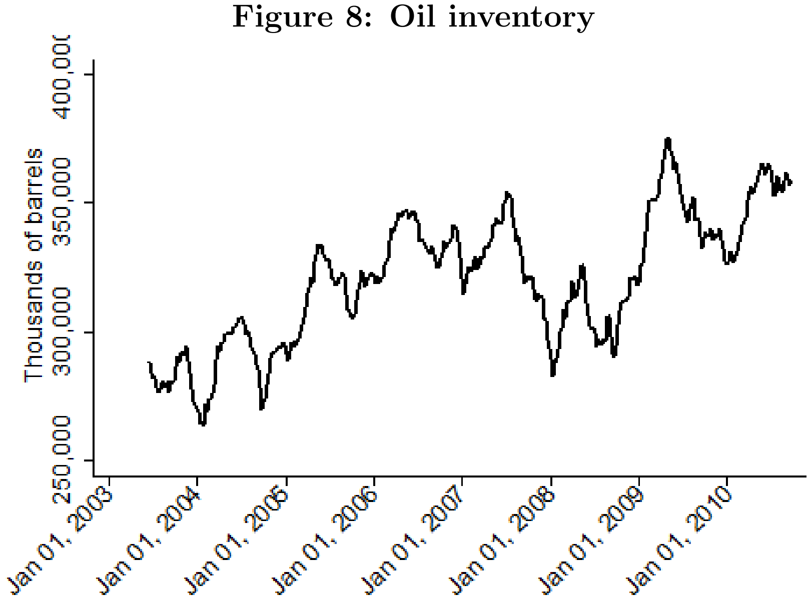
5.3.4 Oil and Gas Price Volatility

To check that the results are maintained conditional on the oil and gas price volatility graphed in Figures 4 and 5, ratios of the absolute value of the daily return to the absolute value of average daily return for that day of the week are calculated for oil and gas. The oil ratio is significant for the *oil price volatility* with a coefficient of 0.00030 but it does not affect the coefficients on the other variables. The gas ratio is not significant for the *oil price volatility.* The oil ratio is not significant for the *oil price return.* The gas ratio is significant for the *oil price return* with a coefficient of 0.00004 but it does not affect coefficients on the other variables.

5.3.5 Oil and Gas Inventory

To check that the results are maintained conditional on the *oil inventory* graphed in Figure 8, a dummy taking on the value of 1 for oil inventory below its mean and 0 otherwise is created. This dummy is significant for the *oil price volatility* with the coefficient of 0.00067. The oil shortage and glut coefficients decrease slightly (by less than 10%) but the gas shortage and glut coefficients are unaffected. The dummy is significant for the *oil price return* with the coefficient of 0.00093 but it does not affect coefficients on the other variables.

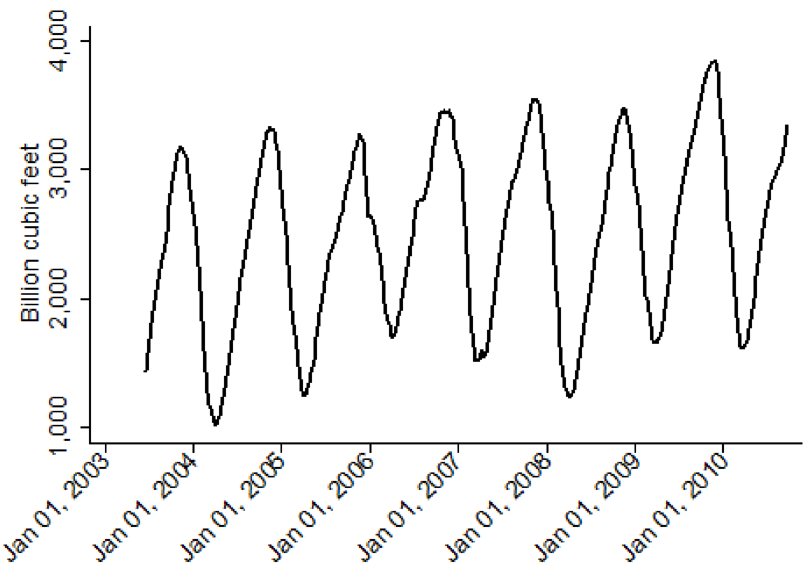
Figure 8: Oil inventory



Source: Weekly Petroleum Status Report published by the Energy Information Administration of the U.S. Department of Energy

To check that the results are maintained conditional on the *gas inventory* graphed in Figure 9, a dummy taking on the value of 1 for gas inventory below its mean and 0 otherwise is created. This dummy is not significant in the *oil price volatility* regression. It is significant with the coefficient of 0.00109 in the *oil price return* regression without affecting the oil shortage, oil glut and gas shortage coefficients. The gas glut coefficient increases by 33%, indicating a higher sensitivity of the oil price return to the gas inventory when the gas inventory is low.

Figure 9: Gas inventory



Source: Weekly Natural Gas Storage Report published by the Energy Information Administration of the U.S. Department of Energy

Since the gas inventory is highly seasonal, building up in the summer and drawing down in the winter, a dummy taking on the value of 1 during the build-up periods is created. This dummy is significant with the coefficient of -.00008 in the *oil price volatility* regression but it does not affect coefficients on the other variables. It is not significant in the *oil price return* regression.

As another check of seasonality, a dummy is added taking on the value of 1 in the winter (defined as the beginning of October until the end of March) and 0 in the summer (defined as the beginning of April until the end of September). This dummy is significant in the *oil price volatility* regression with the coefficient of .00003 but it does not affect the coefficients on the other variables. This dummy is not significant in the *oil price return* regression.

5.3.6 Gasoline and Distillate Inventory

To check that the results are maintained conditional on the gasoline inventory, a dummy taking on the value of 1 for the gas inventory below its mean and 0 otherwise is created. The dummy is significant in the *oil price volatility* regression with the coefficient of -.00065 but it does not affect coefficients on the other variables. It is not significant in the *oil price return* regression. Similar results hold for the distillate inventory.

6 Conclusions

The previous empirical research on the relationship between crude oil and natural gas markets has concluded the oil market affects the gas market but not vice versa despite economic theory indicating there should be a two-way causality. This paper contributes to solving this puzzle.

There are three main results. First, the bi-directional causality suggested by the theory is confirmed for the first time. These cross-commodity effects turn out to be sizeable. The effect of *gas* inventory announcements on *oil* price volatility is more than twice as strong as the effect of *oil* inventory announcements on *gas* price volatility, highlighting the importance of spillovers from the gas market to the oil market.

Second, the model allows for different reactions of the price volatility and return to inventory gluts and shortages. In contrast to the "good news" vs. "bad news" pattern found in stock and bond markets, the response differs by commodity and type of the shock, which highlights the importance of analyzing the data in detail since general statements may miss the fine differences.

Third, in contrast to previous studies that analyzed only the nearby futures contract, this paper includes contracts with all maturities to investigate how current news announcements affect financial instruments at various points in the future. The two way-causality as well as the asymmetry effect hold not only for the nearby contract but also for the following seven months' contracts. The results described above are, therefore, robust to the maturity structure. In general, the coefficients decrease as the length of the contract increases, supporting the Samuelson (1965) theorem.

The differences in results between the previous studies and this paper can be attributed to two improvements in analyzing the linkages between the oil and gas markets. First, this study takes advantage of intraday oil and gas prices, i.e., prices at a higher frequency than the daily, weekly or monthly prices used in the previous papers. Second, in contrast to the previous papers that conduct a general analysis of the relationship using, for example, Granger causality tests, this paper analyzes the effect of specific news announcements: the weekly oil and gas inventory reports published by the EIA. While a general analysis using low-frequency data may indicate that the causality runs only from oil to gas, a more careful examination of the high-frequency data focusing on a specific type of a shock, i.e., the inventory news announcements, shows that the causality between the two markets is bi-directional.

Two promising research ideas follow from this paper. First, the asymmetric effects are quite intriguing and call for more research. Second, the pre-announcement and post-announcement effects suggest that market participants decrease their trading activity while waiting for the inventory report announcement and increase their trading activity once the reports are released. This merits a closer look at the relationship between volatility and trading volume. While this relationship in the oil and gas markets has been studied using daily data, an intraday analysis could reveal new facts about how information arrives in the markets. Instead of focusing only on trading during the day, the night trading period could be included to allow for a comparison between these two periods that may differ from the information arrival standpoint. This analysis should again cover futures of all maturities since the different maturities may behave very differently in terms of information arrival.

Appendix A:

Table 9: Summary of Papers on *Causality* between Oil and Gas Markets

Author	**Period**	**Frequency**	**Futures or Spot Price**
Pindyck (2004)	1990-2003	Daily	Futures (NYMEX)
Villar & Joutz (2006)	1989-2005	Daily	Spot (U.S.)
Asche, Osmundsen & Sandsmark (2007)	1995-1998	Monthly	Spot (U.K.)
Brown & Yucel (2007)	1997-2006	Weekly	Spot (U.S.)
Hartley, Medlock & Rosthal (2008)	1990-2006	Monthly	Spot (U.S.)
Onour (2009)	1996-2008	Weekly	Spot (U.S.)

Table 10: Summary of Papers on *Relationship* between Oil and Gas Markets

Author	**Period**	**Frequency**	**Futures or Spot Price**
Yucel and Guo (1994)	1947-1974	Annual	Spot (U.S.)
Serletis and Rangel-Ruiz (2004)	1991-2001	Daily	Spot (U.S)
Bachmeier and Griffin(2006)	1990-2004	Daily	Spot (U.S)
Panagiotidis and Rutledge (2007)	1996-2003	Monthly	Spot (U.K.)
Marzo and Zagaglia (2008)	1990-2005	Daily	Futures (NYMEX)
Wang, Wu and Yang (2008)	1995-1999	Intraday	Futures (NYMEX)

Appendix B: Holidays

The sample period from June 13, 2003 till September 24, 2010 includes 380 weeks. Several scenarios occurred because of holidays:

- **314 weeks:** The oil inventory report was released on Wednesday at 10:30 am and NYMEX was open on Wednesday. The gas inventory report was released on Thursday at 10:30 am and NYMEX was open on Thursday. This is the normal scenario.

- **50 weeks:** The gas inventory report was released on Thursday at 10:30 am and NYMEX was open on Thursday. The oil inventory report was delayed from 10:30 am on Wednesday until 11 am on Thursday because Monday, Tuesday or Wednesday was a public holiday. These Thursdays are: 09/04/03, 10/16/03, 11/13/03, 01/22/04, 02/19/04, 06/03/04, 07/08/04, 09/09/04, 10/14/04, 02/24/05, 06/02/05, 07/07/05, 09/08/05, 10/13/05, 12/29/05, 01/05/06, 01/19/06, 02/23/06, 06/01/06, 07/06/06, 09/07/06, 10/12/06, 12/28/06, 01/04/07, 01/18/07, 02/22/07, 05/31/07, 07/05/07, 09/06/07, 10/11/07, 11/15/07, 12/27/07, 01/03/08, 01/24/08, 02/21/08, 05/29/08, 09/04/08, 10/16/08, 11/13/08, 01/22/09, 02/19/09, 05/28/09, 09/10/09, 10/15/09, 11/12/09, 01/21/10, 02/18/10, 06/03/10, 07/08/10 and 09/09/10. These weeks are not an issue because the data show the oil inventory reports on the day and interval when they were actually released.

- **1 week:** The oil inventory report was delayed until 11 am on Friday because of a snow emergency. The Wednesday affected was 02/10/10. Again, this is not an issue because the data show the oil inventory report on the day and interval when it was released.

It is not clear when the gas inventory report was released. This paper assumes it was released on Thursday at 10:30 am. NYMEX was open on all days.

- **3 weeks:** The oil inventory report was released on Wednesday at 10:30 am and NYMEX was open on Wednesday. The gas inventory report was released on Thursdays. However, NYMEX was closed on Thursday. Since NYMEX was also closed on the following Friday, the gas inventory report did not affect the market until Monday. The Thursdays affected were 07/03/03, 12/23/04 and 12/30/04. The gas inventory data for these three days is eliminated when the intraday file is merged with the gas inventory file because the intraday file does not contain these days. The effect on the market stays in the data because the effect is felt on the following Monday, which is contained in the intraday file. Since this occurs only in three out of 380 weeks, it does not have a material effect on the results.

- **4 weeks:** The oil inventory report was released on Wednesday at 10:30 am and NYMEX was open on Wednesday. The gas inventory report was *not* released on Thursdays because Thursday was a public holiday. It is not clear when the gas inventory report was released, so this paper assumes the release took place at 10:30 am on the following Friday. The Thursdays affected were 11/11/04, 11/23/06, 11/22/07 and 11/27/08. NYMEX was open on all days.

- **5 weeks:** The oil inventory report was released on Wednesday at 10:30 am and NYMEX was open on Wednesday. The gas inventory report was *not* released on Thursdays because Thursday was a public holiday. It is not clear when the gas inventory report was released, so this paper assumes the release took place at 10:30 am on the following Friday. Since NYMEX was closed on both Thursday and Friday, the release probably affected the market on Monday. The gas inventory data for these five days is eliminated when the intraday file is merged with the gas inventory file because the intraday file does not contain these days. The effect on the market stays in the data because the effect is felt on the following Monday, contained in the intraday file. Since this occurs in only five out of 380 weeks, the effect on results is immaterial.

- **3 weeks:** The oil inventory report was released on Wednesday at 10:30 am but NYMEX was closed on Wednesday. The Wednesdays affected are: 12/24/03, 12/31/03 and 11/23/05. The gas inventory report was *not* released on Thursdays because Thursday was a public holiday. It is not clear when the gas inventory report was released, so this paper assumes the release took place at 10:30 am on the following Friday. The Thursdays affected are 12/25/03, 01/01/04 and 11/24/05. NYMEX was closed on Thursday.

Appendix C: Volatility Signature Plots

Figure 10: Volatility signature plot for oil price

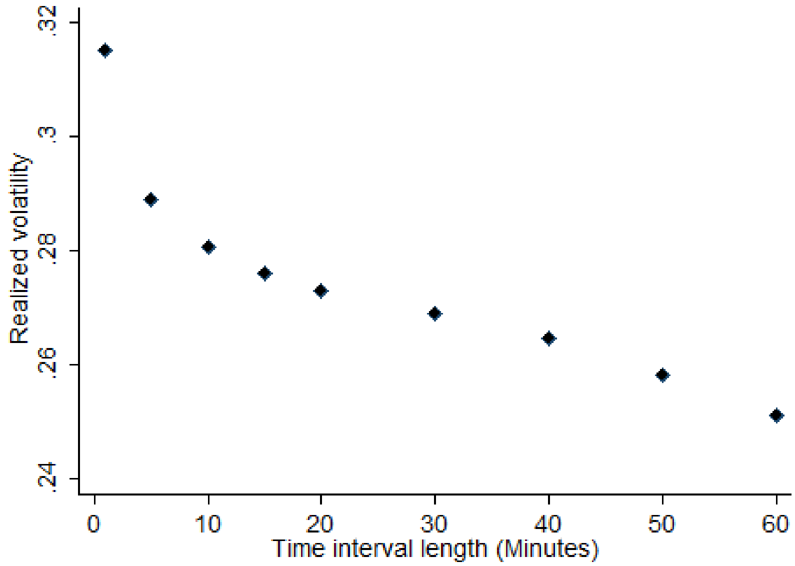
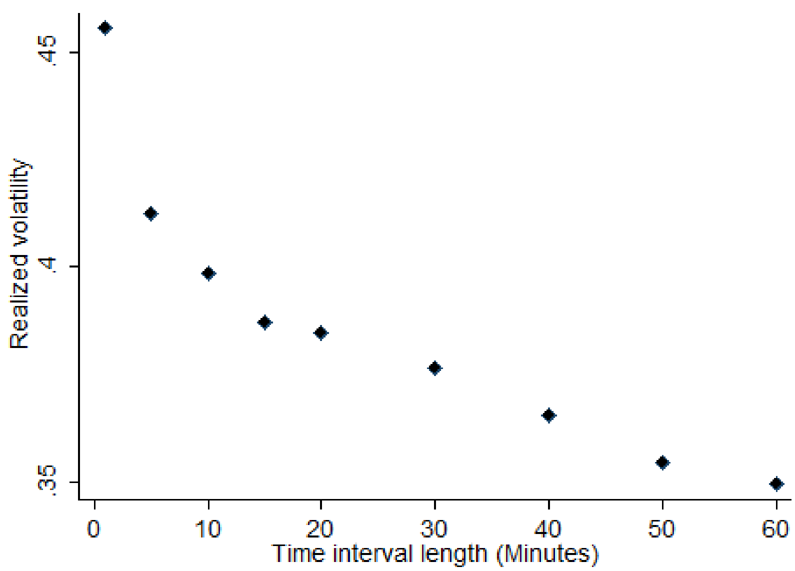


Figure 11: Volatility signature plot for gas price



Appendix D: Price Volatility of Contracts with Longer Maturities

Panel a): Oil contracts

	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6	Contract 7
Oil shortage	***-.00223	***-.00197	***-.00185	**-.00067	***-.00124	*-.00057	-.00005
$S < 0$	(.00030)	(.00026)	(.00026)	(.00028)	(.00030)	(.00031)	(.00031)
Oil glut	***.00428	***.00352	***.00334	***.00208	***.00224	***.00083	***.00116
$S > 0$	(.00031)	(.00027)	(.00027)	(.00029)	(.00031)	(.00031)	(.00032)
Gas shortage	***-.00233	***-.00243	**-.00136	*-.00095	***-.00178	*-.00105	-4.35e-06
$S < 0$	(.00062)	(.00054)	(.00054)	(.00058)	(.00063)	(.00063)	(.00065)
Gas glut	***.00241	***.00223	***.00206	***.00135	***.00247	***.00139	***.00167
$S > 0$	(.00045)	(.00040)	(.00039)	(.00042)	(.00046)	(.00046)	(.00047)
Gasoline shortage	***-.00099	***-.00104	***-.00070	***-.00148	***-.00111	***-.00138	-.00016
$S < 0$	(.00028)	(.00025)	(.00025)	.00026	(.00029)	(.00030)	(.00030)
Gasoline glut	***.00206	***.00205	***.00224	***.00120	***.00120	***.00252	***.00142
$S > 0$	(.00030)	(.00026)	(.00025)	(.00027)	(.00030)	(.00047)	(.00031)
Distillate shortage	***-.00140	***-.00158	***-.00131	***-.00084	-.00006	.00015	-.00015
$S < 0$	(.00026)	(.00023)	(.00023)	(.00025)	(.00027)	(.00027)	(.00028)
Distillate glut	***.00101	***.00102	***.00056	***.00091	-.00025	*.00045	**.00054
$S > 0$	(.00024)	(.00021)	(.00021)	(.00023)	(.00024)	(.00025)	(.00025)
Beg-of-day dummy	***.00877	***.00811	***.00767	***.00699	***.00475	***.00370	***.00205
	(.00008)	(.00007)	(.00007)	(.00007)	(.00008)	(.00008)	(.00008)
End-of-day dummy	***.00136	***.00130	***.00155	***.00150	***.00127	***.00100	***.00078
	(.00008)	(.00007)	(.00007)	(.00007)	(.00008)	(.00008)	(.00008)
Trader composition	***-.00029	***-.00024	***-.00023	.00011	-.00003	-.00002	-.00004
	(.00002)	(.00002)	(.00002)	(.00002)	(.00002)	(.00002)	(.00002)
T-bill rate	***-0.00024	***-.00023	***-.00021	***-.00019	***-.00017	***-.00015	***-.00013
	(9.82e-06)	(8.60e-06)	(8.53e-06)	(9.17e-06)	(9.93e-06)	(9.98e-06)	(.00001)
Volume 1st lag	*6.50e-06	2.15e-06	-.00005	***.00035	***.00048	***.00070	***.00130
	(3.64e-06)	(6.50e-06)	(.00004)	(.00011)	(.00015)	(.00015)	(.00023)
$\|R_j\|$ 1st lag	***0.01658	***-.01132	***-.01568	***-.01375	***-.01249	.00602	.00127
	(.00270)	(.00243)	(.00268)	(.00323)	(.00379)	(.00395)	(.00413)

Panel b): Gas contracts

	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6	Contract 7
Oil shortage	**-.00096	**-.00080	***-.00116	***-.00105	**-.00070	-.00053	-.00030
$S < 0$	(.00043)	(.00041)	(.00038)	(.00035)	(.00035)	(.00033)	(.00034)
Oil glut	**.00100	.00063	**.00086	.00035	.00043	*.00061	.00038
$S > 0$	(.00044)	(.00041)	(.00038)	(.00036)	(.00036)	(.00034)	(.00035)
Gas shortage	***-.02486	***-.02315	***-.02062	***-.01168	***-.01141	***-.01015	***-.00752
$S < 0$	(.00088)	(.00083)	(.00077)	(.00072)	(.00073)	(.00069)	(.00070)
Gas glut	***.02120	***.02059	***.01782	***.01515	***.01220	***.01194	***.00827
$S > 0$	(.00064)	(.00061)	(.00056)	(.00052)	(.00053)	(.00050)	(.00051)
Gasoline shortage	-.00042	-.00049	-4.92e-08	-.00021	-.00039	-.00024	-.00029
$S < 0$	(.00040)	(.00038)	(1.69e-07)	(.00033)	(.00033)	(.00032)	(.00032)
Gasoline glut	.00044	.00033	.00031	.00015	*.00064	.00031	.00028
$S > 0$	(.00042)	(.00040)	(.00041)	(.00034)	(.00035)	(.00033)	(.00033)
Distillate shortage	***-.00104	-.00099	**-.00065	**-.00069	-.00047	**-.00073	-.00019
$S < 0$	(.00038)	(.00036)	(.00033)	(.00031)	(.00031)	(.00029)	(.00030)
Distillate glut	**.00086	***.00101	**.00065	**.00069	*.00050	***.00083	.00002
$S > 0$	(.00034)	(.00033)	(.00030)	(.00028)	(.00028)	(.00027)	(.00027)
Beg-of-day dummy	***.01286	***.01218	***.01121	***.01019	***.00868	***.00707	***.00575
	(.00011)	(.00010)	(.00010)	(.00009)	(.00009)	(.00009)	(.00009)
End-of-day dummy	***.00102	***.00097	***.00114	***.00099	***.00093	***.00080	***.00082
	(.00011)	(.00010)	(.00010)	(.00009)	(.00009)	(.00009)	(.00009)
Trader composition	-.00003	*-.00006	-.00001	.00003	***-.00008	***.00008	***.00010
	(.00003)	(.00003)	(.00003)	(.00003)	(.00003)	(.00003)	(.00003)
T-bill rate	-1.00e-06	-.00001	**-.00002	***-.00003	-.00001	***-.00004	**-.00002
	(.00014)	(.00001)	(.00001)	(.00001)	(.00001)	(.00001)	(.00001)
Volume 1st lag	*5.22e-06	**-.00009	***-.00034	.00016	*.00036	***.00073	**.00070
	(.00001)	(.00003)	(.00012)	(.00017)	(.00021)	(.00028)	(.00031)
$\lvert R_j \rvert$ 1st lag	***0.01141	***-.02173	***-.02790	***-.03666	***-.04523	***-.03379	***-.03087
	(.00272)	(.00251)	(.00265)	(.00282)	(.00309)	(.00333)	(.00355)

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. The number of observations is 54,598. Only the first lags of volume and absolute return are reported to save space.*

Appendix E: Price Return of Contracts with Longer Maturities

Panel a): Oil contracts

	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6	Contract 7
Oil shortage	***-.00425	***-.00396	***-.00388	***-.00250	***-.00245	***-.00240	***-.00132
$S < 0$	(.00043)	(.00039)	(.00037)	(.00037)	(.00036)	(.00034)	(.00034)
Oil glut	***-.00564	***-.00508	***-.00435	***-.00363	***-.00286	***-.00177	***-.00138
$S > 0$	(.00044)	(.00040)	(.00039)	(.00038)	(.00037)	(.00035)	(.00035)
Gas shortage	.00034	.00085	-.00023	.00037	.00108	-.00029	-.00018
$S < 0$	(.00088)	(.00080)	(.00077)	(.00075)	(.00073)	(.00070)	(.00069)
Gas glut	***-.00299	***-.00286	***-.00291	***-.00167	***-.00237	**-.00103	***-.00249
$S < 0$	(.00064)	(.00058)	(.00056)	(.00055)	(.00053)	(.00051)	(.00050)
Gasoline shortage	***-.00177	***-.00185	***-.00146	***-.00120	***-.00121	***-.00127	**-.00067
$S < 0$	(.00041)	(.00037)	(.00036)	(.00035)	(.00034)	(.00033)	(.00032)
Gasoline glut	***-.00420	***-.00399	***-.00401	***-.00242	***-.00304	***-.00173	***-.00179
$S > 0$	(.00043)	(.00039)	(.00037)	(.00036)	(.00030)	(.00034)	(.00033)
Distillate shortage	***-.00333	***-.00318	***-.00249	***-.00238	***-.00212	***-.00081	*-.00059
$S < 0$	(.00040)	(.00036)	(.00035)	(.00034)	(.00033)	(.00032)	(.00031)
Distillate glut	***-.00222	***-.00208	***-.00173	***-.00135	-.00038	**-.00070	***-.00091
$S < 0$	(.00037)	(.00033)	(.00032)	(.00031)	.00030	(.00029)	(.00029)
Ref util shortage	***-.00045	**-.00035	*.00024	*-.00022	**.00029	.00019	*-.00023
$S < 0$	(.00015)	(.00014)	(.00013)	(.00013)	(.00013)	(.00012)	(.00012)
Ref util glut	.00047	.00041	.00026	-.00026	-.00013	-.00012	***.00025
$S < 0$	(.00030)	(.00027)	(.00026)	(.00025)	(.00025)	(.00024)	(.00023)
End-of-day dummy	**.00034	***.00043	***.00050	***.00046	***.00045	***.00037	.00009
	(.00011)	(.00010)	(.00010)	(.00010)	(.00009)	(.00009)	(.00009)
R_j 1st lag	***-.25723	***-.28124	***-.26616	***-.19238	***-.11405	***-.06849	***-.04759
	(.00251)	(.00229)	(.00248)	(.00301)	(.00358)	(.00380)	(.00404)

Panel b): Gas contracts

	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6	Contract 7
Oil shortage	**-.00122	***-.00162	***-.00153	***-.00168	***-.00160	***-.00126	-.00063
$S < 0$	(.00060)	(.00056)	(.00051)	(.00047)	(.00045)	(.00041)	(.00040)
Oil glut	***-.00342	***-.00260	***-.00240	***-.00171	***-.00143	***-.00164	***-.00132
$S > 0$	(.00062)	(.00058)	(.00053)	(.00048)	(.00046)	(.00042)	(.00041)
Gas shortage	***-.01853	***-.01702	***-.01637	***-.00977	***-.00899	***-.00997	***-.00732
$S < 0$	(.00122)	(.00115)	(.00105)	(.00096)	(.00092)	(.00084)	(.00082)
Gas glut	***-.02269	***-.02210	***-.01878	***-.00159	***-.01290	***-.01322	***-.00930
$S < 0$	(.00089)	(.00084)	(.00077)	(.00070)	(.00067)	(.00062)	(.00060)
Gasoline shortage	***-.00187	***-.00150	***-.00139	***-.00130	**-.00100	***-.00121	*-.00066
$S < 0$	(.00057)	(.00054)	(.00049)	(.00045)	(.00043)	(.00040)	(.00038)
Gasoline glut	***-.00161	***-.00162	***-.00138	***-.00154	***-.00165	***-.00134	**-.00093
$S > 0$	(.00059)	(.00055)	(.00051)	(.00046)	(.00044)	(.00041)	(.00040)
Distillate shortage	***-.00201	***-.00182	***-.00139	***-.00112	***-.00144	***-.00109	***-.00120
$S < 0$	(.00055)	(.00052)	(.00047)	(.00043)	(.00040)	(.00038)	(.00037)
Distillate glut	***-.00207	***-.00193	***-.00170	***-.00144	***-.00164	***-.00137	-.00051
$S < 0$	(.00051)	(.00048)	(.00043)	(.00040)	(.00038)	(.00035)	(.00034)
Ref util shortage	-.00029	-.00016	.00006	.00004	-3.83e-06	.00009	*-.00025
$S < 0$	(.00021)	(.00020)	(.00018)	(.00017)	(.00016)	(.00015)	(.00014)
Ref util glut	**.00081	**.00086	**.00084	*-.00057	***.00081	*.00050	.00028
$S < 0$	(.00041)	(.00039)	(.00035)	(.00032)	(.00024)	(.00029)	(.00028)
End-of-day dummy	*.00025	***.00037	***.00045	**.00027	***.00028	**.00024	*.00019
	(.00015)	(.00014)	(.00013)	(.00012)	(.00011)	(.00011)	(.00010)
R_j 1st lag	***-.23732	***-.24094	***-.24090	***-.23095	***-.21414	***-.17926	***-.14071
	(.00257)	(.00241)	(.00251)	(.00268)	(.00290)	(.00312)	(.00343)

*Notes: ***, ** and * represent 99%, 95% and 90% significance levels, respectively. Standard errors are shown in parenthesis. The number of observations is 54,598. Only the first lag of return is reported to save space.*

References

[1] Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P. (2000): Great Realizations, *Risk*, March, 105-109.

[2] Andersen, T.G., Bollerslev, T., Diebold, F.X. and Vega, C. (2003): Micro Effects of Macro Announcements: Real-Time Price Discovery in Foreign Exchange, *The American Economic Review*, 93(1), 38-62.

[3] Asche, F., Osmundsen, P. and Sandsmark, M. (2007): The UK Market for Natural Gas, Oil And Electricity: Are the Prices Decoupled?, *The Energy Journal*, 27(2), 27-40.

[4] Ates, A. and Wang, G.H.K. (2007): Price Dynamics in Energy Spot and Futures Markets: The Role of Inventory and Weather, *Akdeniz University and George Mason University Manuscript*.

[5] Bachmeier, L.J. and Griffin, J.M. (2006): Testing for Market Integration: Crude Oil, Coal, and Natural Gas, *The Energy Journal*, 27(2), 55-71.

[6] Balduzzi, P., Elton, E.J. and Green, T.C. (2001): Economic News and Bond Prices: Evidence from the US Treasury Market, *Journal of Financial and Quantitative Analysis*, 36(4), 523-543.

[7] Barsky, R. B. and Kilian L. (2002): Do We Really Know that Oil Caused the Great Stagflation? A Monetary Alternative, in: Ben S. Bernanke and Ken Rogoff (eds), *NBER Macroeconomics Annual 2001*, MIT Press: Cambridge, MA, pp. 137-183.

[8] Barsky, R. B. and Kilian L. (2004): Oil and the Macroeconomy Since the 1970s, *Journal of Economic Perspectives*, 18(4), 115-134.

[9] Basu, P. and Gavin, W.T. (2011): What Explains the Growth in Commodity Derivatives?, *Federal Reserve Bank of St. Louis Review*, 93(1), 37-48.

[10] Becker, K.G., Finnerty, J.E. and Kopecky, K.J. (1993): Economic News and Intraday Volatility in International Bond Markets, *Financial Analysts Journal*, 49(3), 81-86.

[11] Bjursell, J., Gentle, J.E. and Wang, G.H.K. (2009): Inventory Announcements, Jump Dynamics and Volatility in U.S. Energy Futures Markets, *George Mason University Manuscript*.

[12] Bollerslev, T., Cai, J. and Song, F.M. (2000): Intraday Periodicity, Long Memory Volatility, and Macroeconomic Announcement Effects in the US Treasury Bond Market, *Journal of Empirical Finance*, 7(1), 37-55.

[13] Brown, S.P.A. (2005): Natural Gas Pricing: Do Oil Prices Still Matter?, *The Southwest Economy*, July/August, 9-11.

[14] Brown, S.P.A. and Yucel, M.K. (2007): What Drives Natural Gas Prices?, *Federal Reserve Bank of Dallas*, Working Paper 0703.

[15] Buyuksahin, B., Haigh, M.S., Harris, J.H., Overdahl, J.A. and Robe, M.A. (2008): Fundamentals, Trader Activity and Derivative Pricing, *SSRN Working Paper*.

[16] Chang, C., Daouk, H. and Wang, A. (2009): Do Investors Learn about Analyst Accuracy? A Study of the Oil Futures Market, *The Journal of Futures Markets*, 29(5), 414-429.

[17] Cifarelli, G. and Paladino, G. (2010): Oil Price Dynamics and Speculation: A Multivariate Financial Approach, *Energy Economics*, 32(2), 363-372.

[18] CME Group website *www.cmegroup.com* (September 6, 2011).

[19] Dacorogna, M.M., Gancay, R., Muller, U., Olsen, R.B. and Pictet, O.V. (2001): An Introduction to High-Frequency Finance, *Academic Press*.

[20] Ding, Z., Granger, C.W.J. and Engle, R.F. (1993): A Long Memory Property of Stock Market Returns and a New Model, *Journal of Empirical Finance*, 1(1), 83-106.

[21] Downey, M. (2009): Oil 101, *Wooden Table Press LLC*.

[22] Ederington, L.H. and Guan, W. (2005): Forecasting Volatility, *The Journal of Futures Markets*, 25(5), 465-490.

[23] Ederington, L.H. and Lee, J.H. (1993): How Markets Process Information: News Releases and Volatility, *The Journal of Finance*, 48(4), 1161-1191.

[24] Elder, J. and Serletis, A. (2010): Oil Price Uncertainty, *Journal of Money, Credit and Banking*, 42(6), 1137-1159.

[25] Energy Information Administration of the U.S. Department of Energy website *www.eia.doe.gov* (September 6, 2011).

[26] Foster, A.J. (1995): Volume-Volatility Relationships for Crude Oil Futures Markets, *The Journal of Futures Markets*, 15(8), 929-951.

[27] Fujihara, R.A. and Mougoue M. (1997): Linear Dependence, Nonlinear Dependence and Petroleum Futures Market Efficiency, *The Journal of Futures Markets*, 17(1), 75-99.

[28] Futures Industry Magazine Annual Volume Survey (2008): 2008 A Wild Ride.

[29] Gay, G.D., Simkins, B.J. and Turac, M. (2009): Analyst Forecasts and Price Discovery in Futures Markets: The Case of Natural Gas Storage, *The Journal of Futures Markets*, 29(5), 451-477.

[30] Ghura, D. (1990): How Commodity Prices Respond to Macroeconomic News, *World Bank Policy, Research and External Affairs*, Working Paper No. 354.

[31] Girard, E., Sinha, A.K. and Biswas, R. (2008): The Relationship between Information Flow and Energy Futures Volatility, *The Review of Futures Markets*, 16(3), 271-300.

[32] Glosten L.R., Jagannathan, R. and Runkle, D.E. (1993): On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, *The Journal of Finance*, 48(5), 1779-1801.

[33] Gregoire, P. and Boucher, M. (2008): Maturity Effect and Storage Announcements: The Case of Natural Gas, *International Journal of Business Forecasting and Marketing Intelligence*, 1(1), 21-29.

[34] Guidi, M.G.D., Russell, A. and Tarbert, H. (2006): The Effect of OPEC Policy Decisions on Oil and Stock Prices, *OPEC Review*, 30(1), 1-18.

[35] Gwilym, O.A., McMillan, D. and Speight, A. (1999): The Intraday Relationship between Volume and Volatility in LIFFE Futures Markets, *Applied Financial Economics*, 9(6), 593-604.

[36] Hamilton, J. (1985): Historical Causes of Postwar Oil Shocks and Recessions, *The Energy Journal*, 6(1), 97-116.

[37] Hamilton, J. (2008): Oil and the Macroeconomy, *The New Palgrave Dictionary of Economics*, 2nd edition, in: S.N. Durlauf and L.E. Blume (eds), Palgrave Macmillan. The New Palgrave Dictionary of Economics Online, Palgrave Macmillan, http://www.dictionaryofeconomics.com (October 23, 2011).

[38] Hammoudeh, S.M., Yuan, Y. and McAleer, M. (2010): Exchange Rate and Industrial Commodity Volatility, *Kyoto Institute of Economic Research Discussion Paper Series*, No. 751.

[39] Hansen, B.E. (2001): The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity, *Journal of Economic Perspectives*, 15(4), 117-128.

[40] Hartley, P.R., Medlock III, K.B. and Rosthal, J.E. (2008): The Relationship of Natural Gas to Oil Prices, *The Energy Journal*, 29(3), 47-65.

[41] Herbert, J. H. (1995): Trading Volume, Maturity and Natural Gas Futures Price Volatility, *Energy Economics*, 17(4), 293-299.

[42] Hess, D., Huang, H. and Niessen, A. (2008): How Do Commodity Futures Respond to Macroeconomic News?, *Financial Markets and Portfolio Management*, 22(2), 127-146.

[43] Kilian, L. (2008): The Economic Effects of Energy Price Shocks, *Journal of Economic Literature*, 46(4), 871-909.

[44] Kilian, L. (2009): Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market, *American Economic Review*, 99(3), 1053-69.

[45] Kilian, L. and Park, C. (2009): The Impact of Oil Price Shocks on the U.S. Stock Market, *International Economic Review*, 50(4), 1267-1287.

[46] Kilian, L. and Vega, C. (2008): Do Energy Prices Respond to U.S. Macroeconomic News? A Test of the Hypothesis of Predetermined Energy Prices, *The Review of Economics and Statistics*, 93(2), 660-671.

[47] Kocagil, A.E. and Shachmurove Y. (1998): Return-Volume Dynamics in Futures Markets, *The Journal of Futures Markets*, 18(4), 399-426.

[48] Krichene, N. (2007): An Oil And Gas Model, *International Monetary Fund*, Working Paper 07/135.

[49] Kuper, G.H. and van Soest, D.P. (2006): Does Oil Price Uncertainty Affect Energy Use?, *The Energy Journal*, 27(1), 55-78.

[50] Leinert, L. (2010): How Do Unanticipated Discoveries Affect the Oil Price?, *CER-ETH Center of Economic Research at ETH Zurich*, Working Paper 10/140.

[51] Lin, S.X. and Tamvakis, M. (2010): OPEC Announcements and Their Effects on Crude Oil Prices, *Energy Policy*, 38(2), 1010-1016.

[52] Linn, S.C. and Zhu, Z. (2004): Natural Gas Prices and the Gas Storage Report: Public News and Volatility in Energy Futures Markets, *The Journal of Futures Markets*, 24(3), 283-313.

[53] Marzo, M. and Zagaglia, P. (2008): A Note on the Conditional Correlation between Energy Prices: Evidence from Future Markets, *Energy Economics*, 30(5), 2454-2458.

[54] Meyer, G. (September 21, 2011): Crude Gains on Big Stocks Draw, *Financial Times*, www.ft.com.

[55] McKenzie, M.D. (1999): Power Transformation and Forecasting the Magnitude of Exchange Rate Changes, *International Journal of Forecasting*, 15(1), 49-55.

[56] Mu, X. (2007): Weather, Storage, and Natural Gas Price Dynamics: Fundamentals and Volatility, *Energy Economics*, 29(1), 46-63.

[57] National Bureau of Economic Research website www.nber.org (September 6, 2011).

[58] Ohana, S. (2010): Modeling Global and Local Dependence in a Pair of Commodity Forward Curves with an Application to the US Natural Gas and Heating Oil Markets, *Energy Economics*, 32(2), 373-388.

[59] Onour, I. (2009): Natural Gas Markets: How Sensitive Are They to Crude Oil Price Changes?, *OPEC Energy Review*, 33(2), 111-124.

[60] Panagiotidis, T. and Rutledge, E. (2007): Oil and Gas Markets in the UK: Evidence for a Cointegrating Approach, *Energy Economics*, 29(2), 329-347.

[61] Pindyck, R.S. (2004): Volatility in Natural Gas and Oil Markets, *The Journal of Energy and Development*, 30(1), 1-19.

[62] Roache, S.K. and Rossi, M. (2009): The Effects of Economic News on Commodity Prices: Is Gold Just Another Commodity?, *International Monetary Fund*, Working Paper No. 09/140.

[63] Samuelson, P.A. (1965): Proof That Properly Anticipated Price Changes Fluctuate Randomly, *Industrial Management Review*, 6(2), 41-49.

[64] Schmidbauer, H. and Rosch, A. (2009): OPEC News Announcements: Effects on Oil Price Expectation and Volatility, In: The International Institute of Forecasters (ed.), *Proceedings of the 29th International Symposium on Forecasting ISF2009*, Hong Kong, June 21-24, 2009.

[65] Serletis, A. and Herbert, J. (1999): The Message in North American Energy Prices, *Energy Economics*, 21(5), 471-483.

[66] Serletis, A. and Rangel-Ruiz, R. (2004): Testing for Common Features in North American Energy Markets, *Energy Economics*, 26(3), 401-414.

[67] Susmel, R. and Thompson, A. (1997): Volatility, Storage and Convenience: Evidence from Natural Gas Markets, *The Journal of Futures Markets*, 17(1), 17-43.

[68] Switzer, L.N. and El-Khoury, M. (2007): Extreme Volatility, Speculative Efficiency, and the Hedging Effectiveness of the Oil Futures Markets, *The Journal of Futures Markets*, 27(1), 61-84.

[69] Tick Data website *www.tickdata.com* (September 6, 2011).

[70] Yucel, M.K. and Guo, S. (1994): Fuel Taxes and Cointegration of Energy Prices, *Contemporary Economic Policy*, 12(3), 33-41.

[71] Villar, J.A. and Joutz, F.L. (2006): The Relationship between Crude Oil and Natural Gas Prices, *Energy Information Administration, Office of Oil and Gas, Washington DC*, October.

[72] Wang, T., Wu, J. and Yang, J. (2008): Realized Volatility and Correlation in Energy Futures Markets, *The Journal of Futures Markets*, 28(10), 993-1011.

CHAPTER 2

RELATIONSHIP BETWEEN INTRADAY VOLATILITY AND VOLUME IN OIL AND GAS FUTURES MARKETS

1 Introduction

Do volatility and trading volume evolve in a unidirectional or bidirectional, contemporaneous or lagged relationship? This paper answers this question for the crude oil ("oil") and natural gas ("gas") futures, the two largest energy futures markets.

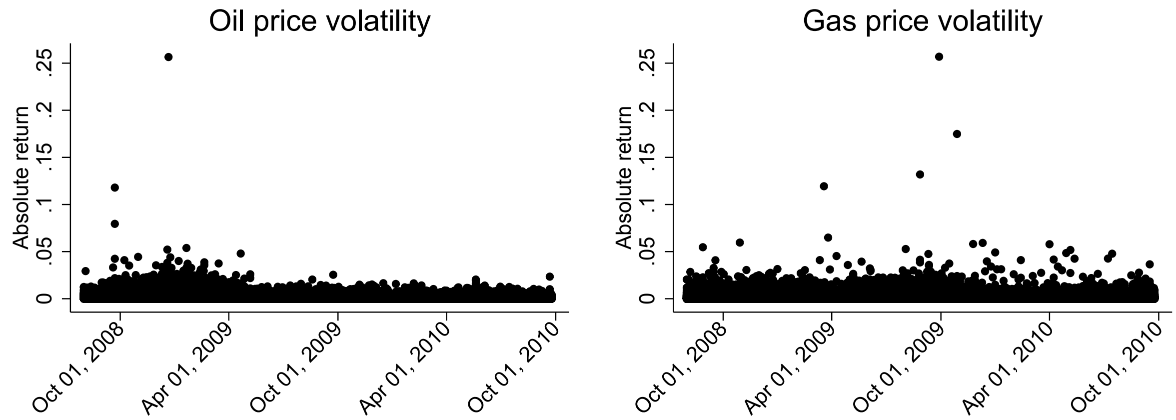
This relationship is important because it affects trading and government regulation. In October 2011, the U.S. Commodity Futures Trade Commission (CFTC) approved limits on the size of positions in the futures and swaps for 28 commodities, including oil and gas, as part of the Dodd-Frank Wall Street Reform and Consumer Protection Act that regulates financial markets.[1] These position limits were proposed to address concerns that recent spikes in volatility resulted from speculative traders suddenly flooding the commodities markets with large volumes.[2] See Figure 1 for oil and gas price volatility.

The new rule generated an intense debate among the public leading to 15,116 comments as well as within the CFTC where it passed with a narrow 3-2 majority. The dissenters highlighted insufficient evidence substantiating the regulation, a lack of costs and benefits

[1]Dodd-Frank Wall Street Reform and Consumer Protection Act (Pub.L. 111-203, H.R. 4173), Section 737.

[2]CFTC Final Regulations on Position Limits for Futures and Swaps, CFTC Q & A Position Limits for Derivatives, and CFTC Position Limits for Futures and Swaps: Final Rule and Interim Final Rule.

Figure 1: Crude oil and natural gas futures price volatility



Notes: These figures show the price volatility of the NYMEX crude oil and natural gas nearby futures contracts defined as absolute return.

analysis, and possible adverse effects on liquidity of the markets. The Securities Industry and Financial Markets Association and the International Swaps and Derivatives Association, two financial industry groups, challenged the rule by filing a lawsuit in December 2011 that was dismissed by a U.S. Court of Appeals in Washington, D.C. in January 2012. The rules are expected to come into effect in August 2012 after the CFTC finalizes the definition of swaps and collects one year of open interest data. In the meantime, the discussion is raging on, most recently with 19 senators filing a friend-of-the-court brief in support of the CFTC in April 2012.[3]

Underlying the reasoning behind the CFTC regulation is the assumption that volume drives volatility. However, in spite of the disagreements and the considerable effect the CFTC regulation may have on the commodities markets, we know surprisingly little about the volume-volatility relationship in the oil and gas futures markets as evidenced by the review of the empirical research in Section 2. The main shortcoming of the previous studies

[3]Wutkowski and Alper (2012).

has to do with the frequency of data used in the analysis. All previous papers used daily or weekly data even though such a low frequency does not capture the volatility-volume relationship accurately because it assumes that information arrives on the markets only once a day or once a week.

This paper improves on the previous studies in three ways. First, it uses high-frequency, *intraday* New York Mercantile Exchange (NYMEX) oil and gas futures prices and volume. This allows studying the volatility-volume relationship at a frequency at which information really arrives on the market. Second, this paper includes trading not only during the day but also during the night as opposed to previous studies that focus only on trading during the day. Third, in contrast to previous studies that analyze primarily the nearby futures contract for the following month, this paper includes contracts with all maturities. Analyzing the entire maturity structure is important since the CFTC rule imposes different position limits on the nearby month contract and the longer maturities contracts.

For the nearby oil and gas futures contracts, the results show that past values of volume help explain volatility. Past values of volatility have explanatory power for volume only when absolute return is used as the volatility measure. When the conditional variance from GARCH models is used as the volatility measure, this Granger-causality disappears. Therefore, the absolute return model shows bidirectional Granger-causality whereas the conditional variance model suggests only a uni-directional causality.

Although the two volatility measures give different answers for the direction from volatility to volume, they agree on lagged values of volume Granger-causing volatility. While Granger causality does not imply true causality between the variables, this dynamic relationship suggests that volume may be driving volatility which is relevant for regulators and policy-makers.

To highlight the importance of analyzing the volatility-volume relationship using intraday data rather than end-of-the-day data, the analysis is repeated with end-of-the-day data. The results differ substantially. These differences show that relying on an analysis that uses end-of-day data could lead to misguided policy and regulations. The complexity of the volatility-volume relationship is highlighted by showing that the Granger-causality changes for contracts with longer maturities.

In addition to providing guidance for regulators and policy-makers, studying the volatility-volume relationship is important for several other reasons. First, it improves our understanding of how information arrives on the market, one of the fundamental processes in economics. Second, it shows whether markets are efficient: if past volume can be used to forecast volatility, markets are not efficient. Third, if volume predicts volatility, it can be used to design trading strategies. Fourth, from a methodology standpoint, it shows whether contemporaneous or lagged volume should be used in models, such as Halova (2011), to account for unobservable sources of volatility.

The remainder of this paper is organized as follows. Section 2 reviews the literature, Section 3 describes the data, Section 4 outlines the methodology, Section 5 presents the results, and Section 6 summarizes the conclusions.

2 Literature Review

In spite of the importance of the volatility-volume relationship, previous research has come to conflicting conclusions on the dynamics between these two variables. Fourteen studies have investigated the volatility-volume relationship in the oil and/or gas futures markets. See Table 1 for an overview of the data used in these papers.[4]

Table 1: Studies on relationship between volatility and volume

Author	**Oil/Gas**	**Period**	**Frequency**	**Contracts**
Serletis (1992)	O	1987-1990	Daily	Contract-by-contract
Foster (1995)	O	1984-1994	Daily	1
Herbert (1995)	G	1990-1994	Daily	1
Fujihara & Mougoue (1997)	O	1984-1993	Daily	1
Kocagil & Shachmurove (1998)	O	1980-1995	Daily	1
Moosa & Korczak (1999)	O	1985-1996	Daily	1, 3
Moosa & Silvapulle (2000)	O	1985-1996	Daily	3, 6
Moosa et al (2003)	O	1985-1996	Daily	3, 6
Bhar & Hamori (2005)	O	1990-2000	Daily	1
Serletis & Shahmoradi (2006)	G	1990-2002	Daily	1
Girard, Sinha & Biswas (2008)	O&G	1992-2006	Daily	1
Ripple & Moosa(2009)	O	1995-2005	Daily	1, Contract-by-contract
Buyuksahin & Harris (2011)	O	2000-2009	Daily	1, All contracts combined
Bohl & Stephan (2012)	O&G	1992-2010	Weekly	All contracts combined

Notes: In the Contracts column, the number indicates the the futures contract analyzed. For example, 1 stands for the first month contract, i.e., nearby contract. Contract-by-contract indicates that an individual contract is analyzed from inception to expiry (or for some shorter period) which is common when studying the effect of maturity on futures prices.

Six of these studies concluded that past changes in volume help explain changes in volatility and past changes in volatility help explain changes in volume, indicating a bidirectional relationship: Foster (1995), Herbert (1995), Kocagil and Shachmurove (1998), Moosa and

[4]In a related study, Girma and Mougoue (2002) investigate the relationship between volatility and volume for petroleum futures spreads, i.e., various spreads between crude oil, heating oil and unleaded gasoline.

Silvapulle (2000), Moosa, Silvapulle and Silvapulle (2003), and Girard, Sinha and Biswas (2008). Two studies concluded that past changes in volatility help explain changes in volume but past changes in volume do not help explain changes in volatility, indicating a unidirectional relationship from volatility to volume: Bhar and Hamori (2005), and Buyuksahin and Harris (2011).[5] One study indicated a unidirectional relationship from volume to volatility: Serletis (1992). Two studies found effects from volume to volatility but did not investigate feedbacks in the opposite direction: Ripple and Moosa (2009), and Bohl and Stephan (2012).[6]

The main shortcoming of the previous studies has to do with the frequency of data used in the analysis. All previous papers used daily or weekly data even though such a low frequency does not capture the volatility-volume relationship accurately because it assumes that information arrives on the markets only once a day or once a week. To our knowledge, this paper is the first study to take advantage of high-frequency, *intraday* New York Mercantile Exchange (NYMEX) oil and gas futures prices and volume.[7] This allows studying the volatility-volume relationship at a frequency at which information really arrives on the market which is the relevant frequency for regulations such as the CFTC position limits.

[5] In contrast to the other studies, Buyuksahin and Harris (2011) use trader positions rather than trading volume.

[6] The remaining three studies focused solely on the contemporaneous relationship between the two variables: Fujihara and Mougoue (1997), Moosa and Korczak (1999), and Serletis and Shahmoradi (2006).

[7] The only exception is an unpublished study by Chevalier and Sevi (2011) that suffers from several methodological issues. Their contemporaneous ordinary least squares regression with volatility modeled as a function of contemporaneous volume suffers from endogeneity, runs only in one direction and precludes investigating intertemporal dynamics. In addition, the authors included only the nearby contract traded during the day.

This study also adds to the literature on the intraday volume-volatility relationship in other assets by, among others, Jain and Joh (1988) for the S&P500 stock index, Chan and Chung (1993) for the Major Market Index (MMI) and the MMI futures contract, Gwilym, McMillans and Speight (1999) for the FTSE-100 stock index, Short Sterling 3-month interest rate futures contract and Long Gilt government bond futures contract, Xu and Wu (1999) for 141 stocks traded on the New York Stock Exchange (NYSE), Darrat, Rahman and Zhong (2003) for 30 stocks comprising the Dow Jones Industrial Average, Huang and Masulis (2003) for 100 stocks comprising the FTSE-100 stock index, McMillan and Speight (2004) for the FTSE-100 futures contracts, Chen (2007) for live cattle, pork bellies, German mark and Swiss franc futures, Darrat, Zhong and Cheng (2007) for 205 stocks traded on NYSE, Hatrick, So, Chung and Deng (2010) for the HSBC stock, and Hussain (2011) for the German blue-chip index DAX30.

3 Data

Oil and gas futures contracts are traded on two main exchanges: the NYMEX in Chicago and the InterContinental Exchange (ICE) in London. This paper focuses on the NYMEX futures data because NYMEX is more liquid than ICE.[8] This proprietary data is provided by Tick Data, Inc., a company that specializes in intraday time series data for equities, futures and options.[9] The data are transaction data, i.e., not bid-ask quotes.

Light Sweet Crude Oil futures started trading on NYMEX under the symbol CL on March 30, 1987. The standardized contract is for 1,000 barrels with the price quoted in U.S.

[8]See Downey (2009) for description of the oil market.

[9]See www.tickdata.com.

dollars per barrel. Natural gas futures started trading on NYMEX under the symbol NG on January 4, 1993. The standardized contract is for 10,000 million British thermal units (mmBTU) with the price quoted in U.S. dollars per mmBTU.[10]

The trading takes place on Sundays through Fridays from 6:00 p.m. until 5:15 p.m. Eastern Time on the CME Globex and CME ClearPort electronic platforms and on Mondays through Fridays from 9:00 a.m. until 2:30 p.m. in the open outcry session.[11] The "day" trading is defined as trading on Mondays through Fridays from 9:00 a.m. until 2:30 p.m. Eastern Time. The "night" trading is defined as trading outside of this period.

The 10-minute interval is used following Halova (2011). For example, the 9:10 interval contains trades from 9:00:00 until 9:09:59. The sample period is limited to August 1, 2008 through September 24, 2010 because comprehensive information on volume is not available prior to this period. The sample period contains 77,051 10-minute intervals on 669 days. Missing prices are set equal to the previous prices.[12]

Futures contracts with different maturities are available. The shortest maturity, called the nearby contract, is for the following month. The longest maturity for oil and gas is nine and twelve years, respectively.

[10]See website of CME Group that owns NYMEX: www.cmegroup.com.

[11]Only trades that took place on electronic platforms, i.e., Globex and ClearPort, are included. Trades that occurred in the open outcry, i.e., in the pit, are excluded because volume data is not available for them. These trades comprise a small portion of total trades, accounting for only 0.01% and 0.05% of all oil and gas trades, respectively.

[12]There are no such missing observations for oil trades during the day. There are only 3 missing observations for gas trades during the day. There are 1,445 and 9,837 missing observations for oil and gas trades during the night, respectively.

For oil, trading ceases on the third business day prior to the twenty-fifth calendar day of the month preceding the delivery. For gas, trading ceases three business days prior to the first day of the delivery month. At expiration, oil and gas have to be physically delivered to Cushing, OK and Henry Hub, LA, respectively.[13] Very few market participants choose to make physical delivery at contract expiration. Instead, they roll over their positions into a new contract. Two methodologies are used in this paper to create a continuous file of the futures contracts. In the first methodology, the current contract is used until the expiration date. However, since market participants begin rolling over their positions on the preceding days, trading may be thin during the last days before the contract expiration date, causing unusual price movements. Therefore, the second methodology switches to the next contract as soon as its daily contract volume exceeds the current contract volume. The results do not materially differ between the two methods. Therefore, only results using the expiration date method are reported. This contract is referred to as "nearby contract".[14]

4 Methodology

4.1 Measures of Volatility

Let P_j be the price at the end of 10-minute interval j. Since both oil and gas price series are non-stationary, the return, i.e., the difference between the log price at the end of interval j

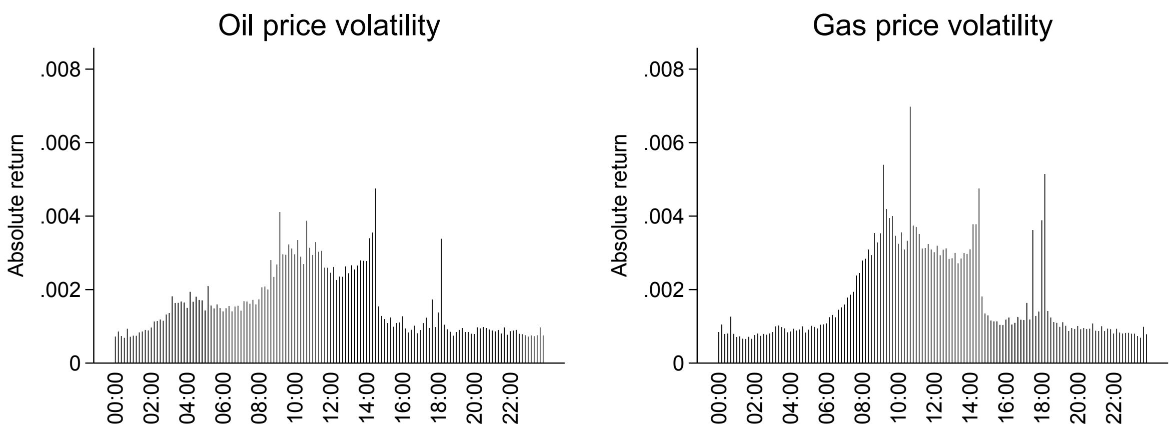
[13] See website of CME Group that owns NYMEX: www.cmegroup.com.

[14] Another method chooses a particular date, e.g., the 15th calendar day of each month to switch from the nearby contract to the second nearby contract in an attempt to model the market participants' behavior. This approach is not used because it is arbitrary.

and the log price at the end of interval *j-1*, $R_j \equiv \ln(P_j) - \ln(P_{j-1})$, is used. The resulting return series are stationary as gauged by both the augmented Dickey-Fuller test and the Elliott, Rothenberg, and Stock (1996) DF-GLS test.

Two measures of price volatility are used. First, the volatility is defined as the absolute value of price returns, $|R_j|$, following, for example, Kocagil and Shachmurove (1998), and Moosa and Silvapulle (2000).[15] This absolute return is stationary for both oil and gas.

Figure 2: Intraday pattern of crude oil and natural gas futures price volatility



Notes: The individual bars represent absolute returns averaged across all days for each 10-minute interval.

Second, as the oil and gas prices exhibit time-varying volatility depicted not only in Figure 1 but also in Figure 2 that displays the intraday volatility pattern, generalized autoregressive conditional heteroskedasticity (GARCH) models lend themselves as tools for analyzing the data.[16] A model with one ARCH term, one GARCH term and Gaussian error

[15]Absolute return has also been used as a measure of volatility in other contexts, such as, to analyze the effect of news announcements on price volatility by, for example, Ding, Granger and Engle (1993), Ederington and Lee (1993), Gwilym, McMillan and Speight (1999), McKenzie (1999), Bollersev, Cai and Song (2000), and Ederington and Guan (2005).

[16]The volatility spikes correspond to the intervals at the beginning and at the end of the 9:00 am - 2:30 pm

distribution that allows for an asymmetric reaction to positive and negative innovations is implemented:

$$R_j = \alpha + \beta R_{j-1} + \varepsilon_j, \tag{1}$$

$$h_j^2 = \nu + \phi \varepsilon_{j-1}^2 + \rho h_{j-1}^2 + \omega \varepsilon_{j-1}, \tag{2}$$

where equation (1) is the mean equation, equation (2) is the conditional variance equation, and the distribution of the error conditional on an information set at time j, Ψ_j, is assumed to be $\varepsilon_j | \Psi_j \sim N[0, h_j^2]$. The term $\omega \varepsilon_{j-1}$ in equation (2) captures the asymmetry because positive innovations $\varepsilon_j > 0$ are allowed to have different effects on the conditional variance than negative innovations $\varepsilon_j < 0$. The conditional variance from this model is the second measure of volatility.[17]

4.2 Measures of Volume

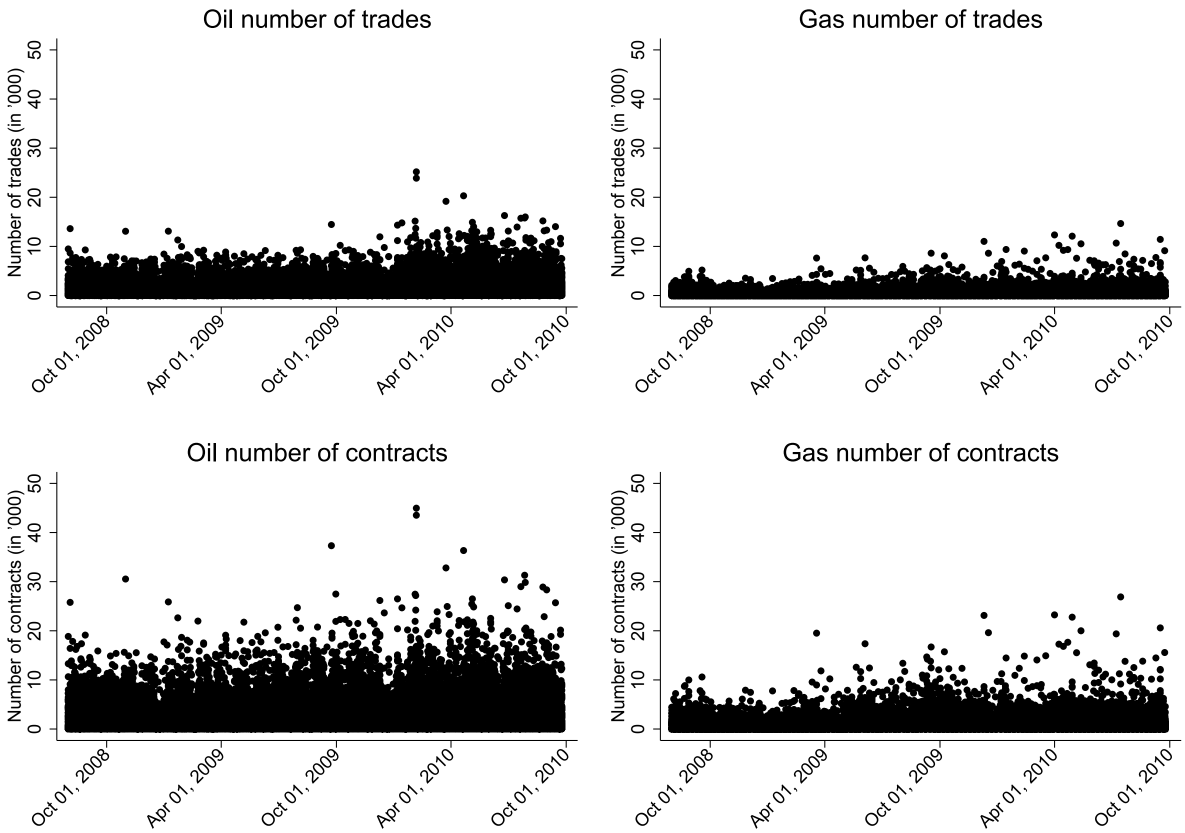
Two measures of volume are used for the analysis. First, the number of trades is used to reflect the number of executed trades. Second, the number of contracts is used to reflect the total volume traded because a trade may involve more than one contract. The number of contracts per trade for oil ranges from 1 to 100 with a mean of 1.7. The number of contracts per trade for gas ranges from 1 to 281 with a mean of 1.8.

trading session, the first interval after the 5:15 p.m. - 6:00 p.m. break, and the 10:40 a.m. interval that reflects the effect of the weekly oil and gas inventory reports released by the Energy Information Administration of the Department of Energy as described in Halova (2011).

[17] As a benchmark, the standard symmetric GARCH(1,1) model that includes only one ARCH term and one GARCH term with the Gaussian distribution of errors is also used. The results do not differ from those reported in Section 5 for the asymmetric GARCH.

Both the number of trades and the number of contracts are stationary for oil as well as gas. Both variables are divided by 1,000 to scale their values. See Figure 3 for the time series of these volume measures.

Figure 3: Crude oil and natural gas futures volume



Notes: This figure shows the number of trades and number of contracts in thousands for the NYMEX crude oil and natural gas nearby futures contracts.

4.3 Granger Causality Tests

A bivariate vector autoregression (VAR) represented by equations (3) and (4) is used to model the dynamic relationship between volatility and volume. This model is run separately for oil and gas using the above two measures of volatility and two measures of volume.

$$Volatility_j = \alpha_1 + \sum_{i=1}^{I} \beta_i Volatility_{j-i} + \sum_{i=1}^{I} \gamma_i Volume_{j-i} + \sum_{k=1}^{139} \delta_k D_{j,k} + \varepsilon_{1,j} \tag{3}$$

$$Volume_j = \alpha_2 + \sum_{i=1}^{I} \beta_i Volume_{t-j} + \sum_{i=1}^{I} \gamma_i Volatility_{j-i} + \sum_{k=1}^{139} \delta_k D_{j,k} + \varepsilon_{2,j}, \tag{4}$$

where $D_{j,k}$ is a vector of dummies for each 10-minute interval to account for intraday seasonality evidenced in Figure 2.[18]

For each specification, the optimal lag length I is determined by the Akaike's information criterion (AIC) and Schwarz's Bayesian information criterion (SBIC) to account for autocorrelation in the series.

A Granger causality test is then used to assess whether lagged values of volume have explanatory power for volatility and whether lagged values of volatility have explanatory power for volume. The null hypothesis in this Granger causality test is that volume does not cause volatility, i.e., $\gamma_1 = ... = \gamma_I = 0$ in Equation (3), and volatility does not cause volume, i.e., $\gamma_0 = ... = \gamma_I = 0$ in Equation (4). The alternative hypothesis is that $\gamma_i \neq 0$ for at least one i.

5 Results

5.1 Absolute Return as Measure of Volatility

Table 2 reports results of the bivariate VAR with the absolute return measuring volatility and the number of trades measuring volume. 21 lags corresponding to 210 minutes and 19

[18]In 24 hours, there are 144 intervals of 10 minutes. Since the market is closed during the intervals from 5:15 p.m. until 6 p.m. and one dummy is left out to avoid multicollinearity, there are 139 dummies.

lags corresponding to 190 minutes are used as the optimal lag lengths per the SBIC criterion for oil and gas, respectively.[19]

Table 2: Granger causality tests with absolute return as measure of volatility

Commodity	Causality direction	Lags	Chi-square	P-value	Causality
Crude oil	Volatility → Volume	21	228.18	0.000	Yes
Crude oil	Volume → Volatility	21	443.64	0.000	Yes
Natural gas	Volatility → Volume	19	148.42	0.000	Yes
Natural gas	Volume → Volatility	19	345.50	0.000	Yes

The Chi-square and p-values show that lagged values of volume have explanatory power for volatility and lagged values of volatility have explanatory power for volume. Therefore, Granger causality runs in both directions as observed by Foster (1995), Herbert (1995), Kocagil and Shachmurove (1998), Moosa and Silvapulle (2000), Moosa, Silvapulle and Silvapulle (2003), and Girard, Biswas and Sinha (2008).

The bivariate VAR is then run with the number of contracts as the volume measure. The results do not change, indicating robustness against different measures of volume.

[19] The AIC criterion indicates 79 and 41 lags as the optimum lag lengths for oil and gas, respectively. As a robustness check, the VARs are also run with these longer lag lengths. However, since the results do not differ, they are not reported. Also, the dummies for the 10-minute intervals are jointly significant indicating intraday seasonality in all specifications. Both the AIC and SBIC optimal lengths are higher when the VAR excludes the 10-minute interval dummies.

5.2 Conditional Variance as Measure of Volatility

Table 3 reports results for the asymmetric GARCH model. All ARCH terms are significant, suggesting that the asymmetric GARCH model is appropriate for the intraday volatility data. Although small in magnitude, the asymmetric term shows a negative sign for both oil and gas, indicating that volatility of the commodity prices responds more strongly to negative price changes.

Table 3: Asymmetric GARCH model

Commodity	**Variable**	**Coefficient**	**Standard error**
Crude oil	1^{st} lag of return	-.00723**	.00308
	ARCH term	.12719***	.00061
	GARCH term	.88222***	.00023
	Asymmetric term	-.00003***	2.59e-06
Natural gas	1^{st} lag of return	-.00723**	.00308
	ARCH term	.12719***	.00061
	GARCH term	.88222***	.00023
	Asymmetric term	-.00003***	2.59e-06

*Note: *** and ** represent 99% and 95% significance levels, respectively.*

Table 4 shows results of the bivariate VAR with the conditional variance from the asymmetric GARCH model measuring volatility and the number of trades measuring volume. 9 lags corresponding to 90 minutes and 11 lags corresponding to 110 minutes are used as the optimal lag lengths per the SBIC criterion for oil and gas, respectively.[20]

[20]The AIC criterion indicates 22 and 24 lags as the optimum lag lengths for oil and gas, respectively. As

Table 4: Granger causality tests with conditional variance as measure of volatility

Commodity	Causality direction	Lags	Chi-square	P-value	Causality
Crude oil	Volatility → Volume	9	10.10	0.343	No
Crude oil	Volume → Volatility	9	87.21	0.000	Yes
Natural gas	Volatility → Volume	11	13.74	0.247	No
Natural gas	Volume → Volatility	11	966.55	0.000	Yes

The Chi-square and p-values show that lagged values of volume have explanatory power for volatility but lagged values of volatility do not have explanatory power for volume which agrees with Serletis (1992). Therefore, in contrast to the bivariate VAR where volatility is measured by the absolute return, Granger causality is unidirectional when volatility is measured by the conditional variance.

Although the two measures of volatility give different answers for the direction from volatility to volume, they agree on lagged values of volume Granger-causing volatility. While Granger causality does not imply true causality, this dynamic relationship suggests that volume may be driving volatility which is of importance to regulators and policy-makers considering position limits as a tool for controlling volatility.

The bivariate VAR is then run with the number of contracts as the measure of volume. Again, the results do not change, indicating robustness against different volume measures.

---

a robustness check, the VARs are also run with these longer lag lengths. However, since the results do not differ, they are not reported. Also, the dummies for the 10-minute intervals are jointly significant indicating intraday seasonality in all specifications. Both the AIC and SBIC optimal lengths are higher when the VAR excludes the 10-minute interval dummies.

5.3 Comparison to Low-Frequency Data Results

To underscore the importance of analyzing the volatility-volume relationship using intraday rather than end-of-the-day data, the same bivariate VARs are run with end-of-the-day data. Table 5 reports results for the VAR that uses absolute return as the measure of volatility. In comparison to the intraday analysis in Table 2 that showed a bidirectional relationship for both commodities, gas now shows only a unidirectional relationship from volume to volatility.

Table 5: Granger causality tests with absolute return as measure of volatility using end-of-day data

Commodity	Causality direction	Lags	Chi-square	P-value	Causality
Crude oil	Volatility → Volume	2	5.76	0.000	Yes
Crude oil	Volume → Volatility	2	18.88	0.000	Yes
Natural gas	Volatility → Volume	2	21.62	0.303	No
Natural gas	Volume → Volatility	2	40.24	0.000	Yes

Similarly, Tables 4 and 6 allow comparing results between intraday and end-of-the-day data for the bivariate VAR that uses asymmetric GARCH conditional variance as the measure of volatility. Again, the results differ. For oil, the intraday data shows unidirectional causality from volume to volatility whereas the end-of-the-day data indicates a unidirectional causality in the opposite direction. For gas, the intraday data also shows unidirectional causality from volume to volatility but the end-of-the-day data suggests bidirectional causality.

These differences show that relying on an analysis that uses end-of-day data could lead to misguided policy and regulations.

Table 6: Granger causality tests with conditional variance as measure of volatility using end-of-day data

Commodity	Causality direction	Lags	Chi-square	P-value	Causality
Crude oil	Volatility → Volume	1	9.25	0.002	Yes
Crude oil	Volume → Volatility	1	0.27	0.600	No
Natural gas	Volatility → Volume	2	6.97	0.031	Yes
Natural gas	Volume → Volatility	2	4.77	0.092	Yes

5.4 Longer Maturities

As Table 1 shows, the previous studies focused primarily on the nearby futures contract for the following month. In contrast, this paper includes contracts of longer maturities. Analyzing the entire maturity structure is important since the CFTC rule imposes different position limits on the nearby month contract and the longer maturities contracts.

In Table 7, Contract 1 represents the nearby contract reported in Table 2. Contract 2 then stands for the second nearby contract, etc. This analysis reveals a complex relationship between volatility and volume that changes with the length of the futures contracts.[21]

[21] This result may partially be due to contracts with longer maturities trading less frequently, causing more missing observations than the nearby contract. Oil contracts 2 through 6 have 0.42%, 6.79%, 25.69%, 46.43% and 60.76% missing observations, respectively. Gas contracts 2 through 6 have 1.19%, 5.83%, 11.83%, 17.97% and 24.81% missing observations, respectively. As in the nearby contract, the missing observations are set equal to the previous observation.

Table 7: Granger causality tests with absolute return as measure of volatility for longer maturities

Commodity	**Causality direction**	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6
Crude oil	Volatility → Volume	Yes	Yes	Yes	No	No	No
Crude oil	Volume → Volatility	Yes	Yes	No	No	No	No
Natural gas	Volatility → Volume	Yes	Yes	Yes	No	No	Yes
Natural gas	Volume → Volatility	Yes	Yes	No	Yes	Yes	Yes

6 Conclusion

This paper studies whether volatility and trading volume evolve in a unidirectional or bidirectional, contemporaneous or lagged relationship in the crude oil and natural gas futures markets. Previous research has come to conflicting conclusions. The main shortcoming of the previous studies has to do with the frequency of data used in the analysis. All previous papers used daily or weekly data even though such a low frequency does not capture the volatility-volume relationship accurately because it assumes that information arrives on the markets only once a day or once a week. This paper overcomes this limitation by using high-frequency, intraday data.

For the nearby oil and gas futures contracts, the results show that past values of volume help explain volatility. Past values of volatility have explanatory power for volume only when absolute return is used as the volatility measure. When the conditional variance from GARCH models is used as the volatility measure, this Granger-causality disappears. Therefore, the absolute return model shows bidirectional Granger-causality whereas the conditional variance model suggests only a uni-directional causality.

Although the two volatility measures give different answers for the direction from volatility to volume, they agree on lagged values of volume Granger-causing volatility. While Granger causality does not imply true causality between the variables, this dynamic relationship suggests that volume may be driving volatility. This is relevant since regulators and policy-makers both in the U.S. and Europe are considering position limits as a tool for controlling volatility.

To highlight the importance of analyzing the volatility-volume relationship using intraday data, the analysis is repeated with end-of-the-day data and it is shown that results differ substantially. The complexity of the volatility-volume relationship is highlighted by showing that the Granger-causality changes for contracts with longer maturities.

In addition to providing guidance for regulators and policy-makers, analysis of the volatility-volume relationship is important for several reasons. First, it improves our understanding of how information arrives on the market, one of the fundamental processes in economics. There are two main theories on this subject. The Mixture of Distributions Hypothesis (MDH) proposed by Clark (1973), and Epps and Epps (1976) states that volatility and volume change simultaneously when new information arrives on the market. Therefore, the relationship between volatility and volume is contemporaneous. Past values of volume cannot be used to forecast current volatility and past values of volatility cannot be used to forecast current volume. The Sequential Information Arrival Hypothesis (SIAH) by Copeland (1976) states that traders receive information sequentially. In the initial equilibrium, all traders have the same information. As traders receive information, they execute their trades, creating a series of intermediate equilibria. After all traders receive the information, the final equilibrium is reached. As a result, lagged volume affects current volatility.

Results from this paper support the SIAH for the oil and gas futures markets.

Second, the relationship between volatility and volume shows whether markets are efficient: if past volume can be used to forecast volatility, markets are not efficient. Therefore, the lagged volume having explanatory power for volatility indicates some market inefficiency, at least at the 10-minute interval frequency.

Third, since volume predicts volatility, it can be used to design trading strategies. Fourth, from a methodology standpoint, the results indicate that lagged volume can be used in models such as Halova (2011) to control for unobservable sources of volatility.

In future research, two additional robustness checks will be performed. First, the Hansen and Lunde (2006) estimator of intraday volatility will be implemented as another measure of volatility. Second, as McMillan and Speight (2004) as well as Hussain (2011) point out, GARCH models assume a geometric decay in the autocorrelation structure and may not account for intraday periodicity in data. Therefore, as a further robustness check, the GARCH models will be run in two steps. In the first step, an ordinary least squares regression will be run where the return, R_j, will be regressed only on the 10-minute interval dummies to remove the intraday periodicity effect. In the second step, residuals from this deseasonalized return regression will be used in the asymmetric GARCH mean equation. The conditional volatility from this model will then become the volatility measure in the bivariate VAR.

The results from this paper offer three interesting extensions using the same data set. First, since the information arrival process may differ between the day and night trading sessions, an analysis will be completed to compare them. Second, since the Granger causality tests address only lagged relationships, the contemporaneous relationship will be studied using a triangular system building on results from the bivariate VAR that showed a unidirectional causality from volume to volatility. In this system, the contemporaneous volatility will be a function of lagged volatility and contemporaneous as well as lagged volume. However, the contemporaneous volume will be a function only of lagged volume and volatility. Third, the analysis will be repeated with intervals of other lengths, for example, 15 and 30 minutes. This will serve not only as a robustness check but also as a market efficiency test. Explanatory power of lags of one variable for the other variable implies market inefficiency since the markets have not absorbed all the available information. Applying the same analysis to longer intervals will show how market efficiency varies with longer time horizons.

The results also lead to three applications to new data. First, once the CFTC position limits come into effect, the sample period could be extended to compare the results before and after the regulation implementation. Second, the CFTC collects daily data on trader composition to distinguish between trader categories, such as oil and gas companies versus speculators. This data could be incorporated into the existing analysis to see if the relationship between volume and volatility differs on days when speculative trading is high. Finally, the analysis could be extended to the other 26 agricultural, energy and metal commodities affected by the regulation to provide a comprehensive analysis of the commodity futures markets.

References

[1] Bhar, R. and Hamori, S. (2005): Causality in Variance and the Type of Traders in Crude Oil Futures, *Energy Economics*, 27(3), 527-539.

[2] Bohl, M.T. and Stephan, P.M. (2012): Does Futures Speculation Destabilize Spot Prices? New Evidence for Commodity Markets, *Unpublished*.

[3] Bollerslev, T., Cai, J. and Song, F.M. (2000): Intraday Periodicity, Long Memory Volatility, and Macroeconomic Announcement Effects in the US Treasury Bond Market, *Journal of Empirical Finance*, 7(1), 37-55.

[4] Buyuksahin, B. and Harris, J.H. (2011): Do Speculators Drive Crude Oil Futures Prices?, *The Energy Journal*, 32(2), 167-202.

[5] Chan, K. and Chung, Y.P. (1993): Intraday Relationships among Index Arbitrage, Spot and Futures Price Volatility, and Spot Market Volume: A Transactions Data Test, *Journal of Banking and Finance*, 17(4), 663-687.

[6] Chen, H. (2007): Intraday Trading by Floor Traders and Customers in Futures Markets: Whose Trades Drive the Volatility-Volume Relation?, *Quarterly Journal of Business & Economics*, 46(4), 45-61.

[7] Chevalier, J. and Sevi, B. (2011): On the Volatility-Volume Relationship in Energy Futures Markets Using Intraday Data, *Universite de Paris Ouest Nanterre La Defense Working Paper*, No. 2011-16.

[8] Clark, P.K. (1973): A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices, *Econometrica*, 41(1), 135-156.

[9] CME Group website *www.cmegroup.com* (September 6, 2011).

[10] Commodity Futures Trading Commission Position Limits for Futures and Swaps: Final Rule and Interim Final Rule (November 18, 2011), *Federal Register*, 76(223).

[11] Commodity Futures Trading Commission website www.cftc.org (October 18, 2011): CFTC Final Regulations on Position Limits for Futures and Swaps.

[12] Commodity Futures Trading Commission website www.cftc.org (October 18, 2011): CFTC Q & A Position Limits for Derivatives.

[13] Copeland, T.E. (1976): A Model of Asset Trading Under the Assumption of Sequential Information Arrival, *The Journal of Finance*, 31(4), 1149-1168.

[14] Darrat, A.F., Rahman, S. and Zhong, M. (2003): Intraday Trading Volume and Return Volatility of the DJIA Stocks: A Note, *Journal of Banking* & *Finance*, 27(10), 2035-2043.

[15] Darrat, A.F., Zhong, M. and Cheng, L.T.W. (2007): Intraday Volume and Volatility Relations with and without Public News, *Journal of Banking* & *Finance*, 31(9), 2711-2729.

[16] Ding, Z., Granger, C.W.J. and Engle, R.F. (1993): A Long Memory Property of Stock Market Returns and a New Model, *Journal of Empirical Finance*, 1(1), 83-106.

[17] Dodd-Frank Wall Street Reform and Consumer Protection Act (Pub.L. 111-203, H.R. 4173).

[18] Downey, M. (2009): Oil 101, *Wooden Table Press LLC*.

[19] Ederington, L.H. and Guan, W. (2005): Forecasting Volatility, *The Journal of Futures Markets*, 25(5), 465-490.

[20] Ederington, L.H. and Lee, J.H. (1993): How Markets Process Information: News Releases and Volatility, *The Journal of Finance*, 48(4), 1161-1191.

[21] Elliott, G., Rothenberg, T.J. and Stock, J.H. (1996). Efficient Ttests for an Aautoregressive Unit Root, *Econometrica*, 64(4), 813-836.

[22] Epps, T.W. and Epps, M.L. (1976): The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distributions Hypothesis, *Econometrica*, 44(2), 305-321.

[23] Foster, A.J. (1995): Volume-Volatility Relationships for Crude Oil Futures Markets, *The Journal of Futures Markets*, 15(8), 929-951.

[24] Fujihara, R.A. and Mougoue M. (1997): Linear Dependence, Nonlinear Dependence and Petroleum Futures Market Efficiency, *The Journal of Futures Markets*, 17(1), 75-99.

[25] Girard, E., Sinha, A.K. and Biswas, R. (2008): The Relationship between Information Flow and Energy Futures Volatility, *The Review of Futures Markets*, 16(3), 271-300.

[26] Girma, P.B. and Mougoue, M. (2002): An Empirical Examination of the Relation between Futures Spreads Volatility, Volume, and Open Interest, *The Journal of Futures Markets*, 22(11), 1083-1102.

[27] Gwilym, O.A., McMillan, D. and Speight, A. (1999): The Intraday Relationship between Volume and Volatility in LIFFE Futures Markets, *Applied Financial Economics*, 9(6), 593-604.

[28] Halova, M.W. (2011): Gas Does Affect Oil: Evidence from Intraday Prices and Inventory Reports, *To be published one day, hopefully.*

[29] Hansen, P.R. and Lunde, A. (2006): Realized Variance and Market Microstructure Noise, *Journal of Business and Economic Statistics*, 24(2), 127-161.

[30] Hatrick, K., So, M.K.P., Chung, S.W. and Deng R. (2010): Dynamic Relationship among Intraday Realized Volatility, Volume and Number of Trades, *Asia-Pacific Financial Markets*, 18(3), 291-317.

[31] Herbert, J. (1995): Trading Volume, Maturity and Natural Gas Futures Price Volatility, *Energy Economics*, 17(4), 293-299.

[32] Huang, R.D. and Masulis, R.W. (2003): Trading Activity and Stock Price Volatility: Evidence from the London Stock Exchange, *Journal of Empirical Finance*, 10(3), 249-269.

[33] Hussain, S.M. (2011): The Intraday Behaviour of Bid-Ask Spreads, Trading Volume and Return Volatility: Evidence from DAX30, *International Journal of Economics and Finance*, 3(1), 23-34.

[34] Jain, P.C. and Jon, G. (1988): The Dependence between Hourly Prices and Trading Volume, *The Journal of Financial and Quantitative Analysis*, 23(3), 269-283.

[35] Kocagil, A.E. and Shachmurove Y. (1998): Return-Volume Dynamics in Futures Markets, *The Journal of Futures Markets*, 18(4), 399-426.

[36] McKenzie, M.D. (1999): Power Transformation and Forecasting the Magnitude of Exchange Rate Changes, *International Journal of Forecasting*, 15(1), 49-55.

[37] McMillan, D.G. and Speight, A.E.H. (2004): Intra-Day Periodicity, Temporal Aggregation and Time-to-Maturity in FTSE-100 Index Futures Volatility, *Applied Financial Economics*, 14(4), 253-263.

[38] Moosa, I. and Korczak, M. (1999): Is the Price-Volume Relationship Symmetric in the Futures Markets?, *Journal of Financial Studies*, 7(1), 1-15.

[39] Moosa, I.A. and Silvapulle, P. (2000): The PriceVolume Relationship in the Crude Oil Futures Market: Some Results Based on Linear and Nonlinear Causality Testing, *International Review of Economics and Finance*, 9(1), 11-30.

[40] Moosa, I.A., Silvapulle, P. and Silvapulle, M. (2003): Testing for Temporal Asymmetry in the Price-Volume Relationship, *Bulletin of Economic Research*, 55(4), 373-389.

[41] Ripple, R.D. and Moosa, I.A. (2009): The Effect of Maturity, Trading Volume, and Open Interest on Crude Oil Futures Price Range-Based Volatility, *Global Finance Journal*, 20 (3), 209-219.

[42] Serletis, A. (1992): Maturity Effects in Energy Futures, *Energy Economics*, 14(2), 150-157.

[43] Serletis, A. and Shahmoradi, A. (2006): Returns and Volatility in the NYMEX Henry Hub Natural Gas Futures Market, *OPEC Review*, 30(3), 171-186.

[44] Tick Data website *www.tickdata.com*.

[45] Wutkowski, K. and Alper, A. (April 13, 2012): Senators Defend CFTC in Court over Position Limits, *www.reuters.com*.

[46] Xu, X.E. and Wu, C. (1999): The Intraday Relation between Return Volatility, Transactions and Volume, *International Review of Economics and Finance*, 8(4), 375-397.

CHAPTER 3

MULTINATIONAL PRODUCTION, OPTIMAL PORTFOLIOS AND HOME EQUITY BIAS

1 Introduction

Does the production structure of firms affect international optimal portfolios, risk-sharing, and response of terms of trade (TOT) to shocks? The answer to this question would enhance our understanding of the empirically observed and much researched home equity bias, i.e., a situation in which individuals hold a higher share of home assets in their portfolios than diversification theory would suggest. If investing in shares of multi-national companies (MNCs) provides international diversification, then the home equity bias may be lower than it appears to be in the data.

Empirical research of the home equity bias has led to conflicting conclusions as the literature review in the Appendix shows, creating a long-standing puzzle of whether investing in shares of MNCs provides investors with international diversification. Since the effect of firms' production structure on optimal portfolios and risk sharing has not been studied theoretically, this paper seeks to fill this gap to gain a theoretical understanding that produces implications for empirical research.

This theoretical investigation uses a two-country dynamic stochastic general equilibrium (DSGE) model with endogenous portfolio allocation. The model is based on Ghironi, Lee, and Rebucci (2009) (GLR). Households supply labor, consume, and hold equity shares in home and foreign firms. Firms are monopolistically competitive, which is necessary to generate profits and equity price dynamics. Asset markets are incomplete. In contrast to GLR, in which firms operate in their home countries and serve the foreign markets by exporting, i.e., exporter-only firms, firms in this model hire labor in both countries and produce locally for sale in both countries, i.e., MNC-extreme firms.

The paper produces five results. First, it shows the optimal portfolios change as the firms' production structure moves from exporter-only firms to MNC-extreme firms. The optimal portfolios are driven by the relative profit of home and foreign firms. It is shown that the relative profit of home and foreign firms equals the relative home and foreign GDP. As the production structure moves from exporter-only firms towards MNC-extreme firms, the home GDP becomes more affected by the foreign technology. At the same time, the foreign GDP becomes more affected by the home technology. This means that technologies come to play a smaller role in the relative GDP. Since changing the firms' production structure eliminates exposure to technology differences, the optimal portfolio of the home household includes more home firm shares and fewer foreign firm shares since the home household can accomplish the same diversification with less foreign equity. The MNC production structure of firms thus provides one possible explanation for the empirically observed home equity bias.

Second, the elasticity of labor supply magnifies the effect of the production structure on the optimal portfolios. When the elasticity of labor supply decreases, the effect of production structure on the optimal portfolios declines and disappears entirely when the labor supply becomes perfectly inelastic. This extends the GLR result that the households select complete markets portfolios that insure only against productivity shocks when the supply of labor is perfectly inelastic. Shocks to government spending do not matter since government spending simply crowds out consumption and, therefore, does not affect production, firms' profits, and equity holdings. Since government spending has no impact on production under perfectly inelastic labor supply, the firms' production structure becomes irrelevant.

Third, the paper shows that the home optimal portfolio becomes indeterminate when the production structure reaches the MNC-extreme because the home GDP is affected by the foreign technology as much as by the home technology. Since this is true also for the foreign GDP, the technologies cease to affect the relative GDP. Therefore, any one portfolio insures against shocks as well as any other portfolio. The optimal portfolios become indeterminate. This is similar to the Cole and Obstfeld (1991) result that optimal portfolios become indeterminate when the elasticity of substitution between home and foreign goods equals one because the TOT move one-for-one with relative productivity, i.e., risks are perfectly shared even without asset trade. This paper maintains the Cole and Obstfeld result, but extends it by showing that the MNC-extreme production structure results in indeterminate optimal portfolios no matter what the elasticity of substitution is between home and foreign goods.

Fourth, the firms' production structure also has implications for international risk sharing. As the production structure changes from exporter-only firms towards MNC-extreme firms, a positive shock to the technology differential leads to less risk sharing, i.e., the consumption differential increases, whereas a positive shock to the government spending differential leads to more risk sharing, i.e., the consumption differential decreases.

Fifth, changes in the firms' production structure have implications for the effect of technology shocks on the TOT. With exporter-only firms, a positive shock to the technology differential causes a deterioration of the home TOT as in the standard international macroeconomic model. With the MNC-extreme firms, however, a positive shock to the technology differential leads to an improvement of the home TOT. This paper, therefore, offers a new explanation for why the home TOT appreciate in response to a technology differential shock, a phenomenon observed in the data but contrary to the standard theory.

In summary, by producing testable predictions, this paper underscores the need to take the firms' production structure into account when analyzing international optimal portfolios, risk sharing, and the response of the TOT to technology shocks in open-economy macro models. This is especially important since the empirical research in these areas has so far generated conflicting results.

2 Model

2.1 Households and Governments

As in GLR, there are two countries: home and foreign. Each country is populated by infinitely lived, atomistic households. The world population equals the continuum $[0, 1]$. Home and foreign households comprise the intervals $[0, a)$ and $[a, 1]$, respectively.

The representative home household maximizes an expected intertemporal utility function that depends on consumption C_t and labor L_t:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \chi \frac{L_s^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right),$$

where $1 > \beta > 0$ and $\sigma, \chi, \varphi > 0$. The labor market is competitive.

The foreign representative household faces the same maximization problem with foreign variables denoted as C_t^* and L_t^*. Labor is immobile between countries.

Home consumption C_t consists of consumption of goods produced by both home and foreign firms denoted as C_{Ht} and C_{Ft}. C_{Ht} and C_{Ft} are aggregated by a CES aggregator:

$$C_t = [a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}},$$

where $\omega > 0$ is the elasticity of substitution between goods produced by home and foreign firms.

This consumption basket C_t has the same format as the GLR consumption basket, but the interpretation differs. In GLR, C_{Ht} and C_{Ft} stand for the home household consumption of goods produced by home firms in the home country and foreign firms in the foreign country. In this model, C_{Ht} and C_{Ft} stand for the home household consumption of goods produced by home and foreign firms in the home country. This interpretation corresponds to the concept of the MNC-extreme firms producing in both markets, described in more detail in Section 2.2.

The consumption sub-baskets C_{Ht} and C_{Ft} aggregate individuals goods produced by home and foreign firms, respectively, using Dixit-Stiglitz CES aggregators:

$$C_{Ht} = [(\tfrac{1}{a})^{\frac{1}{\theta}} \textstyle\int_o^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}} \text{ and } C_{Ft} = [(\tfrac{1}{1-a})^{\frac{1}{\theta}} \textstyle\int_a^1 c_t(z^*)^{\frac{\theta-1}{\theta}} dz^*]^{\frac{\theta}{\theta-1}}.$$

Price indices follow from the above consumption preferences. The CPI index is:

$$P = [aP_{Ht}^{1-\omega} + (1-a)P_{Ft}^{1-\omega}]^{\frac{1}{1-\omega}}.$$

Price indices for goods produced by home and foreign firms, respectively, are:

$$P_{Ht} = [\tfrac{1}{a} \textstyle\int_o^a p_t(z)^{1-\theta} dz]^{\frac{1}{1-\theta}} \text{ and } P_{Ft} = [\tfrac{1}{1-a} \textstyle\int_a^1 p_t(z^*)^{1-\theta} dz*]^{\frac{1}{1-\theta}},$$

where $p_t(z)$ and $p_t(z^*)$ are the prices of the individual goods produced by home and foreign firms, respectively.

Prices and wages are denominated in currency but money is just a unit of account. In the model, the economy is "Woodford cashless". Nominal prices and wages are flexible, i.e., there are no financial frictions in the model.

In addition to supplying labor and consuming, home households hold shares in home and foreign firms. Aggregate per capita holdings of home and foreign firms entering period *t+1* are denoted by x_{t+1} and x^*_{t+1}, respectively. Similarly, foreign households' aggregate per capita holdings of home and foreign firms is denoted by x_{*t+1} and x^*_{*t+1}, respectively.

Governments of each country consume the same consumption baskets as households. Government spending G_t and G^*_t is wasteful. Governments run balanced budgets, so that government spending equals lump-sum taxation of household income.

Equilibrium versions of the budget constraints of home and foreign households are:

$$v_t x_{t+1} + v^*_t x^*_{t+1} + C_t + G_t = (v_t + d_t + d_{t*})x_t + (v^*_t + d^*_t + d^*_{*t})x^*_t + w_t L_t,$$

$$v_t x_{*t+1} + v^*_t x^*_{*t+1} + C^*_t + G^*_t = (v_t + d_t + d_{*t})x_{*t} + (v^*_t + d^*_t + d^*_{*t})x^*_{*t} + w^*_t L^*_t,$$

where w_t and w^*_t are home and foreign real wages and v_t and v^*_t are prices of shares in home and foreign firms. Reflecting the multi-national nature of production, d_t and d_{*t} are profits generated by home firms in the home and foreign countries, and d^*_t and d^*_{*t} are the profits generated by foreign firms in home and foreign countries. The profits are paid as dividends to the households. All these variables are in units of the consumption basket.

Home households choose C_t, L_t, x_{t+1}, and x^*_{t+1}. The maximization problem results in:

$$L_t^{\frac{1}{\varphi}} = \frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi},$$

$$C_t^{-\frac{1}{\sigma}} = \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}} R_{t+1}\},$$

$$E_t\{C_{t+1}^{-\frac{1}{\sigma}} R_{t+1}\} = E_t\{C_{t+1}^{-\frac{1}{\sigma}} R^*_{t+1}\},$$

where $R_{t+1} \equiv (v_{t+1} + d_{t+1} + d_{*t+1})/v_t$ is the gross return from holding home equity and $R^*_{t+1} \equiv (v^*_{t+1} + d^*_{t+1} + d^*_{*t+1})/v^*_t$ is the gross return from holding foreign equity. The first equation gives the optimal labor supply. This is the *total* home labor supply, i.e., labor supplied to both home and foreign firms in the home country. The second equation is the Euler equation for home equity. The third equation says the households are indifferent between holding home and foreign equity at an optimum. Similar equations hold for the foreign households.

As shown in the Technical Appendix A.1, the budget constraints can be used to derive the law of motion for net foreign assets[1]:

$$nfa_{t+1} = R^D_t \alpha_t + R_t nfa_t + (1-a)(y^D_t - C^D_t - G^D_t),$$

where $R^D_t \equiv R^*_t - R_t$ is the excess return on foreign equity and $y^D_t \equiv y_t - y^*_t$ is the difference between home and foreign GDP. Similarly, $C^D_t \equiv C_t - C^*_t$ and $G^D_t \equiv G_t - G^*_t$. Portfolio α_t is defined as $\alpha_t \equiv v^*_{t-1} x^*_t$, i.e., a home household's holdings of foreign firm shares multiplied by the price of the foreign firms' shares. This is the portfolio that will be solved in the steady state in Section 3. Note that a higher α_t represents more foreign equity and less home equity while a lower α_t represents less foreign equity and more home equity.

The above law of motion for net foreign assets is the starting point for the derivation of the steady-state home optimal portfolio α outlined in detail in the Technical Appendix B and summarized in Section 3. Note that the law of motion takes the same form as in GLR but the definitions of R_t and R^*_t, and hence R^D_t, differ from GLR.

Also, note that the optimal portfolio held by foreign households, α^*_t, is defined by a

[1]The Technical Appendix is available upon request.

market-clearing condition $\alpha_t^* = -\frac{a}{1-a}\alpha_t \equiv -\frac{a}{1-a}v_{t-1}^* x_t^*$, as in GLR. This means that lower α_t, i.e., home households owning more home equity, translates into higher α_t^*, i.e., foreign households owning more foreign equity.

2.2 Firms

Firms are monopolistically competitive. Each firm produces a differentiated good z. Firms comprise a continuum $[0, 1]$, with home and foreign firms comprising $z \in [0, a)$ and $z^* \in [a, 1]$.

In contrast to GLR, in which firms produce in their home countries and serve foreign markets by exporting, firms in this model produce in both countries. They hire labor in both markets and sell products locally in the market in which they produce.

The home firm z produces $y_t(z)$ for consumption in the home country using home labor according to the production function $y_t(z) = Z_t L_t(z)$. The home firm also produces $y_t^*(z)$ for consumption in the foreign country using foreign labor according to the production function $y_t^*(z) = Z_t^{\gamma} Z_t^{*1-\gamma} L_t^*(z)$ where Z_t is aggregate home productivity and Z_t^* is aggregate foreign productivity. The foreign firm z^* is similar. It produces $y_{*t}^*(z^*)$ for consumption in the foreign country using foreign labor according to production function $y_{*t}^*(z^*) = Z_t^* L_{*t}^*(z^*)$. It also produces $y_{*t}(z^*)$ for consumption in the home country using home labor according to the production function $y_{*t}(z^*) = Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t}(z^*)$. Table 1 summarizes this setup.

Note that firms produce subject to a "mix" of home and foreign technologies when operating outside their countries. This captures the fact that production of an MNC subsidiary is likely to be affected by the productivity of the MNC parent company's country as well as the productivity of the local labor force.

Table 1: Production Structure of Firms (General Setup)

	Home Firm z	Foreign Firm z^*
Home Country	$y_t(z) = Z_t L_t(z)$	$y_{*t}(z^*) = Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t}(z^*)$
Foreign Country	$y_t^*(z) = Z_t^{\gamma} Z_t^{*1-\gamma} L_t^*(z)$	$y_{*t}^*(z^*) = Z_t^* L_{*t}^*(z^*)$

When $\gamma = 0$, the general specification reduces to the country-specific technology setup that subjects the firms to the technologies of the countries in which they operate. Table 2 summarizes this setup.

Table 2: $\gamma = 0$ Country-Specific Technology

	Home firm z	Foreign firm z^*
Home country	$y_t(z) = Z_t L_t(z)$	$y_{*t}(z^*) = Z_t L_{*t}(z^*)$
Foreign country	$y_t^*(z) = Z_t^* L_t^*(z)$	$y_{*t}^*(z^*) = Z_t^* L_{*t}^*(z^*)$

When $\gamma = 1$, the general specification reduces to the firm-specific technology setup that subjects the firms to the technologies of the countries from which the firms originate. Table 3 summarizes this setup.

As explained in more detail in Section 3, the country-specific technology setup in Table 2 is equivalent to the exporter-only firms in GLR since all production in the home country is generated using the aggregate home productivity, Z_t. This result is very useful because it allows an exploration of how the optimal portfolios change as the production structure moves from the exporter-only assumption in GLR to the MNC-extreme assumption by analyzing

Table 3: $\gamma = 0$ **Firm-Specific Technology**

	Home firm z	Foreign firm z^*
Home country	$y_t(z) = Z_t L_t(z)$	$y_{*t}(z^*) = Z_t^* L_{*t}(z^*)$
Foreign country	$y_t^*(z) = Z_t L_t^*(z)$	$y_{*t}^*(z^*) = Z_t^* L_{*t}^*(z^*)$

how the optimal portfolios change as the production structure moves from the country-specific technology setup to the firm-specific technology setup.

Optimal price-setting implies that the prices set by firms are given by constant markups over marginal costs. The home firm charges $RP_t = \frac{\theta}{\theta-1}\frac{w_t}{Z_t}$ in the home country and $RP_t^* = \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^\gamma Z_t^{*1-\gamma}}$ in the foreign country and the foreign firm charges $RP_{*t} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t^{1-\gamma} Z_t^{*\gamma}}$ in the home country and $RP_{*t}^* = \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*}$ in the foreign country as derived in the Technical Appendix A.2.1. Note that the model assumes goods market segmentation, i.e., a firm can charge different prices in different markets.

The optimal aggregate per capita labor demand in the home country, L_t^d, consists of labor demands of home firms and foreign firms in the home country. As Technical Appendix A.2.2 shows, $L_t^d = y_t^W(\frac{RP_t^{-\omega}}{Z_t} + \frac{RP_{*t}^{-\omega}}{Z_t^{1-\gamma} Z_t^{*\gamma}})$, where RP_t and RP_{*t} are the prices charged in the home country by home firms and foreign firms. Optimal aggregate per capita labor demand in the foreign country, L_t^{*d}, consists of the labor demands of home firms and foreign firms in the foreign country. As Technical Appendix A.2.2 shows, $L_t^{*d} = y_t^W(\frac{RP_t^{*-\omega}}{Z_t^\gamma Z_t^{*1-\gamma}} + \frac{RP_{*t}^{*-\omega}}{Z_t^*})$, where RP_t^* and RP_{*t}^* are prices charged in the foreign country by home firms and foreign firms.

2.3 Shocks

As in GLR, there are four shocks: G_t, G_t^*, Z_t and Z_t^*. Since there are only two assets, i.e., shares of home firms and shares of foreign firms, this guarantees incomplete asset markets, which is necessary to meaningfully discuss international portfolios and risk sharing.

2.4 Some Useful Properties

2.4.1 Relative GDP y_t/y_t^*

For the derivation of the home optimal portfolio α, it will be useful to have an expression for the relative GDP, y_t/y_t^*. The home GDP y_t consists of output produced by home and foreign firms in the home country: $y_t = RP_t Z_t L_t + RP_{*t} Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t}$ where relative prices convert GDP into units of the consumption basket, i.e., the basket consisting of home and foreign goods. As Technical Appendix A.3.1 shows, substituting the expressions for RP_t and RP_{*t} gives $y_t = \frac{\theta}{\theta-1}(w_t L_t + w_t L_{*t})$. Similarly, the foreign GDP y_t^* consists of output produced by home and foreign firms in the foreign country: $y_t^* = RP_t^* Z_t^\gamma Z_t^{*1-\gamma} L_t^* + RP_{*t}^* Z_t^* L_{*t}^*$. Substituting expressions for RP_t^* and RP_{*t}^* gives $y_t^* = \frac{\theta}{\theta-1}(w_t^* L_t^* + w_t^* L_{*t}^*)$. Therefore, $y_t/y_t^* = [w_t(L_t+L_{*t})]/[w_t^*(L_t^*+L_{*t}^*)]$. After substituting expressions for w_t, w_t^*, (L_t+L_{*t}) and $(L_t^*+L_{*t}^*)$, and imposing the labor market equilibrium, the following expression is obtained: $y_t/y_t^* = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}} (\frac{Z_t^{\omega-1}+(Z_t^{1-\gamma}Z_t^{*\gamma})^{\omega-1}}{Z_t^{*\omega-1}+(Z_t^\gamma Z_t^{*1-\gamma})^{\omega-1}})^{\frac{1+\varphi}{\varphi+\omega}}$.

When $\gamma = 0$, i.e., in the country-specific technology setup, this expression reduces to $\frac{y_t}{y_t^*} = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}} (\frac{Z_t}{Z_t^*})^{\frac{(\omega-1)(\varphi+1)}{\varphi+\omega}}$ which is equivalent to the GLR result.

When $\gamma = 1$, i.e., in the firm-specific technology setup, this expression reduces to $\frac{y_t}{y_t^*} = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}}$. Note that the technology terms drop out. As explained in more detail in Section

3, these relative GDP expressions have implications for the optimal portfolios.

2.4.2 Constant Proportions of Income Distribution

For the derivation of the home optimal portfolio α, it will also be useful to take advantage of the fact that income distribution is determined by constant proportions, which is a feature of monopolistic competition models. Income consists of labor income and dividend income. As derived above, home GDP y_t equals $\frac{\theta}{\theta-1}(w_tL_t + w_tL_{*t})$. Therefore, the total home labor income equals $w_tL_t + w_tL_{*t} = y_t\frac{\theta-1}{\theta}$, which shows that the share of labor income in the home GDP is a constant proportion $\frac{\theta-1}{\theta}$. The profit of home firms, i.e., the profit generated by home firms in home and foreign countries, equals $d_t + d_{*t} = y_t - y_t\frac{\theta-1}{\theta} = y_t\frac{1}{\theta}$, which shows that the share of firm profits, i.e., the dividend income, in the home GDP is a constant proportion $\frac{1}{\theta}$. A similar calculation can be performed for the foreign country to obtain $d_t^* + d_{*t}^* = \frac{1}{\theta}y_{*t}$. It is then possible to write $\frac{d_t+d_{*t}}{d_t^*+d_{*t}^*} = \frac{y_t}{y_t^*}$. The home optimal portfolio depends on the relative profit of the home firm and foreign firm, i.e., $\frac{d_t+d_{*t}}{d_t^*+d_{*t}^*}$. Since $\frac{d_t+d_{*t}}{d_t^*+d_{*t}^*} = \frac{y_t}{y_t^*}$, the home optimal portfolio depends on the relative GDP. The expression for the relative GDP y_t/y_t^* obtained in Section 2.3.1. can then be used to derive the home optimal portfolio α.[2]

[2] Note that it is assumed that firms repatriate profits to their countries of origin for distribution to domestic and foreign shareholders. Therefore, the home firms' profits generated in the foreign country become a part of the home country's GDP while the foreign firms' profits generated in the home country become a part of the foreign country's GDP. Relaxing this assumption on profit repatriation is discussed in Section 4 as one of the future extensions of the model.

3 Steady-State Optimal Portfolio α and Results

The portfolio denoted by α_t is defined in Section 2.1 as $\alpha_t \equiv v^*_{t-1}x^*_t$, i.e., the home household holdings of foreign firm shares multiplied by the price of the foreign firm shares. The optimal portfolios are defined as portfolios optimally chosen to insure against shocks Z_t, Z^*_t, G_t and G^*_t. The steady-state home optimal portfolio is derived in Technical Appendix B following the GLR method based on Devereux and Sutherland (2007) and Tille and van Wincoop (2010) as:

$$\alpha = \tfrac{\beta(1-a)}{1-\beta}[1 - \tfrac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}],$$

which compares to the GLR home optimal portfolio:

$$\alpha = \tfrac{\beta(1-a)}{1-\beta}[1 - \tfrac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}].$$

As in GLR, ϕ_Z and ϕ_G denote the persistence of relative productivity and government spending shocks defined as the percentage deviation from the steady state. $\sigma^2_{\varepsilon ZD}$ and $\sigma^2_{\varepsilon GD}$ are the relative variances of i.i.d, zero-mean, normal shocks to productivities Z_t and Z^*_t and government spending G_t and G^*_t. Shocks to productivities are assumed to be uncorrelated with shocks to government spending. Also as in GLR, G is not only the level of government spending but also the ratio of government spending to GDP because GDP is normalized to 1, which gives $1 = C + G$ and imposes $0 \leq G < 1$.

This section discusses implications of the firms' production structure for the optimal portfolios. The goal is to show how the optimal portfolios change as the production structure moves from the GLR exporter-only firms to the MNC-extreme firms. To show this, it is first shown that the country-specific technology setup leads to the same optimal portfolios as the

GLR exporter-only setup. It is then possible to see how the optimal portfolios change as the production structure moves from the GLR exporter-only firms to the MNC-extreme setup by showing how the optimal portfolios change as the production structure moves from the country-specific technology setup to the firm-specific technology setup.

3.1 Why Is GLR Equal to Country-Specific Technology Setup?

The portfolio α_t is driven by the relative profit of home and foreign firms, i.e., the relative dividends $(d_t + d_{*t})/(d_t^* + d_{*t}^*)$. Section 2.4.2 shows that the relative profit $(d_t + d_{*t})/(d_t^* + d_{*t}^*)$ equals the relative GDP y_t/y_t^*. Section 2.4.1 shows the relative GDP y_t/y_t^* depends on the relative wage w_t/w_t^* and the relative labor supply L_t^S/L_t^{*S}. In GLR, the relative wage w_t/w_t^* and the relative labor supply L_t^S/L_t^{*S} expressions contain a relative technology term Z_t/Z_t^* because home variables w_t and L_t^S include only the home technology Z_t whereas foreign variables w_t^* and L_t^{S*} include only the foreign technology Z_t^*.

When the country-specific technology is applied, i.e., $\gamma = 0$, that is still the case: the home variables w_t and L_t^S include only the home technology Z_t whereas the foreign variables w_t^* and L_t^{S*} include only the foreign technology Z_t^*. Therefore, this setup is equivalent to GLR, as shown in Technical Appendix C.

When the firm-specific technology is applied, i.e., $\gamma = 1$, that is no longer the case: the home variables w_t and L_t^S now include the home technology Z_t as well as the foreign technology Z_t^*. Similarly, the foreign variables w_t^* and L_t^{S*} now include both technologies Z_t and Z_t^*. Therefore, the technology terms cancel.

These results follow because the optimal portfolios are affected by *relative* values, i.e., the ratios of home and foreign variables.

3.2 Impact of Firms' Production Structure on Steady-State Optimal Portfolio α and Equity Holdings x, x^*, x_* and x_*^*

This section explains what happens as the production structure changes from the GLR exporter-only firms towards the MNC-extreme firms by increasing γ from 0 towards 1. As noted at the beginning of Section 3, the fact that the GLR exporter-only setup leads to the same optimal portfolios as the country-specific technology setup is very useful because changes in the optimal portfolios as the production structure moves from the GLR exporter-only setup to the MNC-extreme setup can be analyzed by showing how the optimal portfolios change as the production structure moves from the country-specific technology setup to the firm-specific technology setup.

As γ increases, i.e., as the production structure moves toward the firm-specific technology, the home GDP y_t is affected more by the foreign technology Z_t^*. At the same time, the foreign GDP y_t^* is affected more by the home technology Z_t. This means that technologies play a smaller role in the relative GDP y_t/y_t^*. Since changing the firms' production structure eliminates exposure to technology differences, the optimal portfolio of the home households includes more home firm shares and fewer foreign firm shares as the home household can accomplish the same diversification with less foreign equity.

This can be seen by examining the steady-state expression for the home optimal portfolio α derived in Technical Appendix B and the derivative of α with respect to γ derived in Technical Appendix D. Increasing γ from 0 towards 1 lowers α, which means that the home households' holdings of the shares of the foreign firm decrease, i.e., the optimal portfolio of the home households includes more home equity. The MNC production structure of firms thus provides one possible explanation for the empirically observed home equity bias. Note that the effect of increasing γ is not linear: changing γ has a larger impact on α at higher values of γ, as shown in Figure 1.[3]

At the same time, the optimal portfolio of the foreign households includes more foreign equity. This can be seen from the market-clearing condition $\alpha_t^* = -\frac{a}{1-a}\alpha_t \equiv -\frac{a}{1-a}v_{t-1}^* x_t^*$ introduced in Section 2.1, which shows that lower α_t, i.e., home households owning more home equity, translates into higher α_t^*, i.e., foreign households owning more foreign equity.

Technical Appendix E shows that these results hold also for steady-state equity holdings x, x^*, x_* and x_*^*. As γ increases from 0 towards 1, home households choose to hold more home firm equity x and less foreign firm equity x^* while foreign household choose to hold more foreign firm equity x_*^* and less home firm equity x_*.

The Frisch elasticity of labor supply, φ, magnifies the effect of γ on α. As shown in Technical Appendix D, when φ increases, $\frac{\partial \alpha}{\partial \gamma}$ becomes more negative, indicating home households hold less foreign equity and more home equity. Conversely, a decrease in φ leads to a decrease in the effect of γ on α. When φ reaches 0, the effect of γ on α disappears, i.e., the production structure of firms becomes irrelevant when the labor supply becomes perfectly inelastic. This extends the GLR result that $\varphi = 0$ implies that the households select portfo-

[3] As Figure 1 shows, when γ increases to approximately 0.7, the home optimal portfolio α becomes negative. Since the home optimal portfolio is defined as the home households' holdings of foreign firm shares multiplied by the price of the foreign firm shares, this can be interpreted as short-selling. However, the value of α and, therefore, the short-selling scenario depend on other parameter values that are adopted from GLR.

lios that insure only against productivity shocks. As GLR explain, under perfectly inelastic labor supply, relative output is affected only by relative technology, i.e., it is not affected by relative labor. Therefore, shocks to government spending do not matter because the government spending does not affect production and, hence, relative firm profits and relative equity holdings. Since the government spending shocks have no impact on production, the households choose *complete markets* portfolios that insure only against relative productivity shocks. Therefore, the firms' production structure, γ, plays no role in the optimal portfolios when the labor supply is perfectly inelastic.[4] When the labor supply becomes elastic, i.e., $\varphi > 0$, government spending shocks begin to affect relative production and, therefore, the production structure, given by γ, comes into play. Note that the effect of φ on the derivatives of x, x^*, x_* and x_*^* with respect of γ is analogous to the effect of φ on the derivative of α, as shown in Technical Appendix E.

3.3 2 Cases when Optimal Portfolio Is Indeterminate:

This section discusses two cases in which the optimal portfolios are indeterminate. Although the first result has been established in previous studies, the second result is new.

Case 1: Elasticity of substitution between home and foreign goods, ω, equals 1: In GLR, when the elasticity of substitution between home and foreign goods, ω, equals 1, the relative GDP $\frac{y_t}{y_t^*} = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}}(\frac{Z_t}{Z_t^*})^{\frac{(1+\varphi)(\omega-1)}{\omega+\varphi}}$ reduces to $y_t = y_t^*$. This can be understood by analyzing the TOT defined in GLR as the ratio of home country price RP_t^H to foreign

[4] The same result occurs when there is no government spending, i.e., $G = 0$ and $G^* = 0$. In this case, the households also choose complete markets portfolios that insure against relative productivity shocks.

country price RP_t^F:

$$TOT_t \equiv \frac{RP_t^H}{RP_t^F} = \frac{\frac{\theta}{\theta-1}\frac{w_t}{Z_t}}{\frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*}} = \frac{\frac{w_t}{Z_t}}{\frac{w_t^*}{Z_t^*}} = \frac{\frac{C_t^{\frac{\varphi}{\sigma(\varphi+\omega)}}\chi^{\frac{\varphi}{\varphi+\omega}}(Y_t^W)^{\frac{1}{\varphi+\omega}}(\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}}Z_t^{\frac{\omega-1}{\varphi+\omega}}}{Z_t}}{\frac{C_t^{*\frac{\varphi}{\sigma(\varphi+\omega)}}\chi^{\frac{\varphi}{\varphi+\omega}}(Y_t^W)^{\frac{1}{\varphi+\omega}}(\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}}Z_t^{*\frac{\omega-1}{\varphi+\omega}}}{Z_t^*}} = (\frac{C_t}{C_t^*})^{\frac{\varphi}{\sigma(\omega+\varphi)}}(\frac{Z_t}{Z_t^*})^{-\frac{1+\varphi}{\omega+\varphi}}.$$

If the Frisch elasticity of labor supply, φ, equals 0, $TOT_t = \frac{Z_t}{Z_t^*}^{-\frac{1}{\omega}}$. $\omega = 1$ then results in $TOT_t = \frac{Z_t^*}{Z_t}$. The TOT are a mechanism for transferring purchasing power and increasing foreign consumption following a positive home productivity shock. Home output increases which lowers the price of home goods. $TOT_t = \frac{Z_t^*}{Z_t}$ means that the TOT move one-for-one with relative productivity. As GLR point out, this is the main mechanism for the Cole and Obstfeld (1991) result that the TOT adjust in direct proportion to relative productivity, i.e., risks are perfectly shared between countries even without asset trade when $\omega = 1$. Since the TOT provide all the insurance against shocks, it does not matter what the portfolios look like, i.e., the optimal portfolios are indeterminate.

This is still the case in the model with the MNC production structure because relative GDP $\frac{y_t}{y_t^*} = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}}(\frac{Z_t^{\omega-1}+(Z_t^{1-\gamma}Z_t^{*\gamma})^{\omega-1}}{Z_t^{*\omega-1}+(Z_t^{\gamma}Z_t^{*1-\gamma})^{\omega-1}})^{\frac{1+\varphi}{\varphi+\omega}}$ also reduces to $y_t = y_t^*$ when $\omega = 1$. Again, this can be understood by analyzing the TOT derived in the Technical Appendix F:[5]

$$TOT_t = \frac{RP_{t*}}{RP_t^*} = (\frac{C_t}{C_t^*})^{\frac{\varphi}{\sigma(\varphi+\omega)}}\frac{\frac{[Z_t^{\omega-1}+(Z_t^{1-\gamma}Z_t^{*\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{Z_t^{1-\gamma}Z_t^{*\gamma}}}{\frac{[Z_t^{*\omega-1}+(Z_t^{\gamma}Z_t^{*1-\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{Z_t^{\gamma}Z_t^{*1-\gamma}}}$$

When $\gamma = 0$, the TOT reduce to the above GLR expression. Hence, when $\varphi = 0$ and $\omega = 1$, $TOT_t = \frac{Z_t^*}{Z_t}$ which means that the TOT still move one-for-one with the productivity differential. This serves as a check that the model works properly because the Cole and Obstfeld (1991) result should not be lost by changing the assumptions on the firms' production

[5]Note that these are "shadow TOT" because even $\gamma = 0$ is a "no trade" scenario since the model does not feature any trade in goods. The shadow TOT are the relative price of goods produced by the MNCs abroad, i.e., goods that would be delivered via exports in GLR.

structure.

Case 2: Firm-specific technology production structure, i.e., $\gamma = 1$:

When γ reaches 1, i.e., when the technology becomes firm-specific, the home GDP y_t is affected by the foreign technology Z_t^* as much as by the home technology Z_t. Since this is true also for the foreign GDP y_t^*, the technologies cease to affect the relative GDP y_t/y_t^*. Therefore, any one portfolio insures against shocks as well as any other portfolio. The optimal portfolios become indeterminate.

Again, this can be understood by analyzing the TOT. As can be seen from the above expression, $\gamma = 1$ and $\varphi = 0$ result in $TOT_t = \frac{Z_t}{Z_t^*}$, i.e., the TOT again move one-for-one with the productivity differential. Note that it is not necessary for the elasticity of substitution between home and foreign goods, ω, to equal 1. The firm-specific technology production structure results in indeterminate optimal portfolios no matter what the elasticity of substitution is between home and foreign goods.

3.4 Terms of Trade

Even though both $\gamma = 0$ and $\gamma = 1$ production structures result in the TOT moving one-for-one with the productivity differential, the direction of the movement differs. $\gamma = 0$ leads to $TOT_t = \frac{Z_t^*}{Z_t}$ as in GLR. Known as "immiserizing growth", this is the standard result that an increase in the home productivity leads a *deterioration* in the home TOT. $\gamma = 1$, however, gives the opposite result: $TOT_t = \frac{Z_t}{Z_t^*}$. An increase in the home productivity leads to an *improvement* in the home TOT.

This can be seen by examining the TOT that arise under perfectly inelastic labor supply, $\varphi = 0$:

$$TOT_t = \frac{\frac{[Z_t^{\omega-1}+(Z_t^{1-\gamma}Z_t^{*\gamma})^{\omega-1}]^{\frac{1}{\omega}}}{Z_t^{1-\gamma}Z_t^{*\gamma}}}{\frac{[Z_t^{*\omega-1}+(Z_t^{\gamma}Z_t^{*1-\gamma})^{\omega-1}]^{\frac{1}{\omega}}}{Z_t^{\gamma}Z_t^{*1-\gamma}}},$$

which, under $\omega = 1$ yields:

$$TOT_t = \frac{Z_t^{\gamma}Z_t^{*1-\gamma}}{Z_t^{1-\gamma}Z^{*\gamma}}.$$

When the production structure is country-specific, i.e., $\gamma = 0$, goods made by the foreign firm in the home country are produced using the home technology, Z_t, and goods made by the home firm in the foreign country are produced using the foreign technology, Z_t^*. When Z_t increases, goods made in the home country become more abundant and thus cheaper, causing a *deterioration* in the home TOT. However, when the production structure is firm-specific, i.e., $\gamma = 1$, goods made by the foreign firm in the home country are produced using the foreign technology, Z_t^*, and goods made by the home firm in the foreign country are produced using the foreign technology, Z_t. When Z_t increases, goods made in the *foreign* country become more abundant and thus cheaper, causing an *improvement* in the home TOT. Therefore, the production structure parameter, γ, determines the effect of productivity changes on the TOT.

Although the standard models, such as, Backus, Kehoe and Kydland (1994), indicate that a positive home technology shocks leads to an improvement in the home TOT, the empirical evidence is mixed. For example, Corsetti, Dedola and Leduc (2008b) show that the home TOT can worsen in response to a positive home technology shock. Acemoglu and Ventura (2003) and Debaere and Lee (2004) offer evidence consistent with this finding. Several mechanisms have been proposed to account for this empirical evidence that contradicts the standard theory. Corsetti, Dedola and Leduc (2008) show that a positive home technology shock can cause an appreciation of the home TOT if there is a high degree of home bias in absorption, the long-run price elasticity of domestic tradeables is high, and the households can only borrow and lend in international markets. Ghironi and Melitz (2005) allow for entry of new producers and varieties and show that the home TOT will appreciate in response to a positive home technology shock because the shock reduces both the marginal cost of producing goods and the sunk entry cost of setting up new firms. Corsetti, Martin and Pesenti (2007) present a model in which the home TOT deteriorate if the positive home productivity shock decreases the marginal cost of producing goods but improve if the positive home productivity shock decreases the cost of setting up new firms. The model in this paper offers a new mechanism using the production structure of firms to explain the phenomenon of the home TOT appreciating after a positive home technology shock.

3.5 Impulse Response Functions

The production structure of firms also affects responses to shocks, as shown by the impulse response functions in Figures 2 through 7.[6]

3.5.1 Technology Shock

In GLR, a positive shock to the home technology that translates into a positive shock to the technology differential, Z_t^D, increases home output. This depresses the price of goods made in the home country, worsens the home TOT and lowers the home household purchasing

[6]Note that calibration of all parameters is identical to GLR.

power. At the same time, the profit of the home firms increases relative to the foreign firms, which increases the price of the home equity relative to foreign equity and, therefore, decreases net foreign assets. Faced with a lower purchasing power and lower net foreign assets, the home household increases its labor supply. This increases relative income and consumption, leading to an increase in the consumption differential, C^D.

As the production structure changes from exporter-only firms to MNC-extreme firms, a positive shock to the home technology still causes an increase in the home output relative to foreign output but not as much because a part of the production in the home country is produced using the foreign technology, Z_t^*. Therefore, the home TOT do not worsen as much and the home household purchasing power does not decline as much. At the same time, the profit of the home firms relative to the foreign firms does not increase as much, which means the price of the home equity relative to the foreign equity does not increase as much and, therefore, net foreign assets do not decrease as much. Consequently, the increases in the home households' labor supply, relative income, and consumption are mitigated. Since the home TOT do not deteriorate as much, the foreign households do not benefit from the shock as much because their purchasing power does not improve as much. Therefore, foreign consumption, C_t^*, does not increase as much. As a result, the consumption differential, C^D, increases more than in GLR, implying less risk sharing between the home and foreign households.

3.5.2 Government Shock

As GLR explain, a positive shock to home government spending that translates into a positive shock to the government spending differential, G_t^D, crowds out the home consumption because of Ricardian behavior. As the expression for the TOT shows, a decrease in consumption worsens the home TOT, which increases the demand for home output. As a result, the profit of the home firms increases. Home equity becomes more attractive to the foreign households than foreign equity is to the home households because of consumption smoothing. This drives up the price of home equity relative to foreign equity, which lowers net foreign assets. Since the home TOT deteriorate, foreign households enjoy a higher purchasing power and increase their consumption. The consumption differential, C^D, then decreases because home consumption decreases and foreign consumption increases.

As the production structure changes from exporter-only firms to MNC-extreme firms, a positive shock to home government spending still decreases home consumption. The home TOT still worsen, but not as much. Therefore, the demand for the home output and the resulting profit of the home firm do not increase as much, which means that the price of home equity relative to foreign equity does not rise as much and net foreign assets do not decline as much. Since the home TOT do not deteriorate as much, foreign households do not benefit from the shock as much because their purchasing power does not improve as much. Therefore, foreign consumption, C_t^*, does not increase as much. As a result, the consumption differential, C^D, decreases less than in GLR, implying more risk sharing between the home and foreign households.

4 Conclusions and Future Extensions

This paper studies the effect of the firms' production structure on international optimal portfolios, risk-sharing, and response of terms of trade to shocks. It is the first paper to formally investigate this question, filling a gap in the theoretical research and producing implications for empirical research that has so far generated conflicting results. This paper generates five main results.

First, using a two-country DSGE model with endogenous portfolio allocation, it shows that the optimal portfolios change as the firms' production structure moves from exporter-only firms to MNC-extreme firms. The optimal portfolios are driven by the relative profit of home and foreign firms. The home GDP becomes more affected by the foreign technology and the foreign GDP becomes more affected by the home technology. Technologies play a smaller role in the relative GDP and, therefore, in the relative profit of home and foreign firms. Since changing the production structure of firms eliminates exposure to technology differences, the optimal portfolio of the home household includes more home firm shares and fewer foreign firm shares. Home households can accomplish the same diversification with less foreign equity. The MNC production structure of firms thus provides one possible explanation for the empirically observed home equity bias.

Second, the paper concludes the elasticity of labor supply magnifies the effect of the firms' production structure on the optimal portfolios. When the elasticity of labor supply decreases, the effect of the production structure on the optimal portfolios declines and disappears entirely when the labor supply becomes perfectly inelastic. This extends the GLR result that the households select complete markets portfolios that insure only against productivity shocks when the labor supply is perfectly inelastic because shocks to government spending do not matter. Government spending simply crowds out consumption and, therefore, does not affect production, firms' profits, and equity holdings. Since government spending has no impact on production, the firms' production structure becomes irrelevant.

Third, the optimal portfolios become indeterminate when the firms' production structure reaches the MNC-extreme because the home GDP is affected by the foreign technology as much as by the home technology. Since this is true also for the foreign GDP, the technologies cease to affect the relative GDP. Therefore, any one portfolio insures against the shocks as well as any other portfolio. The optimal portfolios become indeterminate. This is similar to the Cole and Obstfeld (1991) result that optimal portfolios become indeterminate when the elasticity of substitution between home and foreign goods equals one because the TOT move one-for-one with relative productivity, i.e., risks are perfectly shared even without asset trade. The paper maintains the Cole and Obstfeld result, but extends it is by showing that the MNC-extreme production structure results in indeterminate optimal portfolios no matter what the elasticity of substitution is between home and foreign goods.

Fourth, the firms' production structure also has implications for international risk sharing. As the production structure changes from exporter-only firms to MNC-extreme firms, a positive shock to the technology differential leads to less risk sharing, i.e., the consumption differential increases, whereas a positive shock to the government spending differential leads to more risk sharing, i.e., the consumption differential decreases.

Fifth, changes in the firms' production structure have implications for the effect of technology shocks on the TOT. With exporter-only firms, a positive shock to the technology differential causes a deterioration of the home TOT, which is the standard result in the international macroeconomic theory. With MNC-extreme firms, however, a positive shock to the technology differential leads to an improvement of the home TOT, offering an explanation for why the home TOT appreciate in response to a shock to the technology differential, a phenomenon observed in the data but contrary to the standard theory.

In summary, the testable predictions of this paper underscore the need to take the firms' production structure into account when analyzing international optimal portfolios, risk sharing, and the response of the TOT to technology shocks in open-economy macro models. This is especially important since the empirical research has so far generated conflicting results in these areas.

To highlight the impact of the firms' production structure, the model on purpose abstracts from many other aspects of the economy. Several extensions should be made to enhance our understanding of consequences of the production structure on international optimal portfolios, risk sharing, and TOT. First, to account for non-traded goods, a domestic firm should be added that produces only in the domestic country for consumption in the domestic market following, for example, Backus and Smith (1993). Second, trade should be introduced, for example, by exploring a setup where a portion of the output, $1-\eta$, made by the home firm to be sold abroad is produced in the home country and exported while the other portion, η, is produced in the foreign country where the home firm operates as an MNC. This could be achieved by modeling the foreign demand for the product made by the home firm as $C_t^*(z) = Z_t L_t^*(z) + \eta Z_t L_t(z)$. Introducing trade would also allow the usual definition of the TOT as the relative price of exports and imports, as opposed to the "shadow TOT" defined as the ratio of the price of goods made by the foreign firm in the home country to the price of goods made by the home firm in the foreign country. Third, the model assumes all profits are repatriated to the firms' country of origin. In reality, this is not always the case as documented, for instance, by Dharmapala, Foley and Forbes (2009). This assumption should be relaxed, for example, to explore the effect of unequal tax policies. Lastly, an endogenous production structure, i.e., the firm's decision to export or undertake foreign direct investment, should be incorporated following, for example, Contessi (2007). This could be achieved by introducing trade costs.

References

[1] Acemoglu, D. and Ventura, J. (2003): The World Income Distribution, Quarterly *Journal of Economics*, 117(2), 659-694.

[2] Adler, M. and Dumas, B. (1983): International Portfolio Choice and Corporate Finance: A Synthesis, *Journal of Finance*, 38(3), 925-984.

[3] Aggarwal, R., Berrill, J. and Kearney, C. (2011): What is a Multinational Company? Classifying the Degree of Firm-Level Multinationality, *International Business Review*, 20(5), 557-577.

[4] Agmon, T. and Lessard D. (1977): Investor Recognition of Corporate International Diversification, *Journal of Finance*, 32(4), 1049-1055.

[5] Ahearne, A., Griever, W. and Warnock, F. (2004): Information Costs and Home Bias: Analysis of US Holdings of Foreign Equity, *Journal of International Economics*, 62(2), 313-336.

[6] Amadi, A. (2004): Equity Home Bias: A Disappearing Phenomenon, *University of California at Davis*, *Unpublished*.

[7] Amadi, A. and Bergin P. (2008): Understanding International Portfolio Diversification and Turnover Rates, *Journal of International Financial Markets, Institutions and Money*, 18(2), 191-206.

[8] Backus, D. K., Kehoe, P.J. and Kydland, F.E. (1994): Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?, *The American Economic Review*, 84(1), 84-103.

[9] Backus, D.K. and Smith, G.W. (1993): Consumption and Real Exchange Rates in Dynamics Economies with Non-Traded Goods, *Journal of International Economics*, 35(3-4), 297-316.

[10] Baxter, M. and Jermann, U.J. (1997): The International Diversification Puzzle Is Worse than You Think, *The American Economic Review*, 87, 170-180.

[11] Baxter, M., Jermann, U.J. and King, R.G. (1998): Non-Traded Goods, Non-Traded Factors and International Non-Diversification, *Journal of International Economics*, 44(2), 211-229.

[12] Bekaert, G. and Urias, M. (1996): Diversification, Integration and Emerging Market Closed-End Funds, *Journal of Finance*, 51(3), 835-869.

[13] Benartzi, S. (2001): Excessive Extrapolation and the Allocation of 401(k) Accounts to Company Stock, *Journal of Finance*, 56(5), 1747-1764.

[14] Benigno, P. and Nistico, S. (2009): International Portfolio Allocation under Model Uncertainty, *NBER Working Paper*, No. 14734.

[15] Berrill, J. and Kearney, C. (2008): Firm-Level Internationalisation and the Home Bias Puzzle, *Journal of Economics and Business*, 62(4), 235-256.

[16] Black, F. (1974): International Capital Market Equilibrium with Investment Barriers, *Journal of Financial Economics*, 1(4), 337-352.

[17] Bottazzi, L., Pesenti, P. and van Wincoop E.(1996): Wages, Profits and the International Portfolio Puzzle, *European Economic Review*, 40(2), 219-254.

[18] Brennan, M. and Cao, H. (1997): International Portfolio Investment Flows, *Journal of Finance*, 52(5), 1851-1880.

[19] Brewer, H. (1981): Investor Benefits from Corporate International Diversification, *Journal of Financial and Quantitative Analysis*, 16(1), 113-126.

[20] Cai, F. and Warnock, F. (2004): International Diversification at Home and Abroad, *Board of Governors of the Federal Reserve System, International Finance Discussion Papers*, No. 793.

[21] Coen, A. (2001): Home Bias and International Capital Asset Pricing Model with Human Capital, *Journal of Multinational Financial Management*, 11(4), 497-513.

[22] Coeurdacier, N. (2009) Do Trade Costs in Goods Markets Lead to Home Bias in Equities?, *Journal of International Economics*, 77(1), 86-100.

[23] Coeurdacier, N. and Gourinchas, P.O. (2008): When bonds matter: Home Bias in Goods and Assets, *London Business School and University of California-Berkeley Working Paper*.

[24] Cole, H.L. and Obstfeld, M. (1991): Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?, *Journal of Monetary Economics*, 28(1), 3-24.

[25] Contessi, S. (2007): International Macroeconomic Dynamics, Endogenous Tradedness and FDI with Heterogeneous Firms, *Federal Reserve Bank of St. Louis*, *Unpublished*.

[26] Cooper, I. and Kaplanis, E. (1994): Home Bias in Equity Portfolios, Inflation Hedging and International Capital Market Equilibrium, *Review of Financial Studies*, 7(1), 45-60.

[27] Corsetti, G., Dedola, L. and Leduc S. (2008a): International Risk-Sharing and the Transmission of Productivity Shocks, *Review of Economics Studies*, 75(2), 443-473.

[28] Corsetti, G., Dedola, L. and Leduc, S. (2008b): Productivity, External Balance, and Exchange Rates: Evidence on the Transmission Mechanism among the G7 Countries, in Reichlin, L. and West, K. (eds), *NBER International Seminar on Macroeconomics 2006*, Cambridge, MIT Press, 117-178.

[29] Corsetti, G., Martin, P. and Pesenti, P. (2007): Productivity Spillovers, Terms of Trade, and the "Home Market Effect", *Journal of International Economics*, 73(1), 99-127.

[30] Coval, J. and Moskowitz, T. (1999): Home Bias at Home: Local Equity Preference in Domestic Portfolios, *Journal of Finance*, 54(6), 2045-2073.

[31] Dahlquist, M., Pinkowitz, L., Stulz, R.M. and Williamson, R. (2003): Corporate Governance and the Home Bias, *Journal of Financial and Quantitative Analysis*, 38(1), 87-110.

[32] Debaere, P. and Lee, H. (2004): The Real-Side Determinants of Countries' Terms of Trade: A Panel Data Analysis, *University of Texas Working Paper*.

[33] Devereux, M.B. and Sutherland, A. (2007): Solving for Country Portfolios in Open Economy Macro Models, *International Monetary Fund Working Paper Series*, WP/07/284.

[34] Dharmapala, D., Foley, C.F. and Forbes, K.J. (2009): Watch What I Do, Not What I Say: The Unintended Consequences of the Homeland Investment Act, *The Journal of Finance*, 66(3), 753-787.

[35] Dumas, B., Lewis, K.K., and Osambela, E. (2009): Differences in Opinion in an International Financial Market Equilibrium, *NBER Working Paper*, No. 16726.

[36] Eldor, R., Pines, D. and Schwartz, A. (1988): Home Asset Preferences and Productivity Shocks, *Journal of International Economics*, 25(1-2), 165-176.

[37] Errunza, V.R. (1984): A New Opportunity for Improving Global Portfolio Performance, *Financial Analysts Journal*, 39(5), 51-58.

[38] Errunza, V., Hogan, K. and Hung M. (1999): Can the Gains from International Diversification Be Achieved Without Trading Abroad?, *Journal of Finance*, 54(6), 2075-2107.

[39] Fatemi, A. (1984): Shareholder Benefits from Corporate International Diversification, *Journal of Finance*, 39(5), 1325-1344.

[40] Fellner, G. and Maciejovsky, B. (2003): The Equity Home Bias: Contrasting an Institutional with a Behavioral Explanation, *Max Planck Institute for Research into Economic Systems Working Paper*, No. 3-2003.

[41] Fidora, M., Fratzscher, M. and Thimann, C. (2006): Home Bias in Global Bond and Equity Markets: The Role of Real Exchange Rate Volatility, *European Central Bank Research Paper Series*, No. 685.

[42] Gehrig, T.P. (1993): An Information Based Explanation of the Domestic Bias International Equity Investment, *Scandinavian Journal of Economics*, 95(1), 97-109.

[43] Ghironi, F., Lee, J. and Rebucci, A. (2009): The Valuation Channel of External Adjustment, *Boston College Working Paper Series*, No. 722.

[44] Ghironi, F. and Melitz, M.J. (2005): International Trade and Macroeconomic Dynamics with Heterogenous Firms, *Quarterly Journal of Economics*, 120(3), 865-915.

[45] Gianetti, M. and Koskinen, Y. (2003): Investor Protection and Equity Holdings: An Explanation for the Two Puzzles?, *CEPR Discussion Paper*, No. 4017.

[46] Glassman, D.A. and Riddick, L.A. (2001): What Causes Home Asset Bias and How Should It Be Measured?, *Journal of Empirical Finance*, 8(1), 35-54.

[47] Grinblatt, M. and Keloharju, M. (2000): The Investment Behavior and Performance of Various Investor Types: A Study of Finland's Unique Data Set, *Journal of Financial Economics*, 55(1), 43-67.

[48] Grinblatt, M. and Keloharju, M. (2001): How Distance, Language and Culture Influence Stockholdings and Trades, *Journal of Finance*, 56(3), 1053-1073.

[49] Hasan, I. and Simaan, Y. (2000): A Rational Explanation of Home Country Bias, *Journal of International Money and Finance*, 19(3), 331-361.

[50] Heathcote, J. and Perri, F. (2007): The International Diversification Puzzle Is Not as Bad as You Think, *NBER Working Paper*, No. 13483.

[51] Huberman, G. (1998): Familiarity Breeds Investment, *Review of Financial Studies*, 14(3), 659-680.

[52] Hughes, J., Logue, D. and Sweeney, R. (1975): Corporate International Diversification and Market Assigned Measures of Risk and Diversification, *Journal of Financial and Quantitative Analysis*, 10(4), 627-637.

[53] Jacquillat, B. and Solnik, B.H. (1978): Multinationals Are Poor Tools for Diversification, *Journal of Portfolio Management*, 4(2), 8-12.

[54] Jeske, K. (2001): Equity Home Bias: Can Information Cost Explain the Puzzle?, *Federal Reserve of Atlanta Economic Review*, 3rd Quarter, 31-42.

[55] Julliard, C. (2002): The International Diversification Puzzle Is Not Worse than You Think, *London School of Economics, Unpublished*.

[56] Kang, J.K. and Stulz, R.M. (1997): Why Is There Home Bias? An Analysis of Foreign Portfolio Equity Ownership in Japan, *Journal of Financial Economics*, 46, 3-28.

[57] Kilka, M. and Weber, M. (2000): Home Bias in International Stock Return Expectations, *Journal of Psychology and Financial Markets*, 1(3), 176-192.

[58] Kollmann, R. (2006): International Portfolio Equilibrium and the Current Account, *CERP Discussion Paper Series*, No. 5512.

[59] Lauterbach, B. and Reisman, H. (2004) Keeping up with the Joneses and the home bias, *European Financial Management*, 10(2), 225-234.

[60] Lewis, K. (1999): Trying to Explain Home Bias in Equities and Consumption, *Journal of Economic Literature*, 37(2), 571-608.

[61] Logue, D. (1982): An Experiment in International Diversification, *Journal of Portfolio Management*, 9(1), 22-27.

[62] Lutje, T. and Menkhoff, L. (2007): What Drives Home Bias? Evidence from Fund Managers' Views, *International Journal of Finance and Economics*, 12(1), 21-35.

[63] Mathur, I., Singh, M. and Gleason, K. (2001): The Evidence from Canadian Firms on Multinational Diversification and Performance, *Quarterly Review of Economics and Finance*, 41(4), 561-578.

[64] Michel, A. and Shaked, I. (1986): Multinational Corporations versus Domestic Corporations: Financial Performance and Characteristics, *Journal of International Business Studies*, 17(3), 89-100.

[65] Mikhail, A. and Shawky H. (1979): Investment Performance of US Based Multinational Corporations, *Journal of International Business Studies*, 10(1), 53-66.

[66] Morck, R. and Yeung, B. (1991): Why Investors Value Multinationality, *Journal of Business*, 64(2), 165-187.

[67] Morse, A. and Shive, S. (2011): Patriotism in Your Portfolio, *Journal of Financial Markets*, 14(2), 411-440.

[68] Nieuwerburgh van, S. and Veldkamp, L. (2005): Information Immobility and the Home Bias Puzzle, *The Journal of Finance*, 64(3), 1187-1215.

[69] Obstfeld, M. and Rogoff, K. (2000): The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?, *NBER Macroeconomics Annual 2000*, Bernanke, B.S. and Rogoff, K. (eds), Volume 15, MIT Press.

[70] Pesenti, P. and van Wincoop, E. (2002): Can Nontradables Generate Substantial Home Bias?, *Journal of Money, Credit and Banking*, 34(1), 25-49.

[71] Rowland, P.F. (1999): Transaction Costs and International Portfolio Diversification, *Journal of International Economics*, 49(1), 145-170.

[72] Rowland, P. and Tesar L. (2004): Multinationals and the Gains from International Diversification, *Review of Economic Dynamics*, 7(4), 789-82.

[73] Salehizadeh, M. (2003): U.S. Multinational and the Home Bias Puzzle: An Empirical Analysis, *Global Finance Journal*, 14(3), 303-318.

[74] Seasholes, M.S. and Zhu, N. (2005): Is There Information in the Local Portfolio Choices of Individuals? *The Journal of Finance*, 65(5), 1987-2010.

[75] Senchack, A. and Beedles, W. (1980): Is Direct International Diversification Desirable?, *Journal of Portfolio Management*, 6(2), 49-57.

[76] Severn, A.K. (1974): Investor Evaluation of Foreign and Domestic Risk, *Management Science*, 29(2), 545-550.

[77] Strong, N. and Xu, X. (2003): Understanding the Equity Home Bias: Evidence from Survey Data, *Review of Economics and Statistics*, 85(2), 307-312.

[78] Stulz, R.M. (1981): On the Effects of Barriers to International Investment, *Journal of Finance*, 36(4), 923-934.

[79] Tesar, L. (1993): International Risk-Sharing and Non-Traded Goods. *Journal of International Economics*, 35(1-2), 68-89.

[80] Tesar, L. and Werner, I.M. (1995): Home Bias and High Turnover, *Journal of International Money and Finance*, 14(4), 476-492.

[81] Tille, C. and van Wincoop, E. (2010): International Capital Flows, *Journal of International Economics*, 80(2), 157-175.

[82] Warnock, F.E. (2001): Home Bias and High Turnover Reconsidered, *Journal of International Money and Finance*, 21(6), 795-805.

[83] Wei, S. (2000): How Taxing Is Corruption on International Investors?, *Review of Economics and Statistics*, 82(1), 1-11.

[84] Wincoop van, E. and Warnock, F.E. (2006): Is Home Bias in Assets Related to Home Bias in Goods?, *NBER Working Paper*, No. 12728.

Figure 1:

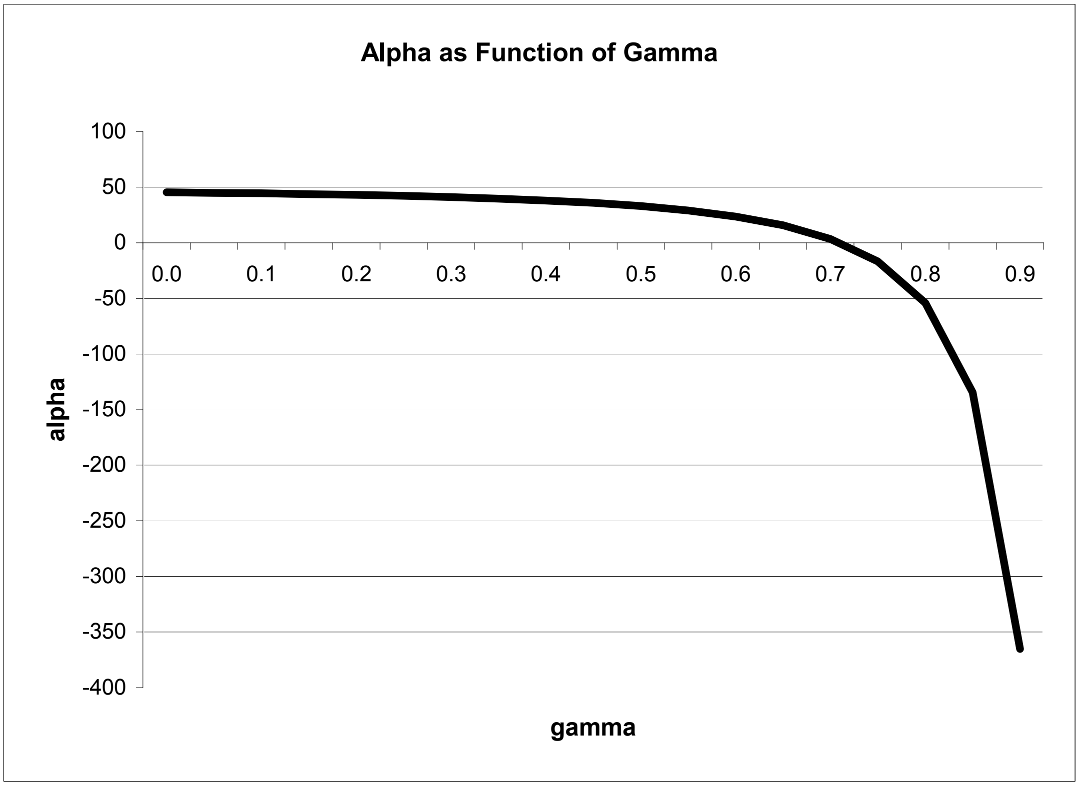

Alpha as Function of Gamma
100
50
0
-50
-100
-150
-200
-250
-300
-350
-400
alpha
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
gamma

Figure 2:
Consumption Differential, Z Shock

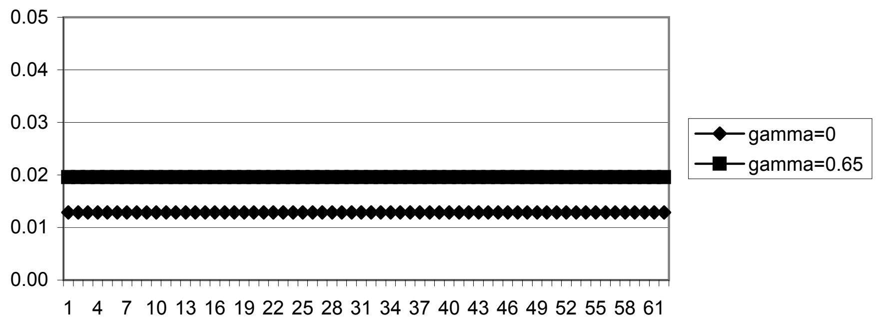


Figure 3:
Equity Price Differential, Z Shock

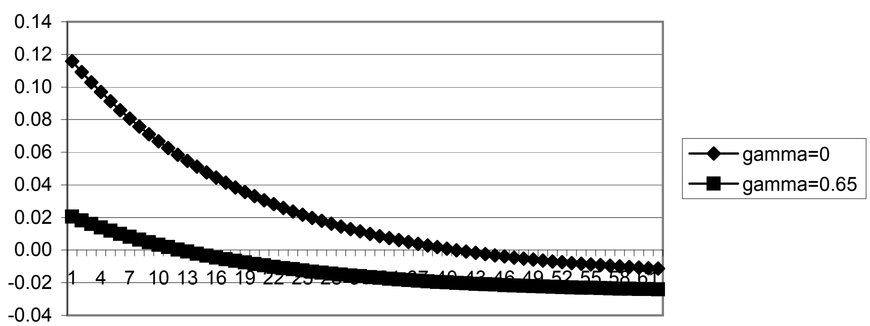


Figure 4:
Net Foreign Assets Differential, Z Shock

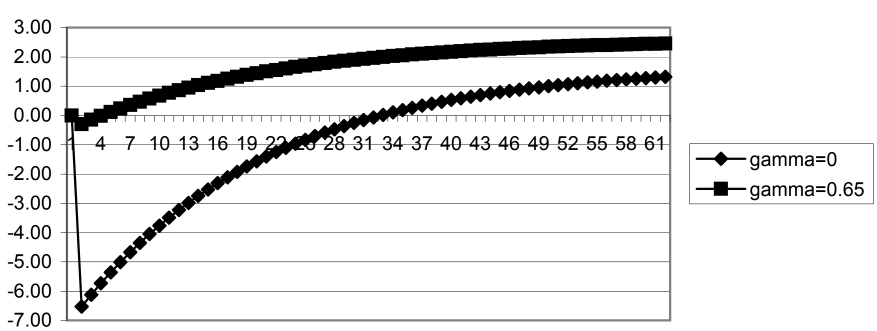


Figure 5:
Consumption Differential, G Shock

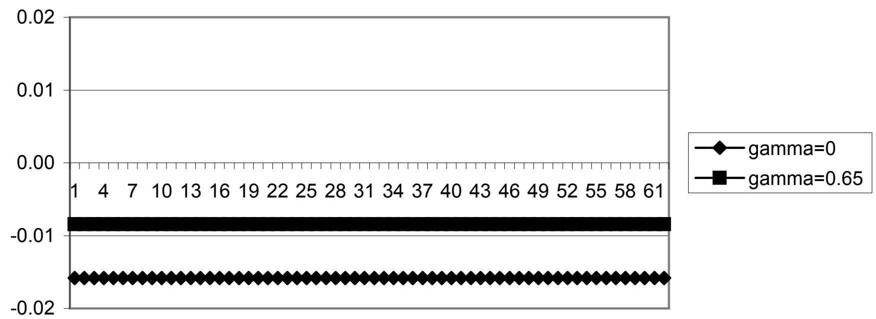
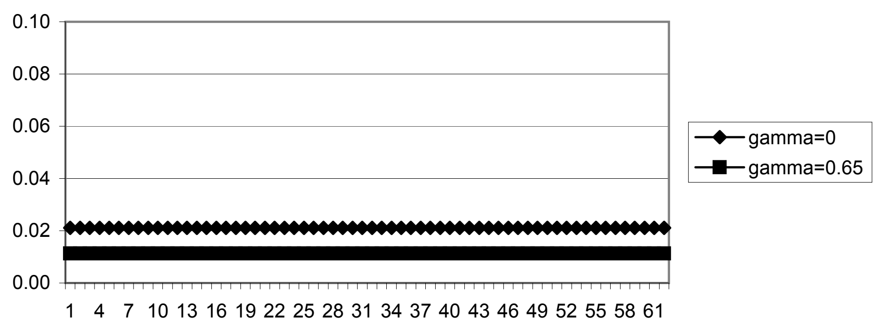
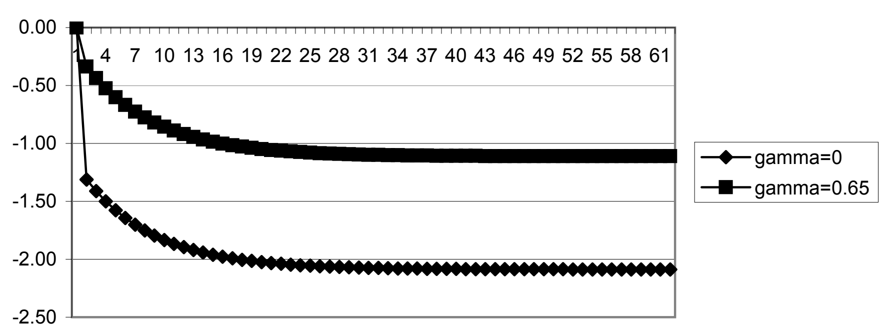


Figure 6:
Equity Price Differential, G Shock



Graph 7:
Net Foreign Assets Differential, G Shock



Appendix: Review of Empirical Research on the Role of MNCs in Portfolio Home Bias

This Appendix reviews empirical research on the role of MNCs in portfolio home bias.[7] The following papers have concluded that the investors *can* diversify internationally by investing in the MNCs instead of purchasing foreign equity.

Severn (1974) provides a very basic discussion of the topic. Hughes, Logue and Sweeney

[7]Other explanations of the home bias discussed in the literature include information asymmetries and geographic proximity (Gehrig (1993), Brennan and Cao (1997), Kang and Stulz (1997), Huberman (1998), Coval and Moskowitz (1999), Hasan and Simaan (2000), Benartzi (2001), Grinblatt and Keloharju (2001), Ahearne, Griever and Warnock (2004), Amadi (2004), and Nieuwerburgh and Veldkamp (2005)), domestic country risk including non-tradeable goods and trade costs (Eldor, Pines, and Schwartz (1988), Tesar (1993), Baxter, Jermann and King (1998), Obstfeld and Rogoff (2000), and Coeurdacier (2009)), immobile human capital (Bottazzi, Pesenti and van Wincoop (1996), Baxter and Jermann (1997), Coen (2001), Julliard (2002), Pesenti and van Wincoop (2002), and Heathcote and Perri (2007)) and inflation (Adler and Dumas (1983), and Cooper and Kaplanis (1994)), exchange rate volatility (Cooper and Kaplanis (1994), Fidora, Fratzscher and Thimann (2006), Benigno and Nistico (2009), and Coeurdacier and Gourinchas (2009)), home consumption bias (Lewis (1999), Kollmann (2006), and van Wincoop and Warnock (2006)), transaction costs (Black (1974), Stulz (1981), Tesar and Werner (1995), Rowland (1999), Glassman and Riddick (2001), Warnock (2001), and Amadi and Bergin (2008)), corporate governance (Wei (2000), Dahlquist, Pinkowitz, Stulz and Williamson (2003), and Gianetti and Koskinen (2003)), relative return optimism (Grinblatt and Keloharju (2000), Kilka and Weber (2000), Jeske (2001), Strong and Xu (2003), Lutje and Menkhoff (2007), and Seasholes and Zhu (2005)), differences in beliefs (Dumas, Lewis and Osambela (2009)), patriotism (Morse and Shive (2011)), social identity (Fellner and Maciejovsky (2003)), and "keeping up with the Joneses" (Lauterbach and Reisman (2004)).

(1975) use data from 1970 to 1973 for 46 US MNCs and 50 US domestic firms to compare Treynor's risk-adjusted returns on domestic and diversified portfolios. They conclude that the MNCs have higher returns and lower unsystematic, systematic and total risk than the domestic firms. Therefore, investors can diversify via the MNCs.

Agmon and Lessard (1977) use 217 US firms from 1959 to 1972 to analyze the relationship between share price behavior and international involvement of the firm measured by percentage of international sales. They find that firms with more international involvement are more correlated with the rest-of-the-world index. The firms with less international involvements are more correlated with the domestic market index. Therefore, diversification offers diversification benefits.

Mikhail and Shawky (1979) use 30 US MNCs from 1968 to 1975 to compare returns, the standard deviation, and the coefficient of variation of these firms and the S&P500 index. They conclude that MNCs have higher returns and thus outperform the domestic market.

Logue (1982) uses the US stock market returns for 18 countries and 50 US MNCs from 1955 to 1975 to construct efficient frontiers. The optimal portfolio is the one with the highest ratio of expected return to standard deviation. He compares the optimal portfolios of the MNCs and the foreign market indices, and concludes that the MNCs outperform foreign indices. Therefore, US investors are better off investing in MNCs than in foreign equity.

Errunza (1984) uses data for 14 developed and 15 emerging markets from 1976 to 1980 and concludes that emerging markets provide a diversification opportunity.

Errunza, Hogan and Hung (1999) revisit the topic with data for 7 developed and 9 emerging markets from 1976 to 1993 to run mean-variance spanning tests, return correlations, and Sharpe ratio tests, and conclude that international diversification can be achieved without investing in foreign equity.

Bekaert and Urias (1996) use data on 43 U.S. closed-end mutual funds and 37 U.K. closed-end mutual funds from 1986 to 1993 and find benefits to diversifying into emerging markets for the U.K. funds but not for the U.S. funds.

Cai and Warnock (2004) analyze US equity holdings in 1994 and 2000 and find that both home and foreign investors prefer the MNCs. They then construct a measure of home bias that accounts for foreign operations of the US firms and suggests that the US investors have 20% rather than the usually stated 13% exposure to foreign markets.

However, several papers disagree and conclude that investors *cannot* diversify internationally by investing in MNCs instead of purchasing foreign equity.

Jacquillat and Solnik (1978) use data from about 300 European and 100 US firms from 1966 to 1974 to compare the standard deviations of three portfolios: US domestic firm portfolio, US MNCs portfolio, and equally weighted major national stock market portfolio. They conclude that US MNCs provide some diversification benefits but cannot replace purchasing foreign equity because MNCs are more influenced by the domestic markets than the foreign markets.

Senchack and Beedles (1980) come to the same conclusion using data on 284 US firms from 1963 to 1976.

Brewer (1981) uses the domestic CAPM of 151 US MNCs and 137 US domestic firms from 1963 to 1975 to compare the risk-adjusted performance of shares in the MNCs and domestic firms. He concludes that the two types of firms have similar results and, therefore, MNCs do not offer diversification benefits.

Fatemi (1984) uses returns for 84 US MNCs and 52 US domestic firms from 1976 to

1980 to perform several tests and concludes that the MNCs do not facilitate international portfolio diversification because risk-adjusted returns are similar for the MNCs and domestic firms.

Michel and Shaked (1986) use data for 58 US MNCs and 43 US domestic firms from 1973 to 1982 to calculate Jensen, Treynor, and Sharpe measures and compare the performance of the two types of firms. They conclude that the domestic firms outperform the MNCs.

Morck and Yeung (1991) use 1978 data for 1,644 firms to examine the effect of multinationality on the firm's market value and conclude that MNCs cannot be used to diversify.

Mathur, Singh and Hanagan (2001) use Canadian data on 180 MNCs and 226 domestic firms from 1992-1994 and 1997 to test whether multinationality affects firm performance and conclude that the MNCs do not outperform the Canadian firms.

Rowland and Tesar (2004) use data for 411 MNCs from 7 countries from 1984 to 1995 to conduct mean-variance spanning tests and conclude that MNCs provide negligible diversification in most countries.

Salehizadeh (2003) uses data on 48 MNCs from 1995 to 2001 to compare their returns to the S&P index and concludes that the correlations do not justify diversifying by using MNCs instead of foreign equity.

The latest and most comprehensive paper on the topic is Berrill and Kearney (2008). They analyze 7 countries and provide an excellent summary of the previous research.[8] Noting that previous studies have inconsistently classified MNCs, they apply the Aggarwal, Berrill and Kearney (2011) classification of MNCs and find that investors *can* diversify by investing in MNCs.

[8]Their summary was used in this literature review.

Technical Appendix

Appendix A: Model Details

This Appendix shows derivations for Section 2.

A.1 Households

Derivation of price indices:

min $P_{Ht}C_{Ht} + P_{Ft}C_{Ft}$ subject to $C_t = 1$ where $C_t = [a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}}$

$\ell = P_{Ht}C_{Ht} + P_{Ft}C_{Ft} - P_t[[a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}} - 1]$

$\frac{\partial \ell}{\partial C_{Ht}} = P_{Ht} - P_t \frac{\omega}{\omega-1}[a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}-1} a^{\frac{1}{\omega}} \frac{\omega-1}{\omega} C_{Ht}^{\frac{\omega}{\omega-1}-1} = 0$

$P_{Ht} = P_t \frac{\omega}{\omega-1}[a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}-1} a^{\frac{1}{\omega}} \frac{\omega-1}{\omega} C_{Ht}^{\frac{\omega}{\omega-1}-1}$

$\frac{\partial \ell}{\partial C_{Ft}} = P_{Ft} - P_t \frac{\omega}{\omega-1}[a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}-1} a^{\frac{1}{\omega}} \frac{\omega-1}{\omega} C_{Ft}^{\frac{\omega}{\omega-1}-1} = 0$

$P_{Ft} = P_t \frac{\omega}{\omega-1}[a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}-1} a^{\frac{1}{\omega}} \frac{\omega-1}{\omega} C_{Ft}^{\frac{\omega}{\omega-1}-1}$

Since $[a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}} \equiv C_t$, it is possible to write $[a^{\frac{1}{\omega}} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} C_{Ft}^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}-1} = C_t^{\frac{1}{\omega}} = 1$

$P_{Ht} = P_t a^{\frac{1}{\omega}} C_{Ht}^{-\frac{1}{\omega}}$, so $C_{Ht}^{-\frac{1}{\omega}} = \frac{P_{Ht}}{P_t} a^{-\frac{1}{\omega}}$, so $C_{Ht} = (\frac{P_t}{P_{Ht}})^{\omega} a$

$P_{Ft} = P_t (1-a)^{\frac{1}{\omega}} C_{Ft}^{-\frac{1}{\omega}}$, so $C_{Ft}^{-\frac{1}{\omega}} = \frac{P_{Ft}}{P}(1-a)^{-\frac{1}{\omega}}$, so $C_{Ft} = (\frac{P_t}{P_{Ft}})^{\omega}(1-a)$

Substitute into $C_t = 1$

$[a^{\frac{1}{\omega}} (\frac{P_t}{P_{Ht}})^{\omega-1} a^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} (\frac{P_t}{P_{Ft}})^{\omega-1} (1-a)^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}} = 1$

$a^{\frac{\omega}{\omega-1}} (\frac{P_t}{P_{Ht}})^{\omega} + (1-a)^{\frac{\omega}{\omega-1}} (\frac{P_t}{P_{Ft}})^{\omega} = 1$

$a(\frac{P_t}{P_{Ht}})^{\omega-1} + (1-a)(\frac{P_t}{P_{Ft}})^{\omega-1} = 1$

$\frac{aP_{Ht}^{1-\omega}+(1-a)P_{Ft}^{1-\omega}}{P_t^{1-\omega}} = 1$

$P_t = [aP_{Ht}^{1-\omega} + (1-a)P_{Ft}^{1-\omega}]^{\frac{1}{1-\omega}}$

min $p_t(z)c_t(z)$ subject to $C_{Ht} = 1$ where $C_{Ht} = [(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}}$

$\ell = p_t(z)c_t(z) - P_{Ht}[[(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}} - 1]$

$\frac{\partial \ell}{\partial c_t(z)} = p_t(z) - P_{Ht}\frac{\theta}{\theta-1}[(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}-1}(\frac{1}{a})^{\frac{1}{\theta}}\frac{\theta-1}{\theta}c_t(z)^{\frac{\theta-1}{\theta}-1} = 0$

$p_t(z) = P_{Ht}[(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{1}{\theta-1}}(\frac{1}{a})^{\frac{1}{\theta}}c_t(z)^{-\frac{1}{\theta}}$

$c_t(z)^{-\frac{1}{\theta}} = \frac{p_t(z)}{P_{Ht}}a^{\frac{1}{\theta}}$

$c_t(z) = (\frac{p_t(z)}{P_{Ht}})^{-\theta}\frac{1}{a} = \frac{1}{a}(\frac{P_{Ht}}{p_t(z)})^{\theta}$

Substitute into $C_{Ht} = 1$

$C_{Ht} = [(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a (\frac{P_{Ht}}{p_t(z)})^{\theta-1}(\frac{1}{a})^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}} = 1$

$[(\frac{1}{a})^{\frac{1}{\theta}}(\frac{1}{a})^{\frac{\theta-1}{\theta}} \int_o^a (\frac{P_{Ht}}{p_t(z)})^{\theta-1} dz]^{\frac{\theta}{\theta-1}} = 1$

$P_{Ht}^{\theta}[(\frac{1}{a}) \int_o^a (\frac{1}{p_t(z)})^{\theta-1} dz]^{\frac{\theta}{\theta-1}} = 1$

$[\frac{1}{a} \int_o^a p_t(z)^{1-\theta} dz]^{\frac{\theta}{\theta-1}} = P_{Ht}^{-\theta}$

$[\frac{1}{a} \int_o^a p_t(z)^{1-\theta} dz]^{\frac{-1}{\theta-1}} = P_{Ht}$

$[\frac{1}{a} \int_o^a p_t(z)^{1-\theta} dz]^{\frac{1}{1-\theta}} = P_{Ht}$

$[\frac{1}{1-a} \int_a^1 p_t(z*)^{1-\theta} dz*]^{\frac{1}{1-\theta}} = P_{Ft}$ can be derived by following the same steps.

Note that these expressions are identical to GLR.

Household demand:

Demand for output produced by home firm z in home country is:

$Y_t^d(z) = (\frac{p_t(z)}{P_{Ht}})^{-\theta}(\frac{P_{Ht}}{P_t})^{-\omega}Y_t^W \equiv (RP_t(z))^{-\theta}(RP_t)^{\theta-\omega}Y_t^W$ where

$RP_t(z)$ is the price of good z in units of world consumption basket,

RP_t is the price of the home sub-basket of goods in units of world consumption basket and

Y_t^W is aggregate world demand for the consumption basket.

This can be written as:

$Y_t^d(z) = RP_t^{-\omega}Y_t^W$ because $RP_t(z) = RP_t$ at optimum.

Demand for output produced by foreign firm z^* in home country: $Y_{*t}^d(z^*) = RP_{*t}^{-\omega}Y_t^W$

Demand for output produced by home firm z in foreign country: $Y_t^{*d}(z) = RP_t^{*-\omega}Y_t^W$

Demand for output produced by foreign firm z^* in foreign country: $Y_{*t}^{*d}(z^*) = RP_{*t}^{*-\omega}Y_t^W$

Note that these expressions are identical to GLR.

FOCs:

$\ell = E_t \sum_{s=t}^{\infty} \beta^{s-t}(\frac{C_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \chi\frac{L_s^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}) +$

$+ \lambda_t[(v_t + d_t + d_{*t})x_t + (v_t^* + d_t^* + d_{*t}^*)x_t^* + w_tL_t - v_tx_{t+1} - v_t^*x_{t+1}^* - C_t - G_t]$

This Lagrangian is identical to the GLR Lagrangian with the exception of the additional terms d_{*t} and d_{*t}^*.

With respect to C_t:

$\frac{\partial\ell}{\partial C_t} = (1-\frac{1}{\sigma})\frac{C_t^{\frac{\sigma-1}{\sigma}-1}}{1-\frac{1}{\sigma}} + \lambda_t(-1) = 0$

$C_t^{-\frac{1}{\sigma}} = \lambda_t$

which is the same as in GLR.

With respect to L_t:

$$\frac{\partial \ell}{\partial L_t} = -\chi(1+\frac{1}{\varphi})\frac{L_t^{\frac{\varphi+1}{\varphi}-1}}{1+\frac{1}{\varphi}} + \lambda_t w_t = 0$$

$$\chi L_t^{\frac{\psi+1-\psi}{\psi}} = \lambda_t w_t$$

$$\chi L_t^{\frac{1}{\varphi}} = C_t^{-\frac{1}{\sigma}} w_t$$

$$L_t^{\frac{1}{\varphi}} = \frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi}$$

$$L_t = (\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi}$$

which is the same as in GLR.

With respect to x_{t+1}:

$$\frac{\partial \ell}{\partial x_{t+1}} = \lambda_t(-v_t) + \beta E_t\{\lambda_{t+1}(v_{t+1} + d_{t+1} + d_{*t+1})\} = 0$$

$$C_t^{-\frac{1}{\sigma}} v_t = \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}}(v_{t+1} + d_{t+1} + d_{*t+1})\}$$

$$C_t^{-\frac{1}{\sigma}} = \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}} \frac{v_{t+1}+d_{t+1}+d_{*t+1}}{v_t}\} \equiv \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}} R_{t+1}\}$$

where the definition of R_{t+1} differs from GLR due to the additional term d_{*t+1}.

With respect to $x_{t+1}*$:

$$\frac{\partial \ell}{\partial x^*_{t+1}} = \lambda_t(-v^*_t) + \beta E_t\{\lambda_{t+1}(v^*_{t+1} + d^*_{t+1} + d^*_{*t+1})\} = 0$$

$$C_t^{-\frac{1}{\sigma}} v^*_t = \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}}(v^*_{t+1} + d^*_{t+1} + d^*_{*t+1})\}$$

$$C_t^{-\frac{1}{\sigma}} = \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}} \frac{v^*_{t+1}+d^*_{t+1}+d^*_{*t+1}}{v^*_t}\} \equiv \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}} R^*_{t+1}\}$$

where the definition of R^*_{t+1} differs from GLR due to the additional term d^*_{*t+1}.

NFA law of motion:

Start with household budget constraint:

$$v_t x_{t+1} + v^*_t x^*_{t+1} + C_t + G_t = (v_t + d_t + d_{*t})x_t + (v^*_t + d^*_t + d^*_{*t})x^*_t + w_t L_t$$

$v_t x_{t+1} + nfa_{t+1} + \frac{1-a}{a} v_t x_{*t+1} + C_t + G_t = \frac{(v_t + d_t + d_{*t}) v_{t-1}}{v_{t-1}} x_t + \frac{(v_t^* + d_t^* + d_{*t}^*) v_{t-1}^*}{v_{t-1}^*} x_t^* + w_t L_t$

where nfa_{t+1} is defined as $nfa_{t+1} \equiv v_t^* x_{t+1}^* - \frac{1-a}{a} v_t x_{*t+1}$, i.e., home holdings of foreign shares minus foreign holdings of home shares adjusted for population sizes of home and foreign countries, i.e., a and $1-a$ as in GLR.

$v_t x_{t+1} + nfa_{t+1} + \frac{1-a}{a} v_t x_{*t+1} + C_t + G_t = R_t v_{t-1} x_t + R_t^* v_{t-1}^* x_t^* + w_t L_t$

$nfa_{t+1} = -v_t x_{t+1} - \frac{1-a}{a} v_t x_{*t+1} + R_t v_{t-1} x_t + R_t^* v_{t-1}^* x_t^* + w_t L_t - C_t - G_t$

$nfa_{t+1} = -v_t (x_{t+1} + \frac{1-a}{a} x_{*t+1}) + R_t v_{t-1} x_t + R_t^* v_{t-1}^* x_t^* + w_t L_t - C_t - G_t$

$nfa_{t+1} = -v_t + R_t v_{t-1} x_t + R_t^* v_{t-1}^* x_t^* + w_t L_t - C_t - G_t$

where market clearing condition $a x_{t+1} + (1-a) x_{*t+1} = a$ was used to obtain $x_{t+1} = 1 - \frac{1-a}{a} x_{*t+1}$ as in GLR.

$nfa_{t+1} = -v_t + R_t^* v_{t-1}^* x_t^* + R_t v_{t-1} (1 - \frac{1-a}{a} x_{*t}) + w_t L_t - C_t - G_t$

$nfa_{t+1} = -v_t + R_t^* v_{t-1}^* x_t^* + R_t v_{t-1} - R_t v_{t-1} \frac{1-a}{a} x_{*t} + w_t L_t - C_t - G_t$

$nfa_{t+1} = -v_t + R_t^* v_{t-1}^* x_t^* + v_t + d_t + d_{*t} - R_t v_{t-1} \frac{1-a}{a} x_{*t} + w_t L_t - C_t - G_t$

$nfa_{t+1} = R_t^* v_{t-1}^* x_t^* - R_t v_{t-1} \frac{1-a}{a} x_{*t} + y_t - C_t - G_t$

where $y_t \equiv d_t + d_{*t} + w_t L_t$ which differs from GLR due to the additional term d_{*t}

$nfa_{t+1} = R_t v_{t-1}^* x_t^* - R_t v_{t-1}^* x_t^* + R_t^* v_{t-1}^* x_t^* - R_t v_{t-1} \frac{1-a}{a} x_{*t} + y_t - C_t - G_t$

Define excess return from holding foreign equity $R_t^D = R_t^* - R_t$ and portfolio holding

$\alpha_t = v_{t-1}^* x_t$:

$nfa_{t+1} = R_t^D \alpha_t + R_t v_{t-1}^* x_t^* - R_t v_{t-1} \frac{1-a}{a} x_{*t} + y_t - C_t - G_t$

where definition $nfa_t = v_{t-1}^* x_t^* - \frac{1-a}{a} v_{t-1} x_{*t}$

$nfa_{t+1} = R_t^D \alpha_t + R_t nfa_t + y_t - C_t - G_t$

was used which is identical to GLR except the definitions of R_t and R_t^*, and hence R_t^D, differ as explained above.

Similar derivations can be done to obtain NFA law of motion for the foreign household:

$nfa^*_{t+1} = R^D_t \alpha^*_t + R_t nfa^*_t + y^*_t - C^*_t - G^*_t$

Subtracting the home and foreign NFA laws of motions and using the superscript D to denote the difference between home and foreign variables gives:

$nfa_{t+1} - nfa^*_{t+1} = R^D_t \alpha_t + R_t nfa_t + y_t - C_t - G_t - R^D_t \alpha^*_t - R_t nfa^*_t - (y^*_t - C^*_t - G^*_t)$

$nfa_{t+1} - (-\frac{a}{1-a}) nfa_{t+1} = R^D_t \alpha_t - (-\frac{a}{1-a}) R^D_t \alpha_t + R_t nfa_t - (-\frac{a}{1-a}) R_t nfa_t + y^D_t - C^D_t - G^D_t$

$nfa_{t+1} \frac{1}{1-a} = R^D_t \alpha_t \frac{1}{1-a} + R_t nfa_t \frac{1}{1-a} + y^D_t - C^D_t - G^D_t$

$nfa_{t+1} = R^D_t \alpha_t + R_t nfa_t + (1-a)(y^D_t - C^D_t - G^D_t)$

where market clearing conditions $nfa^*_{t+1} = -\frac{a}{1-a} nfa_{t+1}$ and $\alpha^*_t = -\frac{a}{1-a} \upsilon^*_{t-1} x^*_t = -\frac{a}{1-a} \alpha_t$ were used as in GLR.

A.2 Firms

A.2.1 Derivation of prices:

Derivation of price of good produced by home firm z:

min $w_t L_t(z) + w^*_t L^*_t(z) - mc_t(Z_t L_t(z) + Z^\gamma_t Z^{*1-\gamma}_t L^*_t(z) - Y_t(z))$

$\frac{\partial \ell}{\partial L_t(z)} = w_t - mc_t Z_t = 0$

$\frac{w_t}{Z_t} = mc_t = RP_t \frac{\theta-1}{\theta}$

$RP_t = \frac{\theta}{\theta-1} \frac{w_t}{Z_t}$ which is the price charged in home country. This is identical to GLR.

$\frac{\partial \ell}{\partial L^*_t(z)} = w^*_t - mc_t Z^\gamma_t Z^{*1-\gamma}_t = 0$

$\frac{w^*_t}{Z^\gamma_t Z^{*1-\gamma}_t} = mc_t = RP^*_t \frac{\theta-1}{\theta}$

$RP_t^* = \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^{\gamma} Z_t^{*1-\gamma}}$ which is the price charged in foreign country. This differs from GLR.

Derivation of price of good produced by foreign firm z^*:

$\min\, w_t L_{*t}(z^*) + w_t^* L_{*t}^*(z^*) - mc_{*t}(Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t}(z^*) + Z_t^* L_{*t}^*(z^*) - Y_t^*(z^*))$

$\frac{\partial \ell}{\partial L_{*t}(z^*)} = w_t - mc_{*t} Z_t^{1-\gamma} Z_t^{*\gamma} = 0$

$\frac{w_t}{Z_t^{1-\gamma} Z_t^{*\gamma}} = mc_{*t} = RP_{*t}\frac{\theta-1}{\theta}$

$RP_{*t} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t^{1-\gamma} Z_t^{*\gamma}}$ which is the price charged in home country. This differs from GLR.

$\frac{\partial \ell}{\partial L_{*t}^*(z^*)} = w_t^* - mc_{*t} Z_t^* = 0$

$\frac{w_t^*}{Z_t^*} = mc_{*t} = RP_{*t}^*\frac{\theta-1}{\theta}$

$RP_{*t}^* = \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*}$ which is the price charged in foreign country. This is identical to GLR.

A.2.2 Derivation of optimum labor demands:

Note that indexation of labor by z and z^* showing whether the labor is hired by home firm z or foreign firm z^* is dropped to simplify the notation.

Derivation of optimal labor demand by home firm z in home country:

Production function is given as $Y_t^s = Z_t L_t$

Demand for output was derived from household optimization as: $Y_t^d = RP_t^{-\omega} Y_t^W$.

Since $Y_t^s = Y_t^d$, it is possible to write $Z_t L_t = RP_t^{-\omega} Y_t^W$.

From this, $L_t = \frac{RP_t^{-\omega} Y_t^W}{Z_t}$ follows. This is identical to GLR.

Derivation of optimal labor demand by foreign firm z^* in home country:

Production function is given as $Y_{*t}^s = Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t}$

Demand for output was derived from household optimization as: $Y_{*t}^{d} = RP_{*t}^{-\omega} Y_t^W$

Since $Y_{*t}^{s} = Y_{*t}^{d}$, we can write $Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t} = RP_{*t}^{-\omega} Y_t^W$.

From this, $L_{*t} = \frac{RP_{*t}^{-\omega} Y_t^W}{Z_t^{1-\gamma} Z_t^{*\gamma}}$ follows. This differs from GLR.

Derivation of total optimal labor demand in home country:

$L_t^d = L_t + L_{*t} = \frac{RP_t^{-\omega} Y_t^W}{Z_t} + \frac{RP_{*t}^{-\omega} Y_t^W}{Z_t^{1-\gamma} Z_t^{*\gamma}} = Y_t^W \left(\frac{RP_t^{-\omega}}{Z_t} + \frac{RP_{*t}^{-\omega}}{Z_t^{1-\gamma} Z_t^{*\gamma}}\right)$ which differs from GLR.

This expression contains Y_t^W, i.e., aggregate world demand for the consumption basket. Aggregate *per capita* world demand for the consumption basket can be defined as $y_t^W \equiv aC_t + (1-a)C_t^* + aG_t + (1-a)G_t^*$. The aggregate *per capita* labor demand in home country can then be written as $L_t^d = y_t^W \left(\frac{RP_t^{-\omega}}{Z_t} + \frac{RP_{*t}^{-\omega}}{Z_t^{1-\gamma} Z_t^{*\gamma}}\right)$ which differs from GLR.

Derivation of optimal labor demand by home firm z in foreign country:

Production function is given as $Y_t^{*s} = Z_t^{\gamma} Z_t^{*1-\gamma} L_t^*$

Demand for output was derived from household optimization as: $Y_t^{*d} = RP_t^{*-\omega} Y_t^W$

Since $Y_t^{*s} = Y_t^{*d}$, it is possible to write $Z_t^{\gamma} Z_t^{*1-\gamma} L_t^* = RP_t^{*-\omega} Y_t^W$.

From this, $L_t^* = \frac{RP_t^{*-\omega} Y_t^W}{Z_t^{\gamma} Z_t^{*1-\gamma}}$ follows. This differs from GLR.

Derivation of optimal labor demand by foreign firm z^* in foreign country:

Production function is given as $Y_{*t}^{*s} = Z_t^* L_{*t}^*$

Demand for output was derived from household optimization as: $Y_{*t}^{*d} = RP_{*t}^{*-\omega} Y_t^W$

Since $Y_{*t}^{*s} = Y_{*t}^{*d}$, it is possible to write $Z_t^* L_{*t}^* = RP_{*t}^{*-\omega} Y_t^W$.

From this, $L_{*t}^{*} = \frac{RP_{*t}^{*-\omega} Y_t^W}{Z_t^*}$ follows. This is identical to GLR.

Derivation of total optimal labor demand in foreign country:

$L_t^{*d} = L_t^* + L_{*t}^* = \frac{RP_t^{*-\omega} Y_t^W}{Z_t^{\gamma} Z_t^{*1-\gamma}} + \frac{RP_{*t}^{*-\omega} Y_t^W}{Z_t^*} = Y_t^W \left(\frac{RP_t^{*-\omega}}{Z_t^{\gamma} Z_t^{*1-\gamma}} + \frac{RP_{*t}^{*-\omega}}{Z_t^*} \right)$ which differs from GLR.

Aggregate *per capita* labor demand in foreign country is: $L_t^{*d} = y_t^W \left(\frac{RP_t^{*-\omega}}{Z_t^{\gamma} Z_t^{*1-\gamma}} + \frac{RP_{*t}^{*-\omega}}{Z_t^*} \right)$ which differs from GLR.

A.3 Some Useful Properties

A.3.1 To get $\frac{y_t}{y_t^*}$:

Derivation of home GDP y_t, i.e., output produced by home and foreign firms in the home country:

$y_t = RP_t Z_t L_t + RP_{*t} Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t} = \frac{\theta}{\theta-1} \frac{w_t}{Z_t} Z_t L_t + \frac{\theta}{\theta-1} \frac{w_t}{Z_t^{1-\gamma} Z_t^{*\gamma}} Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t} = \frac{\theta}{\theta-1}(w_t L_t + w_t L_{*t})$

Derivation of foreign GDP y_t^*, i.e., output produced by home and foreign firms in the foreign country:

$y_t^* = RP_t^* Z_t^{\gamma} Z_t^{*1-\gamma} L_t^* + RP_{*t}^* Z_t^* L_{*t}^* = \frac{\theta}{\theta-1} \frac{w_t^*}{Z_t^{\gamma} Z_t^{*1-\gamma}} Z_t^{\gamma} Z_t^{*1-\gamma} L_t^* + \frac{\theta}{\theta-1} \frac{w_t^*}{Z_t^*} Z_t^* L_{*t}^* =$

$= \frac{\theta}{\theta-1}(w_t^* L_t^* + w_t^* L_{*t}^*)$

Note that the relative prices convert GDP into units of consumption basket, i.e., basket consisting of home and foreign goods.

Expression for $\frac{y_t}{y_t^*}$:

$\frac{y_t}{y_t^*} = \frac{RP_t Z_t L_t + RP_{*t} Z_t^{1-\gamma} Z_t^{*\gamma} L_{*t}}{RP_t^* Z_t^{\gamma} Z_t^{*1-\gamma} L_t^* + RP_{*t}^* Z_t^* L_{*t}^*} = \frac{\frac{\theta}{\theta-1}(w_t L_t + w_t L_{*t})}{\frac{\theta}{\theta-1}(w_t^* L_t^* + w_t^* L_{*t}^*)} = \frac{w_t(L_t + L_{*t})}{w_t^*(L_t^* + L_{*t}^*)}$

Even though the production structure differs from GLR, the technology terms cancel in this expression, so this result is identical to GLR.

Next, expressions for w_t, w_t^*, $(L_t + L_{*t})$ and $(L_t^* + L_{*t}^*)$ are obtained:

To get w_t, home labor supply and home labor demand are equated.

Home labor supply was derived above from home household FOCs as $L_t^s = (\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi}$.

Home labor demand was derived above from firm FOCs as $L_t^d = Y_t^W (\frac{RP_t^{-\omega}}{Z_t} + \frac{RP_{*t}^{-\omega}}{Z_t^{1-\gamma} Z_t^{*\gamma}})$

Since $L_t^s = L_t^d$, it is possible to write: $(\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi} = Y_t^W (\frac{RP_t^{-\omega}}{Z_t} + \frac{RP_{*t}^{-\omega}}{Z_t^{1-\gamma} Z_t^{*\gamma}})$

Plug in expressions for RP_t and RP_{*t}:

$(\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi} = Y_t^W (\frac{(\frac{\theta}{\theta-1}\frac{w_t}{Z_t})^{-\omega}}{Z_t} + \frac{(\frac{\theta}{\theta-1}\frac{w_t}{Z_t^{1-\gamma} Z_t^{*\gamma}})^{-\omega}}{Z_t^{1-\gamma} Z_t^{*\gamma}})$

$(\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi} = Y_t^W [(\frac{\theta}{\theta-1})^{-\omega} w_t^{-\omega} Z_t^{\omega-1} + (\frac{\theta}{\theta-1})^{-\omega} w_t^{-\omega} (Z_t^{1-\gamma} Z_t^{*\gamma})^{\omega-1}]$

$(\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi} = Y_t^W (\frac{\theta}{\theta-1})^{-\omega} w_t^{-\omega} [Z_t^{\omega-1} + (Z_t^{1-\gamma} Z_t^{*\gamma})^{\omega-1}]$

$w_t^{\varphi+\omega} = C_t^{\frac{\varphi}{\sigma}} \chi^{\varphi} Y_t^W (\frac{\theta}{\theta-1})^{-\omega} [Z_t^{\omega-1} + (Z_t^{1-\gamma} Z_t^{*\gamma})^{\omega-1}]$

$w_t = C_t^{\frac{\varphi}{\sigma(\varphi+\omega)}} \chi^{\frac{\varphi}{\varphi+\omega}} (Y_t^W)^{\frac{1}{\varphi+\omega}} (\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}} [Z_t^{\omega-1} + (Z_t^{1-\gamma} Z_t^{*\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}$

To get w_t^*, equate foreign labor supply and foreign labor demand are equated.

Foreign labor supply could be derived in the same way as home labor supply from household FOCs as $L_t^{*s} = (\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi}$.

Foreign labor demand was derived above from firm FOCs as $L_t^{*d} = Y_t^W (\frac{RP_t^{*-\omega}}{Z_t^{\gamma} Z_t^{*1-\gamma}} + \frac{RP_{*t}^{*-\omega}}{Z_t^*})$

Since $L_t^{*s} = L_t^{*d}$, it is possible write: $(\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi} = Y_t^W (\frac{RP_t^{*-\omega}}{Z_t^{\gamma} Z_t^{*1-\gamma}} + \frac{RP_{*t}^{*-\omega}}{Z_t^*})$

Plug in expressions for RP_t^* and RP_{*t}^*:

$$(\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi} = Y_t^W (\frac{(\frac{\theta}{\theta-1} \frac{w_t^*}{Z_t^{\gamma} Z_t^{*1-\gamma}})^{-\omega}}{Z_t^{\gamma} Z_t^{*1-\gamma}} + \frac{(\frac{\theta}{\theta-1} \frac{w_t^*}{Z_t^*})^{-\omega}}{Z_t^*})$$

$$(\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi} = Y_t^W [(\frac{\theta}{\theta-1})^{-\omega} w_t^{*-\omega} (Z_t^{\gamma} Z_t^{*1-\gamma})^{*\omega-1} + (\frac{\theta}{\theta-1})^{-\omega} w_t^{*-\omega} (Z_t^*)^{\omega-1}]$$

$$(\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi} = Y_t^W (\frac{\theta}{\theta-1})^{-\omega} w_t^{*-\omega} [Z_t^{*\omega-1} + (Z_t^{\gamma} Z_t^{*1-\gamma})^{\omega-1}]$$

$$w_t^{*\varphi+\omega} = C_t^{*\frac{\varphi}{\sigma}} \chi^{\varphi} Y_t^W (\frac{\theta}{\theta-1})^{-\omega} [Z_t^{*\omega-1} + (Z_t^{\gamma} Z_t^{*1-\gamma})^{\omega-1}]$$

$$w_t^* = C_t^{*\frac{\varphi}{\sigma(\varphi+\omega)}} \chi^{\frac{\varphi}{\varphi+\omega}} (Y_t^W)^{\frac{1}{\varphi+\omega}} (\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}} [Z_t^{*\omega-1} + (Z_t^{\gamma} Z_t^{*1-\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}$$

To get $(L_t + L_{*t})$ i.e., total home labor supply L_t^S:

As derived above from home household FOC: $L_t^s = (\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi}$

Plug in w_t: $L_t^s = (\frac{C_t^{-\frac{1}{\sigma}} C_t^{\frac{\varphi}{\sigma(\varphi+\omega)}} \chi^{\frac{\varphi}{\varphi+\omega}} (Y_t^W)^{\frac{1}{\varphi+\omega}} (\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}} [Z_t^{\omega-1} + (Z_t^{1-\gamma} Z_t^{*\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{\chi})^{\varphi}$

To get $(L_t^* + L_{*t}^*)$i.e., total foreign labor supply L_t^{*s}:

From foreign household FOC: $L_t^{*s} = (\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi}$

Plug in w_t^*: $L_t^{*s} = (\frac{C_t^{*-\frac{1}{\sigma}} C_t^{*\frac{\varphi}{\sigma(\varphi+\omega)}} \chi^{\frac{\varphi}{\varphi+\omega}} (Y_t^W)^{\frac{1}{\varphi+\omega}} (\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}} [Z_t^{*\omega-1} + (Z_t^{\gamma} Z_t^{*1-\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{\chi})^{\varphi}$

Then, $\frac{y_t}{y_t^*} = \frac{w_t(L_t + L_{*t})}{w_t^*(L_t^* + L_{*t}^*)} = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}} (\frac{Z_t^{\omega-1} + (Z_t^{1-\gamma} Z_t^{*\gamma})^{\omega-1}}{Z_t^{*\omega-1} + (Z_t^{\gamma} Z_t^{*1-\gamma})^{\omega-1}})^{\frac{1+\varphi}{\varphi+\omega}}$

When $\gamma = 0$, this expression reduces to $\frac{y_t}{y_t^*} = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}} (\frac{Z_t}{Z_t^*})^{\frac{(\omega-1)(\varphi+1)}{\varphi+\omega}}$ which is the "country-specific technology" setup. It is equivalent to the result in GLR.

When $\gamma = 1$, this expression reduces to $\frac{y_t}{y_t^*} = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}}$ which is the "firm-specific technology" setup.

This validates the point made in Section 2.2 about this specification being a "mix" of the "country-specific technology" and "firm-specific technology" setups.

Next, this expression is log-linearized. Note the log-linearization uses the fact that in steady state $Z = Z^* = 1$.

$dy_t - dy_t^* =$

$= -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} dC_t - (-\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}) dC_t^* + \frac{1+\varphi}{\varphi+\omega}\{[(\omega-1)dZ_t + (\omega-1)((1-\gamma)dZ_t + \gamma dZ_t^*)](1+1)/(1+1)^2 - [(\omega-1)dZ_t^* + (\omega-1)(\gamma dZ_t + (1-\gamma)dZ_t^*)](1+1)/(1+1)^2\} =$

$= -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} dC_t - (-\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}) dC_t^* + \frac{1+\varphi}{\varphi+\omega}\{\frac{(\omega-1)dZ_t}{2} + \frac{(\omega-1)(1-\gamma)dZ_t}{2} + \frac{(\omega-1)\gamma dZ_t^*}{2} - \frac{(\omega-1)dZ_t^*}{2} - \frac{(\omega-1)\gamma dZ_t}{2} - \frac{(\omega-1)(1-\gamma)dZ_t^*}{2}\}$

$\widehat{y}_t^D = -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}_t^D + \frac{1+\varphi}{\varphi+\omega}\{\frac{(\omega-1)\widehat{Z}_t^D}{2} + \frac{(\omega-1)(1-\gamma)\widehat{Z}_t^D}{2} - \frac{(\omega-1)\gamma\widehat{Z}_t^D}{2}\} =$

$= -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}_t^D + \frac{1+\varphi}{\varphi+\omega}\frac{\omega-1}{2}(\widehat{Z}_t^D + (1-\gamma)\widehat{Z}_t^D - \gamma\widehat{Z}_t^D) =$

$= -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}_t^D + \frac{1+\varphi}{\varphi+\omega}\frac{\omega-1}{2}(\widehat{Z}_t^D + \widehat{Z}_t^D - \gamma\widehat{Z}_t^D - \gamma\widehat{Z}_t^D) =$

$= -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}_t^D + \frac{1+\varphi}{\varphi+\omega}\frac{\omega-1}{2}(2\widehat{Z}_t^D - 2\gamma\widehat{Z}_t^D) =$

$= -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}_t^D + \frac{1+\varphi}{\varphi+\omega}\frac{\omega-1}{2}2(\widehat{Z}_t^D - \gamma\widehat{Z}_t^D) =$

$= -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}_t^D + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)\widehat{Z}_t^D$

When $\gamma = 0$, this expression reduces to $\widehat{y}_t^D = -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}_t^D + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}\widehat{Z}_t^D$ which is the "country-specific technology" setup. It is equivalent to the result in GLR.

When $\gamma = 1$, this expression reduces to $\widehat{y}_t^D = -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}_t^D$ which is the "firm-specific technology" setup.

Appendix B: Steady-State Optimal Portfolio α

This Appendix shows derivations for Section 3.

B.1 Log-linearize NFA LOM

Start with NFA LOM derived in Section A.1: $nfa_{t+1} = R^D_t\alpha_t + R_t nfa_t + (1-a)(y^D_t - C^D_t - G^D_t)$

Log-linearize it: $dnfa_{t+1} = dR^D_t\alpha + R^D d\alpha_t + dR_t nfa + Rdnfa_t + (1-a)(dy^D_T - dC^D_t - dG^D_t)$

Use $R^D = 0$ and $nfa = 0$: $dnfa_{t+1} = dR^D_t\alpha + Rdnfa_t + (1-a)(dy^D_t - dC^D_t - dG^D_t)$

Divide by C. Note that $C = 1 - G$ due to $y = C + G$ and assumption $y = y^* = L = L^* = 1$, so $1 = C + G$.

$\frac{dnfa_{t+1}}{C} = \frac{dR^D_t\alpha}{1-G} + \frac{Rdnfa_t}{C} + (1-a)(\frac{dy^D_t}{1-G} - \frac{dC^D_t}{C} - \frac{dG^D_t}{1-G})$

$\frac{dnfa_{t+1}}{C} = \frac{dR^D_t\alpha R}{(1-G)R} + \frac{\frac{1}{\beta}dnfa_t}{C} + (1-a)(\frac{dy^D_t y}{(1-G)y} - \widehat{C}_t - \frac{dG^D_t G}{(1-G)G})$

$n\widehat{f}a_{t+1} = \frac{1}{\beta}n\widehat{f}a_t + \frac{\alpha}{\beta(1-G)}\widehat{R}^D_t + \frac{1-a}{1-G}\widehat{y}^D_t - (1-a)\widehat{C}^D_t - \frac{(1-a)G}{1-G}\widehat{G}^D_t$

Define $\widehat{\xi}_t \equiv \frac{\alpha}{\beta(1-G)}\widehat{R}^D_t$ as in GLR.

$n\widehat{f}a_{t+1} = \frac{1}{\beta}n\widehat{f}a_t + \widehat{\xi}_t + \frac{1-a}{1-G}\widehat{y}^D_t - (1-a)\widehat{C}^D_t - \frac{(1-a)G}{1-G}\widehat{G}^D_t$

Note that this expression is **identical to GLR** derivation.

The expression for relative GDP $\frac{y_t}{y^*_t}$ above is log-linearized as:

$\widehat{y}^D_t = -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\widehat{C}^D_t + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)\widehat{Z}^D_t$

Note that this expression is **not identical to GLR** because there is an additional term $(1-\gamma)$ in front of $\widehat{Z}^D$.

Plug into the log-linearized NFA LOM:

$$n\widehat{f}a_{t+1} = \frac{1}{\beta}n\widehat{f}a_t + \widehat{\xi}_t + \frac{1-a}{1-G}[-\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\widehat{C}_t^D + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)\widehat{Z}_t^D] - (1-a)\widehat{C}_t^D - \frac{(1-a)G}{1-G}\widehat{G}_t^D =$$

$$= \frac{1}{\beta}n\widehat{f}a_t + \widehat{\xi}_t - (1-a)(1 + \frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})\widehat{C}_t^D + \frac{1-a}{1-G}\frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)\widehat{Z}_t^D - \frac{(1-a)G}{1-G}\widehat{G}_t^D$$

Note that this expression is **not identical to GLR** because there is an additional term $(1-\gamma)$ in front of $\widehat{Z}^D$.

B.2 Log-linearize Euler Equations for Consumption

Log-linearize Euler equation for consumption:

$$-\frac{1}{\sigma}\widehat{C}_t = -\frac{1}{\sigma}E_t\widehat{C}_{t+1} + E_t\widehat{R}_{t+1}$$

Same for foreign: $-\frac{1}{\sigma}\widehat{C}_t^* = -\frac{1}{\sigma}E_t\widehat{C}_{t+1}^* + E_t\widehat{R}_{t+1}$

Subtract the home and foreign equations:

$$-\frac{1}{\sigma}(\widehat{C}_t - \widehat{C}_t^*) = -\frac{1}{\sigma}E_t(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)$$

$$\widehat{C}_t^D = E_t\widehat{C}_{t+1}^D$$

Note that this expression is **identical to GLR**.

B.3 Find Elasticities of $\widehat{C}_t^D$ and $n\widehat{f}a_{t+1}$

The first step is to guess a solution. There are two variables of interest: $\widehat{C}_t^D$ and $n\widehat{f}a_{t+1}$. These variables are expected to depend on a state variable $n\widehat{f}a_t$ and exogenous shocks $\widehat{Z}_t^D$, $\widehat{G}_t^D$ and $\widehat{\xi}_t$.

$$\widehat{C}_t^D = \eta_{C^D a}n\widehat{f}a_t + \eta_{C^D Z^D}\widehat{Z}_t^D + \eta_{C^D G^D}\widehat{G}_t^D + \eta_{C^D \xi}\widehat{\xi}_t$$

$$n\widehat{f}a_{t+1} = \eta_{aa}n\widehat{f}a_t + \eta_{aZ^D}\widehat{Z}_t^D + \eta_{aG^D}\widehat{G}_t^D + \eta_{a\xi}\widehat{\xi}_t$$

Use method of undetermined coefficients to solve for elasticities:

Plug $\widehat{C}_t^D$ into $n\widehat{f}a_{t+1}$:

$n\widehat{f}a_{t+1} = \frac{1}{\beta} n\widehat{f}a_t + \widehat{\xi}_t - (1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})(\eta_{C^D a} n\widehat{f}a_t + \eta_{C^D Z^D}\widehat{Z}_t^D + \eta_{C^D G^D}\widehat{G}_t^D + \eta_{C^D \xi}\widehat{\xi}_t) +$
$\frac{1-a}{1-G}\frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z_t^D - \frac{(1-a)G}{1-G}\widehat{G}_t^D$

Match coefficient on $n\widehat{f}a_t$ in this equation to the coefficient on $n\widehat{f}a_t$ in $n\widehat{f}a_{t+1}$ equation. The coefficient on $n\widehat{f}a_t$ in $n\widehat{f}a_{t+1}$ equation, i.e., η_{aa}, is $= 1$ because of non-stationarity.

$\frac{1}{\beta} - (1-a)[1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)}]\eta_{C^d a} = 1$

$\eta_{C^d a} = \frac{1-\beta}{\beta}\frac{\sigma(1-G)(\omega+\varphi)}{(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}$

To get $\eta_{a\xi}$:

$\eta_{a\xi} = 1 - (1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})\eta_{C^D\xi}$

From above, is is possible to obtain $\widehat{C}_t^D = \eta_{C^D a} n\widehat{f}a_t + \eta_{C^D G^D}\widehat{G}_t^D + \eta_{C^D\xi}\widehat{\xi}_t$ which can be rolled forward by 1 period to get:

$E_t\widehat{C}_{t+1}^D = \eta_{C^D a} E_t n\widehat{f}a_{t+1} + \eta_{C^D G^D} E_t\widehat{G}_{t+1}^D + \eta_{C^D\xi} E_t\widehat{\xi}_{t+1} =$

$= \eta_{C^D a}(\eta_{aa} n\widehat{f}a_t + \eta_{aG^D}\widehat{G}_t^D + \eta_{a\xi}\widehat{\xi}_t) + \eta_{C^D G^D}(\phi_G\widehat{G}_t^D) + 0$

using $E_t\widehat{G}_{t+1}^D = \phi_G\widehat{G}_t^D$ and $E_t\widehat{\xi}_{t+1} = 0$

Match coefficients on $\widehat{\xi}_t$:

$\eta_{C^D\xi} = \eta_{C^D a}\eta_{a\xi}$

Plug into $\eta_{a\xi}$:

$\eta_{a\xi} = 1 - (1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})\eta_{C^D a}\eta_{a\xi} =$

$= 1 - (1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})(\frac{1-\beta}{\beta}\frac{\sigma(1-G)(\omega+\varphi)}{(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)})\eta_{a\xi}$

$1 - (\frac{\sigma(1G)(\omega+\varphi)+\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})(\frac{1-\beta}{\beta}\frac{\sigma(1G)(\omega+\varphi)}{\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)})\eta_{a\xi} = \eta_{a\xi}$

$1 - \frac{1-\beta}{\beta}\eta_{a\xi} = \eta_{a\xi}$

$\eta_{a\xi} = \beta$

To get $\eta_{C^D\xi}$:

$\eta_{C^D\xi} = \eta_{C^Da}\eta_{a\xi} = \frac{1-\beta}{\beta}\frac{\sigma(1-G)(\omega+\varphi)}{(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}\beta = \frac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}$

To get $\eta_{C^DZ^D}$ and η_{aZ^D}:

$\eta_{aZ^D} = -(1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})\eta_{C^DZ^D} + \frac{1-a}{1-G}\frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)$

$\eta_{C^DZ^D} = \eta_{C^Da}\eta_{aZ^D} + \eta_{C^DZ^D}\phi_Z$

$\eta_{C^DZ^D} - \eta_{C^DZ^D}\phi_Z = \eta_{C^Da}\eta_{aZ^D} =$

$= [\frac{1-\beta}{\beta}\frac{\sigma(1-G)(\omega+\varphi)}{(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}][-(1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})\eta_{C^DZ^D} + \frac{1-a}{1-G}\frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)]$

$\eta_{C^DZ^D} - \eta_{C^DZ^D}\phi_Z + \frac{(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}{\sigma(1-G)(\omega+\varphi)}\frac{(1-\beta)\sigma(1-G)(\omega+\varphi)}{\beta(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}\eta_{C^DZ^D} =$

$= \frac{(1-a)(1-\beta)\sigma(1-G)(\omega+\varphi)}{(1-G)\beta(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}\frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)$

$\eta_{C^DZ^D}(1-\phi_Z+\frac{1-\beta}{\beta}) = \frac{(1-\beta)\sigma(1+\varphi)(\omega-1)}{\beta(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}(1-\gamma)$

$\eta_{C^DZ^D} = \frac{\sigma(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)$

$\eta_{aZ^D} = -(1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})\eta_{C^DZ^D} + \frac{1-a}{1-G}\frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma) =$

$= \frac{-(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}{\sigma(1-G)(\omega+\varphi)}\eta_{C^DZ^D} + \frac{1-a}{1-G}\frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma) =$

$= \frac{-(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}{\sigma(1-G)(\omega+\varphi)}\frac{\sigma(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma) + \frac{1-a}{1-G}\frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma) =$

$= \frac{-(1-a)(1-\beta)(1+\varphi)(\omega-1)(1-\gamma)}{(1-G)(\omega+\varphi)(1-\beta\phi_Z)} + \frac{(1-a)(1+\varphi)(\omega-1)(1-\gamma)}{(1-G)(\varphi+\omega)} =$

$= \frac{-(1-a)(1-\beta)(1+\varphi)(\omega-1)(1-\gamma)+(1-a)(1+\varphi)(\omega-1)(1-\gamma)(1-\beta\phi_Z)}{(1-G)(\omega+\varphi)(1-\beta\phi_Z)} =$

$= \frac{(1-a)(1+\varphi)(\omega-1)(1-\gamma)(1-\beta\phi_Z-1+\beta)}{(1-G)(\omega+\varphi)(1-\beta\phi_Z)} =$

$= \frac{(1-a)(1+\varphi)(\omega-1)\beta(1-\phi_z)}{(1-G)(\omega+\varphi)(1-\beta\phi_Z)}(1-\gamma)$

To get $\eta_{C^DG^D}$ and η_{aG^D}:

$\eta_{aG^D} = -(1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})\eta_{C^DG^D} - \frac{(1-a)G}{1-G}$

$\eta_{C^DG^D} = \eta_{C^Da}\eta_{aG^D} + \eta_{C^DG^D}\phi_G$

$\eta_{C^DG^D} - \eta_{C^DG^D}\phi_G = \eta_{C^Da}\eta_{aG^D} =$

$= [\frac{1-\beta}{\beta}\frac{\sigma(1-G)(\omega+\varphi)}{(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}][-(1-a)(1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)})\eta_{C^DG^D} - \frac{(1-a)G}{1-G}]$

$\eta_{C^DG^D} - \eta_{C^DG^D}\phi_G + \frac{(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}{\sigma(1-G)(\omega+\varphi)}\frac{(1-\beta)\sigma(1-G)(\omega+\varphi)}{\beta(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}\eta_{C^DG^D} =$

$= -\frac{(1-a)(1-\beta)\sigma(1-G)(\omega+\varphi)G}{(1-G)\beta(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}$

$\eta_{C^DG^D}(1-\phi_G+\frac{1-\beta}{\beta}) = \frac{-(1-\beta)\sigma(\omega+\varphi)G}{\beta(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1))}$

$\eta_{C^DG^D} = -\frac{\sigma(1-\beta)G(\omega+\varphi)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}$

$\eta_{aG^D} = -\frac{\beta(1-\phi_G)G(1-a)}{(1-\beta\phi_G)(1-G)}$

Note that the above elasticities are identical to GLR except for elasticities $\eta_{C^DZ^D}$ and η_{aZ^D} that include additional $(1-\gamma)$ terms.

B.4 Find Elasticities of v

First, $\widehat{v}_t$ is derived. The steps are identical to GLR but definitions of some variables differ from GLR, so the derivations are done in detail to highlight the difference.

This is how it is done in GLR. Log-linearize Euler equations for home firm and foreign firm shares. Starting with home firm shares:

$C_t^{-\frac{1}{\sigma}}v_t = \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}}R_{t+1}\} = \beta E_t\{C_{t+1}^{-\frac{1}{\sigma}}(v_{t+1}+d_{t+1})\}$

$dv_t = \beta E_t dv_{t+1} + \beta E_t dd_{t+1}$

Divide by v:

$\frac{dv_t}{v} = \frac{\beta dE_t v_{t+1}}{v} + \frac{\beta dE_t d_{t+1}}{v}$

$\widehat{v}_t = \beta E_t \widehat{v}_{t+1} + \beta E_t \widehat{d}_{t+1} \frac{d}{v}$

From Useful Properties, the following holds: $d_t = \frac{1}{\theta} y_t$. Due to the assumption $y_t = 1$, it is possible to write: $d_t = \frac{1}{\theta}$ which in steady state becomes $d = \frac{1}{\theta}$. In steady state, Euler equation becomes $v = \beta v + \beta d$ which can be written as $v = \beta v + \beta \frac{1}{\theta}$ which can be written as $v(1-\beta) = \frac{\beta}{\theta}$ which can be written as $v = \frac{\beta}{\theta(1-\beta)}$.

$\widehat{v}_t = \beta E_t \widehat{v}_{t+1} + \beta E_t \widehat{d}_{t+1} \frac{\frac{1}{\theta}}{\frac{\beta}{(1-\beta)\theta}}$

$\widehat{v}_t = \beta E_t \widehat{v}_{t+1} + E_t \widehat{d}_{t+1}(1-\beta)$

Following the same calculations for foreign firm shares:

$\widehat{v}_t^* = \beta E_t \widehat{v}_{t+1}^* + E_t \widehat{d}_{t+1}^*(1-\beta)$

Subtracting calculations for home firm and foreign firm shares:

$\widehat{v}_t^D = E_t[\beta \widehat{v}_{t+1}^D + (1-\beta)\widehat{d}_{t+1}^D]$ which is equation (43) on p.A-4 of GLR.

In this model, the first step is also starting with log-linearizing Euler equations for home firm and foreign firm shares. Starting with home firm shares:

$C_t^{-\frac{1}{\sigma}} v_t = \beta E_t \{ C_{t+1}^{-\frac{1}{\sigma}} (v_{t+1} + d_{t+1} + d_{*t+1}) \}$

$dv_t = \beta E_t dv_{t+1} + \beta E_t dd_{t+1} + \beta E_t dd_{*t+1}$

Divide by v:

$\frac{dv_t}{v} = \frac{\beta d E_t v_{t+1}}{v} + \frac{\beta d E_t d_{t+1}}{v} + \frac{\beta d E_t d_{*t+1}}{v}$

$\widehat{v}_t = \beta E_t \widehat{v}_{t+1} + \beta E_t \widehat{d}_{t+1} \frac{d}{v} + \beta \widehat{d}_{*t+1} \frac{d_*}{v}$

From Useful Properties, the following holds: $d_t + d_{*t} = \frac{1}{\theta} y_t$. Due to the assumption $y_t = 1$, it is possible to write: $d_t + d_{*t} = \frac{1}{\theta}$. In steady state, Euler equation for home and foreign shares becomes $v = \beta v + \beta d + \beta d_*$ which becomes $v = \beta v + \beta \frac{1}{\theta}$ which can be written as $v(1-\beta) = \frac{\beta}{\theta}$ which can be written as $v = \frac{\beta}{\theta(1-\beta)}$.

$\widehat{v}_t = \beta E_t \widehat{v}_{t+1} + \beta E_t \widehat{d}_{t+1} \frac{d}{\frac{\beta}{1-\beta}\frac{1}{\theta}} + \beta E_t \widehat{d}_{*t+1} \frac{d_*}{\frac{\beta}{1-\beta}\frac{1}{\theta}} = \beta E_t \widehat{v}_{t+1} + (1-\beta) E_t \widehat{d}_{t+1} d\theta + (1-\beta) E_t \widehat{d}_{*t+1} d_* \theta = \beta E_t \widehat{v}_{t+1} + (1-\beta) E_t \widehat{d}_{t+1} d\theta + (1-\beta) E_t \widehat{d}_{*t+1}(1-d\theta)$ which does not lead to a convenient format as in GLR.

Therefore, I define total home firm profit as $\overline{d}_{t+1} = d_{t+1} + d_{*t+1}$.

I can then write the Euler equation for home firm shares as: $C_t^{-\frac{1}{\sigma}} v_t = \beta E_t \{C_{t+1}^{-\frac{1}{\sigma}} R_{t+1}\} = \beta E_t \{C_{t+1}^{-\frac{1}{\sigma}} (v_{t+1} + \overline{d}_{t+1})\}$

$dv_t = \beta E_t dv_{t+1} + \beta E_t d\overline{d}_{t+1}$

Divide by v:

$\frac{dv_t}{v} = \frac{\beta d E_t v_{t+1}}{v} + \frac{\beta d E_t \overline{d}_{t+1}}{v}$

$\widehat{v}_t = \beta E_t \widehat{v}_{t+1} + \beta E_t \widehat{d}_{t+1} \frac{\overline{d}}{v}$

From Useful Properties, the following holds: $d_t + d_{*t} \equiv \overline{d}_t = \frac{1}{\theta} y_t$. Due to the assumption $y_t = 1$, it is possible to write: $\overline{d}_t = \frac{1}{\theta}$ which in steady state becomes $\overline{d} = \frac{1}{\theta}$. In steady state, Euler equation becomes $v = \beta v + \beta \overline{d}$ which becomes $v = \beta v + \beta \frac{1}{\theta}$ which can be written as $v(1-\beta) = \frac{\beta}{\theta}$ which can be written as $v = \frac{\beta}{\theta(1-\beta)}$.

$\widehat{v}_t = \beta E_t \widehat{v}_{t+1} + \beta E_t \widehat{\overline{d}}_{t+1} \frac{\frac{1}{\theta}}{\frac{\beta}{(1-\beta)\theta}}$

$\widehat{v}_t = \beta E_t \widehat{v}_{t+1} + E_t \widehat{\overline{d}}_{t+1}(1-\beta)$

Following the same calculations for foreign firm shares:

$\widehat{v}_t^* = \beta E_t \widehat{v}_{t+1}^* + E_t \widehat{\overline{d}}_{t+1}^*(1-\beta)$ where $\overline{d}_t^*$ is defined as $d_t^* + d_{*t}^*$, i.e., total profit of the foreign firm.

Subtracting calculations for home firm and foreign firm shares:

$\widehat{v}_t^D = E_t[\beta \widehat{v}_{t+1}^D + (1-\beta)\widehat{\overline{d}}_{t+1}^D]$ where the log-linearized difference between total profits generated by home firm and total profits generated by foreign firm $\widehat{\overline{d}}_{t+1}^D$ is defined as $\widehat{\overline{d}}_{t+1} - \widehat{\overline{d}}_{t+1}^* = (d_{t+1} \widehat{+} d_{*t+1}) - (d_{t+1}^* \widehat{+} d_{*t+1}^*)$. Note that this is similar to equation (43) on p.A-4 of GLR.

Next, an expression for $\widehat{\overline{d}}^{D}_{t+1}$ is obtained. An expression for $\widehat{y}^{D}_{t}$ was obtained above. Since $\overline{d}_t = d_t + d_{*t} = \frac{1}{\theta} y_t$ and $\overline{d}^{*}_t = d^{*}_t + d^{*}_{*t} = \frac{1}{\theta} y_{*t}$, it is possible to write $\frac{\overline{d}_t}{\overline{d}^{*}_t} = \frac{d_t + d_{*t}}{d^{*}_t + d^{*}_{*t}} = \frac{\frac{1}{\theta} y_t}{\frac{1}{\theta} y^{*}_t}$ which means $\frac{\overline{d}_t}{\overline{d}^{*}_t} = \frac{y_t}{y^{*}_t}$.

Roll it forward by 1 period: $\frac{\overline{d}_{t+1}}{\overline{d}^{*}_{t+1}} \equiv \frac{d_{t+1} + d_{*t+1}}{d^{*}_{t+1} + d^{*}_{*t+1}} = \frac{y_{t+1}}{y^{*}_{t+1}}$.

Log-linearizing gives $\widehat{\overline{d}}^{D}_{t+1} = \widehat{y}^{D}_{t+1}$.

It is then possible to write $\widehat{v}^{D}_t = E_t[\beta \widehat{v}^{D}_{t+1} + (1-\beta)\widehat{y}^{D}_{t+1}]$ and using the expression for $\widehat{y}^{D}_t$ rolled forward by 1 period, i.e., $\widehat{y}^{D}_{t+1} = -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}^{D}_{t+1} + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)\widehat{Z}^{D}_{t+1}$:

$\widehat{v}^{D}_t = E_t \beta \widehat{v}^{D}_{t+1} + E_t(1-\beta)\widehat{y}^{D}_{t+1}$

$\widehat{v}^{D}_t = E_t \beta \widehat{v}^{D}_{t+1} + E_t(1-\beta)[-\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}^{D}_{t+1} + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)\widehat{Z}^{D}_{t+1}]$

$\widehat{v}^{D}_t = E_t \beta \widehat{v}^{D}_{t+1} - E_t \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}^{D}_{t+1} + E_t \frac{(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)\widehat{Z}^{D}_{t+1}$

Use $E_t \widehat{C}^{D}_{t+1} = \widehat{C}^{D}_t$ and assumption $\widehat{Z}^{D}_{t+1} = \phi_Z \widehat{Z}^{D}_t + \varepsilon^{Z^D}_{t+1}$. Plug in:

$\widehat{v}^{D}_t = E_t \beta \widehat{v}^{D}_{t+1} - \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}^{D}_t + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)\widehat{Z}^{D}_t$

Use method of undetermined coefficients:

Guess the solution takes the form:

$\widehat{v}^{D}_t = \eta_{v^D a} n\widehat{f}a_t + \eta_{v^D Z^D} \widehat{Z}^{D}_t + \eta_{v^D G^D} \widehat{G}^{D}_t + \eta_{v^D \xi} \widehat{\xi}_t$

Roll it forward by 1 period:

$\widehat{v}^{D}_{t+1} = \eta_{v^D a} n\widehat{f}a_{t+1} + \eta_{v^D Z^D} \widehat{Z}^{D}_{t+1} + \eta_{v^D G^D} \widehat{G}^{D}_{t+1} + \eta_{v^D \xi} \widehat{\xi}_{t+1}$

Plug in expressions for $\widehat{v}^{D}_t$ and $\widehat{v}^{D}_{t+1}$ into:

$\widehat{v}^{D}_t = E_t \beta \widehat{v}^{D}_{t+1} - \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}^{D}_t + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z^{D}_t$:

$\eta_{v^D a} n\widehat{f}a_t + \eta_{v^D Z^D} \widehat{Z}^{D}_t + \eta_{v^D G^D} \widehat{G}^{D}_t + \eta_{v^D \xi} \widehat{\xi}_t =$

$= \beta \eta_{v^D a} n\widehat{f}a_{t+1} + \beta \eta_{v^D Z^D} \widehat{Z}^{D}_{t+1} + \beta \eta_{v^D G^D} \widehat{G}^{D}_{t+1} + \beta \eta_{v^D \xi} \widehat{\xi}_{t+1} - \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)} \widehat{C}^{D}_t + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}(1-$

$\gamma)Z^{D}_t =$

$= \beta\eta_{v^D a}(n\widehat{f}a_t + \eta_{aZ^D}\widehat{Z}_t^D + \eta_{aG^D}\widehat{G}_t^D + \eta_{a\xi}\widehat{\xi}_t) + \beta\eta_{v^D Z^D}\phi_Z\widehat{Z}_t^D + \beta\eta_{v^D G^D}\phi_G\widehat{G}_t^D + 0 -$

$- \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}(\eta_{C^D a}n\widehat{f}a_t + \eta_{C^D Z^D}\widehat{Z}_t^D + \eta_{C^D G^D}\widehat{G}_t^D + \eta_{C^D\xi}\widehat{\xi}_t) + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z_t^D =$

$= \beta\eta_{v^D a}n\widehat{f}a_t + \beta\eta_{v^D a}\frac{\beta(1-\phi_Z)(1-a)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(1-G)(\omega+\varphi)}(1-\gamma)\widehat{Z}_t^D - \beta\eta_{v^D a}\frac{\beta(1-\phi_G)G(1-a)}{(1-\beta\phi_G)(1-G)}\widehat{G}_t^D + \beta\eta_{v^D a}\beta\widehat{\xi}_t +$

$+ \beta\eta_{v^D Z^D}\phi_Z\widehat{Z}_t^D + \beta\eta_{v^D G^D}\phi_G\widehat{G}_t^D - \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}\frac{1-\beta}{\beta}\frac{\sigma(1-G)(\omega+\varphi)}{(1-a)(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}n\widehat{f}a_t -$

$- \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_t^D - \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}\frac{\sigma(1-\beta)G(\omega+\varphi)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}\widehat{G}_t^D -$

$- \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}\frac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}\widehat{\xi}_t + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z_t^D$

To get $\eta_{v^D a}$:

$\eta_{v^D a}n\widehat{f}a_t - \beta\eta_{v^D a}n\widehat{f}a_t = -\frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}\frac{1-\beta}{\beta}\frac{\sigma(1-G)(\omega+\varphi)}{(1-a)[(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}n\widehat{f}a_t$

$\eta_{v^D a}n\widehat{f}a_t(1-\beta) = -\frac{(1-\beta)\varphi(\omega-1)(1-\beta)\sigma(1-G)(\omega+\varphi)}{\sigma(\omega+\varphi)\beta(1-a)[(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}n\widehat{f}a_t$

$\eta_{v^D a}n\widehat{f}a_t = -\frac{(1-\beta)\varphi(\omega-1)\sigma(1-G)(\omega+\varphi)}{\sigma(\omega+\varphi)\beta(1-a)[(\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}n\widehat{f}a_t$

$\eta_{v^D a} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}$

To get $\eta_{v^D Z^D}$:

$\eta_{v^D Z^D}\widehat{Z}_t^D = \beta\eta_{v^D a}\frac{\beta(1-\phi_Z)(1-a)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(1-G)(\omega+\varphi)}(1-\gamma)\widehat{Z}_t^D +$

$+\beta\eta_{v^D Z^D}\phi_Z\widehat{Z}_t^D - \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_t^D + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z_t^D$

$\eta_{v^D Z^D}\widehat{Z}_t^D - \beta\eta_{v^D Z^D}\phi_Z\widehat{Z}_t^D =$

$= \beta(\frac{-\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]})\frac{\beta(1-\phi_Z)(1-a)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(1-G)(\omega+\varphi)}(1-\gamma)\widehat{Z}_t^D -$

$- \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_t^D + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z_t^D$

$\eta_{v^D Z^D}\widehat{Z}_t^D(1-\beta\phi_Z) =$

$= [\beta(\frac{-\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]})\frac{\beta(1-\phi_Z)(1-a)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(1-G)(\omega+\varphi)} - \frac{(1-\beta)\varphi(\omega-1)}{\sigma(\omega+\varphi)}\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} +$

$\frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}] \times$

$\times (1-\gamma)Z_t^D =$

$= [\frac{-\varphi(\omega-1)(1-\beta)}{\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}\frac{\beta(1-\phi_Z)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(\omega+\varphi)} - \frac{(1-\beta)\varphi(\omega-1)}{\omega+\varphi}\frac{(1-\beta)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\varphi+\omega}]\times$

$\times (1-\gamma)Z_t^D =$

$= [\frac{-\varphi(\omega-1)(1-\beta)\beta(1-\phi_Z)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} - \frac{(1-\beta)\varphi(\omega-1)(1-\beta)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} +$

$+ \frac{\phi_z(1-\beta)(1+\varphi)(\omega-1)(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}](1-\gamma)Z_t^D =$

$= [\frac{(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\{-\varphi\beta(1-\phi_Z)(\omega-1)-(1-\beta)\varphi(\omega-1)+\phi_Z(1-\beta\phi_Z)[\sigma(1-$

$G)(\omega+\varphi)+\varphi(\omega-1)]\}](1-\gamma)Z_t^D =$

$= [\frac{(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\{\varphi(\omega-1)\beta\phi_Z-\varphi(\omega-1)+\phi_Z(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+$

$\varphi(\omega-1)]\}](1-\gamma)Z_t^D =$

$= [\frac{(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\{\varphi(\omega-1)\beta\phi_Z-\varphi(\omega-1)+\phi_Z(1-\beta\phi_Z)\sigma(1-G)(\omega+\varphi)+$

$\phi_Z(1-\beta\phi_Z)\varphi(\omega-1)\}](1-\gamma)Z_t^D =$

$= [\frac{(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\{\varphi(\omega-1)\beta\phi_Z-\varphi(\omega-1)+\phi_Z(1-\beta\phi_Z)\sigma(1-G)(\omega+\varphi)+$

$\phi_Z\varphi(\omega-1)-\phi_Z\beta\phi_Z\varphi(\omega-1)\}](1-\gamma)Z_t^D =$

$= [\frac{(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\{\varphi(\omega-1)\beta\phi_Z(1-\phi_Z)-\varphi(\omega-1)(1-\phi_Z)+\phi_Z(1-\beta\phi_Z)\sigma(1-$

$G)(\omega+\varphi)\}](1-\gamma)Z_t^D =$

$= [\frac{(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\{-\varphi(\omega-1)(1-\phi_Z)(1-\beta\phi_Z)+\phi_Z(1-\beta\phi_Z)\sigma(1-G)(\omega+$

$\varphi)\}](1-\gamma)Z_t^D =$

$= [\frac{(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\{(1-\beta\phi_Z)[-\varphi(\omega-1)(1-\phi_Z)+\phi_Z\sigma(1-G)(\omega+\varphi)]\}](1-\gamma)Z_t^D$

$\eta_{v^D Z^D} = \frac{(1-\beta)(1+\varphi)(\omega-1)[\sigma\phi_Z(1-G)(\omega+\varphi)-\varphi(\omega-1)(1-\phi_Z)]}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)$

To get $\eta_{v^D G^D}$:

$\eta_{v^D G^D}\widehat{G}_t^D = \beta\eta_{v^D a}\eta_{aG^D}\widehat{G}_t^D + \beta\eta_{v^D G^D}\phi_G\widehat{G}_t^D - \frac{\varphi(\omega-1)(1-\beta)}{\sigma(\omega+\varphi)}\eta_{C^D G^D}\widehat{G}_t^D$

$\eta_{v^D G^D}\widehat{G}_t^D - \beta\eta_{v^D G^D}\phi_G\widehat{G}_t^D = \beta\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\frac{\beta(1-\phi_G)G(1-a)}{(1-\beta\phi_G)(1-G)}\widehat{G}_t^D +$

$+ \frac{\varphi(\omega-1)(1-\beta)}{\sigma(\omega+\varphi)}\frac{\sigma(1-\beta)G(\omega+\varphi)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}\widehat{G}_t^D$

$\eta_{v^D G^D}\widehat{G}_t^D(1-\beta\phi_G) = \frac{\varphi(\omega-1)(1-\beta)G(\beta(1-\phi_G)+(1-\beta))}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\widehat{G}_t^D$

$\eta_{v^D G^D} = \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}$

To get $\eta_{v^D\xi}$:

$\eta_{v^D\xi}\widehat{\xi}_t = \beta\eta_{v^Da}\eta_{a\xi}\widehat{\xi}_t - \frac{\varphi(\omega-1)(1-\beta)}{\sigma(\omega+\varphi)}\eta_{C^D\xi}\widehat{\xi}_t =$

$= -\beta\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\beta\widehat{\xi}_t - \frac{\varphi(\omega-1)(1-\beta)}{\sigma(\omega+\varphi)}\frac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\widehat{\xi}_t =$

$= \frac{-\beta\varphi(\omega-1)(1-\beta)(1-G)-\varphi(\omega-1)(1-\beta)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\widehat{\xi}_t$

$\eta_{v^D\xi} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}$

Note that the above elasticities are identical to GLR except for the elasticity $\eta_{v^DZ^D}$ that has an additional term $(1-\gamma)$.

B.5 Find Elasticities of d:

From above: $\widehat{y}_t^D = -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\widehat{C}_t^D + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z_t^D$

Since $\widehat{y}_t^D = \widehat{d}_t^D$, it is possible to write $\widehat{d}_t^D = -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\widehat{C}_t^D + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z_t^D$

The solution for $\widehat{C}_t^D$ above is: $\widehat{C}_t^D = \eta_{C^Da}n\widehat{f}a_t + \eta_{C^DZ^D}\widehat{Z}_t^D + \eta_{C^DG^D}\widehat{G}_t^D + \eta_{C^D\xi}\widehat{\xi}_t$

Plugging this in:

$\widehat{d}_t^D = -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}(\eta_{C^Da}n\widehat{f}a_t + \eta_{C^DZ^D}\widehat{Z}_t^D + \eta_{C^DG^D}\widehat{G}_t^D + \eta_{C^D\xi}\widehat{\xi}_t) + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)Z_t^D =$

$= -\frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\eta_{C^Da}n\widehat{f}a_t - \frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\eta_{C^DG^D}\widehat{G}_t^D - \frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\eta_{C^D\xi}\widehat{\xi}_t + [\frac{-\varphi(\omega-1)}{\sigma(\omega+\varphi)}\eta_{C^DZ^D} + \frac{(1+\varphi)(\omega-1)}{\varphi+\omega}(1-\gamma)]\widehat{Z}_t^D$

Use method of undetermined coefficients:

Guess the solution takes the form:

$\widehat{d}_t^D = \eta_{d^Da}n\widehat{f}a_t + \eta_{d^DZ^D}\widehat{Z}_t^D + \eta_{d^DG^D}\widehat{G}_t^D + \eta_{d^D\xi}\widehat{\xi}_t$

To get η_{d^Da}:

$\eta_{d^Da}n\widehat{f}a_t = -\frac{\varphi(\omega-1)\sigma(1-\beta)(1-G)(\omega+\varphi)}{\sigma(\omega+\varphi)\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}n\widehat{f}a_t$

$\eta_{d^Da} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} = \eta_{v^Da}$

To get $\eta_{d^DZ^D}$:

$\eta_{d^DZ^D}\widehat{Z}_t^D = \frac{-\varphi(\omega-1)\sigma(1-\beta)(\omega-1)(1+\varphi)(1-\gamma)}{\sigma(\omega+\varphi)(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\widehat{Z}_t^D + \frac{(1+\varphi)(\omega-1)(1-\gamma)}{\varphi+\omega}\widehat{Z}_t^D$

$\eta_{d^DZ^D} = \frac{-\varphi(\omega-1)\sigma(1-\beta)(\omega-1)(1+\varphi)(1-\gamma)+(1+\varphi)(\omega-1)(1-\gamma)\sigma(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}{\sigma(\omega+\varphi)(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} =$

$= \frac{(1+\varphi)(\omega-1)[-\varphi(\omega-1)(1-\beta)+(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]]}{(\omega+\varphi)(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma) =$

$= \frac{(1+\varphi)(\omega-1)[(1-\beta\phi_Z)\sigma(1-G)(\omega+\varphi)+(1-\beta\phi_Z)\varphi(\omega-1)-\varphi(\omega-1)(1-\beta)]}{(\omega+\varphi)(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma) =$

$= \frac{(1+\varphi)(\omega-1)[(1-\beta\phi_Z)\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)(1-\beta\phi_Z-(1-\beta)]}{(\omega+\varphi)(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma) =$

$= \frac{(1+\varphi)(\omega-1)[(1-\beta\phi_Z)\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\beta(1-\phi_Z)]}{(\omega+\varphi)(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)$

To get $\eta_{d^DG^D}$:

$\eta_{d^DG^D}\widehat{G}_t^D = -\frac{G\varphi(\omega-1)\sigma(1-\beta)(\omega+\varphi)}{\sigma(\omega+\varphi)(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\widehat{G}_t^D$

$\eta_{d^DG^D} = -\frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} = \eta_{v^DG^D}$

To get $\eta_{d^D\xi}$:

$\eta_{d^D\xi}\widehat{\xi}_t = -\frac{\varphi(\omega-1)\sigma(1-\beta)(1-G)(\omega+\varphi)}{\sigma(\omega+\varphi)(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\widehat{\xi}_t$

$\eta_{d^D\xi} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} = \eta_{v^D\xi}$

Note that the above elasticities are identical to GLR except for the elasticity $\eta_{d^DZ^D}$ that has an additional term $(1-\gamma)$.

B.6 Show that Excess Return $\widehat{R}_t^D$ is a Linear Function of Innovations to Relative Productivity and Government Spending:

Recall: $\widehat{R}_t^D = -\beta\widehat{v}_t^D - (1-\beta)\widehat{d}_t^D + \widehat{v}_{t-1}^D = -[\beta\widehat{v}_t^D + (1-\beta)\widehat{d}_t^D] + \widehat{v}_{t-1}^D$

Note that the following hold:

$\beta\eta_{v^Da} + (1-\beta)\eta_{d^Da} = \eta_{v^Da} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}$

$\beta\eta_{v^DZ^D} + (1-\beta)\eta_{d^DZ^D} =$

$= \frac{\beta(1-\beta)(1+\varphi)(\omega-1)[\sigma\phi_Z(1-G)(\omega+\varphi)-\varphi(\omega-1)(1-\phi_Z)]}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)+$

$+\frac{(1-\beta)(1+\varphi)(\omega-1)[(1-\beta\phi_Z)\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\beta(1-\phi_Z)]}{(\omega+\varphi)(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma) =$

$= \frac{(1-\beta)(1+\varphi)(\omega-1)\beta\sigma\phi_Z(1-G)+(1-\beta)(1+\varphi)(\omega-1)(1-G)(\sigma-\sigma\beta\phi_Z)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma) =$

$= \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)$

$\beta\eta_{v^D G^D} + (1-\beta)\eta_{d^D G^D} = \eta_{v^D G^D} = \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}$

$\beta\eta_{v^D \xi} + (1-\beta)\eta_{d^D \xi} = \eta_{v^D \xi} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}$

Therefore:

$\widehat{R}_t^D = -\eta_{v^D a} n\widehat{f}a_t - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_t^D - \eta_{v^D G^D}\widehat{G}_t^D - \eta_{v^D \xi}\widehat{\xi}_t + \widehat{v}_{t-1}^D$

Roll forward by 1 period:

$\widehat{R}_{t+1}^D = -\eta_{v^D a} n\widehat{f}a_{t+1} - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_{t+1}^D - \eta_{v^D G^D}\widehat{G}_{t+1}^D - \eta_{v^D \xi}\widehat{\xi}_{t+1} + \widehat{v}_t^D$

Plug in equation: $\widehat{v}_t^D = \eta_{v^D a} n\widehat{f}a_t + \eta_{v^D Z^D}\widehat{Z}_t^D + \eta_{v^D G^D}\widehat{G}_t^D + \eta_{v^D \xi}\widehat{\xi}_t$

$\widehat{R}_{t+1}^D = -\eta_{v^D a} n\widehat{f}a_{t+1} - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_{t+1}^D - \eta_{v^D G^D}\widehat{G}_{t+1}^D - \eta_{v^D \xi}\widehat{\xi}_{t+1} + \eta_{v^D a} n\widehat{f}a_t +$

$+\eta_{v^D Z^D}\widehat{Z}_t^D + \eta_{v^D G^D}\widehat{G}_t^D + \eta_{v^D \xi}\widehat{\xi}_t =$

$= -\eta_{v^D a} n\widehat{f}a_{t+1} + \eta_{v^D a} n\widehat{f}a_t - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_{t+1}^D + \eta_{v^D Z^D}\widehat{Z}_t^D - \eta_{v^D G^D}\widehat{G}_{t+1}^D +$

$+\eta_{v^D G^D}\widehat{G}_t^D - \eta_{v^D \xi}\widehat{\xi}_{t+1} + \eta_{v^D \xi}\widehat{\xi}_t =$

$= -\eta_{v^D a}(n\widehat{f}a_{t+1} - n\widehat{f}a_t) - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_{t+1}^D - \eta_{v^D Z^D}\widehat{Z}_t^D -$

$- \eta_{v^D G^D}(\widehat{G}_{t+1}^D - \widehat{G}_t^D) - \eta_{v^D \xi}(\widehat{\xi}_{t+1} - \widehat{\xi}_t)$

Use $n\widehat{f}a_{t+1} = n\widehat{f}a_t + \eta_{aZ^D}\widehat{Z}_t^D + \eta_{aG^D}\widehat{G}_t^D + \eta_{a\xi}\widehat{\xi}_t$

$\widehat{R}_{t+1}^D = -\eta_{v^D a}(n\widehat{f}a_t + \eta_{aZ^D}\widehat{Z}_t^D + \eta_{aG^D}\widehat{G}_t^D + \eta_{a\xi}\widehat{\xi}_t - n\widehat{f}a_t) -$

$- \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_{t+1}^D - \eta_{v^D Z^D}\widehat{Z}_t^D - \eta_{v^D G^D}(\widehat{G}_{t+1}^D - \widehat{G}_t^D) - \eta_{v^D \xi}(\widehat{\xi}_{t+1} - \widehat{\xi}_t) =$

$= -\eta_{v^D a}(\eta_{aZ^D}\widehat{Z}_t^D + \eta_{aG^D}\widehat{G}_t^D + \eta_{a\xi}\widehat{\xi}_t) - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\widehat{Z}_{t+1}^D - \eta_{v^D Z^D}\widehat{Z}_t^D -$

$-\eta_{v^D G^D}(\widehat{G}_{t+1}^D - \widehat{G}_t^D) - \eta_{v^D \xi}(\widehat{\xi}_{t+1} - \widehat{\xi}_t)$

Use $\widehat{G}_t^D = \phi_G\widehat{G}_{t-1}^D + \varepsilon_t^{G^D}$ and $\widehat{Z}_t^D = \phi_Z\widehat{Z}_{t-1}^D + \varepsilon_t^{Z^D}$

$\widehat{R}_{t+1}^D = -\eta_{v^D a}(\eta_{aZ^D}\widehat{Z}_t^D + \eta_{aG^D}\widehat{G}_t^D + \eta_{a\xi}\widehat{\xi}_t) - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\varepsilon_{t+1}^{Z^D} -$

$-\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\phi_Z\widehat{Z}_t^D + \eta_{v^D Z^D}\widehat{Z}_t^D - \eta_{v^D G^D}[\varepsilon_{t+1}^{G^D} - (1-\phi_G)\widehat{G}_t^D] - \eta_{v^D \xi}(\widehat{\xi}_{t+1} - \widehat{\xi}_t)$

Use the following:

$-\eta_{v^D a}\eta_{aZ^D} + \eta_{v^D Z^D} - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\phi_Z =$

$= [-\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}][\frac{(1-a)(1+\varphi)(\omega-1)\beta(1-\phi_z)}{(1-G)(\omega+\varphi)(1-\beta\phi_Z)}(1-\gamma)] -$

$- \frac{(1-\beta)(1+\varphi)(\omega-1)[\sigma\phi_Z(1-G)(\omega+\varphi)-\varphi(\omega-1)(1-\phi_Z)]}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma) - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\phi_Z = 0$

$-\eta_{v^D a}\eta_{aG^D} + \eta_{v^D G^D}(1-\phi_G) =$

$= [-\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}][-\frac{\beta(1-\phi_G)G(1-a)}{(1-\beta\phi_G)(1-G)}] + \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\phi_G) = 0$

$-\eta_{v^D a}\eta_{a\xi} + \eta_{v^D \xi} =$

$= [-\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}][\beta] + -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} = 0$

Then:

$\widehat{R}^D_{t+1} = -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\varepsilon^{Z^D}_{t+1} - \eta_{v^D G^D}\varepsilon^{G^D}_{t+1} - \eta_{v^D\xi}\widehat{\xi}_{t+1}$

Plug in elasticities:

$\widehat{R}^D_{t+1} = -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\varepsilon^{Z^D}_{t+1} - \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{G^D}_{t+1} +$

$+ \frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\widehat{\xi}_{t+1}$

Since $\widehat{\xi}_{t+1} \equiv \frac{\alpha}{\beta(1-G)}\widehat{R}^D_{t+1}$, it is possible to write:

$\widehat{R}^D_{t+1} =$

$= -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\varepsilon^{Z^D}_{t+1} - \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{G^D}_{t+1} +$

$+ \frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\frac{\alpha}{\beta(1-G)}\widehat{R}^D_{t+1} =$

$= -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\varepsilon^{Z^D}_{t+1} - \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{G^D}_{t+1} +$

$+ \frac{\varphi(\omega-1)(1-\beta)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\frac{\alpha}{\beta}\widehat{R}^D_{t+1}$

Combine $\widehat{R}^D_{t+1}$ terms::

$\widehat{R}^D_{t+1}(1 - \frac{\varphi(\omega-1)(1-\beta)\alpha}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]\beta}) = -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\varepsilon^{Z^D}_{t+1} -$

$- \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{G^D}_{t+1}$

$\widehat{R}^D_{t+1}\frac{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]\beta-\varphi(\omega-1)(1-\beta)\alpha}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]\beta} =$

$= -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)\varepsilon^{Z^D}_{t+1} - \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\varepsilon^{G^D}_{t+1}$

$$\widehat{R}^D_{t+1}\frac{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]\beta-\varphi(\omega-1)(1-\beta)\alpha}{(1-a)\beta}=-\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)}(1-\gamma)\varepsilon^{Z^D}_{t+1}-\frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)}\varepsilon^{G^D}_{t+1}$$

This allows us to solve for $\widehat{R}^D_{t+1}$ as:

$\widehat{R}^D_{t+1}=\eta_{R^D\varepsilon^{Z^D}}\varepsilon^{Z^D}_{t+1}+\eta_{R^D\varepsilon^{G^D}}\varepsilon^{G^D}_{t+1}$ where

$$\eta_{R^D\varepsilon^{Z^D}}=-\frac{\beta(1-a)\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}(1-\gamma)$$

$$\eta_{R^D\varepsilon^{G^D}}=-\frac{\beta G\varphi(1-a)(1-\beta)(\omega-1)}{(1-\beta\phi_G)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}$$

This result is identical to GLR except for the additional $(1-\gamma)$ term in $\eta_{R^D\varepsilon^{Z^D}}$.

B.7 2nd-Order Approximation of the Portfolio Part of the Model:

From household FOCs for :

$C_t^{-\frac{1}{\sigma}}=\beta E_t\{C_{t+1}^{-\frac{1}{\sigma}}R_{t+1}\}$ which can be written as: $\frac{C_t^{-\frac{1}{\sigma}}}{\beta}=E_t\{C_{t+1}^{-\frac{1}{\sigma}}R_{t+1}\}$

$C_t^{-\frac{1}{\sigma}}=\beta E_t\{C_{t+1}^{-\frac{1}{\sigma}}R^*_{t+1}\}$ which can be written as: $\frac{C_t^{-\frac{1}{\sigma}}}{\beta}=E_t\{C_{t+1}^{-\frac{1}{\sigma}}R^*_{t+1}\}$

Equating these two expressions gives us: $E_t(C_{t+1}^{-\frac{1}{\sigma}}R_{t+1})=E_t(C_{t+1}^{-\frac{1}{\sigma}}R^*_{t+1})$ which can be written as $E_t(C_{t+1}^{-\frac{1}{\sigma}}R_{t+1})-E_t(C_{t+1}^{-\frac{1}{\sigma}}R^*_{t+1})=0$

The same derivation can be done for foreign: $E_t(C_{t+1}^{*-\frac{1}{\sigma}}R_{t+1})=E_t(C_{t+1}^{*-\frac{1}{\sigma}}R^*_{t+1})$ which can be written as $E_t(C_{t+1}^{*-\frac{1}{\sigma}}R_{t+1})-E_t(C_{t+1}^{*-\frac{1}{\sigma}}R^*_{t+1})=0$

Take second-order approximation to the home FOC and evaluate it at steady state:

$$E_t(-\tfrac{1}{\sigma}C_{t+1}^{-\frac{1}{\sigma}-1}dC_{t+1}R_{t+1})+E_t(C_{t+1}^{-\frac{1}{\sigma}}dR_{t+1})-E_t(-\tfrac{1}{\sigma}C_{t+1}^{-\frac{1}{\sigma}-1}dC_{t+1}R^*_{t+1})-E_t(C_{t+1}^{-\frac{1}{\sigma}}dR^*_{t+1})+$$

$$+\tfrac{1}{2}[-\tfrac{1}{\sigma}(-\tfrac{1}{\sigma}-1)C_{t+1}^{-\frac{1}{\sigma}-1-1}d^2C_{t+1}R_{t+1}+C_{t+1}^{-\frac{1}{\sigma}}0+2(-\tfrac{1}{\sigma})C_{t+1}^{-\frac{1}{\sigma}-1}dC_{t+1}dR_{t+1}]-$$

$$-\tfrac{1}{2}[-\tfrac{1}{\sigma}(-\tfrac{1}{\sigma}-1)C_{t+1}^{-\frac{1}{\sigma}-1-1}d^2C_{t+1}R^*_{t+1}-C_{t+1}^{-\frac{1}{\sigma}}0-2(-\tfrac{1}{\sigma})C_{t+1}^{-\frac{1}{\sigma}-1}dC_{t+1}dR^*_{t+1}]=$$

$$=E_t(-\tfrac{1}{\sigma}C^{-\frac{1}{\sigma}-1}dC_{t+1}\tfrac{1}{\beta})+E_t(C^{-\frac{1}{\sigma}}dR_{t+1})-E_t(-\tfrac{1}{\sigma}C^{-\frac{1}{\sigma}-1}dC_{t+1}\tfrac{1}{\beta})-E_t(C^{-\frac{1}{\sigma}}dR^*_{t+1})+$$

$$+\tfrac{1}{2}[-\tfrac{1}{\sigma}(-\tfrac{1}{\sigma}-1)C^{-\frac{1}{\sigma}-1-1}d^2C_{t+1}\tfrac{1}{\beta}+2(-\tfrac{1}{\sigma})C^{-\frac{1}{\sigma}-1}dC_{t+1}dR_{t+1}]-$$

$$-\tfrac{1}{2}[-\tfrac{1}{\sigma}(-\tfrac{1}{\sigma}-1)C^{-\frac{1}{\sigma}-1-1}d^2C_{t+1}\tfrac{1}{\beta}+2(-\tfrac{1}{\sigma})C^{-\frac{1}{\sigma}-1}dC_{t+1}dR^*_{t+1}]=$$

$= E_t(C^{-\frac{1}{\sigma}}dR_{t+1}) - E_t(C^{-\frac{1}{\sigma}}dR^*_{t+1}) + \frac{1}{2}[2(-\frac{1}{\sigma})C^{-\frac{1}{\sigma}-1}dC_{t+1}dR_{t+1}] - \frac{1}{2}[2(-\frac{1}{\sigma})C^{-\frac{1}{\sigma}-1}dC_{t+1}dR^*_{t+1}]$

$= E_t(C^{-\frac{1}{\sigma}}dR_{t+1}) - E_t(C^{-\frac{1}{\sigma}}dR^*_{t+1}) + [-\frac{1}{\sigma}C^{-\frac{1}{\sigma}-1}dC_{t+1}dR_{t+1}] - [-\frac{1}{\sigma}C^{-\frac{1}{\sigma}-1}dC_{t+1}dR^*_{t+1}]$

Divide by $C^{-\frac{1}{\sigma}}$ and R.

$\widehat{R}_{t+1} - \widehat{R}^*_{t+1} + (-\frac{1}{\sigma}\widehat{C}_{t+1}\widehat{R}_{t+1}) - (-\frac{1}{\sigma}\widehat{C}_{t+1}\widehat{R}^*_{t+1}) = 0$

Same derivation for the foreign country gives:

$\widehat{R}_{t+1} - \widehat{R}^*_{t+1} + (-\frac{1}{\sigma}\widehat{C}^*_{t+1}\widehat{R}_{t+1}) - (-\frac{1}{\sigma}\widehat{C}^*_{t+1}\widehat{R}^*_{t+1}) = 0$

Subtract expressions for the home and foreign countries:

$\widehat{R}_{t+1} - \widehat{R}^*_{t+1} + (-\frac{1}{\sigma}\widehat{C}_{t+1}\widehat{R}_{t+1}) - (-\frac{1}{\sigma}\widehat{C}_{t+1}\widehat{R}^*_{t+1}) - [\widehat{R}_{t+1} - \widehat{R}^*_{t+1} + (-\frac{1}{\sigma}\widehat{C}^*_{t+1}\widehat{R}_{t+1}) - (-\frac{1}{\sigma}\widehat{C}^*_{t+1}\widehat{R}^*_{t+1})] = 0$

$(-\frac{1}{\sigma}\widehat{C}_{t+1}\widehat{R}_{t+1}) - (-\frac{1}{\sigma}\widehat{C}_{t+1}\widehat{R}^*_{t+1}) - [(-\frac{1}{\sigma}\widehat{C}^*_{t+1}\widehat{R}_{t+1}) - (-\frac{1}{\sigma}\widehat{C}^*_{t+1}\widehat{R}^*_{t+1})] = 0$

$\widehat{C}_{t+1}\widehat{R}_{t+1} - \widehat{C}_{t+1}\widehat{R}^*_{t+1} - [\widehat{C}^*_{t+1}\widehat{R}_{t+1} - \widehat{C}^*_{t+1}\widehat{R}^*_{t+1}] = 0$

$E_t(\widehat{C}_{t+1}\widehat{R}^D_{t+1}) - E_t(\widehat{C}^*_{t+1}\widehat{R}^D_{t+1}) = 0$

$E_t(\widehat{C}^D_{t+1}\widehat{R}^D_{t+1}) = 0$

This results is the same as in GLR.

B.8 Solve for Steady-State Portfolio α:

Plug in expressions for $\widehat{C}^D_{t+1}$ from Section B.3 and $\widehat{R}^D_{t+1}$ from Section B.6:

$E_t\{(\eta_{C^D a}n\widehat{f}a_{t+1} + \eta_{C^D Z^D}\widehat{Z}^D_{t+1} + \eta_{C^D G^D}\widehat{G}^D_{t+1} + \eta_{C^D \xi}\widehat{\xi}_{t+1})(\eta_{R^D \varepsilon^{Z^D}}\varepsilon^{Z^D}_{t+1} + \eta_{R^D \varepsilon^{G^D}}\varepsilon^{G^D}_{t+1})\} = 0$

$E_t\{[(\eta_{C^D a}(n\widehat{f}a_t + \eta_{aZ^D}\widehat{Z}^D_t + \eta_{aG^D}\widehat{G}^D_t + \eta_{a\xi}\widehat{\xi}_t) + \eta_{C^D Z^D}\phi_Z\widehat{Z}^D_t + \eta_{C^D Z^D}\varepsilon^{Z^D}_{t+1} + \eta_{C^D G^D}\phi_G\widehat{G}^D_t +$

$+ \eta_{C^D G^D}\varepsilon^{G^D}_{t+1} + \eta_{C^D \xi}\frac{\alpha}{\beta(1-G)}\eta_{R^D \varepsilon^{G^D}}\varepsilon^{G^D}_{t+1}](\eta_{R^D \varepsilon^{Z^D}}\varepsilon^{Z^D}_{t+1} + \eta_{R^D \varepsilon^{G^D}}\varepsilon^{G^D}_{t+1})\} = 0$

$E_t\{\eta_{C^D Z^D}\varepsilon^{Z^D}_{t+1}(\eta_{R^D \varepsilon^{Z^D}}\varepsilon^{Z^D}_{t+1}) + \eta_{C^D G^D}\varepsilon^{G^D}_{t+1}(\eta_{R^D \varepsilon^{G^D}}\varepsilon^{G^D}_{t+1}) + \eta_{C^D \xi}\frac{\alpha}{\beta(1-G)}(\eta_{R^D \varepsilon^{Z^D}}\varepsilon^{Z^D}_{t+1} + \eta_{R^D \varepsilon^{G^D}}\varepsilon^{G^D}_{t+1})^2\}$

$= 0$

$$\eta_{C^D Z^D}\eta_{R^D{}_{\varepsilon}Z^D}\sigma^2_{\varepsilon^{ZD}} + \eta_{C^D G^D}\eta_{R^D{}_{\varepsilon}G^D}\sigma^2_{\varepsilon^{GD}} + \tfrac{\alpha}{\beta(1-G)}\eta_{C^D\xi}\eta^2_{R^D{}_{\varepsilon}Z^D}\sigma^2_{\varepsilon^{ZD}} + \tfrac{\alpha}{\beta(1-G)}\eta_{C^D\xi}\eta^2_{R^D{}_{\varepsilon}G^D}\sigma^2_{\varepsilon^{GD}}$$

$$= 0$$

Plug in elasticities to solve for steady-state portfolio α:

$$[\tfrac{\sigma(1-\beta)(1+\varphi)(\omega-1)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}(1-\gamma)][-\tfrac{\beta(1-a)\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}(1-\gamma)]\sigma^2_{\varepsilon^{ZD}}+$$

$$+[-\tfrac{\sigma(1-\beta)G(\omega+\varphi)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}][-\tfrac{\beta G\varphi(1-a)(1-\beta)(\omega-1)}{(1-\beta\phi_G)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}]\sigma^2_{\varepsilon^{GD}} +$$

$$+ \tfrac{\alpha}{\beta(1-G)}[\tfrac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}][-\tfrac{\beta(1-a)\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}(1-\gamma)]$$

$$[-\tfrac{\beta(1-a)\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}(1-\gamma)]\sigma^2_{\varepsilon^{ZD}} +$$

$$+ \tfrac{\alpha}{\beta(1-G)}[\tfrac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}][-\tfrac{\beta G\varphi(1-a)(1-\beta)(\omega-1)}{(1-\beta\phi_G)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}]$$

$$[-\tfrac{\beta G\varphi(1-a)(1-\beta)(\omega-1)}{(1-\beta\phi_G)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}]\sigma^2_{\varepsilon^{GD}} = 0$$

$$[\tfrac{1+\varphi}{1-\beta\phi_Z}(1-\gamma)][-\tfrac{\sigma(\omega-1)(1+\varphi)(1-G)}{1-\beta\phi_Z}(1-\gamma)]\sigma^2_{\varepsilon^{ZD}} + [\tfrac{G(\omega+\varphi)}{1-\beta\phi_G}][\tfrac{G\varphi}{1-\beta\phi_G}]\sigma^2_{\varepsilon^{GD}} +$$

$$+\tfrac{\alpha}{\beta(1-G)}[\tfrac{(1-G)(\omega+\varphi)}{1-a}][\tfrac{\sigma(1+\varphi)(1-G)}{1-\beta\phi_Z}(1-\gamma)][\tfrac{\beta(1-a)\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}(1-\gamma)]\sigma^2_{\varepsilon^{ZD}}$$

$$+ \tfrac{\alpha}{\beta(1-G)}[\tfrac{(1-G)(\omega+\varphi)}{1-a}][\tfrac{G\varphi}{1-\beta\phi_G}][\tfrac{\beta G\varphi(1-a)(1-\beta)(\omega-1)}{(1-\beta\phi_G)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}]\sigma^2_{\varepsilon^{GD}} = 0$$

$$[\tfrac{1+\varphi}{1-\beta\phi_Z}(1-\gamma)][-\tfrac{\sigma(\omega-1)(1+\varphi)(1-G)}{1-\beta\phi_Z}(1-\gamma)]\sigma^2_{\varepsilon^{ZD}} + [\tfrac{G(\omega+\varphi)}{1-\beta\phi_G}][\tfrac{G\varphi}{1-\beta\phi_G}]\sigma^2_{\varepsilon^{GD}} +$$

$$+ \alpha(\omega+\varphi)[\tfrac{\sigma(1+\varphi)(1-G)}{1-\beta\phi_Z}(1-\gamma)][\tfrac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}(1-\gamma)]\sigma^2_{\varepsilon^{ZD}} +$$

$$+ \alpha(\omega+\varphi)[\tfrac{G\varphi}{1-\beta\phi_G}][\tfrac{G\varphi(1-\beta)(\omega-1)}{(1-\beta\phi_G)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}]\sigma^2_{\varepsilon^{GD}} = 0$$

$$\frac{-(1+\varphi)(1-\gamma)\sigma(\omega-1)(1+\varphi)(1-G)(1-\gamma)\sigma^2_{\varepsilon^{ZD}}}{(1-\beta\phi_Z)(1-\beta\phi_Z)} + \frac{G(\omega+\varphi)G\varphi\sigma^2_{\varepsilon^{GD}}}{(1-\beta\phi_G)(1-\beta\phi_G)} +$$

$$+ \frac{\alpha(\omega+\varphi)\sigma(1+\varphi)(1-G)(1-\gamma)\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)(1-\gamma)\sigma^2_{\varepsilon^{ZD}}}{(1-\beta\phi_Z)(1-\beta\phi_Z)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}} +$$

$$+ \frac{\alpha(\omega+\varphi)G\varphi G\varphi(1-\beta)(\omega-1)\sigma^2_{\varepsilon^{GD}}}{(1-\beta\phi_G)(1-\beta\phi_G)\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}} = 0$$

$$\frac{-(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}}{(1-\beta\phi_Z)^2}+\frac{G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}}{(1-\beta\phi_G)^2}+$$
$$+\frac{\alpha(\omega+\varphi)\sigma^2(1+\varphi)^2(1-G)^2(1-\gamma)^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon ZD}}{(1-\beta\phi_Z)^2\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}+$$
$$+\frac{\alpha(\omega+\varphi)G^2\varphi^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon GD}}{(1-\beta\phi_G)^2\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}=0$$

$$\frac{\alpha(\omega+\varphi)\sigma^2(1+\varphi)^2(1-G)^2(1-\gamma)^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon ZD}}{(1-\beta\phi_Z)^2\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}+$$
$$+\frac{\alpha(\omega+\varphi)G^2\varphi^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon GD}}{(1-\beta\phi_G)^2\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}=\frac{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}}{(1-\beta\phi_Z)^2}-\frac{G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}}{(1-\beta\phi_G)^2}$$

$$\frac{\alpha(\omega+\varphi)\sigma^2(1+\varphi)^2(1-G)^2(1-\gamma)^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2+\alpha(\omega+\varphi)G^2\varphi^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2}{(1-\beta\phi_Z)^2(1-\beta\phi_G)^2\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}}=$$
$$=\frac{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2}{(1-\beta\phi_Z)^2(1-\beta\phi_G)^2}$$

$$\frac{\alpha(\omega+\varphi)\sigma^2(1+\varphi)^2(1-G)^2(1-\gamma)^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2+\alpha(\omega+\varphi)G^2\varphi^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)}=$$
$$=(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2$$

$\alpha(\omega+\varphi)\sigma^2(1+\varphi)^2(1-G)^2(1-\gamma)^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2+\alpha(\omega+\varphi)G^2\varphi^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2=$

$=\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]-\alpha\varphi(\omega-1)(1-\beta)\}$

$\alpha(\omega+\varphi)\sigma^2(1+\varphi)^2(1-G)^2(1-\gamma)^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2+$

$+\alpha(\omega+\varphi)G^2\varphi^2(1-\beta)(\omega-1)\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2+$

$+\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\{\alpha\varphi(\omega-1)(1-\beta)\}=$

$=\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]\}$

$$\alpha(1-\beta)[(\omega+\varphi)\sigma^2(1+\varphi)^2(1-G)^2(1-\gamma)^2(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2+$$

$$+(\omega+\varphi)G^2\varphi^2(\omega-1)\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2+$$

$$+\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\varphi(\omega-1)]=$$

$$=\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]$$

$$\alpha(1-\beta)[(\omega+\varphi)\sigma^2(1+\varphi)^2(1-G)^2(1-\gamma)^2(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2+$$

$$+(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2\varphi(\omega-1)]=$$

$$=\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]$$

$$\alpha(1-\beta)[(1+\varphi)^2(1-\gamma)^2(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2\sigma(1-G)][(\omega+\varphi)\sigma(1-G)+\varphi(\omega-1)]=$$

$$=\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]$$

$$\alpha(1-\beta)[(1+\varphi)^2(1-\gamma)^2(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2\sigma(1-G)]=$$

$$=\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\beta(1-a)$$

$$\alpha(1-\beta)=\frac{\{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2\}\beta(1-a)}{(1+\varphi)^2(1-\gamma)^2(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2\sigma(1-G)}$$

$$\alpha=\frac{\beta(1-a)}{1-\beta}\frac{(1+\varphi)^2(1-\gamma)^2\sigma(\omega-1)(1-G)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2-G^2(\omega+\varphi)\varphi\sigma^2_{\varepsilon GD}(1-\beta\phi_Z)^2}{(1+\varphi)^2(1-\gamma)^2(\omega-1)\sigma^2_{\varepsilon ZD}(1-\beta\phi_G)^2\sigma(1-G)}$$

$$\alpha=\frac{\beta(1-a)}{1-\beta}[1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]$$

Compared to the GLR optimal portfolio $\alpha = \frac{\beta(1-a)}{1-\beta}[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]$, there is an additional term $(1-\gamma)$ in the denominator. As γ increases from 0 towards 1, the steady-state portfolio α decreases, i.e., portfolio held by home household contains more home equity.

Appendix C: "Country-Specific Technology" Setup Leads to the Same Result as the GLR "Exporter-Only" Setup

This Appendix shows that the "country-specific technology" setup leads to the same result as the GLR "exporter-only" setup. The "country-specific technology" subjects the firms to the technologies of the countries in which they operate. This setup is represented by setting $\gamma = 0$ in Table 1. The home firm z still produces according to $y_t(z) = Z_t L_t(z)$ in the home country, but it now produces $y^*_t(z)$ according to $y^*_t(z) = Z^*_t L^*_t(z)$ in the foreign country. Similarly, the foreign firm z^* still produces $y^*_{*t}(z^*)$ according to $y^*_{*t}(z^*) = Z^*_t L^*_{*t}(z^*)$ in the foreign country, but it now produces $y_{*t}(z^*)$ according to $y_{*t}(z^*) = Z_t L_{*t}(z^*)$ in the home country. Table 2 summarizes this setup.

C.1 Households

Since the household part of the model is unaffected by changing the assumption on firm production structure, the derivations in Appendix A.1 remain the same. However, the derivations for the firm part of the model in Appendix A.2 differ.

C.2 Firms

C.2.1 Derivation of prices:

Derivation of price of good produced by home firm z:

$\min\ w_t L_t(z) + w_t^* L_t^*(z) - mc_t(Z_t L_t(z) + Z_t^* L_t^*(z) - Y_t(z))$

$\frac{\partial \ell}{\partial L_t(z)} = w_t - mc_t Z_t = 0$

$\frac{w_t}{Z_t} = mc_t = RP_t \frac{\theta-1}{\theta}$

$RP_t = \frac{\theta}{\theta-1}\frac{w_t}{Z_t}$ which is the price charged in home country. This is identical to Appendix A.2.1.

$\frac{\partial \ell}{\partial L_t^*(z)} = w_t^* - mc_t Z_t^* = 0$

$\frac{w_t^*}{Z_t^*} = mc_t = RP_t^* \frac{\theta-1}{\theta}$

$RP_t^* = \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*}$ which is the price charged in foreign country. This differs from Appendix A.2.1.

Derivation of price of good produced by foreign firm z^*:

$\min\ w_t L_{*t}(z^*) + w_t^* L_{*t}^*(z^*) - mc_{*t}(Z_t L_{*t}(z^*) + Z_t^* L_{*t}^*(z^*) - Y_t^*(z^*))$

$\frac{\partial \ell}{\partial L_{*t}(z^*)} = w_t - mc_{*t} Z_t = 0$

$\frac{w_t}{Z_t} = mc_{*t} = RP_{*t} \frac{\theta-1}{\theta}$

$RP_{*t} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t}$ which is the price charged in home country. This differs from Appendix A.2.1.

$\frac{\partial \ell}{\partial L_{*t}^*(z^*)} = w_t^* - mc_{*t} Z_t^* = 0$

$\frac{w_t^*}{Z_t^*} = mc_{*t} = RP_{*t}^* \frac{\theta-1}{\theta}$

$RP_{*t}^* = \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*}$ which is the price charged in foreign country. This is identical to Appendix A.2.1.

C.2.2 Derivation of optimum labor demands:

Note that indexation of labor by z and z^* showing whether the labor is hired by home firm z or foreign firm z^* is dropped to simplify the notation.

Derivation of optimal labor demand by home firm z in home country:

Production function is given as $Y_t^s = Z_t L_t$

Demand for output was derived from household optimization as: $Y_t^d = RP_t^{-\omega} Y_t^W$.

Since $Y_t^s = Y_t^d$, it is possible to write $Z_t L_t = RP_t^{-\omega} Y_t^W$.

From this, $L_t = \frac{RP_t^{-\omega} Y_t^W}{Z_t}$ follows. This is identical to Appendix A.2.2.

Derivation of optimal labor demand by foreign firm z^* in home country:

Production function is given as $Y_{*t}^s = Z_t L_{*t}$

Demand for output was derived from household optimization as: $Y_{*t}^d = RP_{*t}^{-\omega} Y_t^W$

Since $Y_{*t}^s = Y_{*t}^d$, it is possible to write $Z_t L_{*t} = RP_{*t}^{-\omega} Y_t^W$.

From this, $L_{*t} = \frac{RP_{*t}^{-\omega} Y_t^W}{Z_t}$ follows. This differs from Appendix A.2.2.

Derivation of total optimal labor demand in home country:

$L_t^d = L_t + L_{*t} = \frac{RP_t^{-\omega} Y_t^W}{Z_t} + \frac{RP_{*t}^{-\omega} Y_t^W}{Z_t} = Y_t^W \left(\frac{RP_t^{-\omega} + RP_{*t}^{-\omega}}{Z_t}\right)$ which differs from differs from Appendix A.2.2.

Derivation of optimal labor demand by home firm z in foreign country:

Production function is given as $Y_t^{*s} = Z_t^* L_t^*$

Demand for output was derived from household optimization as: $Y_t^{*d} = RP_t^{*-\omega} Y_t^W$

Since $Y_t^{*s} = Y_t^{*d}$, it is possible to write $Z_t^* L_t^* = RP_t^{*-\omega} Y_t^W$.

From this, $L_t^* = \frac{RP_t^{*-\omega}Y_t^W}{Z_t^*}$ follows. This differs from Appendix A.2.2.

Derivation of optimal labor demand by foreign firm z^* in foreign country:

Production function is given as $Y_{*t}^{*s} = Z_t^* L_{*t}^*$

Demand for output was derived from household optimization as: $Y_{*t}^{*d} = RP_{*t}^{*-\omega}Y_t^W$

Since $Y_{*t}^{*s} = Y_{*t}^{*d}$, it is possible to write $Z_t^* L_{*t}^* = RP_{*t}^{*-\omega}Y_t^W$.

From this, $L_{*t}^* = \frac{RP_{*t}^{*-\omega}Y_t^W}{Z_t^*}$ follows. This is identical to Appendix A.2.2.

Derivation of total optimal labor demand in foreign country:

$L_t^{*d} = L_t^* + L_{*t}^* = \frac{RP_t^{*-\omega}Y_t^W}{Z_t^*} + \frac{RP_{*t}^{*-\omega}Y_t^W}{Z_t^*} = Y_t^W(\frac{RP_t^{*-\omega}+RP_{*t}^{*-\omega}}{Z_t^*})$ which differs from Appendix A.2.2.

Note that these expressions are similar to GLR where aggregate per capita labor demands in home and foreign countries are $L_t = \frac{RP_t^{-\omega}y_t^W}{Z_t}$ and $L_t^* = \frac{RP_t^{*-\omega}y_t^W}{Z_t^*}$, respectively. The only difference is that the above expressions contain two prices.

C.3 Useful Properties

C.3.1 To get $\frac{y_t}{y_t^*}$:

Derivation of home GDP y_t, i.e., output produced by home and foreign firms in the home country:

$y_t = RP_t Z_t L_t + RP_{*t} Z_t L_{*t} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t} Z_t L_t + \frac{\theta}{\theta-1}\frac{w_t}{Z_t} Z_t L_{*t} = \frac{\theta}{\theta-1}(w_t L_t + w_t L_{*t})$

Derivation of foreign GDP y_t^*, i.e., output produced by home and foreign firms in the foreign country:

$y_t^* = RP_t^* Z_t^* L_t^* + RP_{*t}^* Z_t^* L_{*t}^* = \frac{\theta}{\theta-1}\frac{w_t}{Z_t^*} Z_t^* L_t^* + \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*} Z_t^* L_{*t}^* = \frac{\theta}{\theta-1}(w_t^* L_t^* + w_t^* L_{*t}^*)$

Expression for $\frac{y_t}{y_t^*}$:

$\frac{y_t}{y_t^*} = \frac{RP_t Z_t L_t + RP_{*t} Z_t L_{*t}}{RP_t^* Z_t^* L_t^* + RP_{*t}^* Z_t^* L_{*t}^*} = \frac{\frac{\theta}{\theta-1}(w_t L_t + w_t L_{*t})}{\frac{\theta}{\theta-1}(w_t^* L_t^* + w_t^* L_{*t}^*)} = \frac{w_t(L_t + L_{*t})}{w_t^*(L_t^* + L_{*t}^*)}$

As explained in Appendix A.3, even though the production structure differs from GLR, the technology terms cancel in this expression, so this result is identical to GLR.

Next, expressions for w_t, w_t^*, $(L_t + L_{*t})$ and $(L_t^* + L_{*t}^*)$ are obtained:

To get w_t, home labor supply and home labor demand are equated.

Home labor supply was derived above from home household FOCs as $L_t^s = (\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi}$.

Home labor demand was derived above from firm FOCs as $L_t^d = Y_t^W(\frac{RP_t^{-\omega} + RP_{*t}^{-\omega}}{Z_t})$

Since $L_t^s = L_t^d$, it is possible to write: $(\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi} = Y_t^W(\frac{RP_t^{-\omega} + RP_{*t}^{-\omega}}{Z_t})$

Plug in expressions for RP_t and RP_{*t}:

$(\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi} = Y_t^W(\frac{(\frac{\theta}{\theta-1}\frac{w_t}{Z_t})^{-\omega}}{Z_t} + \frac{(\frac{\theta}{\theta-1}\frac{w_t}{Z_t})^{-\omega}}{Z_t})$

$(\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi} = Y_t^W[(\frac{\theta}{\theta-1})^{-\omega} w_t^{-\omega} Z_t^{\omega-1} + (\frac{\theta}{\theta-1})^{-\omega} w_t^{-\omega} Z_t^{\omega-1}]$

$w_t^{\varphi+\omega} = C_t^{\frac{\varphi}{\sigma}} \chi^{\varphi} 2Y_t^W(\frac{\theta}{\theta-1})^{-\omega} Z_t^{\omega-1}$

$w_t = C_t^{\frac{\varphi}{\sigma(\varphi+\omega)}} \chi^{\frac{\varphi}{\varphi+\omega}} (2Y_t^W)^{\frac{1}{\varphi+\omega}} (\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}} Z_t^{\frac{\omega-1}{\varphi+\omega}}$

To get w_t^*, equate foreign labor supply and foreign labor demand are equated.

Foreign labor supply could be derived in the same way as home labor supply from household

FOCs as $L_t^{*s} = (\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi}$.

Foreign labor demand was derived above from firm FOCs as $L_t^{*d} = Y_t^W(\frac{RP_t^{*-\omega}+RP_{*t}^{*-\omega}}{Z_t^*})$

Since $L_t^{*s} = L_t^{*d}$, it is possible to write: $(\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi} = Y_t^W(\frac{RP_t^{*-\omega}+RP_{*t}^{*-\omega}}{Z_t^*})$

Plug in expressions for RP_t^* and RP_{*t}^*:

$$(\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi} = Y_t^W(\frac{(\frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*})^{-\omega}}{Z_t^*} + \frac{(\frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*})^{-\omega}}{Z_t^*})$$

$$(\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi} = Y_t^W[(\frac{\theta}{\theta-1})^{-\omega} w_t^{*-\omega} Z_t^{*\omega-1} + (\frac{\theta}{\theta-1})^{-\omega} w_t^{*-\omega} Z_t^{*\omega-1}]$$

$$w_t^{*\varphi+\omega} = C_t^{*\frac{\varphi}{\sigma}} \chi^{\varphi} 2Y_t^W (\frac{\theta}{\theta-1})^{-\omega} Z_t^{*\omega-1}$$

$$w_t^* = C_t^{*\frac{\varphi}{\sigma(\varphi+\omega)}} \chi^{\frac{\varphi}{\varphi+\omega}} (2Y_t^W)^{\frac{1}{\varphi+\omega}} (\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}} Z_t^{*\frac{\omega-1}{\varphi+\omega}}$$

To get $(L_t + L_{*t})$ i.e., total home labor supply L_t^S:

As derived above from home household FOC: $L_t^s = (\frac{C_t^{-\frac{1}{\sigma}} w_t}{\chi})^{\varphi}$

Plug in w_t: $L_t^s = (\frac{C_t^{-\frac{1}{\sigma}} C_t^{\frac{\varphi}{\sigma(\varphi+\omega)}} \chi^{\frac{\varphi}{\varphi+\omega}} (2Y_t^W)^{\frac{1}{\varphi+\omega}} (\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}} Z_t^{\frac{\omega-1}{\varphi+\omega}}}{\chi})^{\varphi}$

To get $(L_t^* + L_{*t}^*)$i.e., total foreign labor supply L_t^{*s}:

From foreign household FOC: $L_t^{*s} = (\frac{C_t^{*-\frac{1}{\sigma}} w_t^*}{\chi})^{\varphi}$

Plug in w_t^*: $L_t^{*s} = (\frac{C_t^{*-\frac{1}{\sigma}} C_t^{*\frac{\varphi}{\sigma(\varphi+\omega)}} \chi^{\frac{\varphi}{\varphi+\omega}} (2Y_t^W)^{\frac{1}{\varphi+\omega}} (\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}} Z_t^{*\frac{\omega-1}{\varphi+\omega}}}{\chi})^{\varphi}$

Then, $\frac{y_t}{y_t^*} = \frac{w_t(L_t+L_{*t})}{w_t^*(L_t^*+L_{*t}^*)} = (\frac{C_t}{C_t^*})^{\frac{-\varphi(\omega-1)}{\sigma(\varphi+\omega)}} (\frac{Z_t}{Z_t^*})^{\frac{(\omega-1)(\varphi+1)}{\varphi+\omega}}$

As opposed to Appendix A.3, this result **is identical to GLR** derivation. Applying the "country-specific technology" to firm production gives the same result for relative GDPs $\frac{y_t}{y_t^*}$ as the GLR setup with "exporter-only" firms using technology of their respective countries.

Following from this result, the steady-state portfolio α will be identical to the GLR steady-state portfolio $\alpha = \frac{\beta(1-a)}{1-\beta}[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]$.

This is a very useful result because it allows modeling the GLR "exporter-only" setup as the "country-specific technology" MNC setup. It is then possible to see see how the optimal portfolio changes as the production structure moves from "exporter-only" setup in GLR to "MNC-only" setup by showing how the optimal portfolio changes as the production structure moves from the "country-specific technology" setup to the "firm-specific technology" setup.

Appendix D: Derivative of α with Respect to γ

This Appendix shows the derivative of α with respect to γ and the role played by the Frisch elasticity of labor supply, φ.

$$\alpha = \frac{\beta(1-a)}{1-\beta}[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]$$

$$\frac{\partial\alpha}{\partial\gamma} = -\frac{\beta(1-a)}{1-\beta}\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}(-2)(1-\gamma)^{-2-1}(-1) =$$

$$= -\frac{\beta(1-a)}{1-\beta}\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}2(1-\gamma)^{-3} = -\frac{2\beta(1-a)}{1-\beta}\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^3\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}$$

This expression is negative for parameter values assumed by GLR. This means that an increase in γ leads to a decrease in α, i.e., home household foreign equity decreases and home equity increases.

Next, the role of φ is analyzed in more detail. When φ increases, the above expression for $\frac{\partial\alpha}{\partial\gamma}$ becomes more negative. This means that increasing φ magnifies the effect of γ on α. Conversely, a decrease in φ leads to a decrease in the effect of γ on α. When φ reaches 0, the effect of γ on α disappears, i.e., firm production structure becomes irrelevant when labor supply becomes inelastic. This can be seen by taking a derivative of $\frac{\partial\alpha}{\partial\gamma}$ with respect to φ.

$$\frac{\partial\frac{\partial\alpha}{\partial\gamma}}{\partial\varphi} = \frac{[-2\beta(1-a)G^2(\omega+2\varphi)(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}][(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]}{[(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]^2} -$$

$$\frac{[-2\beta(1-a)G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}][(1-\beta)(1-\gamma)^3\sigma(\omega-1)2(1+\varphi)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]}{[(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]^2} =$$

$$= \frac{[-(\omega+2\varphi)(1+\varphi)^2+(\omega+\varphi)\varphi2(1+\varphi)][2\beta(1-a)G^2(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]}{(1+\varphi)^4[(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]^2} =$$

$$= \frac{[-(\omega+2\varphi)(1+\varphi)+(\omega+\varphi)\varphi2][2\beta(1-a)G^2(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]}{(1+\varphi)^3[(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]^2} =$$

$$= \frac{[\varphi(\omega-2)-\omega][2\beta(1-a)G^2(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]}{(1+\varphi)^3[(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}]^2} =$$

$$= \frac{[\varphi(\omega-2)-\omega][2\beta(1-a)G^2(1-\beta\phi_Z)^2\sigma^2_{\varepsilon}{}_{GD}]}{(1+\varphi)^3[(1-\beta)(1-\gamma)^3\sigma(\omega-1)(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon}{}_{ZD}]}$$

For GLR calibration values $\omega = 2$ and $\varphi = 4$, this expression is negative which means an increase in φ leads to a decrease in the value of $\frac{\partial\alpha}{\partial\gamma}$. Since $\frac{\partial\alpha}{\partial\gamma}$ is negative, the absolute value of $\frac{\partial\alpha}{\partial\gamma}$ increases. This means that the effect of γ on α becomes larger with increasing φ.

Note that G has a similar effect on the relationship between γ and α. As G increases, the effect of γ on α is magnified.

Appendix E: Steady-State Equity Holdings x, x^*, x_* & x_*^*

This Appendix shows how γ affects steady-state equity holdings x, x^*, x_* and x_*^*. The derivations are identical to GLR except for the last steps where the expression for steady-state portfolio α is substituted. Also, derivatives of x, x^*, x_* & x_*^* with respect to γ are shown.

x^*, i.e., home household holdings of foreign firm shares:

In the symmetric steady state, $v = v^* = \frac{\beta}{(1-\beta)\theta}$ as derived in Appendix B.4 and $\alpha \equiv v^* x^*$ as defined in Section 2.1. Therefore, $x^* = \frac{\alpha}{v^*} = \frac{\alpha\theta(1-\beta)}{\beta}$. Substituting the expression for steady-state portfolio α derived in Appendix B.8 gives:

$$x^* = \frac{\theta(1-\beta)}{\beta}\frac{\beta(1-a)}{1-\beta}[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}] =$$
$$= (1-a)\theta[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]$$

As γ increases from 0 towards 1, x^* decreases which means the home household holds fewer foreign firm shares.

x_*^*, i.e., foreign household holdings of foreign firm shares:

Using equity market clearing condition $ax^* + (1-a)x_*^* = 1-a$, it is possible to write $x_*^* = \frac{1-a-ax^*}{1-a} = \frac{1-a-a\frac{\alpha\theta(1-\beta)}{\beta}}{1-a}$. Substituting the expression for α gives:

$$x_*^* = \frac{1-a-a\frac{\theta(1-\beta)}{\beta}\frac{\beta(1-a)}{1-\beta}[1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]}{1-a} =$$
$$= \frac{1-a-a\theta(1-a)[1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]}{1-a} =$$
$$= 1 - a\theta[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]$$

As γ increases from 0 towards 1, x_*^* increases which means the foreign household holds more foreign firm shares.

x_*, i.e., foreign household holdings of home firm shares:

Using the definition of net foreign assets from Appendix A.1 $nfa \equiv v^*x^* - \frac{1-a}{a}vx_*$ combined with the fact that $nfa = 0$ in steady state, it is possible to write $v^*x^* = \frac{1-a}{a}vx_*$. Since $\alpha \equiv v^*x^*$, $\alpha = \frac{1-a}{a}vx_*$. Then, $x_* = \frac{a\alpha}{(1-a)v} = \frac{a\theta(1-\beta)\alpha}{(1-a)\beta}$. Substituting the expression for α gives:

$$x_* = \frac{a\theta(1-\beta)}{(1-a)\beta}\frac{\beta(1-a)}{1-\beta}[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}] =$$
$$= a\theta[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]$$

As γ increases from 0 towards 1, x_* decreases which means the foreign household holds fewer home firm shares.

x, i.e., home household holdings of home firm shares:

Using equity market clearing condition $ax + (1-a)x_* = a$ introduced in Appendix A.1, it is possible to write $x = \frac{a-(1-a)x_*}{a} = \frac{a-(1-a)\frac{a\theta(1-\beta)\alpha}{(1-a)\beta}}{a} = \frac{a-\frac{a\theta(1-\beta)\alpha}{\beta}}{a}$. Substituting the expression for α gives:

$$x = \frac{a-\frac{a\theta(1-\beta)}{\beta}\frac{\beta(1-a)}{1-\beta}[1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]}{a} =$$
$$= \frac{a-a\theta(1-a)[1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}]}{a}$$
$$= 1 - \theta(1-a)[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^2\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}].$$

As γ increases from 0 towards 1, x increases which means the home household holds more home firm shares.

These results show that the conclusion derived in Appendix B.8 and discussed in Section 3.2, i.e., steady-state portfolio α becoming more home-biased as γ increases, extends to the steady-state equity holdings x, x^*, x_* & x_*^*. The home household will hold more home firm equity and less foreign firm equity whereas the foreign household will hold more foreign firm equity and less home firm equity.

Derivatives of x, x^*, x_* & x_*^* with respect to γ:

$$\frac{\partial x}{\partial \gamma} = \theta(1-a)\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}(-2)(1-\gamma)^{-2-1}(-1) =$$
$$= \theta(1-a)2\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^3\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}$$

$$\frac{\partial x^*}{\partial \gamma} = -\theta(1-a)2\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^3\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}$$

$$\frac{\partial x_*}{\partial \gamma} = -a\theta 2\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^3\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}$$

$$\frac{\partial x_*^*}{\partial \gamma} = a\theta 2\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{(1-\gamma)^3\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}$$

The effect of φ on $\frac{\partial x}{\partial \gamma}$, $\frac{\partial x^*}{\partial \gamma}$, $\frac{\partial x_*}{\partial \gamma}$ and $\frac{\partial x_*^*}{\partial \gamma}$ is consistent with the effect of φ on $\frac{\partial \alpha}{\partial \gamma}$ explained in Appendix D.

Appendix F: Terms of Trade TOT_t

This Appendix shows derivations of terms of trade TOT_t.

In GLR, $TOT_t \equiv \frac{RP_t^H}{RP_t^F}$.

In this paper, $TOT_t = \frac{PriceOfGoodsMadeByForeignFirmInHomeCountry}{PriceOfGoodsMadeByHomeFirmInForeignCountry} = \frac{RP_{t*}}{RP^{t*}} = \frac{\frac{\theta}{\theta-1}\frac{w_t}{Z_t^{1-\gamma}Z_t^{*\gamma}}}{\frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^{\gamma}Z_t^{*1-\gamma}}} =$

$$\frac{\frac{C_t^{\frac{\varphi}{\sigma(\varphi+\omega)}}\chi^{\frac{\varphi}{\varphi+\omega}}(Y_t^W)^{\frac{1}{\varphi+\omega}}(\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}}[Z_t^{\omega-1}+(Z_t^{1-\gamma}Z_t^{*\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{Z_t^{1-\gamma}Z_t^{*\gamma}}}{\frac{C_t^{*\frac{\varphi}{\sigma(\varphi+\omega)}}\chi^{\frac{\varphi}{\varphi+\omega}}(Y_t^W)^{\frac{1}{\varphi+\omega}}(\frac{\theta}{\theta-1})^{-\frac{\omega}{\varphi+\omega}}[Z_t^{*\omega-1}+(Z_t^{\gamma}Z_t^{*1-\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{Z_t^{\gamma}Z_t^{*1-\gamma}}} =$$

$$= \frac{\frac{C_t^{\frac{\varphi}{\sigma(\varphi+\omega)}}[Z_t^{\omega-1}+(Z_t^{1-\gamma}Z_t^{*\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{Z_t^{1-\gamma}Z_t^{*\gamma}}}{\frac{C_t^{*\frac{\varphi}{\sigma(\varphi+\omega)}}[Z_t^{*\omega-1}+(Z_t^{\gamma}Z_t^{*1-\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{Z_t^{\gamma}Z_t^{*1-\gamma}}} = \left(\frac{C_t}{C_t^*}\right)^{\frac{\varphi}{\sigma(\varphi+\omega)}}\frac{\frac{[Z_t^{\omega-1}+(Z_t^{1-\gamma}Z_t^{*\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{Z_t^{1-\gamma}Z_t^{*\gamma}}}{\frac{[Z_t^{*\omega-1}+(Z_t^{\gamma}Z_t^{*1-\gamma})^{\omega-1}]^{\frac{1}{\varphi+\omega}}}{Z_t^{\gamma}Z_t^{*1-\gamma}}}$$

When $\gamma = 0$, the expression reduces to the GLR expression $TOT_t = (\frac{C_t}{C_t^*})^{\frac{\varphi}{\sigma(\varphi+\omega)}}(\frac{Z_t}{Z_t^*})^{\frac{-(1+\varphi)}{\varphi+\omega}}$. When additionally $\varphi = 0$ and $\omega = 1$, $TOT_t = \frac{Z_t^*}{Z_t}$ which means the terms of trade move one-for-one with productivity differential. As GLR explain, this is the main mechanism for Cole and Obstfeld (1991) result that states the terms of trade adjust directly proportionately to relative productivity.

When $\gamma = 1$, the expression reduces to $TOT_t = (\frac{C_t}{C_t^*})^{\frac{\varphi}{\sigma(\varphi+\omega)}}\frac{Z_t}{Z_t^*}$. When additionally $\varphi = 0$, $TOT_t = \frac{Z_t}{Z_t^*}$, i.e., the terms of trade again move one-for-one with the productivity differential. This result holds regardless of the value of ω.

Note that these are "shadow TOT" because even $\gamma = 0$ is a "no trade" scenario since this model does not feature any trade in goods.