Essays in International Macroeconomics

Author: Mikhail Dmitriev

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Essays in International Macroeconomics

Mikhail Dmitriev

Advised by Professor Susanto Basu (Co-Chair), Professor Fabio Ghironi (Co-Chair), Professor Peter Ireland, Professor Fabio Schiantarelli and Assistant Professor Ryan Chahrour

Abstract

My dissertation develops a set of tools for thinking about heterogeneity in economic models in an analytically tractable way. Many models use the representative agent framework, which greatly simplifies macroeconomic aggregation but abstracts from the heterogeneity we see in the real world. Models with heterogeneity in general equilibrium have too many moving parts, so that it is hard to disentangle cause and effect. First, my work in international macroeconomics incorporates heterogeneity via idiosyncratic shocks across countries in a simple and analytical way. Second, my work on financial frictions helps to understand the role of asymmetric information between lenders and borrowers in different contractual environments. Crucially, these insights can be incorporated into the models currently used by academics and central banks for policy analysis.

The first chapter of my dissertation, "Price Stability in Small Open Economies," joint work with Jonathan Hoddenbagh, studies the conduct of optimal monetary policy in a continuum of small open economies. We obtain a novel closed-form solution that does not restrict the elasticity of substitution between home and foreign goods to one. Using this global closed-form solution, we give an exact characterization of optimal monetary policy and welfare with and without international policy cooperation. We consider the cases of internationally complete asset markets and financial autarky, producer currency pricing and local currency pricing. Under producer currency pricing, it is always optimal to mimic the flexible-price equilibrium through a policy of price stability. Under local currency pricing, policy should fix the exchange rate. Even though countries have monopoly power, the continuum of small open economies implies that policymakers cannot affect world income. This inability to influence world income removes the incentive to deviate from price stability under producer currency pricing or a fixed exchange rate under local currency pricing, and prevents gains from international monetary cooperation in all cases examined. Our results contrast with those for large open economies, where interactions between home policy and world income drive optimal policy away from price stability or fixed exchange rates, and gains from cooperation are present.

The second chapter of my dissertation, "The Optimal Design of a Fiscal Union", joint work with Jonathan Hoddenbagh, examines the role of fiscal policy cooperation and financial market integration in an open economy setting, motivated by the recent crisis in the euro area. I show that the optimal design of a fiscal union is governed by the degree of substitutability between the export goods of different countries. When countries produce goods that are imperfect substitutes they should harmonize their income taxes to prevent large terms of trade externalities. On the other hand, when countries produce goods that are close substitutes, they should organize a contingent fiscal transfer scheme to insure against idiosyncratic shocks. The welfare gains from the optimal fiscal union are as high as 5% of permanent consumption when countries are able to trade safe government bonds, and approach 20% of permanent consumption when countries lose access to international financial markets. These gains are especially large for countries like Greece that produce highly substitutable export goods and who cannot raise funds on international financial markets to insure against downside risk. The results illustrate why federal currency unions such as the U.S., Canada and Australia, with income tax harmonization and built-in fiscal transfer arrangements, withstand asymmetric shocks across regions much better than the euro area, which lacks these ingredients at the moment.

The third chapter of my dissertation, joint work with Jonathan Hoddenbagh, studies macro-financial linkages and the impact of financial frictions on real economic activity in some of my other work. Beginning with the Bernanke-Gertler-Gilchrist (1999) financial accelerator model, a large literature has shown that financial frictions amplify business cycles. Using this framework, Christiano, Motto and Rostagno (AER, 2013) show that shocks to financial frictions can explain business cycle fluctuations quite well. However, this literature relies on two ad hoc assumptions. When these assumptions are relaxed and agents have access to a broader set of lending contracts, the financial accelerator disappears, and shocks to financial frictions have little to no impact on the economy. In addition, under the ad hoc lending contract inflation targeting eliminates the financial accelerator. These results provide guidance for monetary policymakers and present a puzzle for macroeconomic theory.

Dedication

To Fabio Ghironi and Susanto Basu,

who nurtured my love for what I am doing.

We have not succeeded in answering all our problems. The answers we have found only serve to raise a whole set of new questions. In some ways we feel we are as confused as ever, but we believe we are confused on a higher level and about more important things.

Posted outside the mathematics reading room, Tromsø University

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Chapter 1

Price Stability in Small Open Economies

1.1 Introduction

Price stability is widely viewed as a benchmark monetary policy for central banks, and has been found to be optimal in many closed economy studies.[1] However, there is a longstanding debate over the desirability of price stability in the open economy. Asymmetric shocks, current account imbalances, cross-border pricing decisions, terms of trade movements, policy cooperation and other elements unique to the open economy introduce complications that are not present in the closed economy. These additional ingredients present a challenge for understanding optimal policy in the open economy.

[1]In this paper, we refer to price stability as the policy which implements the flexible price allocation. A non-exhaustive list of papers finding price stability to be optimal in closed economies includes Goodfriend and King (1997, 2001), King and Wolman (1998), and Woodford (2000, 2002). See Schmitt-Grohé and Uribe (2011) for a recent survey. A counterexample is provided by multi-sector models such as that in Aoki (2001).

In the classic case of producer currency pricing (PCP), exchange rate fluctuations pass-through into imported goods prices one-for-one. Under PCP, the first generation of micro-founded models proved that a central bank should mimic the flexible price allocation, emphasizing the isomorphism between optimal monetary policy in closed and open economies (Clarida, Gali and Gertler (2002) and Obstfeld and Rogoff (2002)). But the optimality of price stability hinged on two crucial assumptions: unitary elasticity of substitution between home and foreign goods, and PCP. Unitary elasticity implies complete risk-sharing across countries via terms-of-trade movements, so that households face no idiosyncratic consumption risk in complete markets, incomplete markets or financial autarky (Cole and Obstfeld (1991)). Counterfactually, under unitary elasticity export revenues are constant and immune to exchange rate fluctuations and productivity shocks. No expenditure switching occurs, so that household consumption does not shift to goods from "cheaper" countries (countries with a depreciated currency for example).

Benigno and Benigno (2003) relaxed this assumption, and showed that price stability is no longer optimal for PCP under non-unitary elasticity. Their work affirmed a central bank's incentive to manipulate its terms of trade, removing the isomorphism between optimal monetary policy in closed and open economies.[2] In addition, a number of studies have

[2]Benigno and Benigno (2003) hearkened back to the earlier predictions of non-microfounded models, which established a deflationary bias in the open economy due to a central bank's incentive to appreciate its terms of trade (Rogoff (1985), Oudiz and Sachs (1984), and Canzoneri and Henderson (1990)). Benigno and Benigno (2006) later showed that price stability is optimal for large open economies under non-unitary elasticity when central banks cooperate and markets are complete. Corsetti, Dedola and Leduc (2010) also examine optimal monetary policy under financial autarky, and show that price stability is never optimal in financial autarky for large open economies. This contrasts with our results for small open economies, where price stability is optimal under PCP in all cases, even in financial autarky.

shown that under local currency pricing (LCP), when exchange rate fluctuations do not pass-through into imported goods prices, price stability is not optimal (Devereux and Engel (2003), Corsetti and Pesenti (2005), Corsetti, Dedola and Leduc (2010)).[3]

In this paper, we study the theoretical conditions under which price stability is optimal in a continuum of *small* open economies. We solve the model globally in closed-form without restricting the elasticity of substitution between home and foreign goods to one. To the best of our knowledge, we derive the first closed-form solution for an open economy model that allows substitutability to differ from one, enabling us to consider cases away from perfect risk-sharing, including financial autarky. As in Benigno and Benigno (2003, 2006), Corsetti and Pesenti (2001, 2005), Devereux and Engel (2003), Sutherland (2004) and others, we assume one-period in advance price setting and no home bias.

While previous studies of optimal monetary policy in small open economies focus on PCP with non-cooperative policymakers, we give an exact characterization of optimal monetary policy and welfare with and without international policy cooperation for PCP and LCP, in complete markets and financial autarky.[4] The continuum framework allows us to examine

[3]Devereux and Engel (2003) study optimal monetary policy for two large open economies under PCP and LCP in a complete markets model with unitary elasticity and find that a fixed exchange rate is the optimal policy under LCP. Corsetti, Dedola and Leduc (2010) examine LCP under non-unitary elasticity in complete markets and financial autarky for cooperative policymakers, and show that a fixed exchange rate is not optimal. We do the same exercise for small open economies and find that a fixed exchange rate is optimal for LCP in all cases examined, for any elasticity.

[4]A non-exhaustive list of studies of optimal monetary policy in small open economies includes Gali and Monacelli (2005), Faia and Monacelli (2008), De Paoli (2009a) and De Paoli (2009b), Catao and Chang (2012), Hevia and Nicolini (2012) and Monacelli (2012). Generally, these papers allow for a richer specification and show various results depending on the emphasis and specific ingredients. All focus on PCP and do not examine potential gains from cooperation. Modeling LCP in a continuum of small open economies is quite difficult because the law of one price and purchasing power parity no longer hold. With two economies, it is not hard to keep track of exchange rate policy, but in the continuum this proves more challenging.

the implications of international policy cooperation and solve for Nash equilibria, which is not possible in the standard small open economy setup where there is only one measure zero small open economy and one "rest of the world" block. Combined with our closed-form solution, we obtain exact welfare gains resulting from cooperation amongst a continuum of small, interdependent economies.[5]

Our results point to the importance of country size in the conduct of optimal monetary policy. We prove that for small open economies implementing the flexible price allocation through a policy of price stability is optimal under PCP, while a fixed exchange rate is optimal under LCP. Both results hold for any elasticity of substitution between home and foreign goods, in complete markets and financial autarky, with or without international policy cooperation. We also show that international monetary cooperation does not improve welfare for small open economies. Under central bank commitment, the cooperative and non-cooperative solutions exactly coincide under any combination of PCP and LCP, complete markets and financial autarky for any elasticity of substitution.[6]

Even though countries have monopoly power and can manipulate their terms of trade

[5]To the best of our knowledge, we are the first to examine cooperation within the continuum framework. Our analysis does not face the problems encountered when conducting such an exercise in a log-linear model. In a log-linear model, the steady state will be different in cooperative and non-cooperative equilibria. As such, it is not clear how to make an accurate comparison of welfare between cooperative and non-cooperative regimes. This is one reason why there was such an emphasis on closed-form solutions in the early micro-founded literature on international policy cooperation. See Corsetti and Pesenti (2001, 2005) and Obstfeld and Rogoff (2001, 2002). In this paper we focus only on monetary policy cooperation, but in a related paper we consider optimal fiscal policy and fiscal cooperation across countries (Dmitriev and Hoddenbagh (2013)).

[6]This matches results for large economies under unitary elasticity (Obstfeld and Rogoff (2002)), where gains from cooperation are absent, but contrasts with results for large open economies under non-unitary elasticity (Benigno and Benigno (2006), Corsetti *et al* (2010) and Sutherland (2004)), where gains are present.

in our model, the continuum of small open economies implies that policymakers cannot affect world income. This inability to influence world income removes the incentive to deviate from price stability under producer currency pricing or a fixed exchange rate under local currency pricing, and prevents gains from international monetary cooperation in all cases examined.[7] Small open economies are analogous to firms in a model of monopolistic competition: policymakers have market power but cannot influence the behavior of other countries in the model.

Policymakers' inability to influence world income has very different implications under PCP and LCP. Under PCP, the policy decisions of small open economies have no effect on consumption baskets in foreign countries, which shuts down the expenditure switching channel abroad. Unable to induce expenditure switching or influence foreign income, policymakers charge a constant terms of trade markup, utilizing monopoly power at the country level. Monopolistic producers in our model also charge a constant markup utilizing their monopoly power at the firm level within each country, but policymakers remove this domestic markup via production subsidies. Since monetary policy can only affect the variance and not the level of these constant markups, the optimal solution for the policymaker under PCP is to mimic the flexible price allocation.

[7] In our model there is monopoly power at the firm and the country level. Both fiscal and monetary policymakers can utilize country-level monopoly power to manipulate their terms of trade. But because economies are small and cannot affect world income or induce expenditure switching abroad, central banks will choose not to utilize this monopoly power as they find it optimal to replicate the flexible-price equilibrium. On the other hand, optimizing fiscal authorities will choose to apply a constant terms of trade markup on exports. We focus on the role of fiscal policy in another paper (Dmitriev and Hoddenbagh (2013)). To be clear: in our model central banks are able to manipulate the terms of trade but will choose not to, whereas fiscal authorities will charge a constant terms of trade markup equal to $\frac{\gamma-1}{\gamma}$.

In contrast, when economies are large, policymakers must internalize the effect of their decisions on the rest of the world. When domestic policymakers charge higher markups, foreign households become poorer and shift consumption toward cheaper products. Under PCP, expenditure switching abroad forces policymakers in large economies to behave more cooperatively and charge lower markups. Since the relative size of an economy changes stochastically with productivity shocks, the domestic policymaker in a large open economy finds it optimal to deviate from flexible prices and charge a variable markup rather than a constant one.

Under LCP, prices are fixed one period in advance in the currency of the importer, so the proportion of income allocated to country-specific goods is insulated from monetary policy and exogenous shocks. As a result, a constant proportion of world income is spent on each country's unique variety, and expenditure switching does not occur. In addition, small open economies cannot affect world income, so domestic output and labor are unaffected by monetary policy. The central bank thus maximizes welfare by stabilizing domestic consumption via a fixed exchange rate. With a fixed exchange rate, labor fluctuates with productivity shocks while consumption remains constant.

On the other hand, monetary policy in large open economies can influence domestic labor under LCP. Expansionary monetary policy at home will increase wages, while goods prices remain fixed. This induces home and foreign households to work more, since domestic and foreign firms face higher demand for their goods. Higher real wages and an increase in hours worked lead to higher domestic consumption. Policymakers in large open economies thus face a trade-off between labor and consumption volatility. In such cases, a fixed exchange rate is no longer optimal.[8]

Is the Optimality of Price Stability Robust to Alternative Assumptions on Price-Setting?

Up to this point we have assumed that prices are set one period in advance. If we change this assumption and use quadratic costs of adjustment on prices or Calvo pricing, is price stability still the optimal policy for central banks under PCP?

Optimal monetary policy should always set a fixed distance between the desired markup and the actual markup, and this distance should be as small as possible. If the desired markup is fixed, then there is no role for cyclical monetary policy, regardless of all other conditions. If there are adjustment costs for prices, then the central bank will use them to minimize the average distance between the actual and desired markup.

For small open economies without home bias, the desired markup is fixed over the business cycle because all production is exported: with a CES demand structure, a constant terms of trade markup is optimal. Under quadratic adjustment costs for prices, the central bank will use steady-state deflation to tilt the terms of trade in its favor and impose a constant terms of trade markup on its export goods. A policy of constant steady-state deflation will thus be the optimal monetary policy. One can view this as optimal "deflation targeting". In this case, monetary policy cooperation will lead to gains in the steady-state, as the

[8] Unless labor enters the utility function as a linear term, as in Devereux and Engel (2003), in which case a fixed exchange rate is optimal. In our model, labor enters the utility function as a non-linear term but we still find that a fixed exchange rate is optimal under LCP.

distortionary impact of the terms of trade markup imposed through steady-state deflation will be removed when central banks internalize the impact of their monetary policy on other countries. Note however that there is no incentive for the policymaker to deviate from the policy of constant steady-state deflation over the business cycle, because the desired markup is fixed. As a result, there are no gains from cooperation over the business cycle.

Under Calvo pricing on the other hand, steady-state inflation or deflation will have no impact on the actual markup because agents in the economy anticipate the central bank's policy under commitment and prices are set in a forward looking manner. Unlike with quadratic adjustment costs, central banks cannot affect the markup in the steady-state because of the forward looking nature of the price setting assumption. As a result, under Calvo pricing and one period in advance pricing central banks are unable to tilt the terms of trade in their favor through monetary policy. Mimicking the flexible price equilibrium through a policy of price stability is thus optimal under Calvo pricing as well as one period in advance pricing, and there are no gains from cooperation in the steady-state or over the business cycle.

Economy Size and the Continuum

Table 1.1: Economy Size Measured By Contribution to World Trade

Country	**Exports***	**Imports***
EU†	11.99	13.02
China	10.68	9.69
US	8.50	12.86
Japan	4.50	4.42
South Korea	3.13	2.92
Russia	2.93	1.38
UK	2.79	3.64
Canada	2.53	2.61

*Exports and imports as a % of world exports and imports (Source: WTO, 2011).
† Excludes internal trade.

Although the continuum enables very sharp predictions, is it a realistic assumption? The data show that it is. For the purposes of this paper, we proxy for an economy's size by calculating its contribution to world trade – a measure of both size *and* openness. Table 1 shows the top eight trading nations measured by exports and imports to the rest of the world as a percentage of total world exports and imports, respectively.[9] Using this metric, the continuum is a reasonable assumption. The largest economy in the world, the U.S., accounts for only about ten percent of world trade, which demonstrates the importance of openness when evaluating relative country size in the world economy. A country with a

[9] EU trade data excludes intra-EU trade because we are trying to capture the influence of a central bank (in this case the ECB) on the global economy. Thus, any trade *within* a currency union is excluded from our calculation. This is why we look at the EU as a whole, rather than considering Germany, France, Italy, etc. as separate countries. From the perspective of evaluating optimal monetary policy, these countries share the same central bank. We recognize that the euro area is smaller than the EU, but we could not find adequate data at the euro area level that disaggregated internal and external trade, so we were forced to consider the EU as a whole. In this vein, we naturally exclude intra-U.S. trade in our calculation.

large GDP may make up a small percentage of world trade if it is relatively closed to the rest of the world.

In line with this empirical evidence, Gali and Monacelli (2005, 2008) introduced a model that considered a continuum of small open economies. They solved for optimal policy in the aforementioned Cole-Obstfeld case of log utility and unitary elasticity. In their own words:

> Domestic price stability (along with fully flexible exchange rates) stands out as the welfare maximizing policy in the particular case of log utility and unitary elasticity of substitution ... The derivation of the relevant welfare function for the small open economy in the case of more general preferences as well as that of uncorrected steady state distortions would allow a more thorough analysis and quantitative evaluation of the optimal monetary policy and should certainly be the object of future research.

Our paper answers this call. We evaluate optimal policy away from the Cole-Obstfeld case, considering both incomplete risk-sharing and a general preference specification, without having to approximate a welfare function around a steady state (whether distorted or otherwise). This enables us to consider cases with uncorrected steady state distortions, such as a constant terms of trade markup, as well as international monetary cooperation and non-cooperative Nash equilibria. All of these possibilities demonstrate the power of the global closed-form solution.

The paper proceeds as follows. We begin by laying out the model for PCP in Section 2. In Section 3, we examine optimal monetary policy in the closed economy to develop intuition, and then move to the open economy where we solve for optimal monetary policy under PCP, first in complete markets and then in financial autarky. Finally, we lay out the model for LCP in Section 4, and solve for optimal monetary policy in complete markets and financial autarky. Section 5 concludes.

1.2 The Model

We consider a continuum of small open economies represented by the unit interval, as popularized in the literature by Gali and Monacelli (2005, 2008).[10] Each economy consists of a representative household and a representative firm. All countries are identical ex-ante: they have the same preferences, technology, and price-setting. Ex-post, economies will differ depending on the realization of their technology shock. Households are immobile across countries, however goods can move freely across borders. Each economy produces one final good, over which it exercises a degree of monopoly power. This is crucially important: countries are able to manipulate their terms of trade even though they are measure zero. However, because countries are small they will be unable to influence world income and induce expenditure switching to their goods. Policymakers will thus charge a constant terms of trade markup on their exports.

We use one-period-in-advance price setting to introduce nominal rigidities.[11] Monopolistic firms set next-period's nominal prices, in terms of domestic currency, prior to next-

[10] A similar version of this model appears in Dmitriev and Hoddenbagh (2013) where we employ wage rigidity instead of price rigidity and study the optimal design of a fiscal union within a currency union.

[11] Assuming rigid wages or prices has no impact on the results: they yield identical policy implications.

period's production and consumption decisions. These firms will charge a constant markup in the flex-price equilibrium, utilizing their monopoly power at the firm level. Given this preset price, firms supply as much output as demanded by households.

We lay out a general framework below, and then hone in on two specific cases: complete markets and financial autarky. To avoid additional notation, we ignore time subindices unless absolutely necessary. When time subindices are absent, we are implicitly referring to period t.

Households In each economy $i \in [0, 1]$, there is a representative household with lifetime expected utility

$$\mathbb{E}_{t-1}\left\{\sum_{k=0}^{\infty}\beta^k\left(\frac{C_{it+k}^{1-\sigma}}{1-\sigma}-\chi\frac{N_{it+k}^{1+\varphi}}{1+\varphi}\right)\right\} \tag{1.1}$$

where $\beta < 1$ is the household discount factor, C is the consumption basket or index, and N is household labor effort (think of this as hours worked). Households face a general budget constraint that nests both complete markets and financial autarky; we will discuss the differences between the two in subsequent sections. For now, it is sufficient to simply write out the most general form of the budget constraint:

$$C_{it} = (1-\tau_i)\left(\frac{W_{it}}{P_{it}}\right)N_{it} + \mathcal{D}_{it} + \mathcal{T}_{it}. \tag{1.2}$$

The distortionary tax rate on household labor income in country i is denoted by τ_i, while $\mathcal{T}_{it}$ is a lump-sum tax rebate to households. Net taxes equal zero in the model, as any amount of government revenue is rebated lump-sum to households. The consumer price index corresponds to P_{it}, while the nominal wage is W_{it}. $\mathcal{D}_{it}$ denotes state-contingent portfolio payments expressed in real consumption units, and can be written in more detail as:

$$\mathcal{D}_{it}P_{it} = \int_0^1 \mathcal{E}_{ijt}B_{ijt}dj, \tag{1.3}$$

where B_{ijt} is a state-contingent payment in currency j.[12] $\mathcal{E}_{ijt}$ is the exchange rate in units of currency i per one unit of currency j; an increase in $\mathcal{E}_{ijt}$ signals a depreciation of currency i relative to currency j. When international asset markets are complete, households perform all cross-border trades in contingent claims in period 0, insuring against all possible states in all future periods. The transverality condition simply states that all period 0 transactions must be balanced: payment for claims issued must equal payment for claims received. Leaving the details in the appendix, we use the following relationship as the transversality condition for complete markets:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t C_{it}^{-\sigma} \mathcal{D}_{it} \right\} = 0, \tag{1.4}$$

while in financial autarky

$$\mathcal{D}_{it} = 0.$$

[12]Equation (2.6) holds in all possible states in all periods. Details are provided in Appendix A.1.

Intuitively, the transversality condition (2.7) stipulates that the present discounted value of future earnings should be equal to the present discounted value of future consumption flows. Under complete markets, consumers choose a state contingent plan for consumption, labor supply and portfolio holdings in period 0.

Consumption and Price Indices Households in each country consume a basket of imported goods. This consumption basket is an aggregate of all of the varieties produced by different countries. The consumption basket for a representative small open economy i, which is common across countries, is defined as follows:

$$C_i = \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \tag{1.5}$$

where c_{ij} is the consumption by country i of the final good produced by country j, and γ is the elasticity of substitution between domestic and foreign goods (the Armington elasticity). Because there is no home bias in consumption, countries will export all of the output of their unique variety, and import varieties from other countries to assemble the consumption basket.

Prices are defined as follows: upper case P_{ij} denotes the price in country i (in currency i) of the unique final good produced in country j, while CPI_i is the aggregate consumer price index in country i. Given the above consumption index, the consumer price index will be:

$$CPI_i = \left(\int_0^1 P_{ij}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}. \tag{1.6}$$

Consumption by country i of the unique variety produced by country j is:

$$c_{ij} = \left(\frac{P_{ij}}{CPI_i}\right)^{-\gamma} C_i. \tag{1.7}$$

We assume that producer currency pricing (PCP) holds, and that the law of one price (LOP) holds, so that the price of the same good is equal across countries when converted into a common currency. We define the nominal bilateral exchange rate between countries i and j, $\mathcal{E}_{ij}$, as units of currency i per one unit of currency j. LOP requires that:

$$P_{ij} = \mathcal{E}_{ij} P_{jj}. \tag{1.8}$$

Given LOP and identical preferences across countries, PPP will also hold for all i, j country pairs:

$$CPI_i = \mathcal{E}_{ij} CPI_j, \tag{1.9}$$

The terms of trade for country j will be:

$$TOT_j = \frac{P_{\text{Exports from } j}}{P_{\text{Imports to } j}} = \frac{P_{jj}}{CPI_j}, \tag{1.10}$$

where TOT_j is defined as the home currency price of exports over the home currency price of imports. Since the domestic consumption basket is entirely composed of foreign inputs, the domestic CPI will be equal to the home currency price of imports. Now we can take

(2.10), and using (2.11) and (2.12), solve for demand for country j's unique variety:

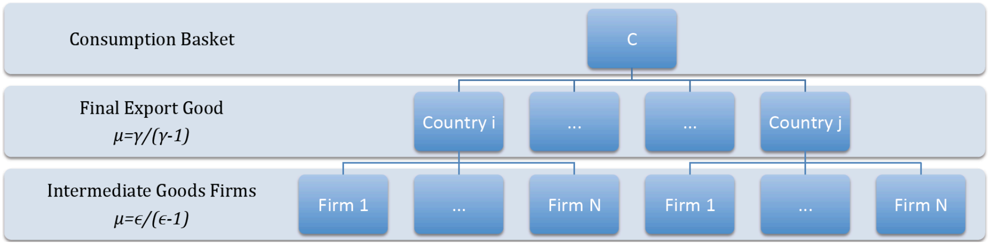
$$Y_j = \int_0^1 c_{ij} di = \int_0^1 \left(\frac{P_{ij}}{CPI_i}\right)^{-\gamma} C_i di \overset{(2.11)+(2.12)}{=} \left(\frac{P_{jj}}{CPI_j}\right)^{-\gamma} \int_0^1 C_i di = TOT_j^{-\gamma} C_w. \tag{1.11}$$

where C_w is defined as the average world consumption across all i economies, $C_w = \int_0^1 C_i di$.

Production Each economy i consists of a group of intermediate goods producers, $h \in [0, 1]$, who exercise monopoly power over their unique variety, and a perfectly competitive final goods exporter who aggregates the intermediates in CES fashion into a final export good. For simplicity, we assume that intermediates are non-tradable. Thus, each country will bundle its intermediates into one final export good.[13] Figure 1 below illustrates the bundling of intermediates into a final export good, and the bundling of those exports into the household consumption basket. We differentiate between the markup on intermediate domestic goods ($\mu = \frac{\varepsilon}{\varepsilon - 1}$) and the markup on the final export good ($\mu_\gamma = \frac{\gamma}{\gamma - 1}$), which is imposed only if policymakers decide to manipulate their terms of trade.

[13]We assume non-tradable intermediates with a final tradable consumption good that aggregates those intermediates for simplicity. In Gali and Monacelli's (2005, 2008) setup, intermediate goods are tradable, such that every country's import consumption basket is made up of an infinite number of varieties imported from an infinite number of countries. This requires integrating over two continuums. While it is straightforward for us to maintain their setup, we prefer the tractable alternative: a final goods exporter bundles the domestically produced intermediates for export. In this way, each country produces only one unique variety, and we only need integrate over one continuum. This assumption does not change the results in any way. In both cases the household consumption basket in each country is made up of imported goods from all i countries, which are themselves made up of intermediates produced domestically.

Figure 1



Production of intermediates requires technology Z_i, which is common across firms within a country, and labor $N_i(h)$, which is unique to each firm. We do not need to assume a specific distribution for technology, a luxury afforded by our closed-form solution. We do assume that technology is independent across time and across countries, but is identical across firms within the same country. Given this, the production function of a representative intermediate goods firm h in country i will be:

$$y_i(h) = Z_i n_i(h). \tag{1.12}$$

Because intermediate goods firms produce differentiated varities, they have monopoly power over their output, which leads to a markup in intermediate goods. Perfectly competitive final goods exporters aggregate the intermediate input of each firm, so that production of the representative final exporter in a specific country is:

$$Y_i = \left[\int_0^1 y_i(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1.13}$$

where ε is the elasticity of substitution between different intermediates, and $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the markup. The price of the final good in country i, P_{ii}, will be a function of the nominal price for intermediate goods, $p_i(h)$:

$$P_{ii} = \left[\int_0^1 p_i(h)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}}.$$

Cost minimization by the perfectly competitive final goods exporter leads to the following demand for intermediates:

$$y_i(h) = \left[\frac{p_i(h)}{P_{ii}}\right]^{-\varepsilon} Y_i. \tag{1.14}$$

In summary, monopoly power may be exercised at the firm and the country level: at the firm level because of the production of differentiated domestic varieties, and at the country level because each economy produces a unique variety for export. Households and final goods exporters have no monopoly power and are perfectly competitive.[14]

Intermediate goods firms will price their unique good one-period-in-advance according to the following condition, which results from profit maximization:

$$p_{it}(h) = \mu \frac{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma} y_{it}(h) \frac{W_{it}}{Z_{it} CPI_i}\right\}}{\mathbb{E}_{t-1}\left\{\frac{C_{it}^{-\sigma} y_{it}(h)}{CPI_{it}}\right\}}. \tag{1.15}$$

[14] It is entirely plausible to shift the country-level monopoly power from the policymaker to the final goods exporter. All results will be exactly the same: terms of trade markups will be constant — just as in a model of monopolistic competition. As such, price stability will remain optimal under PCP.

Households maximize utility (2.4) subject to their budget constraint (2.5). The FOC with respect to labor will give the following household labor supply condition:

$$\frac{W_{it}}{CPI_{it}} = \left(\frac{\chi}{1-\tau_i}\right) N_{it}^{\varphi} C_{it}^{\sigma}. \tag{1.16}$$

Firms are identical, so that in equilibrium $p_i(h) = P_{ii}$ and $y_i(h) = Z_i n_i(h) = Z_i N_i = Y_i$. Using the labor demand condition (2.16) and the labor supply condition (2.15), and the fact that prices are preset at time $t-1$, the labor market clearing condition will be:

$$1 = \left(\frac{\chi\mu}{1-\tau_i}\right) \frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma} Y_{it} TOT_i\right\}}. \tag{1.17}$$

This is the general labor market clearing condition; it holds in the closed economy and in the open economy for PCP. Under PCP, the demand for the unique variety (2.14) will give the following labor market clearing condition:

$$1 = \left(\frac{\chi\mu}{1-\tau_i}\right) \frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}\right\}}. \tag{1.18}$$

Taking the expectations operator out of (2.18) will give the flexible price equilibrium for PCP.

We now turn our attention to the difference between complete markets and financial autarky.

1.2.1 Complete Markets

In this section, we assume that agents in each economy trade a full set of domestic and foreign state-contingent assets. Households in all countries will maximize their utility (2.4), choosing consumption, leisure, money holdings, and a complete set of state-contingent nominal bonds, subject to (2.5).

Risk-Sharing Complete markets and PPP imply the following risk-sharing condition:

$$\frac{C_{it}^{-\sigma}}{C_{it+1}^{-\sigma}} = \frac{C_{jt}^{-\sigma}}{C_{jt+1}^{-\sigma}} \quad \forall i, j \tag{1.19}$$

which states that the ratio of the marginal utility of consumption at time t and $t+1$ must be equal across all countries. Importantly, this condition does not imply that consumption is equal across countries. Consumption in country i will depend on its initial asset position, monetary policy, the distribution of country-specific shocks, the covariance of global and local shocks, and other factors.

When the central bank in economy i changes its policy, the consumption allocation in country i may change as well. For example, monetary policy can affect the covariance between home production and world consumption, and this covariance will influence the level of household consumption even under complete markets. The risk-sharing condition (2.19) and the transversality condition (2.7) are both robust to changes in monetary policy. If we combine the two, the resulting goods market clearing condition will also be robust to changes in monetary policy. When (2.7),(2.17), and (2.19) hold, consumption in country i can be expressed as a function of world consumption:

$$C_{it} = \frac{\mathbb{E}_{t-1}\left\{\sum_{t=1}^{\infty}\beta^t\left[Y_{it}C_{wt}^{-\sigma}TOT_{it}\right]\right\}}{\mathbb{E}_{t-1}\left\{\sum_{t=1}^{\infty}\beta^t C_{wt}^{1-\sigma}\right\}}C_{wt}. \tag{1.20}$$

We solve for this expression explicitly in Appendix A.1.

Using the fact that Z_{it} is independent across time and and across countries, and prices are preset, (2.20) is equivalent to

$$C_{it} = \mathbb{E}_{t-1}\left\{Y_{it}TOT_{it}\right\} = C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}. \tag{1.21}$$

1.2.2 Financial Autarky

The aggregate resource constraint under financial autarky specifies that the nominal value of output in the home country (exports) must equal the nominal of consumption in the home country (imports). That is, trade in goods must be balanced. In a model with cross-border lending, bonds would also show up in this condition, but in financial autarky, they are obviously absent. The primary departure from complete markets lies in the household and economy-wide budget constraints.

$$\underbrace{P_{ii}\cdot Y_i}_{\text{Exports}} = \underbrace{CPI_i\cdot C_i}_{\text{Imports}} \tag{1.22}$$

Using the fact that (2.14) holds under both complete markets and financial autarky, and substituting this into (2.22), one can show that demand for country i's good in financial autarky will be

$$C_{it} = C_{wt}^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}}. \tag{1.23}$$

Complete markets and autarky differ only by goods market clearing. In complete markets consumption is equal to expected domestic output expressed in consumption baskets; in autarky consumption is equal to realized domestic output expressed in consumption baskets.

1.3 Optimal Monetary Policy

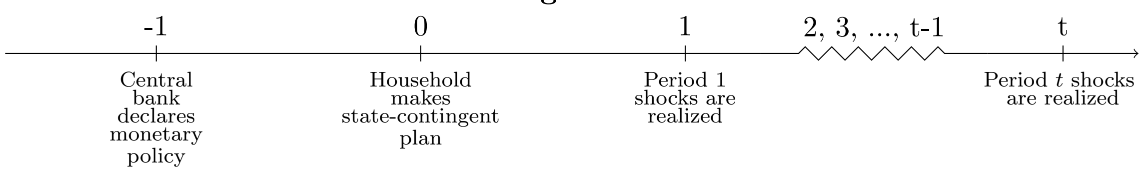
Now that we've laid out the model in detail for both complete markets and financial autarky, we consider optimal policy for a variety of scenarios. Without loss of generality, we assume a cashless limiting economy. Central banks will optimize by choosing labor instead of money supply or an interest rate rule, but all three are equivalent in this model: money supply will determine the interest rate, which will in turn determine labor. We prove this in Appendix F.[15] One can easily write down a money supply rule or interest rate rule that exactly implements the allocation resulting from optimization over labor. But for the sake of simplicity, we assume the central bank optimizes over labor.

The timing of the model is described in Figure 2 below. Before any shocks are realized, national central banks declare their policy for all states of the world. With this knowledge

[15] Benigno and Benigno (2003) also describe a cashless-limiting economy in detail in their appendix, pp.756-758.

in hand, households lay out a state-contingent plan for consumption, labor, money and asset holdings. After that, shocks hit the economy. Note that under financial autarky, no international asset trading will occur.

Figure 2



We begin with an analysis of optimal monetary policy in a closed economy version of our model, and then proceed to the open economy. In all cases, we consider optimal policy under commitment.

1.3.1 Closed Economy

The Flexible Price Allocation

To solve for the flexible price allocation in the closed economy, simply take expectations out of (2.17), set $TOT_i = 1$, and use goods market clearing ($Y_t = C_t = N_t Z_t$). This will give us a system of two equations in two unknowns, N_t and C_t:

$$1 = \left(\frac{\chi\mu}{1-\tau}\right)\frac{N_t^{1+\varphi}}{C_t^{1-\sigma}},$$

$$C_t = N_t Z_t.$$

The solution to this two equation system is the flexible price allocation for the closed economy: $C_t = \left(\frac{1-\tau}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}$.

Optimal Monetary Policy

The central bank will choose labor to maximize the expected utility of the representative agent, given the closed economy labor market and goods market clearing constraints.

$$\max_{N_t} \mathbb{E}_{t-1}\left\{\frac{C_t^{1-\sigma}}{1-\sigma} - \chi\frac{N_t^{1+\varphi}}{(1+\varphi)}\right\} \tag{1.24}$$

s.t.

$$1 = \left(\frac{\chi\mu}{1-\tau}\right)\frac{\mathbb{E}_{t-1}\left\{N_t^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_t^{1-\sigma}\right\}} \tag{1.25}$$

$$C_t = Z_t N_t \tag{1.26}$$

Although it is standard practice to use a welfare-loss function for optimal policy evaluation, we can simply use the household utility function because of our global, closed-form solution.[16]

Proposition 1 *In the closed economy under ex ante commitment, the central bank will maximize (1.24) subject to (1.25) and (1.26). The solution to this problem is:* $C_t = \left(\frac{1-\tau}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}$. *The central bank replicates the flexible price allocation via a policy of*

[16] The only reason to use a welfare-loss function is if the model in question must be approximated around a steady state. Here, no such approximation is required, and thus a welfare-loss function is not needed.

price stability.

Proof See Appendix B. ■

Optimal Policy Under A Social Planner

Proposition 2 *In the closed economy, the social planner will maximize (1.24) subject to (1.26), ignoring the labor condition (1.25). The solution to this problem is:* $C_t = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}$.

Proof See Appendix B. ■

In comparing the solutions described in Proposition 1 and 2, notice that the social planner mimics the flexible price allocation while eliminating the monopolistic markup μ (Proposition 2), while the markup and labor tax remain when we only consider optimal monetary policy (Proposition 1). In the case of ex ante commitment in Proposition 1, a fiscal authority may introduce subsidies to exactly offset the markup and replicate the social planner equilibrium. It is straightforwad to show that $\tau = 1 - \mu$ will get rid of the monopolistic distortion on labor inputs and give the Pareto efficient allocation.

We've studied optimal policy in the closed economy, and proved the optimality of price stability. The divine coincidence holds, a well known result in the closed economy. One already sees the link between stable monopolistic markups and a desire to mimic the flexible price allocation. We now turn our attention to the open economy, where we prove that optimal monetary policy in closed and small open economies is isomorphic in both complete markets and financial autarky.

1.3.2 Global Social Planner

The global social planner is a benevolent "dictator" that distributes goods across countries in order to maximize aggregate world utility. This scenario is analogous to perfect cooperation among the social planner's of all i countries. The global social planner solution defines the Pareto efficient allocation. Since the economies in our model are identical ex-ante, the global social planner will maximize a weighted utility function over all i countries,

$$\int_0^1 \left[\frac{C_i^{1-\sigma}}{1-\sigma} - \chi \frac{N_i^{1+\varphi}}{(1+\varphi)} \right] di, \tag{1.27}$$

subject to the consumption basket and the aggregate resource constraint:

$$C_i = \left[\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}, \tag{1.28}$$

$$Y_i = Z_i N_i = \int_0^1 c_{ji} dj. \tag{1.29}$$

Proposition 3 *The global social planner will maximize utility weighted over all i countries*

(2.24), subject to (2.25) and (2.26). The solution to this optimization problem is:

$$
\begin{aligned}
\mathbb{E}\{U_i\} &= C_i^{1-\sigma}\left(\frac{1}{1-\sigma}-\frac{1}{1+\varphi}\right),\\
C_i &= \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}},\\
N_i &= \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}},\\
Y_i &= \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma(1+\varphi)}{1+\gamma\varphi}},\\
Z_w &= \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\,di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}.
\end{aligned}
$$

Proof See Appendix C. ■

The global social planner allocation above is a traditional benchmark for the evaluation of different policy regimes. Because this is the Pareto efficient allocation, it provides a natural marker with which to compare various policies under commitment. Notice that there are no markups in the Pareto efficient allocation: the benevolent global social planner has eliminated the markup on intermediate goods μ, and resisted the temptation to manipulate the terms of trade. In the next sections we will look closely at optimal monetary policy for central banks and see what conditions are necessary to replicate the global social planner allocation.

1.3.3 Complete Markets

In this section we examine the optimal monetary policy for cooperative and non-cooperative central banks in complete markets. The objective functions for cooperative and non-cooperative central banks are below.

$$\max_{N_{it}} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \tag{1.30a}$$

$$\max_{\forall N_{it}} \int_0^1 \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} di \tag{1.30b}$$

Non-cooperative central banks will choose the optimal amount of labor to maximize their domestic welfare (1.30a), while cooperative central banks will choose the optimal amount of labor in order to maximize the welfare of all i economies (1.30b). In both cases, policymakers will maximize the appropriate objective function subject to labor market clearing (2.29a) and goods market clearing (2.29b) constraints, and production (2.29c) and aggregate world consumption (2.29d):

$$1 = \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\right\}}, \tag{1.31}$$

$$C_{it} = C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}, \tag{1.32}$$

$$Y_{it} = Z_{it}N_{it}, \tag{1.33}$$

$$C_{wt} = \left(\int_0^1 Y_{it}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{1.34}$$

Proposition 4 *In complete markets, non-cooperative central banks will maximize (1.30a) and cooperative central banks will maximize (1.30b), subject to (2.29a), (2.29b), (2.29c) and (2.29d). The solution under commitment for both cooperative and non-cooperative central banks in complete markets is:*

$$\mathbb{E}\{U_i\} = C_i^{1-\sigma}\left(\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu(1+\varphi)}\right)$$

$$C_i = \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}}$$

$$N_i = \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}}$$

$$Y_i = \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma(1+\varphi)}{1+\gamma\varphi}}$$

$$Z_w = \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}$$

The resulting equilibrium allocation exactly coincides with the flexible price allocation. Mimicking the flexible price allocation is thus the dominant strategy for non-cooperative central banks in complete markets, and is the optimal policy under cooperation. If the government corrects the distortions due to market power with a non-contingent tax $\tau_i = 1 - \mu$*, then the flexible price allocation in complete markets is identical to the global social planner solution.*

Proof See Appendix D. ■

There are a few important points to note from this exercise. First of all, note that consumption in the optimal allocation is not subject to idiosyncratic technology shocks. Because we are in complete markets, consumption is simply a function of average world technology. Second, note that cooperative and non-cooperative equilibria are identical: both yield the flexible price allocation.

In complete markets, we've shown that small open economy central banks will mimic the flexible price allocation under both cooperative and non-cooperative regimes, for any elasticity of substitution between home and foreign goods. In addition, we've demonstrated that international monetary cooperation has no impact on welfare. This is because monetary policy has no power against non-contingent distortions like markups, and can only address the contingent price rigidity distortion. In the next section, we conduct the same exercise for financial autarky.

1.3.4 Financial Autarky

In financial autarky, the objective functions for cooperative and non-cooperative central banks will be identical to those in complete markets. The only difference in the optimization problem will be in the goods market constraint. In complete markets home consumption is a function of expected output (2.29b), while in autarky home consumption is a function of actual output (2.31).

$$1 = \left(\frac{\chi\mu}{1-\tau_i} \right) \frac{\mathbb{E}_{t-1}\left\{ N_{it}^{1+\eta} \right\}}{\mathbb{E}_{t-1}\left\{ C_{it}^{-\sigma} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{w,t}^{\frac{1}{\gamma}} \right\}} \tag{1.35}$$

$$C_{it} = C_{w,t}^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}} \tag{1.36}$$

$$Y_{it} = Z_{it} N_{it} \tag{1.37}$$

$$C_{wt} = \left(\int_0^1 Y_{it}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}. \tag{1.38}$$

Proposition 5 *In financial autarky, non-cooperative central banks will maximize (1.30a) and cooperative central banks will maximize (1.30b), subject to (1.35), (2.31), (1.37) and (1.38). The solution under commitment for both cooperative and non-cooperative central* *banks in financial autarky is:*

$$
\begin{aligned}
\mathbb{E}\{U_i\} &= C_i \left(\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu(1+\varphi)} \right), \\
C_i &= \left(\frac{1-\tau_i}{\chi\mu} \right)^{\frac{1}{\sigma+\varphi}} \left(Z_i^{\gamma-1} Z_w^{\frac{1+\varphi}{\sigma+\varphi}} \right)^{\frac{1+\varphi}{1-\sigma+\gamma(\varphi+\sigma)}}, \\
N_i &= \left(\frac{1-\tau_i}{\chi\mu} \right)^{\frac{1}{\sigma+\varphi}} \left(Z_i^{\gamma-1} Z_w^{\frac{1+\varphi}{\sigma+\varphi}} \right)^{\frac{1-\sigma}{1-\sigma+\gamma(\varphi+\sigma)}}, \\
Y_i &= \left(\frac{1-\tau_i}{\chi\mu} \right)^{\frac{1}{\sigma+\varphi}} \left(Z_i^{\gamma-1} Z_w^{\frac{1+\varphi}{\sigma+\varphi}} \right)^{\frac{1-\sigma}{1-\sigma+\gamma(\varphi+\sigma)}} Z_i, \\
Z_w &= \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1-\sigma+\gamma(\sigma+\varphi)}} di \right)^{\frac{1-\sigma+\gamma(\sigma+\varphi)}{(\gamma-1)(1+\varphi)}}.
\end{aligned}
$$

The resulting equilibrium allocation exactly coincides with the flexible price allocation. Mimicking the flexible price allocation is thus the dominant strategy for non-cooperative central banks in financial autarky, and is the optimal policy under cooperation.

Proof See Appendix D. ■

Note that in financial autarky, consumption is a function of idiosyncratic technology Z_i, reflecting the lack of international risk-sharing. As in complete markets, the optimal policy for cooperative and non-cooperative central banks is to mimic the flexible price equilibrium. We thus demonstrate the isomorphism between optimal monetary policy in closed and small open economies for both complete markets and financial autarky.

As we've stated before, the key to this isomorphism stems from the inability of small open economies to affect foreign consumption or income. The optimal terms of trade markup on exports chosen by domestic policymakers is thus constant. Combined with monopolistic firms charging a constant markup, the optimal policy response is to remove price rigidities and return to the flexible price allocation. Similarly, when elasticity is unitary and economies are large, as in Clarida, Gali and Gertler (2002) and Obstfeld and Rogoff (2002), countries are unable to influence world income because terms of trade movements no longer lead to expenditure switching. As a result, price stability is optimal.

1.4 Local Currency Pricing

We now turn our attention to the case of LCP. Modeling LCP in a continuum of small open economies is difficult because the law of one price and purchasing power parity no longer hold. In two economy models, keeping track of exchange rate policy is trivial, but in the continuum this becomes challenging. Although we will gloss over many of the methodological nuances necessary to deal with LCP in the continuum, all modeling details can be found in Appendix E. To simplify expressions, we assume log utility ($\sigma = 1$), but allow φ to vary as before.

Under LCP, firms price their export good one-period-in-advance in the currency of the importing country. As such, there is zero exchange rate pass-through into import prices. Each country's consumer price index will thus be fixed one-period-in-advance, as exchange rate movements will have no impact on import prices. When firms price in this way, the labor market and goods market clearing constraints will differ from those under PCP. In complete markets and financial autarky, goods and labor market clearing conditions for

LCP are:

$$1 = \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-1}Y_{it}TOT_{it}\right\}}, \tag{1.39}$$

$$C_{it} = \frac{P_{0it}}{CPI_{it}}\mathcal{E}_{i0t}Y_{it}. \tag{1.40}$$

To keep track of exchange rate policy in the continuum, we introduce the concept of a numeraire currency, which we assume is the currency of country 0. Thus, $\mathcal{E}_{i0t}$ is the exchange rate between country i and the numeraire country, and P_{0it} is the import price of the numeraire country's good in currency i.

Proposition 6 *Under LCP, non-cooperative central banks will maximize (1.30a) subject to (1.39) and (1.40). A fixed exchange rate will be the Nash equilibrium policy for a non-cooperative central bank in both complete markets and financial autarky under LCP.*

Proof See Appendix E. ■

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}\mathcal{E}_{i0t}\right\} = \frac{\chi\mu}{1-\tau_i}\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}\mathbb{E}_{t-1}\left\{C_{jt}\right\}}{\mathbb{E}_{t-1}\left\{\mathcal{E}_{0jt}C_{jt}\right\}}, \tag{1.41}$$

$$C_{it} = \mathcal{E}_{i0t}\int_0^1 C_{jt}\mathcal{E}_{0jt}dj. \tag{1.42}$$

Proposition 7 *Under LCP, cooperative central banks will maximize (1.30b) subject to (1.41) and (1.42). The optimal policy for cooperative central banks in both complete markets and financial autarky will be a fixed exchange rate.*

Proof See Appendix E. ■

Price stability is no longer optimal under LCP. Instead, central banks should fix the exchange rate. Why is a fixed exchange rate optimal under LCP? As we discussed in the introduction, LCP insulates household consumption baskets in each country from exchange rate fluctuations, guaranteeing a constant proportion of world income spent on each unique import good because exchange rate fluctuations do not pass-through into imported goods prices. Policymakers in small open economies are also unable to influence world income. Therefore domestic output and labor are unaffected by monetary policy. Stabilizing consumption becomes the central bank's goal, and the policy mechanism to achieve this goal is a fixed exchange rate. With a fixed exchange rate, labor fluctuates with productivity shocks but consumption remains constant. In contrast, when economies are large, monetary policy can influence domestic labor under LCP. In such cases, a fixed exchange rate is no longer optimal as shown by Corsetti, Dedola and Leduc (2010). The reasons for this are outlined in the introduction.

1.5 Conclusion

We derive the first closed-form solution for an open economy model that does not restrict substitutability between home and foreign goods to one. Different from the standard modeling framework in the literature, we assume a continuum of small open economies interacting in general equilibrium, rather than two large open economies. The tractability of our framework requires simplifying assumptions along other dimensions. Prices are set one period in advance, and each country exports all of its production and imports varieties from all other countries to aggregate into a final consumption basket. Using this setup, we answer the following question: What are the theoretical conditions under which it is optimal for a central bank to mimic the flexible price allocation?

We prove that for PCP, small open economy central banks should mimic the flexible price allocation for any elasticity of substitution between home and foreign goods, in cooperative and non-cooperative equilibria, in both complete markets and financial autarky. Policymakers should focus solely on eliminating domestic price rigidities and ignore external factors such as exchange rate movements, even in the presence of monopolistic distortions in goods markets and terms of trade distortions which are not internalized by domestic producers. Under LCP, we show that small open economy central banks should fix their exchange rate. Our results contrast with those in the literature for large open economies, where it is not optimal to mimic the flexible price allocation under PCP or to fix the exchange rate under LCP when substitutability differs from one.

We also calculate the welfare gains from cooperation among national monetary authorities. We show that for any elasticity in PCP and LCP, in both complete markets and financial autarky, international monetary cooperation has no impact on welfare. Price stability is the optimal policy for central banks under PCP in both non-cooperative and cooperative scenarios, while a fixed exchange rate optimal under LCP in both non-cooperative and cooperative scenarios. As a result, no benefits arise from cross-country monetary policy cooperation.

Our goal in this paper has been to provide a unifying framework for the analysis of optimal monetary policy in small open economies. We focus on the classic benchmark cases, PCP and LCP in both complete markets and financial autarky, to try and gain intuition and understanding about what differentiates policymaking in small and large open economies, particularly when risk sharing is not provided by terms of trade movements.

Chapter 2

The Optimal Design of a Fiscal Union

2.1 Introduction

The recent crisis in the euro area has prompted much debate about the need for greater fiscal cooperation across member countries. Although not to the same extent, fiscal crises at the state and municipal level have begun to raise similar questions in federal currency unions like the United States.[1] This debate has longstanding roots in the economics profession, dating back to Kenen (1969), who emphasized the importance of fiscal integration in a currency union.

While the concept of a fiscal union has received a great deal of attention in policy circles, there is considerable uncertainty about how to design such a union. We study the optimal design of a fiscal union in an open economy model where countries face three distortions:

[1] For example, the $14 billion-plus bankruptcy of the city of Detroit is on par with the roughly €10 billion bank bailout of Cyprus by the euro area governments.

nominal rigidities, incomplete financial markets, and terms of trade externalities. We show that the negative welfare impact of these distortions is highly sensitive to the elasticity of substitution between goods produced in different countries. As such, we find that the magnitude of this elasticity — the Armington elasticity — governs the optimal design of a fiscal union.

When the Armington elasticity is equal to one, a common assumption in the literature known as the Cole-Obstfeld specification, terms of trade movements provide complete international risk-sharing through offsetting income and substitution effects, even in financial autarky.[2] If a country produces less output and exports fewer goods under unitary elasticity, its terms of trade will improve and exactly offset the decline in quantity produced so that export revenues are constant. In this case, there is no need for a fiscal union to improve international risk-sharing. At the same time, when goods are imperfect substitutes countries are exposed to a relatively high degree of monopoly power at the export level. Domestic fiscal policymakers use this monopoly power to impose a large markup on their exports — what the literature refers to as a terms of trade externality. The optimal fiscal union will force domestic fiscal policymakers to internalize this externality and prevent countries from exploiting their monopoly power and manipulating their terms of trade. We find that when the elasticity is close to one countries should cooperate in setting steady state domestic income tax rates to ameliorate large terms of trade externalities — what we call a *tax union.*

[2]Cole and Obstfeld (1991) were the first to demonstrate the provision of complete international risk-sharing via terms of trade movements under unitary elasticity.

As the Armington elasticity increases and exports become closer substitutes, the degree of international risk sharing provided by terms of trade movements declines, as does each country's monopoly power. Because country-level monopoly power falls, the welfare gains from a tax union fall as well. On the other hand, there is now a role for a fiscal union to improve international risk-sharing. We find that when the elasticity is high and goods are close substitutes countries should organize a contingent cross-country transfer scheme to provide international risk-sharing — what we call a *transfer union*.

The Armington elasticity thus governs the optimal design of a fiscal union: the welfare gains from a tax union *decrease* in the Armington elasticity; the welfare gains from a transfer union *increase* in the Armington elasticity. We prove that this is true for countries outside of and within currency unions, in financial autarky and incomplete markets.

Empirical estimates of the Armington elasticity range from one to twelve or higher, depending on the estimation method, country and time period being examined.[3] In Section 2.8 we compute the welfare gains from a fiscal union for a wide range of elasticities, including country-specific estimates for European countries from Corbo and Osbat (2013). For standard calibrations, the welfare gains from a tax union are as high as 3% of permanent consumption when the elasticity is close to one; the welfare gains from a transfer union

[3]Using highly disaggregated data, Eaton and Kortum (2002) estimate the elasticity to be 9.28, Broda and Weinstein (2006) find an unweighted median of 3.1 and mean of 12.6, while Romalis finds a range of 4 to 13. Imbs and Majean (2011) find a mean of 6.7 with a standard deviaion of 4.9, and a median of 5.1. Lai and Trefler (2002) estimate a range between 5 and 8. More recently, Simonovska and Waugh (2011) find a range between 3.38 and 5.42, while Feenstra, Obstfeld and Russ (2012) find a median estimate of the elasticity between foreign countries of 3.1 for the U.S. In a survey of the literature on elasticity estimates, Anderson and Van Wincoop (2004) conclude a range of five to ten is reasonable. Ruhl (2008) explains why the international macro and trade literatures have quite different estimates of the Armington elasticity.

are as high as 5% of permanent consumption when the elasticity is five or higher, and can approach 20% when countries lose access to international financial markets. These gains are 75 to 400 times larger than Lucas' (2003) estimates of the welfare cost of business cycles.[4]

Although a tax union removes terms of trade externalities and a transfer union improves international risk-sharing, the distortive effects of nominal rigidities remain unchecked by international fiscal cooperation. If countries control their own monetary policy this is not a problem, as the central bank can move the economy toward the efficient level of output. However within a currency union, the union-wide central bank cannot eliminate nominal rigidities in the presence of asymmetric shocks across countries, which prevents efficient adjustment of the economy through changes in relative prices.[5] National fiscal authorities therefore have a role to play in implementing contingent policies that move the economy toward the efficient level of output and eliminate nominal rigidities. Importantly, such policies do not require international fiscal cooperation. We show that the negative welfare impact of nominal rigidities and hence the necessity of contingent domestic fiscal policy is increasing in the Armington elasticity.

In addition to studying optimal fiscal policies, we examine the implications of labor mobility within a currency union. We show that labor mobility completely removes the negative welfare impact of wage rigidity and also facilitates international risk-sharing. In

[4]Lucas (2003) estimated the welfare cost of business cycle fluctuations to be 0.05% of permanent consumption.

[5]This role is fulfilled by national central banks under flexible exchange rates, but a common union-wide central bank has only one instrument to fight many idiosyncratic shocks. Note that if shocks are symmetric across countries, the union wide central bank is able to eliminate nominal rigidities and mimic the flexible price equilibrium.

so doing, labor mobility eliminates the need for contingent domestic fiscal policy as well as the need for a transfer union. For standard calibrations, the welfare gains from labor mobility are as high as 5% of steady state consumption when elasticity is five or higher. While the construction of a fiscal union is fraught with political and legal hurdles, labor mobility is already guaranteed as one of the four pillars of the European Union. From a policy perspective, efforts to improve labor mobility may be more effective than efforts to agree on some type of fiscal union. At the very least, we show that the benefits of contingent fiscal policy and a transfer union are redundant if labor is fully mobile across borders.

We also study the welfare implications of joining a currency union. One of the arguments advanced by Mundell (1961, 1973) and others in favor of a currency union is that countries who join such a union experience deeper financial integration. We thus compare the welfare of a country outside of a currency union with no access to international financial markets versus a country in a currency union with full access to international financial markets. Even in this extreme scenario, the welfare benefits of entering a currency union are not large enough to overcome the loss of independent monetary policy for standard calibrations. Although these losses can be partially offset by contingent domestic fiscal policy or labor mobility, it is telling that from the perspective of deeper financial integration, the benefits of a currency union are relatively small compared with the costs of losing monetary independence.

How We Differ From the Literature: Non-Cooperative Fiscal Policy, Imperfect Risk-Sharing and a Global Closed-Form Solution

This paper is related to the literature on the conduct of optimal monetary and fiscal policy among interdependent economies, particularly within a currency union.[6] Beetsma and Jensen (2005), Gali and Monacelli (2008) and Ferrero (2009) focus primarily on the case of cooperative policy with internationally complete asset markets. These papers show that monetary policy should stabilize inflation at the union level and that cooperative fiscal policy in the form of government spending has a country-specific stabilization role. Farhi and Werning (2012) study cooperative fiscal policy in a transfer union in the aforementioned Cole-Obstfeld specification with unitary elasticity and log utility.[7] They demonstrate that even when private asset markets are complete internationally, there is a role for contingent cross country transfers to provide consumption insurance. We differ from all of these papers in two important ways. First, we analyze non-cooperative equilibria between fiscal policymakers in a currency union. Second, we relax the assumption of perfect risk-sharing via

[6] Early non-microfounded contributions in this area include Canzoneri and Henderson (1990) and Eichengreen and Ghironi (2002). Another strand of the literature focuses on the division of seignorage within a currency union. Sims (1999) and Bottazzi and Manasse (2002) examine the interaction between monetary and fiscal policy when seignorage is distributed by a common central bank. While this is an important issue, we abstract from the role of seignorage and focus on the potential for fiscal policy cooperation to improve welfare. Benigno and De Paoli (2010) emphasize the international dimension of fiscal policy for the case of a small open economy in a flexible exchange rate regime, abstracting from the role of strategic interactions between countries as well as from the role of a currency union, both focuses of our analysis. Evers (2012) takes a more quantitative approach and estimates the welfare gains from a variety of "transfer rules", in essence running a horse race between different types of fiscal regimes within a currency union. Our exact analytical solution for optimal policy and our consideration of flexible exchange rate allocations stand in contrast with this approach.

[7] We nest the Cole-Obstfeld calibration as a special case in the closed-form version of our model in Section 2.2, as well as in our extended model in Section 2.8. We show that the welfare gains from a transfer union are extremely small in the Cole-Obstfeld specification due to the provision of complete cross-country risk-sharing via terms of trade movements.

complete markets or unitary elasticity. Our model thus introduces two additional distortions that are empirically relevant: terms of trade externalities and incomplete international risk sharing. We also analyze the welfare implications of a tax union and of labor mobility, elements which are absent in the literature.

Our methodology provides us with a unique perspective on the role of currency unions, fiscal unions and labor mobility. We obtain a novel global closed-form solution for an open economy model that does not restrict the elasticity of substitution between the goods of different countries to one. This provides a tractable framework to analyze optimal policy and enables us to accurately compare welfare across a variety of risk-sharing regimes, which is not possible under unitary elasticity. In the closed-form model, we consider two financial market regimes: internationally complete asset markets with cross-border trade in bonds and equities (perfect risk sharing) and financial autarky (no risk sharing).

Our closed-form model requires two simplifying assumptions: complete openness in consumption for all economies in the model as well as one period in advance nominal rigidities. We relax both of these assumptions in Section 2.8 and evaluate the welfare gains from a fiscal union in a model with consumption home bias and Calvo wage rigidities. As in the closed-form case, we solve the extended model away from the Cole-Obstfeld calibration so that financial market structure matters. In the extended model, we consider incomplete markets with cross-border trade in safe government bonds (incomplete risk sharing) as well as financial autarky. In this setup, we find that home bias *increases* the welfare gains from a transfer union, while home bias *decreases* the welfare gains from a tax union. We also find that Calvo rigidities do not impact the results, yielding similar welfare consequences as one period in advance rigidities. As such, our closed-form results *underestimate* the welfare benefits of a transfer union, contingent fiscal policy and labor mobility, but *overestimate* the welfare benefits of a tax union. We focus on the closed-form model for the first half of the paper as it generates analytically tractable and intuitive results, and then shift to the extended model.

2.2 The Model in Closed-Form

We consider a continuum of small open economies represented by the unit interval, as popularized in the literature by Gali and Monacelli (2005, 2008). Our model is based on Dmitriev and Hoddenbagh (2013), although here we consider wage rigidity rather than price rigidity and extend the closed-form solution for flexible exchange rates to the case of a currency union.

Each economy consists of a representative household and a representative firm. All countries are identical ex-ante: they have the same preferences, technology, and wage-setting. Ex-post, economies will differ depending on the realization of their technology shock. Households are immobile across countries, however goods can move freely across borders. Each economy produces one final good, over which it exercises a degree of monopoly power. This is crucially important: countries are able to manipulate their terms of trade even though they are measure zero. As in Corsetti and Pesenti (2001, 2005) and Obstfeld and Rogoff (2000, 2002), we use one-period-in-advance wage setting to introduce nominal rigidities.

Workers set next period's nominal wages, in terms of domestic currency, prior to next-period's production and consumption decisions. Given this preset wage, workers supply as much labor as demanded by firms. We lay out a general framework below, and then hone in on the specific case of complete markets and financial autarky. To avoid additional notation, we ignore time subindices unless absolutely necessary. When time subindices are absent, we are implicitly referring to period t.

Production Each economy i produces a final good, which requires technology, Z_i, and aggregated labor, N_i. We assume that technology is independent across time and across countries. We need not impose any particular distributional requirement on technology at this point. The production function of each economy will be:

$$Y_i = Z_i N_i. \tag{2.1}$$

Households, indexed by h, each have monopoly power over their differentiated labor input, which will lead to a markup on wages. A perfectly competitive, representative final goods producer aggregates differentiated labor inputs from households in CES fashion into a final good for export. Production of the representative final goods firm in a specific country is:

$$N_i = \left(\int_0^1 N_i(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.2}$$

where ε is the elasticity of substitution between different types of labor, and $\mu_\varepsilon = \frac{\varepsilon}{\varepsilon-1}$ is the markup on labor.

The aggregate labor cost index, W, defined as the minimum cost to produce one unit of output, will be a function of the nominal wage for household h, $W(h)$:

$$W_i = \left(\int_0^1 W_i(h)^{1-\varepsilon} dh \right)^{\frac{1}{1-\varepsilon}}.$$

Cost minimization by the firm leads to demand for labor from household h:

$$N_i(h) = \left(\frac{W_i(h)}{W_i} \right)^{-\varepsilon} N_i. \tag{2.3}$$

In the open economy, monopoly power is exercised at both the household and the country level: at the household level because of differentiated labor, and at the country level because each economy produces a unique good. We show in Section 2.4 that optimizing non-cooperative policymakers will remove the household markup on labor but will introduce a terms of trade markup through the income tax rate. Just to be clear, firms have no monopoly power and are perfectly competitive.

Households In each economy, there is a household, h, with lifetime expected utility

$$\mathbb{E}_{t-1} \left\{ \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{it+k}(h)^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it+k}(h)^{1+\varphi}}{1+\varphi} \right) \right\} \tag{2.4}$$

where $\beta < 1$ is the household discount factor, $C(h)$ is the consumption basket or index, $N(h)$ is household labor effort (think of this as hours worked). Households face a general budget constraint that nests both complete markets and financial autarky; we will discuss the differences between the two in subsequent sections. For now, it is sufficient to simply write out the most general form of the budget constraint:

$$C_{it}(h) = (1 - \tau_i)\left(\frac{W_{it}(h)}{P_{it}(h)}\right) N_{it}(h) + \mathcal{D}_{it}(h) + \mathcal{T}_{it}(h) + \Gamma_{it}(h). \tag{2.5}$$

The distortionary tax rate on household labor income in country i is denoted by τ_i, while Γ_{it} is a domestic lump-sum tax rebate to households. $\mathcal{T}$ refers to lump-sum cross-country transfers. In the absence of a fiscal union, these cross-country transfers will equal zero ($\mathcal{T} = 0$). Net taxes equal zero in the model, as any amount of government revenue is rebated lump-sum to households. The consumer price index corresponds to P_{it}, while the nominal wage is W_{it}. $\mathcal{D}_{it}$ denotes state-contingent portfolio payments expressed in real consumption units, and can be written in more detail as:

$$\mathcal{D}_{it} P_{it} = \int_0^1 \mathcal{E}_{ijt} B_{ijt} dj, \tag{2.6}$$

where B_{ijt} is a state-contingent payment in currency j.[8] $\mathcal{E}_{ijt}$ is the exchange rate in units of currency i per one unit of currency j; an increase in $\mathcal{E}_{ijt}$ signals a depreciation of currency i

[8]Equation (2.6) holds in all possible states in all periods. Details are provided in Appendix A.1.

relative to currency j. In a currency union, $\mathcal{E}_{ijt} = 1$ for all i, j, t. When international asset markets are complete, households perform all cross-border trades in contingent claims in period 0, insuring against all possible states in all future periods. The transverality condition simply states that all period 0 transactions must be balanced: payment for claims issued must equal payment for claims received. Leaving the details in the appendix, we use the following relationship as the transversality condition for complete markets:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t C_{it}^{-\sigma} \mathcal{D}_{it} \right\} = 0, \tag{2.7}$$

while in financial autarky

$$\mathcal{D}_{it} = 0.$$

Intuitively, the transversality condition (2.7) stipulates that the present discounted value of future earnings should be equal to the present discounted value of future consumption flows. Under complete markets, consumers choose a state contingent plan for consumption, labor supply and portfolio holdings in period 0.

Consumption and Price Indices Households in each country consume a basket of imported goods. This consumption basket is an aggregate of all of the varieties produced by different countries. The consumption basket for a representative small open economy i, which is common across countries, is defined as follows:

$$C_i = \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \tag{2.8}$$

where lower case c_{ij} is the consumption by country i of the final good produced by country j, and γ is the elasticity of substitution between domestic and foreign goods (the Armington elasticity). Because there is no home bias in consumption, countries will export all of the output of their unique variety, and import varieties from other countries to assemble the consumption basket.

Prices are defined as follows: lower case p_{ij} denotes the price in country i (in currency i) of the unique final good produced in country j, while upper case P_i is the aggregate consumer price index in country i. Given the above consumption index, the consumer price index will be:

$$P_i = \left(\int_0^1 p_{ij}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}. \tag{2.9}$$

Consumption by country i of the unique variety produced by country j is:

$$c_{ij} = \left(\frac{p_{ij}}{P_i} \right)^{-\gamma} C_i. \tag{2.10}$$

We assume that producer currency pricing (PCP) holds, and that the law of one price (LOP) holds, so that the price of the same good is equal across countries when converted into a common currency. We define the nominal bilateral exchange rate between countries

i and j, $\mathcal{E}_{ij}$, as units of currency i per one unit of currency j. LOP requires that:

$$p_{ij} = \mathcal{E}_{ij} p_{jj}. \tag{2.11}$$

Given LOP and identical preferences across countries, PPP will also hold for all i, j country pairs:

$$P_i = \mathcal{E}_{ij} P_j, \tag{2.12}$$

The terms of trade for country j will be:

$$TOT_j = \frac{p_{jj}}{P_j}, \tag{2.13}$$

where TOT_j is defined as the home currency price of exports over the home currency price of imports. Now we can take (2.10), and using (2.11) and (2.12), solve for demand for country j's unique variety:

$$Y_j = \int_0^1 c_{ij} di = \int_0^1 \left(\frac{p_{ij}}{P_i}\right)^{-\gamma} C_i di \overset{(2.11)+(2.12)}{=} \left(\frac{p_{jj}}{P_j}\right)^{-\gamma} \int_0^1 C_i di = TOT_j^{-\gamma} C_w. \tag{2.14}$$

where C_w is defined as the average world consumption across all i economies, $C_w = \int_0^1 C_i di$.

Labor Market Clearing Households maximize (2.4) subject to (2.5). The first order condition for labor will give the optimal preset wage (that is, the labor supply condition):

$$W_{it} = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{\frac{C_{it}^{-\sigma}N_{it}}{P_{it}}\right\}}. \tag{2.15}$$

The optimization problem of the representative firm in country i is standard. It maximizes profit choosing the appropriate amount of aggregate labor.

$$\max_{N_i} Y_i p_i - W_i N_i \Rightarrow \frac{W_i}{p_i} = \frac{Y_i}{N_i} = Z_i \tag{2.16}$$

This labor demand condition equates the real wage at time t with the marginal product of labor, Z_{it}. Using the labor demand condition ($N_{it} = Y_{it}p_{it}/W_{it}$) from (2.16), and the fact that the wage is preset at time $t-1$, the labor market clearing condition will be:

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}\frac{p_{it}}{P_{it}}\right\}}. \tag{2.17}$$

This is the general labor market clearing condition; it holds for the closed economy and in the open economy for producer currency pricing and local currency pricing. Under producer currency pricing, our focus in this paper, the demand for the unique variety (2.14) will give the following labor market clearing condition:

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\right\}}. \tag{2.18}$$

Taking the expectations operator out of (2.18) will give the flexible wage equilibrium.

We now turn our attention to the difference between complete markets and financial autarky.

2.2.1 Complete Markets

In complete markets, agents in each economy have access to a full set of domestic and foreign state-contingent assets. Households in all countries will maximize (2.4), choosing consumption, leisure, money holdings, and a complete set of state-contingent nominal bonds, subject to (2.5). Complete markets and PPP imply the following risk-sharing condition:

$$\frac{C_{it}^{-\sigma}}{C_{it+1}^{-\sigma}} = \frac{C_{jt}^{-\sigma}}{C_{jt+1}^{-\sigma}} \quad \forall i, j \tag{2.19}$$

which states that the ratio of the marginal utility of consumption at time t and $t+1$ must be equal across all countries. Importantly, this condition does not imply that consumption is equal across countries. Consumption in country i will depend on the initial asset position, fiscal and monetary policy, the distribution of country-specific shocks, the covariance of global and local shocks, and other factors.[9]

When (2.7),(2.17), and (2.19) hold, consumption in country i can be expressed as a

[9] A policy change in economy i may lead to a change in consumption. For example, monetary policy affects the covariance between home production and world consumption, which in turn influences home consumption, even in complete markets. Fiscal policy can tax consumption and cause a lower level of consumption in the long-run relative to the rest of the world. In spite of this, it is still possible to characterize an optimal consumption plan that is robust to changes in monetary and fiscal policy.

function of world consumption:

$$C_{it} = \frac{\mathbb{E}_{t-1}\left\{\sum \beta^s \left[Y_{it+s} C_{wt+s}^{-\sigma} TOT_{it+s}\right]\right\}}{\mathbb{E}_{t-1}\left\{\sum \beta^s C_{wt+s}^{1-\sigma}\right\}} C_{wt}. \tag{2.20}$$

This defines the optimal consumption allocation for country i in complete markets.[10] Using the fact that Z_{it} is independent across time and across countries, and wages are preset, (2.20) is equivalent to

$$C_{it} = \mathbb{E}_{t-1}\left\{Y_{it} TOT_{it}\right\} + \mathcal{T}_{it} = C_w^{\frac{1}{\gamma}} \mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\} + \mathcal{T}_{it} \tag{2.21}$$

where transfers will equal zero in all states of the world because risk-sharing is provided by trade in contingent claims.

2.2.2 Financial Autarky

The aggregate resource constraint under financial autarky specifies that the nominal value of output in the home country (exports) must equal the nominal of consumption in the home country (imports). That is, trade in goods must be balanced. In a model with cross-border lending, bonds would also show up in this condition, but in financial autarky they are obviously absent. The primary departure from complete markets lies in the household

[10] Details are found in the appendix of Dmitriev and Hoddenbagh (2013).

and economy-wide budget constraints,

$$\underbrace{P_i \cdot C_i}_{\text{Imports}} = \underbrace{p_{ii} \cdot Y_i}_{\text{Exports}} + \underbrace{\mathcal{T}_{it}}_{\text{Transfers}} \tag{2.22}$$

where transfers will be zero unless countries form a transfer union, in which case transfers will provide risk-sharing. Using the fact that (2.14) holds under both complete markets and financial autarky, and substituting this into (2.22), one can show that demand for country i's good in financial autarky will be

$$C_{it} = C_w^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}} + \mathcal{T}_{it}. \tag{2.23}$$

Complete markets and autarky differ only by goods market clearing. In complete markets consumption is equal to expected domestic output expressed in consumption baskets; in autarky consumption is equal to realized domestic output expressed in consumption baskets.

2.3 Global Social Planner

We begin by describing the maximization problem faced by a benevolent global social planner who has complete control over the monetary and fiscal policies of each country. Since the economies in our model are identical ex-ante, the global social planner will maximize a weighted utility function over all i countries,

$$\int_0^1 \left[\frac{C_i^{1-\sigma}}{1-\sigma} - \chi \frac{N_i^{1+\varphi}}{(1+\varphi)}\right] di, \tag{2.24}$$

subject to the consumption basket and the aggregate resource constraint:

$$C_i = \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}}, \tag{2.25}$$

$$Y_i = N_i Z_i = \int_0^1 c_{ji} dj. \tag{2.26}$$

Proposition 8 *The global social planner will maximize (2.24), subject to (2.25) and (2.26). The solution to the global social planner problem is:*

$$C_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}}, \tag{2.27a}$$

$$N_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}}, \tag{2.27b}$$

$$Z_w = \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}. \tag{2.27c}$$

Proof See Appendix C in Dmitriev and Hoddenbagh (2013). ■

The global social planner solution characterizes the Pareto efficient allocation. From (A.11a), we see that domestic consumption depends on average world technology Z_w, which is a constant because technology shocks are identically and independently distributed. Consumption is thus stabilized at the country level, insuring risk averse households from consumption risk. On the other hand, (A.11b) shows that labor will fluctuate with technology shocks, increasing in booms and decreasing in recessions. There are no distortions in the efficient allocation: wage ridigity, incomplete risk-sharing, and the terms of trade externality are absent.

The efficient allocation provides a natural benchmark to evaluate different policy regimes. In Sections 2.4 and 2.5 we look closely at optimal monetary and fiscal policy in non-cooperative and cooperative settings and see what conditions are necessary to replicate the Pareto efficient allocation outside of and within a currency union. In Section 2.6 we study the effect of labor mobility and see if it can replicate the Pareto efficient allocation.

2.4 Non-Cooperative Policy

In order to study the benefits of international policy cooperation, we must first understand the non-cooperative Nash equilibrium. What outcomes naturally arise when policymakers do not cooperate? Our goal in this section is to illuminate the various distortions that are present in the non-cooperative Nash equilibrium, and to compare and contrast with the global social planner equilibrium defined in Proposition 8. We can then pinpoint specific areas of policy cooperation that ameliorate welfare decreasing distortions, leading us to the optimal design of a fiscal union. We begin with the Nash equilibrium under flexible exchange rates and then move to the case of a currency union.

2.4.1 Flexible Exchange Rates

When exchange rates are flexible, each country has its own central bank and its own fiscal authority. Before any shocks are realized, national fiscal authorities declare non state-contingent taxes, and then national central banks declare monetary policy for all states of the world. With this knowledge in hand, households lay out a state-contingent plan for consumption and labor as well asset holdings when markets are complete. After that, shocks hit the economy. A detailed timeline is provided in Figure 2.1.

Without loss of generality, we assume a cashless limiting economy.[11] Central banks set monetary policy in each period by optimally choosing the amount of labor. Although central banks optimize by choosing labor instead of money or an interest rate, the three are equivalent in this model.[12] We can write down an interest rate rule that gives the exact same allocation. Domestic fiscal authorities choose the optimal labor tax rate τ_i. The objective function for non-cooperative domestic policymakers will be

$$\max_{N_{it}} \max_{\tau_i} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\}, \tag{2.28}$$

where the fiscal authority acts first and chooses τ_i and the central bank then chooses N_{it}.

We first examine the Nash equilibrium for non-cooperative policymakers when international asset markets are complete. Policymakers in complete markets will maximize their objective function subject to the labor market clearing (2.29a) and goods market clearing

[11] Benigno and Benigno (2003) describe a cashless-limiting economy in detail in their appendix, pp.756-758.
[12] We demonstrate the equivalence Appendix F of Dmitriev and Hoddenbagh (2013).

(2.29b) constraints, and production (2.29c) and aggregate world consumption (2.29d):

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\right\}}, \tag{2.29a}$$

$$C_{it} = C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\} \tag{2.29b}$$

$$Y_{it} = Z_{it}N_{it}, \tag{2.29c}$$

$$C_{wt} = \left(\int_0^1 Y_{it}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{2.29d}$$

Proposition 9 ***Flexible Exchange Rates + Complete Markets*** *When international asset markets are complete and exchange rates are flexible, non-cooperative policymakers will maximize (2.28) subject to (2.29a), (2.29b), (2.29c) and (2.29d). The solution under commitment for non-cooperative policymakers in complete markets is:*

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}}\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(\sigma+\varphi)}}, \tag{2.30a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}}\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}di\right)^{\frac{1-\gamma\sigma}{(\gamma-1)(\sigma+\varphi)}}Z_i^{\frac{\gamma-1}{1+\gamma\varphi}}. \tag{2.30b}$$

It is optimal for non-cooperative central banks under commitment to mimic the flexible wage allocation. The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix B.1. ■

The above allocation replicates the global social planner allocation with the addition of a terms of trade markup, $\mu_\gamma = \frac{\gamma}{\gamma-1}$, that lowers consumption and output. It is optimal for central banks to mimic the flexible wage allocation through a policy of price stability.[13] Optimizing fiscal authorities internalize the negative welfare impact of the domestic markup on differentiated labor inputs (μ_ε), and thus choose an income tax rate that cancels out the labor markup. However, fiscal authorities also want to use their country-level monopoly power. Because each country in the continuum is measure zero, policymakers do not internalize the impact of charging a higher markup for their export good on the welfare of other countries. This leads fiscal authorities to set an income tax rate which reduces hours worked and restricts production, $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$, so that exports from each country are subject to a terms of trade markup (μ_γ). The terms of trade externality leads to lower welfare outcomes because households in each country must pay a higher price on each import good in the consumption basket. Even though asset markets are complete, the non-cooperative allocation under flexible exchange rates yields lower welfare than the global social planner allocation due to the imposition of the terms of trade markup. The need for some sort of international fiscal cooperation that would force domestic fiscal authorities to internalize this externality is clear.

Now that we've examined the complete markets equilibrium when exchange rates are flexible, we turn our attention to the case of financial autarky. The objective function in financial autarky will be identical to the complete markets case. Domestic fiscal authorities will first choose the optimal tax rate, and then central banks will set the optimal monetary

[13] In a related paper (Dmitriev and Hoddenbagh 2013), we prove that mimicking the flexible price allocation is a dominant strategy for small open economy central banks. This result is robust to changes in elasticity between domestic and foreign goods, the degree of cooperation between policymakers in different countries, and the degree of financial integration across countries.

policy by choosing labor. However, there is a slight difference in the constraints faced by policymakers in complete markets and financial autarky. In complete markets, home consumption is a function of expected output (2.29b), while in autarky home consumption is a function of actual output

$$C_{it} = C_{wt}^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}}. \tag{2.31}$$

Aside from (2.31), all other constraints are identical in complete markets and financial autarky.

Proposition 10 ***Flexible Exchange Rates + Financial Autarky*** *Non-cooperative policymakers in financial autarky will maximize (2.28) subject to (2.29a), (2.29c), (2.29d) and (2.31). The solution under commitment for non-cooperative policymakers in financial autarky is:*

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1-\sigma+\gamma(\sigma+\varphi)}} di\right)^{\frac{(1+\varphi)}{(\gamma-1)(\sigma+\varphi)}} Z_i^{\frac{(\gamma-1)(1+\varphi)}{1-\sigma+\gamma(\sigma+\varphi)}}, \tag{2.32a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1-\sigma+\gamma(\sigma+\varphi)}} di\right)^{\frac{(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}} Z_i^{\frac{(\gamma-1)(1-\sigma)}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{2.32b}$$

It is optimal for non-cooperative central banks to mimic the flexible wage allocation. The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix B.1. ■

As in complete markets, central banks find it optimal to mimic the flexible wage equilibrium through a policy of price stability in financial autarky. On the fiscal side, policymakers again eliminate the domestic markup μ_ε, but impose a terms of trade markup on their unique export good μ_γ via the steady state income tax rate. Financial autarky removes cross-country consumption insurance, as households no longer have the ability to trade in international contingent claims. This can be seen most clearly in (2.32a), where equilibrium consumption is a function of idiosyncratic productivity, Z_i, and will fluctuate with country-specific shocks to technology.

2.4.2 Currency Union

Within a currency union, a single central bank sets monetary policy for the union as a whole. Countries no longer control their domestic monetary policy as they do when exchange rates are flexible. In the presence of aggregate shocks, the union-wide central bank will stabilize inflation at the union level, a result shown in Gali and Monacelli (2008). However, for tractability we assume no aggregate shocks, only asymmetric country-specific shocks. With only one policy instrument, the union-wide central bank cannot eliminate wage rigidity at the country level in the presence of asymmetric shocks. As a result, the union-wide central bank does nothing in our model. None of our results change if we add aggregate shocks: these shocks would simply be counteracted by the union-wide central bank.

In a currency union each country retains control over it's own fiscal policy. The objective function for non-cooperative fiscal policymakers in a currency union is:

$$\max_{\tau_i} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \tag{2.33}$$

The constraints faced by policymakers within a currency union are identical to those faced by policymakers under flexible exchange rates, with the addition of a fifth constraint unique to currency unions. Thus, relative to the optimization problem under flexible exchange rates, we add one constraint and subtract one FOC. We know that demand for country i's good is $Y_i = TOT_i^{-\gamma} C_w = \left(\frac{p_{ii}}{P_i}\right)^{-\gamma} C_w$ from (2.14) and that $p_{ii} = \frac{W_i}{Z_i}$ from (2.16). Plugging (2.16) into (2.14) gives:

$$Y_{it} = \underbrace{\left(\frac{W_{it}}{P_{it}}\right)^{-\gamma} C_w}_{A} Z_{it}^{\gamma} = A Z_{it}^{\gamma} \tag{2.34}$$

where A is a constant. (2.34) is the additional constraint faced by the policymaker in a currency union.

Proposition 11 ***Currency Union + Complete Markets*** *Non-cooperative policymakers in a currency union will maximize (2.33) subject to (2.29a), (2.29b), (2.29c), (2.29d) and (2.34). The solution under commitment for non-cooperative policymakers within a currency union in complete markets is:*

$$C_i = C_w = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left[\frac{\left(\int_0^1 Z_i^{\gamma-1} di\right)^{\frac{\gamma(1+\varphi)}{\gamma-1}}}{\int_0^1 Z_i^{(\gamma-1)(1+\varphi)} di}\right]^{\frac{1}{\sigma+\varphi}}, \tag{2.35a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left[\frac{\left(\int_0^1 Z_i^{\gamma-1} di\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}}}{\int_0^1 Z_i^{(\gamma-1)(1+\varphi)} di}\right]^{\frac{1}{\sigma+\varphi}} Z_i^{\gamma-1}. \tag{2.35b}$$

The resulting equilibrium allocation does not replicate the flexible wage equilibrium. The *optimal tax rate for non-cooperative fiscal authorities is* $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix B.2. ■

Within a currency union, the inability of the union-wide central bank to alleviate asymmetric shocks across countries leads to the presence of wage rigidity in the optimal allocation. In addition, non-cooperative fiscal authorities exploit their country-level monopoly power and impose a terms of trade markup via income tax policy. We thus see the presence of two distortions in the equilibrium allocation: wage rigidity and a terms of trade markup. As in the flexible exchange rate allocation, there is no idiosyncratic technology risk in consumption under complete markets, so consumption will be equalized across countries in equilibrium. However, welfare will be lower when wages are rigid than when they are flexible, as one can notice by comparing the above allocation with the Pareto efficient allocation.[14]

Proposition 12 ***Currency Union + Financial Autarky*** *In financial autarky, non-cooperative policymakers in a currency union will maximize (2.33) subject to (2.29a), (2.29c), (2.29d), (2.31) and (2.34). The optimal allocation in financial autarky given by a non-*

[14]We calculate explicit welfare differences between allocations in Section 6.

contingent policymaker in a currency union is:

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left[\frac{\left(\int_0^1 Z_i^{(\gamma-1)(1-\sigma)}di\right)\left(\int_0^1 Z_i^{\gamma-1}di\right)^{\frac{1+\varphi}{\gamma-1}}}{\int_0^1 Z_i^{(\gamma-1)(1+\varphi)}di}\right]^{\frac{1}{\sigma+\varphi}} Z_i^{\gamma-1}, \tag{2.36a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left[\frac{\left(\int_0^1 Z_i^{(\gamma-1)(1-\sigma)}di\right)\left(\int_0^1 Z_i^{\gamma-1}di\right)^{\frac{1-\sigma}{\gamma-1}}}{\int_0^1 Z_i^{(\gamma-1)(1+\varphi)}di}\right]^{\frac{1}{\sigma+\varphi}} Z_i^{\gamma-1}. \tag{2.36b}$$

The resulting equilibrium allocation does not replicate the flexible wage allocation. The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix B.2. ■

In the autarky Nash equilibrium described in Proposition 12, members of a currency union face three welfare decreasing distortions: wage rigidity resulting from the absence of country-specific monetary policy; idiosyncratic consumption risk, caused by lack of access to international financial markets; and a terms of trade markup, imposed by non-cooperative fiscal authorities in other countries. The potential for cooperative measures to ameliorate these distortions is evident, and will be the focus of Section 2.5. Before we broach the topic of a fiscal union however, we study the implications of contingent fiscal policy in the non-cooperative setup.

2.4.3 Contingent Fiscal Policy

Up to this point we have assumed that fiscal policy is non-contingent, so that fiscal authorities can only set a constant income tax rate. If we relax this assumption so that fiscal policymakers can adjust tax rates over the business cycle, the objective function under flexible exchange rates is

$$\max_{N_{it}} \max_{\tau_{it}} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\}, \tag{2.37}$$

and within a currency union is

$$\max_{\tau_{it}} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\}. \tag{2.38}$$

As we showed in Proposition 9 and 10, when exchange rates are flexible national central banks will mimic the flexible wage allocation and a constant labor tax rate will be optimal for both contingent and non-contingent fiscal policymakers. The role of fiscal policy under flexible exchange rates is simply to ameliorate the monopolistic markup on differentiated labor inputs and impose a terms of trade markup in the non-cooperative case. In other words, contingent fiscal policy is redundant when exchange rates are flexible because national central banks adjust monetary policy over the business cycle to counteract wage rigidity.

In contrast, the role of fiscal policy in a currency union is twofold: to impose a terms of trade markup, but also to eliminate wage rigidity at the national level resulting from asymmetric shocks. Optimal contingent fiscal policy within a currency union will not set a constant tax rate. Within a currency union the union-wide central bank has only one policy instrument at its disposal and cannot offset the effect of asymmetric shocks across countries. Contingent national fiscal policy can fill the void, setting domestic tax rates in each period to remove domestic wage rigidity and mimic the flexible wage equilibrium. One already begins to see that fiscal policy is more important within a currency union than outside of one. Given that contingent fiscal policy is only necessary in a currency union, we ignore flexible exchange rate allocations in Proposition 13.

Proposition 13 ***Contingent Fiscal Policy*** *Contingent non-cooperative policymakers within a currency union will maximize (2.38), subject to (2.29a), (2.29b), (2.29c), (2.29d), and (2.34) in complete markets and subject to (2.29a), (2.29c), (2.29d), (2.31) and (2.34) in autarky. The optimal allocation will exactly coincide with (2.30a) in complete markets and (2.32a) in autarky, replicating the flexible wage allocation with a terms of trade markup.*

Proof See Appendix B.3. ■

The optimal contingent labor tax within a currency union will be equal to $\tau_{it} = 1 - \mu_{it}$. The realized markup μ_{it} is defined by

$$MPL_{it} = \mu_{it} \cdot MRS_{it},$$

where $MPL_{it} = Z_{it}$ is the marginal product of labor and MRS_{it} is the marginal rate of substitution. A positive productivity shock increases MPL_{it}, but since wages remain fixed, there will be a rise in demand for labor. The rise in demand for labor will induce households to work more hours and cause an even larger increase in MRS_{it}. As a result, the markup will be countercylical and the optimal contingent labor tax will be procyclical: taxes will increase when productivity shocks are positive and decline when productivity shocks are negative.

Note that international policy cooperation is not necessary to eliminate the wage rigidity distortion: non-cooperative contingent fiscal policy is all that is required. For the remainder of the paper, we will assume that fiscal policy is non-contingent. However, keep in mind that contingent fiscal policy within a currency union can play the same role as monetary policy outside of a currency union, eliminating nominal rigidities and mimicking the flexible-wage allocation. While the ability of contingent fiscal policy to eliminate nominal rigidities has been shown in other closed and open economy studies, we are the first to emphasize that the distortionary impact of wage rigidity and hence the importance of contingent fiscal policy is increasing in the Armington elasticity γ.[15]

2.4.4 Summary of Non-Cooperative Policy

In sections 2.4.1 and 2.4.2 we solved for Nash equilibria under flexible exchange rates and within a currency union, for complete markets and financial autarky. Policymakers maximized the welfare of their domestic households without internalizing the impact of their policy decisions on other countries. The non-cooperative allocations featured three distortions: wage rigidity, lack of access to international financial markets, and terms of trade externalities. Proposition 9 and 10 proved that domestic monetary policy is sufficient to eliminate wage rigidity outside of a currency union, while Proposition 13 proved that con-

[15] A non-exhaustive list of papers demonstrating the importance of contingent fiscal policy includes Chugh (2006), Correia et al (2013), Schmitt-Grohe and Uribe (2004) and Siu (2004).

tingent fiscal policy is sufficient to eliminate wage rigidity within a currency union. In both cases, policy cooperation was unnecessary. In Section 2.5, we will show that the remaining two distortions — incomplete risk-sharing and terms of trade externalities — can be remedied with appropriate international policy cooperation via the construction of a tax union and a transfer union.

2.5 Cooperative Policy in a Fiscal Union

In this section we analyze fiscal policy cooperation. Although the concept of fiscal cooperation may be quite broad, we focus here on two specific types — a *tax union* and a *transfer union*. In a tax union, fiscal policymakers in each country cooperatively set steady state income tax rates to maximize the welfare of the union as a whole. A tax union may be viewed as a cross-country agreement on income tax harmonization between domestic fiscal authorities, or as a tax rate chosen by a supranational (or federal) fiscal authority. In a transfer union, fiscal policymakers in each country arrange contingent cross-country transfers to maximize the welfare of the union as a whole. The transfer scheme may derive from an agreement between national fiscal authorities or from a supranational (or federal) fiscal authority.

Each arrow in Figure 2.2 denotes cooperation between countries in a fiscal union. Complete cooperation (all arrows), no cooperation (no arrows), or cooperation along only one axis are all possible.[16]

[16]Because our focus is on the optimal design of a fiscal union, we ignore the implications of monetary

The objective functions for the various types of fiscal union described in Figure 2.2 are below.

$$\max_{\forall \tau_i} \int_0^1 \left[\max_{N_{it}} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{2.39a}$$

$$\max_{\forall \tau_i} \int_0^1 \left[\mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{2.39b}$$

Objective functions (2.39a) and (2.39b) refer to a tax union outside of and within a currency union, respectively. Here, the fiscal authorities in each country jointly maximize the welfare of all countries in the union by choosing the same steady state income tax rate.

$$\max_{\forall \mathcal{T}_{it}} \int_0^1 \left[\max_{N_{it}} \max_{\tau_i} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{2.39c}$$

$$\max_{\forall \mathcal{T}_{it}} \int_0^1 \left[\max_{\tau_i} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{2.39d}$$

Objective functions (2.39c) and (2.39d) refer to a transfer union outside of and within a currency union, respectively. Here, a supranational (or federal) fiscal body optimally

policy cooperation. With one common central bank, monetary cooperation within a currency union is not possible. In another paper (Dmitriev and Hoddenbagh (2013)), we show that in a continuum of small open economies monetary cooperation yields no welfare gains, as non-cooperative and cooperative equilibria exactly coincide.

chooses cross-country transfers in order to maximize union-wide welfare.

$$\max_{\forall \tau_i, \mathcal{T}_{it}} \int_0^1 \left[\max_{N_{it}} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{2.39e}$$

$$\max_{\forall \tau_i, \mathcal{T}_{it}} \int_0^1 \left[\mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{2.39f}$$

Finally, (2.39e) and (2.39f) refer to a tax *and* transfer union outside of and within a currency union, respectively. Here, countries not only agree on income tax rates, but also agree to send contingent cash transfers across countries.

Proposition 14 ***Tax Unions*** *Policymakers in a tax union will internalize the impact of their income tax rate on all union members. As a result, a tax union will remove the incentive for policymakers to manipulate their terms of trade. The optimal tax rate in a tax union is $\tau_i = 1 - \mu_\varepsilon$, which will remove the markup on domestic production in each country, μ_ε, while preventing the imposition of a terms of trade markup on exports, μ_γ, from all equilibrium allocations.*

Proof See Appendix B.4. ■

A tax union forces domestic fiscal authorities to internalize the impact of their terms of trade externality on union wide welfare. As a result, policymakers will not impose a terms of trade markup on the export of their country's unique good in a tax union. This improves welfare for the entire union as well as for each individual country, particularly for low values of elasticity when countries have a high degree of monopoly power. The distortive impact of the terms of trade externality increases as the degree of substitutability decreases, which will become clear in Section 2.7 when we calculate the welfare gains from a tax union. In other words, the benefits of a tax union are increasing in the degree of country-level monopoly power.

Members of a transfer union agree to send contingent cash transfers across countries in order to insure against idiosyncratic consumption risk. In complete markets the presence of cross-country transfers will alter the goods market clearing constraint, so that (2.29b) is replaced by the following two conditions:

$$C_{it} = C_{wt}^{\frac{1}{\gamma}} \mathbb{E}_{t-1} \left\{ Y_{it}^{\frac{\gamma-1}{\gamma}} \right\} + \mathcal{T}_{it}, \tag{2.40}$$

$$\text{where} \int_0^1 \mathcal{T}_{it} di = 0. \tag{2.41}$$

In financial autarky the presence of cross-country transfers will alter the goods market clearing constraint, so that (2.31) is replaced by the following two conditions:

$$C_{it} = C_{wt}^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}} + \mathcal{T}_{it}, \tag{2.42}$$

$$\text{where} \int_0^1 \mathcal{T}_{it} di = 0. \tag{2.43}$$

Proposition 15 ***Transfer Unions*** *Policymakers in a transfer union agree to send contingent cash transfers across countries in order to insure against idiosyncratic consumption risk. The equilibrium allocation within a transfer union will be identical with the equilibrium* *allocation under complete markets. As a result, transfer unions are redundant when international asset markets are complete or when substitutability is one, but yield large welfare gains when markets are incomplete.*

Proof See Appendix B.5. ■

As Proposition 15 states, a transfer union guarantees complete cross-country consumption insurance and thus replicates the effect of complete markets. The welfare benefits of a transfer union are increasing in the Armington elasticity: as goods become closer substitutes, the natural risk-sharing role played by the terms of trade begins to disappear. This will be seen more clearly in Section 2.7 when we calculate the welfare gains from a transfer union.

As shown in Figure 2.2, it is also possible to have a tax *and* a transfer union. If countries agree to both, they will enjoy complete risk-sharing and eliminate the distortive impact of the terms of trade externality. Proposition 14 and 15 show that a tax union, a transfer union or a combination of the two will move countries toward the Pareto efficient allocation.

Proposition 16 ***Pareto Optimum*** *The Pareto efficient allocation is achieved through a combination of: (1) independent monetary policy outside of a currency union or contingent fiscal policy within a currency union; (2) internationally complete asset markets or a transfer union; and (3) a tax union.*

Proof (1) eliminates wage rigidity, (2) provides cross-country risk-sharing, and (3) prevents terms of trade manipulation. Any combination of (1), (2) and (3), for example a tax and transfer union whose members control their own monetary policy outside of a currency union, will yield the Pareto efficient allocation. ■

Although each of these ingredients is necessary to achieve the Pareto efficient allocation, the *relative importance* of these ingredients is highly sensitive to the elasticity of substitution between products from different countries. In Section 2.7, we show that the optimal design of a fiscal union, and the emphasis given to a tax versus a transfer union, will depend on the value of the elasticity parameter.

2.6 Labor Mobility

While a fiscal union provides large economic benefits, it may be politically difficult to achieve in practice. This is especially true within the euro area, where the construction of a fiscal union would require sovereign governments to cede at least partial control over national fiscal policy. If deeper fiscal integration is not possible, what should governments do? Our answer, heralding back to James Meade (1957), is to pursue policies that increase labor mobility. Discussing the creation of a common currency area in Western Europe, Meade argued that without the free movement of goods, capital and labor, the idea was doomed to failure. Meade didn't prove the necessity of labor mobility rigorously, but we do so here using our analytical closed-form model.

Although we've assumed that labor is immobile up to this point, it is quite plausible that labor will be mobile across borders within a currency union. When labor is mobile, non-cooperative policymakers in a currency union in financial autarky maximize the familiar objective function, but face a new set of constraints (found in Appendix B.6). As each economy in the currency union is hit with idiosyncratic shocks, labor will shift from low demand bust countries to high demand boom countries, equalizing wages across countries and acting as a natural shock absorber that enables efficient adjustment of the economy without any policy actions taken by the monetary or fiscal authority.

Proposition 17 ***Labor Mobility*** *Labor mobility will eliminate two distortions: wage rigidity and the lack of risk-sharing in financial autarky. The resulting equilibrium allocations yield higher welfare than the flexible wage allocations in complete markets and financial autarky. When labor can move freely across borders, the solution under commitment for non-cooperative policymakers outside of or within a currency union, in complete markets or financial autarky, is:*

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\gamma-1} di\right)^{\frac{1+\varphi}{(\sigma+\varphi)(\gamma-1)}} \tag{2.44a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1+\varphi-\gamma(\sigma+\varphi)}{(\gamma-1)(\sigma+\varphi)}} Z_j^{\gamma-1}. \tag{2.44b}$$

The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix B.6. ■

Labor mobility thus plays the role of a transfer union by generating perfect international risk sharing, as well as the role of monetary policy under flexible exchange rates or contingent fiscal policy within a currency union by removing wage rigidity. In addition to facilitating consumption risk sharing, labor mobility has the added advantage of facilitating labor risk sharing — so that both consumption *and* hours worked are stabilized. Although the distortive effect of the terms of trade externality remains so there is still a need for a tax union, the benefits of labor mobility are potentially massive and ease the burden on fiscal policy greatly. We quantify these gains in Sections 2.7 and 2.8.

How mobile is labor in the data? Legally, labor mobility is guaranteed as one of the four pillars of economic integration within the European Union. EU citizens are free to migrate to any other EU country to seek employment (Kahanec 2012, Zimmermann 2005). Workers are also free to move across state borders in the U.S., as well as provincial borders in Canada and state borders in Australia. Despite similar legal environments, actual labor mobility is much higher within the U.S., Canada and Australia than within the EU.

Figure 2.3 plots the extent of labor mobility across countries in the EU as well as across U.S. states, Canadian provinces, and Australian states and territories. Over and above regulatory and legal barriers to mobility, language seems to rule: linguistic and cultural differences across countries make emigration much more difficult. For example, notice the high degree of labor mobility within unilingual currency unions (Australia and the U.S.), the slightly lower degree of labor mobility in a bilingual currency union (Canada), and the much lower degree of labor mobility in a multilingual currency union (euro area). One can see the importance of language most clearly by focusing on the much higher degree of mobility within EU countries (0.95%) where languages are uniform than across EU countries (0.29%) where they differ, as well as the the high mobility across Canada as a whole (0.98%) versus the low degree of mobility between French-speaking Quebec and the English-speaking provinces (0.39%). These data reinforce the notion that labor mobility is vital to the sound functioning of a currency union: high mobility in the US, Australia and Canada dampens internal wage rigidity, lowers unemployment and improves risk sharing. On the other hand, low mobility in the EU leads to high unemployment and overvalued wages in areas that are hit by large negative shocks.

2.7 Welfare Analysis in the Closed-Form Model

In this section we analyze the welfare gains resulting from a tax union, a transfer union, labor mobility, and from eliminating wage rigidity via flexible exchange rates or contingent fiscal policy within a currency union. The advantage of the closed-form solution is most apparent here. Rather than approximating a quadratic welfare function around a particular steady state, we can calculate welfare explicitly at any steady state, whether distorted or otherwise. This is particularly important when we focus on the welfare gains from a tax union, which eliminates the terms of trade markup from the steady state allocation. In a log-linear model, comparing welfare between the tax union and no tax union cases is infeasible because of the different steady states.

To begin, we take consumption and labor from the five allocations: flexible exchange rates with complete risk-sharing (Proposition 9); flexible exchange rates in financial autarky (Proposition 10); currency union in complete markets (Proposition 11); currency union in financial autarky (Proposition 12); and labor mobility (Proposition 17). Using consumption and labor from each allocation, we then derive the expected utility. Allocations that eliminate wage rigidity (via flexible exchange rates or contingent fiscal policy) are denoted by $flex$, while those that do not are denoted by $fixed$. We assume that there is no contingent fiscal policy in the currency union allocations, so that wages are rigid. Similarly, allocations with complete international risk-sharing (via complete markets or a transfer union) are denoted by *complete*, while autarky allocations with no risk-sharing are denoted by $autarky$.

Below we calculate the log of expected utility for the five allocations. We ignore the constant terms and focus only on the exponents of Z. Details on how we compute welfare analytically for each allocation are found in Appendix B.7. We assume technology is log-normally distributed in all countries, $\log(Z_i) \sim N(0, \sigma_Z^2)$, and is independent across time and across countries.

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} = \frac{(\gamma-1)(1-\sigma)(1+\varphi)^2}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2 \tag{2.45a}$$

$$\log \mathbb{E}\left\{U_{fixed,complete}\right\} = \frac{(\gamma-1)(1-\sigma)(1+\varphi)(1+\varphi-\gamma\varphi)}{(\sigma+\varphi)}\sigma_Z^2 \tag{2.45b}$$

$$\log \mathbb{E}\left\{U_{flex,autarky}\right\} = \frac{(\gamma-1)(1-\sigma)(1+\varphi)^2\left[1+\varphi+(\gamma-1)(1-\sigma)(\sigma+\varphi)\right]}{(\sigma+\varphi)[1-\sigma+\gamma(\sigma+\varphi)]^2}\sigma_Z^2 \tag{2.45c}$$

$$\log \mathbb{E}\left\{U_{fixed,autarky}\right\} = \frac{(\gamma-1)(1-\sigma)(1+\varphi)\left[1-(\gamma-1)(\sigma+\varphi)\right]}{\sigma+\varphi}\sigma_Z^2 \tag{2.45d}$$

$$\log \mathbb{E}\left\{U_{labor\ mobility}\right\} = \frac{(\gamma-1)(1+\varphi)}{\sigma+\varphi}\sigma_Z^2 \tag{2.45e}$$

Using these expected utilities, and the fact that any constant terms will cancel out when subtracted from each other, we calculate the welfare differences for four scenarios: (1)

complete markets vs. autarky for flexible wages; (2) complete markets vs. autarky for fixed wages; (3) flexible vs. fixed wages for complete markets; and (4) flexible vs. fixed wages for autarky. When comparing welfare across different allocations, it is important to keep in mind that as risk-aversion decreases, (i.e. as $\sigma \to 1$), the welfare differences expressed in logarithms also decrease but the *absolute values* of utility increase. In other words, when risk aversion is low, the welfare differences shown in (2.46a) – (2.46d) will shrink, but this does not mean that the welfare differences are decreasing in absolute value.

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{flex,autarky}\right\} = \frac{\sigma(\gamma-1)^2(1-\sigma)(1+\varphi)^2}{(\sigma+\varphi)(1+\gamma\varphi)[1-\sigma+\gamma(\sigma+\varphi)]}\sigma_Z^2 \tag{2.46a}$$

$$\log \mathbb{E}\left\{U_{fixed,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} = \frac{\sigma(\gamma-1)^2(1-\sigma)(1+\varphi)}{\sigma+\varphi}\sigma_Z^2 \tag{2.46b}$$

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,complete}\right\} = \frac{\gamma\varphi^2(\gamma-1)^2(1-\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2 \tag{2.46c}$$

$$\log \mathbb{E}\left\{U_{flex,autarky}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} = \frac{(\gamma-1)^2(1-\sigma)(1+\varphi)[\gamma(\sigma+\varphi)-\sigma]}{1+\gamma(\sigma+\varphi)-\sigma}\sigma_Z^2 \tag{2.46d}$$

Not surprisingly, equations (2.46a) – (2.46d) prove that for non-unitary Armington elasticity: (1) improved risk-sharing always has positive welfare consequences; and (2) moving from fixed to flexible exchange rates always has positive welfare consequences. We also see that in the special case of unitary elasticity the expected utility for all policy coalitions is identical. Under this special assumption, there is no difference in welfare between a fixed and flexible exchange rate, nor is there any benefit from improved risk-sharing across countries. Equations (2.46a) – (2.46d) thus demonstrate the restrictive nature of assuming unitary elasticity. In particular, as we've mentioned above, unitary elasticity of substitution between home and foreign goods: (i) leads to complete risk-sharing, eliminating any difference between allocations in complete markets and financial autarky *and* (ii) eliminates wage rigidities, removing the difference between allocations under flexible exchange rates and within a currency union as well as between non-contingent and contingent domestic fiscal policy in a currency union. In both cases, risk-sharing and the elimination of nominal rigidities occur via movements in the terms of trade. This explains why Obstfeld and Rogoff (2002) and others found such small gains from cooperation: when the elasticity is unitary, there are simply no gains from cooperation available as movements in the terms of trade fill the role of cross-country risk-sharing and negate the influence of nominal rigidities.

One can see this explicitly by comparing consumption in all allocations when $\gamma = 1$:

$$C_{flex,complete|\gamma=1} = C_{fixed,complete|\gamma=1} = C_{flex,autarky|\gamma=1} = C_{fixed,autarky|\gamma=1} = C_{labor\ mobility|\gamma=1}.$$

Under unitary elasticity, consumption is equalized across all risk-sharing regimes, and across all exchange rate regimes. This demonstrates the importance of considering welfare away from the unitary elasticity case. The closed-form solution allows us to evaluate the optimal design of a fiscal union under non-unitary elasticity and isolate the impact of risk sharing, wage rigidity and labor mobility from a welfare perspective.

Another interesting welfare comparison concerns the gains from improved risk-sharing outside of and within currency unions. Using (2.46a) – (2.46d), one can easily show that the gains from improved risk sharing are higher within a currency union than outside of one (which matches the results of Farhi and Werning (2012)),

$$\log \mathbb{E}\left\{U_{fixed,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} \geq \log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{flex,autarky}\right\},$$

and that the gains from a flexible exchange rate are higher in autarky than under complete risk-sharing

$$\log \mathbb{E}\left\{U_{flex,autarky}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} \geq \log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,complete}\right\}.$$

In other words, the benefits of a transfer union are higher in a currency union, and the benefits of independent monetary policy are higher in financial autarky.

2.7.1 Calibration in the Closed-Form Model

After deriving analytical expressions for welfare, we examine the benefits of a tax union, a transfer union, a flexible exchange rate, contingent fiscal policy and labor mobility. Our calibration for the closed-form model is reported in Table 2.1. We calibrate our parameters at quarterly frequency according to standard benchmarks given in Gali and Monacelli (2005, 2008), Rotemberg and Woodford (1997) and others.

In our welfare analysis, we allow the Armington elasticity to vary while fixing the other parameters of the model. Our aim is to calculate welfare exactly for a wide range of

Armington elasticity values. Each plot below shows the loss in consumption as a percentage of the Pareto optimal allocation for a range of γ.

2.7.2 Tax Union

We begin by comparing the welfare of a country in a tax union (denoted by tax) with the welfare of a country outside of a tax union (denoted by $notax$), assuming that the two countries are identical in all other respects. Aside from the terms of trade externality, both countries are subject to the same distortions, whether that be incomplete risk-sharing or wage rigidity. They differ only in the fact that one country faces a terms of trade markup in it's steady state allocation (the country outside of a tax union) and the other does not (the country within a tax union). The log difference in expected utility between a country inside a tax union and a country outside of a tax union is:

$$\log \mathbb{E}\left\{U_{tax}\right\} - \mathbb{E}\left\{U_{notax}\right\} = \left(\frac{1-\sigma}{\sigma+\varphi}\right)\log \mu_{\gamma} = \left(\frac{1-\sigma}{\sigma+\varphi}\right)\log\left(\frac{\gamma}{\gamma-1}\right). \tag{2.47}$$

Equation (2.47) shows that the welfare gains from a tax union are decreasing in γ, the degree of substitutability between products across countries. As goods become closer substitutes, country level monopoly power falls and the distortionary impact of the terms of trade externality decreases due to the declining markup on exports. In the limit, as $\gamma \to \infty$ and goods become perfect substitutes, the terms of trade markup will go to zero, and a tax union will be unnecessary. On the other hand, as the Armington elasticity decreases, countries gain a higher degree of monopoly power over their export good and thus increase their terms of trade markup μ_γ. In the limit, as $\gamma \to 1$, the terms of trade markup approaches infinity and the benefits of a tax union dwarf the gains from other policy measures.

Figure 2.4 illustrates the importance of a tax union for low values of the Armington elasticity and plots the loss in consumption from the terms of trade distortion relative to the Armington elasticity. The figure makes it clear that as $\gamma \to 1$, a tax union becomes imperative. In the absence of a tax union, optimizing non-cooperative fiscal authorities charge extremely large markups on their export good, leading to a dismal equilibrium for all countries. This is one of the most important insights we glean from the closed-form model: non-cooperative fiscal policy inflicts extremely large welfare losses on other countries under low Armington elasticity. Because there are very few papers that consider non-cooperative fiscal policy, this dynamic is generally absent in the literature. As we've mentioned before, it is important to point out that the benefits of a tax union occur in the steady state because the terms of trade markup is present in the steady state. Business cycle fluctuations and shocks have no bearing on the welfare gains from a tax union. The gains from a tax union are a steady state phenomenon, and as such, cannot be analyzed in a log-linear model.

2.7.3 Transfer Union, Flexible Wages and Labor Mobility

Having established the relative importance of a tax union with respect to the Armington elasticity, we now turn to the welfare gains achieved through improved risk-sharing (via a transfer union or deeper financial integration), the elimination of wage rigidity (via optimal monetary policy outside of a currency union or contingent fiscal policy within a currency union) and labor mobility. In what follows, we assume the presence of a tax union, which removes the constant terms of trade markup.

Figure 2.5 plots the consumption in each allocation as a percentage of the Pareto optimum. The negative impact of the wage rigidity distortion dominates the negative impact of financial autarky, as both fixed wage allocations perform quite poorly relative to the flexible wage allocations, particularly as the degree of substitutability increases. The relative similarity of all flexible wage allocations (Flex Complete, Flex Autarky, and Labor Mobility) is quite striking. Even in financial autarky, when wages are flexible households are able to stabilize consumption with small movements in their labor hours. As a result, the benefit of consumption risk-sharing is very small when wages are flexible. The gains from flexible wages approach 2% of permanent consumption under complete risk-sharing and 4% of permanent consumption under financial autarky for $\gamma = 10$. On the other hand, the welfare gains from perfect risk-sharing via a transfer union or complete markets equal 2% of permanent consumption within a currency union when $\gamma = 10$.

2.7.4 Are countries better off in a currency union?

One of the arguments in support of a currency union, advanced by Mundell (1961, 1973) among others, is that the formation of such a union will lead to deeper financial integration and improve cross-country risk sharing. Using this logic, we conduct a thought experiment on the potential benefits of a currency union. We compare the welfare of a country outside a currency union in financial autarky with a member of a currency union in complete markets. Is a country better off with a flexible exchange rate and no risk sharing, or in a currency union with perfect risk sharing? The answer depends on the degree of risk aversion as defined by σ, the inverse Frisch elasticity of labor supply φ, as well as the Armington elasticity γ. Details are found in Appendix B.8. Here we focus on the intuition.

When households are completely risk neutral ($\sigma = 0$), they prefer a country with independent monetary policy in financial autarky if $\gamma > 1$. As households become more risk averse, they prefer a country that is a member of a currency union with full risk-sharing for a wider range of values of γ. In the limit, as $\sigma \to \infty$, households prefer a currency union if $\gamma \in \left(1, \frac{1+2\varphi}{\varphi}\right)$. For standard calibrations of φ, this means households will prefer a currency union when γ is between 1 and 2. So there is a very small range of γ for which households are better off in a currency union in complete markets than outside of one in financial autarky. Even under extreme risk aversion, deeper financial integration is not worth the loss of independent monetary policy.

As export goods become closer substitutes, the welfare losses from financial autarky fall relative to the gains from independent monetary policy. What causes this? Assume country i is hit with a negative productivity shock. If country i is a member of a currency union, wage rigidity will force its producers to charge a higher price. With a flexible exchange rate, the higher domestic price would be offset by a depreciated currency, but in a currency union this effect is absent. Given the higher price, consumers in country i and in other countries will switch to cheaper substitutes. If the elasticity of substitution is very high, demand for country i's good will collapse, and country i will produce almost nothing. If markets within the currency union are complete or a transfer union is in place, consumption must be equal across countries. However only a few countries will produce any output, and households in those few countries will have to work long hours to supply goods for the whole currency union. As a result, average consumption and welfare will fall. This effect is exacerbated as goods become closer substitutes. In the limit, when goods are perfect substitutes ($\gamma = \infty$) and shocks are asymmetric, only one country in the currency union will produce any output, and consumption and welfare will equal zero for all countries in the union. This illustrates the challenges that arise within a currency union when countries produce similar goods and face wage rigidity.

On the other hand, when substitutability is close to one, the welfare losses from wage rigidity and the gains from risk-sharing go to zero. Terms of trade movements will provide risk-sharing and insulate economies from the negative impact of asymmetric productivity shocks and nominal rigidities. In this case, a country will be indifferent between remaining outside a currency union in financial autarky and joining a currency union with full risk-sharing.

In reality of course, membership in a currency union does not guarantee perfect risk sharing through access to complete markets, nor does lack of membership in a currency union prevent countries from accessing international financial markets. Whether countries enjoy some degree of cross-border risk sharing seems to be largely unrelated to their membership in a currency union, although it is true that the introduction of the euro led to an increase in cross-border lending within the euro area, as well as an initial convergence of borrowing rates within the union. However, from a theoretical standpoint it is hard to argue that the potential risk sharing benefits of a currency union outweigh the loss of independent monetary policy.

2.8 Extended Model: Home Bias, Calvo Rigidities and Incomplete Markets

In this section we relax the assumption of complete openness and conduct welfare analyses in a model with home bias. We also employ Calvo wage rigidities in place of one period in advance wage setting and consider incomplete markets where cross-border trading in safe government bonds is permitted. The model presented here, which we refer to as the extended model, is identical in all other respects to the closed-form model described in Section 2.2. The extended model is laid out in full detail in Appendix B.9.

In each country i, the consumption basket consists of home (C_{it}^H) and foreign (C_{it}^F) goods,

$$C_{it} = \left[(1-\alpha)^{\frac{1}{\eta}}(C^H it)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{it}^F)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.48}$$

where C_{it}^F and C_{it}^H are defined as

$$C_{it}^F = \left(\int (C_{ijt}^F)^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}} \quad \text{and} \quad C_{it}^H = \left(\int (C_{it}^H(h))^{\frac{\varepsilon-1}{\varepsilon}} dh\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.49}$$

C^F_{ijt} denotes consumption by households in country i of the variety produced by country j, while $C^H_{it}(h)$ denotes consumption by households in country i of the domestic variety produced by intermediate firm h. The elasticity of substitution between home and foreign products is defined by η, while the elasticity of substitution between the goods of different countries remains γ. The relative weight of these goods in the consumption basket is defined by the degree of home bias, $1-\alpha$. When $\alpha = 0$, home bias is complete and households only consume domestic goods. In the opposite extreme, when $\alpha = 1$, the economy is fully open and households will consume a basket made up entirely of imports from all other countries in the world.

imilarly the price index will consist of goods prices of both home and foreign products:

$$P_{it} = \left[(1-\alpha)P^{1-\eta}_{H,it} + \alpha P^{1-\eta}_{F,it}\right]^{\frac{1}{1-\eta}}. \tag{2.50}$$

Relative demand for home and foreign products is given by

$$C_{H,it} = (1-\alpha)\left(\frac{P_{H,it}}{P_{it}}\right)^{-\eta} C_{it}, \tag{2.51}$$

$$C_{F,it} = \alpha\left(\frac{P_{F,it}}{P_{it}}\right)^{-\eta} C_{it}, \tag{2.52}$$

where $C_{F,it}$ and $C_{H,it}$ are defined as

$$C_{F,it} = \left(\int C_{F,ijt}^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}}, \tag{2.53}$$

$$C_{H,it} = \left(\int C_{H,it}(h)^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{2.54}$$

$C_{F,ijt}$ denotes consumption by households in country i of the variety produced by country j. $C_{H,it}(h)$ denotes consumption by households in country i of the domestic variety produced by intermediate firm h.

Calvo wage setting can be expressed as

$$W_t^{1-\varepsilon} = (1-\theta_W)\tilde{W}_t^{1-\varepsilon} + \theta_W W_{t-1}^{1-\varepsilon} \tag{2.55}$$

where W_t is the actual wage, $\tilde{W}$ is the optimal reset wage and θ_W is the fraction of households who are able to reset wages in each period.

There is a nominal government bond that pays in units of the import basket C_F. Households will maximize utility from (2.4) subject to the following budget constraint:

$$C_{it}(h) + \frac{B_{it}(h)}{P_{it}} = (1-\tau_i)\left(\frac{W_{it}(h)}{P_{it}(h)}\right) N_{it}(h) + \mathcal{D}_{it}(h) + \mathcal{T}_{it}(h) + \Gamma_{it}(h) + (1+i_{t-1})\left(\frac{B_{it-1}(h)}{P_{it}}\right). \tag{2.56}$$

The domestic interest rate i_t equals the world interest rate plus a country specific interest rate premium $p()$ that is strictly increasing in the amount of debt B_t:

$$i_t = i^* + p(B_t). \tag{2.57}$$

Financial autarky is the case for which p goes to infinity. The interest rate premium is necessary to ensure stationarity, a feature demonstrated in Schmitt-Grohe and Uribe (2003).

2.8.1 Calibration in the Extended Model

As in the closed-form case, we calibrate the extended model at a quarterly frequency. All parameter values are found in Table 2.2. We conduct a number of experiments to measure the robustness of our closed-form results for varying degrees of openness (α) and substitutability (γ). We calibrate $\eta = 1$, the elasticity between home and foreign goods, to match Gali and Monacelli (2005, 2008) and Farhi and Werning (2012), although later we consider cases where $\eta > 1$. The assumption of $\eta = 1$ leads to a Cobb-Douglas consumption basket: $C_{it} = C_{H,it}^{1-\alpha} C_{F,it}^{\alpha}$. We still allow for non-unitary elasticity between the products of different countries defined by γ. In our baseline calibration, we set $\theta_W = 0.93$, which implies that wages are reset every three years. This is a conservative parameterization relative to recent estimates by Schmitt-Grohe and Uribe (2011 and 2012), who estimate the degree of downward wage rigidity for a number of economies in Europe from 2008-2011. They find complete downward wage rigidity in a number of countries, including Greece, Portugal and Spain. Although wages are more flexible in the upward direction, our focus here is on the negative effect of downward wage rigidity and the large welfare losses that accrue in a currency union under this scenario. We consider a wider range of wage rigidity parameterizations in Figure 2.9.

2.8.2 Welfare Analysis in the Extended Model

We first analyze the welfare gains from a tax union. We compare the difference in steady state consumption between a set of countries outside of a tax union with those inside a tax union. Figure 2.6 plots the loss in permanent consumption from the terms of trade externality as a function of openness and the Armington elasticity. The distortionary impact of the terms of trade markup is increasing in both openness and the Armington elasticity. One can see this by examining the optimal non-cooperative income tax rate in the extended model, $\tau_i = 1 - \mu_\varepsilon \left(\frac{\gamma - 1 + (1-\alpha)\eta}{\gamma - (1-\alpha)(1-\eta)} \right)$, which we derive in Appendix B.10. This tax rate is increasing in openness and both elasticities, η and γ.

The intuition is as follows. As the degree of home bias increases, optimizing non-cooperative fiscal policymakers find it less desirable to impose a large terms of trade markup on their export good because home consumers pay the markup when they consume home products. This is a result of the law of one price: if the fiscal authority taxes workers in order to reduce supply and increase the price of its unique final good, households in all countries will suffer but home households will suffer more because the home good makes up (1-α) fraction of the consumption basket. In contrast, when economies are completely open, home households consume measure zero of their domestic good, and the non-cooperative fiscal authority is no longer concerned about reducing home welfare through the terms of trade markup. As we saw earlier in the closed-form version of the model, the optimal income tax rate converges to $\tau_i = 1 - \frac{\gamma}{\gamma-1}$ when economies are completely open. Within a tax union, the optimal tax rate remains $\tau_i = 1 - \mu_\varepsilon$.

Why does the welfare loss from the terms of trade externality increase in the degree of openness? When an economy is more open the household consumption basket consists of a larger percentage of imports. These imports are subject to a terms of trade markup. As a result, the more a country needs to import, the greater the negative welfare impact of the terms of trade externality. In addition, as goods become less substitutable (as the Armington elasticity decreases), fiscal authorities have an incentive to impose larger terms of trade markups on their exports through income tax setting. Country-level monopoly power is thus increasing in both openness and the Armington elasticity.

Next, we calculate the welfare losses from business cycle fluctuations in financial autarky and incomplete markets. We follow Lucas (2003) and estimate the utility from a deterministic consumption path and a risky consumption path with the same mean. We then calculate the amount of consumption necessary to make a risk averse household indifferent between the deterministic and risky consumption streams.

Figure 2.7 plots the loss in permanent consumption from business cycle fluctuations in financial autarky when wage rigidity $\theta_W = 0.93$. We ignore the impact of the terms of trade externality and assume that there is no steady state terms of trade markup here. Figure 2.7 shows that home bias lowers welfare for every value of the Armington elasticity. In other words, home bias exacerbates the negative welfare impact of wage rigidity in the absence of risk-sharing, demonstrating that our closed-form results actually provide a *lower bound* estimate of the welfare benefits of a transfer union under financial autarky. The losses in permanent consumption in financial autarky in a currency union are as high as 25% when economies are completely open ($\alpha = 1$) and 26% under full home bias ($\alpha \to 0$). Home bias thus adds about 1% to the loss in permanent consumption from rigid wages. In contrast, depending on the value of the Armington elasticity, the losses in permanent consumption can be as low as 0% ($\gamma = 1$) and as high as 26% ($\gamma = 20$). The Armington elasticity is far more important than the degree of openness in determining welfare losses.

Why does home bias increase the welfare losses from business cycle fluctuations in financial autarky? To better understand the intuition, consider the following thought experiment. Assume that wages are completely rigid, that $\alpha = 0.01$ so the economy is almost closed, and that home and foreign consumption baskets are perfect complements. In this case, one percent of consumption always goes to imports. Under financial autarky in a currency union, the cash value of imports must equal the cash value of exports. Therefore, total consumption is equal to the value of exports multiplied by 100. Fluctuations in total household consumption are thus equal to fluctuations in export revenues when the home and foreign consumption baskets are perfect complements. This effect is only strengthened when home and foreign goods are imperfect complements or substitutes. A negative shock raises the price of the home good, which will lead domestic households to substitute home goods for foreign goods in their consumption basket. The share of imports in total consumption will thus increase. But the value of imports must equal the value of exports in financial autarky, and exports are now uncompetitive on the world market due to the rise in price caused by the negative shock. As a result, total home consumption must fall.

Figure 2.8 plots the loss in permanent consumption from business cycle fluctuations in incomplete markets. The ability to trade bonds greatly improves welfare for countries in a currency union who are exposed to asymmetric shocks. Different from financial autarky, an increase in home bias improves the ability of countries to stabilize business cycles when markets are incomplete. If a country is completely open, bonds allow households to stabilize consumption but not labor, because exports are not competitive following a negative technology shock and wages are rigid. When home bias increases, stabilization of both consumption and labor is possible, because firms supply goods mainly to the home market. On the other hand, under financial autarky consumption is never stabilized, so that an increase in home bias has the opposite effect.

As a robustness check, we plot the loss in permanent consumption from business cycle fluctuations for varying levels of Calvo wage rigidity in Figure 2.9. We set openness equal to the euro area average ($\alpha = 0.35$). As in Figure 2.7, we ignore the effects of the terms of trade externality. Again, it is not simply wage rigidity that leads to large welfare losses, but rather the combination of wage rigidity *and* a high Armington elasticity. Simply put, when a country in a currency union produces exports that are easily substitutable, the welfare consequences are dramatic. When wages are completely rigid, as the evidence in Schmitt-Grohe and Uribe (2011) suggests for some European countries, the losses in permanent consumption are small for low values of elasticity, but approach 80% for high values of elasticity. Even for conservative estimates of wage rigidity, the losses in permanent consumption are quite large when the Armington elasticity is high.

Overall our findings in this section confirm our closed-form results. The Armington elasticity remains an essential parameter that governs the optimal design of a fiscal union. A tax union is more important when exports are imperfect substitutes, while a transfer union is more important when exports are close substitutes. We also find that home bias increases the distortionary impact of imperfect risk sharing, but decreases the impact of the terms of trade externality because monopoly power at the export level declines when domestic consumption baskets consist of a significant portion of domestically produced goods. In the limit, when countries only consume domestic goods, there are no terms of trade externalities. Home bias thus strengthens the need for a transfer union but decreases the need for a tax union. In addition, home bias strengthens the distortionary impact of wage rigidity, raising the importance of contingent fiscal policy or labor mobility within a currency union. Our welfare analysis also demonstrates that under the commonly assumed Cole-Obstfeld calibration, which sets $\sigma = \eta = \gamma = 1$, the welfare losses from business cycle fluctuations are extremely small. We nest the Cole-Obstfeld calibration as a special case, and show that the the distortionary impact of imperfect risk-sharing and wage rigidity is much larger as the Armington elasticity moves away from unity and goods become more substitutable.

2.8.3 Country-Specific Elasticity Estimates and Welfare Losses

In this section we move from general welfare analysis to country-specific analysis. In Table 2.3 and 2.4 we compute the welfare losses under financial autarky and incomplete markets for a number of European countries using the elasticity estimates calculated by Corbo and Osbat (2013). Their paper is one of the few to actually estimate the aggregate elasticity of substitution between home and foreign products (η) and the aggregate elasticity of substitution between the products of different countries (γ). Most trade papers focus only on sector-specific estimates. We set the Calvo wage rigidity parameter $\theta_W = 0.93$ for all countries in this section.

Table 2.3 plots the loss in permanent consumption for the mean elasticity estimates. The results are striking: the losses in permanent consumption from financial autarky are as high as 33% for Greece. Large losses in financial autarky also occur in Austria (24.78%), France (23.25%), Germany (23.90%) and Sweden (31.86%). The benefits of incomplete markets become clear here, as the losses drop to a range of 3.71% (Italy) to 17.98% (Greece) when countries are able to trade a safe government bond. The importance of maintaining access to international financial markets is quite apparent in the last column of the table, where we calculate the welfare gain when a country moves from financial autarky to incomplete markets. The values in the last column can also be viewed as a lower bound estimate of the welfare gain from the optimal transfer union, which enables perfect risk-sharing.

Table 2.4 plots the loss in permanent consumption for the median elasticity estimates. The median elasticity estimates are lower than the mean estimates. As a result, the welfare losses here are not as large, ranging from 2.8% (UK) to 5.49% (Sweden). But the losses in permanent consumption are still incredibly large relative to Lucas' (2003) estimates of the welfare cost of business cycles. These results suggest that the potential gains from international fiscal cooperation are economically significant for a number of countries in the euro area, but particularly for countries that lose access to international financial markets.

2.9 Conclusion

In this paper we provide a unique perspective on the welfare gains from international fiscal policy cooperation. We first derive a global closed-form solution for an open economy model in which countries are subject to three distortions: nominal rigidities, imperfect risk sharing and terms of trade externalities. Using this global closed-form solution, we study the benefits of a fiscal union in complete markets and financial autarky, for varying degrees of substitutability between traded goods produced in different countries, both within and outside of currency unions. This setup allows us to examine the optimal structure of a fiscal union and analytically calculate the exact welfare gains from cooperation among national policymakers for a broad set of scenarios. We then compute the welfare gains from the optimal fiscal union in a larger model that incorporates home bias and Calvo wage rigidities.

We show that the optimal design of a fiscal union depends crucially on the Armington elasticity, which defines the degree of substitutability between the products of different countries. When substitutability is low (around one), risk-sharing occurs naturally via terms of trade movements so that a transfer union is unnecessary. Terms of trade externalities are large however, and optimal policy will implement a tax union to prevent terms of trade manipulation. The welfare gains from a tax union can be as high as 3% of permanent consumption for standard calibrations. When substitutability is high (above one), risk-sharing no longer occurs naturally via terms of trade movements. If financial markets do not provide complete risk-sharing across countries, there is a role for a transfer union to insure against idiosyncratic shocks. The relative importance of a transfer union increases as goods become more substitutable. The welfare gains from a transfer union are as high as 5% of permanent consumption when countries are able to trade safe government bonds, and approach 20% of consumption when countries lose access to international financial markets. We also show that contingent domestic fiscal policy can make up for the lack of national monetary policy within a currency union and eliminate nominal rigidities. Finally, we prove that labor mobility eliminates nominal rigidities and enhances international risk-sharing.

Our results illustrate why federal currency unions such as the U.S., Canada and Australia, with relatively high labor mobility, income tax harmonization and built-in fiscal transfer arrangements, can withstand asymmetric shocks across regions much better than the euro area, which lacks many of these ingredients at the moment. The potential welfare gain from implementing such policies in the euro area is quite large, particularly for countries that produce highly substitutable export goods and that cannot raise funds on international financial markets to insure against downside risk.

Table 2.1: Calibration of the Closed-Form Model

Parameter	Value	Description
σ	2	Risk aversion parameter
φ	3	Inverse labor supply elasticity (Gali and Monacelli (2005, 2008))
χ	1	Following Gali and Monacelli (2005, 2008)
ε	6	Elasticity between different types of labor
σ_Z	0.01	Standard deviation of technology
β	0.99	Household discount factor
γ	Varies	Armington elasticity

Table 2.2: Calibration of the Extended Model with Home Bias and Calvo Wage Rigidity

Parameter	Value	Description
σ	2	Risk aversion parameter
φ	3	Inverse labor supply elasticity (Gali and Monacelli (2005, 2008))
χ	1	Following Gali and Monacelli (2005, 2008)
ε	6	Elasticity between different types of labor
θ_W	Varies	Calvo parameter for wage rigidity
α	0.25	Openness (Gali and Monacelli (2005, 2008))
η	1	Elasticity between home and foreign goods
ρ_Z	0.95	Persistence of technology shock
σ_Z	0.01	Standard deviation of technology
β	0.99	Household discount factor
γ	Varies	Armington elasticity

Table 2.3: Losses in Permanent Consumption from Business Cycle Fluctuations for Mean Country-Specific Elasticity Estimates

	α	η	γ	Welfare Loss Autarky	Welfare Loss Incomp. Mkts	Welfare Gain from Access to Financial Markets	Welfare Loss Autarky	Welfare Loss Incomp. Mkts	Welfare Gain from Access to Financial Markets
Austria	0.55	24.5	5.3	24.78	12.27	12.51	4.88	3.04	1.84
Czech	0.7	13.3	9.5	23.29	10.82	12.47	4.53	3.09	1.44
Denmark	0.4	8.6	4.2	15.03	6.58	8.45	3.72	1.71	2.01
Finland	0.32	10.9	4.6	18.16	7.03	11.13	4.09	1.91	2.18
France	0.31	13.2	8.9	23.25	9.85	13.4	4.72	2.54	2.18
Germany	0.34	8.2	13.3	23.9	10.54	13.36	4.79	2.69	2.1
Greece	0.35	28.5	22	33.67	17.98	15.69	5.6	3.99	1.61
Hungary	0.49	9.6	7.7	19.55	9.28	10.27	4.28	2.44	1.84
Italy	0.22	4.6	4.7	13.57	3.71	9.86	3.38	1.06	2.32
Netherlands	0.62	9.5	5.3	16.48	8.04	8.44	3.85	2.17	1.68
Portugal	0.41	6.8	5.4	15.67	6.32	9.35	3.72	1.75	1.97
Slovakia	0.3	8.5	7.8	19.87	7.67	12.2	4.32	2.02	2.3
Spain	0.3	5.5	6.7	16.67	5.88	10.79	3.88	1.64	2.24
Sweden	0.42	15.8	25.5	31.86	17.74	14.12	5.48	3.96	1.52
UK	0.37	4.4	6	14.64	5.51	9.13	3.56	1.55	2.01
European Avg	0.35	11.46	9.126666667	22.39	9.73	12.66	4.62	2.52	2.1

All losses are in percent. Openness (α) is taken from Balta and Delgado (2009). The elasticity of substitution between home and foreign products (η) and the elasticity of substitution between the products of different countries (γ) for European countries is taken from Table 4 and 5 of Corbo and Osbat (2013).

Table 2.4: Losses in Permanent Consumption from Business Cycle Fluctuations for Median Country-Specific Elasticity Estimates

	α	η	γ	Welfare Loss Autarky	Welfare Loss Incomplete Mkts	Welfare Gain from Access to Financial Markets
Austria	0.55	5.7	4.0	12.23	4.77	7.46
Czech Rep.	0.70	3.8	4.3	11.46	4.12	7.34
Denmark	0.40	3.9	3.6	9.83	4.05	5.78
Finland	0.32	4.0	3.5	10.59	3.23	7.36
France	0.31	4.3	4.4	12.47	3.97	8.50
Germany	0.34	4.2	5.3	13.61	4.73	8.88
Greece	0.35	3.1	4.6	11.38	3.75	7.63
Hungary	0.49	3.8	4.5	11.46	4.72	6.74
Italy	0.22	3.4	3.4	10.03	2.36	7.67
Netherlands	0.62	4.1	3.7	9.87	4.47	5.4
Portugal	0.41	3.6	4.1	10.87	3.90	6.97
Slovakia	0.30	4.1	4.3	12.13	3.75	8.38
Spain	0.30	3.8	3.5	10.43	3.04	7.39
Sweden	0.42	5.0	5.2	13.89	5.49	8.40
UK	0.37	3.1	3.3	8.95	2.80	6.15
European Avg	0.35	3.9	4.1	11.35	3.74	7.61

All losses are in percent. Openness (α) is taken from Balta and Delgado (2009). The elasticity of substitution between home and foreign products (η) and the elasticity of substitution between the products of different countries (γ) for European countries is taken from Table 4 and 5 of Corbo and Osbat (2013).

Figure 2.1: Model Timeline

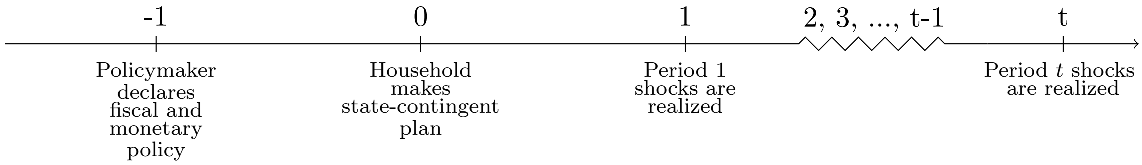


Figure 2.2: Types of Fiscal Union

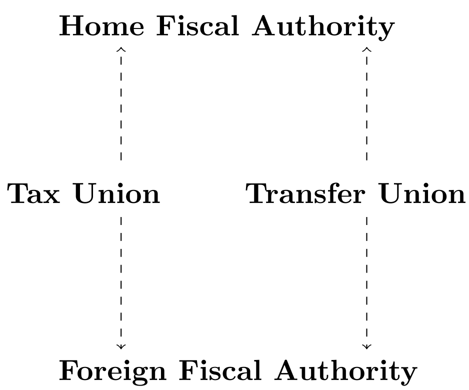
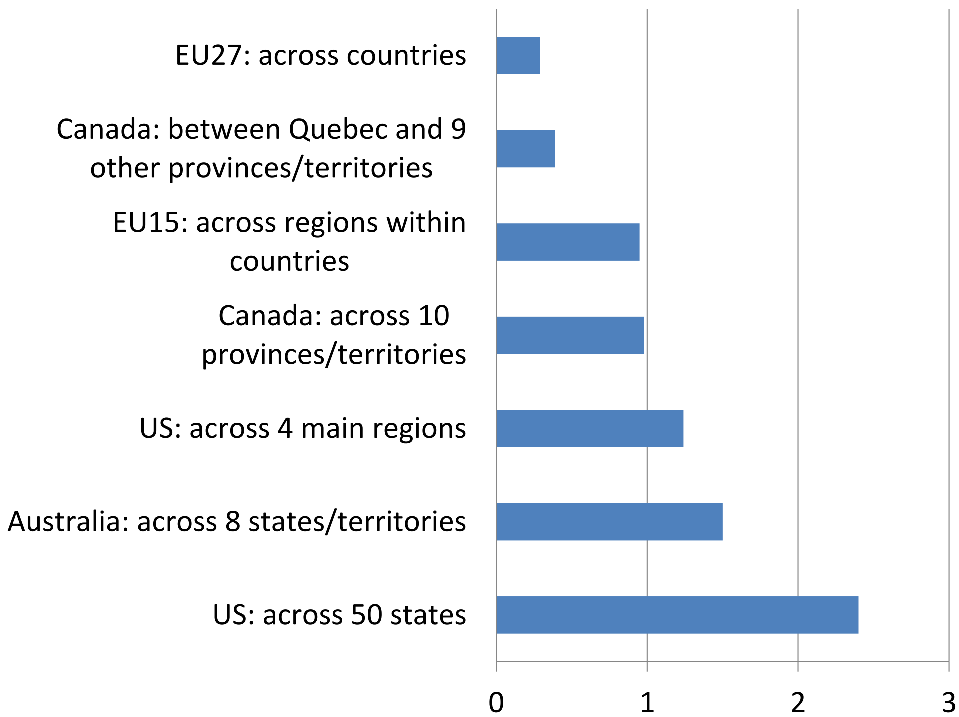


Figure 2.3: Annual Cross-Border Mobility, % of Population (2010)



Source: OECD

Figure 2.4: Welfare Losses From Terms of Trade Externalities in the Closed-Form Model

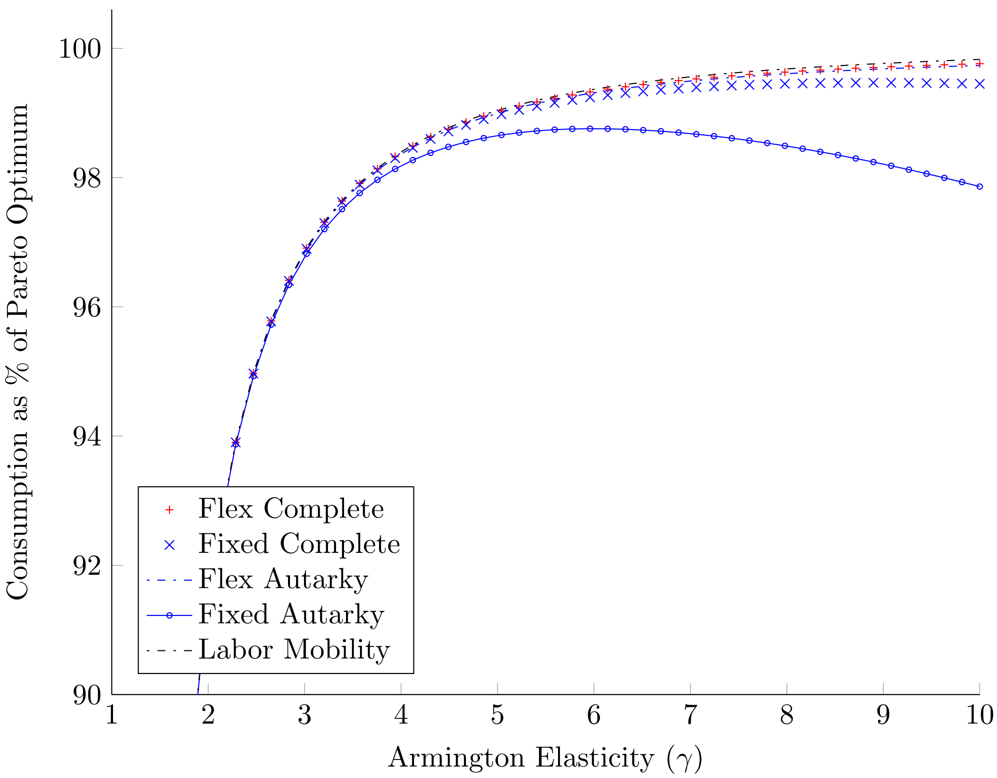


Figure 2.5: Welfare Losses from Business Cycle Fluctuations in the Closed-Form Model

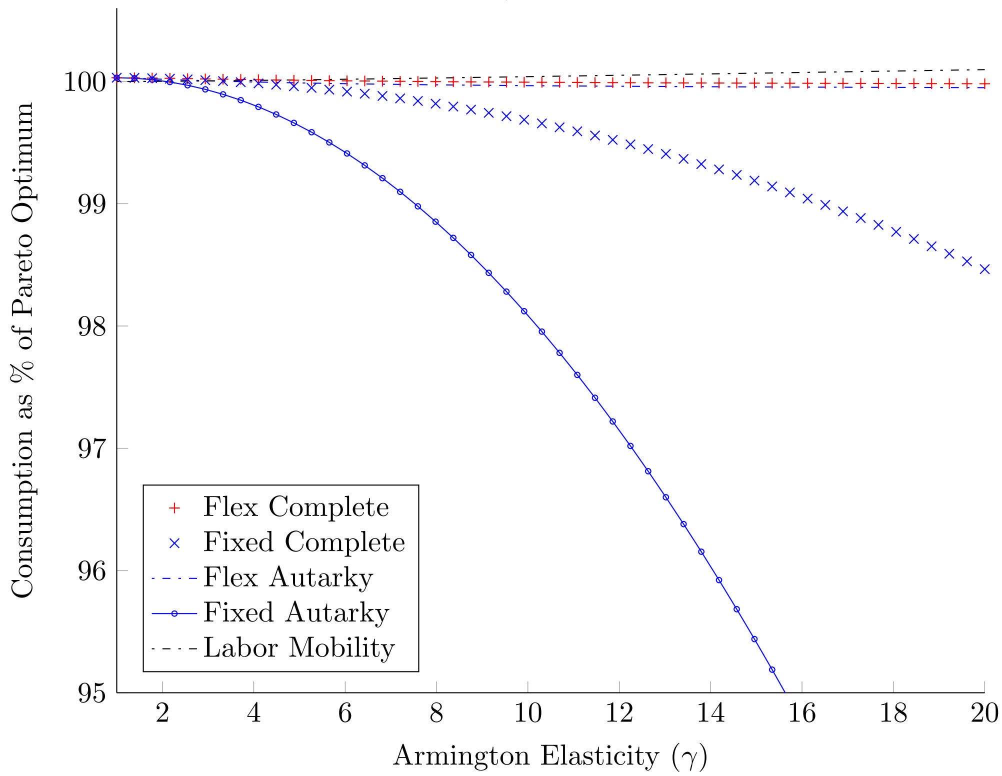


Figure 2.6: Welfare Losses from Terms of Trade Externalities

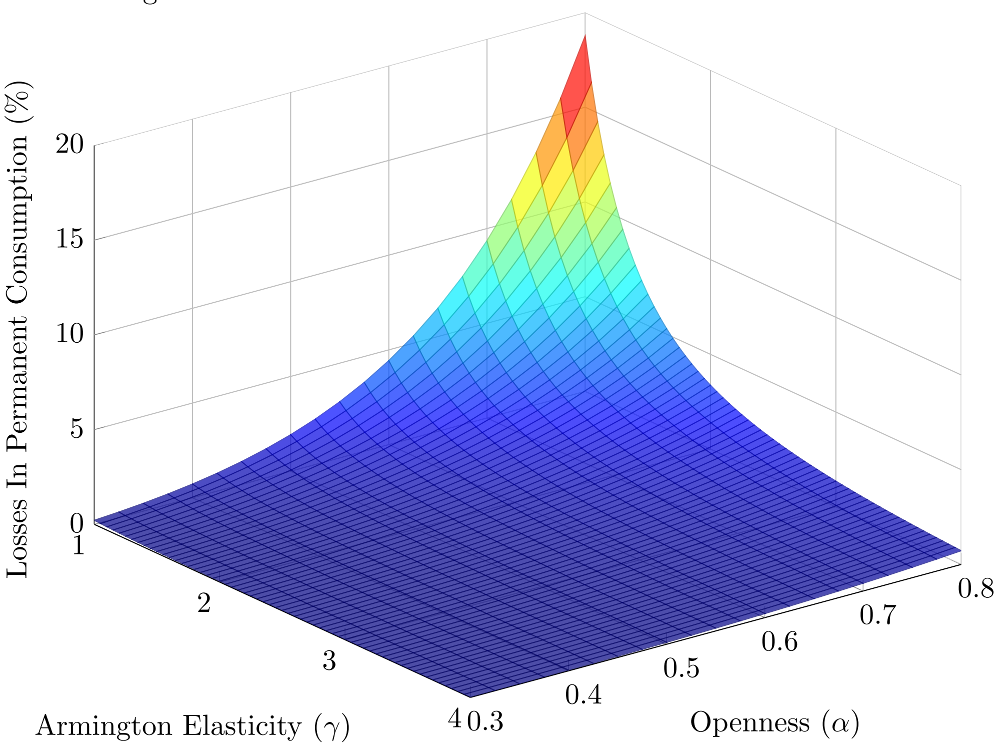


Figure 2.7: Welfare Losses from Business Cycle Fluctuations in Financial Autarky

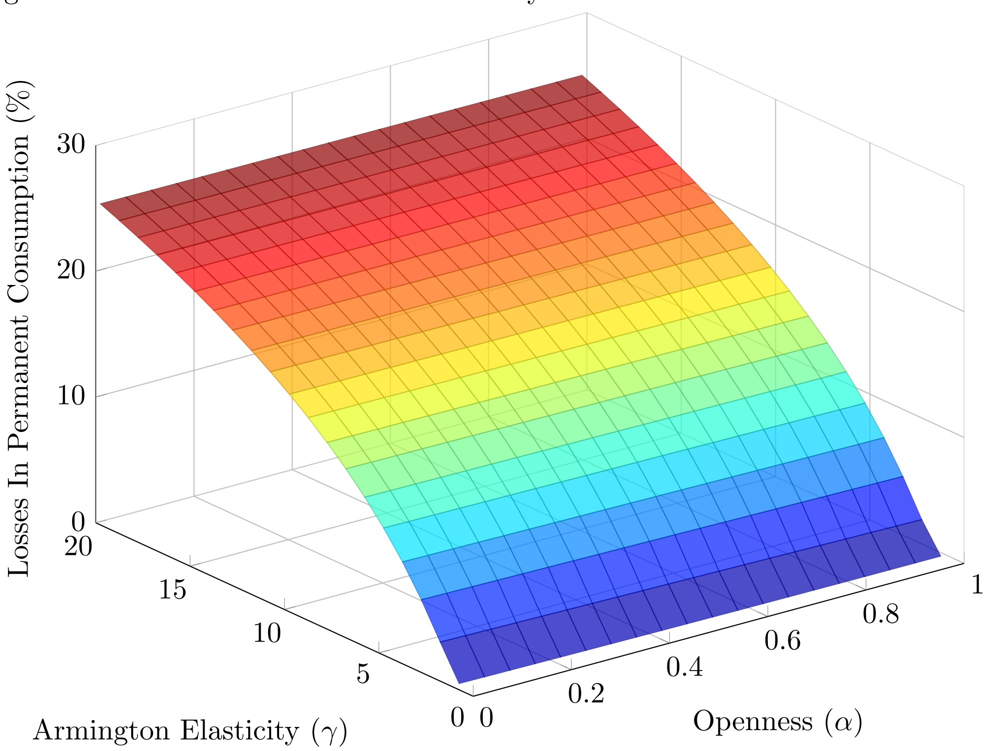


Figure 2.8: Welfare Losses from Business Cycle Fluctuations in Incomplete Markets

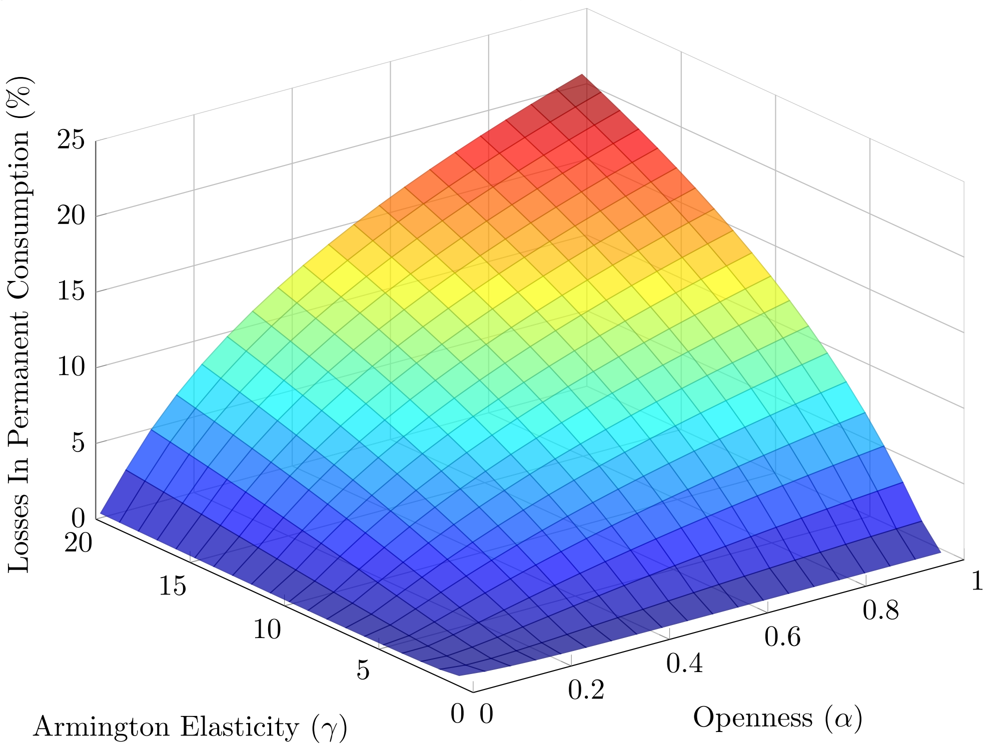
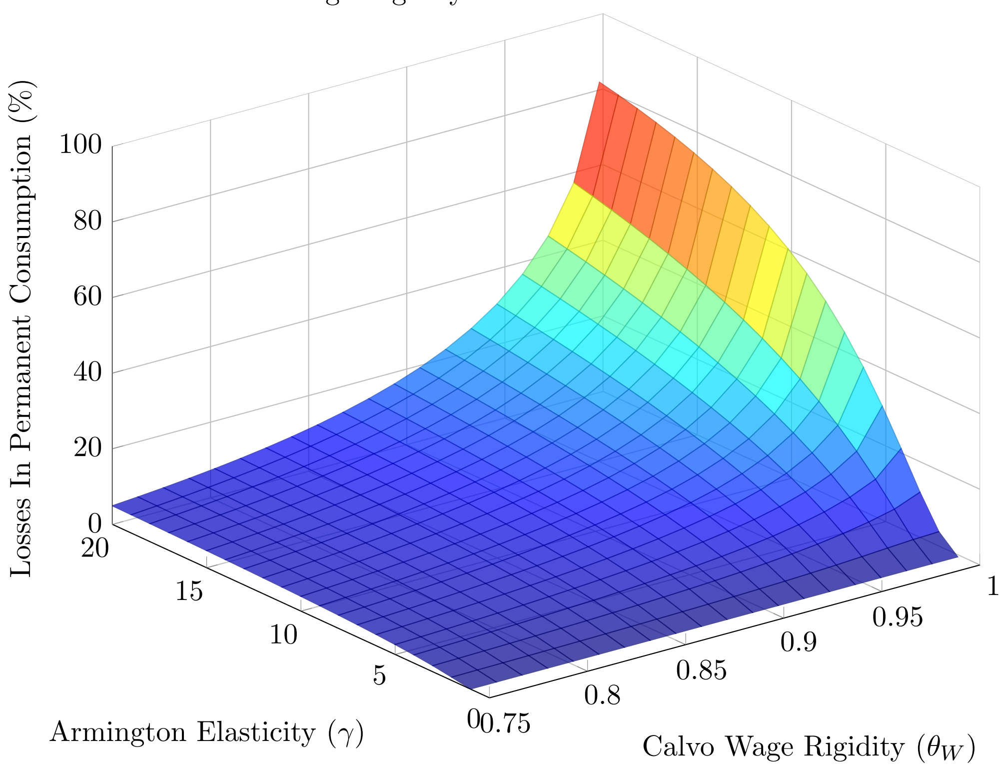


Figure 2.9: Welfare Losses from Business Cycle Fluctuations in Financial Autarky for Different Levels of Wage Rigidity



Chapter 3

The Financial Accelerator and the Optimal Lending Contract

3.1 Introduction

In one of the foundational papers in the literature on financial frictions in macroeconomic models, Bernanke, Gertler and Gilchrist (1999) derive a contract between risk averse lenders and risk neutral borrowers in the costly state verification (CSV) framework of Townsend (1979).[1] Although this loan contract has become the standard contract for CSV models of financial frictions, it is not optimal because it assumes returns for lenders are predetermined and borrowers are myopic.

In this paper we relax these two assumptions and derive the optimal history-independent

[1] A non-exhaustive list of some important early contributions in this literature include Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997).

loan contract in the CSV model.[2] We allow returns to the lender to be contingent on the aggregate state of the economy following early criticism of Bernanke, Gertler and Gilchrist (1999) by Chari (2003). Chari's concerns were later formalized by Carlstrom, Fuerst and Paustian (2012). We also introduce forward looking entrepreneurs who maximize the present discounted value of all future consumption instead of next period expected consumption.[3]

Our analysis provides three main results. First, under the optimal contract we find that financial frictions do not amplify business cycles. Relative to a model with financial frictions, monetary and technology shocks generate much larger output responses when frictions are absent. Second, shocks to the cross-sectional variance of entrepreneurs' idiosyncratic productivity — so-called "risk" shocks — have little to no impact on the real economy, in contrast with the standard Bernanke, Gertler and Gilchrist contract (hereafter BGG). This is particularly important as recent work by Christiano, Motto and Rostagno (2013) emphasizes the importance of risk shocks in driving business cycles. We show here that risk shocks provide amplification only when the lending contract is suboptimal. Third, we show that the financial accelerator in the original BGG framework is dependent on three key characteristics: a suboptimal contract, loose monetary policy and extremely persistent technology shocks. We conduct a number of robustness checks in Section 3.5 and find

[2]To be precise, we derive the optimal one-period contract with deterministic monitoring. An excellent list of references for partial equilibrium multi-period contracts includes Monnett and Quintin (2005) for stochastic monitoring, Wang (2005) for deterministic monitoring, and Cole (2013) for self-enforcing stochastic monitoring.

[3]In an October 2013 version of their paper, Carlstrom, Fuerst and Paustian independently derive the dynamically optimal contract for forward looking households without risk shocks. Different from their paper, we introduce a frictionless benchmark to evaluate amplification and also consider the effect of risk shocks.

that the removal of any one of these characteristics significantly weakens or eliminates the financial accelerator. Overall, our results cast doubt on the qualitative and quantitative importance of the financial accelerator in the costly state verification framework.

Our model is standard and consists of a risk averse representative household and risk neutral entrepreneurs. Entrepreneurs borrow money from the representative household and purchase capital to use in production. Entrepreneurs are identical ex ante but differ depending on the ex post realization of an idiosyncratic productivity shock. Both agents have full information about the distribution of idiosyncratic shocks ex ante, so there is no adverse selection problem. Borrowers observe the realization of their idiosyncratic shock, but lenders do not: they need to pay monitoring costs to observe it.

In the BGG contract borrowers guarantee a constant safe rate of return to lenders in order to maximize returns on their equity. As a result, borrowers absorb all risk in the economy. It should be noted that this is an assumption and not an equilibrium condition. Because of this assumption, negative shocks cause a decline in entrepreneurs' net worth which leads to a tightening of financial constraints. The subsequent fall in investment and output is stronger than the effect from the initial shock. This results in the financial accelerator: the BGG contract amplifies macroeconomic fluctuations in a dynamic stochastic general equilibrium (DSGE) model.

Recent work by Carlstrom, Fuerst and Paustian (2012), hereafter CFP, shows that the BGG contract is suboptimal because the predetermined deposit rate does not provide appropriate consumption insurance for risk averse households. In CFP, risk neutral entrepreneurs find it optimal to offer lenders a contract with a negative covariance between the rate of return and the lenders' consumption. During a recession when consumption is low, entrepreneurs pay a higher borrowing rate in order to provide household consumption insurance. Entrepreneurs thus have to pay a higher interest rate on their loans exactly when their net worth is already low. Because the quantity of capital in the economy is a function of net worth, the dynamics of net worth directly affect investment, leading to a sharp decline in investment during recessions. The stabilizing effect of the insurance channel on household consumption is outweighed by a much stronger decline in investment, leading to higher volatility in the economy at large.

In BGG, CFP and the entire CSV literature entrepreneurs are myopic: they maximize their expected *next period* consumption, but expected utility depends on the expected discounted stream of *all future* consumption. We depart from the literature and embed forward looking entrepreneurs into an otherwise standard CSV framework. Our analysis provides a number of results that call the robustness of the financial accelerator into question under optimal and suboptimal contracts, for myopic and non-myopic entrepreneurs.

The intuition is as follows. When lenders' returns are predetermined, we find that to a first order approximation the lending contract is identical regardless of whether entrepreneurs are forward looking or myopic. In period t, the predetermined lending rate is chosen to satisfy the lender's Euler equation in that specific period without the possibility of revisions in period $t+1$. As a result, it does not matter whether entrepreneurs are forward looking or not, as the lender's stochastic discount factor determines the rate of return. In order to generate amplification however, this suboptimal contract must be combined with other ingredients. In our robustness exercise in Section 3.5, we show that contracts with a predetermined deposit rate only generate a financial accelerator when monetary policy deviates from price stability and when technology shocks are stationary.

On the other hand, when lender's returns are chosen optimally and vary with the aggregate state of the economy, the presence of forward looking entrepreneurs or myopic entrepreneurs matters greatly. In CFP, entrepreneurs sell as much insurance to the household as they can because insurance does not effect their next period expected consumption. During a recession, the provision of insurance leads to very tight financial constraints for entrepreneurs, as they face a higher lending rate due to the fall in household consumption. During a boom the opposite occurs: myopic entrepreneurs have too much capital and earn small returns on their capital. In other words myopic entrepreneurs miss good investment opportunities on a consistent basis because they do not take the future flow of capital returns into account when making investment decisions. Under the optimal contract however, forward looking entrepreneurs sell less insurance because they are concerned not only about next period expected consumption but also expected consumption in all future periods, which is impacted by insurance claims. In particular, forward looking entrepreneurs desire high net worth in states of the world where the financial premium is also high.

To provide more intuition on the role of forward looking entrepreneurs, consider the following example. Assume that ex-post there is a shock which suddenly decreases the entrepreneur's net worth. Lower net worth today means that the financial premium today and in the future will be higher. The entrepreneur desires more net worth in states with a higher financial premium because capital returns are higher and borrowing is more costly. Forward looking entrepreneurs thus find it profitable to enter into an ex-ante agreement that stipulates a lower lending rate in these states. Correspondingly, entrepreneurs prefer to pay a higher lending rate when a shock increases net worth, because the financial premium will be lower in these states. This interplay between movements in net worth and the financial premium leads risk-neutral entrepreneurs to behave in a "risk averse" manner because they want to avoid borrowing in states with a high financial premium. In contrast, if there is no costly state verification so that financial frictions are absent, non-myopic entrepreneurs will ignore concerns about the financial premium and provide as much insurance as possible, generating large amplification.

We also find that risk shocks have little effect on the real economy and give the wrong comovement between macroeconomic aggregates when contracts are optimal. This contrasts with Christiano, Motto and Rostagno (2013), who employ the BGG contract and emphasize the importance of risk shocks in generating business cycle fluctuations. Under the BGG contract, increased idiosyncratic variance causes an increase in defaults leading to a decline in the price of capital and consequently net worth. However, if returns to lenders are not predetermined and entrepreneurs are forward looking, they realize that lower net worth implies higher financial premiums and more costly borrowing in the future. Therefore, forward looking entrepreneurs desire more net worth in these states and thus negotiate lower returns to lenders, which stabilizes the response of net worth to the shock. As a result, under the optimal contract the financial accelerator is severely dampened for risk shocks.

3.2 The Optimal Lending Contract in Partial Equilibrium

Our main theoretical contribution in this paper is to introduce forward looking entrepreneurs into an otherwise standard CSV model of financial frictions. In this section we outline the key differences between the dynamically optimal loan contract chosen by utility maximizing entrepreneurs and the alternative loan contracts in BGG and CFP in a partial equilibrium setting. Here we assume that entrepreneurs take the price of capital and the expected return to capital as given. In Section 3.3 we endogenize these variables in general equilibrium.

At time t, entrepreneur j purchases capital $K_t(j)$ at a unit price of Q_t. At time $t+1$, the entrepreneur rents this capital to perfectly competitive wholesale goods producers. The entrepreneur uses his net worth $N_t(j)$ and a loan $B_t(j)$ from the representative lender to purchase capital:

$$Q_t K_t(j) = N_t(j) + B_t(j). \tag{3.1}$$

After buying capital, the entrepreneur is hit with an idiosyncratic shock $\omega_{t+1}(j)$ and an aggregate shock R^k_{t+1}, so that entrepreneur j is able to deliver $Q_t K_t(j) R^k_{t+1} \omega_{t+1}(j)$ units of assets. The idiosyncratic shock $\omega(j)$ is a log-normal random variable with distribution $\log(\omega(j)) \sim \mathcal{N}(-\frac{1}{2}\sigma^2_\omega, \sigma^2_\omega)$ and mean of one.

Following BGG, we assume entrepreneurs are risk neutral and die with constant probability $1-\gamma$. Upon dying, entrepreneurs consume all operational equities, which are equal to net worth minus wages. If entrepreneurs survive they do not consume anything, and they supply labor and earn wages which they later reinvest. Entrepreneur j's value function is

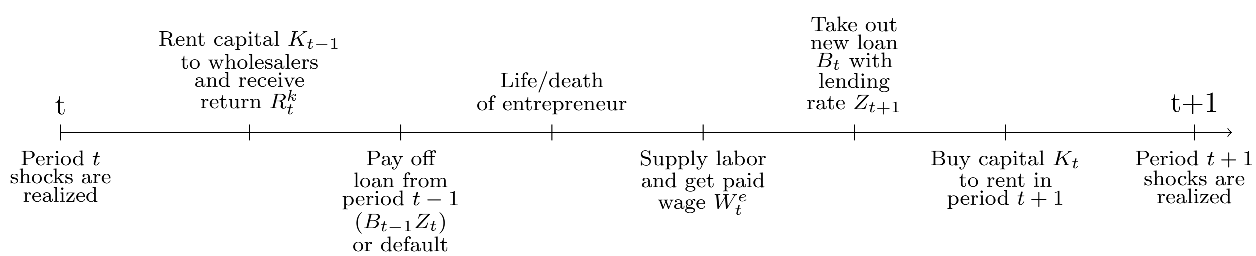
$$V_t^e(j) = (1-\gamma)\sum_{s=1}^{\infty}\gamma^s C_{t+s}^e \tag{3.2}$$

where C_{t+s}^e is the entrepreneur's consumption,

$$C_t^e(j) = N_t(j) - W_t^e \tag{3.3}$$

defined as wealth accumulated from operating firms, equal to net worth without entrepreneurial real wages W_t^e. The timeline for entrepreneurs is plotted in Figure 3.1.

Figure 3.1: Timeline for Entrepreneurs



3.2.1 Borrower and Lender Payoffs

The contract between the lender and borrower follows the familiar CSV framework. We assume that the lender cannot observe the realization of idiosyncratic shocks to entrepreneurs unless he pays monitoring costs μ which are a fixed percentage of total assets. Given this friction, the risk neutral borrower offers the risk averse lender a contract with an state-contingent interest rate Z_{t+1} subject to macroeconomic conditions.

The entrepreneur repays the loan only when it is profitable to do so. In particular, the entrepreneur will repay the loan only if, after repayment, he has more assets than liabilities. We define the cutoff productivity level $\bar{\omega}_{t+1}$, also known as the bankruptcy threshold, as the minimum level of productivity necessary for an entrepreneur to repay the loan:

$$\underbrace{B_t(j)Z_{t+1}(j)}_{\text{Cost of loan repayment}} = \underbrace{\bar{\omega}_{t+1}R^k_{t+1}Q_tK_t(j).}_{\text{Minimum revenue for loan repayment}} \tag{3.4}$$

If $\omega_{t+1}(j) < \bar{\omega}_{t+1}$ the entrepreneur defaults and enters bankruptcy; if $\omega_{t+1}(j) \geq \bar{\omega}_{t+1}$ he repays the loan. The cutoff productivity level allows us to express the dynamics of net worth for a particular entrepreneur j:

$$N_{t+1}(j) = Q_tK_t(j)R^k_{t+1} \max\left\{\omega_{t+1}(j) - \bar{\omega}_{t+1}, 0\right\} + W^e_{t+1}. \tag{3.5}$$

The gross rate of return for the lender, R_{t+1}, also depends on the productivity cutoff. For idiosyncratic realizations above the cutoff, the lender will be repaid the full amount of the loan $B_t(j)Z_{t+1}(j)$. For idiosyncratic realizations below the cutoff, the entrepreneur will enter bankruptcy and the lender will pay monitoring costs μ and take over the entrepreneur's assets, ending up with $(1-\mu)K_t(j)R^k_{t+1}h(\omega_{t+1}(j))$. More formally, the lender's ex post return is

$$B_t(j)R_{t+1}(j) = \begin{cases} B_t(j)Z_{t+1} & \text{if } \omega_{t+1}(j) \geq \bar{\omega}_{t+1} \\ (1-\mu)K_t(j)R^k_{t+1}\omega_{t+1}(j) & \text{if } \omega_{t+1}(j) < \bar{\omega}_{t+1} \end{cases} \tag{3.6}$$

Taking into account that loans to entrepreneurs are perfectly diversifiable, the lenders return on a loan R_{t+1} to entrepreneur j is defined as

$$B_t(j)R_{t+1} \equiv Q_t K_t(j) R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}), \tag{3.7}$$

where $h(\bar{\omega}_{t+1}, \sigma_{\omega,t})$ is the share of total returns to capital that go to the lender. We define this share as

$$h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = \left\{ \underbrace{\bar{\omega}_{t+1}\left[1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t})\right]}_{\text{Share to lender if loan pays}} + \underbrace{(1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega}_{\text{Share to lender if loan defaults}} \right\} \tag{3.8}$$

where f is the probability density function and F is the cumulative distribution function of the log-normal distribution of idiosyncratic productivity.

In order to simplify the entrepreneur's optimization problem, we introduce the concept of leverage, κ_t, defined as the value of the entrepreneur's capital divided by net worth:

$$\kappa_t(j) \equiv Q_t K_t(j)/N_t(j). \tag{3.9}$$

3.2.2 Loan Contracts: BGG, CFP and the Optimal Contract

The differences between the BGG contract, the CFP contract and the optimal contract arise from two sources: the lender's participation constraint and the borrower's objective function.

First, the lender's participation constraint in BGG differs from CFP and the optimal contract. The participation constraint arises from the household Euler equation and stipulates the minimum rate of return that entrepreneurs must offer to lenders to receive a loan. In BGG, the participation constraint has the following form:

$$\mathbb{E}_t\Big\{\Lambda_{t,t+1}\Big\}R_{t+1} = 1, \tag{3.10}$$

where

$$\Lambda_{t,s} \equiv \beta^s \frac{U_{C,t+s}}{U_{C,t}} \tag{3.11}$$

is the household (*i.e.* shareholder) intertemporal marginal rate of substitution, also known as the household stochastic discount factor. Under this participation constraint, entrepreneurs pay a constant safe rate of return to the lenders, R_{t+1}, which ignores the risk averse representative household's desire for consumption insurance. In contrast, the participation constraint in CFP and the optimal contract is:

$$\mathbb{E}_t\Big\{\Lambda_{t,t+1}R_{t+1}\Big\} = 1. \tag{3.12}$$

As CFP show, the above expression implies that households prefer a state contingent rate of return that is negatively correlated with household consumption. Quite simply, households like consumption insurance. In recessions, households desire a higher rate of return because their marginal utility of consumption is high, and vice versa in booms.

Second, the borrower's objective function in BGG and CFP differs from the optimal contract. Entrepreneurs in BGG and CFP maximize next period net worth, defined in equation (3.5). If we substitute the expression for leverage from (3.9) into (3.5), we have the entrepreneur's objective function in BGG and CFP:

$$\kappa_t(j)N_t(j)\mathbb{E}_t\left\{R^k_{t+1}\max\left[\omega_{t+1}(j)-\bar{\omega}_{t+1},0\right]\right\}. \tag{3.13}$$

In contrast, under the dynamically optimal contract entrepreneurs maximize utility, given by (3.2). As we have mentioned before, utility maximizing entrepreneurs are concerned not only about current capital returns but also future capital returns and future financial premiums.

We now have all of the ingredients necessary to set up the entrepreneur's optimization problem and solve for the three different loan contracts: (1) the BGG contract; (2) the CFP contract; and, (3) the optimal contract.

Proposition 18 *To solve for the BGG contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize next period net worth (3.13) subject to (3.5),* *(3.7) and (3.10). The solution to this problem is given by*

$$\kappa_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\mathbb{E}_t \left\{ \frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \right\} \frac{1}{\mathbb{E}_t \Lambda_{t,t+1}}. \tag{3.14}$$

where $g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = \int_{\bar{\omega}_{t+1}}^{\infty} \omega f(\omega, \sigma_{\omega,t}) d\omega - \bar{\omega}_{t+1} \Big[1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Big]$.

Proof See Appendix C.2. ■

Corollary 1 *Log-linearization of the BGG optimality condition (3.14) and the BGG participation constraint (3.10) gives*

$$\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{3.15}$$

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = 0 \tag{3.16}$$

where the constant $\nu_\kappa = \frac{\frac{h_{\omega\omega}}{h_\omega} - \frac{g_{\omega\omega}}{g_\omega}}{-\frac{g_\omega}{g} + \frac{h_\omega}{h} + \frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}} \frac{1}{\kappa - 1}$ *and* $\nu_\sigma = \frac{-\frac{h_{\sigma\omega}}{h}\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right) + \frac{h_\omega}{h}\left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_{\sigma\omega}}{g}\right)}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}} \sigma_\omega$.

Proof See Appendix C.7. ■

Equation (3.15) shows that in the BGG contract the entrepreneur's leverage depends on next period's expected financial premium while (3.16) shows that lenders returns (deposit rate) are predetermined. We prove in Appendix C.5 that when lenders' returns are predetermined, to a first order approximation the lending contract is identical regardless of whether entrepreneurs are forward looking or myopic.[4]

[4]This is also true when the lending rate Z is predetermined, rather than lenders returns R. The general

Proposition 19 *To solve for the CFP contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize (3.13) subject to (3.5), (3.7) and (3.12). The solution to this problem is given by*

$$\kappa_t \mathbb{E}_t \Big\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Big\} = -\frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{1}{\Lambda_{t,t+1}}. \tag{3.17}$$

Proof See Appendix C.3. ■

Corollary 2 *Log-linearization of the CFP optimality condition (3.17) and the CFP participation constraint (3.12) gives*

$$\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{3.18}$$

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_t R^k_{t+1} - \tilde{\alpha}\sigma(\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1}) \tag{3.19}$$

where $\tilde{\alpha} = -\frac{\frac{h_\omega}{h}}{\frac{h_{\omega\omega}}{h_\omega} - \frac{g_{\omega\omega}}{g_\omega}}$.

Proof See Appendix C.7. ■

Corollary 2 clearly illustrates the differences between the BGG contract and the CFP contract. In equation (3.19), lender's returns depend on capital returns and household consumption, both elements which are missing in the BGG contract. For standard calibrations, $\tilde{\alpha}$ takes a value between five and six and the risk aversion parameter σ is equal to one, so that lender's returns are very sensitive to the consumption level and the consumption in-

equilibrium behavior of the model under predetermined R and predetermined Z is similar, so we only report results for predetermined R.

surance channel dominates the response to capital returns. When consumption is high, the lending rate declines; when consumption is low the lending rate increases. The negative covariance between the lender's consumption and returns reflects the nature of insurance, which amplifies the impact of shocks to the economy. Note that as entrepreneurs become more risk averse (as σ decreases), the impact of the consumption insurance channel declines.

Now that we have described the BGG and CFP contracts in detail, we turn our attention to the optimal contract. As we discussed above, the optimal contract takes the consumption insurance channel from CFP and adds forward looking entrepreneurs.

Proposition 20 *To solve for the optimal contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize (3.2) subject to (3.3), (3.5), (3.7) and (3.12). The solution to this problem is given by*

$$\kappa_t \mathbb{E}_t \left\{ \Psi_{t+1} R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{\Psi_{t+1}}{\Lambda_{t,t+1}} \tag{3.20}$$

where

$$\Psi_t = 1 + \gamma \kappa_t \mathbb{E}_t \left\{ g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R^k_{t+1} \Psi_{t+1} \right\} \tag{3.21}$$

Proof See Appendix C.4. ■

Corollary 3 *Log-linearization of the optimal contract, (3.20) and (3.21), and the partici-*

pation constraint (3.12) gives

$$\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = \nu_\kappa\hat{\kappa}_t + \nu_\sigma\hat{\sigma}_{\omega,t} \tag{3.22}$$

$$\hat{R}_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_t R^k_{t+1} - \tilde{\alpha}\Big[\sigma(\hat{C}_{t+1} - \mathbb{E}_t\hat{C}_{t+1}) + \hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1}\Big] \tag{3.23}$$

$$\hat{\Psi}_{t+1} = \epsilon_N\mathbb{E}_{t+1}\Big\{(\kappa-1)(\hat{R}^k_{t+2} - \hat{R}_{t+2}) + \hat{R}^k_{t+2} + \nu_\Psi\hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2}\Big\} \tag{3.24}$$

where $\nu_\Psi = \frac{g_\sigma - h_\sigma\frac{g_\omega}{h_\omega}}{g}\sigma_\omega$.

Proof See Appendix C.7. ■

We see from (3.23) that under the optimal contract, the surprise to lender's returns depends not only on surprises to capital returns and consumption, as in the CFP contract, but *future* capital returns and *future* financial premiums as well. If entrepreneurs are more optimistic about expected future financial premiums or future returns to capital, they prefer to pay the lender a lower interest rate because one unit of net worth becomes more valuable. Corollary 3 thus illustrates the strong stabilizing mechanism of the optimal contract. When a crisis hits and decreases entrepreneur's net worth, expected future financial premiums will rise. But entrepreneurs will also pay lenders a smaller deposit rate, which stabilizes their net worth. As a result, the main channel for the financial accelerator, the volatility in net worth, is diminished when entrepreneurs are forward looking.

Although we have taken a partial equilibrium view here, Corollaries 1-3 are identical in the general equilibrium setting. In both partial and general equilibrium, leverage and the deposit rate are determined by the paths of capital returns and consumption. Therefore, the intuition provided by Corollaries 1-3 holds in general equilibrium.

3.3 The Model in General Equilibrium

We now embed the three loan contracts in a standard dynamic New Keynesian model. There are six agents in our model: households, entrepreneurs, financial intermediaries, capital producers, wholesalers and retailers. Entrepreneurs buy capital from capital producers and then rent it out to perfectly competitive wholesalers, who sell their goods to monopolistically competitive retailers. Retailers costlessly differentiate the wholesale goods and sell them to households at a markup over marginal cost. Retailers have price-setting power and are subject to Calvo price rigidities. Households bundle the retail goods in CES fashion into a final consumption good. A graphical overview of the model is provided in Figure 3.2. The dotted lines denote financial flows, while the solid lines denote real flows (goods, labor, and capital).

3.3.1 Households

The representative household maximizes its utility by choosing the optimal path of consumption, labor and money

$$\max \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \zeta \log\left(\frac{M_{t+s}}{P_{t+s}}\right) - \chi \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\}, \tag{3.25}$$

where C_t is household consumption, M_t/P_t denotes real money balances, and H_t is household labor effort. The budget constraint of the representative household is

$$C_t = W_t H_t - T_t + \Pi_t + R_t \frac{D_t}{P_t} - \frac{D_{t+1}}{P_t} + \frac{M_{t-1} - M_t}{P_t} + \frac{B_{t-1} R_t^n - B_t}{P_t} \tag{3.26}$$

where W_t is the real wage, T_t is lump-sum taxes, Π_t is profit received from household ownership of final goods firms distributed in lump-sum fashion, and D_t are deposits in financial intermediaries (banks) that pay a contingent nominal gross interest rate R_t, and B_t are nominal bonds that pay a gross nominal non-contingent interest rate R_t^n.

Households maximize their utility (3.25) subject to the budget constraint (3.26) with respect to deposits, labor, nominal bonds and money, yielding four first order conditions:

$$U_{C,t} = \beta \mathbb{E}_t \left\{ R_{t+1} U_{C,t+1} \right\}, \tag{3.27}$$

$$U_{C,t} = \beta R_t^n \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\} \tag{3.28}$$

$$W_t U_{C,t} = \chi H_t^{\eta}, \tag{3.29}$$

$$U_{C,t} = \zeta \frac{1}{m_t} + \beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\}. \tag{3.30}$$

We define the gross rate of inflation as $\pi_{t+1} = P_{t+1}/P_t$, and real money balances as $m_t = M_t/P_t$.

3.3.2 Retailers

The final consumption good is made up of a basket of intermediate retail goods which are aggregated together in CES fashion by the representative household:

$$C_t = \left(\int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3.31}$$

Demand for retailer i's unique variety is

$$c_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\varepsilon} C_t, \tag{3.32}$$

where p_{it} is the price charged by retail firm i. The aggregate price index is defined as

$$P_t = \left(\int_0^1 p_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{3.33}$$

Each retail firm chooses its price according to Calvo (1979) in order to maximize net discounted profit. With probability $1-\theta$ each retailer is able to change its price in a particular period t. Retailer i's objective function is

$$\max_{p_{it}^*} \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t+s} \frac{p_{it}^* - P_{t+s}^w}{P_{t+s}} \left(\frac{p_{it}^*}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \right\}, \tag{3.34}$$

where P_t^w is the wholesale goods price. The first order condition with respect to the retailer's price p_{it}^* is

$$\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} (p_{it}^*/P_{t+s})^{-\varepsilon} Y_{t+s} \left[p_{it}^* - \frac{\varepsilon}{\varepsilon - 1} P_{t+s}^w \right] \right\} = 0. \tag{3.35}$$

From this condition it is clear that all retailers which are able to reset their prices in period t will choose the same price $p_{it}^* = P_t^* \ \ \forall i$. The price level will evolve according to

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{3.36}$$

Dividing the left and right hand side of (3.36) by the price level gives

$$1 = \left[\theta \pi_{t-1}^{\varepsilon-1} + (1-\theta)(p_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{3.37}$$

where $p_t^* = P_t^*/P_t$. Using the same logic, we can normalize (3.35) and obtain:

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} p_{t+s}^w \right\}}{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} \right\}}, \tag{3.38}$$

where $p_{t+s}^w = \frac{P_{t+s}^w}{P_t}$ and $p_{t+s} = P_{t+s}/P_t$.

3.3.3 Wholesalers

Wholesale goods are produced by perfectly competitive firms and then sold to monopolistically competitive retailers who costlessly differentiate them. Wholesalers hire labor from households and entrepreneurs in a competitive labor market at real wage W_t and W_t^e and rent capital from entrepreneurs at rental rate R_t^r. Note that capital purchased in period t is used in period $t+1$. Following BGG, the production function of the representative wholesaler is given by

$$Y_t = A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)}, \tag{3.39}$$

where A_t denotes aggregate technology, K_t is capital, H_t is household labor, H_t^e is entrepreneurial labor, and Ω defines the relative importance of household labor and entrepreneurial labor in the production process. Entrepreneurs inelastically supply one unit of labor, so that the production function simplifies to

$$Y_t = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega}. \tag{3.40}$$

One can express the price of the wholesale good in terms of the price of the final good. In this case, the price of the wholesale good will be

$$\frac{P_t^w}{P_t} = \frac{1}{\mathcal{X}_t}, \tag{3.41}$$

where $\mathcal{X}_t$ is the variable markup charged by final goods producers. The objective function for wholesalers is then given by

$$\max_{H_t, H_t^e, K_{t-1}} \frac{1}{\mathcal{X}_t} A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)} - W_t H_t - W_t^e H_t^e - R_t^r K_{t-1}. \tag{3.42}$$

Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are

$$\frac{1}{\mathcal{X}_t}\alpha\frac{Y_t}{K_{t-1}} = R_t^r, \tag{3.43}$$

$$\frac{\Omega}{\mathcal{X}_t}(1-\alpha)\frac{Y_t}{H_t} = W_t, \tag{3.44}$$

$$\frac{\Omega}{\mathcal{X}_t}(1-\alpha)\frac{Y_t}{H_t^e} = W_t^e. \tag{3.45}$$

3.3.4 Capital Producers

The perfectly competitive capital producer transforms final consumption goods into capital. Capital production is subject to adjustment costs, according to

$$K_t = I_t + (1-\delta)K_{t-1} - \frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1}, \tag{3.46}$$

where I_t is investment in period t, δ is the rate of depreciation and ϕ_K is a parameter that governs the magnitude of the adjustment cost. The capital producer's objective function is

$$\max_{I_t} K_t Q_t - I_t, \tag{3.47}$$

where Q_t denotes the price of capital. The first order condition of the capital producer's optimization problem is

$$\frac{1}{Q_t} = 1 - \phi_K\left(\frac{I_t}{K_{t-1}} - \delta\right). \tag{3.48}$$

3.3.5 Lenders

One can think of the representative lender in the model as a perfectly competitive bank which costlessly intermediates between households and borrowers. The role of the lender is to diversify the household's funds among various entrepreneurs. The bank takes nominal household deposits D_t and loans out nominal amount B_t to entrepreneurs. In equilibrium, deposits will equal loanable funds ($D_t = B_t$). Households, as owners of the bank, receive a state contingent real rate of return R_{t+1} on their "deposits" — which equals the rate of return on loans to entrepreneurs.[5] Households choose the optimal lending rate according to their first order condition with respect to deposits:

$$\beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} R_{t+1} \right\} = \mathbb{E}_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\} = 1.$$

As we discussed above, the lender prefers a return that co-varies negatively with household consumption, which amplifies the financial accelerator.

3.3.6 Entrepreneurs

We have already described the entrepeneur's problem in detail in Section 3.2. Entrepreneurs choose their cutoff productivity level and leverage according to: (3.14) in BGG; (3.17) in CFP; and (3.20) and (3.21) in the dynamically optimal contract.

Wholesale firms rent capital at rate $R^r_{t+1} = \frac{\alpha Y_t}{\mathcal{X}_t K_{t-1}}$ from entrepreneurs. After production

[5] Note that lenders are not necessary in the model, but we follow BGG and CFP in positing a perfectly competitive financial intermediary between households and borrowers.

takes place entrepreneurs sell undepreciated capital back to capital goods producers for the unit price Q_{t+1}. Aggregate returns to capital are then given by

$$R^k_{t+1} = \frac{\frac{1}{\mathcal{X}_t}\frac{\alpha Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t}. \tag{3.49}$$

Consistent with the partial equilibrium specification, entrepreneurs die with probability $1-\gamma$, which implies the following dynamics for aggregate net worth:

$$N_{t+1} = \gamma N_t \kappa_t R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + W^e_{t+1}. \tag{3.50}$$

3.3.7 Goods market clearing

We have goods market clearing

$$Y_t = C_t + I_t + G_t + C^e_t + \mu G(\bar{\omega}_t, \sigma_{\omega,t-1}) R^k_t Q_{t-1} K_{t-1}, \tag{3.51}$$

where $\mu G(\bar{\omega}) = \int_0^{\bar{\omega}} \mu f(\omega)\omega d\omega$ is the fraction of capital returns that go to monitoring costs, paid by lenders.

3.3.8 Monetary Policy

We assume that there is a central bank which conducts monetary policy by choosing the nominal interest rate R_t^n. In Section 3.4 we employ the nominal interest rate rule in BGG:

$$\log(R_t^n) - \log(R^n) = \rho^{R^n}\Big(\log(R_{t-1}^n) - \log(R)\Big) + \xi\pi_{t-1} + \epsilon_t^{R^n} \tag{3.52}$$

where ρ^{R^n} and ξ determine the relative importance of the past interest rate and past inflation in the central bank's interest rate rule. Shocks to the nominal interest rate are given by ϵ^{R^n}. It should be noted that the interest rule in BGG differs from the conventional Taylor rule, which targets current inflation rather than past inflation.

In Section 3.5, we consider the conventional Taylor rule with interest rate smoothing

$$\log(R_t^n) - log(R^n) = \rho^{R^n}\Big(\log(R_{t-1}^n) - \log(R)\Big) + \xi\pi_t + \rho^Y\Big(\log(Y_t) - \log(Y_{t-1})\Big) + \epsilon_t^{R^n}. \tag{3.53}$$

nd an inertial Taylor rule with interest rate smoothing

$$\log(R_t^n) - \log(R^n) = 0.5(\log(R_{t-1}^n) - \log(R)) + 0.5[1.5\pi_t + 0.5(\log(Y_t) - \log(Y_{t-1}))] + \epsilon_t^{R^n}. \tag{3.54}$$

3.3.9 Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) processes for technology, government spending and idiosyncratic volatility are given by

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon_t^A, \tag{3.55}$$

$$\log(G_t/Y_t) = (1-\rho^G)\log(G_{ss}/Y_{ss}) + \rho^G \log(G_{t-1}/Y_{t-1}) + \epsilon_t^G, \tag{3.56}$$

$$\log(\sigma_{\omega,t}) = (1-\rho^{\sigma_\omega})\log(\sigma_{\omega,ss}) + \rho^{\sigma_\omega}\log(\sigma_{\omega,t-1}) + \epsilon_t^{\sigma_\omega} \tag{3.57}$$

where ϵ^A, ϵ^G and ϵ^{σ_ω} denote exogenous shocks to technology, government spending and idiosyncratic volatility, and G_{ss} and $\sigma_{\omega,ss}$ denote the steady state values for government spending and idiosyncratic volatility respectively. Recall that σ_ω^2 is the variance of idiosyncratic productivity, so that σ_ω is the standard deviation of idiosyncratic productivity. Nominal interest rate shocks are defined by the BGG Rule in (3.52) or the Taylor rule in (3.53).

3.3.10 Equilibrium

The model has 20 endogenous variables and 20 equations. The endogenous variables are: Y, H, C, Λ, C^e, W, W^e, I, Q, K, R^n, R^k, R, p^*, $\mathcal{X}$, π, N, $\bar{\omega}$, k and Z. The equations defining these endogenous variables are: (3.9), (3.27), (3.29), (3.30), (3.11), (3.37), (3.38), (3.41), (3.40), (3.44), (3.45), (3.46), (3.48), (3.49), (3.50), (3.51), (3.52), (3.28), (B.8) and (B.18). The exogenous processes for technology, government spending and idiosyncratic volatility follow (3.55), (3.56) and (3.57) respectively. Nominal interest rate shocks are defined by the Taylor rule in (3.52).

3.4 Quantitative Analysis

3.4.1 Calibration

Our baseline calibration largely follows BGG. We set the discount factor $\beta = 0.99$, the risk aversion parameter $\sigma = 1$ so that utility is logarithmic in consumption, and the elasticity of labor is 3 ($\eta = 1/3$). The share of capital in the Cobb-Douglas production function is $\alpha = 0.35$. Investment adjustment costs are $\phi_k = 10$ to generate an elasticity of the price of capital with respect to the investment capital ratio of 0.25. Quarterly depreciation is $\delta = .025$. Monitoring costs are $\mu = 0.12$. The death rate of entrepreneurs is $1 - \gamma = .0272$, yielding an annualized business failure rate of three percent. The idiosyncratic productivity term, $\log(\omega(j))$, is assumed to be log-normally distributed with variance of 0.28. The weight of household labor relative to entrepreneurial labor in the production function is $\Omega = 0.99$.

For price-setting, we assume the Calvo parameter $\theta = 0.75$, so that only 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the BGG monetary policy rule and set the autoregressive parameter on the nominal interest rate to $\rho^{R^n} = 0.9$ and the parameter on past inflation to $\xi = 0.11$. Note that in Section 3.5 we also consider a conventional Taylor rule where the central bank targets current inflation rather than past inflation. For the conventional Taylor rule, we set $\rho^{R^n} = 0$, $\xi = 1.5$ and $\rho^Y = 0.5$ as a benchmark, and consider an inertial interest rate rule with smoothing parameter $\rho^{R^n} = 0.5$, $\xi = 0.75$ and $\rho^Y = 0.25$. We follow BGG and set the persistence of the shocks to technology and government spending at $\rho^A = 0.999$ and $\rho^G = 0.95$. We follow Christiano, Motto and Rostagno (2013) and set the persistence of idiosyncratic volatility at $\rho^{\sigma_\omega} = 0.9706$ and the distribution of the shocks equal to $\epsilon_t^{\sigma_\omega} \sim N(0, 0.0283)$.

Following BGG, we consider a one percent technology shock and a 25 basis point shock (in annualized terms) to the nominal interest rate. For the risk shock, we allow the standard deviation of idiosyncratic productivity to increase by one percentage point, from 0.28 to 0.29.

3.4.2 Quantitative Comparison: BGG, CFP and the Optimal Contract

In our quantitative analysis we compare three allocations: the competitive equilibrium under the BGG contract; the competitive equilibrium under the CFP contract; and the competitive equilibrium under the optimal contract. Impulse responses for shocks to technology, the nominal interest rate and idiosyncratic volatility are found in this section.

Figure 3.3 shows impulse responses for a extremely persistent one percent technology shock when prices are sticky. Notice the impact of consumption insurance. Lenders in the CFP contract allocations will settle for a lower rate of return in a boom in order to ensure a higher rate of return in a recession, which amplifies the response of the economy. However, this does not occur under the optimal contract because entrepreneurs are forward looking: they act as a stabilizing influence on the economy. Forward looking entrepreneurs are reluctant to invest in new capital when a positive technology shock hits because financial premiums will be low. Asset prices will decline back to their steady state value, so entrepreneurs offer higher deposit rates to lender's in order utilize financial resources in states that promise higher capital returns. The stabilizing influence of forward looking entrepreneurs cancels out the consumption insurance channel under this calibration, such that the optimal contract and BGG output responses coincide almost exactly. In general, this coincidence does not hold outside of the particular calibration employed here.

The difference between the three allocations is very noticeable in Figure 3.4, which plots impulse responses for a one percent shock to the nominal interest rate when prices are sticky. Because the monetary shock is less persistent than the technology shock, the price of capital depreciates back to its steady state value very quickly after an initial rise. As a result, capital returns are positive in the first period, but negative thereafter. This leads to an even sharper difference between the response of entrepreneurs in the three models. Under the BGG contract the deposit rate does not respond to the shock at all because it is predetermined; under the CFP contract the deposit rate falls because household consumption increases in response to the shock; and under the optimal contract the deposit rate increases, because the financial premium goes down after the shock. Forward looking entrepreneurs thus stabilize consumption and output, leading to small amplification. In contrast, the CFP contract with consumption insurance leads to a decline in the lending rate following the rise in consumption, which amplifies the response of output, consumption and other macroeconomic aggregates to the interest rate shock.

In Figure 3.5 we plot impulse responses for a one standard deviation increase in unobserved idiosyncratic volatility σ_ω. This is what we defined earlier as a risk shock. In all three models, the household consumption response on impact is close to zero, but slightly positive for CFP and BGG and slightly negative for the optimal contract. The consumption insurance channel in CFP and the optimal contract leads to a decline in the lending rate following a risk shock. An additional factor is at work under the optimal contract: the financial premium rises because, other things equal, higher idiosyncratic variance makes default more likely. Therefore, borrowing is more expensive and returns to capital are higher. Net worth thus increases on impact under the optimal contract. Overall, risk shocks have a very small impact on the real economy in the optimal contract equilibrium, and may even boost output over a longer time horizon. Also note the negative correlation between output and consumption under the optimal contract, unlike the BGG contract where output and consumption both fall.

3.5 How Robust is the Financial Accelerator? Comparison with the Frictionless Model

To truly measure the strength of the financial accelerator, we need to compare the CSV model with financial frictions against a frictionless benchmark. As our frictionless benchmark, we take the model described in Section 3.3 and set monitoring costs and idiosyncratic productivity to zero.[6]

Our frictionless model is similar to Carlstrom and Fuerst's (1997), which also sets monitoring costs equal to zero, but different from the BGG frictionless model. BGG's frictionless model assumes a constant positive financial premium. We choose our definition because a constant positive financial premium in different aggregate states implies different profits for entrepreneurs, which distorts their decisions. We focus here only on the frictionless model with zero monitoring costs, but all of our main results hold relative to both frictionless cases.

One might ask at this point, why not use the basic New Keynesian model as a frictionless benchmark? Our reasoning is as follows. The basic New Keynesian sticky price model deviates from the CSV framework in *two* dimensions: (1) it abstracts from heterogeneity between lenders and borrowers because there are no entrepreneurs, *and* (2) it has no CSV frictions. As a result, if we use the basic New Keynesian model as a frictionless benchmark, it is impossible to isolate the impact of the CSV friction on volatility from the impact of heterogeneity. In order to isolate these two effects, we need a model that incorporates heterogeneity between lenders and borrowers but which eliminates the CSV friction. Our frictionless benchmark does exactly that, providing an exact characterization of the role of the CSV friction in generating volatility.

Before we proceed to the frictionless benchmark, let us compare the amplification response

[6]The model without monitoring costs generates a different steady state relative to the model with monitoring costs, but the difference between the steady states is small for all variables except leverage. We correct steady state leverage in the frictionless model by increasing the share of entrepreneurs in the production function from 0.01 to 0.1. These modifications have a very small effect on equilibrium dynamics and do not alter our conclusions in any way.

of the model with frictions to the basic New Keynesian model. Figure 3.6 shows that all three models with frictions generate more amplification than the basic New Keynesian model for very persistent technology shocks. In this case, forward looking entrepreneurs forecast higher capital returns in the future, which makes one unit of net worth more valuable today, leading to a large increase in net worth following the shock. For less persistent technology shocks ($\rho^A = 0.99$ for example) amplification under the optimal contract is actually lower than in the basic New Keynesian model.

Figure 3.6 also shows that the optimal contract delivers slightly smaller volatility than the New Keynesian model for monetary shocks. The intuition is very simple. In the wake of a positive monetary shock net worth increases and cash is abundant, so one additional unit of net worth generates a smaller consumption flow. Therefore, entrepreneurs want to increase their payments to lenders and pay out back their increase in net worth. As a result net worth does not react to the shock, which stabilizes expenditures relative to the basic New Keynesian model. Overall then, we see that amplification under the optimal contract is slightly smaller for technology shocks with persistence equal to or lower than $\rho^A = 0.99$ as well as for monetary shocks.

Now, let us consider the amplification response of the model with frictions relative to the frictionless benchmark with no monitoring costs. As we discussed earlier, the amplifying effect of the CSV framework depends on three characteristics: a suboptimal lending contract, extremely persistent technology shocks and loose monetary policy. The removal of any one of these characteristics eliminates the financial accelerator or even reverses the accelerator, such that financial frictions stabilize the economy in the presence of shocks.

Figure 3.7 plots the output response to a variety of technology shocks in a CSV model with and without monitoring costs and demonstrates the fragility of the financial accelerator to these three characteristics. The first row of plots in Figure 3.7 shows the response of output to an extremely persistent technology shock when prices are sticky. The model with frictions provides slightly more amplification in the suboptimal BGG lending contract, while the frictionless model generates more amplification for the CFP contract and the optimal contract.

Why do financial frictions stabilize business cycles in the latter two cases? First, entrepreneurs sell insurance to the household in order to smooth household consumption. The resulting decline in household consumption volatility leads to a rise in the volatility of entrepreneurial consumption and net worth, and entrepreneurs become a driver of the business cycle. Second, when entrepreneurs are forward looking they behave in a risk averse manner by trying to tighten the financial constraint during booms, when the financial premium is low, in order to relax it during recessions, when the financial premium is high. States with positive technology shocks promise falling asset prices in the short run after the initial reaction of asset prices on impact, and higher dividends in the long run from non-stationarity. This leads entrepreneurs to lever up and increase their net worth by a large amount, generating massive amplification in the frictionless model under the optimal contract. We also find that under the CFP contract, financial frictions stabilize business cycle, although the stabilizing effect is much smaller than under the optimal contract.

The sensitivity of the financial accelerator to stationary technology shocks is illustrated clearly in the second row of Figure 3.7, where we consider technology shocks with lower persistence ($\rho^A = 0.95$). In this calibration, financial frictions stabilize business cycles not only for the optimal contract and CFP but also for BGG. Why is it the case? We know, that for flexible prices model with and without financial friction deliver very similar results under BGG. Therefore, amplification should exacerbate fluctuation of markups. However, we know that for stationary technology shocks even in standard New Keynesian models, markups move procyclically and stabilize business cycles. If financial frictions exacerbate fluctuation for markups, they stabilize the model response to stationary technology shocks, since markups become even more procyclical.

Rows three and four of Figure 3.7 demonstrate the output response for extremely persistent technology shocks with a conventional Taylor rule and under flexible prices, respectively. The impulse responses show that the financial accelerator is not robust to more conservative monetary policy or flexible prices. As in the previous cases, the frictionless model for the CFP and optimal contracts generates higher amplification than the model with CSV frictions. Under the BGG contract, the accelerator disappears when the central bank follows a conventional Taylor rule. It is still present under flexible prices, but is extremely small quantitatively. In other words, the magnitude of the financial accelerator in the BGG case is negligible when monetary policy is more aggressive or when prices are flexible.

3.5.1 Sensitivity of the Financial Accelerator to Different Monetary Policy Rules

How sensitive is the financial accelerator to different monetary policy rules?[7] Figure 3.9 plots output responses to a 25 basis point shock to the nominal interest rate for the BGG monetary policy rule (Row 1), the inertial Taylor rule (Row 2) and the conventional Taylor rule (Row 3). Here we see the sensitivity of the financial accelerator to different monetary policy specifications. Under the BGG policy rule, the coefficient on *past* inflation is $\xi = 0.11$, while the interest rate smoothing parameter is $\rho^{R^n} = 0.9$. Following the initial 25 basis point decrease in the nominal interest rate, there is little subsequent change in the interest rate under the BGG policy rule because the central bank is targeting *past* inflation, and also smoothing the interest rate. Any increase in inflation on impact is not taken into account until the next quarter. Under the BGG policy rule, monetary shocks are thus quite persistent, and entrepreneur's increase their net worth in the first period, which amplifies the shock. On the other hand, under conservative monetary policy asset prices and net worth are more stable, and there is no amplification.

We also calculate the quarterly inflation response to a monetary shock for the BGG policy rule and the conventional Taylor rule. For the conventional Taylor rule with a weight $\rho^{R^n} = 0.5$ on the previous interest rate, a two percent surprise to the Fed funds rate in annual terms leads to a one percent inflation response, while for the BGG monetary policy

[7] Gilchrist and Leahy (2002) investigate the role of monetary policy for non-contingent contracts in the BGG framework.

rule a one percent surprise to the Fed funds rate will lead to four percent inflation response, which significantly deviates from the flexible price equilibrium.

Overall, our simulations show that under the CFP contract and the optimal contract, financial frictions do not amplify business cycles for any calibration, while under the BGG contract they amplify business cycles only when technology shocks are extremely persistent and monetary policy is loose.

3.5.2 Sensitivity of the Financial Accelerator to Household Risk Aversion

One might expect that an increase in household risk aversion would shift aggregate risk onto the balance sheet of entrepreneurs, causing the financial accelerator to reappear. However, we find that households sell insurance to entrepreneurs and use labor supply to smooth their consumption regardless of the degree of risk aversion.[8] Although this may seem counterintuitive, it is natural for entrepreneurs to increase their returns by shifting aggregate risk to households, as households can smooth their consumption through variation in labor while enjoying higher returns from insuring aggregate risk. The only way to motivate households to buy insurance from entrepreneurs is to make labor supply elasticity very small, which counterfactually reduces business cycle fluctuations to a minimum. Thus, we find that financial frictions with state-contingent contracts and forward looking entrepreneurs stabilize business cycles even when households are extremely risk averse.

[8] Although not reported here, we conduct experiments for coefficients of risk-aversion, σ, from 1 to 500 and find that the financial accelerator is not present for all values.

3.6 Related Literature

There is a large literature on the role of financial frictions in macroeconomics, particularly on how such frictions amplify and propagate shocks, which is the idea of the financial accelerator. We focus here on the CSV framework, but there are other ways of modeling financial frictions in general equilibrium models. In much of the literature, returns to lenders are predetermined by assumption and lenders are unable to use state-contingent contracts, despite the fact that they desire to do so.

One early example is Kiyotaki and Moore (1997), who show that feedback between collateral prices and loans leads to amplification. However, Krishnamurthy (2003) later proved that the amplification in Kiyotaki and Moore (1997) disappears when agents are able to use collaterilized state-contingent contracts.[9]

There is also a rich literature on pecuniary externalities which applies Kiyotaki and Moore (1997) type constraints in different environments. Topics in this vein include sudden stops for emerging economies, such as Caballero and Krishnamurthy (2001, 2003, 2004), Jeanne and Korinek (2013) and Bianchi (2011), as well as research on macroprudential policies, including Stein (2012), Jeanne and Korinek (2010) and Bianchi and Mendoza (2011). Krishnamurthy's (2003) critique of Kiyotaki and Moore (1997) also applies to this literature.

[9]Krishnamurthy (2003) restores financial amplification in a three period version of the model by introducing large aggregate shocks that reverses the role of lenders and borrowers, so that lenders have to post collateral. He also introduces constraints on aggregate collateral in the spirit of Holmstrom and Tirole (1998). This is a very different channel however, which has not been investigated in quantitative general equilibrium.

Similar concerns about the financial accelerator arise in the costly state enforcement literature. In Kiyotaki and Gertler (2010) and all other examples we are aware of, lender's returns are predetermined. Jermann and Quadrini (2012) allow both debt and equity, however they introduce adjustment costs between these instruments and rule out other instruments.

Adverse selection is another way to model financial frictions. House (2006) extends the Stiglitz and Weiss (1981) framework and shows that financial frictions amplify business cycles only when returns for lenders are predetermined. When contracts allow both debt and equity, financial frictions actually stabilize business cycles. Our results in the CSV framework are consistent with House (2006).

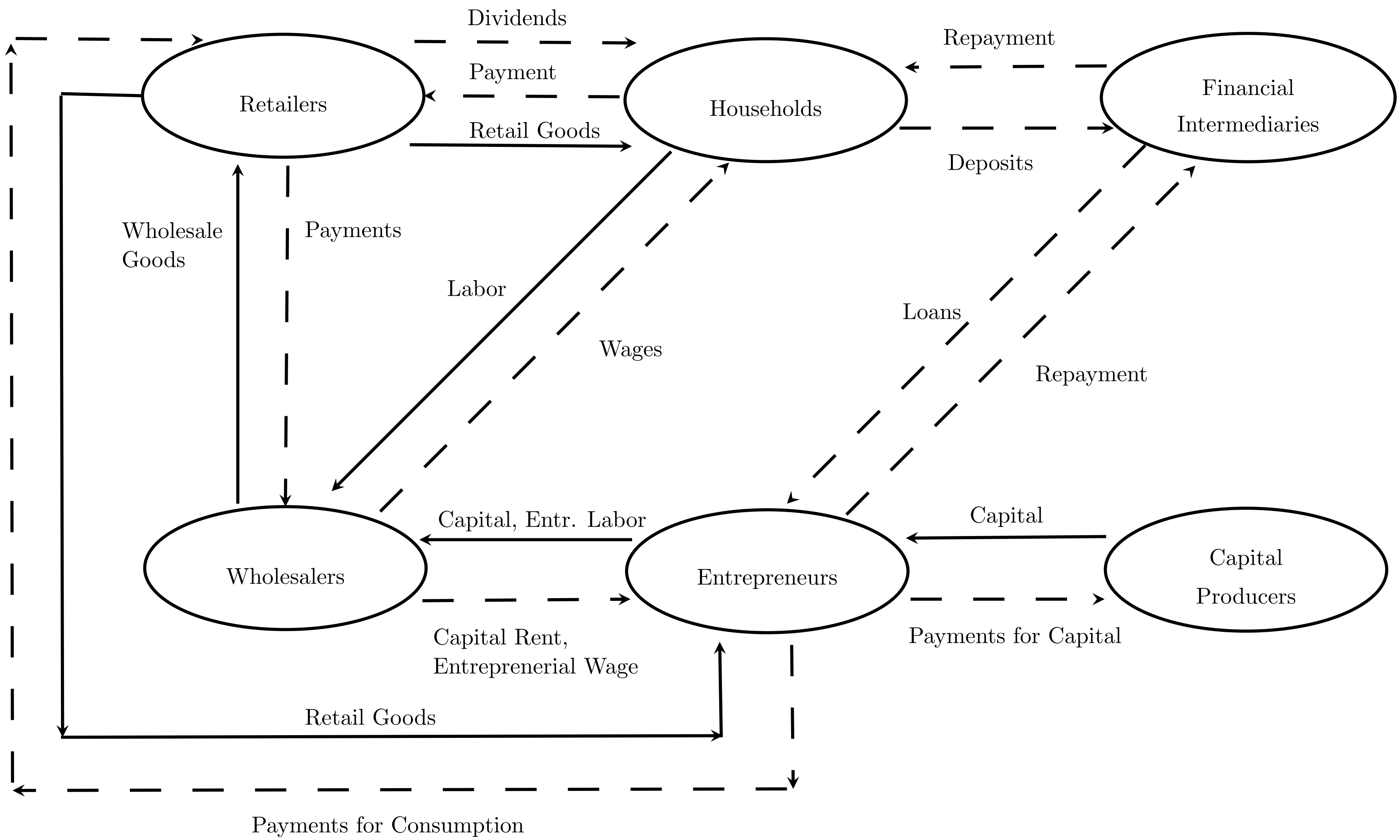
Our paper is also related to the growing body of medium sized DSGE models with financial frictions. To the best of our knowledge, the literature follows the BGG framework and employs myopic entrepreneurs with suboptimal contracts. Examples include Villaverde (2009, 2010) and Christiano, Motto and Rostagno (2013). Again, our results differ from the conclusions of this literature, as risk shocks under the optimal contract with non-myopic entrepreneurs have very little impact on the real economy.

3.7 Conclusion

This paper contributes to the literature on financial frictions in macroeconomics by introducing forward looking entrepreneurs into the costly state verification framework. In the literature, lending contracts are suboptimal and entrepreneurs are myopic. We solve for the optimal contract with forward looking entrepreneurs and show that financial frictions neither amplify nor propagate business cycles when lending contracts are optimal. In addition, we show that shocks to the variance of the unobserved productivity of entrepreneurs — so-called "risk shocks" — have little effect on the economy and generate the wrong comovement between macroeconomic aggregates when contracts are optimal.

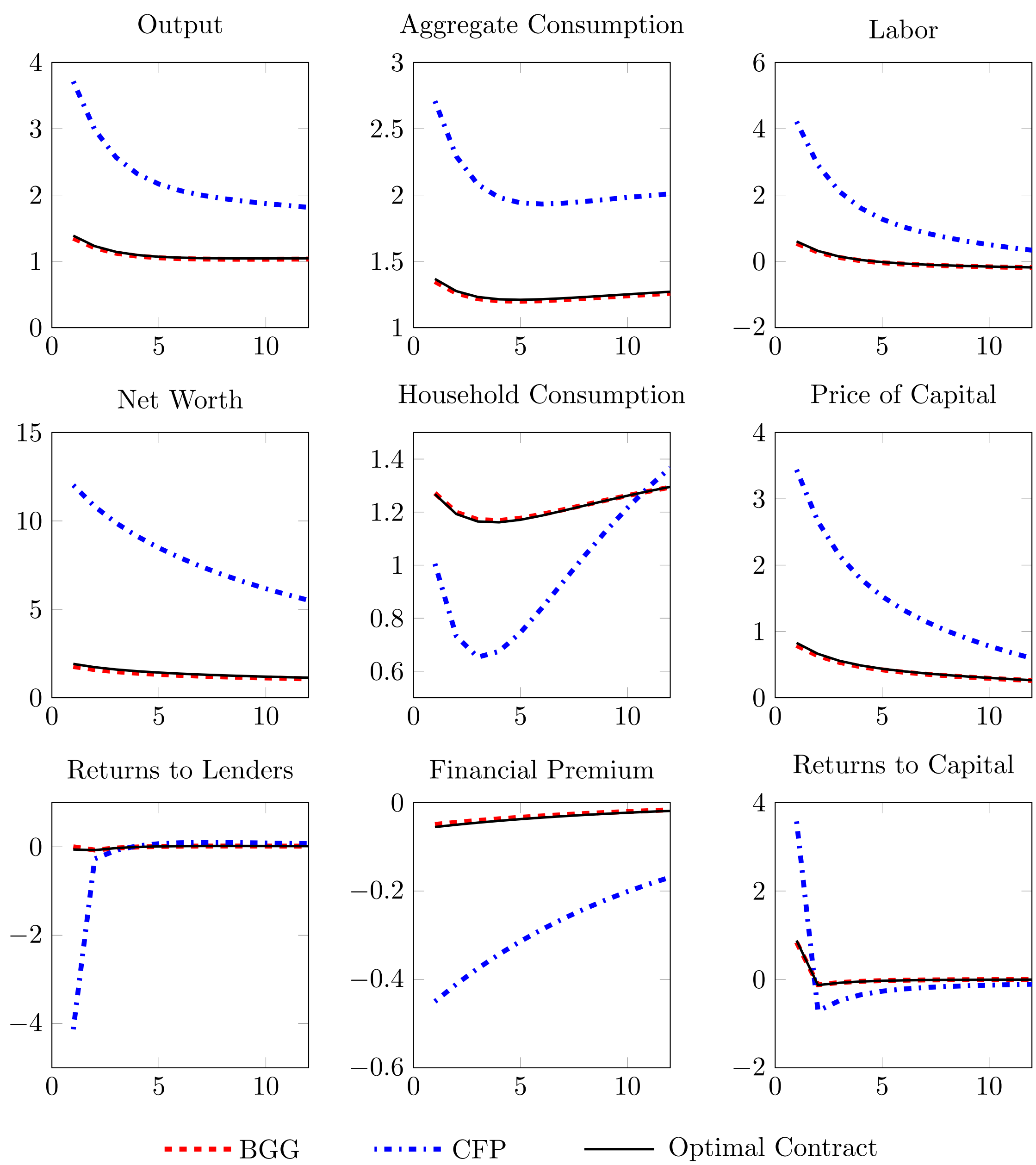
We also investigate the robustness of the financial accelerator under the standard BGG contract, which assumes that lenders receive a constant safe rate of return and borrowers are myopic. In this setup, we find that the accelerator depends on a combination of three things: a suboptimal lending contract, extremely persistent technology shocks, and loose monetary policy. Stationary technology shocks or a standard Taylor rule eliminate the financial accelerator or even reverse the accelerator such that financial frictions stabilize macroeconomic fluctuations. We thus conclude that the amplifying effect of financial frictions is present only under very restrictive conditions in costly state verification models.

Figure 3.2: Overview of the Model



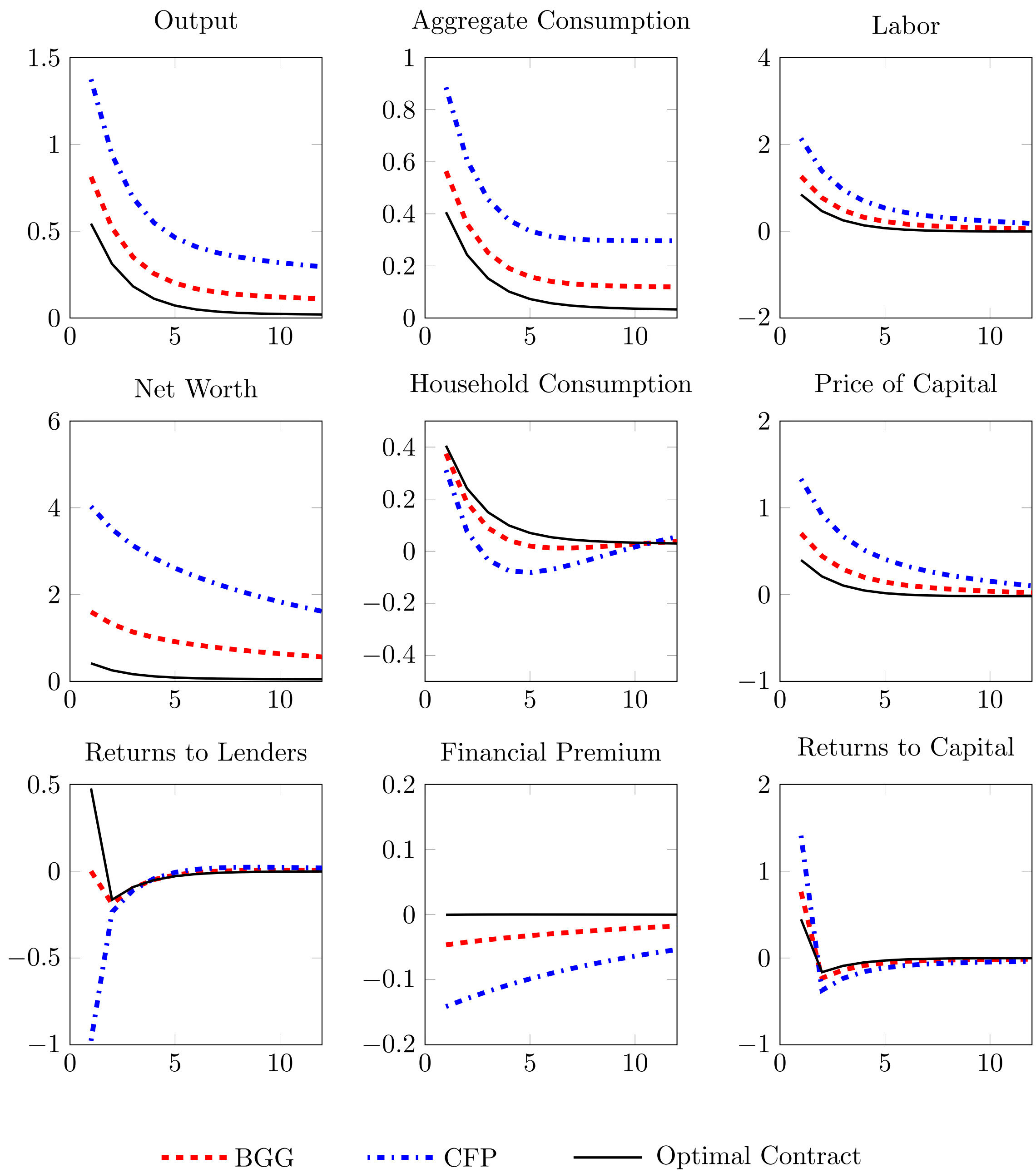
[scale=0.8]

Figure 3.3: Technology Shock



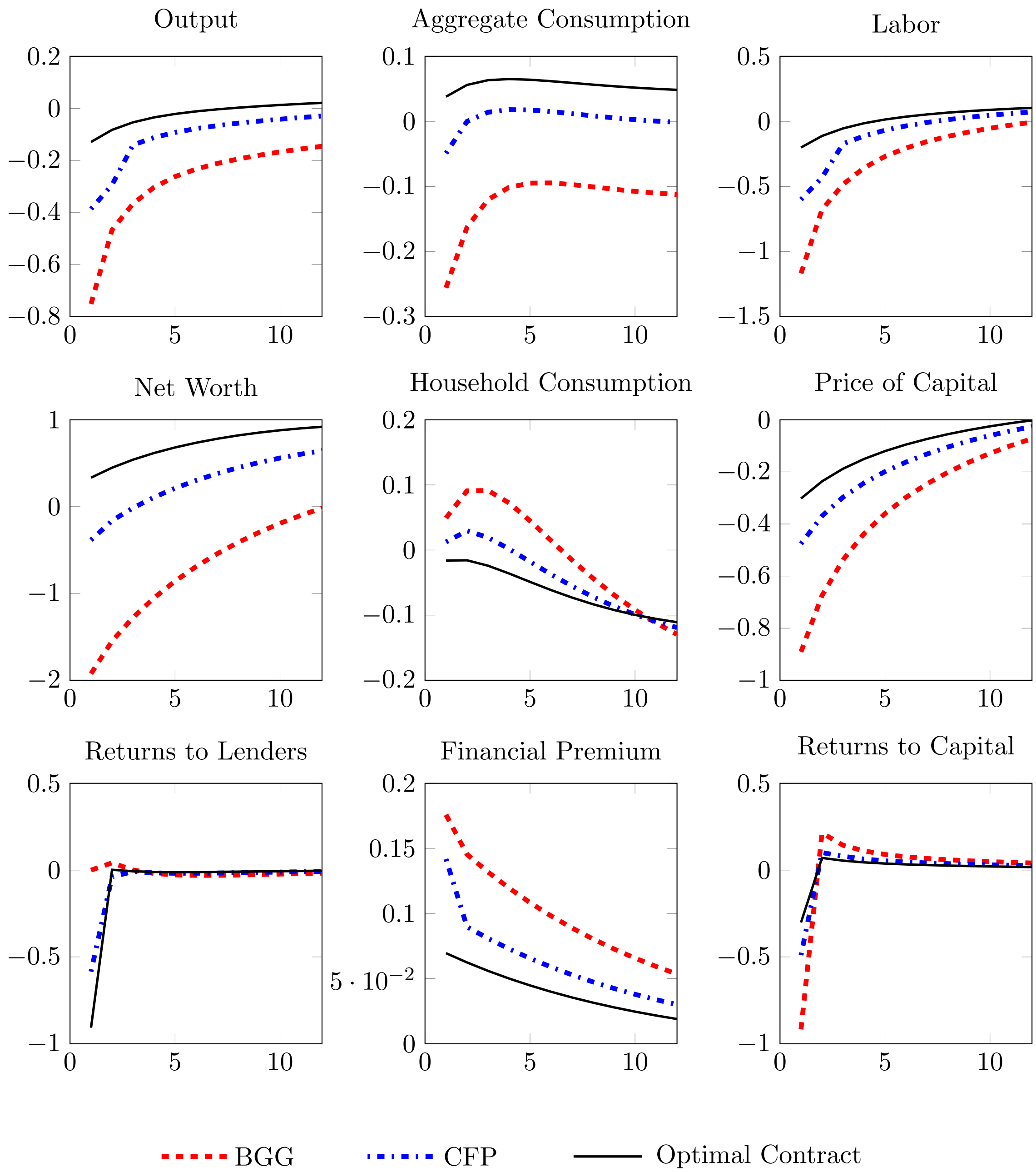
Note: All impulse responses are plotted as percent deviations from steady state.

Figure 3.4: Monetary Policy Shock



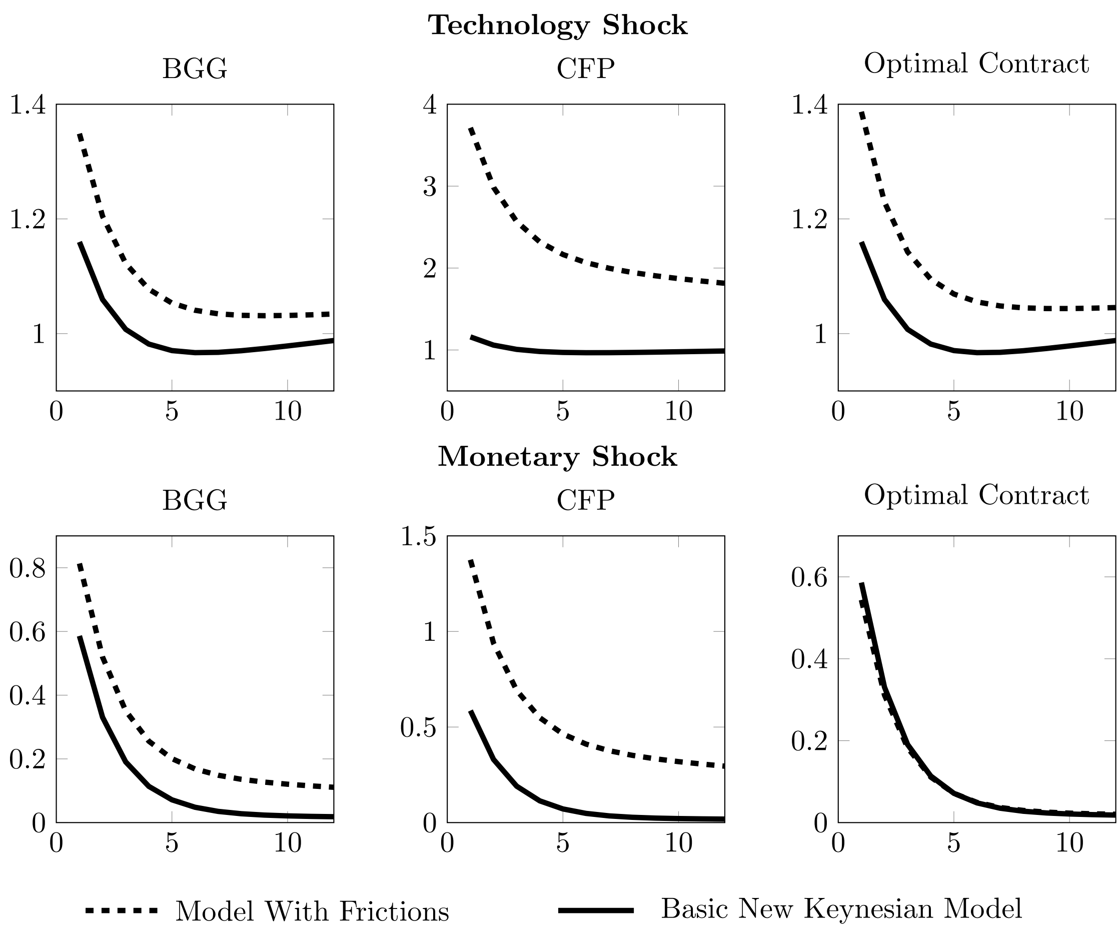
Note: All impulse responses are plotted as percent deviations from steady state.

Figure 3.5: Idiosyncratic Volatility Shock



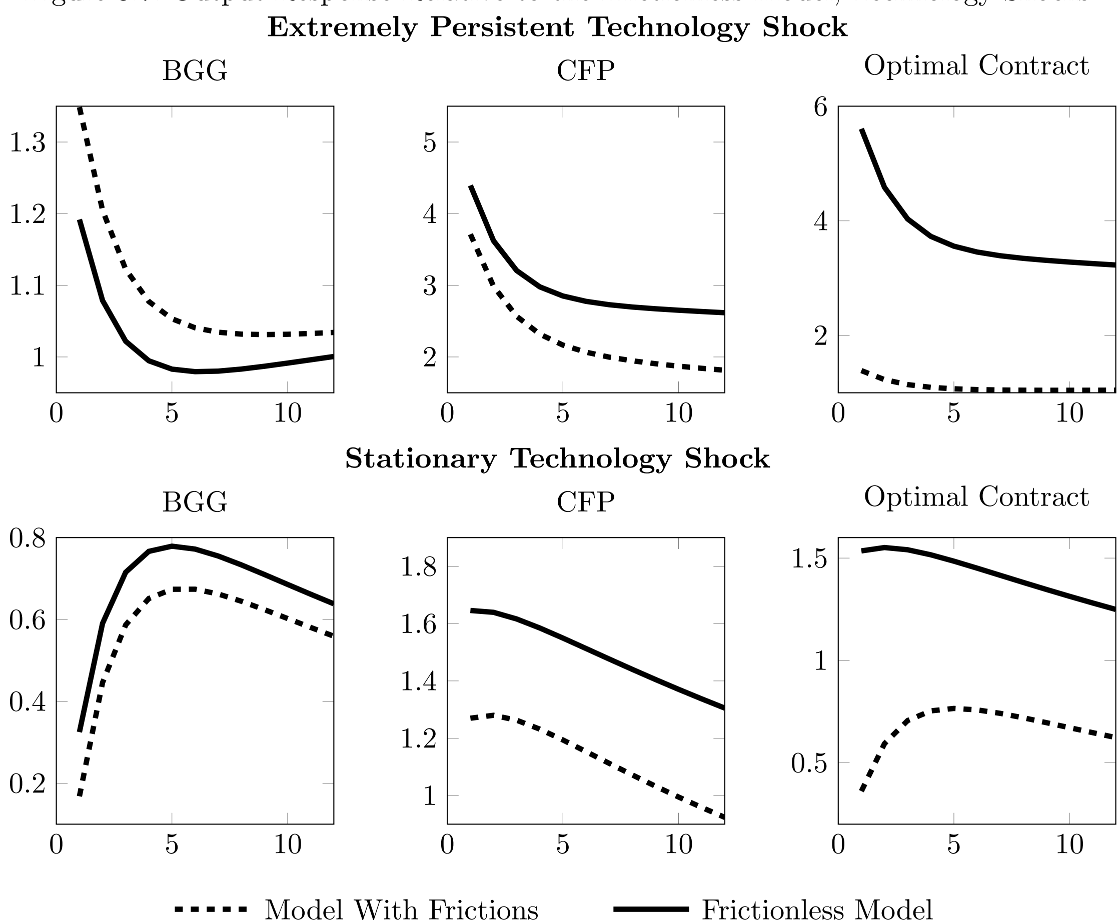
Note: All impulse responses are plotted as percent deviations from steady state.

Figure 3.6: Output Response Relative to the Basic New Keynesian Model



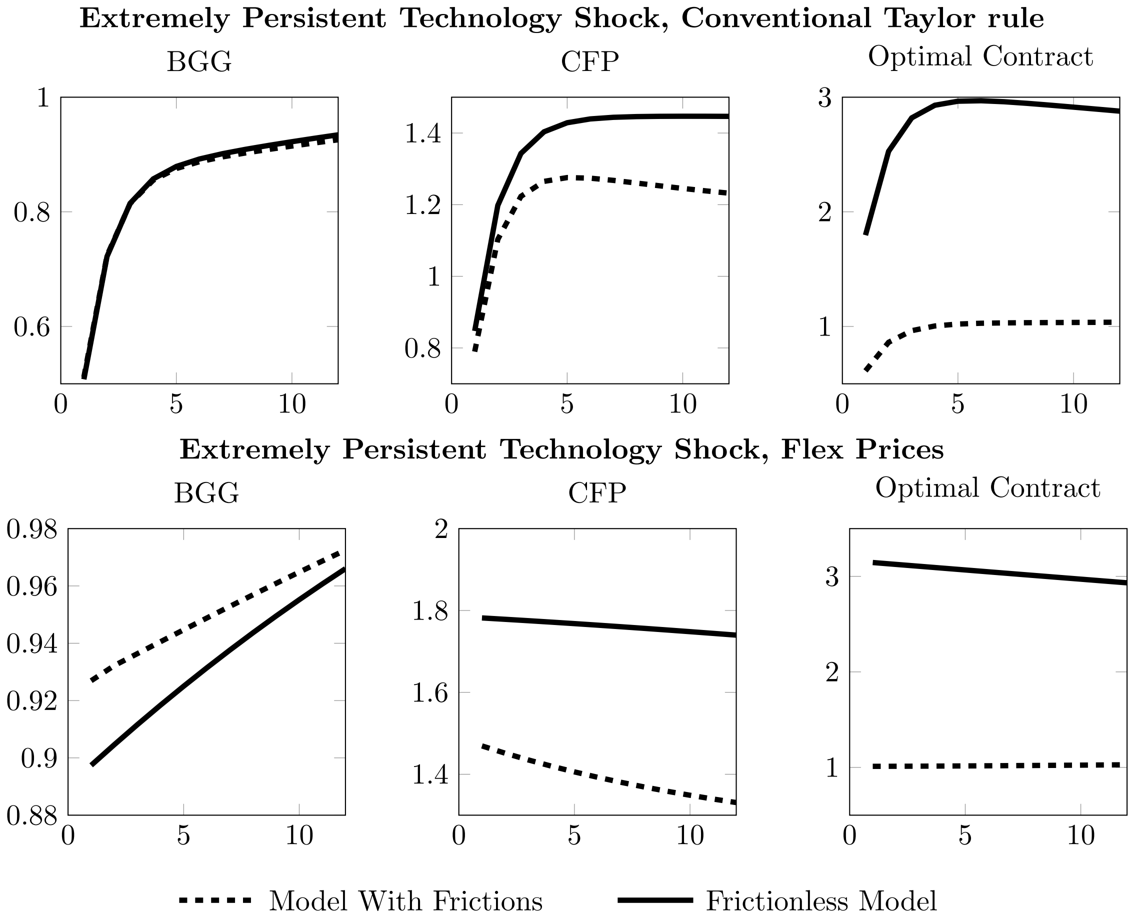
Note: All impulse responses are plotted as percent deviations from steady state.

Figure 3.7: Output Response Relative to the Frictionless Model, Technology Shocks



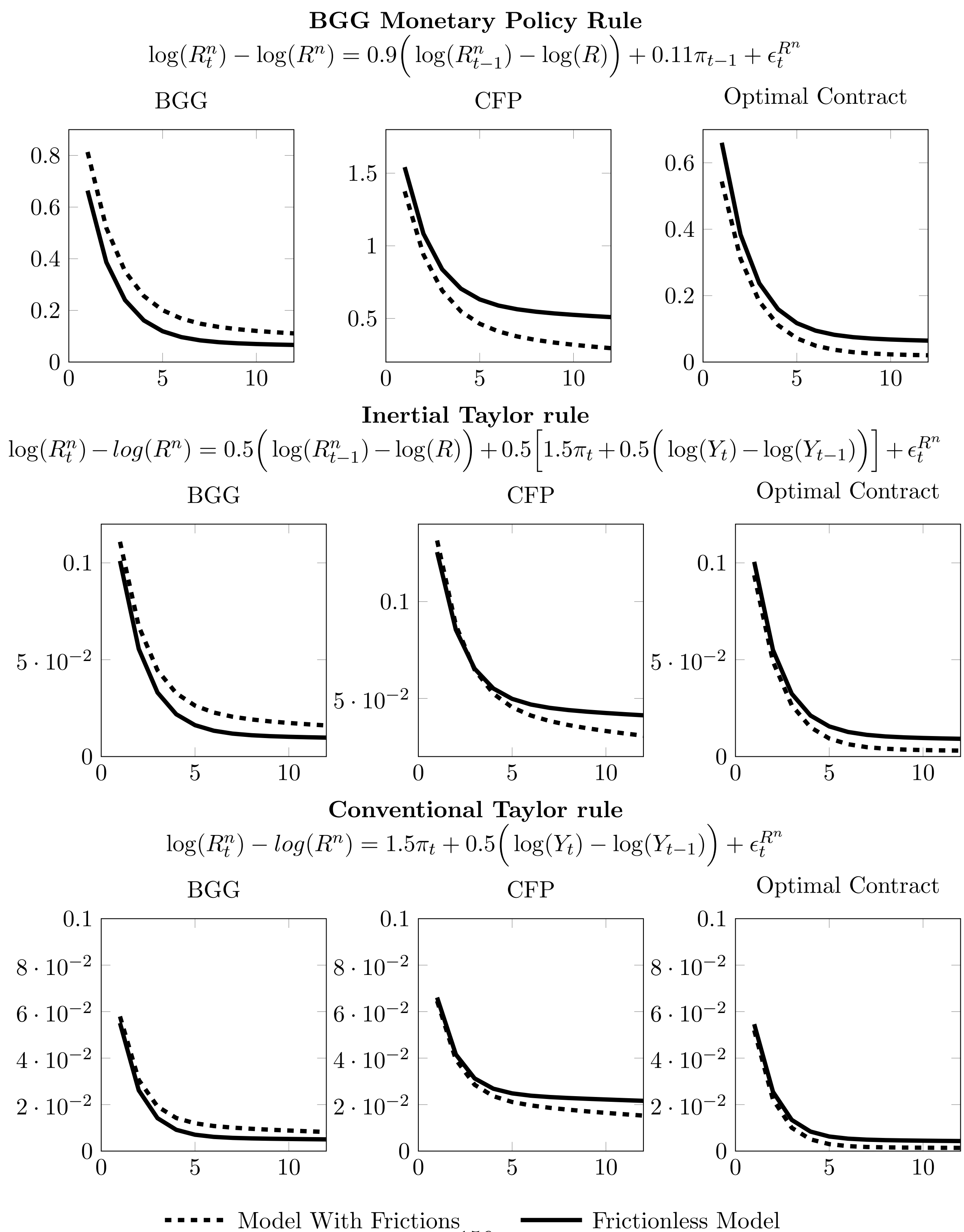
Note: All impulse responses are plotted as percent deviations from steady state.

Figure 3.8: Output Response Relative to the Frictionless Model, Technology Shocks



Note: All impulse responses are plotted as percent deviations from steady state.

Figure 3.9: Output Response Relative to the Frictionless Model, Monetary Shocks



Note: All impulse responses are plotted as percent deviations from steady state.

Appendix A

Price Stability in Small Open Economies

A.1 Risk-Sharing

The household in country i will maximize lifetime utility (2.4), subject to the following budget constraint and transversality condition:

$$C_i(s_t)P_i(s_t) = W_i(s_t)N_i(s_t) + \int_0^1 \mathcal{E}_{ij}(s_t)B_{ij}(s_t)dj, \tag{A.1}$$

$$\sum_{t=1}^{\infty}\sum_{s_t}\int_0^1 q_j(s_t)B_{ij}(s_t)dj = 0. \tag{A.2}$$

$B_{ij}(s_t)$ denotes the state-contingent bond that pays in currency j in state s_t; $q_j(s_t)$ is the price of that bond in period 0 (when all trading occurs), $q_j(s_t)$ is arbitrary up to a constant. Household in period 0 cares about relative price of claims across states and currencies. The transverality condition stipulates that all period 0 transactions must be balanced: payment for claims issued must equal payment for claims received. The household Lagrangian is:

$$\mathcal{L}_i = \sum_{t=1}^{\infty} \beta^t Pr(s_t) \Bigg\{ U_i(C(s_t)) - V_i(N(s_t)) + \frac{\lambda_i(s_t)}{P_i(s_t)} \Bigg[W_i(s_t) N_i(s_t) + \int_0^1 \mathcal{E}_{ij} B_{ij}(s_t)(s_t) dj - C_i(s_t) P_i(s_t) \Bigg] \Bigg\}$$
$$- \lambda_{i0} \sum_{t=1}^{\infty} \sum_{s_t} \int_0^1 q_j(s_t) B_{ij}(s_t) dj, \tag{A.3}$$

Now take the FOC with respect to state contingent bonds $B_{ij}(s_t)$:

$$\frac{\partial \mathcal{L}_i}{\partial B_{ij}(s_t)} = \lambda_{i0} q_j(s_t) + \frac{\beta^t \lambda_i(s_t) Pr(s_t) \mathcal{E}_{ij}(s_t)}{P_i(s_t)} = 0, \tag{A.4}$$

which gives price of the state-contingent bond,

$$q_j(s_t) = \beta^t \frac{\lambda_i(s_t) Pr(s_t) \mathcal{E}_{ij}(s_t)}{\lambda_{i0} P_i(s_t)}. \tag{A.5}$$

The analogous FOC for country j, $\frac{\partial \mathcal{L}_j}{\partial B_{jj}(s_t)} = 0$ will yield:

$$q_j(s_t) = \beta^t \frac{\lambda_j(s_t) Pr(s_t) \mathcal{E}_{jj}(s_t)}{\lambda_{j0} P_j(s_t)}. \tag{A.6}$$

Using $\mathcal{E}_{jj}(s_t) = 1$ and setting (A.5) equal to (A.6), we get the risk-sharing condition

$$\frac{\lambda_i(s_t)}{\lambda_j(s_t)} = \frac{\lambda_{i0}}{\lambda_{j0}} \frac{P_i(s_t)}{P_j(s_t) \mathcal{E}_{ij}(s_t)}. \tag{A.7}$$

When PPP holds, $\frac{P_i(s_t)}{P_j(s_t)\mathcal{E}_{ij}(s_t)} = 1$, and the risk-sharing condition simplifies to $\frac{\lambda_i(s_t)}{\lambda_j(s_t)} = \left(\frac{C_i(s_t)}{C_j(s_t)}\right)^{-\sigma} = \frac{\lambda_{i0}}{\lambda_{j0}}$. When the consumption ratio is constant across countries, $C_{it} = A_i C_{wt}$.

In order to solve for A_i, we substitute (A.5) into the transversality condition.

$$
\begin{aligned}
\sum_{t=1}^{\infty}\sum_{s_t}\int_0^1 q_j(s_t)B_{ij}(s_t)dj &= \sum_{t=1}^{\infty}\sum_{s_t}\int_0^1 \beta^t \frac{\lambda_i(s_t)Pr(s_t)\mathcal{E}_{ij}(s_t)}{\lambda_{i0}P_i(s_t)} B_{ij}(s_t)dj \\
&= \sum_{t=1}^{\infty}\sum_{s_t}\beta^t \frac{\lambda_i(s_t)Pr(s_t)}{\lambda_{i0}P_i(s_t)} \int_0^1 \mathcal{E}_{ij}(s_t)B_{ij}(s_t)dj \\
&\overset{(A.1)}{=} \sum_{t=1}^{\infty}\sum_{s_t}\beta^t \frac{\lambda_i(s_t)Pr(s_t)}{\lambda_{i0}P_i(s_t)} \Big(P_i(s_t)C_i(s_t) - W_i(s_t)N_i(s_t)\Big) \\
&= 0
\end{aligned}
$$

We substitute $C_i(s_t) = A_i C_w(s_t)$ into the above equation, and solve for A_i.

$$
\begin{aligned}
A_i &= \frac{\sum_{t=1}^{\infty}\sum_{s_t}\beta^t \frac{W(s_t)N(s_t)}{P(s_t)}\lambda_i(s_t)Pr(s_t)}{\sum_{t=1}^{\infty}\sum_{s_t}\beta^t C_w(s_t)\lambda_i(s_t)Pr(s_t)} \\
&= \frac{\sum_{t=1}^{\infty}\beta^t \mathbb{E}_{t-1}\left\{\frac{W_t N_t}{P_t}\lambda_i(s_t)\right\}}{\sum_{t=1}^{\infty}\beta^t \mathbb{E}_{t-1}\left\{C_{wt}\lambda_i(s_t)\right\}} \\
&= \frac{\sum_{t=1}^{\infty}\beta^t \mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}\lambda_i(s_t)\right\}}{\sum_{t=1}^{\infty}\beta^t \mathbb{E}_{t-1}\left\{C_{wt}\lambda_i(s_t)\right\}} \\
&= \frac{\sum_{t=1}^{\infty}\beta^t \mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}} C_{wt}^{-\sigma}\right\}}{\sum_{t=1}^{\infty}\beta^t \mathbb{E}_{t-1}\left\{C_{wt}C_{wt}^{-\sigma}\right\}}
\end{aligned}
$$

where we used $\lambda_i(s_t) = A_i^{-\sigma}C_w^{-\sigma}(s_t)$. This gives us the definition of complete markets from the text, equation (2.20).

A.2 Closed Economy: Propositions and Proofs

Proposition 1 *In the closed economy under ex ante commitment, the central bank will maximize (1.24) subject to (1.25) and (1.26). The solution to this problem is:* $C_t = \left(\frac{1-\tau}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}$. *The central bank replicates the flexible price allocation via a policy of price stability.*

Proof: The flexible price allocation is obtained from solving this two equation system in two unknowns (C, N):

$$1 = \frac{\mu\chi}{1-\tau} \frac{N_t^{1+\varphi}}{C_t^{1-\sigma}},$$

$$C_t = Z_t N_t.$$

The solution is $C_t = \left(\frac{1-\tau}{\mu\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}$. Now, let's reformulate the central bank's problem by substituting the labor market clearing condition (1.25) and the goods market clearing condition (1.26) into the objective function.

$$\max_{C_t} \mathbb{E}_{t-1} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{C_t^{1+\varphi} Z_t^{-1-\varphi}}{(1+\varphi)} \right\}$$

s.t.

$$1 = \frac{\mu\chi}{1-\tau} \frac{\mathbb{E}_{t-1} \left\{ C_t^{1+\varphi} Z_t^{-(1+\varphi)} \right\}}{\mathbb{E}_{t-1} C_t^{1-\sigma}}.$$

The Lagrangian for this constrained optimization problem is

$$\mathcal{L} = \mathbb{E}_{t-1}\left\{\frac{C_t^{1-\sigma}}{1-\sigma} - \chi\frac{C_t^{1+\varphi}Z_t^{-1-\varphi}}{(1+\varphi)}\right\} + \lambda\left(\mathbb{E}_{t-1}C_t^{1-\sigma} - \frac{\mu\chi}{1-\tau}\mathbb{E}_{t-1}\left\{C_t^{1+\varphi}Z_t^{-(1+\varphi)}\right\}\right). \tag{A.8}$$

The first order condition with respect to consumption is[1]

$$\frac{\partial\mathcal{L}}{\partial C_t} = (1+\lambda(1-\sigma))\,C_t^{-\sigma} - \chi\left(1+\frac{\lambda\mu}{1-\tau}\right)C_t^{\varphi}Z_t^{-1-\varphi} = 0. \tag{A.9}$$

which is equivalent to

$$C_t^{1-\sigma} = \chi\left(\frac{1+\frac{\lambda\mu}{1-\tau}}{1+\lambda(1-\sigma)}\right)N_t^{1+\varphi}. \tag{A.10}$$

Given that λ is a constant and not a variable, the first order condition and the budget constraint are satisfied only under the flexible price equilibrium. Thus, the central bank's optimal policy is to mimic the flexible price equilibrium in the closed economy. $\diamondsuit$

Proposition 2 *In the closed economy, the social planner will maximize (1.24) subject to (1.26), ignoring the labor condition (1.25). The solution to this problem is:* $C_t = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}$.

Proof: Insert the aggregate goods market clearing constraint (1.26) directly into the

[1] One can easily carry out the same exercise by optimizing with respect to labor.

objective function, replacing N_{t+1}, and maximize this objective function.

$$\max_{C_{t+1}} \left[\frac{C_{t+1}^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} \left(\frac{C_{t+1}}{Z_{t+1}} \right)^{1+\varphi} \right]$$

The solution to this optimization problem is $C_t = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}$. $\diamondsuit$

A.3 Global Social Planner: Proposition and Proof

Proposition 8 *The global social planner will maximize (2.24), subject to (2.25) and (2.26). The solution to the global social planner problem is:*

$$C_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}}, \tag{A.11a}$$

$$N_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}}, \tag{A.11b}$$

$$Z_w = \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}. \tag{A.11c}$$

Proof: If we substitute (2.25) and (2.26) directly into the objective function (2.24), then we can reformulate the problem as follows:

$$\max_{\forall c_{ij}} \int_0^1 \left[\frac{\left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}}}{1-\sigma} - \frac{\chi}{1+\varphi}\left(\frac{\int_0^1 c_{ji} dj}{Z_i}\right)^{1+\varphi} \right] di. \tag{A.12}$$

Rearranging, we have

$$\max_{c_{ij}} \frac{1}{1-\sigma} \int_0^1 \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}} di - \frac{\chi}{1+\varphi} \int_0^1 \frac{\left(\int_0^1 c_{ji} dj\right)^{1+\varphi}}{Z_i^{1+\varphi}} di. \tag{A.13}$$

The FOC with respect to c_{ij} is

$$0 = \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}-1} c_{ij}^{\frac{-1}{\gamma}} - \chi \frac{\left(\int_0^1 c_{ji} dj\right)^{\varphi}}{Z_j^{1+\varphi}}. \tag{A.14}$$

This is equivalent to

$$\begin{aligned}
0 &= \underbrace{\left[\left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{1-\sigma\gamma}{\gamma-1}}\right]}_{=C_i^{\frac{1-\sigma\gamma}{\gamma}}} c_{ij}^{\frac{-1}{\gamma}} - \chi \underbrace{\left(\frac{\int_0^1 c_{ji} dj}{Z_j}\right)^{\varphi}}_{=N_j^{\varphi}} \frac{1}{Z_j}, \\
\Rightarrow 0 &= C_i^{\frac{1-\sigma\gamma}{\gamma}} c_{ij}^{-\frac{1}{\gamma}} - \chi \frac{N_j^{\varphi}}{Z_j},
\end{aligned}$$

and solving for c_{ij} we have:

$$c_{ij} = \frac{Z_j^{\gamma} C_i^{1-\gamma\sigma}}{\chi^{\gamma} N_j^{\gamma\varphi}}. \tag{A.15}$$

The consumption basket in country i (C_i) can then be expressed as:

$$\begin{aligned}
C_i &= \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}}, \\
&= \left[\int_0^1 \left(\frac{Z_j^{\gamma} C_i^{1-\gamma\sigma}}{\chi^{\gamma} N_j^{\gamma\varphi}}\right)^{\frac{\gamma-1}{\gamma}} dj\right]^{\frac{\gamma}{\gamma-1}}, \\
&= \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma}} \left[\int_0^1 \left(\frac{Z_j}{N_j^{\varphi}}\right)^{(\gamma-1)} dj\right]^{\frac{1}{\sigma(\gamma-1)}}.
\end{aligned} \tag{A.16}$$

So C_i does not depend on its own technology Z_i. Now, let's solve for labor (N_i) and output

(Y_i).

$$
\begin{aligned}
N_i &= \frac{Y_i}{Z_i} && \text{from goods market clearing} \\
&= \frac{\int_0^1 c_{ji} dj}{Z_i} && \text{from (2.26)} \\
&= \frac{\int_0^1 \left(\frac{Z_i^{\gamma} C_j^{1-\gamma\sigma}}{\chi^{\gamma} N_i^{\gamma\varphi}} \right) dj}{Z_i} && \text{from } (A.15) \\
&= \frac{Z_i^{\gamma-1}}{\chi^{\gamma} N_i^{\gamma\varphi}} \int_0^1 C_j^{1-\gamma\sigma} dj && \text{(A.17)}
\end{aligned}
$$

From (A.16), we know that $C_i = C_j = C$ for all i, j. So we can take C_j outside of the integral in (A.17) and solve for N_i:

$$
\begin{aligned}
N_i &= \frac{Z_i^{\gamma-1} C_j^{1-\gamma\sigma}}{\chi^{\gamma} N_i^{\gamma\varphi}} \\
\Rightarrow N_i &= \left(\frac{Z_i^{\gamma-1} C^{1-\gamma\sigma}}{\chi^{\gamma}} \right)^{\frac{1}{1+\gamma\varphi}}. \qquad \text{(A.18)}
\end{aligned}
$$

Similarly, output will be:

$$
Y_i = \left(\frac{Z_i^{\gamma(1+\varphi)} C^{1-\gamma\sigma}}{\chi^{\gamma}} \right)^{\frac{1}{1+\gamma\varphi}}. \qquad \text{(A.19)}
$$

Substitute (A.18) and (A.19) back into the definition of the consumption basket (A.16), and solve for the consumption basket C in each country, which will be identical:

$$C = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma}} \left\{ \int_0^1 \left[\left(\frac{Z_j^{\gamma-1} C^{1-\gamma\sigma}}{\chi^{\gamma}} \right)^{\frac{1}{1+\gamma\varphi}} \right]^{-(\gamma-1)\varphi} Z_j^{\gamma-1} dj \right\}^{\frac{1}{\sigma(\gamma-1)}},$$

$$C^{\frac{\sigma+\varphi}{(1+\gamma\varphi)\sigma}} = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma}} \left[\int_0^1 \left(\frac{Z_j^{\gamma-1}}{\chi^{\gamma}} \right)^{-\frac{(\gamma-1)\varphi}{1+\gamma\varphi}} Z_j^{\gamma-1} dj \right]^{\frac{1}{\sigma(\gamma-1)}},$$

$$\Rightarrow C = C_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} dj \right)^{\frac{1+\gamma\varphi}{(\sigma+\varphi)(\gamma-1)}}. \tag{A.20}$$

Solve for labor and output by substituting (A.20) into (A.18) and (A.19) respectively:

$$N_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} dj \right)^{\frac{1-\gamma\sigma}{(\sigma+\varphi)(\gamma-1)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}}, \tag{A.21}$$

$$Y_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} dj \right)^{\frac{1-\gamma\sigma}{(\sigma+\varphi)(\gamma-1)}} Z_i^{\frac{\gamma(1+\varphi)}{1+\gamma\varphi}}. \tag{A.22}$$

This is the Pareto efficient allocation. When $\gamma \to \infty$, the flexible price allocation and the global social planner allocation become identical. Consumption is identical to the first order between social planner and flexible price allocation. However, it is not true for labor. $\diamondsuit$

A.4 Producer Currency Pricing: Propositions and Proofs

Proposition 4 *In complete markets, non-cooperative central banks will maximize (1.30a) and cooperative central banks will maximize (1.30b), subject to (2.29a), (2.29b), (2.29c) and (2.29d). The solution under commitment for both cooperative and non-cooperative central banks in complete markets is:*

$$
\begin{aligned}
\mathbb{E}\{U_i\} &= C_i^{1-\sigma}\left(\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu(1+\varphi)}\right) \\
C_i &= \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}} \\
N_i &= \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}} \\
Y_i &= \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma(1+\varphi)}{1+\gamma\varphi}} \\
Z_w &= \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\, di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}
\end{aligned}
$$

The resulting equilibrium allocation exactly coincides with the flexible price allocation. Mimicking the flexible price allocation is thus the dominant strategy for non-cooperative central banks in complete markets, and is the optimal policy under cooperation. If the government corrects the distortions due to market power with a non-contingent tax $\tau_i = 1-\mu$, then the flexible price allocation in complete markets is identical to the global social planner solution.

Proposition 5 *In financial autarky, non-cooperative central banks will maximize (1.30a) and cooperative central banks will maximize (1.30b), subject to (1.35), (2.31), (1.37) and (1.38). The solution under commitment for both cooperative and non-cooperative central banks in financial autarky is:*

$$
\begin{aligned}
\mathbb{E}\{U_i\} &= C_i\left(\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu(1+\varphi)}\right),\\
C_i &= \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}}\left(Z_i^{\gamma-1}Z_w^{\frac{1+\varphi}{\sigma+\varphi}}\right)^{\frac{1+\varphi}{1-\sigma+\gamma(\varphi+\sigma)}},\\
N_i &= \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}}\left(Z_i^{\gamma-1}Z_w^{\frac{1+\varphi}{\sigma+\varphi}}\right)^{\frac{1-\sigma}{1-\sigma+\gamma(\varphi+\sigma)}},\\
Y_i &= \left(\frac{1-\tau_i}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}}\left(Z_i^{\gamma-1}Z_w^{\frac{1+\varphi}{\sigma+\varphi}}\right)^{\frac{1-\sigma}{1-\sigma+\gamma(\varphi+\sigma)}} Z_i,\\
Z_w &= \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1-\sigma+\gamma(\sigma+\varphi)}}\,di\right)^{\frac{1-\sigma+\gamma(\sigma+\varphi)}{(\gamma-1)(1+\varphi)}}.
\end{aligned}
$$

The resulting equilibrium allocation exactly coincides with the flexible price allocation. Mimicking the flexible price allocation is thus the dominant strategy for non-cooperative central banks in financial autarky, and is the optimal policy under cooperation.

Proof: The objective function for non-cooperative and cooperative central banks will be, respectively:

$$\max_{N_{it}} \mathbb{E}_{t-1}\left\{\frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi\frac{N_{it}^{1+\varphi}}{1+\varphi}\right\}, \tag{A.23a}$$

$$\max_{\forall N_{it}} \int_0^1 \mathbb{E}_{t-1}\left\{\frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi\frac{N_{it}^{1+\varphi}}{1+\varphi}\right\} di. \tag{A.23b}$$

Policymakers in each scenario will maximize their objective function subject to the following constraints:

$$1 = \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\right\}}, \tag{A.24}$$

$$Y_{it} = Z_{it}N_{it}, \tag{A.25}$$

$$C_{wt} = \left(\int_0^1 Y_{it}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{A.26}$$

$$C_{it} = C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\} \tag{A.27a}$$

$$C_{it} = C_{wt}^{\frac{1}{\gamma}}Y_{it}^{\frac{\gamma-1}{\gamma}} \tag{A.27b}$$

where (A.26) is a constant, (A.27a) refers to goods market clearing and risk-sharing under complete markets, while (A.27b) refers to goods market clearing and risk-sharing under financial autarky.

Notice that under complete markets we can substitute (A.27a) into (A.24) so that:

$$
\begin{aligned}
1 &= \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\right\}} \\
&\overset{\text{Plug in } C_{it}}{=} \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{\left[C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}\right]^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\right\}}, \\
&\overset{C_{wt}\text{ is constant}}{=} \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{C_{wt}^{\frac{1-\sigma}{\gamma}}\mathbb{E}_{t-1}\left\{\left[\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}\right]^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}}, \\
&\overset{\text{LIE}}{=} \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{C_{wt}^{\frac{1-\sigma}{\gamma}}\left[\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}\right]^{-\sigma}\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}}, \\
&= \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{C_{wt}^{\frac{1-\sigma}{\gamma}}\left[\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}\right]^{1-\sigma}} \qquad \text{(A.28)}
\end{aligned}
$$

where LIE denotes the law of iterated expectations. The denominator of (A.28) is constant, so its expectation will also be constant. We now take the expectation of the entire denominator so that this expression will be identical in complete markets and financial autarky. Continuing on, we can rewrite (A.28) as

$$
\begin{aligned}
\left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{C_{wt}^{\frac{1-\sigma}{\gamma}}\left[\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}\right]^{1-\sigma}} &= \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{\left[C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\}\right]^{1-\sigma}\right\}} \\
\Rightarrow 1 &= \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\}}, \qquad \text{(A.29)}
\end{aligned}
$$

Again, note that in complete markets it is not necessary to include the expectation in the denominator of (A.29) because the denominator is constant, but we do so in order that our expression for price-setting is identical in complete markets and financial autarky. It is trivial to verify that under financial autarky the price-setting condition equal to (A.29) by substituting (A.27b) into (A.24).

Using (A.29) as our constraint in complete markets and in financial autarky, we can formulate a Lagrangian for the non-cooperative and cooperative cases:

$$\mathcal{L} = \frac{\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\}}{1-\sigma} - \chi\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{1+\varphi} + \lambda_i\left(\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\} - \frac{\chi\mu}{1-\tau_i}\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}\right) \quad \text{(A.30a)}$$

$$\mathcal{L} = \int_0^1 \mathbb{E}_{t-1}\left[\frac{\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\}}{1-\sigma} - \chi\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{1+\varphi} + \lambda_i\left(\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\} - \frac{\chi\mu}{1-\tau_i}\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}\right)\right] di \quad \text{(A.30b)}$$

Using $C_{it} = C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{N_{it}^{\frac{\gamma-1}{\gamma}} Z_{it}^{\frac{\gamma-1}{\gamma}}\right\}$ for complete markets, or $C_{it} = C_{wt}^{\frac{1}{\gamma}} N_{it}^{\frac{\gamma-1}{\gamma}} Z_{it}^{\frac{\gamma-1}{\gamma}}$ for financial autarky, we can take the first order condition with respect to N_{it}.[2] The FOC will be identical in both cases.

$$\frac{\partial\mathcal{L}}{\partial N_{it}} = C_{it}^{-\sigma}\Big[1+\lambda_i(1-\sigma)\Big]\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}}{N_{it}}\right) - \chi\left[1+\lambda\frac{\mu}{1-\tau_i}(1+\varphi)\right]\left(\frac{1}{N_{it}}\right) N_{it}^{1+\varphi} = 0 \quad \text{(A.31)}$$

[2]Remember that we are optimizing given the fact that state s_t is realized. Expectations in our context thus refer to a summation over all possible states multiplied by the probability of each state occuring. For example, $\mathbb{E}_{t-1}\{C_{it}^{1-\sigma}\} = \sum_{s_t} C_i^{1-\sigma}(s_t)\Pr(s_t)$.

In equilibrium, this equals:

$$1 = \underbrace{\chi\left(\frac{1+\frac{\lambda_i\mu(1+\varphi)}{1-\tau_i}}{1+\frac{\lambda_i(1-\sigma)(\gamma-1)}{\gamma}}\right)}_{\text{Constant}}\left(\frac{N_{it}^{1+\varphi}}{C_{it}^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}}\right). \tag{A.32}$$

This equation holds in both complete markets and financial autarky, and differs from the flexible price equilibrium only by the constant term. However, subject to labor market clearing, this constant will coincide with the flexible price equilibrium. The flexible price equilibrium in complete markets and financial autarky is found by taking expectations out of the labor market clearing condition (A.24) and substituting in goods market clearing (A.25):

$$1 = \left(\frac{\chi\mu}{1-\tau_i}\right)\frac{Y_{it}^{\frac{1+\varphi\gamma}{\gamma}}}{C_{it}^{-\sigma}C_{wt}^{\frac{1}{\gamma}}Z_{it}^{1+\varphi}}. \tag{A.33}$$

For complete markets, we can express output as a function of technology and a constant term by substituting (A.27a) into (B.2): $Y_{it} = A_i Z_{it}^{\frac{\gamma(1+\varphi)}{1+\gamma\varphi}}$. (We can do the same for exercise for autarky by substituting (A.27b) into (B.2), but leave that to the reader). Using this expression for output, consumption in complete markets in country i can be expressed as

$$C_{it} = A_i^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}. \tag{A.34}$$

Now substitute (B.3) back into the flexible price equilibrium (B.2)

$$1 = \left(\frac{\chi\mu}{1-\tau_i}\right)C_{wt}^{\frac{\sigma}{\gamma}}A_i^{\frac{(\gamma-1)\sigma}{\gamma}}\mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}^{\sigma}A_i^{\frac{1+\varphi\gamma}{\gamma}}C_{wt}^{-\frac{1}{\gamma}}, \tag{A.35}$$

and rearrange and solve for A_i:

$$A_i = \left[\left(\frac{1-\tau_i}{\chi\mu} \right)^{\gamma} C_{wt}^{1-\sigma} \mathbb{E}_{t-1} \left\{ Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} \right\}^{-\sigma\gamma} \right]^{\frac{1}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{A.36}$$

Now, substitute the solution for A_i, (B.5), into (B.3):

$$C_{it} = \left[\left(\frac{\chi\mu}{1-\tau_i} \right)^{1-\gamma} C_{wt}^{\varphi+1} \mathbb{E}_{t-1} \left\{ Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} \right\}^{1+\gamma\varphi} \right]^{\frac{1}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{A.37}$$

Using the fact that $C_{wt} = \int_0^1 C_{it} di$ and setting $Z_w = \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di \right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}$, consumption for country i in complete markets is:

$$C_{it} = \left(\frac{1-\tau_i}{\chi\mu} \right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}}. \tag{A.38}$$

Solving for labor and output using (B.7) is a straightforward exercise. The solution to the central bank's problem in complete markets and financial autarky for cooperative and non-cooperative equilibria, coincides exactly with the flexible price allocation. Here we've explicitly outlined the proof for complete markets. The proof for financial autarky is identical up to (B.2). We simply substitute (A.27b) into (B.2) to get the optimal allocation under financial autarky. $\diamondsuit$

A.5 Local Currency Pricing: Propositions and Proofs

Proposition 6 *Under LCP, non-cooperative central banks will maximize (1.30a) subject to (1.39) and (1.40). A fixed exchange rate will be the Nash equilibrium policy for a non-cooperative central bank in both complete markets and financial autarky under LCP.*

Proof:

In this section we explain the case of LCP in the continuum framework. First, let another country j import goods from firm h in country i.

Price Setting

Profits from the exports of firm h in country i to country j in domestic currency will be:

$$P_{jit}(h)\mathcal{E}_{ijt}C_{jit}(h) - W_{it}N_{jit}(h) \tag{B.1}$$

where $P_{jit}(h)$ denotes the price charged by firm h in country j which is located in country i. This price will be denominated in currency j. $N_{jit}(h)$ is the amount of labor used by firm h (which is manufacturing in country i) in the production of its exports to country j. Given this, firm h will choose the price that maximizes its expected profit:

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\left(\frac{P_{jit}(h)\mathcal{E}_{ijt}C_{jit}(h) - W_{it}N_{jit}(h)}{CPI_{it}}\right)\right\}. \tag{B.2}$$

We also have demand for firm h's good:

$$C_{jit}(h) = \left(\frac{P_{jit}(h)}{P_{jit}}\right)^{-\varepsilon} C_{jit}. \tag{B.3}$$

In order to solve the maximization problem, we substitute the demand for firm h's good (B.3) into the expected profit function (B.2) and take the FOC with respect to $P_{jit}(h)$. The FOC will be:

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\left(\frac{P_{jit}(h)\mathcal{E}_{ijt}C_{jit}(h)}{P_{jit}(h)CPI_{it}}\right)\right\} = \frac{\varepsilon}{\varepsilon-1}\mathbb{E}_{t-1}\left\{\frac{W_{it}N_{jit}(h)}{P_{jit}(h)CPI_{it}}\right\} \tag{B.4}$$

$P_{jit}(h)$ on the LHS will cancel out, and we will be left with:

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\frac{\mathcal{E}_{ijt}C_{jit}(h)}{CPI_{it}}\right\} = \mu\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\frac{W_{it}N_{jit}(h)}{P_{jit}(h)CPI_{it}}\right\} \tag{B.5}$$

Labor Market Clearing

We can take out the h index for firms because they are all identical. Now, substituting labor supply $\frac{\chi}{1-\tau_i}\frac{W_{it}}{CPI_{it}} = N_{it}^{\varphi}C_{it}^{\sigma}$ into the above equation, we get

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\frac{\mathcal{E}_{ijt}C_{jit}}{CPI_{it}}\right\} = \frac{\chi\mu}{1-\tau_i}\mathbb{E}_{t-1}\left\{\frac{N_{it}^{\varphi}N_{jit}}{P_{jit}}\right\}. \tag{B.6}$$

Substituting the identities $N_{jit} = C_{jit} Z_{it}^{-1}$ and $C_{jit} = \left(\frac{P_{jit}}{CPI_{jt}}\right)^{-\gamma} C_{jt}$ into the above equation gives:

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\frac{\mathcal{E}_{ijt}P_{jit}^{-\gamma}CPI_{jt}^{\gamma}C_{jit}}{CPI_{it}}\right\} = \frac{\chi\mu}{1-\tau_i}\mathbb{E}_{t-1}\left\{\frac{N_{it}^{\varphi}P_{jit}^{-\gamma}CPI_{jt}^{\gamma}C_{jit}}{Z_{it}P_{jit}}\right\}. \tag{B.7}$$

Now we use the fact that CPI_{it}, P_{jit} and N_{it} are predetermined:

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\frac{\mathcal{E}_{ijt}Y_{it}P_{jit}C_{jt}}{CPI_{it}}\right\} = \frac{\chi\mu}{1-\tau_i}\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}C_{jt}\right\}. \tag{B.8}$$

The next step in our derivation requires that we assume there is a common numeraire country, call it 0, and all countries fix their currency with respect to this numeraire currency. This means that $\mathcal{E}_{ijt} = \mathcal{E}_{i0t}\mathcal{E}_{0jt}$. When combined with predetermined prices and output, the following condition emerges:

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\frac{Y_{it}P_{jit}}{CPI_{it}}\mathcal{E}_{i0t}\right\} = \frac{\chi\mu}{1-\tau_i}\frac{\mathbb{E}_{t-1}\left\{C_{jt}\right\}\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{\mathcal{E}_{0jt}C_{jt}\right\}} \tag{B.9}$$

A.5.1 Non-Cooperative Equilibrium

Below we describe the non-cooperative Nash equilibrium under LCP. Here we assume that all countries, with the expection of country i, have a fixed exchange rate with the numeraire currency, such that $\mathcal{E}_{0jt} = 1$. Given this assumption, the labor clearing condition becomes

$$\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}\frac{Y_{it}P_{jit}}{CPI_{it}}\mathcal{E}_{i0t}\right\} = \frac{\chi\mu}{1-\tau_i}\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}. \tag{B.10}$$

Let us now write out the same equation for the numeraire country exporting to i and j respectively:

$$\mathbb{E}_{t-1}\left\{C_{0t}^{-\sigma}\frac{Y_{0t}P_{i0t}}{CPI_{0t}}\mathcal{E}_{00t}\right\} = \frac{\chi\mu}{1-\tau_0}\mathbb{E}_{t-1}\left\{N_{0t}^{1+\varphi}\right\}\frac{\mathbb{E}_{t-1}C_{it}}{\mathbb{E}_{t-1}[\mathcal{E}_{0it}C_{it}]}, \tag{B.11}$$

$$\mathbb{E}_{t-1}\left\{C_{0t}^{-\sigma}\frac{Y_{0t}P_{j0t}}{CPI_{0t}}\mathcal{E}_{00t}\right\} = \frac{\chi\mu}{1-\tau_0}\mathbb{E}_{t-1}\left\{N_{0t}^{1+\varphi}\right\}\frac{\mathbb{E}_{t-1}C_{jt}}{\mathbb{E}_{t-1}[\mathcal{E}_{0jt}C_{jt}]}. \tag{B.12}$$

If we divide (B.11) by (B.12), and use the fact that prices are predetermined and $\mathcal{E}_{0jt} = 1$, we get

$$\frac{P_{i0t}}{P_{j0t}} = \frac{CPI_{it}}{CPI_{jt}} = \frac{CPI_{it}}{CPI_{0t}} = \frac{\mathbb{E}_{t-1}C_{it}}{\mathbb{E}_{t-1}[\mathcal{E}_{0it}C_{it}]}. \tag{B.13}$$

Financial Autarky

In autarky, we assume all economies in the rest of the world $(-i)$ are ex-ante identical and that the actions of country i will not influence their policy decisions. Therefore, prices and price indices will be equalized across all $-i$ countries, so that $P_{0it} = P_{jit}$ and $CPI_{0t} = CPI_{jt}$ respectively. This will lead to the following set of identities:

$$C_{it} = \frac{\int_0^1 C_{jit}P_{0it}\mathcal{E}_{ijt}dj}{CPI_{it}} = \frac{P_{0it}}{CPI_{it}}\int_0^1 C_{jit}\mathcal{E}_{ijt}dj = \frac{P_{0it}}{CPI_{it}}\mathcal{E}_{i0t}\int_0^1 C_{jit}\mathcal{E}_{0jt}dj = \frac{P_{0it}}{CPI_{it}}\mathcal{E}_{i0t}Y_{it}. \tag{B.14}$$

Output is standard:

$$Y_{it} = \int_0^1 C_{jit} dj = \int_0^1 \left(\frac{P_{jit}}{CPI_{jt}}\right)^{-\gamma} C_{jt} dj = \left(\frac{P_{0it}}{CPI_{0t}}\right)^{-\gamma} C_{wt}, \tag{B.15}$$

$$\frac{P_{0it}}{CPI_{0t}} = Y_{it}^{\frac{-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}. \tag{B.16}$$

Now we can plug these equations into the labor market clearing equation (1.39).

$$\mathbb{E}_{t-1} C_{it}^{1-\sigma} = \frac{\chi\mu}{1-\tau_i} \mathbb{E}_{t-1} N_{it}^{1+\varphi} \tag{B.17}$$

Goods market clearing will be the following

$$C_{it} = \frac{P_{0it}}{CPI_{it}} \mathcal{E}_{i0t} Y_{it} = \frac{P_{0it}}{CPI_{0t}} \frac{CPI_{0t}}{CPI_{it}} \mathcal{E}_{i0t} Y_{it} = \frac{\mathcal{E}_{i0t} CPI_{0t}}{CPI_{it}} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}. \tag{B.18}$$

Using (B.13), this can be rewritten as:

$$C_{it} = \frac{\mathbb{E}_{t-1}\left\{\mathcal{E}_{0it} C_{it}\right\}}{\mathcal{E}_{0it} \mathbb{E}_{t-1}\left\{C_{it}\right\}} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}. \tag{B.19}$$

The optimization problem of the non-cooperative central bank in country i will then be:

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\} - \chi Y_{it}^{1+\varphi} \mathbb{E}_{t-1}\left\{Z_{it}^{-1-\varphi}\right\} + \lambda_1 \left[\mathbb{E}_{t-1} C_{it}^{1-\sigma} - \frac{\chi\mu}{1-\tau_i} \mathbb{E}_{t-1} N_{it}^{1+\varphi}\right] \\ &+ \lambda_2 \left[C_{it} - \frac{\mathbb{E}_{t-1}[\mathcal{E}_{0it} C_{it}]}{\mathcal{E}_{0it} \mathbb{E}_{t-1} C_{it}} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}\right]\end{aligned} \tag{B.20}$$

Maximization with respect to $\mathcal{E}_{0it}$ will yield the following FOC:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \mathcal{E}_{0it}} &= -\frac{1}{\mathcal{E}_{0it}} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}} + \frac{\mathbb{E}_{t-1}[\mathcal{E}_{0it} C_{it}]}{\mathcal{E}_{0it}^2 \mathbb{E}_{t-1} C_{it}} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}} = 0 \\
\Rightarrow \mathbb{E}_{t-1}\left\{\mathcal{E}_{0it} C_{it}\right\} &= \mathcal{E}_{0it} \mathbb{E}_{t-1}\left\{C_{it}\right\} \qquad \text{(B.21)}
\end{aligned}
$$

This proves that the optimal exchange rate chosen by the central bank must not be state-contingent. In other words, the central bank will choose to fix its exchange rate.

Complete Markets

In this section, we assume that the degree of exchange rate pass-through is governed by parameter $\eta \in [0,1]$, where $\eta = 1$ is perfect pass-through (PCP) and $\eta = 0$ is zero pass-through (LCP). Maximization of (A.3) with respect to state contingent bonds yields the following FOC:

$$
\frac{\lambda_i(s_t)}{\lambda_j(s_t)} = \frac{\lambda_{i0}}{\lambda_{j0}} \frac{P_i(s_t)}{P_j(s_t)\mathcal{E}_{ij}(s_t)} = \frac{\lambda_{i0}}{\lambda_{j0}} \frac{\mathcal{E}_{i0t}^{\eta}(s_t)}{\mathcal{E}_{j0t}^{\eta}(s_t)\mathcal{E}_{ij}(s_t)} = \frac{\lambda_{i0}}{\lambda_{j0}} \mathcal{E}_{ijt}^{\eta-1} \qquad \text{(B.22)}
$$

Using marginal utility, this will become:

$$
\frac{C_{it}^{-\sigma}}{C_{jt}^{-\sigma}} = \mathcal{E}_{ijt}^{\eta-1}. \qquad \text{(B.23)}
$$

So we can express consumption in the following way: $C_{it} = A_i \mathcal{E}_{ijt}^{\frac{1-\eta}{\sigma}} C_{jt}$. Using the fact that $C_{jt} = C_{wt}$, this will become $C_{it} = A_i \mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}} C_{wt}$. To find A_i we plug this expression into the transversality condition.

$$\begin{aligned}
\sum_{t=1}^{\infty}\sum_{s_t}\int_0^1 q_j(s_t)B_{ij}(s_t)dj &= \sum_{t=1}^{\infty}\sum_{s_t}\int_0^1 \beta^t \frac{\lambda_i(s_t)Pr(s_t)\mathcal{E}_{ij}(s_t)}{\lambda_{i0}P_i(s_t)}B_{ij}(s_t)dj \\
&= \sum_{t=1}^{\infty}\sum_{s_t}\beta^t \frac{\lambda_i(s_t)Pr(s_t)}{\lambda_{i0}P_i(s_t)}\int_0^1 \mathcal{E}_{ij}(s_t)B_{ij}(s_t)dj \\
&\overset{(A.1)}{=} \sum_{t=1}^{\infty}\sum_{s_t}\beta^t \frac{\lambda_i(s_t)Pr(s_t)}{\lambda_{i0}P_i(s_t)}\left(P_i(s_t)C_i(s_t) - W_i(s_t)N_i(s_t) - \Pi_i(s_t)\right) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
A_i &= \frac{\sum_{t=0}^{\infty}\sum_{s_t}\beta^t \frac{W(s_t)N(s_t)}{P(s_t)}\lambda_i(s_t)Pr(s_t)}{\sum_{t=0}^{\infty}\sum_{s_t}\beta^t \mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}C_w(s_t)\lambda_i(s_t)Pr(s_t)} \\
&= \frac{\sum_{t=0}^{\infty}\beta^t \mathbb{E}_t\left\{\frac{W_tN_t}{P_t}\lambda_i(s_t)\right\}}{\sum_{t=0}^{\infty}\beta^t \mathbb{E}_{t-1}\left\{\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}C_{wt}\lambda_i(s_t)\right\}} \\
&= \frac{\sum_{t=0}^{\infty}\beta^t \mathbb{E}_t\left\{Y_{it}TOT_{it}\lambda_i(s_t)\right\}}{\sum_{t=0}^{\infty}\beta^t \mathbb{E}_t\left\{\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}C_{wt}\lambda_i(s_t)\right\}} \\
&= \frac{\sum_{t=0}^{\infty}\beta^t \mathbb{E}_t\left\{Y_{it}TOT_{it}C_{wt}^{-\sigma}\right\}}{\sum_{t=0}^{\infty}\beta^t \mathbb{E}_t\left\{\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}C_{wt}C_{wt}^{-\sigma}\right\}}
\end{aligned}$$

Given the solution for A_i, it follows that:

$$C_{it} = \frac{\sum_{t=0}^{\infty}\beta^t \mathbb{E}_0\left\{Y_{it}TOT_{it}C_{wt}^{-\sigma}\right\}}{\sum_{t=0}^{\infty}\beta^t \mathbb{E}_0\left\{\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}C_{wt}^{1-\sigma}\right\}}\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}C_{wt}. \tag{B.24}$$

If technology shocks are independent across time, then $C_{it} = \frac{\mathbb{E}_{t-1}\left\{Y_{it}TOT_{it}C_{wt}^{-\sigma}\right\}}{\mathbb{E}_{t-1}\left\{\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}C_{wt}^{1-\sigma}\right\}}\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}C_{wt}$. Assuming independence of shocks across countries as well, we obtain $C_{it} = \frac{\mathbb{E}_{t-1}\{Y_{it}TOT_{it}\}}{\mathbb{E}_{t-1}\left\{\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}\right\}}\mathcal{E}_{i0t}^{\frac{1-\eta}{\sigma}}$.

Under LCP, if all countries except i have a fixed exchange rate, we have

$$\begin{aligned}
C_{it} =& \mathbb{E}_{t-1}\left\{\frac{\int_0^1 C_{jit}P_{0it}\mathcal{E}_{ijt}dj}{CPI_{it}}\right\}\frac{\mathcal{E}_{i0t}^{\frac{1}{\sigma}}}{\mathbb{E}_{t-1}\left\{\mathcal{E}_{i0t}^{\frac{1}{\sigma}}\right\}} \\
=& \mathbb{E}_{t-1}\left\{\frac{P_{0it}}{CPI_{it}}\mathcal{E}_{i0t}\int_0^1 C_{jit}\mathcal{E}_{0jt}dj\right\}\frac{\mathcal{E}_{i0t}^{\frac{1}{\sigma}}}{\mathbb{E}_{t-1}\left\{\mathcal{E}_{i0t}^{\frac{1}{\sigma}}\right\}} \\
=& \mathbb{E}_{t-1}\left\{\frac{P_{0it}}{CPI_{it}}\mathcal{E}_{i0t}Y_{it}\right\}\frac{\mathcal{E}_{i0t}^{\frac{1}{\sigma}}}{\mathbb{E}_{t-1}[\mathcal{E}_{i0t}^{\frac{1}{\sigma}}]}
\end{aligned}$$

Now we assume $\sigma = 1$, so that we have log utility. The goods market clearing constraint in complete markets will then be:

$$C_{it} = \frac{P_{0it}}{CPI_{it}}\mathcal{E}_{i0t}Y_{it} \tag{B.25}$$

Notice that this expression is identical to (1.40), the goods market clearing constraint in financial autarky. Since the labor equilibrium condition is also identical to the labor equilibrium condition in autarky, and the objective function does not change, the solution to the central bank's optimization problem in complete markets will be identical to that in autarky.

◇

A.5.2 Cooperative Equilibrium

Below we describe the cooperative equilibrium under LCP.

Proposition 7 *Under LCP, cooperative central banks will maximize (1.30b) subject to (1.41) and (1.42). The optimal policy for cooperative central banks in both complete markets and financial autarky will be a fixed exchange rate.*

Proof:

Financial Autarky

$$C_{it} = \frac{\int_0^1 C_{jit} P_{jit} \mathcal{E}_{ijt} dj}{CPI_{it}} \tag{B.26}$$

Now we assume $\sigma = 1$, and $P_{jit} = P_{0it} = CPI_{0t} = CPI_{it}$ because of ex-ante symmetry, so the above equation becomes:

$$C_{it} = \mathcal{E}_{i0t} \int_0^1 C_{jit} \mathcal{E}_{0jt} dj. \tag{B.27}$$

Complete Markets

$$C_{it} = \mathbb{E}_{t-1} \left\{ \frac{\int_0^1 C_{jit} P_{0it} \mathcal{E}_{ijt} dj}{CPI_{it}} \right\} \frac{\mathcal{E}_{i0t}^{\frac{1}{\sigma}}}{\mathbb{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{\frac{1}{\sigma}} \right\}} = \mathbb{E}_{t-1} \left\{ \frac{P_{0it}}{CPI_{it}} \mathcal{E}_{i0t} \int_0^1 C_{jit} \mathcal{E}_{0jt} dj \right\} \frac{\mathcal{E}_{i0t}^{\frac{1}{\sigma}}}{\mathbb{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{\frac{1}{\sigma}} \right\}} \tag{B.28}$$

Again, we assume $\sigma = 1$, and $P_{jit} = P_{0it} = CPI_{0t} = CPI_{it}$ because of ex-ante symmetry, so the above equation becomes:

$$C_{it} = \mathcal{E}_{i0t} \int_0^1 C_{jit} \mathcal{E}_{0jt} dj. \tag{B.29}$$

Notice that this expression is identical to autarky. Since the labor equilibrium condition is also identical to autarky, the optimization problem for cooperative central banks under LCP will be identical in complete markets and autarky.

We know that output will be predetermined because prices are predetermined. As a result, exchange rate policy can affect only consumption, but not labor. Therefore the maximization problem faced by cooperative central banks will be as follows:

$$\mathcal{L} = \int_0^1 C_{it}^{1-\sigma} di = \int_0^1 C_{it}^{1-\sigma} di + \int_0^1 \lambda_{it} \left(C_{it} - \mathcal{E}_{i0t} \int_0^1 C_{jt} \mathcal{E}_{j0t}^{-1} dj \right) di. \tag{B.30}$$

The FOC with respect to $\mathcal{E}_{i0t}$ is

$$\frac{\partial \mathcal{L}}{\partial \mathcal{E}_{i0t}} = -\lambda_{it} \frac{C_{it}}{\mathcal{E}_{i0t}} + \int_0^1 \lambda_{jt} \mathcal{E}_{j0t} C_{it} \mathcal{E}_{i0t}^{-2} dj = 0, \tag{B.31}$$

which yields the following condition:

$$\lambda_{it} \mathcal{E}_{i0t} = \int_0^1 \lambda_{jt} \mathcal{E}_{j0t} dj. \tag{B.32}$$

The above equation holds for all i, j pairs, so that $\lambda_{it}\mathcal{E}_{i0t} = \lambda_{jt}\mathcal{E}_{j0t}$. This equation holds when exchange rates are fixed. $\diamondsuit$

A.6 Cashless Economy

In this section we demonstrate the equivalence of the central bank optimizing over labor or the money supply. We begin with the closed economy and then proceed to the open economy. We will follow the notation employed in the rest of the paper.

A.6.1 Closed Economy

When we have money in the model, the household will maximize the following utility function:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ \frac{C_{t+k}^{1-\sigma}}{1-\sigma} + \frac{\rho}{1-b} \left(\frac{M_{t+k}}{P_{t+k}} \right)^{1-b} - \chi \frac{N_{t+k}^{1+\eta}}{1+\eta} \right\}$$

where M/P denotes the household's real money balances. The household budget constraint is:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = (1-\tau)\frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1}) \left(\frac{B_{t-1}}{P_t} \right) + \Pi_t. \tag{B.33}$$

The FOC with respect to M_t is:

$$C_t^{-\sigma} = \rho \left(\frac{M_t}{P_t} \right)^{-b} + \frac{P_t}{P_{t+1}} \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \right\}. \tag{B.34}$$

Labor market clearing implies

$$1 = \left(\frac{\chi\mu}{1-\tau} \right) \frac{\mathbb{E}_{t-1}\{N_t^{1+\varphi}\}}{\mathbb{E}_{t-1}\{C_t^{1-\sigma}\}}, \tag{B.35}$$

while goods market clearing gives

$$C_t = N_t Z_t. \tag{B.36}$$

We can now formulate the full optimization problem by the central bank, which maximizes the utility of the household by choosing a state contingent plan for the money supply.

$$\max_{M_t \ldots} \lim_{\rho \to 0} \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ \frac{C_{t+k}^{1-\sigma}}{1-\sigma} + \frac{\rho}{1-b} \left(\frac{M_{t+k}}{P_{t+k}} \right)^{1-b} - \chi \frac{N_{t+k}^{1+\eta}}{1+\eta} \right\} \tag{B.37}$$

subject to

$$C_t = N_t Z_t, \tag{B.38}$$

$$1 = \left(\frac{\chi \mu}{1-\tau} \right) \frac{\mathbb{E}_{t-1}\{N_t^{1+\varphi}\}}{\mathbb{E}_{t-1}\{C_t^{1-\sigma}\}}, \tag{B.39}$$

$$C_t^{-\sigma} = \rho \left(\frac{M_t}{P_t} \right)^{-b} + \frac{P_t}{P_{t+1}} \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \right\}. \tag{B.40}$$

Notice that the price level P is undefined in these equations because prices are set one-period-in-advance.

The solution to (B.37)-(B.40) will be inferior to that of the following optimization problem, where the central bank chooses labor.

$$\max_{N_t \ldots} \lim_{\rho \to 0} \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\rho}{1-b} \left(\frac{M_{t+k}}{P_{t+k}} \right)^{1-b} - \chi \frac{N_{t+k}^{1+\eta}}{1+\eta} \right\} \tag{B.41}$$

subject to

$$C_t = N_t Z_t \tag{B.42}$$

$$1 = \left(\frac{\chi \mu}{1-\tau} \right) \frac{\mathbb{E}_{t-1}\{N_t^{1+\varphi}\}}{\mathbb{E}_{t-1}\{C_t^{1-\sigma}\}}. \tag{B.43}$$

Why is the solution to the problem characterized by (B.37)-(B.40) inferior to the problem characterized by (B.41)-(B.43)? In the first problem money supply and inflation define the dynamics of consumption, and labor and goods market clearing constraints have to be satisfied. On the other hand, in the second problem the consumption path is arbitrary as long as it does not violate labor and goods market clearing.

As we demonstrated in Appendix B, the solution to (B.41)-(B.43) is

$$C_t = \left(\frac{1-\tau}{\chi \mu} \right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}. \tag{B.44}$$

We can construct a money supply rule that replicates this allocation by substituting (B.44) into (B.40) and solving for M.

The following money supply rule will implement the consumption allocation given by (B.44):

$$\left[\left(\frac{1-\tau}{\chi \mu} \right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}} \right]^{-\sigma} - \frac{P_t}{P_{t+1}} \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \right\} = \rho \left(\frac{M_t}{P_t} \right)^{-b}. \tag{B.45}$$

Thus, in equilibrium money supply would follow

$$\left[\left(\frac{1-\tau}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}\right]^{-\sigma} - \frac{P_t}{P_{t+1}}\mathbb{E}_t\left\{\left[\left(\frac{1-\tau}{\chi\mu}\right)^{\frac{1}{\sigma+\varphi}} Z_{t+1}^{\frac{1+\varphi}{\sigma+\varphi}}\right]^{-\sigma}\right\} = \rho\left(\frac{M_t}{P_t}\right)^{-b} \tag{B.46}$$

Notice, that the money supply rule (B.45) is different from (B.46) in the sense that it ensures only one equilibrium consumption path.

A.6.2 Open Economy

In the open economy, the central bank's money supply path is again subject to goods and labor market clearing constraints. For simplicity we will concentrate on the case of financial autarky, but the derivation for complete markets is quite similar. Also we ignore i subindices denoting individual countries to eliminate unnecessary notation. A central bank optimizing over money supply M in the open economy will face the following problem:

$$\max_{M_t\ldots}\lim_{\rho\to 0}\mathbb{E}_t\sum_{k=0}^{\infty}\left\{\frac{C_{t+k}^{1-\sigma}}{1-\sigma}+\frac{\rho}{1-b}\left(\frac{M_{t+k}}{P_{t+k}}\right)^{1-b}-\chi\frac{N_{t+k}^{1+\eta}}{1+\eta}\right\}, \tag{B.47}$$

subject to (B.48)

$$C_t = C_{wt}^{\frac{1}{\gamma}} N_t^{\frac{\gamma-1}{\gamma}} Z_t^{\frac{\gamma-1}{\gamma}} \tag{B.49}$$

$$1 = \left(\frac{\chi\mu}{1-\tau}\right)\frac{\mathbb{E}_{t-1}\{N_t^{1+\varphi}\}}{\mathbb{E}_{t-1}\{C_t^{1-\sigma}\}}, \tag{B.50}$$

$$C_t = \left[\rho\left(\frac{M_t}{CPI_t}\right)^{-b} + \mathbb{E}_t\left\{\frac{CPI_t}{CPI_{t+1}}C_{t+1}^{-\sigma}\right\}\right]^{\sigma}. \tag{B.51}$$

The optimization problem faced by a central bank in the open economy choosing labor N is given by:

$$\max_{C_t \dots} \lim_{\rho \to 0} \mathbb{E}_t \sum_{k=0}^{\infty} \left[\frac{C_{t+k}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+k}}{CPI_{t+k}} \right)^{1-b} - \chi \frac{N_{t+K}^{1+\eta}}{1+\eta} \right], \tag{B.52}$$

$$\text{subject to} \tag{B.53}$$

$$C_t = C_{wt}^{\frac{1}{\gamma}} N_t^{\frac{\gamma-1}{\gamma}} Z_t^{\frac{\gamma-1}{\gamma}}, \tag{B.54}$$

$$1 = \left(\frac{\chi\mu}{1-\tau} \right) \frac{\mathbb{E}_{t-1}\{N_t^{1+\varphi}\}}{\mathbb{E}_{t-1}\{C_t^{1-\sigma}\}}. \tag{B.55}$$

As in the closed economy, when central banks optimize over money (B.47)-(B.51), the solution will be inferior to that when central banks optimize over labor (B.52)-(B.55). As we showed in Appendix D, the solution to (B.52)-(B.55) is:

$$C_t = \left(\frac{1-\tau}{\chi\mu} \right)^{\frac{1}{\sigma+\varphi}} \left(Z_t^{\gamma-1} Z_{wt}^{\frac{1+\varphi}{\sigma+\varphi}} \right)^{\frac{1+\varphi}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{B.56}$$

The money supply rule that implements the same allocation is given by:

$$\left[\left(\frac{1-\tau}{\chi\mu} \right)^{\frac{1}{\sigma+\varphi}} \left(Z_t^{\gamma-1} Z_{wt}^{\frac{1+\varphi}{\sigma+\varphi}} \right)^{\frac{1+\varphi}{1-\sigma+\gamma(\varphi+\sigma)}} \right]^{-\sigma} = \rho \left(\frac{M_t}{CPI_t} \right)^{-b} + \mathbb{E}_t \left\{ \frac{CPI_t}{CPI_{t+1}} C_{t+1}^{-\sigma} \right\} \tag{B.57}$$

In financial autarky, we know that $CPI_t C_t = P_t Y_t$. Assuming that the producer price path P is equal to 1, we can plug in the expression $CPI_t = \frac{Y_t}{C_t}$ and (B.56) to determine the dynamics of real money supply and formulate a money supply rule that exactly replicates the allocation given by B.56).

Appendix B

The Optimal Design of a Fiscal Union

B.1 Proof of Proposition 9 and 10: Flexible Exchange Rate Allocations

Non-cooperative central banks will maximize their objective function (2.28) subject to (2.29a), (2.29b), (2.29c) and (2.29d) in complete markets and (2.29a), (2.29c), (2.29d) and (2.31) in financial autarky. The Lagrangian for the non-cooperative and cooperative cases is:

$$\mathcal{L} = \frac{\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\}}{1-\sigma} - \chi\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{1+\varphi} + \lambda_i\left(\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\} - \frac{\chi\mu_{\varepsilon}}{1-\tau_i}\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}\right)$$

Using $C_{it} = C_{wt}^{\frac{1}{\gamma}} \mathbb{E}_{t-1}\left\{N_{it}^{\frac{\gamma-1}{\gamma}} Z_{it}^{\frac{\gamma-1}{\gamma}}\right\}$ for complete markets, or $C_{it} = C_{wt}^{\frac{1}{\gamma}} N_{it}^{\frac{\gamma-1}{\gamma}} Z_{it}^{\frac{\gamma-1}{\gamma}}$ for financial autarky, we can take the first order condition with respect to N_{it}.[1] The FOC will be identical in both cases.

$$\frac{\partial \mathcal{L}}{\partial N_{it}} = C_{it}^{-\sigma}\left(1+\lambda_i(1-\sigma)\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}}{N_{it}}\right) - \chi\left(1+\lambda\frac{\mu_\varepsilon}{1-\tau_i}(1+\varphi)\right)\frac{1}{N_{it}} N_{it}^{1+\varphi} = 0$$

In equilibrium, this equals:

$$1 = \underbrace{\chi\left(\frac{1+\frac{\lambda_i\mu_\varepsilon(1+\varphi)}{1-\tau_i}}{1+\frac{\lambda_i(1-\sigma)(\gamma-1)}{\gamma}}\right)}_{\text{Constant}}\left(\frac{N_{it}^{1+\varphi}}{C_{it}^{-\sigma} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}}\right). \tag{B.1}$$

This equation holds in both complete markets and financial autarky, and differs from the flexible price equilibrium only by the constant term. However, subject to labor market clearing, this constant will coincide with the flexible price equilibrium. The flexible wage equilibrium in complete markets and financial autarky is found by taking expectations out of the labor market clearing condition (2.18) and substituting in goods market clearing (2.29b):

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)\frac{Y_{it}^{\frac{1+\varphi\gamma}{\gamma}}}{C_{it}^{-\sigma} C_{wt}^{\frac{1}{\gamma}} Z_{it}^{1+\varphi}}. \tag{B.2}$$

For complete markets, we can express output as a function of technology and a constant term by substituting (2.29b) into (B.2): $Y_{it} = A_i Z_{it}^{\frac{\gamma(1+\varphi)}{1+\gamma\varphi}}$. (We can do the same for exercise

[1] Remember that we are optimizing given the fact that state s_t is realized. Expectations in our context thus refer to a summation over all possible states multiplied by the probability of each state occuring. For example, $\mathbb{E}_{t-1}\{C_{it}^{1-\sigma}\} = \sum_{s_t} C_i^{1-\sigma}(s_t)\Pr(s_t)$.

for autarky by substituting (2.31) into (B.2), but leave that to the reader). Using this expression for output, consumption in complete markets in country i can be expressed as

$$C_{it} = A_i^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}} \mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}. \tag{B.3}$$

Now substitute (B.3) back into the flexible price equilibrium (B.2)

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right) C_{wt}^{\frac{\sigma}{\gamma}} A_i^{\frac{(\gamma-1)\sigma}{\gamma}} \mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}^{\sigma} A_i^{\frac{1+\varphi\gamma}{\gamma}} C_{wt}^{-\frac{1}{\gamma}}, \tag{B.4}$$

and rearrange and solve for A_i:

$$A_i = \left[\left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\gamma} C_{wt}^{1-\sigma} \mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}^{-\sigma\gamma}\right]^{\frac{1}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{B.5}$$

Now, substitute the solution (B.5) into (B.3):

$$C_{it} = \left[\left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)^{1-\gamma} C_{wt}^{\varphi+1} \mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}^{1+\gamma\varphi}\right]^{\frac{1}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{B.6}$$

Using the fact that $C_{wt} = \int_0^1 C_{it} di = C_{it}$ in equilibrium, and setting $Z_w = \left(\int_0^1 Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}$, integrate (B.6) over all i and solve for consumption for country i in complete markets:

$$C_{it} = \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}}. \tag{B.7}$$

Solving for labor and output using (B.7) is a straightforward exercise. The solution to the central bank's problem in complete markets and financial autarky for cooperative and non-cooperative equilibria coincides exactly with the flexible wage allocation.

Non-cooperative fiscal authorities will set a labor tax rate of $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$, introducing a terms of trade markup to exploit their country-level monopoly power. ■

B.2 Proof of Proposition 11 and 12: Currency Union Allocations

Non-cooperative policymakers in a currency union in complete markets will maximize (2.33) by choosing a non state contingent income tax rate, subject to (2.29a), (2.29b), (2.29c), (2.29d) and (2.34). From (2.34) we can compute labor using $Y_{it} = Z_{it} N_{it}$:

$$N_{it} = A Z_{it}^{\gamma-1}. \tag{B.8}$$

Given the above, consumption will be

$$C_{it} = C_{wt} = A \left(\int Z_{it}^{\gamma-1} di \right)^{\frac{\gamma}{\gamma-1}}. \tag{B.9}$$

Using labor market clearing (2.18), and substituting in Y_{it}, C_{it}, N_{it} expressed as functions of A and Z_{it} from above, we find:

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right) \frac{A^{1+\varphi}\int_0^1 Z_{it}^{(\gamma-1)(1+\varphi)}di}{A^{1-\sigma}\left(\int_0^1 Z_{it}^{(\gamma-1)}di\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}}} \tag{B.10}$$

Now we can solve for A:

$$A = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)^{\frac{-1}{\sigma+\varphi}} \left(\int_0^1 Z_{it}^{(\gamma-1)(1+\varphi)}di\right)^{\frac{-1}{\sigma+\varphi}} \left(\int_0^1 Z_{it}^{\gamma-1}di\right)^{\frac{\gamma}{\gamma-1}\frac{(1-\sigma)}{\sigma+\varphi}}. \tag{B.11}$$

Given this solution for the constant A, one can solve for C_{it} and N_{it} by substituting A into the expressions above, resulting in (2.35a) for C_{it} and (2.35b) for N_{it}. The same exercise in financial autarky will yield (2.36a) for C_{it} and (2.36b) for N_{it}. ■

B.3 Proof of Proposition 13: Contingent Fiscal Policy Within A Currency Union

We now assume that fiscal policymakers can choose their income tax rate in each period. In both complete markets and financial autarky, the fiscal authority in country i will maximize utility in each period, choosing the optimal contingent labor tax τ_{it}, given the realization of Z_{it} in that period. To solve for the optimal allocation, follow the exact same steps as in Appendix B.2, but use τ_{it} as the policy instrument rather than N_{it}. The optimal allocations will remove the wage rigidity distortion and mimic the flexible exchange rate allocations, given by (2.30a) in complete markets and (2.32a) in financial autarky. ■

B.4 Proof of Proposition 14: Tax Union

Under flexible exchange rates, policymakers in a tax union will maximize (2.39a) if they are not in a transfer union or (2.39e) if they are in a transfer union. In a currency union, policymakers in a tax union will maximize (2.39b) if they are not in a transfer union or (2.39f) if they are in a transfer union. Policymakers face the constraints outlined in Propositions 9 (flexible exchange rates and complete markets), 10 (flexible exchange rates and financial autarky), 11 (currency union and complete markets) or 12 (currency union and financial autarky), respectively.

In the non-cooperative case, policymakers do not internalize the impact of their tax rate on other countries. As a result, non-cooperative policymakers set τ_i to eliminate the markup on domestic intermediates, μ_ε, but also introduce a terms of trade markup, μ_γ, to take advantage of the monopoly power they exercise over their unique export good. The solution to the non-cooperative problem is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

In a tax union policymakers do internalize the impact of their tax rate on other countries. The solution to the cooperative problem defines the optimal tax rate within a tax union: $\tau_i = 1 - \mu_\varepsilon$. The optimal tax rate in a tax union eliminates the markup on domestic intermediate goods without imposing a terms of trade markup.

■

B.5 Proof of Proposition 15: Transfer Union

In a transfer union, policymakers agree to send contingent cash payments across countries. Under flexible exchange rates, policymakers in a transfer union will maximize (2.39c) if they are not in a tax union or (2.39e) if they are in a tax union. In a currency union, policymakers in a transfer union will maximize (2.39d) if they are not in a tax union or (2.39f) if they are in a tax union. Policymakers face the constraints outlined in Propositions 9 (flexible exchange rates and complete markets), 10 (flexible exchange rates and financial autarky), 11 (currency union and complete markets) or 12 (currency union and financial autarky), respectively. Because the transfers are state contingent, countries jointly agree on a state contingent plan to insure households against idiosyncratic consumption risk resulting from asymmetric shocks. Outside of a tax union, the solution to the transfer union optimization problem under flexible exchange rates will replicate the complete markets allocation detailed in Proposition 9. Outside of a tax union, the solution to the optimization problem in a currency union will replicate the complete markets allocation detailed in Proposition 11. Within a tax union, the solution to the transfer union optimization problem under flexible exchange rates will replicate the complete markets allocation detailed in Proposition 9 without a terms of trade markup. Outside of a tax union, the solution to the optimization problem in a currency union will replicate the complete markets allocation detailed in Proposition 11 without a terms of trade markup.

■

B.6 Proof of Proposition 17: Labor Mobility

In the presence of labor mobility, non-cooperative policymakers in a currency union in financial autarky will face the following problem

$$\max_{\tau_i} \frac{C^{1-\sigma}}{1-\sigma} - \chi \frac{N^{1+\varphi}}{1+\varphi} \tag{B.12}$$

s.t.

$$WN(h) = CP \tag{B.13a}$$

$$C = \left(\int_0^1 c_j^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \tag{B.13b}$$

$$\frac{W}{p_j} = Z_j \tag{B.13c}$$

$$P = \left(\int_0^1 p_j^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}} \tag{B.13d}$$

$$c_j = Z_j N_j \tag{B.13e}$$

$$N(h) = \int_0^1 N_j(h) dj \tag{B.13f}$$

$$N_j = \left[\int_0^1 N_j(h)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.13g}$$

where W is equalized across countries because of labor mobility. As each economy in the currency union is hit with idiosyncratic shocks, labor will shift from low demand bust countries to high demand boom countries.

FOCs:

$$c_j = \left(\frac{p_j}{P}\right)^{-\gamma} C \tag{B.14}$$

$$\frac{W}{P} = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right) C^\sigma N^\varphi \tag{B.15}$$

To solve for the optimal allocation, begin with (B.13d)

$$P = \left(\int_0^1 p_j^{1-\gamma} dj\right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\frac{W}{Z_j}\right)^{1-\gamma} dj\right)^{\frac{1}{1-\gamma}} = W\left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1}{1-\gamma}}$$

and solve for the real wage

$$\frac{W}{P} = \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1}{\gamma-1}}. \tag{B.16}$$

Now substitute this expression for the real wage into (B.13a):

$$C = N(h)\frac{W}{P} = N(h)\left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1}{\gamma-1}} = \int_0^1 N_j(h) dj \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1}{\gamma-1}}.$$

Take (B.14) and substitute in the expression for the real wage from (B.13c):

$$c_j = \left(\frac{p_j}{P}\right)^{-\gamma} C = \left(\frac{Z_j}{\frac{W}{P}}\right)^{\gamma} C, \tag{B.17}$$

and then substitute $c_j = Z_j N_j$ in B.17

$$N_j = \left(\frac{W}{P}\right)^{-\gamma} Z_j^{\gamma-1} C. \tag{B.18}$$

Integrating N_j over j will yield

$$\int_0^1 N_j dj = N = C\left(\frac{W}{P}\right)^{-\gamma} \int_0^1 Z_j^{\gamma-1} dj = C\left(\int_0^1 Z_j^{\gamma-1} dj\right)^{-\frac{1}{\gamma-1}}. \tag{B.19}$$

Using (B.15), we can substitute in our expression for N from (B.19) and our expression for W/P from (B.16):

$$\frac{W}{P} = \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1}{\gamma-1}} = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right) C^\sigma N^\varphi.$$

Solving for C, we find:

$$C = \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1+\varphi}{(\sigma+\varphi)(\gamma-1)}}. \tag{B.20}$$

To solve for N_j simply substitute (B.20) into (B.18):

$$N_j = \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1+\varphi-\gamma(\sigma+\varphi)}{(\gamma-1)(\sigma+\varphi)}} Z_j^{\gamma-1}. \tag{B.21}$$

As we've mentioned a number of times, the optimal tax rate outside of a tax union in the decentralized Nash equilibrium will be $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

When labor can move freely across borders, the equilibrium allocation outside of a tax union will be:

$$
\begin{aligned}
C_i &= \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\gamma-1} di\right)^{\frac{1+\varphi}{(\sigma+\varphi)(\gamma-1)}} \\
N_i &= \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1+\varphi-\gamma(\sigma+\varphi)}{(\gamma-1)(\sigma+\varphi)}} Z_j^{\gamma-1}.
\end{aligned}
$$

This allocation holds under flexible exchange rates and within a currency union, in both complete markets and financial autarky. ■

B.7 Welfare Derivations

Below, we outline the steps necessary to derive the expected utility functions contained in Section 2.7 of the paper. Here we only conduct the exercise for flexible exchange rates in complete markets, but following the steps presented here will also yield the expected utility functions for the other allocations.

$$
\begin{aligned}
C_{flex,complete} &= \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(\sigma+\varphi)}} \\
\mathbb{E}\left\{U_{flex,complete}\right\} &= \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] \mathbb{E}\left\{C_{flex,complete}^{1-\sigma}\right\} \\
&= \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1-\sigma}{\sigma+\varphi}} \mathbb{E}\left\{\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}}\right\}
\end{aligned}
$$

For normative analysis, we assume that technology is log-normally distributed and is independent across time and across countries: $\log Z_{it} \sim N(0, \sigma_Z^2)$. The expectation above can then be rewritten as:

$$\begin{aligned}\mathbb{E}\left\{\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}}\right\} &= e^{\left[\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}\right]^2 \frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}\sigma_Z^2} \\ &= e^{\frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2}.\end{aligned}$$

Now, we insert this expression back into the original equation and get:

$$\mathbb{E}\left\{U_{flex,complete}\right\} = \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right]\left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1-\sigma}{\sigma+\varphi}} e^{\frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2}.$$

Taking logarithms, we can rewrite the log of expected utility as:

$$\log\mathbb{E}\left\{U_{flex,complete}\right\} = \log\left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] + \frac{1-\sigma}{\sigma+\varphi}\log\left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right) + \frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2. \tag{B.22}$$

Calculating the expected utility for the other coalitions simply requires that one follow the steps outlined here. Notice that when we calculate welfare differences between allocations, the first and second terms on the right hand side of equation (B.22) will cancel out, leaving only the difference between the remaining term on the right hand side.

B.8 Are countries better off in a currency union?

A country with a flexible exchange rate and no risk-sharing is better off than a country in a currency union with perfect risk-sharing whenever

$$(1+\varphi)\left[1+\varphi+(\gamma-1)(1-\sigma)(\sigma+\varphi)\right]-(1+\varphi-\gamma\varphi)\left[1-\sigma+\gamma(\sigma+\varphi)\right]^2 \geq 0, \quad \text{(B.23)}$$

which can be rewritten in cubic form as

$$\begin{aligned}\Big(\gamma-1\Big)\Bigg\{&\varphi(\sigma+\varphi)^2(\gamma-1)^2+(\gamma-1)\Big[2\varphi(\sigma+\varphi)(1+\varphi)-(\sigma+\varphi)^2\Big]\\ &+\varphi(1+\varphi)^2+(1-\sigma)(1+\varphi)(\sigma+\varphi)-2(\sigma+\varphi)(1+\varphi)\Bigg\}\geq 0.\end{aligned} \quad \text{(B.24)}$$

The roots to this cubic equation are:

$$\gamma=\begin{cases}\frac{\sigma^2+2\sigma^2\varphi-\varphi^2+2\sigma\varphi^2-(\sigma+\varphi)^{\frac{3}{2}}\sqrt{\sigma+\varphi+4\sigma\varphi+4\sigma\varphi^2}}{2(\sigma^2\varphi+2\sigma\varphi^2+\varphi^2)}\\ 1\\ \frac{\sigma^2+2\sigma^2\varphi-\varphi^2+2\sigma\varphi^2+(\sigma+\varphi)^{\frac{3}{2}}\sqrt{\sigma+\varphi+4\sigma\varphi+4\sigma\varphi^2}}{2(\sigma^2\varphi+2\sigma\varphi^2+\varphi^2)}\end{cases} \quad \text{(B.25)}$$

where the first root is less than one, and the third root is greater than one. When γ is less than one or greater than the third root expressed in (B.25), a country will be better off outside of a currency union in financial autarky than as a member of a currency union in complete markets.

In the limiting case as households become completely risk neutral ($\sigma \to 0$), the roots of

(B.24) will be

$$\gamma = \begin{cases} -1 \\ 0 \\ 1 \end{cases} \tag{B.26}$$

while in the opposite limiting case, as households become extremely risk averse ($\sigma \to \infty$), the roots of (B.24) will be

$$\gamma = \begin{cases} 0 \\ 1 \\ \frac{1+2\varphi}{\varphi} \end{cases} \tag{B.27}$$

Internationally traded goods are perfect complements as γ approaches zero and perfect substitutes as γ approaches infinity, so γ must be non-negative. We can safely ignore any negative roots and focus only on positive roots. There is a very small window for which countries are better off as members of a currency union than as non-members. In a standard calibration with $\sigma = 2$ and $\varphi = 3$, countries are better off as members of a currency union only when $1 \leq \gamma \leq 1.12$. For all other values of γ outside this narrow range, households prefer to be outside of a currency union in financial autarky.

B.9 Extended Model with Home Bias and Calvo Wage Rigidity

The consumption basket in the home country is given by (2.48), and consists of both home goods C^H and an import basket defined by C^F. Similarly the price index will consist of goods prices of both home and foreign products:

$$P_{it} = \left[(1-\alpha)(P_{it}^H)^{1-\eta} + \alpha(P_{it}^F)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{B.28}$$

Relative demand for home and foreign products is given by

$$C_{it}^H = (1-\alpha)\left(\frac{P_{it}^H}{P_{it}}\right)^{-\eta} C_{it}, \tag{B.29}$$

$$C_{it}^F = \alpha\left(\frac{P_{it}^F}{P_{it}}\right)^{-\eta} C_{it}, \tag{B.30}$$

where C_{it}^F and C_{it}^H are defined as

$$C_{it}^F = \left(\int (C_{ijt}^F)^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}}, \tag{B.31}$$

$$C_{it}^H = \left(\int (C_{it}^H(h))^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{B.32}$$

C_{ijt}^F denotes consumption by households in country i of the variety produced by country j. $C_{it}^H(h)$ denotes consumption by households in country i of the domestic variety produced by intermediate firm h.

Production in each country i and the demand for country $i's$ goods are given by:

$$Y_{it} = Z_{it} N_{it} \tag{B.33}$$

$$Y_{it} = C_{it}^{H} + \left(P_{it}^{H} / P_{it}^{F}\right)^{-\gamma} C_{t}^{H*} \tag{B.34}$$

where C_t^{H*} represents foreign consumption of the home good.

$$C_t = \frac{\mathbb{E}_0 \left\{ \sum \beta^t Y_{it} \frac{P_{H,it}}{P_{it}^F} \right\}}{\mathbb{E}_0 \left\{ \sum \beta^t \left(\frac{P_{it}}{P_{it}^F} \right)^{\frac{\sigma-1}{\sigma}} \right\}} \left(\frac{P_t^F}{P_t} \right)^{\frac{1}{\sigma}} \tag{B.35}$$

The equations describing Calvo wage setting are:

$$V_{W,t} = N_t^{1+\varphi} + \beta \theta_W \mathbb{E}_t V_{W,t+1} \tag{B.36}$$

$$\tilde{V}_{W,t} = C_t^{-\sigma} \frac{N_t}{P_t} + \beta \theta_W \tilde{V}_{W,t+1} \tag{B.37}$$

$$\tilde{W} = \chi \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{V_{W,t}}{\tilde{V}_{W,t}} \tag{B.38}$$

$$W_t^{1-\varepsilon} = (1 - \theta_W) \tilde{W}_t^{1-\varepsilon} + \theta_W W_{t-1}^{1-\varepsilon} \tag{B.39}$$

where $\tilde{W}$ is the flexible wage and θ_W is the fraction of households who are able to reset wages in each period.

B.10 Non-cooperative Tax Policy in the Extended Model

In this section we solve for the optimal steady state income tax rate for non-cooperative fiscal authorities in the extended model with home bias. Because the optimal income tax rate is a steady state object, we will drop all time subscripts in this section. Using the demand equation from (B.29) and the fact that $C = YP^H/P$, we can write

$$C^H = (1-\alpha)C^{1-\eta}Y^{\eta}. \tag{B.40}$$

We can also rearrange the price index (B.28) to get

$$\left(\frac{P_H}{P}\right)^{\eta-1} + \alpha - 1 = \alpha\left(\frac{P_H}{P_F}\right)^{\eta-1}. \tag{B.41}$$

Substitute $C/Y = P^H/P$ into (B.41) and rearrange so that

$$\frac{P^H}{P^F} = \left[\frac{(C/Y)^{\eta-1} + \alpha - 1}{\alpha}\right]^{\frac{1}{\eta-1}}. \tag{B.42}$$

Now we plug this expression into the demand equation for home products (B.34):

$$Y = (1-\alpha)C^{1-\eta}Y^{\eta} + \left[\frac{(C/Y)^{\eta-1} + \alpha - 1}{\alpha}\right]^{\frac{-\gamma}{\eta-1}} C^{H*} \tag{B.43}$$

Using the implicit function theorem, we define $C = f(Y)$=(B.43), so that the non-cooperative policymaker's objective function becomes

$$\max_{Y} \frac{f(Y)^{1-\sigma}}{1-\sigma} - \chi \frac{Y^{1+\varphi}}{1+\varphi}. \tag{B.44}$$

The policymaker's first order conditions with respect to Y are

$$f'(Y)f(Y)^{-\sigma} - \chi Y^{\varphi} = 0 \tag{B.45}$$

where we've used $Y = N$ because we are in steady state, and $C = f(Y)$. From the implicit function theorem, we know that $f'(Y) = -g_Y/g_C$, where

$$g(C,Y) = -Y + (1-\alpha)C^{1-\eta}Y^{\eta} + \left[\frac{C^{\eta-1}Y^{1-\eta}+\alpha-1}{\alpha}\right]^{\frac{-\gamma}{\eta-1}} C^{H*}. \tag{B.46}$$

Solving for g_C gives

$$g_C = (1-\eta)(1-\alpha)C^{-\eta}Y^{\eta} - \frac{\gamma}{\alpha}\left[\frac{C^{\eta-1}Y^{1-\eta}+\alpha-1}{\alpha}\right]^{\frac{-\gamma}{\eta-1}-1} C^{H*}C^{\eta-2}Y^{1-\eta}. \tag{B.47}$$

In steady state, we know that $\alpha C = \alpha Y = C^{H*}$, so that $g_C = (1-\eta)(1-\alpha) - \gamma$. Similarly, we solve for g_Y:

$$g_Y = -1 + (1-\alpha)\eta C^{1-\eta}Y^{\eta-1} + \frac{\gamma}{\alpha}\left[\frac{C^{\eta-1}Y^{1-\eta}+\alpha-1}{\alpha}\right]^{\frac{-\gamma}{\eta-1}-1} C^{H*}C^{\eta-1}Y^{-\eta}, \tag{B.48}$$

which becomes $g_Y = -1+(1-\alpha)\eta+\gamma$ in steady state. Using these two simplified expressions for g_C and g_Y, we can rewrite the FOC from (B.45):

$$f'(Y)f(Y)^{-\sigma} - \chi Y^{\varphi} = -\frac{g_Y}{g_C}C^{-\sigma} - \chi Y^{\varphi} = \frac{1-(1-\alpha)\eta-\gamma}{(1-\eta)(1-\alpha)-\gamma}Y^{-\sigma} - \chi Y^{\varphi} = 0. \quad \text{(B.49)}$$

Using the implicit function theorem and solving for steady state Y yields:

$$Y = \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} = \left[\frac{1}{\chi}\left(\frac{\gamma-1+(1-\alpha)\eta}{\gamma-(1-\alpha)(1-\eta)}\right)\right]^{-\frac{1}{\sigma+\varphi}} \quad \text{(B.50)}$$

where the optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1-\mu_\varepsilon\left(\frac{\gamma-1+(1-\alpha)\eta}{\gamma-(1-\alpha)(1-\eta)}\right)$. As in the closed-form model, the optimal tax rate in a tax union will be $\tau_i = 1-\mu_\varepsilon$.

Appendix C

The Financial Accelerator and the Optimal Lending Contract

C.1 Value Function Transformation

We can also formulate the optimal contract using normalized variables. For example, one can substitute leverage into the right hand side of equation (3.5) to obtain

$$N_{t+s}(j) = N_{t+s-1}(j)\kappa_{t+s-1}(j)R^k_{t+s}\max\{\omega_{t+s}(j) - \bar{\omega}_{t+s}, 0\} + W^e_{t+s} = N_{t+s-1}(j)R^e_{t+s}(j) + W^e_{t+s}, \tag{B.1}$$

where $R_t^e(j) = \kappa_{t-1}(j) R_t^k \max\{\omega_t(j) - \bar{\omega}_t, 0\}$ is the entrepreneur's ex post realized return. Iterating this equation backward generates

$$\begin{aligned} N_{t+s}(j) &= N_{t+s-1}(j) R_{t+s}^e(j) + W_{t+s}^e \\ &= N_t(j) \tilde{R}_{t,t+s}^e + W_{t+1}^e \tilde{R}_{t+1,t+s}^e ... + W_{t+s}^e \\ &= N_t(j) \tilde{R}_{t,t+s}^e + \sum_{i=1}^{s} W_{t+i}^e \tilde{R}_{t+i,t+s}^e, \end{aligned} \tag{B.2}$$

where $\tilde{R}_{t,t+s}^e = R_{t+1}^e R_{t+2}^e ... R_{t+s}^e$ and $\tilde{R}_{t+s,t+s} = 1$. Intuitively, $\tilde{R}_{t,t+s}^e$ is the entrepreneur's ex post accumulated rate of return on projects from period t through period $t+s$. For example, suppose the entrepreneur invests one dollar in period t and continues to reinvest his profits in new projects in each subsequent period. In period $t+s$, the entrepreneur will have accumulated $\tilde{R}_{t,t+s}^e$ from his initial one dollar investment. We can substitute (B.2) into the value function (3.2) and obtain

$$V_t^e(j) = (1-\gamma)\mathbb{E}_t \left\{ N_t(j) R_{t+1}^e + \sum_{s=2}^{\infty} \gamma^{s-1} \left(N_t(j) \tilde{R}_{t,t+s}^e + \sum_{i=1}^{s-1} W_{t+i}^e \tilde{R}_{t+i,t+s}^e \right) \right\}. \tag{B.3}$$

Because the entrepreneur will optimize with respect to leverage and the productivity cutoff we want to express (B.3) as a function of the leverage and the productivity cutoff. In the first step, we separate terms to get

$$V_t^e(j) = (1-\gamma)N_t(j)\mathbb{E}_t\left\{\tilde{R}^e_{t,t+1}\sum_{s=1}^{\infty}\gamma^{s-1}\tilde{R}^e_{t+1,t+s}\right\} + (1-\gamma)\mathbb{E}_t\left\{\sum_{s=1}^{\infty}\gamma^{s-1}\left(\sum_{i=1}^{s-1}W^e_{t+i}\tilde{R}^e_{t+i,t+s}\right)\right\}, \tag{B.4}$$

where we used $\tilde{R}^e_{t,t+s} = \tilde{R}^e_{t,t+1}\tilde{R}^e_{t+1,t+s}$. Net worth enters the value function as a constant multiplicative term and has no effect on the entrepreneur's choice of leverage $\kappa_t(j)$ or cutoff $\bar{\omega}_{t+1}$; both enter only through $\tilde{R}^e_{t,t+1}$. Using the law of iterated expectations $\mathbb{E}_t(x_{t+1}) = \mathbb{E}_t[\mathbb{E}(x_{t+1}|\Omega_{agg,t+1})]$ and the independence of idiosyncratic productivity from aggregate productivity, we can replace the realizations of idiosyncratic productivity with their expectation and get

$$V_t^e(j) = (1-\gamma)N_t(j)\mathbb{E}_t\left[\tilde{R}^{e,agg}_{t,t+1}\sum_{s=1}^{\infty}\gamma^{s-1}\tilde{R}^{e,agg}_{t+1,t+s}\right] + (1-\gamma)\mathbb{E}_t\left[\sum_{s=2}^{\infty}\gamma^{s-1}\left(\sum_{i=1}^{s-1}W^e_{t+i}\tilde{R}^{e,agg}_{t+i,t+s}\right)\right], \tag{B.5}$$

where $\tilde{R}^{e,agg}_{t,t+s} = R^{e,agg}_{t+1}R^{e,agg}_{t+2}...R^{e,agg}_{t+s}$ with $\tilde{R}^{e,agg}_{t+s,t+s} = 1$, and $R^{e,agg}_{t+1} = \kappa_t R^k_{t+1} g(\bar{\omega}_{t+1})$ is the enterpreneur's ex post realized rate of return expressed as a function of aggregate productivity and leverage, with $g(\bar{\omega}_{t+1}) = \int_{\bar{\omega}_{t+1}}^{\infty}[\omega - \bar{\omega}_{t+1}]f(\omega)d\omega$.

Now we can reexpress value function as

$$V_t^e(j) = (1-\gamma)N_t(j)(\Psi_t - 1) + (1-\gamma)\mathbb{E}_t\left[\sum_{s=1}^{\infty}\gamma^s W_{t+s}^e(\Psi_{t+s} - 1)\right], \tag{B.6}$$

where $\Psi_t = 1 + \gamma\mathbb{E}_t\left[\kappa_t R_{t+1}^k g(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Psi_{t+1}\right]$. If we divide the lender's ex post returns in equation (3.7) by $N_t(j)$ we get

$$\left[\kappa_t(j) - 1\right]R_{t+1}(j) = \kappa_t(j)R_{t+1}^k h(\bar{\omega}_{t+1}, \sigma_{\omega,t}). \tag{B.7}$$

Now, if we substitute (B.7) into the Euler equation for the representative household (3.12), we have

$$\beta\mathbb{E}_t\left\{U_{C,t+1}\kappa_t(j)R_{t+1}^k h(\bar{\omega}_{t+1}, \sigma_{\omega,t})\right\} = \left[\kappa_t(j) - 1\right]U_{C,t}. \tag{B.8}$$

Before looking at the first order conditions to the optimization problem, it is important to notice that all entrepreneurs will choose the same leverage and state-contingent interest rate regardless of their net worth, due to the homotheticity of the problem. Thus, the entrepreneur index (j) is omitted below. We use the following notation: BGG refers to the contract of Bernanke, Gertler and Gilchrist (1999), CFP refers to the contract of Carlstrom, Fuerst and Paustian (2012), and *Optimal* refers to the optimal contract with non-myopic entrepreneurs

C.2 BGG Contract

In the BGG contract, the lender is guaranteed a fixed rate of return. In this case, the entrepreneur's Lagrangian will be:

$$\mathcal{L}^{BGG} = (1-\gamma)\mathbb{E}_t\Big\{N_t\kappa_t R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + \lambda_{t+1}\Big[\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}k_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - (k_t - 1)U_{C,t}\Big]\Big\}.$$

The entrepreneur's first order conditions with respect to κ_t and $\bar{\omega}_{t+1}$ are:

$$\frac{\partial \mathcal{L}^{BGG}}{\partial \kappa_t} = N_t\mathbb{E}_t\Big\{R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Big\} - \mathbb{E}_t\Big\{\lambda_{t+1}\Big\}\frac{U_{C,t}}{\kappa_t} = 0 \tag{B.9}$$

$$\begin{aligned}\frac{\partial \mathcal{L}^{BGG}}{\partial \omega_{t+1}} &= N_t\kappa_t R^k_{t+1} g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + \lambda_{t+1}\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}\kappa_t R^k_{t+1} h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = 0 \\ &= N_t g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + \lambda_{t+1}\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\} h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = 0 \end{aligned} \tag{B.10}$$

If we substitute $\frac{\partial \mathcal{L}^{BGG}}{\partial \omega_{t+1}}$ into $\frac{\partial \mathcal{L}^{BGG}}{\partial \kappa_t}$, we find

$$N_t\mathbb{E}_t\Big\{R^k_{t+1} g(\bar{\omega}_{t+1})\Big\} = \mathbb{E}_t - \left\{\frac{N_t g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1})\beta\mathbb{E}_t U_{c,t+1}}\right\}\frac{U_{C,t}}{\kappa_t}. \tag{B.11}$$

Rearranging, simplifying and substituting in the stochastic discount factor yields:

$$\kappa_t\mathbb{E}_t\Big\{R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Big\} = -\mathbb{E}_t\left\{\frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}\right\}\frac{1}{\mathbb{E}_t\Lambda_{t,t+1}}. \tag{B.12}$$

C.3 CFP Contract

In the CFP contract, the entrepreneur's Lagrangian is:

$$\mathcal{L}^{CFP} = (1-\gamma)\Bigg\{N_t\kappa_t\mathbb{E}_t\Bigg[R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\Bigg] + \lambda_t\Bigg[\mathbb{E}_t\Bigg(\beta U_{C,t+1}k_tR^k_{t+1}h(\bar{\omega}_{t+1},\sigma_{\omega,t})\Bigg) - (k_t-1)U_{C,t}\Bigg]\Bigg\}.$$

The entrepreneur's first order conditions with respect to κ_t and $\bar{\omega}_{t+1}$ are:

$$\frac{\partial\mathcal{L}^{CFP}}{\partial\kappa_t} = (1-\gamma)\left[N_t\mathbb{E}_t\left\{R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\right\} + \lambda_t\left(\mathbb{E}_t\left\{\beta U_{C,t+1}R^k_{t+1}h(\bar{\omega}_{t+1},\sigma_{\omega,t}) - U_{C,t}\right\}\right)\right] = 0,$$
$$\frac{\partial\mathcal{L}^{CFP}}{\partial\bar{\omega}_{t+1}} = (1-\gamma)\left[N_t\kappa_tR^k_{t+1}g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t}) + \lambda_t\beta U_{C,t+1}\kappa_tR^k_{t+1}h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})\right] = 0.$$

Rearranging these first order conditions, solving in terms of λ_t and setting them equal to each other yields:

$$\kappa_t\mathbb{E}_t\Bigg\{R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\Bigg\} = -\frac{g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}\frac{1}{\Lambda_{t,t+1}}. \tag{B.13}$$

C.4 Optimal Contract With Forward Looking Entrepreneurs

Under the optimal contract, the forward looking entrepreneur's Lagrangian has the following form (if we divide the value function by $(1-\gamma)N_t(j)$ as a scaling factor):

$$\begin{aligned}\mathcal{L}^{Optimal} =&(1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)(\Psi_t-1)+\sum_{s=1}^{\infty}\gamma^s W^e_{t+s}(\Psi_{t+s}-1)\\&+\sum_{i=0}^{\infty}\lambda_{t+i}\Big[\beta U_{c,t+i+1}\kappa_{t+i}R_{k,t+i+1}h(\bar{\omega}_{t+i+1},\sigma_{\omega,t+i})-(\kappa_{t+i}-1)U_{c,t+i}\Big]\Bigg\},\end{aligned}$$

where $\Psi_t = 1+\kappa_t\mathbb{E}_t\Big\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}\Big\}$. The entrepreneur's first order condition with respect to leverage κ_t is:

$$\frac{\partial\mathcal{L}_t^{Optimal}}{\partial\kappa_t}=(1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}+\lambda_t\Big(\beta U_{C,t+1}R^k_{t+1}h(\bar{\omega}_{t+1},\sigma_{\omega,t})-U_{C,t}\Big)\Bigg\}=0,$$

where we have used the fact that $\frac{\partial\Psi_t}{\partial\kappa_t}=\mathbb{E}_t\Big\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}\Big\}$ and $\frac{\partial\Psi_{t+i}}{\partial\kappa_t}=0$ for $i=1,2,...$ The entrepreneur's first order condition with respect to the productivity cutoff $\bar{\omega}_{t+1}$ is:

$$\frac{\partial\mathcal{L}_t^{Optimal}}{\partial\bar{\omega}_{t+1}}=(1-\gamma)\Bigg\{N_t(j)\kappa_t g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}+\lambda_t\Big[\beta U_{C,t+1}\kappa_t R_{k,t+1}h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})\Big]\Bigg\}=0. \tag{B.14}$$

where we have used the fact that $\frac{\partial\Psi_t}{\partial\bar{\omega}_{t+1}}=\kappa_t g'(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}$ and $\frac{\partial\Psi_{t+i}}{\partial\omega_{t+1}}=0$ for $i=1,2...$ We then move λ_t to the right hand side of both first order conditions and divide the equations by each other to obtain:

$$\frac{g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})\mathbb{E}_{t+1}\Psi_{t+1}}{\mathbb{E}_t\left\{R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\Psi_{t+1}\right\}} = \frac{\beta\kappa_t U_{C,t+1}h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\beta\mathbb{E}_t\Big\{U_{C,t+1}\kappa_t R^k_{t+1}h(\bar{\omega}_{t+1})\Big\}-\kappa_t U_{C,t}} = \frac{\beta\kappa_t U_{C,t+1}h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}{(\kappa_t-1)U_{C,t}-\kappa_t U_{C,t}}$$

where we utilized the participation constraint for lenders in the final step. After rearranging and simplifying, we get

$$\kappa_t\mathbb{E}_t\Big\{\Psi_{t+1}R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\Big\} = -\frac{g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}\frac{\Psi_{t+1}}{\Lambda_{t,t+1}}. \tag{B.15}$$

C.5 BGG Contract With Forward Looking Entrepreneurs

For a predetermined lending rate, the non-myopic entrepreneur's Lagrangian has the following form if we divide the value function by $(1-\gamma)N_t(j)$ as a scaling factor:

$$\begin{aligned}\mathcal{L} =& (1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)(\Psi_t-1)+\sum_{s=1}^{\infty}\gamma^s W^e_{t+s}(\Psi_{t+s}-1)\\ &+\sum_{i=0}^{\infty}\lambda_{t+i+1}\Big[\beta\mathbb{E}_t\Big\{U_{c,t+i+1}\Big\}\kappa_{t+i}R_{k,t+i+1}h(\bar{\omega}_{t+i+1},\sigma_{\omega,t+i})-(\kappa_{t+i}-1)U_{c,t+i}\Big]\Bigg\},\end{aligned}$$

where $\Psi_t = 1+\kappa_t\mathbb{E}_t\Big\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}\Big\}$. The entrepreneur's first order condition with respect to leverage κ_t is:

$$\frac{\partial\mathcal{L}_t}{\partial\kappa_t} = (1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}+\lambda_{t+1}\Bigg(\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}R^k_{t+1}h(\bar{\omega}_{t+1},\sigma_{\omega,t})-U_{C,t}\Bigg)\Bigg\}=0, \tag{B.16}$$

where we have used the fact that $\frac{\partial \Psi_t}{\partial \kappa_t} = \mathbb{E}_t\Big\{g(\bar{\omega}_{t+1}, \sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}\Big\}$ and $\frac{\partial \Psi_{t+i}}{\partial \kappa_t} = 0$ for $i = 1, 2, ...$ The entrepreneur's first order condition with respect to the productivity cutoff $\bar{\omega}_{t+1}$ is:

$$\frac{\partial \mathcal{L}_t}{\partial \bar{\omega}_{t+1}} =(1-\gamma)\Bigg\{N_t(j)\kappa_t g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})R_{k,t+1}\Psi_{t+1} + \lambda_{t+1}\Bigg[\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}\kappa_t R_{k,t+1}h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Bigg]\Bigg\} = 0. \tag{B.17}$$

where we have used the fact that $\frac{\partial \Psi_t}{\partial \bar{\omega}_{t+1}} = \kappa_t g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}$ and $\frac{\partial \Psi_{t+i}}{\partial \omega_{t+1}} = 0$ for $i = 1, 2...$ One can express λ_{t+1} in the equation $\frac{\partial \mathcal{L}}{\partial \omega_{t+1}} = 0$ as a function of other variables, and substitute the result into $\frac{\partial \mathcal{L}}{\partial \kappa_t} = 0$. Then, using the participation constraint to simplify and after some rearranging, we obtain:

$$\kappa_t\mathbb{E}_t\Bigg\{\Psi_{t+1}R^k_{t+1}g(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Bigg\} = -\mathbb{E}_t\Bigg\{\frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}\frac{\Psi_{t+1}}{\mathbb{E}_t\Lambda_{t,t+1}}\Bigg\}. \tag{B.18}$$

It is trivial to show that log-linearization of the BGG contract with myopic or non-myopic agents gives an identical optimality condition. However, this identity does not hold for higher order approximations.

C.6 Complete Log-Linearized Model

In this section we review the whole model in its log-linearized form.

New Keynesian Components

We begin with the set of equations characterizing the standard New Keynesian components of the model. Equation (B.19) gives the Euler equation for state-contingent assets from the FOC for deposits, while (B.20) is the Euler equation for nominal bonds. The labor market clearing condition is given by (B.21), (B.22) is the New Keynesian Phillips curve, (B.23) is the production function, (B.24) gives the evolution of capital, (B.25) refers to the dynamics of the price of capital, (B.26) gives returns to capital, and (B.27) refers to goods market clearing. Shocks to technology, monetary policy, government spending and idiosyncratic risk are defined by (B.28), (B.29), (B.30) and (B.31).

$$-\sigma\Big(\mathbb{E}_t\hat{C}_{t+1}-\hat{C}_t\Big)+\mathbb{E}_t\hat{R}_{t+1}=0, \tag{B.19}$$

$$\hat{R}^n_t=\mathbb{E}_t\hat{R}_{t+1}+\mathbb{E}_t\hat{\pi}_{t+1} \tag{B.20}$$

$$\hat{Y}_t-\hat{H}_t-\hat{\mathcal{X}}_t-\sigma\hat{C}_t=\eta\hat{H}_t, \tag{B.21}$$

$$\hat{\pi}_t=-\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\mathcal{X}}_t+\beta\mathbb{E}_t\hat{\pi}_{t+1}. \tag{B.22}$$

$$\hat{Y}_t=\hat{A}_t+\alpha\hat{K}_{t-1}+(1-\alpha)(1-\Omega)\hat{H}_t. \tag{B.23}$$

$$\hat{K}_t=\delta\hat{I}_t+(1-\delta)\hat{K}_{t-1}, \tag{B.24}$$

$$\hat{Q}_t=\delta\phi_K(\hat{I}_t-\hat{K}_{t-1}), \tag{B.25}$$

$$\hat{R}^k_{t+1}=(1-\epsilon)(\hat{Y}_{t+1}-\hat{K}_t-\hat{\mathcal{X}}_{t+1})+\epsilon\hat{Q}_{t+1}-\hat{Q}_t \tag{B.26}$$

$$Y\hat{Y}_t=C\hat{C}_t+I\hat{I}_t+G\hat{G}_t+C^e\hat{C}^e_t+\phi_\mu\hat{\phi}_{\mu,t}, \tag{B.27}$$

$$\hat{A}=\rho^A\hat{A}_{t-1}+\epsilon^A_t \tag{B.28}$$

$$\hat{R}^n_t=\rho^{R^n}\hat{R}^n_{t-1}+\xi\hat{\pi}_t+\rho^Y\hat{Y}_t+\epsilon^{R^n}_t \tag{B.29}$$

$$\hat{G}_t=\rho^G\hat{G}_{t-1}+\epsilon^G_t \tag{B.30}$$

$$\hat{\sigma}_{\omega,t}=\rho^{\sigma_\omega}\hat{\sigma}_{\omega,t-1}+\epsilon^{\sigma_\omega}_t \tag{B.31}$$

Entrepreneurial Consumption and Net Worth

The evolution of entrepreneurial net worth is given by (B.32), where (B.33) defines leverage. Entrepreneurial consumption is defined by (B.34) and the financial premium is given by

(B.35).

$$\hat{N}_{t+1} = \epsilon_N(\hat{N}_t + \hat{R}_{t+1} + \kappa(\hat{R}^k_{t+1} - \hat{R}_{t+1}) + \nu_\Psi \hat{\sigma}_{\omega,t}) + (1 - \epsilon_N)(\hat{Y}_t - \hat{\mathcal{X}}_t), \tag{B.32}$$

$$\hat{\kappa}_t = \hat{K}_t + \hat{Q}_t - \hat{N}_t \tag{B.33}$$

$$\hat{C}^e_{t+1} = \hat{N}_t + \hat{R}_{t+1} + \kappa(\hat{R}^k_{t+1} - \hat{R}_{t+1}) + \nu_\Psi \hat{\sigma}_{\omega,t} \tag{B.34}$$

$$\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{B.35}$$

Dynamics of the Lending Rate

The BGG lending rate is defined as (B.36a), the CFP lending rate is defined as (B.36b) and the optimal lending rate is defined as (B.36c). These log-linear expressions are derived in Appendix C.7.

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \begin{cases} 0 & \text{(B.36a)} \\ \hat{R}^k_{t+1} - \mathbb{E}_t R^k_{t+1} - \tilde{\alpha}\sigma(\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1}) & \text{(B.36b)} \\ \hat{R}^k_{t+1} - \mathbb{E}_t R^k_{t+1} - \tilde{\alpha}\Big[\sigma(\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1}) + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}\Big] & \text{(B.36c)} \end{cases}$$

$$\hat{\Psi}_{t+1} = \epsilon_N \mathbb{E}_{t+1}\Big\{(\kappa - 1)(\hat{R}^k_{t+2} - \hat{R}_{t+2}) + \hat{R}^k_{t+2} + \nu_\Psi \hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2}\Big\} \tag{B.37}$$

Monitoring Costs

$$\hat{\phi}_{\mu_t} = \hat{C}^e_{t+1} + \nu_\mu \left(\frac{1}{\kappa - 1}\hat{\kappa}_t - (\hat{R}_{k,t+1} - \hat{R}_{t+1})\right) + \nu_{\sigma,\mu}\hat{\sigma}_{\omega,t} \tag{B.38}$$

C.7 Log-linearization of the Lending Contracts

Log-linearization of the Common Optimality Condition

We begin by log-linearizing the common optimality condition for each contract. The non-linear participation constraint and FOC are, respectively:

$$\beta \mathbb{E}_t \left\{ U_{C,t+1} \right\} k_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - (k_t - 1) U_{C,t} = 0, \tag{B.39}$$

$$\kappa_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\mathbb{E}_t \left\{ \frac{g'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h'(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \right\} \frac{1}{\mathbb{E}_t \Lambda_{t,t+1}}. \tag{B.40}$$

In their linearized form, these become:

$$-\sigma \left(\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t \right) + \hat{R}^k_{t+1} + \frac{h_\omega}{h} \bar{\omega} \hat{\omega}_{t+1} + \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t, \tag{B.41}$$

$$\hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{g_{\sigma_\omega}}{g} \sigma_\omega \hat{\sigma}_{\omega,t} = -\mathbb{E}_t \hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} \right) \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} \right) \sigma_\omega \hat{\sigma}_{\omega,t}. \tag{B.42}$$

Now we take the expected value of the participation constraint (B.41) and obtain:

$$\mathbb{E}_t \hat{\Lambda}_{t,t+1} + \mathbb{E}_t \hat{R}^k_{t+1} + \frac{h_\omega}{h} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t. \tag{B.43}$$

Define $\hat{\triangle}_t = \hat{R}^k_t - \hat{R}_t$, and rewrite the system as:

$$\frac{1}{\kappa-1}\hat{\kappa}_t - \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} - \mathbb{E}_t\hat{\triangle}_{t+1} = \frac{h_\omega}{h}\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1},$$
$$\hat{\kappa}_t + \mathbb{E}_t\hat{\triangle}_{t+1} - \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_{\sigma_\omega}}{g}\right)\sigma_\omega\hat{\sigma}_{\omega,t} = \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right)\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}. \tag{B.44}$$

Now we can set these two equations equal to each other and eliminate ω:

$$\hat{\kappa}_t + \mathbb{E}_t\hat{\triangle}_{t+1} - \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_{\sigma_\omega}}{g}\right)\sigma_\omega\hat{\sigma}_{\omega,t} = \frac{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right)}{\frac{h_\omega}{h}}\left(\frac{1}{\kappa-1}\hat{\kappa}_t - \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} - \mathbb{E}_t\hat{\triangle}_{t+1}\right). \tag{B.45}$$

We can rearrange this to obtain:

$$\mathbb{E}_t\hat{\triangle}_{t+1} = \frac{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}\right)}\frac{1}{\kappa-1}\hat{\kappa}_t + \frac{\left[-\frac{h_{\sigma_\omega}}{h}\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right) + \frac{h_\omega}{h}\left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_{\sigma_\omega}}{g}\right)\right]}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}\right)}\sigma_\omega\hat{\sigma}_{\omega,t}. \tag{B.46}$$

This can be simplified to give the log-linear optimality condition, which is identical for all three lending contracts:

$$\mathbb{E}_t\hat{\triangle}_{t+1} = \nu_\kappa\hat{\kappa}_t + \nu_\sigma\sigma_\omega\hat{\sigma}_{\omega,t} \tag{B.47}$$

where

$$\nu_\kappa = \frac{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}\right)} \frac{1}{\kappa - 1}, \tag{B.48}$$

$$\nu_\sigma = \frac{\left[- \frac{h_{\sigma\omega}}{h}\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right) + \frac{h_\omega}{h}\left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_{\sigma\omega}}{g}\right)\right]}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}\right)}. \tag{B.49}$$

Log-linearization of the BGG Lending Rate

The log-linear lending rate in BGG is given by:

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = 0 \tag{B.50}$$

Log-linearization of the CFP Contract

The non-linear participation constraint and FOC are, respectively:

$$\beta \mathbb{E}_t \left\{ U_{C,t+1} k_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} - (k_t - 1) U_{C,t} = 0, \tag{B.51}$$

$$\kappa_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = - \frac{g'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h'(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{1}{\Lambda_{t,t+1}}. \tag{B.52}$$

In their linearized form, these become:

$$\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1} + \frac{h_\omega}{h}\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1} + \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} = \frac{1}{\kappa-1}\hat{\kappa}_t, \tag{B.53}$$

$$\hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{g_\omega}{g}\bar{\omega}\mathbb{E}_t\bar{\omega}_{t+1} + \frac{g_{\sigma_\omega}}{g}\sigma_\omega\hat{\sigma}_{\omega,t} = -\hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\omega}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t}. \tag{B.54}$$

Now we plug the participation constraint (B.53) into the FOC (B.54) and obtain

$$\begin{aligned}\hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}}\left(\frac{1}{\kappa-1}\hat{\kappa}_t - \mathbb{E}_t R^k_{t+1} + \mathbb{E}_t R_{t+1} - \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t}\right) + \frac{g_{\sigma_\omega}}{g}\sigma_\omega\hat{\sigma}_{\omega,t} = \\ -\hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\omega}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t}.\end{aligned} \tag{B.55}$$

Using $-\frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} = (k-1)$, we can simplify the last expression to:

$$k(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t\hat{\Lambda}_{t,t+1} + \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} = \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\omega}_{t+1}. \tag{B.56}$$

Define lender's returns by log-linearizing $R_{t+1} = \frac{\kappa_t}{\kappa_t - 1}h(\bar{\omega}_{t+1}, \sigma_{\omega,t})R^k_{t+1}$:

$$\hat{R}_{t+1} = -\frac{1}{k-1}\hat{k}_t + \frac{h_\omega}{h}\bar{\omega}\hat{\omega}_{t+1} + \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \tag{B.57}$$

Now substitute in the expression for the cutoff and obtain

$$\hat{R}_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}} \left[k(\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} + \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega} \right) \sigma_\omega \hat{\sigma}_{\omega,t} \right]$$
$$- \frac{1}{k-1} \hat{k}_t + \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \tag{B.58}$$

Finally, substitute in the expression for leverage:

$$\hat{R}_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}} \left[\hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} + \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega} \right) \sigma_\omega \hat{\sigma}_{\omega,t} \right]$$
$$+ \frac{\left[-\frac{h_{\sigma_\omega}}{h} \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} \right) + \frac{h_\omega}{h} \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_{\sigma_\omega}}{g} \right) \right]}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} \right)} \sigma_\omega \hat{\sigma}_{\omega,t}$$
$$+ \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} + (\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}) + \hat{R}^k_{t+1}. \tag{B.59}$$

After some rearranging and canceling out like terms, we obtain the log-linear CFP lending rate:

$$\hat{R}_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}} \left[\hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} \right] - (\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}) + \hat{R}^k_{t+1}. \tag{B.60}$$

This can be rewritten in the same form as Corollary 2 in the text:

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_{t+1} \hat{R}^k_{t+1} - \tilde{\alpha}\sigma \big(\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1} \big) \tag{B.61}$$

where $\tilde{\alpha} = \frac{\frac{h_\omega}{h}}{\frac{h_{\omega\omega}}{h_\omega} - \frac{g_{\omega\omega}}{g_\omega}}$.

Log-linearization of the Optimal Contract

The non-linear participation constraint and FOC are, respectively:

$$\beta \mathbb{E}_t \left\{ U_{C,t+1} k_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} - (k_t - 1) U_{C,t} = 0 \tag{B.62}$$

$$\kappa_t \mathbb{E}_t \left\{ \Psi_{t+1} R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h'(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{\Psi_{t+1}}{\Lambda_{t,t+1}}, \tag{B.63}$$

where Ψ_t is defined as:

$$\Psi_t = 1 + \kappa_t \mathbb{E}_t \{ g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} \}. \tag{B.64}$$

In their linearized form, the participation constraint and FOC become:

$$-\sigma \left(\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t \right) + \mathbb{E}_t \hat{R}^k_{t+1} + \frac{h_\omega}{h} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{h_\sigma}{h} \sigma_\omega \hat{\sigma}_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t, \tag{B.65}$$

$$\begin{aligned} \mathbb{E}_t \Psi_{t+1} + \hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \bar{\omega}_{t+1} + \frac{g_{\sigma_\omega}}{g} \sigma_\omega \hat{\sigma}_{\omega,t} =& \hat{\Psi}_{t+1} - \hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} \right) \bar{\omega} \hat{\omega}_{t+1} \\ &+ \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} \right) \sigma_\omega \hat{\sigma}_{\omega,t}. \end{aligned} \tag{B.66}$$

where

$$\hat{\Psi}_t = \epsilon_N \left(\hat{\kappa}_t + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{g_\sigma}{g} \sigma_\omega \hat{\sigma}_{\omega,t} + \mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{\Psi}_{t+1} \right). \tag{B.67}$$

Now we substitute the participation constraint into the optimality condition and obtain:

$$\begin{aligned}
&\hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}}\left(\frac{1}{\kappa - 1}\hat{\kappa}_t - \mathbb{E}_t R^k_{t+1} + \mathbb{E}_t R_{t+1} - \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t}\right) + \frac{g_{\sigma_\omega}}{g}\sigma_\omega\hat{\sigma}_{\omega,t} \\
&= -\hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\omega}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1}. \qquad \text{(B.68)}
\end{aligned}$$

Using $-\frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} = (k-1)$, we can simplify the previous expression to

$$\begin{aligned}
\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\omega}_{t+1} =& k(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t\hat{\Lambda}_{t,t+1} \\
&+ \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} - (\hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1}).
\end{aligned} \qquad \text{(B.69)}$$

Define the lender's returns by log-linearizing $R_{t+1} = \frac{\kappa_t}{\kappa_t - 1}h(\bar{\omega}_{t+1}, \sigma_{\omega,t})R^k_{t+1}$:

$$\hat{R}_{t+1} = -\frac{1}{k-1}\hat{k}_t + \frac{h_\omega}{h}\bar{\omega}\hat{\omega}_{t+1} + \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \qquad \text{(B.70)}$$

Now substitute in the expression for the cutoff and obtain:

$$\begin{aligned}
\hat{R}_{t+1} =& \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}}\Bigg[k(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t\hat{\Lambda}_{t,t+1} + \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} \\
&- (\hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1})\Bigg] - \frac{1}{k-1}\hat{k}_t + \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \qquad \text{(B.71)}
\end{aligned}$$

Rearranging and canceling out like terms yields:

$$\hat{R}_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}} \left[\hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} - (\hat{\Psi}_{t+1} - \mathbb{E}_t \ \hat{\Psi}_{t+1}) \right] + \hat{R}^k_{t+1} - (\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}). \tag{B.72}$$

This can be rewritten in the same form as Corollary 3 in the text:

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_{t+1} \hat{R}^k_{t+1} - \tilde{\alpha} \Big[\sigma \big(\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1} \big) + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1} \Big] \tag{B.73}$$

where $\tilde{\alpha} = \frac{\frac{h_\omega}{h}}{\frac{h_{\omega\omega}}{h_\omega} - \frac{g_{\omega\omega}}{g_\omega}}$. Finally, we eliminate ω in the expression for $\hat{\Psi}_t$:

$$\hat{\Psi}_t = \epsilon_N \left[\hat{\kappa}_t + \frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} \left(\frac{1}{\kappa - 1} \hat{\kappa}_t - \mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{R}_{t+1} - \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} \right) + \frac{g_\sigma}{g} \sigma_\omega \hat{\sigma}_{\omega,t} + \mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{\Psi}_{t+1} \right] \tag{B.74}$$

and we can rearrange this expression to match Corollary 3 in the text:

$$\hat{\Psi}_{t+1} = \epsilon_N \mathbb{E}_{t+1} \Big\{ (\kappa - 1)(\hat{R}^k_{t+2} - \hat{R}_{t+2}) + \hat{R}^k_{t+2} + \nu_\Psi \hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2} \Big\} \tag{B.75}$$

where $\nu_\Psi = \frac{g_\sigma - h_\sigma \frac{g_\omega}{h_\omega}}{g} \sigma_\omega$.

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