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# Quantum Criticality and Emergence of the $T/B$ Scaling in Strongly Correlated Metals

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**Abstract**

A new type of scaling observed in heavy-electron metal $\beta$-$YbAlB_4$, where the magnetic susceptibility is expressed as a single scaling function of the ratio of temperature $T$ and magnetic field $B$ over four decades, is examined theoretically. We develope the mode-coupling theory for critical Yb-valence fluctuations under a magnetic field, verifying that the $T/B$ scaling behavior appears near the QCP of the valence transition. Emergence of the $T/B$ scaling indicates the presence of the small characteristic temperature of the critical Yb-valence fluctuation due to the strong local correlation effect. It is discussed that the $T/B$ scaling as well as the unconventional criticality is explained from the viewpoint of the quantum valence criticality in a unified way.

*Keywords:* quantum criticality, $\beta$-$YbAlB_4$, $T/B$ scaling, critical valence fluctuation

## 1. Introduction

Quantum critical phenomena not following the conventional spin-fluctuation theory in itinerant-electron systems have attracted much attention in the condensed-matter physics. The heavy-electron metal $\beta$-$YbAlB_4$ has recently attracted great interest since it exhibits not only unconventional quantum criticality in physical quantities such as the magnetic susceptibility $\chi(T)$, the resistivity $\rho(T)$, and the specific heat $C(T)$ (1), but also anomalous scaling behavior in the magnetization data (2). Namely, the magnetic susceptibility is expressed as a single scaling function of the ratio of the temperature $T$ and

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the magnetic field $B$ over four decades in the form like

$$\frac{1}{\chi} = (\mu_{\rm B} B)^{1/2} \varphi \left( \frac{k_{\rm B} T}{\mu_{\rm B} B} \right), \tag{1}$$

where $\mu_{\rm B}$ is the Bohr magneton and $k_{\rm B}$ is the Boltzmann constant. Here, $\varphi$ is the scaling function $\varphi(x) = \Lambda(\Gamma^2 + x^2)^{1/4}$ where $\Lambda$ and $\Gamma$ are constants.

To account for this scaling behavior, Eq. (1), a theoretical proposal based on a lattice model with anisotropic hybridization has been put forth, so far (3). However, this theory requires a fine tuning of the f level at the hybridized band edge and it is unclear whether the quantum criticality observed in the resistivity and the specific heat can be explained by this scenario. Although a theoretical proposal based on the impurity Anderson model with a pseudo-gapped density of states was also reported (4), it has not been shown whether the $T/B$ scaling as well as the unconventional criticality can be explained in the lattice model.

Recently, we have shown theoretically that a new type of quantum criticality emerges near the quantum critical point (QCP) of the first-order valence transition, giving rise to unconventional quantum criticality in the physical quantities such as $\chi(T)$, $\rho(T)$, and $C(T)$ as observed in $\beta$-$YbAlB_4$ (5). Hence, it is interesting to examine whether the theory of critical Yb-valence fluctuations can explain the $T/B$-scaling behavior in Eq. (1). It turns out that the $T/B$ scaling as well as the unconventional quantum criticality in $\beta$-$YbAlB_4$ can be naturally understood from the viewpoint of the quantum valence criticality in a unified way (12).

## 2. Mode-coupling theory based on the extended periodic Anderson model

We start from the extended periodic Anderson model as the simplest minimal model for the Yb- and Ce-based heavy-electron systems.

$$H = H_{\rm PAM} + H_{U_{\rm fc}} + H_{\rm Zeeman}, \tag{2}$$

where

$$H_{\rm PAM} = \sum_{{\bf k}\sigma} \varepsilon_{\bf k} c^{\dagger}_{{\bf k}\sigma} c_{{\bf k}\sigma} + \varepsilon_{\rm f} \sum_{i\sigma} n^{\rm f}_{i\sigma} + \sum_{{\bf k}\sigma} \left( V_{\bf k} f^{\dagger}_{{\bf k}\sigma} c_{{\bf k}\sigma} + {\rm h.c.} \right) + U \sum_{i} n^{\rm f}_{i\uparrow} n^{\rm f}_{i\downarrow}, \tag{3}$$

$$H_{U_{\rm fc}} = U_{\rm fc} \sum_{i\sigma\sigma'} n^{\rm f}_{i\sigma} n_{i\sigma'}, \tag{4}$$

$$H_{\rm Zeeman} \;=\; -h\sum_{i} S_{i}^{{\rm f}z}, \tag{5}$$

with $n_{i\sigma}^{\rm a} \equiv a_{i\sigma}^{\dagger}a_{i\sigma}$. $H_{\rm PAM}$ is the so-called periodic Anderson model. The first and second terms represent the conduction electron's energy band and the f level, respectively. The third term is the hybridization between f and conduction electrons. The fourth term is the onsite Coulomb repulsion between f electrons.

$H_{U_{\rm fc}}$ is the inter-orbital Coulomb repulsion, which plays an important role for the valence transition (6; 7). The first-order valence transition (FOVT) where the f-electron number shows a discontinuous change as a function of parameters such as $\varepsilon_{\rm f}$, $V_{\mathbf{k}}$, and temperature is caused by large $U_{\rm fc}$ (8; 9) since large $U_{\rm fc}$ forces electrons to pour out of the f level into the conduction band or vise versa. As $U_{\rm fc}$ decreases, the magnitude of the jump in the f-electron number decreases at the FOVT and finally disappears at the quantum critical end point of the FOVT at zero temperature, which is defined as the QCP of the valence transition. At the QCP, valence fluctuation diverges, giving rise to unconventional superconductivity (10) and even unconventional quantum criticality (5). As $U_{\rm fc}$ further decreases, the valence crossover occurs. The global phase diagram and the properties are summarized in Ref. (11).

$H_{\rm Zeeman}$ is the Zeeman term with $S_{i}^{{\rm f}z} = (n_{i\uparrow}^{\rm f} - n_{i\downarrow}^{\rm f})/2$.

We extend the theory for critical valence fluctuations developed in Ref. (5) so as to describe the effect of a magnetic field. To discuss the quantum criticality of the valence fluctuations, first we should take into account the effect of the Coulomb repulsion for f electrons as the strongest interaction in Eq. (2). After that, we should construct the mode-coupling theory for the critical valence fluctuations caused by the $H_{U_{\rm fc}}$ term. To construct such a framework, we employ the slave-boson large-$N$ expansion scheme, which enables us to describe the heavy-electron state for $U \to \infty$ by introducing the slave-boson operator $\sqrt{N}b_i$ under the constraint $\sum_{\sigma} n_{i\sigma}^{\rm f} + 2b_i^{\dagger}b_i = 1$ (10). Here we set $N = 2$ since the Kramers-doublet ground state is realized in $\beta$-YbAlB$_4$ (13). The Lagrangian is written as $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$, where $\mathcal{L}_0$ is the Lagrangian for $H_{\rm PAM} + H_{\rm Zeeman}$ with the term $-\sum_i \lambda_i \left(\sum_{\sigma} n_{i\sigma}^{\rm f} + 2b_i^{\dagger}b_i - 1\right)$ with $\lambda_i$ being the Lagrange multiplier and $\mathcal{L}'$ is the Lagrangian for $H_{U_{\rm fc}}$.

For $\exp(-S_0)$ with the action $S_0 = \int_0^{\beta} d\tau \mathcal{L}_0(\tau)$, the saddle-point solution is employed through the stationary condition $\delta S_0 = 0$ by approximating the spatially uniform and time independent solutions, $\lambda_{\mathbf{q}} = \lambda\delta_{\mathbf{q}=\mathbf{0}}$ and $b_{\mathbf{q}} = b\delta_{\mathbf{q}=\mathbf{0}}$. For $\exp(-S')$ with the action $S' = \int_0^{\beta} d\tau \mathcal{L}'(\tau)$, we introduce the identity through a Stratonovich-Hubbard transformation

$$e^{-S'} = \int \mathcal{D}\varphi \exp\left[\sum_{i\sigma}\int_0^\beta d\tau \left\{-\frac{U_{\rm fc}}{2}\varphi_{i\sigma}(\tau)^2 + i\frac{U_{\rm fc}}{\sqrt{2}}(c_{i\sigma}f_{i\sigma}^\dagger - f_{i\sigma}c_{i\sigma}^\dagger)\varphi_{i\sigma}(\tau)\right\}\right]. \quad (6)$$

By executing the Grassmann number integration for $cc^\dagger$ and $ff^\dagger$, the partition function is expressed as $Z = \int \mathcal{D}\varphi \exp(-S[\varphi])$ with $S = S_0 + S'$ where $S$ is expanded as

$$\begin{aligned} S[\varphi] = \sum_\sigma \Bigg[ & \frac{1}{2}\sum_{\bar{q}} \Omega_{2\sigma}(\bar{q})\varphi_\sigma(\bar{q})\varphi_\sigma(-\bar{q}) + \sum_{\bar{q}_1,\bar{q}_2,\bar{q}_3} \Omega_{3\sigma}(\bar{q}_1,\bar{q}_2,\bar{q}_3) \\ & \times\varphi_\sigma(\bar{q}_1)\varphi_\sigma(\bar{q}_2)\varphi_\sigma(\bar{q}_3)\delta\left(\sum_{i=1}^3 \bar{q}_i\right) + \sum_{\bar{q}_1,\bar{q}_2,\bar{q}_3,\bar{q}_4} \Omega_{4\sigma}(\bar{q}_1,\bar{q}_2,\bar{q}_3,\bar{q}_4) \\ & \times\varphi_\sigma(\bar{q}_1)\varphi_\sigma(\bar{q}_2)\varphi_\sigma(\bar{q}_3)\varphi_\sigma(\bar{q}_4)\delta\left(\sum_{i=1}^4 \bar{q}_i\right) + \cdots \Bigg], \end{aligned} \quad (7)$$

where the abbreviation $\bar{q} \equiv (\mathbf{q}, i\omega_\ell)$ with $\omega_\ell = 2\ell\pi T$ is used.

By applying the renormalization-group analysis to the action in Eq. (7), it can be shown that the higher order terms than the Gaussian term are irrelevant for the 3-spatial dimensional system (5). Then we construct the action for the Gaussian fixed point. Taking account of the mode-coupling terms up to the 4th-order in $S[\varphi]$ in Eq. (7), we use the Feynman inequality for the free energy: $F \le F_{\rm eff} + T\langle S - S_{\rm eff}\rangle_{\rm eff} \equiv \tilde{F}(\eta)$, where $S_{\rm eff}$ is the effective action for the best Gaussian, $S_{\rm eff}[\varphi] = \frac{1}{2}\sum_\sigma\sum_{\mathbf{q},l}\chi_{{\rm v}\sigma}(\mathbf{q}, i\omega_\ell)^{-1}|\varphi_\sigma(\mathbf{q}, i\omega_\ell)|^2$ with $\chi_{{\rm v}\sigma}(\mathbf{q}, i\omega_\ell) = [\eta + A_\sigma q^2 + C_\sigma|\omega_l|/q]^{-1}$. The self-consistent renormalization (SCR) equation for critical valence fluctuations is derived by optimizing the free energy $d\tilde{F}(\eta)/d\eta = 0$, as follows.

$$\begin{aligned} & \sum_\sigma A_\sigma q_{{\rm B}\sigma}^4 \frac{T_{0\sigma}}{T_{{\rm A}\sigma}^2}\left(1 + \frac{v_{4\sigma}q_{{\rm B}\sigma}^3}{\pi^2}\frac{T_{0\sigma}}{T_{{\rm A}\sigma}^2}\right)\left[C_{2\sigma} + \frac{x_{\rm c}^3}{3}\frac{t_\sigma}{\tilde{y}_\sigma^2}\int_0^{x_{\rm c}} \frac{x^4}{\left(x + \frac{t_\sigma}{6\tilde{y}_\sigma}\right)^2}dx\right] \\ \times & \left[y_{0\sigma} - \tilde{y}_\sigma + \frac{3}{2}y_{1\sigma}t_\sigma\left\{\frac{x_{\rm c}^3}{6\tilde{y}_\sigma} - \frac{1}{2\tilde{y}_\sigma}\int_0^{x_{\rm c}} \frac{x^3}{x + \frac{t_\sigma}{6\tilde{y}_\sigma}}dx\right\}\right] = 0, \end{aligned} \quad (8)$$

where $\tilde{y}_\sigma = y\frac{A}{A_\sigma}\left(\frac{q_{\rm B}}{q_{{\rm B}\sigma}}\right)^2$, $y \equiv \frac{\eta}{Aq_{\rm B}^2}$, $T_{0\sigma} = \frac{A_\sigma q_{{\rm B}\sigma}^3}{2\pi C_\sigma}$, and $T_{{\rm A}\sigma} = \frac{A_\sigma q_{{\rm B}\sigma}^2}{2}$ with $q_{{\rm B}\sigma}$ being the Brillouin zone for spin $\sigma$. Here, $A$ and $q_{\rm B}$ are the zero-field values of $A_\sigma$ and $q_{\mathrm{B}\sigma}$, respectively. The characteristic temperature of the critical valence fluctuation for the spin $\sigma$ is $T_{0\sigma}$ and the scaled temperature is expressed as $t_\sigma = T/T_{0\sigma}$. Here, $v_{4\sigma}$ is the mode-coupling constant for the 4th order given by $\Omega_{4\sigma} \approx v_{4\sigma}/(\beta N_{\mathrm{s}})$ in Eq. (7). The integral variable and its cutoff are defined as $x \equiv q/q_{\mathrm{B}}$ and $x_{\mathrm{c}} \equiv q_{\mathrm{c}}/q_{\mathrm{B}}$, respectively. The parameters $y_{0\sigma}$ and $y_{1\sigma}$ are given by

$$y_{0\sigma} = \frac{\frac{\eta_{0\sigma}}{A_\sigma q_{\mathrm{B}\sigma}^2} + v_{4\sigma}\frac{T_{0\sigma}}{T_{\mathrm{A}\sigma}^2}\frac{q_{\mathrm{B}\sigma}^3}{\pi^2}C_{1\sigma}}{1 + v_{4\sigma}\frac{T_{0\sigma}}{T_{\mathrm{A}\sigma}^2}\frac{q_{\mathrm{B}\sigma}^3}{\pi^2}C_{2\sigma}}, \quad (9)$$

$$y_{1\sigma} = \frac{v_{4\sigma}\frac{T_{0\sigma}}{T_{\mathrm{A}\sigma}^2}\frac{4q_{\mathrm{B}\sigma}^3}{3\pi^2}}{1 + v_{4\sigma}\frac{T_{0\sigma}}{T_{\mathrm{A}\sigma}^2}\frac{q_{\mathrm{B}\sigma}^3}{\pi^2}C_{2\sigma}}, \quad (10)$$

respectively, where $C_{1\sigma}$ and $C_{2\sigma}$ are constants characterizing the frequency dependence of $\chi_{\mathrm{v}}(\mathbf{q}, i\omega_\ell)$.

The calculation procedure in the present framework is summarized as follows. First, we input the parameters of the Hamiltonian Eq. (3) and a magnetic field: $\varepsilon_{\mathbf{k}}$, $\varepsilon_{\mathrm{f}}$, $V_{\mathbf{k}}$, filling $n \equiv \sum_{i\sigma}(\langle n_{i\sigma}^{\mathrm{f}}\rangle + \langle n_{i\sigma}^{\mathrm{c}}\rangle)/(2N_{\mathrm{s}})$, and $h$. Then, the parameters $y_{0\sigma}$ and $y_{1\sigma}$ are obtained by calculating Eq. (9) and Eq. (10), respectively. Then, by solving the valence SCR equation [Eq. (8)], the solution $y$ is finally obtained.

As shown in Ref. (5), the Gaussian fixed point ensures that the dynamical f-spin susceptibility $\chi(\mathbf{q}, i\omega_\ell)$ has the common structure to the valence susceptibility $\chi_{\mathrm{v}}(\mathbf{q}, i\omega_\ell)$ near the QCP of the valence transition. Hence, the uniform f-spin susceptibility diverges simultaneously with the valence susceptibility, i.e., $\chi(\mathbf{0}, 0) \propto \chi_{\mathrm{v}}(\mathbf{0}, 0) \propto y^{-1}$ at the QCP. Indeed, the divergence of the uniform magnetic susceptibility at the QCP of the valence transition was confirmed by the DMRG calculation in the extended periodic Anderson model, Eq. (3) with Eq. (4) (14).

In $\beta$-$YbAlB_4$, the crystalline electronic Field (CEF) analysis which reproduces the anisotropic behavior of the magnetic susceptibility has suggested an existence of the hybridization node along the $c$ direction (13). Here we consider the anisotropic hybridization in the form of $V_{\mathbf{k}} = V(1 - \hat{k}_z^2)$ with $\hat{\mathbf{k}} \equiv \mathbf{k}/|\mathbf{k}|$ (15) to simulate $\beta$-$YbAlB_4$ most simply.

The result for $y/h^{1/2}$ vs. $T/h$ at the QCP of valence transition is shown in Fig. 1. Here, the QCP is identified to be $D = 1$, $V = 0.65$, $\varepsilon_{\mathrm{f}} = -0.7$, and $U_{\mathrm{fc}} = 0.700328652$ at $n = 0.8$, where $D$ is the half band width of conduction

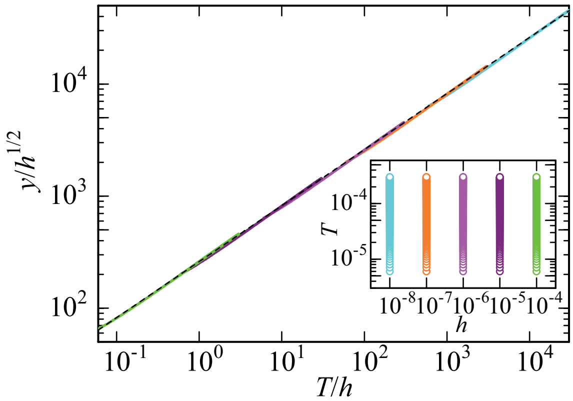


Figure 1: (color online) The solution of the valence SCR equation at the QCP of valence transition is plotted as in the scaled form as $y/h^{1/2}$ vs $T/h$. The inset shows the data used in the main panel are plotted in the $T$-$h$ phase diagram.

electrons given by $\varepsilon_{\mathbf{k}} = k^2/(2m_0) - D$. Here, the mass $m_0$ is set such that the integration from $-D$ to $D$ of the density of states of conduction electrons per spin and site is equal to 1. We note that the energy unit is $D$ and the Kondo temperature within the saddle point solution by usual evaluation manner (e.g., see Ref. (10)) is $T_{\rm K} = 0.02437$, which corresponds to the large characteristic energy scale $\approx 200$ K observed in $\beta$-$YbAlB_4$ (1). In Fig. 1, we find that all the data for $T \leq 3.0 \times 10^{-4}$ and $h \leq 10^{-4}$ (see inset) fall down to a single scaling function. The dashed line represents a function $\varphi(x) = ax^{1/2}$ obtained by the least-square fit for the large $T/h$ range of $10^1 \leq T/h \leq 10^4$. This indicates that the quantum valence criticality dominates in this $T/h$ regime, giving rise to the non-Fermi liquid (NFL) regime. On the other hand, as $T/h$ further decreases, the upward deviation from the dashed line starts to appear around $T/h \approx 10^{-2}$. This reflects suppression of the valence and magnetic susceptibility for a large magnetic field, indicating the crossover to the Fermi-liquid (FL) regime. These results imply that the general tendency of the $T/B$ scaling observed in $\beta$-$YbAlB_4$ can be reproduced by the solutions of the valence SCR equation, Eq. (8).

To analyze how the $T/h$ scaling behavior appears in the present framework, we expressed Eq. (8) in the scaled form as $y/h^{1/2}$ and $t/h$ (12). Then we realized that most terms can be expressed as the scaled forms, except for the denominators of the $x$ integrands. This indicates that the $T/h$ scaling does not hold exactly. However, it turned out that in case of large $t/y$, the denominators become large, which makes the terms of the $x$ integration themselves negligibly small (12).

Our analysis shows that this is the case when the characteristic temperature of the critical valence fluctuation $T_0$ is below or at least comparable to the measured temperature. In the calculation for Fig. 1, $T_0$ is given by $T_0 = 3 \times 10^{-6}$ and the lowest temperature of the data is $T = 6 \times 10^{-6}$. Then the $T/B$ scaling appears. Hence, emergence of the $T/B$ scaling behavior indicates the presence of the small characteristic temperature $T_0$.

Note that the energy unit of our theory is taken as $D$, which is a half band width of conduction electrons. This value is estimated to be in the order of 1 eV, corresponding to the order of 10000 K. Hence, $T_0 = 3 \times 10^{-6}$ is estimated to be in the order of 10 mK. Since the measured lowest temperature is 30 mK in $\beta$-$YbAlB_4$, $T_0$ is considered to be around the lowest temperature.

Here, three points should be remarked. As shown in Ref. (14), the location of the QCP of the valence transition is moved in the ground-state phase diagram of the $\varepsilon_{\rm f}$-$U_{\rm fc}$-$V$ space by applying a magnetic field to $H_{\rm PAM}+H_{U_{\rm fc}}$ (14). In the present calculation, we neglected this effect for simplicity of analysis. If we take into account this effect, the suppression of the magnetic susceptibility is expected to be more prominent, which makes the crossover $T/h$ between the FL and NFL regimes be shifted to the larger direction in Fig. 1. Such a calculation is left for an interesting future study.

The second point is on the experimental data. For the small-$T/B$ regime, a few sets of the data of greatly suppressed susceptibility at $1 \sim 2$ Tesla (at not the lowest temperature) are used to conclude the scaling function in Eq. (1). Namely, the scaling form in the FL regime is proposed to be $\chi = \Lambda\Gamma^{1/2}(\mu_{\rm B}B)^{-1/2}$ in Ref. (2), which was concluded from the middle-$T$ and large-$B$ data, but not the $T \to 0$ limit and small-$B$ data. Hence, the FL regime seems to reflect just the suppression of the magnetic susceptibility under a large magnetic field and it seems unclear whether the scaling form for $T/B \ll 1$ holds in Eq. (1) in the $T \to 0$ limit experimentally.

The third point is on diverging uniform magnetic susceptibility. As mentioned above, $\mathbf{q} = \mathbf{0}$ spin fluctuation diverges at the QCP of the valence transition. The present theory focuses on the vicinity of the QCP in the paramagnetic metal. It is noted that in reality, competition to the other instabilities, such as the antiferromagnetic order (16) and ferromagnetic order by the RKKY interaction, and superconducting order, exists (11). Actually, in $\beta$-$YbAlB_4$, presence of the antiferromagnetic correlation is suggested by the negative Weiss temperature and undergoes the superconducting transition at below 80 mK at zero field (1). The present theory provides a natural explanation for the large Wilson ratio as well as diverging uniform magnetic susceptibility toward zero temperature measured in $\beta$-$YbAlB_4$ from the viewpoint of the enhanced $\mathbf{q} = \mathbf{0}$ spin fluctuation due to the proximity to the QCP of the valence transition.

As shown in Ref. (5), almost dispersionless critical valence fluctuation mode appears near $\mathbf{q} = \mathbf{0}$ in the extended periodic Anderson model $H_{\rm PAM}+H_{U_{\rm fc}}$ because of the strong local correlation effect originating from the onsite-Coulomb repulsion of f electrons. This causes extremely small coefficient $A$ in the valence susceptibility $\chi_{\rm v}$, giving rise to the small characteristic temperature of the critical valence fluctuation, $T_0 = \frac{Aq_{\rm B}^2}{2\pi C}$. Namely, the origin of the small $T_0$, which is greatly reduced from $T_{\rm K}$, is due to the locality of the critical valence-fluctuation mode arising from the strong local correlations.

Hence, even at the low-enough temperature than the Kondo temperature $T_{\rm K}$ of the system, the scaled temperature by $T_0$, $t = T/T_0$, can be "high" temperature. It should be noted that the new type of quantum criticality appears in the $t \gtrsim 1$ regime in the physical quantities such as the resistivity $\rho(t)$, the magnetic susceptibility $\chi(t)$, the specific-heat coefficient $C_{\rm e}(t)/t$, and the NMR/NQR spin-lattice relaxation rate $(T_1T)^{-1}$, which explains the measured unconventional criticality in $\beta$-$YbAlB_4$ (5). Therefore, our result shows that the $T/B$ scaling as well as the unconventional criticality in $\beta$-$YbAlB_4$ can be explained from the viewpoint of the Yb-valence fluctuations in a unified way. It is stressed that although we take into account the anisotropic hybridization to simulate $\beta$-$YbAlB_4$ realistically, our theory does *not* require the pinning of the f-level position at the hybridized band edge as assumed in Ref. (3).

To examine the presence of the small characteristic temperature of the critical valence fluctuation $T_0$ experimentally, the observation of the dynamical valence susceptibility $\chi_{\rm v}(\mathbf{q}, \omega)$ is desirable as a direct measurement. ESR and Mössbauer measurements are considered to be such possible probes to detect $T_0$, which are interesting future subjects.

## 3. Summary

We have developed the mode-coupling theory for critical valence fluctuations under a magnetic field starting from the extended periodic Anderson model, and solved the valence SCR equation by inputting the parameters of the Hamiltonian. This framework describes the hierarchy of energy scales of the system ($D \gg T_{\rm K} \gg T_0$), which makes it possible to compare the theoretical and experimental $T$-$B$ phase diagram quantitatively. By analyzing the solutions of the valence SCR equation derived under consideration of the anisotropic c-f hybridization for $\beta$-$YbAlB_4$, we have shown that the $T$-$B$ scaling behavior appears in the magnetic susceptibility as well as the valence susceptibility near the QCP of the valence transition. The emergence of the $T$-$B$ scaling behavior indicates the presence of the characteristic energy scale of the critical Yb-valence fluctuation, which is smaller than (or at least comparable to) the measured lowest temperature.

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