4. Олійник, А. П. Математичні моделі процесу квазістаціонарного деформування трубопровідних та промислових систем при зміні їх просторової конфігурації / А. П.Олійник // Наукове видання. – Івано-Франківськ: ІФНТУНГ, 2010. – 320 с.

APPLICATION OF THE NEWTON-RAPHSON PROCEDURE FOR THE SEQUENTIAL PARAMETER ESTIMATION USING POLYNOMIAL MAXIMIZATION METHOD

Zabolotnii S.V[1], Warsza Z.L[2], Ivashchenko K.V. [1]

[1] Cherkasy State Technological University 18006, Ukraine, Cherkasy, bul. Shevchenko 460

[2]Industrial Research Institute of Automation and Measurement 02486, Poland, Warszawa, al. Jerozolimskie 202

The peculiarity of the statistical problems arising in the diagnosis of the technical systems and technological processes condition is that the analysis is carried out not on the basis of the fixed volume samples processing, but by the formation of statistics from data that are successively derived from the object of observation.

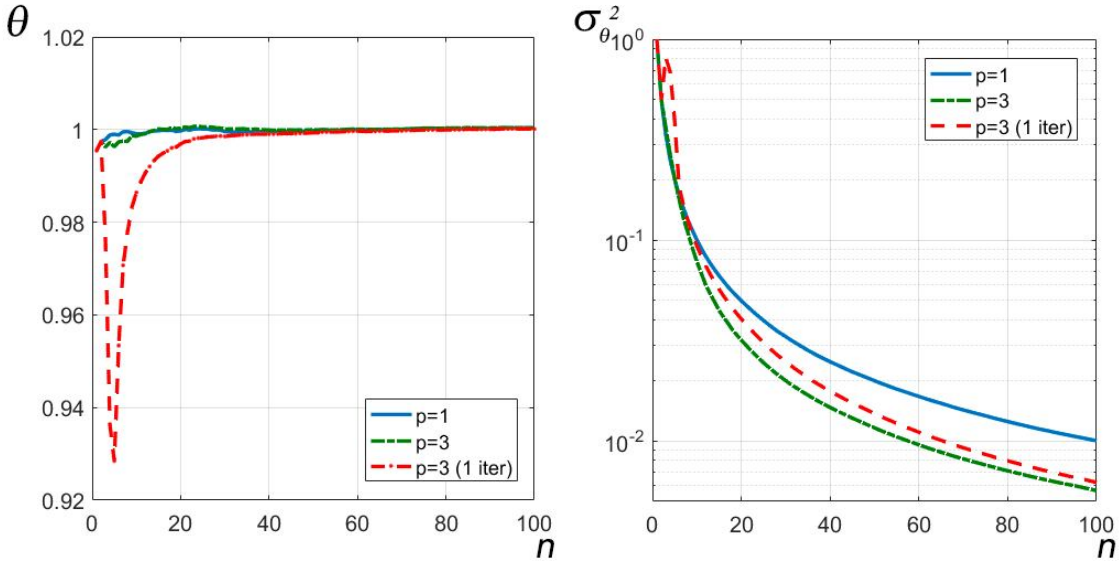
In this research, the problem of estimating the displacement shift parameter is considered, which can also be interpreted as determining the value of a constant component, provided that there are additive random measurement errors. It is known that there are several approaches to solving such problem. The most optimal criterion for accuracy is the use of Maximum Likelihood Method. However, its application requires the availability of a priori information about the form of statistical data distribution law and often leads to fairly complex computational algorithms. A robust approach based on ordinal or rank statistics is also widespread, but its consistent implementation is also characterized by significant computational resources. Therefore, in practice the most commonly used - ordinary statistics in the form of arithmetic mean for a consistent calculation of which there is a simple recursive modification. The main disadvantage of this approach is that the dispersion of the mean arithmetic may significantly outperform the effective values for the difference in the distribution of statistical data from the Gaussian model.

One of the alternative approaches to statistical estimation is the Polynomial Maximizing Method (PMM) [1]. This relatively new method is based on a probabilistic description in the form of statistics of higher orders (moments or cumulants). In the paper [2] the properties were researched and the efficiency of the PMM-estimates of the center of symmetric distributions (arcsines, uniform, trapezoidal, triangular) was analysed. It is shown that the estimation of the arithmetic mean is a partial case of PMM-estimation with polynomial degree $p=1$. An increase of a polynomial degree within non-Gaussian statistical data patterns allows us to reduce the estimate variance of desired parameter on the basis of taking into account the higher orders cumulative coefficients values. A characteristic feature of PMM is that the algorithm for finding estimates reduces to the solution of power equations related to the estimated parameter. The presence of a developed apparatus of numerical methods for the root determination in such equations

opens up possibilities for constructing relatively simple recursive algorithms for the formation of successive PMM-estimates.

In this paper, we investigate the possibility of using the Newton-Raphson numerical procedure for the sequential estimation of the non-Gaussian random sequence shift in the application of PMM using the polynomial degree $p=2$ and $p=3$ (for symmetric distribution).

Fig. 1 shows the example of the results of Monte-Carlo statistical modelling. In this model example, a sequential estimation of a trapezoidal distribution random sequence shift $\theta=1$ (3 with a proportionality parameter $\beta=0.5$) and a normalized dispersion is carried out. In such situation, the use of PMM with polynomial degree $p=3$ theoretically provides an asymptotic (increasing length of the input sequence) reduction of the estimate variance by the coefficient $g_3=0.55$ [2].



a) b)

Figure 1 – Results of statistical simulation of sequential estimation

The dependences of the averaged (according to the results of 10^5 experiments) of the successive parameter estimation (Fig.1,*a*) and their dispersion (Fig.1,*b*), obtained on PMM-statistic (with degree $p=1$ and $p=3$) basis and recurrent one-step modification of the algorithm for the approximate root determination of the cubic the equation ($p=3$ (1 iter)), developed using the Newton-Raphson procedure, confirm the efficiency of proposed approach and the overall effectiveness of PMM application for those situations where the distribution of statistical data is non-Gaussian.

References:

1. Kunchenko Y. (2002), Polynomial Parameter Estimations of Close to Gaussian Random variables. Germany, Aachen: Shaker Verlag.

2. Warsza, Z.L. and Zabolotnii, S.W. (2017), A Polynomial Estimation of Measurand Parameters for Samples of Non-Gaussian Symmetrically Distributed Data BT - Automation 2017: Innovations in Automation, Robotics and Measurement Techniques. Cham: Springer International Publishing, pp. 468–480. doi: 10.1007/978-3-319-54042-9_45.