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Creep Buckling of Reinforced Concrete Columns

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ɪsearch Report No. CE3
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CREEP BUCKLING OF REINFORCED CONCRETE COLUMNS

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Synopsis

The steps taken to develop a rational creep analysis for slender reinforced concrete columns are detailed. An efficient method of short term analysis is selected. Empirical constitutive relationships are developed for concrete. Two methods of creep analysis are described and the more rational of these is discussed in detail. Correlation is provided with experimental tests, both short term and long term, on model columns. The effects of varying material parameters are discussed with regard to column life under sustained load.

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1. INTRODUCTION

This paper is concerned with the buckling of pin-ended reinforced concrete columns. It is a continuation of earlier work by Behan (5) on the short term capacity of columns with initial crookedness. The paper has three main aims:

(a) To develop a simpler short term analysis, directly applicable to long term analysis and to maximise the efficiency of the analysis in terms of computation effort.

(b) To formulate a simple constitutive relationship between concrete stress and strain, and to extend this to time-varying stress conditions.

(c) To develop a mathematical model for column creep analysis at constant axial load; to extend this to a general column with respect to end conditions, varying loads, prestress and a shrinkage history before loading.

A substantial series of short and long term column tests is described, together with associated material tests.

The literature describing concrete stress/strain curves has been summarised by Sargin (3). Many theories have been advocated regarding the calculation of concrete creep at constant stress (1, 2, 4). These are summarised by Brown and Hope (2). The methods of short term analysis discussed in this paper are listed in order in references (5, 6, 7, and 8). Some aspects of the two creep analyses developed were proposed in references (9, 10, and 11). The first method, reducing modulus is advocated in references (11, 12, and 13). The basis of the second method is discussed by Distefano (14) and developed in references (15, 16, and 17). Other theories also have been developed but these are not discussed here. The present work is described also in references (18 and 19).

Some special word usages have been adopted in this paper. Short term stress-dependent strains are called "initial" strains rather than "elastic" strains. Short term failure loads are described as "limiting" loads, P_L, and long term loads as creep capacities, P_{creep}.

Compressive stresses and strains are taken as positive.

2. SHORT TERM ANALYSIS

The planar analysis of columns axially loaded, load P, and bent about the weaker principal axis, moment M, neglecting shear deformations, provides two equations of equilibrium.

$$\Sigma P = \Sigma M = 0 \tag{1}$$

Curvature ϕ may be defined by assuming a linear strain profile, specified by any two distortion parameters which are determined by equilibrium and compatibility.

The short term analysis may be descirbed as follows:

(1) Assume the deflected shape and compute the bending moment M at each section along the column from the axial load P.

(2) At each cross-section find the strain profile which gives stresses whose resultants are P and M and find the curvature ϕ. Typical strain profiles and stress diagrams are shown in Figure 1.

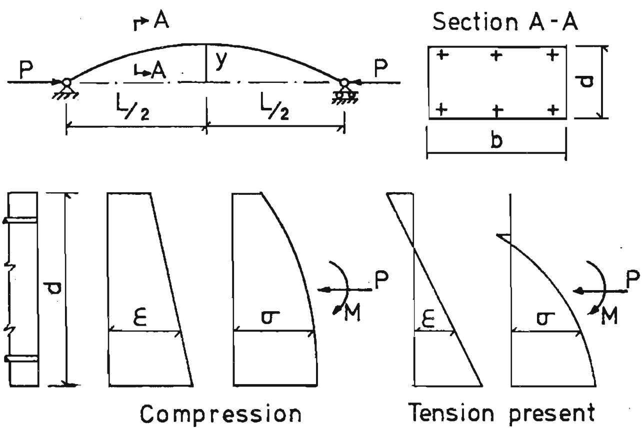

FIGURE 1 : Column cross-section and equilibrium

(3) Integrate curvature along the column to find a new deflected shape.

(4) Iterate to find the correct deflected shape due to P.

(5) Increase P in steps until one of the following limits is reached:

(a) Instability, or lack of convergence in the solution in step 3.
(b) Excessive moment; the moment M is too high for a particular load P.
(c) Excessive deflections of the column occur.
This is a practical limitation; deflections were not allowed to exceed half the column depth d in the plane of bending.

Requirements for this analysis include the following:

(1) The correct strain profile at a section and the correct deflected shape must be predicted efficiently to reduce computation time.

(2) Constitutive relations between stress and strain must be developed for both steel and concrete, defined for the latter by a minimum of material parameters.

(3) The short term analysis should be readily adaptable to creep analysis.

2.1 Equilibrium Analysis

A discrete fibre method of analysis was developed in which the cross-section was divided into layers or fibres of concrete and steel. For each fibre the strain was determined from the linear strain distribution and then the stress from the stress-strain diagram for each material. The resultant force and moment were computed using numerical integration.

As shown in Figure 2, two variables may be used to describe the linear strain profile. Analyses were developed and examined using the pairs (z_o, ϕ), (ε_m, ϕ) and (ε_o, ε_i) (19). The final decision was to use the variables ε_o and ε_i, the greater (compressive) and lesser fibre strains such that, from Figure 1

$$\phi = (\varepsilon_o - \varepsilon_i)/d \tag{2}$$

A Newton-Raphson iteration in two variables was used to obtain the correct strain distribution, with the desired resultants P and M. This process corrected any reasonable trial distribution by using approximate partial derivatives of P and M with respect to ε_o and ε_i. These derivatives were obtained by varying

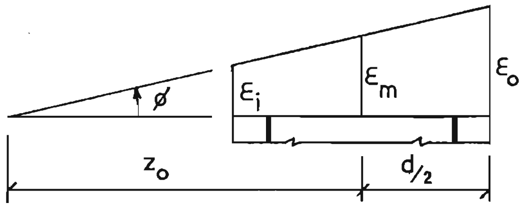

FIGURE 2 : Strain profile definition

ε_o and ε_i in turn by one per cent and calculating the force and moment residuals, i.e. $\frac{\partial P}{\partial \varepsilon_o} \simeq \frac{\Delta P}{\Delta \varepsilon_o}$ etc. The method converges rapidly; accuracies of 0.001 for P and M were obtained in a maximum of three iterations.

The method is general; it does not rely on a linear strain profile, and thus is suitable for extension to long term analysis, where the initial strain profile may be non-linear. The total strain profile always remains linear.

2.2 Compatibility Analysis

The deflected shape was obtained from the curvature distribution by interpolating the curvature distribution at n points along the column to 2n - 1 values and applying numerical integration (Simpson's rule). This was a single pass process; the column was analysed as a cantilever and then compensated for the imcompatible displacement at the free end. Aroni's method (9), based on finite difference techniques was examined but not used. It is not readily applicable to columns other than pin-ended.

Whenever the load P was incremented, the new deflected shape of the column was predicted by the following process.

(1) Secant prediction was applied to the central deflection. This yielded the point Q as shown in Figure 3. The previous deflected shape was retained but was magnified to give the new central deflection.

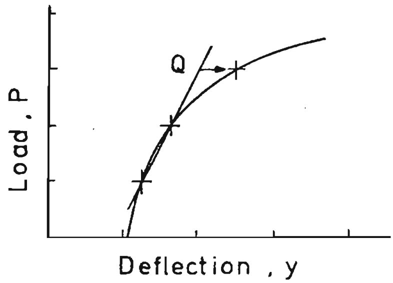

FIGURE 3 : Column load/deflection curve

(2) Starting at point Q, two further trial shapes were calculated.

(3) Figure 4 shows a plot of the next trial value y_{n+1} versus the previous trial y_n, relative to the point Q, these points representing successive approximations. The plot is practically linear, allowing a precise prediction of the correct value of y after two trials. The deflected shape was magnified again and checked.

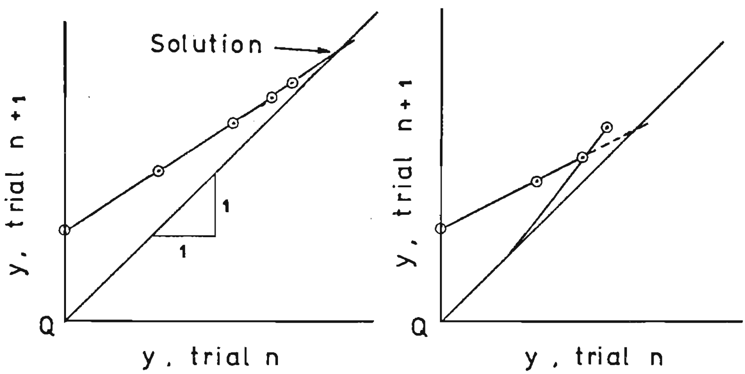

FIGURE 4 : Central deflection trials

(4) The analysis reverts to the use of successive approximations if local divergence is indicated: e.g. as shown in Figure 4 for either steel yielding or for tension initiating in the column.

The process usually required that only three trial shapes be examined, two for prediction and one further as a check.

Failure due to instability was indicated by divergence of the lateral deflection y. This could be obscured by the tolerance chosen, as shown in Figure 5. This was often the case when the method of successive approximations was used; this method converges slowly, particularly for loads near P_L. In addition a spurious indication of failure of this kind could arise if either steel had yielded or tension had initiated in a concrete fibre during the last trial. Techniques were devised (19) for overcoming these problems. Failure was indicated using finite difference techniques to check the curvature of the curve of central deflection versus trial number, as shown in Figure 6.

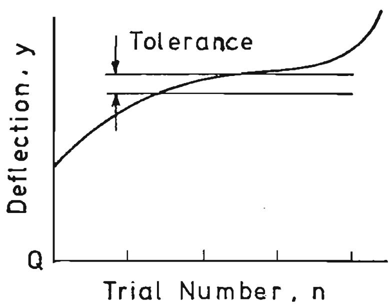

FIGURE 5 : Failure masked by tolerance

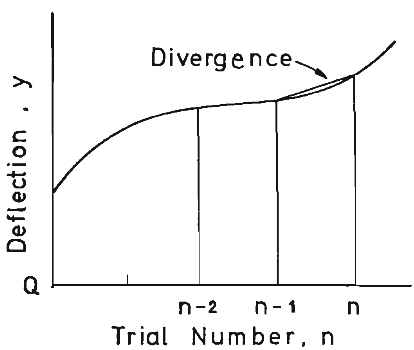

FIGURE 6 : Divergence, indicating failure

2.3 Materials for Tests

The concrete used has a 28 MPa cylinder crushing strength at 28 days and proportions by weight:

Gravel,	passing 3/16" B.S.S.	2.4
Sand,	passing No.7 B.S.S.	1.6
Cement,	normal Portland	1.0
Water,		0.48

The main reinforcement was low carbon welding steel, annealed and straightened to an elasto-plastic tensile condition with properties:

Diameter, area	4.04 mm, 12.8 mm^2
Yield stress, tension	231 MPa
Yield stress, compression	203 MPa
Proportional limit, compression	136 MPa

Typical stress-strain curves are shown in Figure 7. The concrete theoretical curve is defined by the cubic

$$\sigma = A\varepsilon^3 + B\varepsilon^2 + C\varepsilon \tag{3}$$

where $C = E_{inst}$, Hognestad's approximation (Sargin (3)) at the origin.

The other points specified are:

1. $\sigma = F'_c$ at $\varepsilon = 0.002$
2. $\sigma = kF'_c$ at $\varepsilon = 0.003$

where k is a linear factor of F'_c.

This description of the concrete stress-strain curve may be extended for other concrete strengths by varying the specification of two points: the strain ε_u at which F'_c occurs and the value of k defining the stress kF'_c at $\varepsilon = 0.003$. These values could not be determined from tests carried out by Behan (19), due to lack of testing machine stiffness. Tests reported by Sargin (3) were used to establish the expressions:

$$\varepsilon_u = 8.93 \times 10^{-6} F'_c + 1.375 \times 10^{-3} \quad (4)$$

$$k = 0.02 - 2.85 \times 10^{-5} F'_c \quad (5)$$

for F'_c in MPa. Thus the curve can be described completely by specifying F'_c.

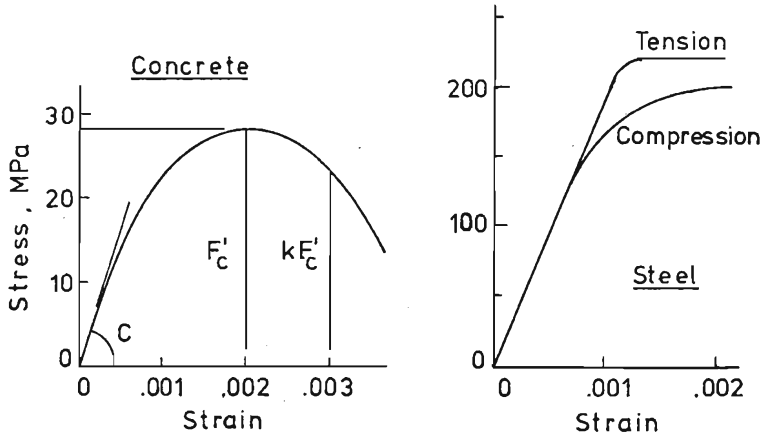

FIGURE 7 : Stress-strain diagrams

2.4 Minimum Number of Fibres

The minimum number of concrete fibres to describe the cross-section is 20, to minimise the discontinuity introduced by tension initiating in the column. Trials were carried out (19) using from 8 to 40 fibres; the optimum also appeared to be 20. The minimum number of positions to specify the ϕ distribution is an inverse function of the slenderness ratio L/d as follows. For L/d > 30, use 8: in the range 30-20, use 12: for L/d < 20, use 20.

2.5 Types of Short Term Tests

Four classes of tests were carried out:

(1) Some 20 stress-strain curves for the concrete used in the columns were obtained at a constant strain rate approximating the normal strain rate developed in the short term column tests. A typical curve is shown in Figure 7.

(2) Series S100, S200. Axial load tests to failure on model columns 76 mm x 38 mm cross-section with lengths 1830, 1520, 1200 and 914 mm, three of each length being tested. These are reported by Behan (5). Subsequently another 30 similar columns were tested in Series S200 (19). These are not commented on further, being merely confirmation of the previous series carried out when loading columns for long term tests. This gave a range of L/d from 48-24.

(3) Series S300. Nine short columns, cross-section 127 mm x 64 mm, lengths 500, 1000 and 1500 mm were tested to check the analysis of short stiff columns. These were cast straight, then loaded eccentrically with equal end eccentricities of L/360. This gave a range of L/d from 24-8.

(4) Series S400. Thirty columns 76 mm x 38 mm x 914 mm long were tested in four sets. Each set had six varying initial central deflections. The sets covered : normal reinforcement and concrete (as 2); different concrete strength; different amount of steel; different steel, i.e. a hard drawn wire.

Columns S401-24 were made from concrete with $F_c' = 35$ MPa; S425-30 used $F_c' = 25$ MPa. Columns S401-8 were made using steel as specified for series S100, S200. S409-30 used hard drawn wire reinforcement, with a yield stress in tension of 376 MPa.

2.6 Discussion of Results

In all cases there was excellent agreement between experimental and calculated

(a) critical loads, and

(b) the shape of the P/y curves.

The results for P_L are tabulated in Tables 1 and 2 for series S100, S300 and S400 respectively. The theoretical prediction is generally within 1% of the test result. While this is essential to indicate correct calibration of the analysis, such accuracy should be expected: much less refined analyses produce similar results.

TABLE 1 : Limiting Short Term Loads, P_L

SERIES: S100
Concrete Section 76 mm x 38 mm
Steel 2.7%

Column Number (1)	Column length in millimetres (2)	Slenderness ratio (3)	Initial central deflection in millimetres (4)	Concrete strength in Megapascals (5)	Limiting Load, P_L in kilonewtons Experimental (6)	Limiting Load, P_L in kilonewtons Theoretical (7)	Load Ratio Theoretical/ Experimental (7)/(6) (8)
S101	1830	48	2.46	30.4	25.8	25.5	0.989
S102	1830	48	2.34	20.1	24.3	24.5	1.008
S103	1830	48	2.39	25.9	25.4	25.2	0.992
S104	1520	40	2.64	28.9	33.9	34.0	1.004
S105	1520	40	2.11	31.1	32.9	32.5	0.988
S106	1520	40	1.96	30.4	35.0	35.2	1.007
S107	1220	32	2.92	29.8	43.7	44.0	1.008
S108	1220	32	2.62	30.7	44.3	44.0	0.993
S109	1220	32	2.39	27.8	40.5	40.8	1.007
S110	910	24	1.98	24.8	50.2	50.4	1.004
S111	910	24	1.96	27.6	55.6	55.2	0.993
S112	910	24	1.98	24.3	49.8	50.1	1.006

SERIES: S300
Concrete Section 127 mm x 63 mm
Steel 3.55%
End Eccentricity, L/360

S301	1500	24	0.00	32.8	220	221	1.005
S302	1500	24	0.000	32.8	219	218	0.995
S303	1500	24	0.46	33.5	222	221	0.996
S304	1000	16	0.38	33.5	271	266	0.982
S305	1000	16	0.33	33.5	262	266	1.015
S306	1000	16	0.38	33.5	263	266	1.011
S307	500	8	0.00	32.1	315	311	0.987
S308	500	8	0.06	32.1	314	311	0.990
S309	500	8	0.13	32.1	313	311	0.994

TABLE 2 : Limiting Short Term Loads, P_L

SERIES: S400
Concrete Section 76 mm x 38 mm

Column Number (1)	Column length in millimetres (2)	Slenderness ratio (3)	Initial central deflection in millimetres (4)	Concrete strength in Megapascals (5)	Limiting Load, P_L in kilonewtons		Load Ratio Theoretical/ Experimental (7)/(6) (8)
					Experimental (6)	Theoretical (7)	
S401	920	24	0.08	33.6	88.5	89.2	1.008
S402	920	24	1.14	36.0	76.6	76.3	0.992
S403	920	24	1.30	33.9	76.5	76.8	1.004
S404	920	24	1.65	33.8	69.8	70.3	1.007
S405	920	24	2.08	36.6	74.3	73.1	0.984
S406	920	24	2.47	35.0	66.7	66.7	1.000
S407	920	24	3.12	36.4	72.5	71.6	0.987
S408	920	24	4.06	36.4	64.9	64.3	0.990
S409	920	24	0.28	34.2	88.1	89.2	1.013
S410	920	24	0.91	36.4	84.5	83.4	0.991
S411	920	24	1.35	34.2	79.2	80.0	1.011
S412	920	24	1.35	35.8	79.3	80.0	1.008
S413	920	24	2.06	36.1	78.7	78.0	0.991
S414	920	24	2.54	35.7	71.6	72.0	1.006
S415	920	24	3.05	35.4	68.1	68.6	1.008
S416	920	24	3.86	36.2	65.4	64.7	0.990
S417	920	24	0.41	36.0	85.9	85.4	0.994
S418	920	24	0.38	35.0	77.2	77.3	1.002
S419	920	24	0.68	34.8	74.3	74.9	1.008
S420	920	24	1.17	36.5	72.5	71.8	0.991
S421	920	24	1.83	34.2	66.7	67.4	1.010
S422	920	24	2.16	34.6	66.0	63.0	1.008
S423	920	24	3.15	36.3	63.2	62.5	0.990
S424	920	24	4.17	36.0	56.9	56.5	0.993
S425	920	24	0.20	25.7	62.3	62.5	1.004
S416	920	24	0.91	24.2	61.0	60.9	0.993
S427	920	24	1.70	25.0	55.6	55.6	1.000
S428	920	24	2.16	26.1	53.4	53.8	1.008
S429	920	24	3.10	25.6	49.1	49.4	1.004
S430	920	24	4.22	26.1	43.7	44.6	1.007

The accurate prediction of the load-deflection curve is far more important. This is a measure of the accuracy of modelling the internal strain distribution, on which any form of long term analysis must be based. The short term analysis reported here can be used with some confidence for long term analysis providing the total strain profile and the initial strain profile can be suitably related at any time. Samples of the experimental curves, together with theoretically predicted curves are shown in Figure 8 as plots of axial load P vs central deflection y. The agreement is generally very satisfactory.

A plot of nondimensional limiting load P_L/P_O vs L/d, where P_O is the ultimate capacity of a very short column, is shown in Figure 9. This indicates the effect of column slenderness L/d on the ultimate capacity. This is compared with the curve prescribed in the Australian Concrete Code AS1480 (20) for compression failure. Observation of the tests indicated the limit of compression failure to be about L/d = 32. The code curve for tension failure is difficult to plot and has not been shown.

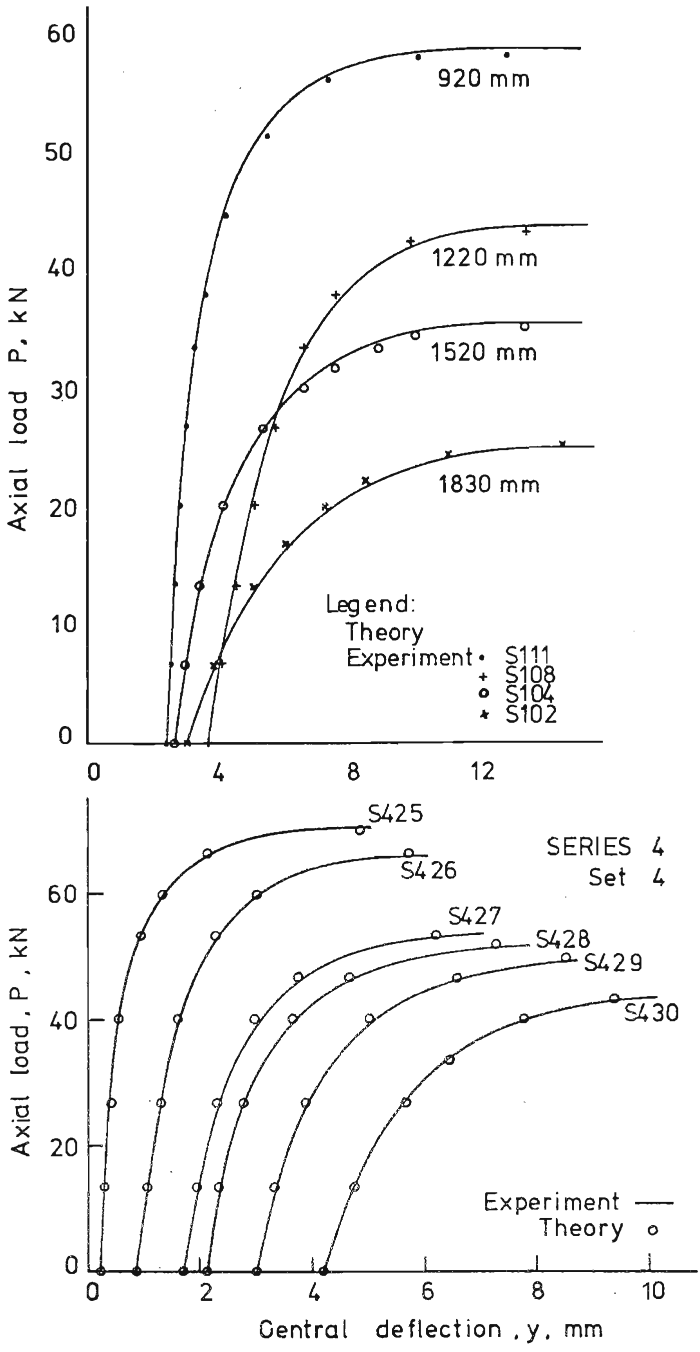

FIGURE 8 : Short-term column tests

3. CREEP ANALYSIS

3.1 Types of Creep Experiments

The following tests were carried out:

(1) Creep and shrinkage tests, on axially loaded cylinders at sustained stresses.

(2) Age Tests, i.e. crushing and compressive stress-strain tests on cylinders at progressive ages.

(3) Column tests, on 76 mm x 38 mm model columns at sustained axial loads.

3.1.1 Creep and Shrinkage Tests

Concrete cylinders were kept at constant load in a machine with soft springs in series. This allowed ε_{ics} - the total initial, creep and shrinkage strain variation with time to be recorded. Concurrent measurements of shrinkage strain ε_s were made on unloaded specimens of similar geometry. By using the normal assumption that shrinkage is not stress-dependent, whereas creep is, records of ε_{ic}, initial plus creep strain and ε_c the creep strain were obtained. The experimental results are shown in Figure 10, in which the shrinkage strain ε_s is plotted against log time in days, and ε_{ic}, the creep plus initial elastic strain, versus time in days as a log-log plot. Results are shown at three stress levels and then related to an arbitrary zero time of 0.03 days after first loading, thus defining the initial elastic strain $\Delta\varepsilon_i$. Since the plots relative to this zero time are coincident, creep at any time T after first load is defined in terms of ε_{ic} as

$$\varepsilon_{ic} = \Delta\varepsilon_i \left(\frac{T-T_L}{0.03}\right)^{0.116} \qquad (6)$$

where $T-T_L$ represents the time in days since the stress change producing initial strain change $\Delta\varepsilon_i$.

The shrinkage has three stages: initial drying out period, 8.3 days; exponential variation; constant shrinkage. An empirical expression for shrinkage ε_s is

$$\varepsilon_s = \log_e (T/8.3) \times 0.0001434 \tag{7}$$

$= 0$ for $T < 8.3$ days

$=$ constant from 400 days.

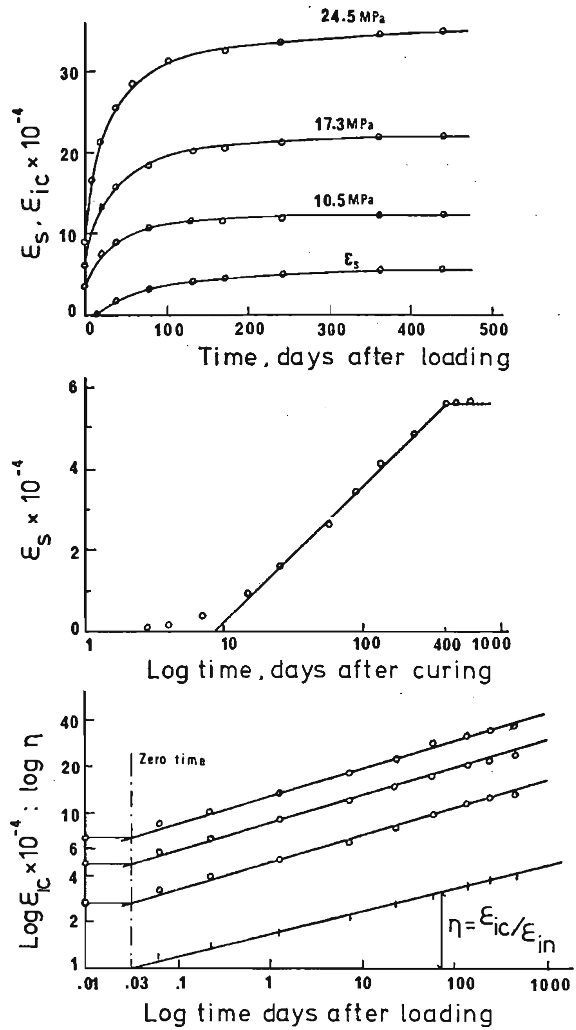

FIGURE 10 : Block tests: Strain variation with time

This expression represents a fast rate of development of shrinkage and a high maximum shrinkage, 0.0006. This is consistent with the small aggregate - rich concrete mix specified earlier, matured in dry laboratory conditions without humidity control. The log term test columns and creep specimens were stored similarly. The analysis does not depend directly on this specification of shrinkage. Any suitable expression for a particular test may be substituted for equation 7.

3.1.2 Age Tests

The instantaneous stress-strain (σ/ε) curve can be described at any time in terms of F'_c at this age, using the cubic expression quoted in equation 3. The factor C is defined by the Hognestad (Sargin (3)) expression as defined in terms of F'_c:

$$C = (12.6 + 0.46\ F'_c) \times 10^3\ \text{MPa}$$

The variation with concrete age of F'_c as a function of F'_{28}, the 28 day cylinder crushing strength, is shown in Figure 11. Since all coefficients of equation 3 may be determined in terms of F'_c only, then the variation of F'_c with age after casting allows the production of the instantaneous stress-strain curve at any specified time.

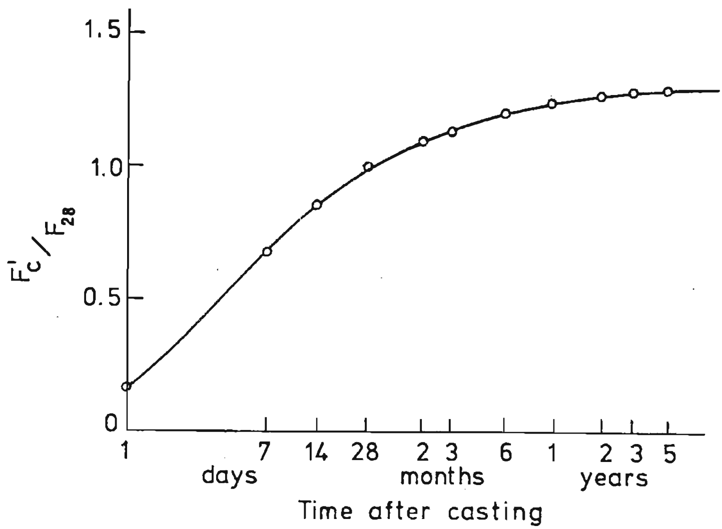

FIGURE 11 : Concrete Strength

3.1.3 Column Tests

Columns 76 mm x 38 mm in the same range of lengths as series S100 and with similar concrete strength and type and identical steel layout were loaded to levels less than the limiting load P_L. This load was maintained in time by using soft springs in series with the column. These tests were called Series L100. Figure 16 shows the variation of central deflection y with log time in days after loading for the four lengths of model column tested. The best fit result from the creep analysis descirbed later is shown on the same curves. The tests lasted until either failure occurred or a stable state was achieved. Several are still proceeding after seven years sustained loading. Correlation between experimental and theoretical column curves is discussed later.

Due to the time involved in each test, only one test was performed at each load. It is to be expected that tests of long duration may show some considerable variation of time to failure under load if repeated.

3.2 Creep Analysis Theories

Analyses based on the three main theories of estimation of creep were considered. These were:

(a) the effective, or reducing modulus theory;

(b) the rate of creep theory; and

(c) the use of the principle of superposition.

The effective modulus theory has the virtue of simplicity. However, it is theoretically unsound for varying stress conditions in that it disregards the prior stress history. In addition, shrinkage is ignored. The rate of creep method assumes that for any age of concrete, the rate of creep is independent of the age at loading, and does not depend on the previous stress history. It is no more accurate than the previous method and is more complex to apply. The principle of superposition has two main bases. For an element with a history of stress change, each stress increment produces a deformation, the effects of which continue indefinitely. A stress decrement is considered as a stress increment with a negative sign in considering the effects of the deformation produced. Thus creep-time curves are required for each age when a stress change occurs. The cumulative effect is produced by the simple addition of the effects of all stress changes.

3.2.1 Reducing Modulus Theory

A practical method of assessing creep effects is to use a reduced concrete stress-strain modulus in a short term analysis. The method gives an approximate load capacity which is uncertain as to safety. Attempts have been made to include shrinkage effects in a logical manner. Several analyses have been developed, e.g. Cederwall (11), Dilger and Neville (12), Goyal and Jackson (13). These are basically attempts to refine the method, and have been claimed accurate enough for design purposes (11), (12).

A similar analysis was developed in terms of a factor η ranging from 1.0 to 5.0, in a manner specified by the ε_{ic}/time plots shown in Figure 10. The factor η represents the ratio total (creep and elastic) strain to original initial strain, being 1.0 at the onset of creep. The logic of superposition of strain effects is suspect with regard to strain effects for changing external loads. Attempts to include shrinkage were only partially successful. In addition, the reaction of the structure to a load change at a significant interval after first load could not be accommodated logically. In defence of the method, it has the virtue of simplicity and gives a fair indication of the maximum load a column may sustain. The method correlates badly in predicting the life of a column at a given load.

A reverse analysis was carried out. Actual deflection/time results were used as data and the values of η to satisfy equilibrium and compatibility were found. This indicated that an illogical creep/time relationship was necessary as shown in Figure 12. The method was abandoned as unsatisfactory.

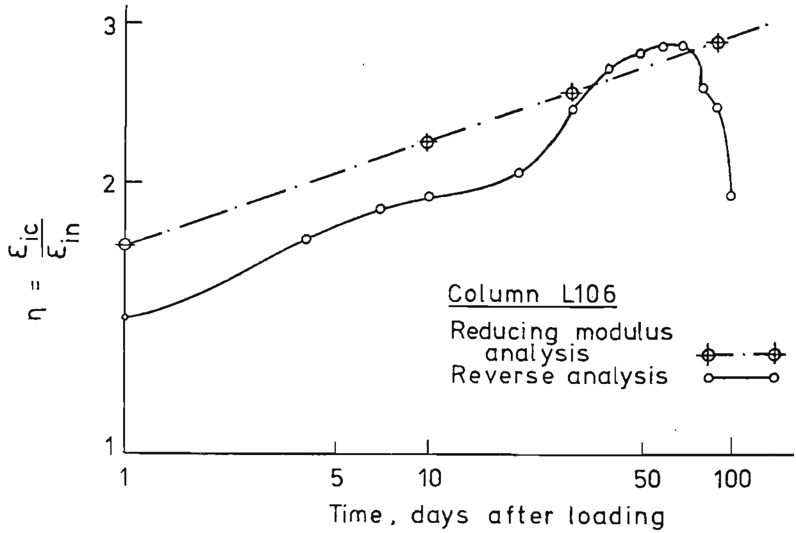

FIGURE 12 : Reducing Modulus Analysis: η

3.2.2 Authors' Creep Hypothesis

This was designed to include: (a) the presence of varying internal stress; (b) varying strength of concrete; (c) modification of strain due to creep at constant stress; (d) provision for shrinkage varying with time only; (e) provision for external force P varying with time; and (f) allowance for tension cracking in the concrete.

Referring to Figure 13, assume that a concrete fibre at time t_o has an initial strain (due to stress only) of $\Delta\varepsilon_o$ and a stress of σ_1. Creep at constant stress σ_1 and free shrinkage take place over a time interval $t_1 - t_o$, yielding a total strain ε_{ics} at t_1. The new σ/ε relationship at t_1 is located on the strain scale by finding the elastic strain ε_1 corresponding to stress σ_1 at time t_1. For any of the following conditions, or any combination,

(1) change of strain due to secondary moments,

(2) change of external load P,

There is an instantaneous strain change $\Delta\varepsilon_1 = \varepsilon - \varepsilon_{ics}$ at time t_1 corresponding to the instantaneous stress change $\sigma_2 - \sigma_1$. For the next time interval $t_2 - t_1$, creep is calculated as the sum of the time variation of strain $\Delta\varepsilon_o$ over interval $t_2 - t_o$ and $\Delta\varepsilon_1$ over interval $t_2 - t_1$. The process continues and may be expressed at time t_n, following n time steps, as

$$(\varepsilon_{ics})t_n = (\varepsilon_s)t_n + \sum_{i=0}^{n-1} \Delta\varepsilon_i \left(\frac{t_n - t_i}{0.03}\right)^{0.116}$$

where t_i denotes the time of first application of strain change $\Delta\varepsilon_i$.

This method of predicting ε_{ics} allows the superposition of creep strains at varying stress levels. Each strain change affects the creep summation indefinitely from its first occurrence. Steel strains are obtained directly from the compatible or total linear strain profile ε. Concrete stresses are calculated from the initial strains ε_e, using the concrete σ/ε relationship at that time. The problem of tension cracks developing in the concrete is handled by specifying a limiting concrete tensile strain of order 0.0001. When this is exceeded the tension part of the σ/ε curve is removed for that fibre for all future analyses. Subsequent load changes may close the "crack" and permit compressive strains, and stresses, to be defined. Figure 14 illustrates the logic.

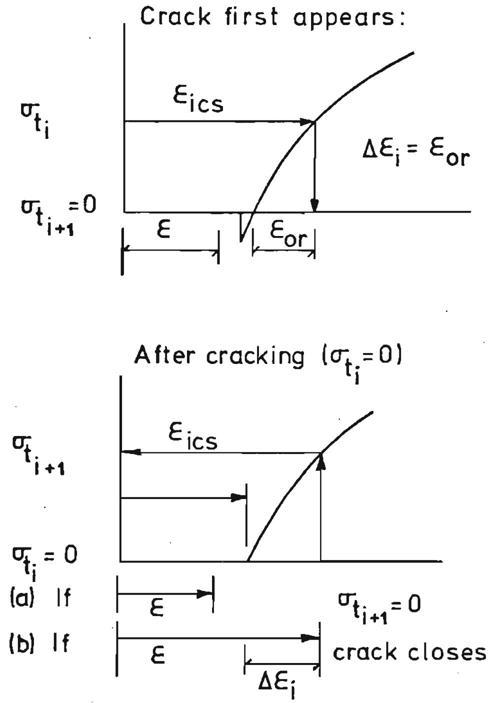

FIGURE 14 : Tension cracking

3.3 Discussion of Results

Initial studies showed that, although the analysis gave reliable estimates of the maximum load a column could take, there was difficulty in estimating accurately the life of a column under a given load. A series of analyses was carried out on one column (L102), varying certain parameters in turn and noting the change in column life and the change in shape of the deflection/ time plots. The effects of these variations are shown in Figure 15 for:

(a) E_{inst} for concrete, with a range 0.75-1.25 times Hognestad's prediction, and F'_c at first loading varying 0.8-1.2 times the measured value (cylinder tests).

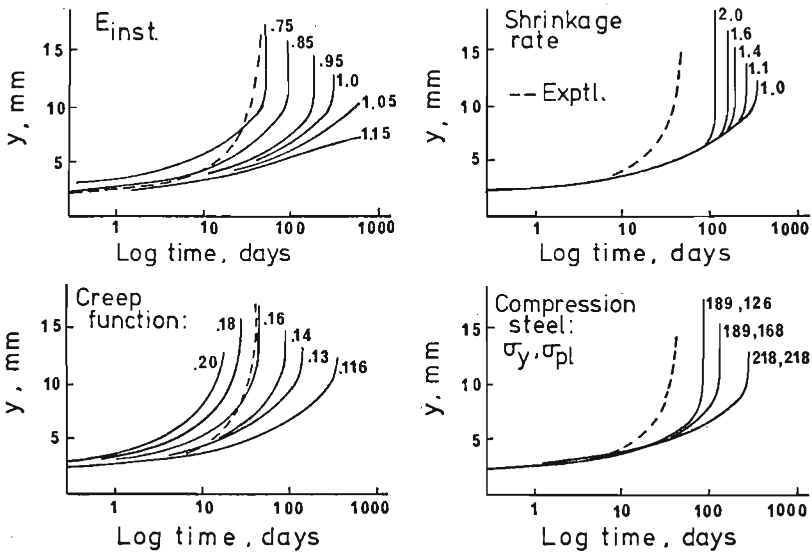

FIGURE 15 : Effects of Varying Material Parameters

(b) Shrinkage strain rate varying from 0.8-2.0 times the experimental rate.

(c) Differing powers of the creep function (equation 6) in the range 0.116-0.200.

The column life under load and the shape of the deflection-time curve were both relatively insensitive to the minor errors ($\not>$ ± 10%) which may have occurred in determining the material parameters. Gross changes were necessary to alter the analysis to such an extent as to make the experimental and theoretical column lives agree. These changes produced, as well, unwanted variations in the deflection-time curves at early ages.

(d) Finally it was recognised that the properties of the steel reinforcement has been determined by tension tests only, whereas the steel was generally in compression in the test columns. Compression tests were performed on the steel reinforcement. This had been annealed and then strain hardened in tension to give an elasto-plastic

σ/ε tensile curve. On strain reversal to compression, a different curve was obtained. The compressive yield stress was found to be 0.88 times the tensile yield stress; a pronounced proportional limit of 0.67 of the compressive yield stress was observed. This is an example of the Bauschinger effect in strain reversal. A typical result for a compression steel test is shown in Figure 7. Inclusion of this latter effect had a pronounced effect on the shape of the deflection/time curves.

The revised theoretical and experimental column lives at particular loads are compared in Table 3. There is reasonable agreement, resulting mainly from the correction for steel behaviour in compression. In Table 3, the material parameters are as measured, for all columns other than L102. For this column, E_{inst} is decreased 0.95 and the shrinkage rate is accelerated 1.10, both within the limits of experimental error anticipated in measuring material parameters.

The theoretical and experimental deflection/time curves are compared in Figure 16. Agreement is generally adequate. It must be noted that a log time base has been used; plotted on a real time base, the lack of agreement is more noticeable. What is worthy of comment among the longer lived columns is the large effect on life of relatively small load changes, e.g. columns L111, L113. A decrease of load from 26.7 to 24.5 kN (- 8.2%) produces an increase in life, experimentally (so far) from 277 to 1890 days (+ 582%).

Figure 17 indicates the effect of creep on the short term capacity P_L. These effects are much more severe for more stocky columns. Further tests on this range of columns, i.e. L/d < 24, would be desirable.

TABLE 3 : Column Lives Under Load; Creep Capacities

Column Number (1)	Slenderness ratio (2)	Initial central deflection in millimetres (3)	Column strength in Megapascals (4)	Axial load in kilonewtons (5)	Column life in days Experimental (6)	Column life in days Theoretical (7)	Estimated creep capacity in kilonewtons (8)
L101	48	1.60	38.4	20.0	18	16	
L102		0.94	41.9	17.8	42	47	
L103		2.41	38.4	15.7	71	81	
L104		3.05	40.5	13.3	675	625	
-							11.5
L105	40	2.25	34.9	26.7	26	29	
L106		2.26	43.3	22.2	95	103	
L107		2.54	44.7	17.8	>900+	1601	
L108		2.69	39.8	16.0	d	$>10^4$	
-							16.0
L109	32	2.34	39.1	35.6	d	3	
L110		2.59	42.1	32.0	16	13	
L111		2.06	41.9	26.7	277	288	
L112		2.82	41.2	24.5	>1890*	4330	
-							21.0
L113	24	0.66	45.4	71.1	19	17	
L114		1.78	45.4	62.3	12	11	
L115		1.35	43.3	53.3	>2301*	4110	
L116		1.80	43.3	45.3	>2574*	4320	
-							35.0

\+ test stopped before failure

* test continuing

d premature failure; accidentally damaged.

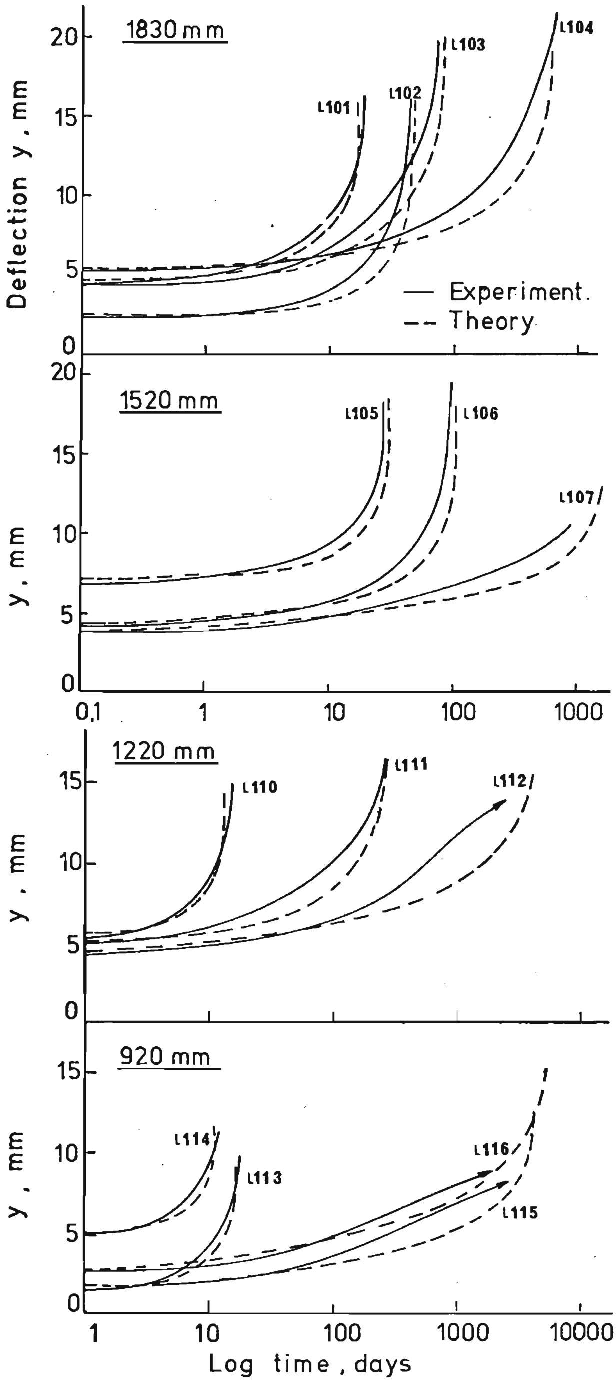

FIGURE 16 : Creep tests on columns

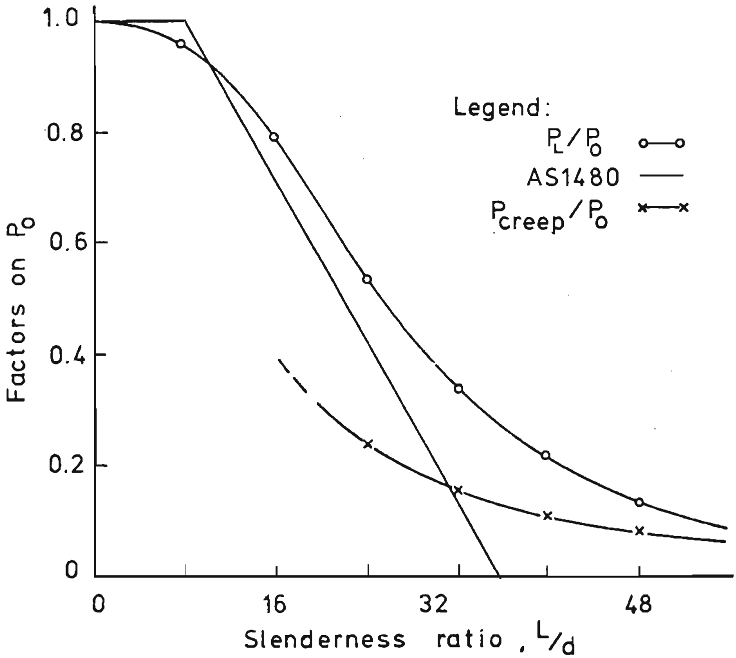

FIGURE 17 : Capacity reduction, long term tests

4. CONCLUSIONS

(1) An efficient short term column analysis, readily adaptable to creep analysis, has been developed. This was proven by comparison with the experimental results for some 85 model columns, ranging in size from 76 mm x 38 mm x 1830 mm long to 127 mm x 64 mm x 500 mm long and containing variations in concrete strengths, amount of reinforcement, type of reinforcement and magnitude of initial imperfections. The agreement was very good in all cases, both in the prediction of the ultimate short term load capacity and the shapes of the deflection/time curves.

(2) Two methods of creep analysis were developed and results compared with those obtained from sustained load tests on 16 model colunns.

(a) The reducing modulus method was discarded - shrinkage effects could not be included logically and the theoretical lives of columns disagreed with those observed from tests.

(b) The method described in the paper gave satisfactory agreement.

(3) This method can be extended to provide for:

(a) time-dependent axial loads
(b) prestressed columns
(c) fixed ended columns
(d) columns with varying cross-sections, and
(e) columns with external end moments.

(4) The use of this analysis to obtain a more extended range of results is described in another paper (21).

APPENDIX A - NOMENCLATURE

Symbol	Meaning
A,B,C	coefficients, equation 3
b	width of column b x d
d	depth of column in plane of bending
E_{inst}	initial concrete modulus; as C
F'_c	concrete crushing strength, general time
F_{PL}	proportional limit, compression steel
F_Y	yield or proof stress, tensile steel
F_{YC}	steel yield stress in compression
F'_{28}	concrete crushing strength at 28 days
L	column length
L/d	slenderness ratio
M	(applied) bending moment at a section
P	axial force in column
P_{creep}	limiting load due to creep
P_L	limiting short term load
P_0	theoretical capacity of short column
t,T	time, in days
y	total deflection of column relative to chord
y_o	initial central imperfection
z_o	distance from point of zero strain to column centroid
$\Delta\varepsilon_i$	strain change due to stress change
ε	strain
ε_e	strain defining initial stress, long term analysis
ε_i	minimum (or tensile) extreme fibre strain
ε_{ic}	initial + creep strain
ε_{ics}	total, i.e. initial + creep + shrinkage, strain
ε_{in}	initial or stress-dependent strain
ε_m	strain at column centroid
ε_o	maximum compressive extreme fibre strain
ε_u	ultimate concrete strain, bending
η	ratio $\varepsilon_{ic}/\varepsilon_{in}$
ϕ	curvature
σ	stress, generally
Σ	summation.

APPENDIX B - REFERENCES

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