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BUCKLING PROPERTIES OF MONOSYMMETRIC I-BEAMS

S. KITIPORNCHAI
and N. S. TRAHAIR

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Department of Civil Engineering,
University of Queensland,
St Lucia, Q 4067, Australia,
[Tel:(07) 377-3342, Telex:UNIVQLD AA40315]

BUCKLING PROPERTIES OF MONOSYMMETRIC I-BEAMS

by

S. Kitipornchai, BE, PhD.
Lecturer in Civil Engineering
University of Queensland

and

N.S. Trahair, B Sc, BE, M Eng Sc, PhD.
Professor of Civil Engineering
University of Sydney

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Synopsis

Approximations are derived for the section properties required for the calculation of the elastic critical loads of monosymmetric I-beams, and are found to be related to the ratio of compression flange and section minor axis second moments of area. Approximations are obtained which are applicable to I-sections with unequal and lipped flanges, and which are in close agreement with accurate calculations of the monosymmetry section properties made for a wide range of cross-sections. An improved design rule is proposed for the elastic critical stress of a monosymmetric I-beam. A comparison is made of the results obtained using the proposed rule and the present rules of the AS 1250, BS 449 and AISC Specification.

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1. INTRODUCTION

The elastic flexural-torsional buckling of beams of doubly symmetric cross-section has been extensively studied, both theoretically and experimentally (5,6,7,9,17). However, there have been relatively few studies made of the elastic buckling of beams of monosymmetric cross-section. Early studies of monosymmetric beams were made by Winter (18); Petterson (15); Hill (8); Kerensky, Flint and Brown (12) and O'Connor (14). The work of Kerensky, Flint and Brown (12) formed the basis for the design of monosymmetric I-beams in the current British Code BS 449 (3) as well as the current Australian Code AS 1250 (16).

More recently, Anderson and Trahair (2) tabulated theoretical results for simply supported monosymmetric I-beams and cantilevers with concentrated loads or uniformly distributed loads. Their study included the effect of the load height above the shear centre of the section.

For a simply supported monosymmetric I-beam under uniform moment, the dimensionless elastic critical moment, γ_c, can be expressed in the form (17)

$$\gamma_c = \frac{M_c L}{\sqrt{EI_y GJ}} = \pi \left[\sqrt{1 + K^2 + \left(\frac{\pi\delta}{2}\right)^2} + \frac{\pi\delta}{2} \right] \qquad (1)$$

where K is the beam parameter,

$$K = \sqrt{\frac{\pi^2 EI_\omega}{GJL^2}} \qquad (2)$$

and EI_y is the minor axis flexural rigidity, GJ is the torsional rigidity, EI_ω is the warping rigidity, L is the length of the beam, and δ is the monosymmetry parameter,

$$\delta = \frac{\beta_x}{L}\sqrt{\frac{EI_y}{GJ}} \qquad (3)$$

in which β_x is the cross-section property

$$\beta_x = \frac{1}{I_x}\left[\int_A x^2 y \, dA + \int_A y^3 \, dA\right] - 2y_o \qquad (4)$$

in which y_o is the coordinate of the shear centre.

The property, β_x, arises from the bending compressive and tensile stresses which may form a resultant torque when the beam twists during buckling. This is sometimes referred to as the "Wagner Effect" (2). For doubly symmetric I-beams, the torque component due to the compressive stresses exactly balances that due to the tensile stresses, and β_x is zero. However, in a monosymmetric beam, there is an imbalance and the resultant torque causes a change in the effective torsional stiffness. When the smaller flange is in compression, there is a reduction in the effective torsional stiffness (β_x is negative), while the reverse is true when the smaller flange is in tension (β_x is positive).

One of the difficulties associated with the calculation of the elastic critical loads for monosymmetric beams is in the determination of the shear centre position y_o, of the warping section constant I_ω, and of the monosymmetry property, β_x. The evaluation of these is not straight forward and the effort required is prohibitive in routine design. Because of this, a number of approximate design methods have been developed, which either avoid these calculations, or replace them by gross simplifications.

The present rules of the AISC Specification (1) for the design of slender monosymmetric I-beams are based on the very simple approximation of compression flange buckling. Thus the section properties used to determine the maximum permissible stresses are those of the compression flange, and the presence of a tension flange is completely ignored.

A more complete basis is used for the rules of the BS 449 (3), and the AS 1250 (16), in which some account is taken of the tension flange. This method was developed by Kerensky, Flint and Brown (12), who started from an approximate theoretical solution of Winter (18) for the elastic buckling of a monosymmetric beam. They showed that Winter's solution tends to overestimate the critical stress when the larger flange is in compression, and introduced a compensating empirical reduction.

More recently, Nethercot and Taylor (13) further developed the approximate formulation of Kerensky, Flint and Brown. They concluded, however, that in view of the degree of approximation of the existing design rules, it would be desirable to permit designers the alternative of basing their designs on the accurate theoretical solution of Equation 1.

It can be seen that a dilemma has arisen in the design of slender monosymmetric beams. On the one hand, the present simple rules, which are based on very crude approximations, lead to significant errors in the predictions of elastic buckling. On the other hand, however, the use of the accurate elastic buckling solutions requires considerable effort to be expended in the evaluation of the section properties.

The purpose of this paper is to present a simple method of determining these section properties, and to develop a more accurate design formula for elastic buckling than those of existing codes (1, 3, 16). The method presented can be used for a wide range of monosymmetric I-sections, including sections with lipped flanges.

2. DERIVATION OF SECTION PROPERTIES

2.1 General

The section properties required for the calculation of the elastic critical moment, M_c, of a monosymmetric I-beam are I_y, J, I_ω and β_x. The values of I_y and J can be calculated from

$$I_y = \int_A x^2 \, dA \tag{5}$$

and

$$J \simeq \Sigma \frac{BT^3}{3} \tag{6}$$

in which B and T are the width and thickness of a typical rectangular element of the section. However, the values of y_o, I_ω and β_x are not so easily calculated.

2.2 Shear Centre Position

It has been suggested (13, 17) that an easily calculated measure of the monosymmetry of the cross-section is given by

$$\rho = \frac{I_{yC}}{I_{yC} + I_{yT}} = \frac{I_{yC}}{I_y} \tag{7}$$

where I_{yC}, I_{yT} are the section minor axis second moments of area of the compression and tension flanges, respectively. The values of ρ thus range from 0 for a tee-beam with the flange in the tension to 1 for a tee-beam with the flange in compression. For an equal flanged beam, $\rho = 0.5$.

The shear centre S of a monosymmetric I-section (see Figs 1 and 2) lies on the web centre line at distances a and b from the compression and tension flange shear centres which are given by (11)

$$a = \frac{I_{yT}}{I_y} h = (1 - \rho)h \tag{8}$$

and

$$b = \frac{I_{yC}}{I_y} h = \rho h \tag{9}$$

in which h is the distance between flange shear centres. The shear centre coordinate y_o is

$$y_o = \bar{y} - a \tag{10}$$

in which $\bar{y}$ is the distance of the centroid C from the compression flange shear centre.

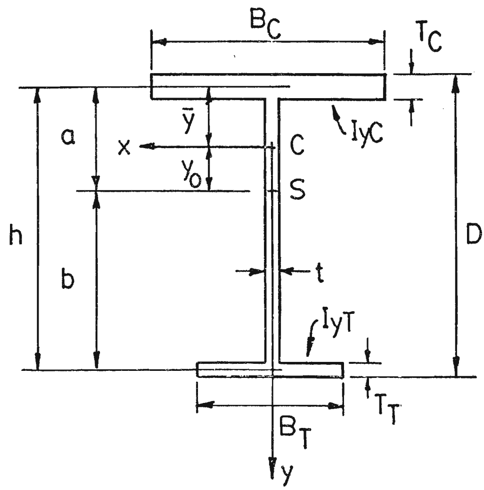

FIGURE 1 : Monosymmetric I-Section

Monosymmetric I-sections with lipped compression flanges (see Fig. 2) are commonly used for crane runway girders. The

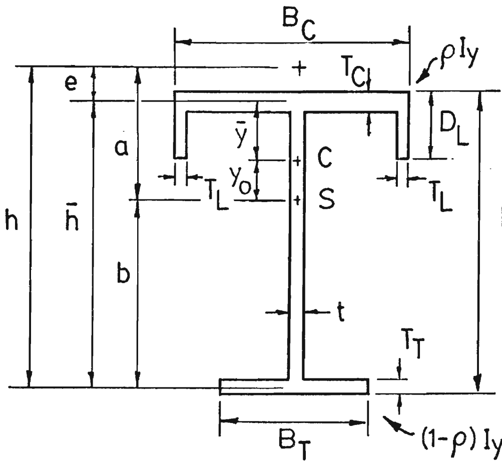

FIGURE 2 : Monosymmetric I-Section with lipped flange

addition of rectangular section lips to a flange increases the minor axis second moment of area and moves the shear centre position towards the flange. The distance between the shear centre of the lipped flange and the centreline of the unlipped flange is given by (12)

$$e = \frac{D_L^2 \, B_C^2 \, T_L}{4\rho \, I_y} \qquad (11)$$

where D_L and T_L are the depth and thickness of the lips respectively. The position of the shear centre for the entire section is then defined by

$$a = (1 - \rho) \, \bar{h} - e\rho = (1 - \rho) h \qquad (12)$$

in which $\bar{h}$ is the distance between flange centre lines (see Fig. 2), that is

$$h = \bar{h} + e \qquad (13)$$

2.3 Warping Section Constant

The warping section constant, I_ω, of a section can be evaluated from (11)

$$I_\omega = a^2 I_{yC} + b^2 I_{yT} \qquad (14)$$

which is exact for an unlipped section, and approximate for a lipped section. If Equations 7, 8 and 9 are substituted, then the warping section constant can be simply expressed as (7, 12)

$$I_\omega = \rho(1 - \rho) I_y h^2 \qquad (15)$$

For a doubly symmetric I-section, $I_\omega = I_y h^2/4$, while for a tee-section, $I_\omega = 0$.

The beam parameter, K, defined in Equation 2 is zero for tee-beams, which leads to computational difficulties in some situations. A more useful parameter is

$$\bar{K} = \sqrt{\frac{\pi^2 E I_y h^2}{4GJL^2}} \qquad (16)$$

If Equation 15 is substituted into Equation 2 for the warping constant, I_ω, then it can be shown that

$$K = \sqrt{4\rho(1 - \rho)}\ \bar{K} \qquad (17)$$

If Ar_y^2 is substituted for I_y, and 2.5 used for the ratio of the moduli E/G, then Equation 16 becomes

$$\bar{K} = 4.47 \frac{h}{\sqrt{J/0.3085A}} \frac{r_y}{L} \qquad (18)$$

It has been found (10) that for a wide range of as-rolled doubly symmetric I-sections (4),

$$\frac{D}{h}\sqrt{\frac{J}{0.3085A}} \approx T \qquad (19)$$

in which D is the section depth and T is the flange thickness. The accuracy of Equation 19 is shown in Fig. 3, for which values of $(D/h)\sqrt{J/0.3085A}$ for as-rolled UB and UC sections (4) are

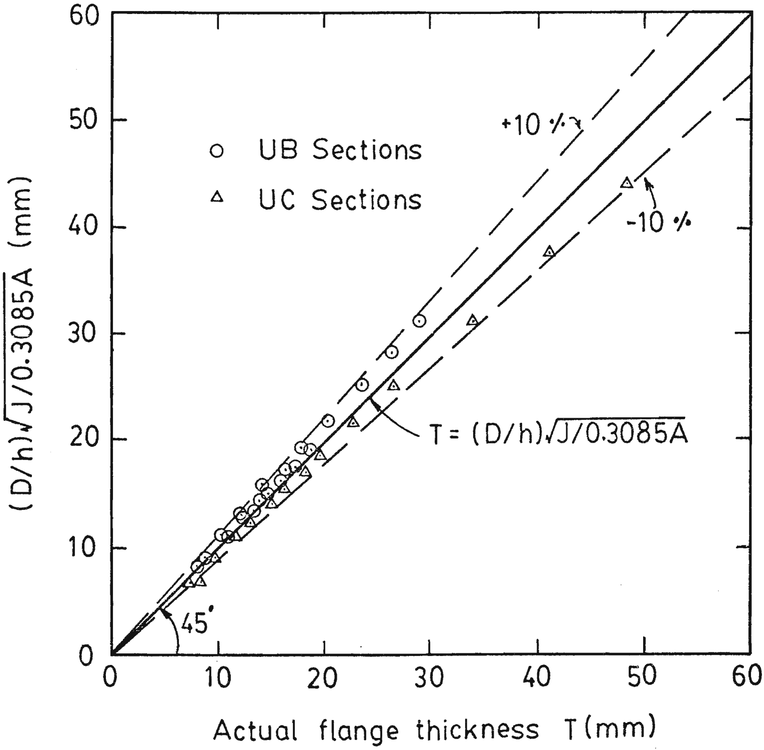

FIGURE 3 : Flange thickness for doubly symmetric as-rolled sections (4)

calculated and plotted against the actual flange thicknesses. It can be seen that the calculated values are slightly lower than the actual values for UB sections, and higher for UC sections, and are within ± 10% of the actual values. Thus, in this case $\bar{K}$ can be approximated by

$$\bar{K} \simeq 4.47 \frac{D/T}{L/r_y} \tag{20}$$

It can be seen that large values of $\bar{K}$ imply short beams and/or deep thin-walled sections for which warping effects are important, whereas small values of $\bar{K}$ are associated with long beams and/or shallow sections for which warping effects are of less importance than those of uniform torsion. The relationships between D/T and L/r_y for various values of K are shown in Fig. 4.

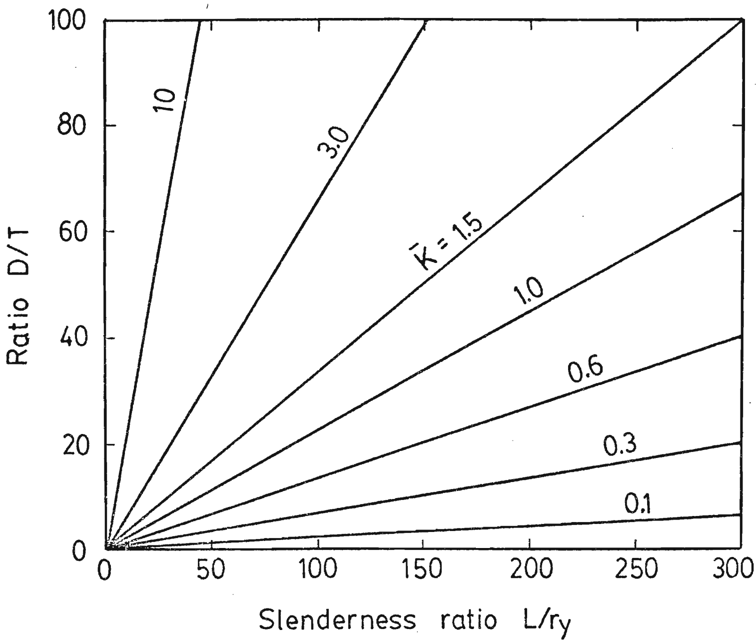

FIGURE 4 : Beam parameter $\bar{K}$

The approximation for $\bar{K}$ given by Equation 20 can also be used for monosymmetric I-sections, provided an effective flange thickness T_e defined by

$$T_e = \frac{D}{h}\sqrt{\frac{J}{0.3085A}} \tag{21}$$

is substituted for T.

2.4 Monosymmetry Property β_x

2.4.1 General

For an unlipped section (see Fig. 1), the monosymmetry section property, β_x, can be expressed in terms of the section dimensions and the coordinate of the shear centre y_o (2, 7, 11) as

$$\beta_x = \frac{1}{I_x}\{(h-\bar{y})[B_T^3T_T/12 + B_TT_T(h-\bar{y})^2 + (h-\bar{y})^3t/4]$$

$$- \bar{y}\,[B_C^3T_C/12 + B_CT_C\bar{y}^2 + \bar{y}^3t/4]\} - 2y_o \qquad (22)$$

2.4.2 Webless I-Beams

The special case of a monosymmetric webless I-section ($t = 0$) is shown in Fig. 5. The position of the centroid C can be defined in terms of the ratio μ

$$\mu = \frac{A_{FC}}{A_{FC} + A_{FT}} = \frac{A_{FC}}{A_F} \qquad (23)$$

where A_{FC} and A_{FT} are the areas of the compression and tension flanges, respectively. Hence, the position of the centroid C is given by

$$\bar{y} = (1 - \mu)h \qquad (24)$$

and

$$h - \bar{y} = \mu h \qquad (25)$$

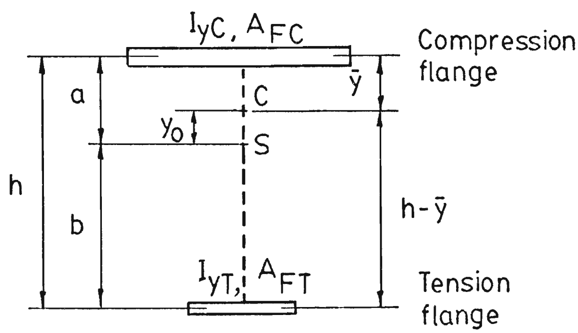

FIGURE 5 : Webless I-Section

Equation 22 can now be rearranged in terms of the flange areas A_{FC}, A_{FT} and the flange second moments of area I_{yC}, I_{yT}, whence

$$\beta_x = \frac{1}{I_x}\{(h - \bar{y})\ [I_{yT} + A_{FT}\ (h - \bar{y})^2] - \bar{y}\ [I_{yC} + A_{FC}\ \bar{y}^2]\} - 2y_o \qquad (26)$$

Now $$I_x = A_{FC}\ (1-\mu)^2h^2 + A_{FT}\mu^2h^2 = \mu(1-\mu)Ah^2 \qquad (27)$$

and $$y_o = a - \bar{y} = (\mu - \rho)h \qquad (28)$$

Substituting Equations 27, 28 into 26, leads to

$$\frac{\beta_x}{h} = (2\rho - 1) + \frac{I_y}{I_x}(\mu - \rho) \qquad (29)$$

When I_y is much less than I_x, then

$$\frac{\beta_x}{h} \approx (2\rho - 1) \qquad (30)$$

which is simple straight line equation varying from -1 for $\rho = 0$ to $+1$ for $\rho = 1$ (see Fig. 6).

2.4.3 Approximations for Real I-Beams

It is not sufficient to neglect the contribution of web in a real I-beam, especially when calculating the major axis second moment of area I_x. To examine the accuracy of the β_x/h approximations given in Equations 29 and 30 for real beams, a full range of as-rolled UB and UC Sections (4) with reduced flange widths or flange thicknesses were investigated. The values of β_x/h calculated by using the accurate formulae (Equation 22) are compared with the straight line approximation (Equation 30) in Fig. 6. It can be seen that Equation 30 provides a reasonable approximation for values of ρ near 0.5 is conservative for ρ less than 0.5, but overestimates β_x/h for ρ greater than 0.5.

To obtain a better approximation for β_x/h, it is necessary to include the effect of the ratio I_y/I_x of the cross-

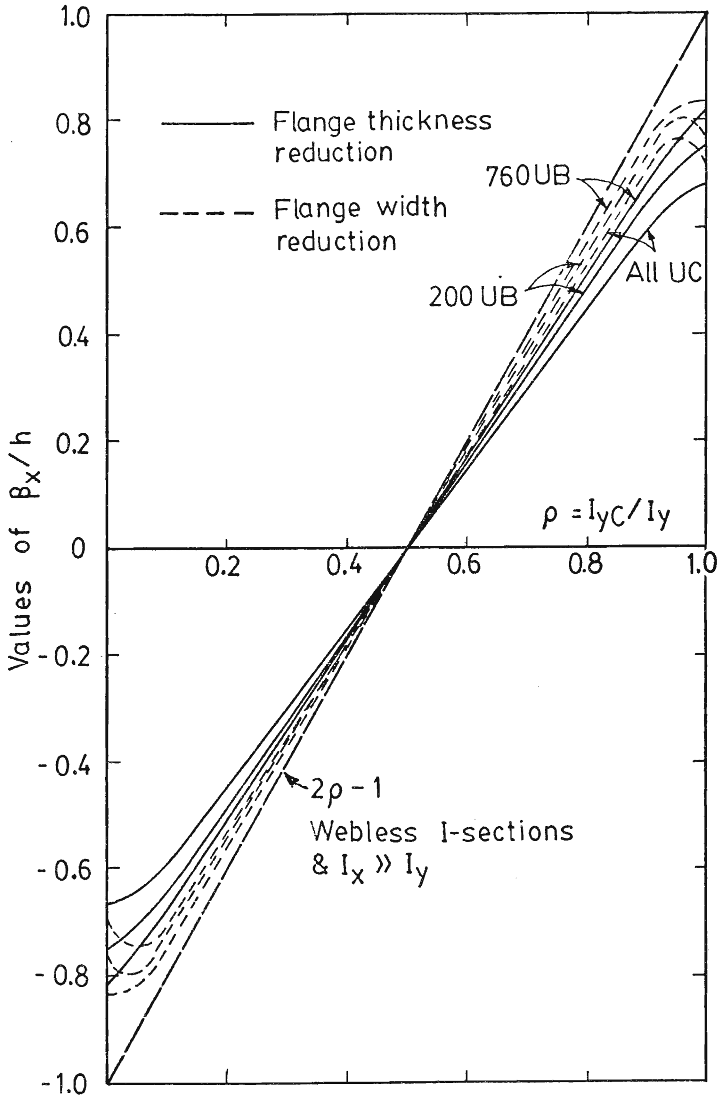

FIGURE 6 : Values of β_x/h for monosymmetric beams made from as-rolled UB and UC sections

section (see Equation 29). Several forms of approximation were tried for a wide range of plate girder dimensions. The combination of cross-sectional dimensions included beams with flange width to thickness ratios in the range $4 \leq B/T \leq 64$, and flange thickness to web thickness in the range $10 \leq D/t \leq 290$. A total of over 3000 beam cross-sections were studied and the errors of each approximation were evaluated. Neglecting those beams for which I_y/I_x exceeds 0.5, the following approximation,

$$\frac{\beta_x}{h} \simeq 0.9(2\rho - 1)\ [1 - (\frac{I_y}{I_x})^2] \qquad (31)$$

was found to give zero mean error with a standard deviation of 0.037 for the range of beams considered. This approximation is shown in Fig. 7.

The monosymmetry parameter, δ, of Equation 3 can now be approximated by using Equations 16 and 31, whence

$$\delta \simeq 0.9(2\rho - 1)\ [1 - (\frac{I_y}{I_x})^2]\ \frac{2\bar{K}}{\pi} \qquad (32)$$

A modified approximate expression for β_x/h for sections with lipped flanges can be obtained by using a similar approach to that above. A total of over 2000 beams with lip depth to section depth ratios in the range $0 \leq D_L/D \leq 1.0$ and flange width ratios in the range $0 \leq B_T/B_C \leq 1.0$ were considered. It was found that

$$\frac{\beta_x}{h} \simeq 0.9(2\rho - 1)\ [1 - (\frac{I_y}{I_x})^2][1 + \frac{D_L}{2D}] \qquad (33)$$

gave good approximation with a mean error of 0.036 and a standard deviation of 0.017 for the range of beams considered.

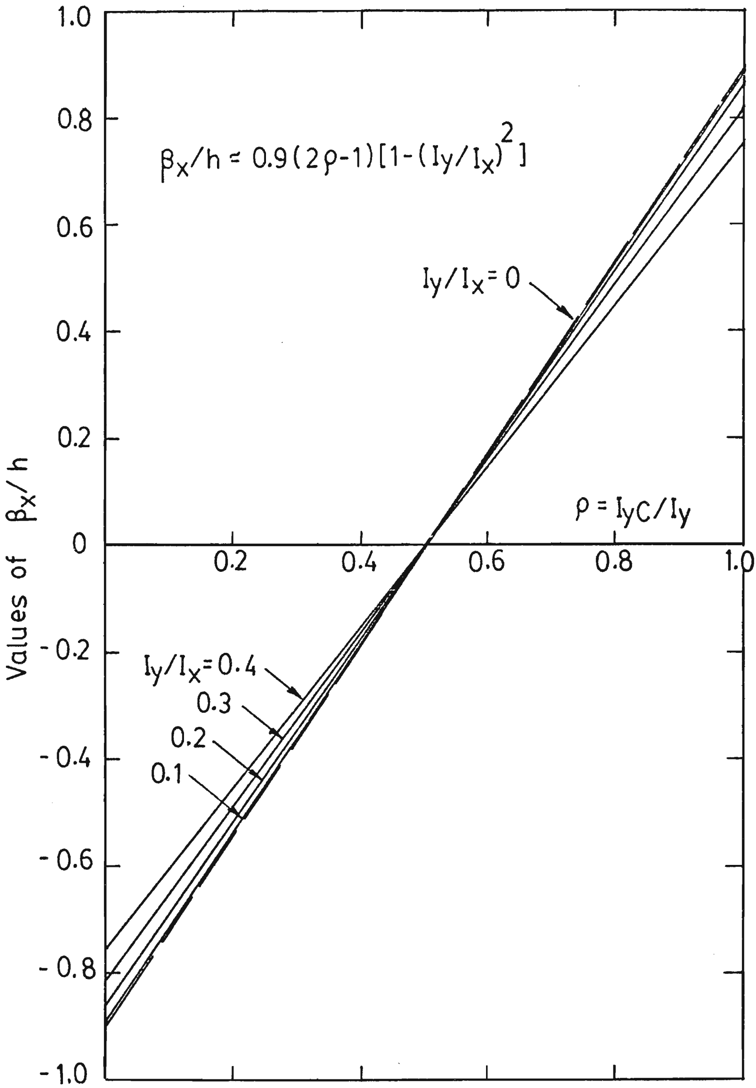

FIGURE 7 : Approximation for β_x/h

3. ELASTIC CRITICAL STRESS RULES

3.1 Present Design Rules

For design purposes, the AS 1250 (16), the BS 449 (3), and the AISC Specification (1) based their maximum permissible stresses on the elastic critical stress

$$F_{ob} = \frac{M_c}{Z_{xmin}} \tag{34}$$

either explicitly or implicitly.

For the AS 1250 (16), an approximation for the elastic critical stress, F_{ob} for monosymmetric I-beams is given by

$$F_{ob} = \frac{2,650,000}{(L/r_y)^2}\left[\sqrt{1 + \frac{1}{20}\left[\frac{L}{r_y}\,\frac{T}{D}\right]^2} + K_2\right]\frac{c_2}{c_1}\ \text{MPa} \tag{35}$$

in which

$$K_2 = 0.5\ (2\rho - 1)\ \text{for}\ \rho > 0.5 \tag{36}$$

$$= (2\rho - 1)\qquad \text{for}\ \rho \leq 0.5 \tag{37}$$

and c_1 and c_2 are the lesser and greater distances from the neutral axis to the extreme fibres. The values of T in Equation 36 is defined as the thickness of the flange which has the greater second moment of area about the minor axis.

An equivalent expression is used for BS 449 (3), except that the factor 2,650,000 is replaced by 2,800,000, and the value of T is defined as the thickness of the compression flange. The BS 449 rules are applicable only to I-beams when the thickness of one flange does not exceed three times the thickness of the other flange. For tee-beams with $\rho = 0$, the value of T is taken as the thickness of the web.

In the AISC Specification (1), no procedures are used for monosymmetric I-beams which are specifically different to those for double symmetric I-beams. It is shown in Ref. 17 that the elastic critical stress, F_{ob}, on which the permissible stress is based is approximated by

$$F_{ob} = \left[\sqrt{\left[\frac{130,400}{LD/BT}\right]^2 + \left[\frac{1,975,000}{(L/r_T)^2}\right]^2}\right] \frac{c_2}{c_1} \text{ MPa} \quad (38)$$

in which r_T is the radius of gyration about the minor axis of the compression flange plus one sixth of the web, and is given by

$$r_T = \sqrt{\frac{B^2/12}{1 + (D - 2T)\ t/6BT}} \quad (39)$$

The values of B and T in Equations 38 and 39 are those of the compression flange. No special procedures for tee-beams are given.

3.2 Proposed Design Rules

In this section, a new method of calculating the elastic critical stresses of monosymmetric beams is proposed which is more accurate and consistent than the present design rules discussed above. This new method is based on the calculation of the dimensionless buckling moment

$$\gamma_c = \frac{M_c L}{\sqrt{EI_y GJ}} \approx \text{fn}\ (\rho, \bar{K}, \frac{I_y}{I_x}) \quad (40)$$

by substituting Equations 17 and 32 into Equation 1, or alternatively, on the use of Fig. 8, which shows values of γ_c for various values of ρ and $\bar{K}$ when $0.1 \leq I_y/I_x \leq 0.3$. The elastic critical stress F_{ob} can be calculated directly from the dimensionless buckling moment γ_c of Equation 40 or Fig. 8 by using Equation 34.

Calculations have been made to assess the errors involved in the approximate dimensionless buckling moments, γ_c, which result from errors of $\pm 8\%$ in the approximation of Equation 31 for β_x/h. The results of these calculations are shown in Fig. 9 for the case when $I_y/I_x = 0.1$. It can be seen that the errors in the approximate dimensionless buckling moments γ_c are quite small, even when $\bar{K}$ is large and the sections are highly monosymmetric.

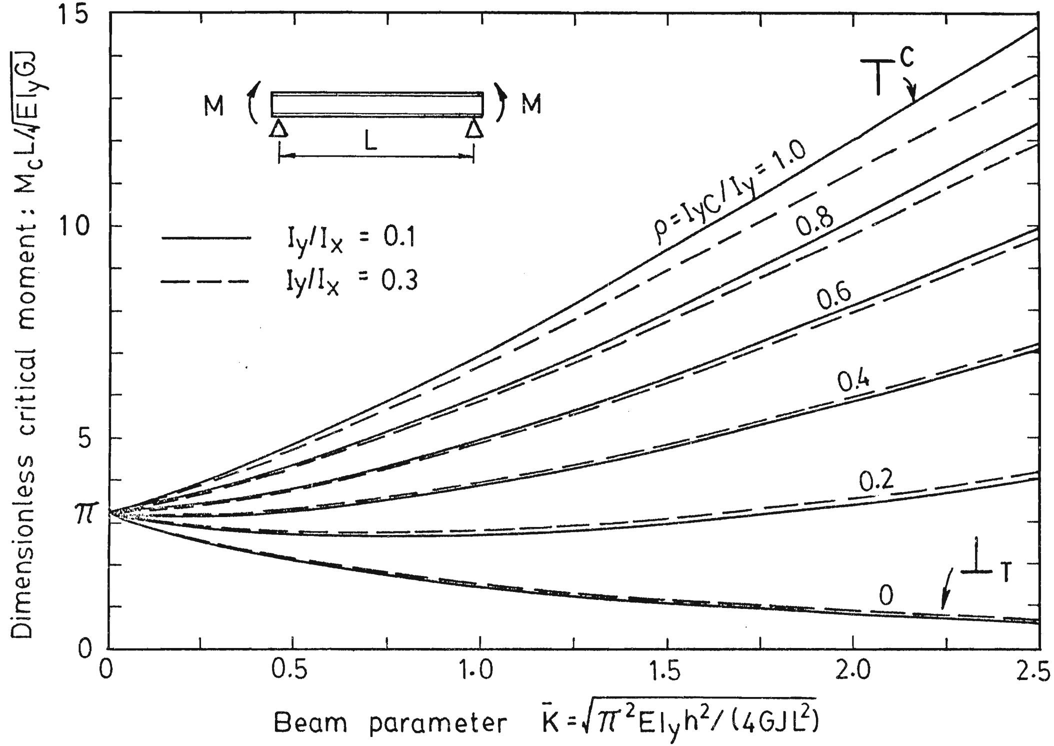

FIGURE 8 : Approximate critical moments of monosymmetric I-beams

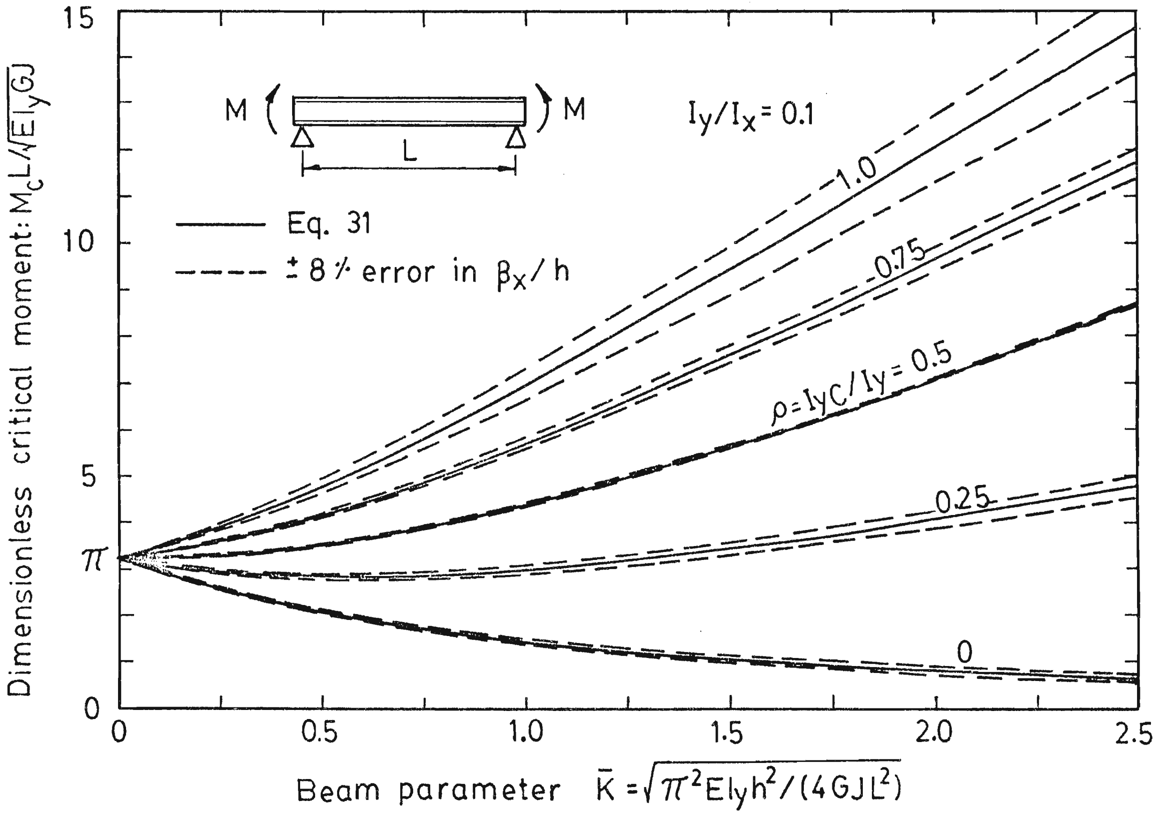

FIGURE 9 : Error range of approximate dimensionless critical moments

As an alternative to this method of calculating the elastic critical stress F_{ob}, Equations 1, 16, 17, 20 and 32 can be substituted into Equation 34, whence

$$F_{ob} = \frac{K_4}{(L/r_y)^2}\left[\sqrt{K_3 + \frac{1}{20}\left(\frac{L}{r_y}\frac{T_e}{D}\right)^2} + K_2\right] \qquad (41)$$

in which $$K_2 = \frac{\beta_x}{h} \qquad (42)$$

$$K_3 = 4\rho(1-\rho) + \left(\frac{\beta_x}{h}\right)^2 \qquad (43)$$

and $$K_4 = \frac{\pi^2 EAh}{2Z_{xmin}} \qquad (44)$$

Equation 41 thus retains the familiar form of the present AS 1250 (16) and BS 449 (3) rules (see Equation 35). Approximate values of K_2 and K_3 are shown in Table 1 (for $I_y/I_x = 0.1$) and are compared with values in BS 449 and AS 1250 in Fig. 10.

The factor K_4 may be rewritten as

$$K_4 = \frac{\pi^2 E}{2}\,\frac{Ah^2}{I_x}\,\frac{c_2}{h} \qquad (45)$$

where c_2 is the maximum distance from neutral axis to the extreme fibre. The second moment of area about the major axis, I_x, in Equation 27 for webless I-beams can be modified to include the effect of the web, whence

$$\frac{I_x}{A} \approx \frac{A_F}{A}\{\mu\bar{y}^2 + (1-\mu)(h-\bar{y})^2\} + \frac{A_W}{A}\left\{\frac{h^2}{12} + \left(\frac{h}{2}-\bar{y}\right)^2\right\} \qquad (46)$$

in which A_F and A_W are cross-sectional area of flanges and web respectively,

and $$A_F + A_W = A \qquad (47)$$

For $\mu \leq 0.5$, c_2 may be approximated by

$$\frac{c_2}{h} \approx \frac{\bar{y}}{h} = \left\{(1-\mu)\frac{A_F}{A} + \frac{1}{2}\frac{A_W}{A}\right\} \qquad (48)$$

When Equations 46, 47 and 48 are substituted into Equation 45, K_4 can be expressed in terms of the ratios μ and A_W/A,

TABLE 1

Values of K_2 and K_3 for Beams with Unequal Flanges

(I_y/I_x = 0.1)

ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K_2	-0.89	-0.71	-0.59	-0.36	-0.18	0	+0.18	+0.36	+0.59	+0.71	+0.89
K_3	0.79	0.86	0.93	0.97	0.99	1.0	0.99	0.97	0.93	0.86	0.79

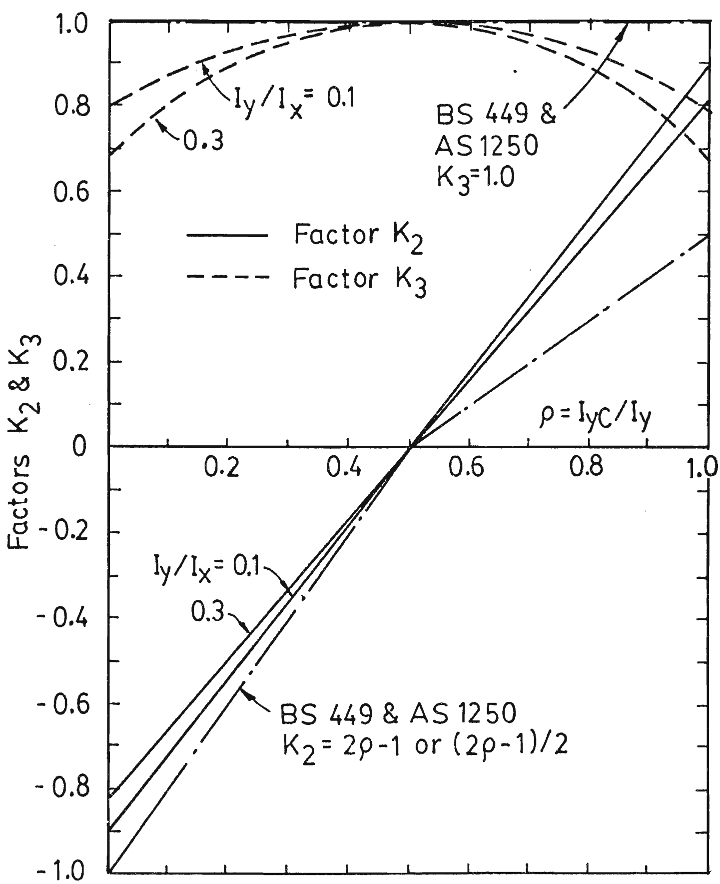

FIGURE 10 : Factors K_2 and K_3

i.e. $$K_4 = fn\left(\mu, \frac{A_w}{A}\right) \qquad (49)$$

The variations of the factor K_4 with μ for various values of the ratio A_w/A are shown in Table 2 and Fig. 11. Also shown in Fig. 11 is the AS 1250 approximation (see Equation 37)

$$K_4 \simeq 2,650,000 \times \frac{c_2}{c_1} \qquad (50)$$

It can be seen that while this approximation is of reasonable accuracy when $A_w/A \simeq 0.5$, it may lead to serious errors otherwise, especially for monosymmetric beams with low values of A_w/A. Because of this, it is suggested that K_4 should be determined either directly from Equations 44 or 45 or from Table 2 or Fig. 11.

TABLE 2

Values of $K_4/10^6$ (MPa)

$\frac{A_w}{A}$	Ratio of flange areas μ					
	0 1.0	0.1 0.9	0.2 0.8	0.3 0.7	0.4 0.6	0.5 0.5
0	∞	9.87	4.93	3.29	2.47	1.97
0.1	30.3	8.18	4.74	3.34	2.59	2.11
0.2	15.7	7.08	4.59	3.41	2.72	2.28
0.3	10.8	6.33	4.49	3.50	2.88	2.47
0.4	8.46	5.81	4.45	3.62	3.08	2.69
0.5	7.10	5.45	4.45	3.78	3.31	2.96
0.6	6.28	5.24	4.51	3.99	3.59	3.29
0.7	5.79	5.15	4.65	4.26	3.95	3.70
0.8	5.55	5.19	4.89	4.63	4.41	4.23

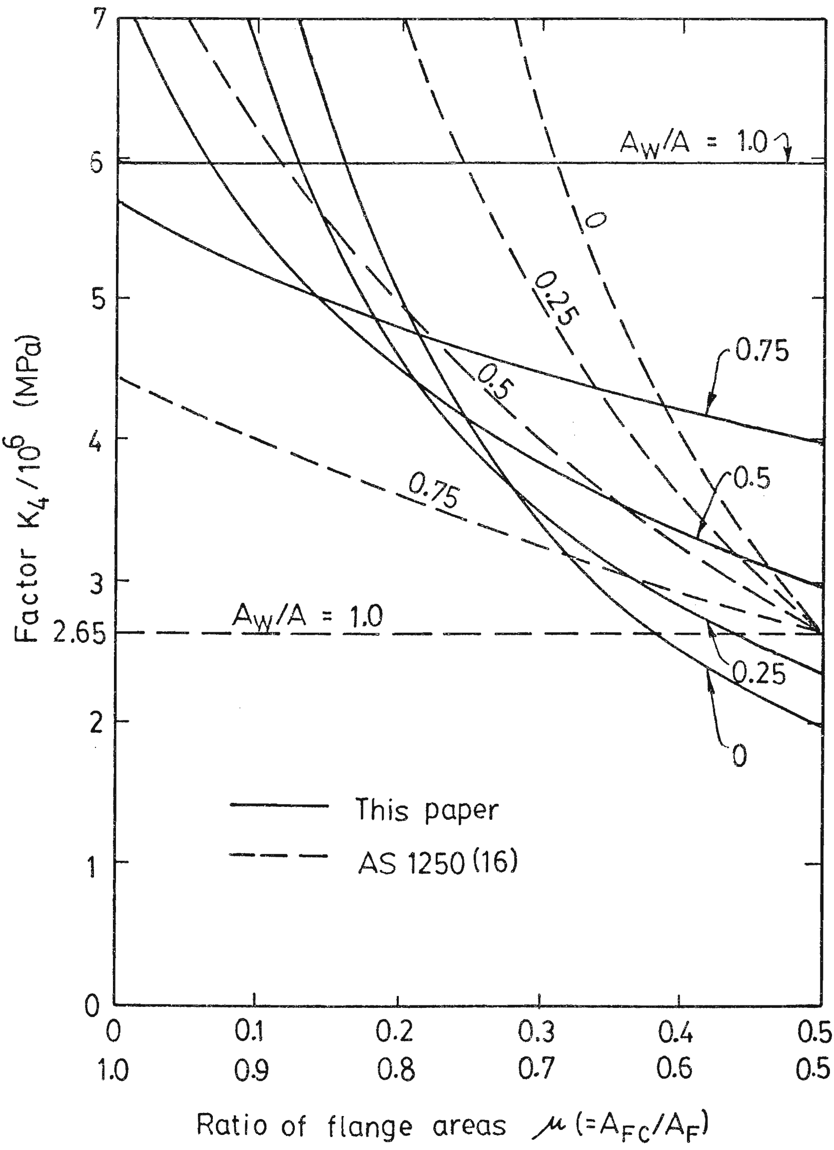

FIGURE 11 : Factor K_4

3.3 Comparison of Present and Proposed Design Rules

Calculations have been made for a number of monosymmetric beams using the present and proposed design rules. The elastic critical stresses of a doubly symmetric I-beam for which B/T = 14, T/t = 2 and D/T = 36 have been calculated and are shown in Fig. 12. The values calculated using the proposed rule virtually coincide with the accurate curve based on Equation 1 and the actual section properties. This is as expected, since the factor K_4 and the effective flange thickness, T_e, were accurately calculated for the proposed rule. The values obtained by using the present design rules are all slightly higher than the accurate curve, the highest being those using BS 449 (because of its high factor 2,800,000), and the lowest (and most accurate) being those of the AISC Specification.

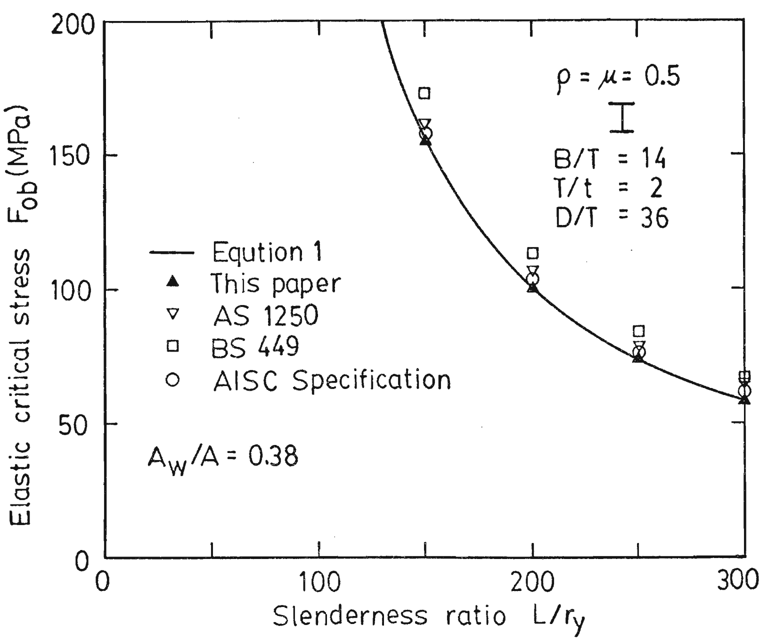

FIGURE 12 : Comparison of elastic critical stresses for doubly symmetric I-beams (ρ = 0.5)

The calculated elastic critical stresses for tee-beams (ρ = 0 and 1.0) made by removing one flange of the same doubly symmetric beam are shown in Fig. 13. When the flange is in compression (ρ = 1.0), the proposed rule gives results which are slightly higher than the accurate curve due to errors in the β_x/h approximation. Of the present rules, the BS 449 again gives the highest values, and the AISC Specification the most accurate. However, when the flange is in tension (ρ = 0), there are considerable differences in the calculated elastic critical stresses, as can be seen in Fig. 13. The proposed rule gives estimates which are slightly lower than the accurate curve, while the results using AS 1250 are approximately 20% higher. Values obtained using the BS 449 rules are considerably lower because the web thickness is used for the flange thickness T in the expression equivalent to Equation 35. The AISC Specification can be interpreted as predicting zero elastic critical stresses because the values of B and T for the compression flange are zero.

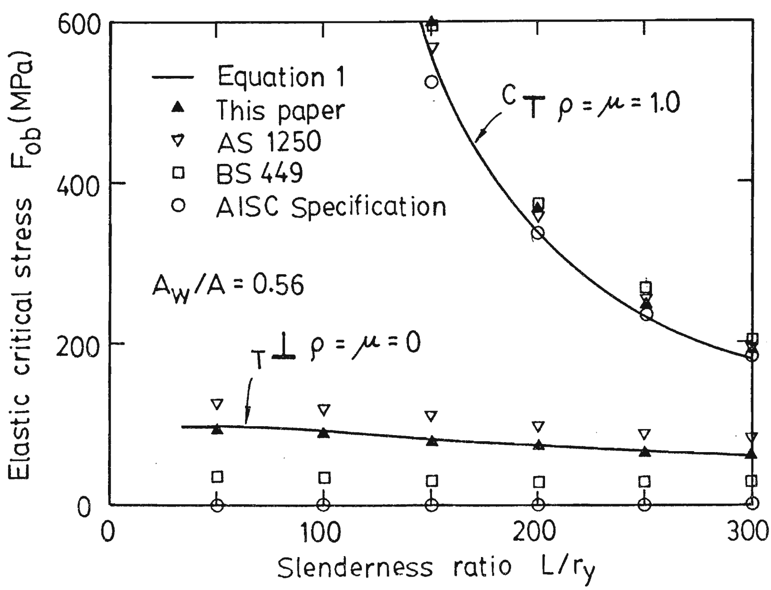

FIGURE 13 : Comparison of elastic critical stresses for tee-beams (ρ = 1.0 and 0)

The elastic critical stresses have also been calculated for beams with unequal flanges, made from the same doubly symmetric I-beam as before. The results shown in Fig. 14 are for beams with equal flange thickness (i.e. the flange width is reduced), while those in Fig. 15 are for beams with equal flange widths (i.e. the flange thickness is reduced). It can be seen that the elastic critical stresses predicted by the proposed rule are very close to the accurate curve. There is, however, considerable disagreement in the values calculated by using the present rules, particularly for monosymmetric I-beams of unequal flange thickness (see Fig. 15). The values using the AS 1250, the BS 449 and the AISC Specification are all higher than the accurate curves, except when the thickness of the compression flange is the lesser, when the predictions using BS 449 are lower than the accurate curves.

It should be pointed out that the comparisons shown in Figs. 12 to 15 are for beams with A_w/A ranging from 0.38 to 0.56. For such beams, the AS 1250 and BS 449 approximation for K_4 are reasonably accurate (see Fig. 11). However, this will

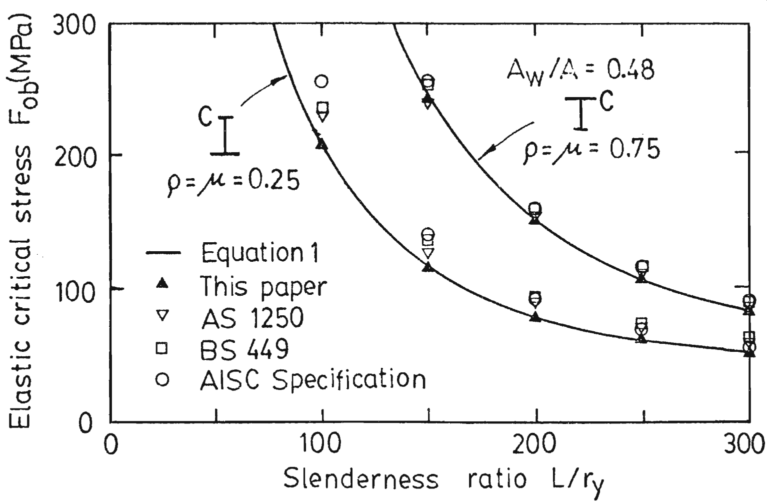

FIGURE 14 : Comparison of elastic critical stresses for monosymmetric I-beams with equal flange thicknesses (ρ = 0.75 and 0.25)

not be the case for beams with more extreme values of A_w/A, and it can be expected that the errors for such beams may be greatly increased.

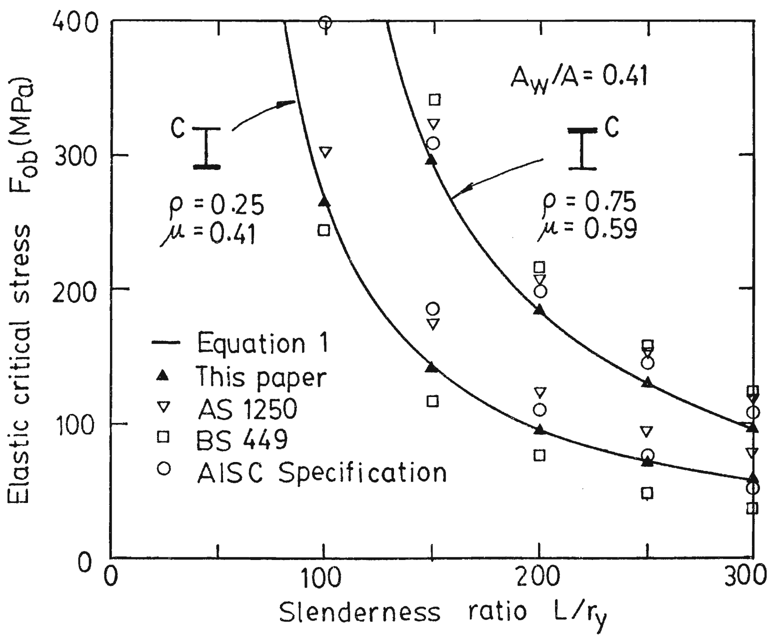

FIGURE 15 : Comparison of elastic critical stresses for monosymmetric I-beams with equal flange widths (ρ = 0.75 and 0.25)

4. CONCLUSIONS

The determination of the section properties required for calculating the elastic critical moment of a monosymmetric I-beam is not straightforward, and the effort required is prohibitive in routine design. Existing design methods either avoid these calculations or replace them by gross simplifications. In this paper it is shown that these properties are related to the easily calculated ratio $\rho = I_{yC}/I_y$ of the compression flange second moments of area to that of the whole section.

Approximate formulae for the monosymmetry section property, β_x, were derived by first considering webless I-sections, and compared with accurate calculations of β_x made for a wide range of monosymmetric cross-sections. The approximate formulae were found to have mean errors of 0 to 0.036 and standard deviations of 0.017 to 0.037. The errors in the elastic critical moments, calculated by using these approximate formulae, were found to be quite small, even when sections were highly monosymmetric.

An improved design rule for determining the elastic critical stresses, F_{ob}, of monosymmetric I-beams has been proposed. The proposed rule retains the familiar form of the existing AS 1250 and BS 449 rules, and is easy to use. While there is at present considerable disagreement on the definition of the flange thickness, T, it is suggested that the effective flange thickness, T_e, should be approximated by $(D/h)\sqrt{J/0.3085A}$ for both doubly symmetric and monosymmetric I-sections, including tee-sections. Comparisons have been made of the calculated elastic critical stresses of doubly symmetric and monosymmetric I-beams using the proposed rule and the present rules of the AS 1250, the BS 449 and the AISC Specification. The values obtained using the proposed rule have been shown to be more accurate than those of the present rules, and are within a few per cent of the accurate values.

5. ACKNOWLEDGEMENTS

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APPENDIX A - NOTATION

Symbol	Meaning
A	cross-sectional area
A_F	area of flanges
A_{FC}, A_{FT}	areas of compression and tension flanges
A_W	area of web
a	distance of section shear centre from compression flange centre line
B	flange width
B_C, B_T	widths of compression and tension flanges
b	distance of section shear centre from tension flange centre line
C	centroid position
c_1, c_2	lesser and greater distances from extreme fibres to neutral axis
D	depth of beam
D_L	depth of lip
E	Young's Modulus of elasticity
e	distance between flange centre line and shear centre of lipped flange
F_{ob}	elastic critical stress
G	shear modulus of elasticity
h	distance between flange shear centres
$\bar{h}$	distance between flange centre lines
I_x	major axis second moment of area
I_y	minor axis second moment of area
I_{yC}, I_{yT}	compression and tension flange second moment of area about minor axis
I_ω	warping section constant
J	section torsion constant

K	$\sqrt{\pi^2 EI_\omega / GJL^2}$
$\bar{K}$	$\sqrt{\pi^2 EI_y h^2 / 4GJL^2}$
K_2	β_x/h
K_3	$4\rho(1 - \rho) + (\beta_x/h)^2$
K_4	$\pi^2 EAh/2Z_{xmin}$
L	length of beam
M_c	elastic critical moment
r_y	minor axis radius of gyration
r_T	radius of gyration about the section minor axis of the compression flange plus one sixth of the web
S	shear centre position
T	flange thickness
T_C, T_T	thicknesses of compression and tension flanges
T_e	effective flange thickness
T_L	thickness of lip
t	web thickness
x, y	major and minor principal axes
y_o	coordinate of shear centre
$\bar{y}$	distance from compression flange centre line to centroid
Z_{xmin}	minimum elastic section modulus
β_x	monosymmetry section property
γ_c	$M_c L/\sqrt{EI_y GJ}$
δ	$(\beta_x/L)\sqrt{EI_y/GJ}$
μ	A_{FC}/A
ρ	I_{yC}/I_y

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