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Buckling and Bracing of Cantilevers

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BUCKLING AND BRACING OF CANTILEVERS

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Synopsis

The elastic flexural-torsional buckling of cantilever I-beams is investigated. The cantilevers have rigid translational and/or rotational restraints at discrete points. The effect of the beam parameter K, the load height, the location of restraint positions along the beam, and the level at which the restraint acts have been studied using the finite integral method. Results are presented graphically as ratios of the increased critical load of the partially braced beam and the corresponding critical load of the unbraced beam. The beam load cases considered are concentrated loads and uniformly distributed loads. The effectiveness of the restraint locations and the types of restraint are investigated. Experiments conducted using extruded high strength aluminium I-section are reported. Test results obtained are in reasonable agreement with the theoretical predictions.

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1. INTRODUCTION

Questions often arising in design are related to the effectiveness of the bracing system used to increase the buckling capacities of the members. While most structural beams may be braced in different ways, most arrangements can be represented by an idealised system consisting of an elastic translational restraint acting at distance $\bar{b}$ above the shear centre of the beam cross section and an elastic rotational restraint (see Figure 1).

A number of studies (3, 4, 6-10) have been made on the effectiveness of the various types of restraint and restraint stiffnesses. Mutton and Trahair (8) investigated stiffness requirements for simply supported beams and columns with mid-span rotational and translational restraint which acted either at the top flange or the shear centre. They calculated the minimum restraint stiffnesses required to cause the member to buckle in its second mode. Kitipornchai and Richter (6, 7, 10) studied the effectivenesses of restraint location along the simply supported beam, and the level of translational restraint within the beam cross-section in relation to the height of application of load. The loading cases considered are end moments, point loads and uniformly distributed load. Optimum braced locations for the various loading are given. They found that translational restraint placed at the tension (bottom) flange level may be effective for long shallow beams for which warping effects are of less importance than those of uniform torsion. This conclusion is confirmed by tests carried out by Roeder and Assadi (5, 11).

Fewer studies have been made on the bracing of cantilever beams. Nethercot (9) studied the effective length factors of cantilevers having two restraint conditions at the end and under concentrated end load and distributed load. He considered full restraint and translational restraint

2. BUCKLING OF PARTIALLY BRACED CANTILEVERS

A cantilever I-beam under general loading and with an intermediate restraint is shown in Figure 2. Loads considered include a concentrated load, P, acting at a distance, a, from the fixed end support and at a level $\bar{a}$ above the shear centre, a uniformly distributed load, w, acting at a level $\bar{w}$ above the shear centre line, and a point moment acting at a distance d, from the fixed end support. The translational restraint is applied to the beam at a distance, b, from the fixed end support and at a level $\bar{b}$ above the shear centre, and provides a force, H_A.

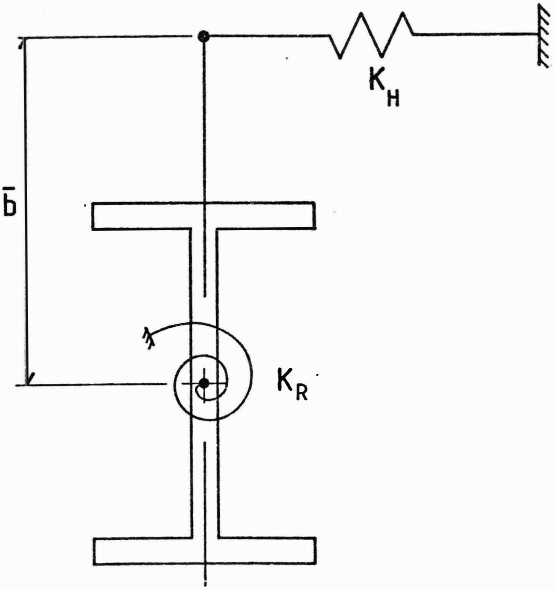


Figure 1 : Idealised translational and rotational restraint

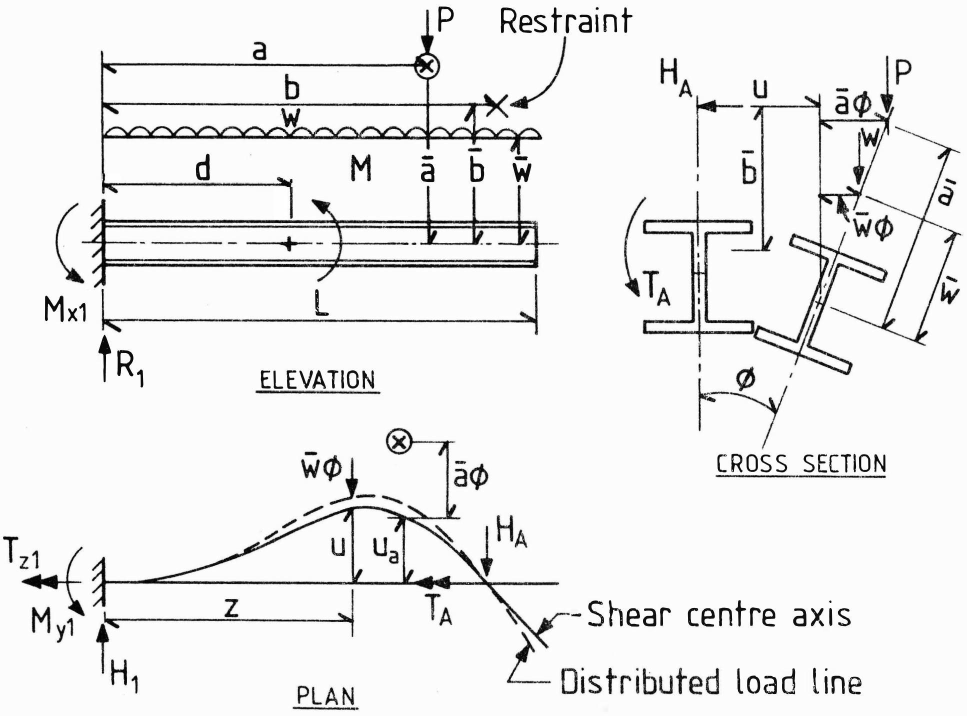


Figure 2 : Cantilever I-beam under general loading and a discrete intermediate restraint

The differential equations of minor axis bending and torsion are

$$M_y = EI_y \frac{d^2u}{dz^2} \qquad (1)$$

$$T_z = GJ \frac{d\phi}{dz} - EI_\omega \frac{d^3\phi}{dz^3} \qquad (2)$$

in which EI_y, GJ and EI_ω are the minor axis bending rigidity, the torsional rigidity and the warping rigidity respectively.

The vertical and horizontal forces R_1 and H_1, the major and minor axis fixed end moments M_{x1} and M_{y1} and the torque reaction T_{z1} at the fixed end support are

$$R_1 = P + wL \qquad (3)$$

$$H_1 = H_A \qquad (4)$$

$$M_{x1} = Pa + \frac{wL^2}{2} - M \qquad (5)$$

at the shear centre, and gave approximate expressions for buckling loads. In this paper, the effectiveness of translational and/or rotational restraints on cantilever beams is examined. The load cases considered are concentrated end load and uniformly distributed load. Tests on high strength extruded aluminium cantilever I-beams have been conducted to validate the theoretical investigation.

$$M_{y_1} = H_A b \tag{6}$$

$$T_{z_1} = P\left(u_a + \bar{a}\phi_a\right) + w\left(\int_0^L udz + \bar{w}\int_0^L \phi dz\right) + T_A + H_A\,\bar{b} \tag{7}$$

where H_A and T_A are the horizontal and torque reactions at the restraint.

The major and minor axis bending moment distribution are

$$M_x = R_1 z = M_{x_1} - P <z - a> - \frac{wz^2}{2} - M <z - d> \tag{8}$$

and

$$M_y = M_{y_1} = M_x\phi - H_1 z + H_A <z - b> \tag{9}$$

where the expressions inside Macauly brackets < > is taken zero if its value is negative.

The axial torque distribution is

$$T_z = T_{z_1} + M_x \frac{du}{dz} - R_1 u + P\,(u - u_a - \bar{a}\phi_a) <z - a>$$

$$+ w\,(uz - \int_0^z udz - \bar{w}\int_0^z \phi\, dz) - T_A <z - b> - H_A\bar{b} <z - b> \tag{10}$$

Combining Equations (1), (2) and (9) the governing differential equations of minor axis bending and torsion become

$$EI_y \frac{d^2u}{dz^2} = M_{y_1} - M_x\,\phi - H_1 z + H_A <z - b> \tag{11}$$

and

$$GJ\,\frac{d\phi}{dz} - EI_{\omega}\,\frac{d^2\phi}{dz^3} = T_1 + M_x\,\frac{du}{dz} - R_1 u + P\,(u - u_a - a\phi_a)\,\langle z - a\rangle$$

$$+ \; w\,(uz - \int_0^z u\,dz - \bar{w}\int_0^z \phi\,dz)$$

$$- \; T_A\,\langle z - b\rangle - H_A\,\bar{b}\,\langle z - b\rangle \qquad (12)$$

The boundary conditions for Equations (11) and (12) are: at the fixed end,

$$z = 0; \quad u = \phi = \frac{du}{dz} = \frac{d\phi}{dz} = 0 \qquad (13)$$

at the free end,

$$z = L; \quad \frac{d^2u}{dz^2} = 0 \qquad (14)$$

and at the restraint,

$$z = b; \quad u = \frac{H_A}{K_H} - \bar{b}\phi_b \qquad (15)$$

and

$$\phi = \frac{T_A}{K_R} \qquad (16)$$

where K_H and K_R are the translational and rotational stiffnesses of the restraint.

The differential equations (Equations 11 and 12) together with the boundary conditions (Equations 13 to 16) may be solved for the elastic critical load factor using the method of finite integrals (2). A computer program has been prepared and the solution technique is similar to that described in previous papers (1, 7, 8).

3. NUMERICAL RESULTS

3.1 General

In practice, the common loadings for a cantilever beam are concentrated load and uniformly distributed load. It is usual for concentrated load points to be also points of restraint and hence the height of application of load does not affect beam buckling capacity. In a crane runway beam, with discrete restraints along its length the load may act at any point. It is not obvious where the optimum restraint locations should be, or whether translational restraint at the level of top flange is fully effective. The uniformly distributed load case is common in roof structures where the load may arise from wind or live loading. The load may act at the top flange, shear centre or bottom flange or at any level. The translational restraint and/or rotational restraint may be applied at any location along the beam.

The effectiveness of a restraint may be measured by, c, the ratio of the buckling load of the cantilever with the restraint arrangement and the buckling load of a similarly unbraced cantilever. Thus the values of c give an indication of the improvement in stability provided by the restraint.

3.2 Position of a Full Restraint Along Cantilever

The influence of the position of a full restraint is investigated. A full restraint is assumed to be capable of preventing both lateral deflection and twisting of the braced cross section. The critical load ratio, c, for values of the beam parameters K = 0.1 to 3.0 are shown in Figures 3 and 4 for cantilevers with a concentrated tip load and uniformly distributed load respectively. The loads are applied at top flange, shear centre and bottom flange. It can be seen that the increases in the buckling

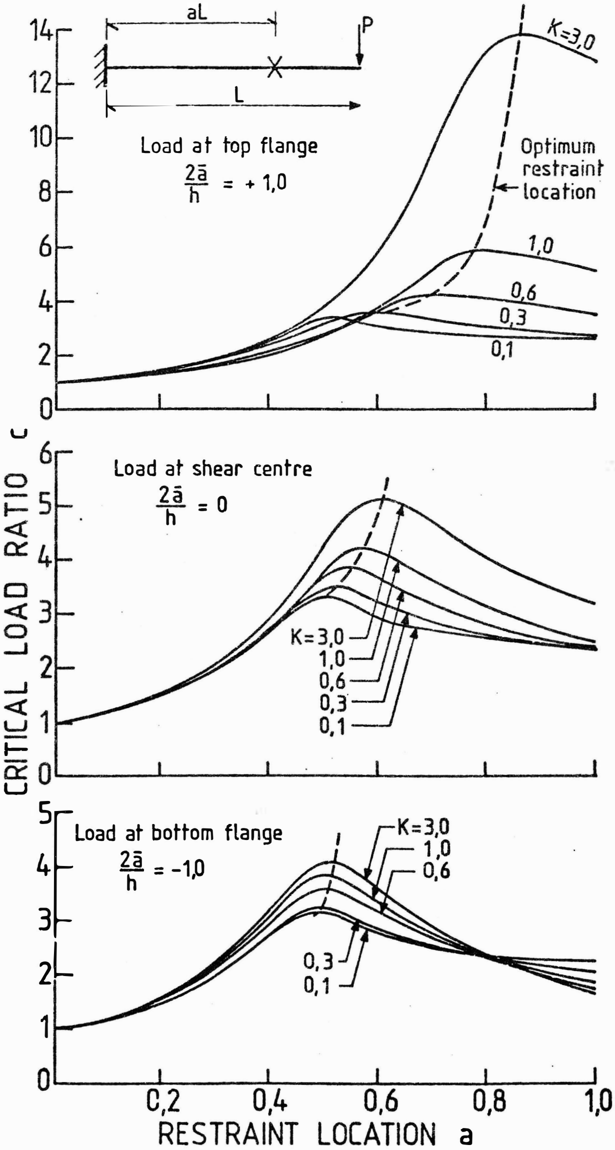


Figure 3 : Buckling load for cantilevers with a concentrated tip load and a full restraint

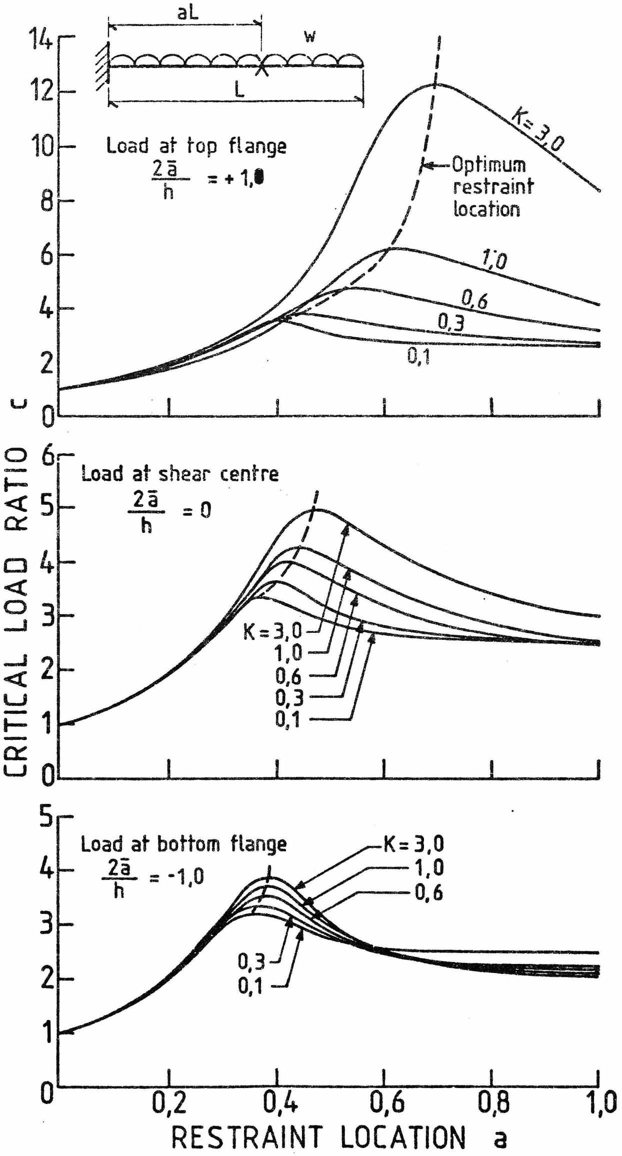


Figure 4 : Buckling load for cantilevers with uniformly distributed load and a full restraint

load are greatest for large values of the beam parameter, K, and more so for top flange loading. The maximum value of c that may be achieved range from 3 for small values of K to 14 for large values of K. However, it is likely that in-plane bending or inelastic buckling will govern the design for cantilevers with large values of K.

The results show that for small values of K, the optimum restraint location is near mid-span for a concentrated tip load and near 0.4 of the length from the fixed end for uniformly distributed load. For higher values of K, the optimum restraint locations move towards the cantilever tip as the height of load application moves toward the top flange. For a concentrated tip load, the optimum location varies between 0.5 and 0.8 and for a uniformly distributed load, it varies between 0.4 and 0.7.

3.3 Effects of Translational and/or Rotational Restraint

The effectiveness of the level of translational restraint is compared with that of rotational restraint and of full restraint for top flange ($2\bar{a}/h = 1$), shear centre ($2\bar{a}/h = 0$) and bottom flange loading ($2\bar{a}/h = -1$) for values of K = 0.6 and 3.0. The results are shown in Figures 5 and 6 for concentrated tip load and in Figures 7 and 8 for uniformly distributed load.

The various types of restraints have different effects on the critical buckling load, depending on the level of load application ($2\bar{a}/h$). In all cases it can be seen that full restraint is by far the best for all K values. The optimum positions have been discussed in the previous section. If full restraint cannot be achieved, rotational restraint is the next best as can be seen from Figures 5 to 8, particularly if the restraint is placed within 0.4 L from the fixed support.

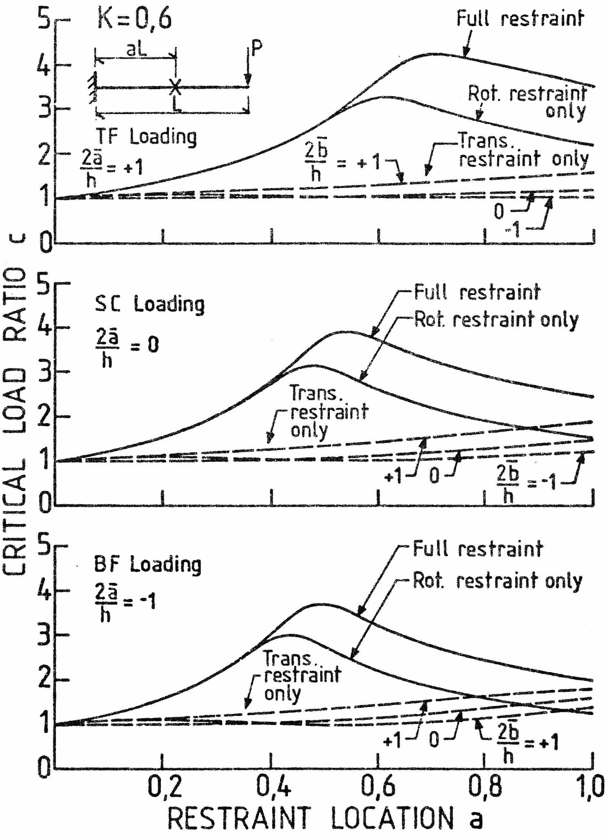


Figure 5 : Comparison of restraint types for cantilevers with a concentrated tip load, K = 0.1

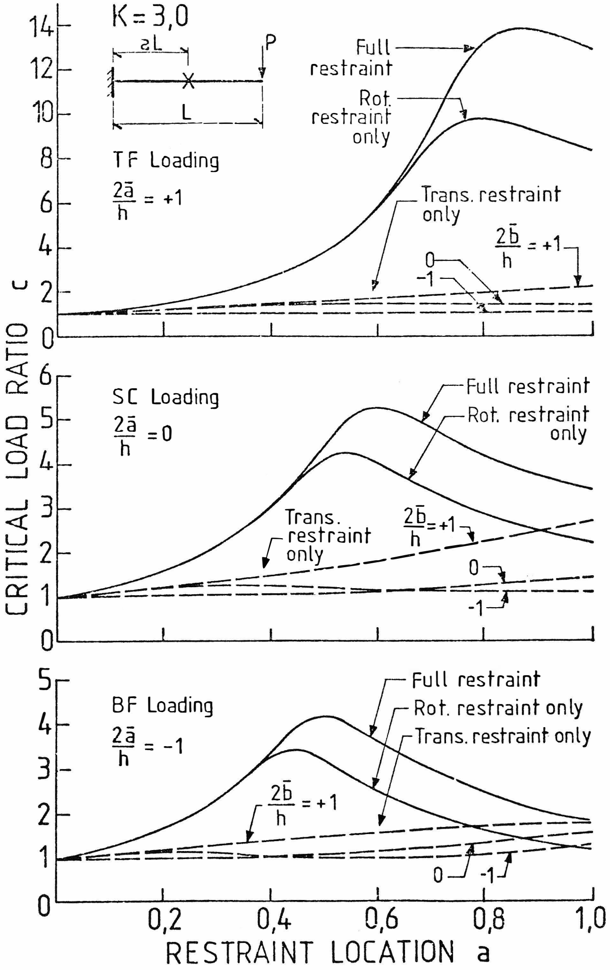


Figure 6 : Comparison of restraint types for cantilevers with a concentrated tip load, K = 3.0

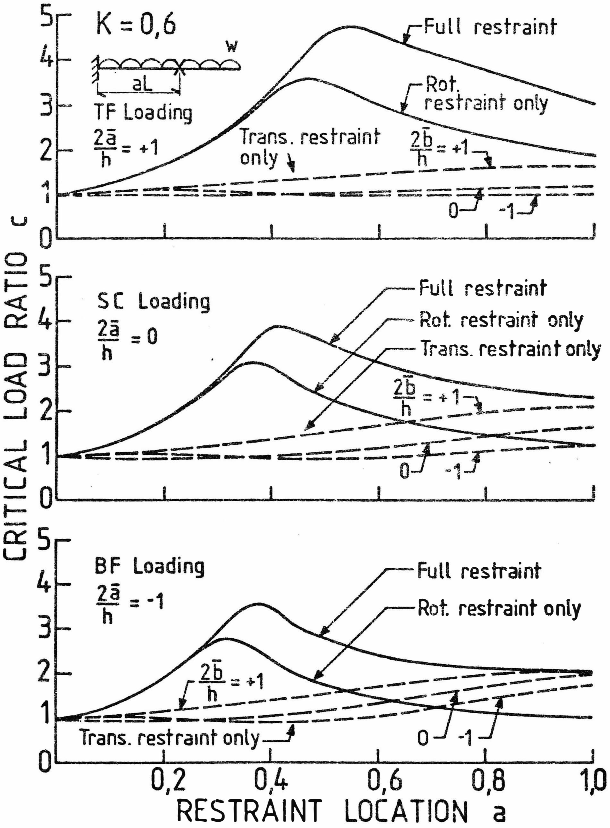


Figure 7 : Comparison of restraint types for cantilevers with a uniformly distributed load, K = 0.1

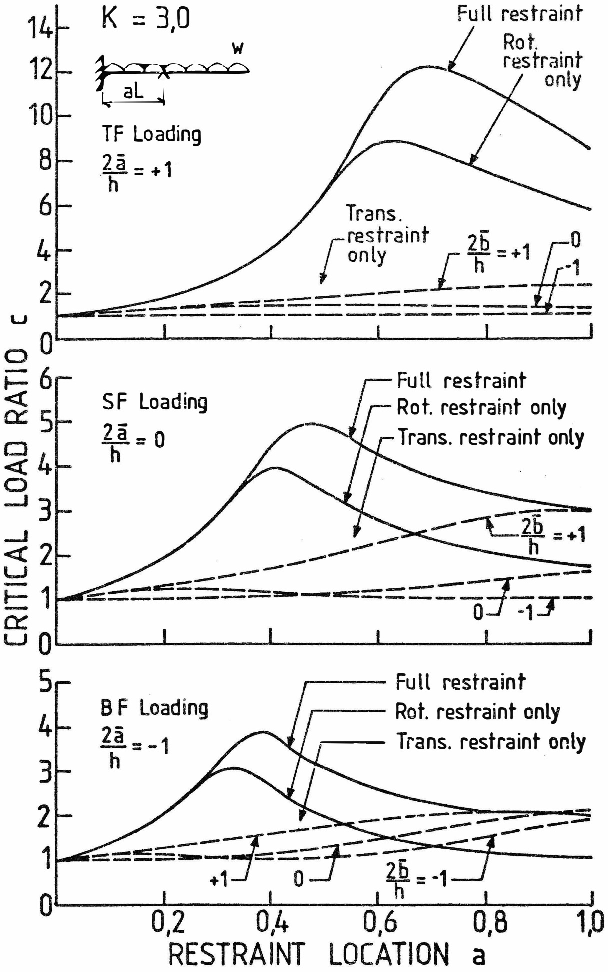


Figure 8 : Comparison of restraint types for cantilevers with a uniformly distributed load, K = 3.0

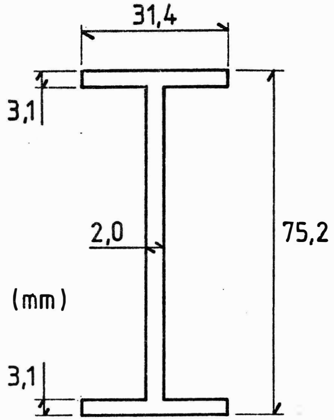
For translational restraints only the critical buckling load ratios increase slowly as the restraint moves towards the free end, irrespective of the level of the restraint. Varying the value of K has only little effect on the maximum value of c for top flange and bottom flange loadings. However, for shear centre loading and with top flange restraint, the effect of increasing K shows a marked improvement in the value of c.

It is recommended that translational restraints be placed as close as possible to the cantilever tip. The effectiveness increases as the level of application of load moves towards the bottom flanges. In all cases if translational restraint alone is used, it should be placed near the top flange and as close as possible to the end of the cantilever. Restraints placed less than 0.4 L from the fixed end are practically useless and therefore are wasted.

4. EXPERIMENTAL INVESTIGATIONS

4.1 General

A series of cantilever I-beams with concentrated loads was tested to verify the theoretical results obtained using the finite integral method of solution. The beams were high strength aluminium extrusions, similar to those used previously in the experimental investigation of elastic simply supported beams (1, 7, 10). The beam cross sectional dimensions and the measured properties are given in Figure 9. The experimental programme consisted of testing five different lengths of cantilevers ranging from 1.0 3.0 metres with varying restraint conditions and locations. All beams were loaded at the level of the top flange by means of a loading yoke (see Figures 10 and 11). The load was applied as close to the tip as practicable. The restraint conditions were full translational restraint at the level of top flange (T), shear centre (S), bottom flange (B) or full restraint (F) against both translational and rotational deformation.



$EI_y = 1040\,\mathrm{Nm}^2$

$GJ = 21{,}15\,\mathrm{Nm}^2$

$EI_\omega = 1{,}347\,\mathrm{Nm}^4$

$\text{Weight} = 0{,}87\,\mathrm{kg/m}$

Figure 9 : Test beam dimensions and cross sectional properties

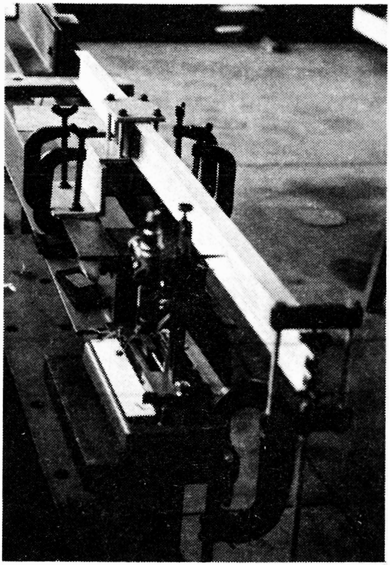


Figure 10: General experimental set-up

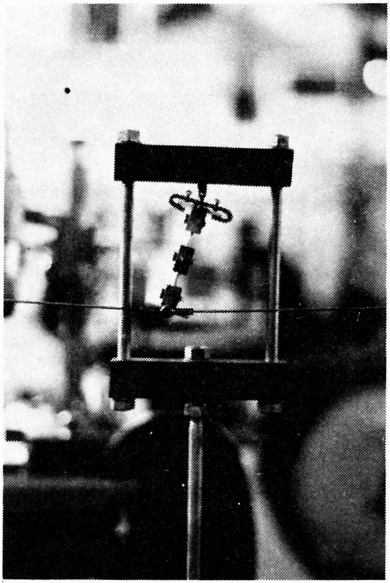


Figure 11: Restraint and loading device

4.2 Equipment and Procedure

The test apparatus and procedure closely followed that used in the previous investigations (1, 7, 10). Figure 10 shows general arrangement of the test set-up. The fixed end support arrangement was similar to that used by Anderson and Trahair (1). It allowed the beam to be moved through the support for variable cantilever length. Once in position, four bolts were used to clamp the 20 mm thick plate, overlying the top flange, down to the base support. Steel blocks cut to shape were fitted on either side of the web between the flanges to avoid web crippling due to the clamp forces. Both lateral displacement and twist were prevented, but this arrangement did not fully restrain warping. This had the effect of reducing the critical buckling load by the order of 1 to 3%.

The restraint device consisted of a brass socket attached to a single wire between two adjustable supports (see Figure 11). The socket slid onto a pin attached to the web. This type of restraint arrangement prevented lateral deflection whilst not providing any twisting restraint. Full restraint was achieved by using two such wires at the top and bottom flange levels.

The test loads were applied to the top flange and shear centre through a loading yoke using a bucket progressively filled with lead shot. Lateral deflections of the beam were measured at the level of the shear centre at a location where maximum lateral deflections were anticipated. These were 0.625, 0.375 and 0.7 of the length from the fixed end for b/L = 0.25, 0.75 and 1.0 respectively. A micrometer connected into an electrical circuit allowed a very sensitive lateral deflection readings to be obtained.

Tests were carried out on the longest cantilevers first in order to prevent any effect on subsequent beams due to damage to the beam near the support. Each experiment was conducted several times in order to ensure repeatability of results and variation was less than 3% in all cases. The modified Southwell plot was used to obtain the experimental critical loads from the load and lateral deflection measurements.

4.3 Results

The experimental results are summarised in Table 1. Also shown in the Table are the theoretical results from using the finite integral methods (1, 7, 10). The predictions have allowed for self weight of the beams and also for the fact that the major axis flexural rigidities EI_x is not infinitely larger than the other rigidities (12). It was found the effect of neglecting both beam self weight together with major axis curvature is for one to approximately cancel the other. The experimental results are in reasonable agreement with the theoretical predictions. The results confirm the theoretical findings that translational restraint at the top flange level is more effective than at other levels, but is not as effective as rotational or full restraint.

TABLE 1: Comparison of Results

Beam	Length (m)	K	*Position of Load	Position of Restraint b/L	*Type of Restraint	Buckling Load (N)		Percentage Difference
						Experiments	Theory	
1	3.0	0.27	TF	1.0	F	181.5	188.3	- 3.6
2	3.0	0.27	TF	1.0	T	104.0	112.7	- 7.7
3	3.0	0.27	TF	1.0	S	89.8	97.1	- 7.5
4	3.0	0.27	TF	1.0	B	80.4	85.0	- 5.4
5	3.0	0.27	SC	0.5	F	297.0	284.8	+ 4.3
6	3.0	0.27	SC	0.5	T	103.9	100.9	+ 3.0
7	3.0	0.27	SC	0.5	B	76.6	74.4	+ 3.0
8	2.5	0.32	SC	0.5	F	439.4	436.4	+ 0.7
9	2.5	0.32	SC	0.5	T	158.6	158.6	0
10	2.5	0.32	SC	0.5	S	130.9	124.6	+ 5.1
11	2.5	0.32	SC	0.5	B	119.3	115.7	+ 3.1
12	2.5	0.32	SC	0.25	F	195.7	206.3	- 5.1
13	2.5	0.32	SC	0.75	F	324.3	337.9	- 3.8
14	2.0	0.40	SC	0.5	F	804.0	738.4	+ 8.9
15	2.0	0.40	SC	0.5	T	277.0	275.4	+ 0.6
16	2.0	0.40	SC	0.5	B	196.5	197.0	- 0.3
17	1.5	0.54	TF	1.0	F	803.2	902.5	- 11.0
18	1.5	0.54	TF	1.0	T	382.0	424.8	- 10.1
19	1.5	0.54	TF	1.0	S	296.0	326.7	- 9.4
20	1.5	0.54	TF	1.0	B	294.0	282.2	+ 4.2
21	1.0	0.79	TF	1.0	T	750.5	801.8	- 6.4
22	1.0	0.79	TF	1.0	S	573.9	565.4	+ 1.5
23	1.0	0.79	TF	1.0	B	519.9	520.2	0

* TF = Load at top flange ($2\bar{a}/h = +1$);

SC = Load at shear centre ($2\bar{a}/h = 0$);

F = Full restraint;

T,S,B = Translational restraint at level of top flange ($2\bar{b}/h = +1$), shear centre ($2\bar{b}/h = 0$) and bottom flange ($2\bar{b}/h = 1$) respectively.

3. CONCLUSIONS

The elastic buckling of cantilever I-beams under general loading and with a variety of restraint conditions is investigated. The governing differential equations together with appropriate boundary conditions are derived. The elastic buckling loads are obtained by solving the differential equations numerically using the method of finite integrals.

The influence of restraint location along the beam, the height of application of load and the types of restraint are studied for the several values of beam parameter, K. The load cases considered are concentrated tip loads and uniformly distributed load. It is found the optimum location of a full restraint for most cases varies between 0.4 to 0.7 from the fixed end support. For beams with a simple translational restraint, the restraint is best placed near the top (tension) flange level. However, this arrangement is not as effective as a rotational restraint or a full restraint.

Experiments on extruded high strength aluminium cantilever I-beams are reported. Eleven beams were tested with lengths varying from 1.0 to 3.0 m. The cantilevers were loaded with concentrated tip load. Restraints placed along the test beams were either a translational restraint at the level of top flange, shear centre, bottom flange or a full restraint. Experimental buckling loads were generally lower than the theoretical predictions. However, results confirm the conclusions from theoretical studies on the order of effectiveness of the different types of restraint.

6. ACKNOWLEDGEMENTS

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APPENDIX B - NOMENCLATURE

Symbol	Meaning
a	location of concentrated load
$\bar{a}$	height of point of application of load above shear centre
b	location of restraint along the beam
$\bar{b}$	height of translational restraint above shear centre
c	ratio of critical load of restrained beam and similar unbraced beam
d	location of point moment
E	Young's modulus of elasticity
G	shear modulus of elasticity
h	distance between flange centroids
H_1	horizontal reaction at fixed end support
H_A	horizontal reaction at restraint
I_x, I_y	major and minor second moment of area
I_ω	warping section constant
J	torsion section constant
K	beam parameter = $\sqrt{\pi^2 E I_\omega / G J L^2}$
K_H	lateral restraint stiffness
K_R	rotational restraint stiffness
L	length of beam
M	applied point moment
M_x	major axis bending moment
M_{x1}	major axis moment reaction at fixed end support
M_y	minor axis bending moment
M_{y1}	minor axis moment reaction at fixed end support
P	concentrated load
R_1	vertical reaction at fixed end support
T_A	torque reaction at restraint
T_z	torque distribution along the beam

Symbol	Meaning
T_{z_1}	torque reaction at fixed end support
u	lateral deflection of shear centre
u_a	lateral deflection at distance $z = a$
w	uniformly distributed load
$\bar{w}$	height of distributed load above shear centre
z	centroidal axis with origin at fixed end support
ϕ	angle of twist
ϕ_a	angle of twist at $z = a$
ϕ_b	angle of twist at $z = b$

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4	Buckling Properties of Monosymmetric I-Beams	KITIPORNCHAI, S. & TRAHAIR, N.S.	May, 1979
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