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# Nonuniform Alongshore Currents and Sediment Transport — A One Dimensional Approach

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# NONUNIFORM ALONGSHORE CURRENTS AND SEDIMENT TRANSPORT
# A ONE DIMENSIONAL APPROACH

by

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## Synopsis

*The theory of nonuniform alongshore currents is reviewed for the one dimensional case neglecting convective acceleration terms and assuming variations in surf zone width are negligible. An equation for the alongshore current velocity which takes account of the relative effects of the breaker angle and an alongshore wave height gradient is obtained. The magnitudes of the two constants* $K_\theta$ *and* $K_{\Delta H}$ *in this equation are found to depend upon both the particular representative velocity selected and the amount of lateral mixing within the surf zone. Recent field and laboratory data are used to obtain the magnitude of lateral mixing factors for various representative surf zone velocities and to estimate the average value of the friction factor* $f_{wc}$ *as well as of the ratio* $(\tan \alpha)/f_{wc}$.

*The CERC-Scripps formula for alongshore transport is modified to include the effect of an alongshore wave height gradient as well as the effect of breaker angle. The resulting equation has two constants, a scale factor* $K_s$ *for the alongshore transport rate and the constant* $K_{\Delta H}$ *which determines the proper balance between the two driving force terms. The magnitude of* $K_s$ *is a function of breaker type. Potential causes of error in computing the alongshore sediment transport from this one dimensional "black box" type of equation are discussed.*

CONTENTS

# 1. INTRODUCTION

Mathematical models of changes in the alignment of sedimentary coasts and beaches are being increasingly used today. The reliability of such models depends upon a number of factors among which one of the most significant is the particular formula adopted for predicting the sediment transport capacity of the alongshore current system causing the realignment of the beach.

In a recent paper Ozasa and Brampton (1980) describe a simple one-dimensional mathematical model for computing the changes in plan shape of a beach backed by a sea wall. Their method includes an equation for predicting the alongshore transport of sediment under conditions where an alongshore gradient of breaking wave height modifies the transport rate caused by the wave crests breaking at an angle to the shoreline. In deriving their equation Ozasa and Brampton use an equation for the velocity of the nonuniform alongshore current derived by Bakker (1971) which was also used by Motyka and Willis (1975) in an earlier formulation of a similar alongshore sediment transport equation.

The influence of an alongshore gradient of wave height upon the magnitude of alongshore currents has been recognised for some time. It is inherent in the work of Bowen (1967 and 1969) which explained the mechanism by which rip currents are formed and has been considered by Komar (1971, 1972 and 1975) with regard to rip current systems associated with large scale "beach cusps". The author has previously shown that in the case of an offshore island or offshore breakwater it is possible for waves to generate current systems which are completely dominated by the driving force of an alongshore gradient of wave set-up caused by an alongshore gradient of breaking wave height which itself is the result of wave diffraction behind the offshore obstacle (Gourlay 1975 and 1977). In the course of these latter studies an equation for sediment transport by a nonuniform alongshore current was derived. The form of this equation is similar to those of both Ozasa and Brampton and Motyka and Willis, but its derivation from the radiation stress theory of alongshore currents (Longuet-Higgins 1970, 1972a) is more straightforward and it avoids some questionable assumptions assocated with Bakker's derivation. Recent work by Kraus (1981) uses a similar approach to that adopted by the author.

## 2. NONUNIFORM ALONGSHORE CURRENTS

### 2.1 Basic Equation and Assumptions

The complete derivation of the basic equations for both uniform and nonuniform alongshore currents is reviewed by the author elsewhere (Gourlay 1978); its major features are given here. For the case of a steady nonuniform alongshore current, caused by waves with a constant breaker angle and an alongshore gradient of breaker height, breaking on a beach with straight parallel bottom contours, the basic equation of motion involves a balance between the driving forces caused by the waves and the forces resisting the alongshore flow. Hence,

$$\tau_\theta + \tau_{\Delta H} + \tau_b + \tau_L = 0 \qquad (1)$$

where $\tau_\theta$ is the breaker angle driving force term,

$\tau_{\Delta H}$ is the alongshore wave height gradient driving force term,

$\tau_b$ is the bottom friction term,

$\tau_L$ is the lateral friction term.

The terms $\tau_\theta$, $\tau_{\Delta H}$, $\tau_b$ and $\tau_L$ in Equation (1) have the dimensions of force per unit area and hence have the appearance of shear stresses applied to a horizontal surface.

The driving force term $\tau_\theta$ is evaluated using the radiation stress theory of Longuet-Higgins and Stewart (1964) as outlined in Longuet-Higgins (1972b) (Appendix C), while the driving force term $\tau_{\Delta H}$ is obtained from an analysis by Komar (1975) which is consistent with the derivation of $\tau_\theta$.

The two resistance terms $\tau_b$ and $\tau_L$ play different roles in the dynamics of the surf zone. The bottom friction largely determines the order of magnitude of the alongshore current velocity while the lateral friction, which results from turbulent mixing processes diffusing alongshore momentum laterally across the surf zone, determines the shape of the horizontal velocity profile within the surf zone.

In order to obtain a reasonably simple analytical formula for the nonuniform alongshore current velocity certain simplifications have to be made. In the first instance the lateral friction term will be omitted and its effects upon the final equation allowed for by an arbitrary mixing coefficient. Shallow water conditions are assumed to prevail in the surf zone with relatively flat plane beaches and so within the surf zone it is assumed that

$$H = \gamma \; (\bar{\eta} + h) \qquad (2)$$

where H is the wave height; h is the still water depth and $\bar{\eta}$ is the mean water level displacement or wave set-up. The breaker index $\gamma$ is assumed to be constant for any given situation. Furthermore it will be assumed that the influence of the alongshore breaker height gradient is not so large that convective acceleration terms and alongshore variations in surf zone width have to be considered. The latter assumption implies that the alongshore wave height gradient is small in comparison with the onshore wave height gradient through the surf zone, i.e. $\partial H/\partial y \ll \partial H/\partial x$. This is consistent with the assumptions made by Bakker. It may not be applicable to situations such as that behind an offshore breakwater where convective acceleration terms can be very much greater than the bottom friction term (Gourlay 1977, 1982).

## 2.2 Breaker Angle Driving Force Term

Following Longuet-Higgins (1972b), it is assumed that outside the surf zone dissipation will be negligible. Consequently, the energy flux towards the shore is constant and there is no driving force for any wave-generated current. Within the surf zone, if there is no reflection, all the wave energy is dissipated and the energy flux at the shore is zero.

Thus outside the surf zone

$$F_x = E\, C_G \cos\theta = \text{constant}$$

$$= E_o\, C_{Go} \cos\theta_o \qquad (3)$$

where $F_x$ is the energy flux or wave power per unit length of coastline, E is the energy density (= $\rho g\ H^2/8$), $C_G$ is the group velocity of the wave train, and $\theta$ is the angle a wave crest makes with the shoreline (Figure 1). Inside the surf zone

$$\frac{\partial F_x}{\partial x} = - D \qquad (4)$$

where D is the rate of energy dissipation per unit time and unit horizontal area. At the shoreline $F_x = 0$.

Considering the momentum flux, the flux of y momentum across a line x = constant, parallel to the shoreline, is obtained from the radiation stress theory (Appendix C) as

$$S_{xy} = n\ E \cos \theta \sin \theta$$

$$= F_x \frac{\sin C}{C} \qquad (5)$$

where $(\sin \theta)/C$ is constant according to Snell's law for wave refraction.

Outside the surf zone, $F_x$ is constant and hence

$$S_{xy} = \text{constant} = T_\theta \qquad (6)$$

while at the shoreline $S_{xy} = 0$ since $F_x = 0$ there.

The term $T_\theta$ thus represents the total alongshore thrust exerted by the waves on the water in the surf zone. It is also the total force available for moving sand or other sediments parallel to the shore over a rough bottom. $T_\theta$ can be expressed in terms of deep water conditions using Equations (3) and (5) as follows:

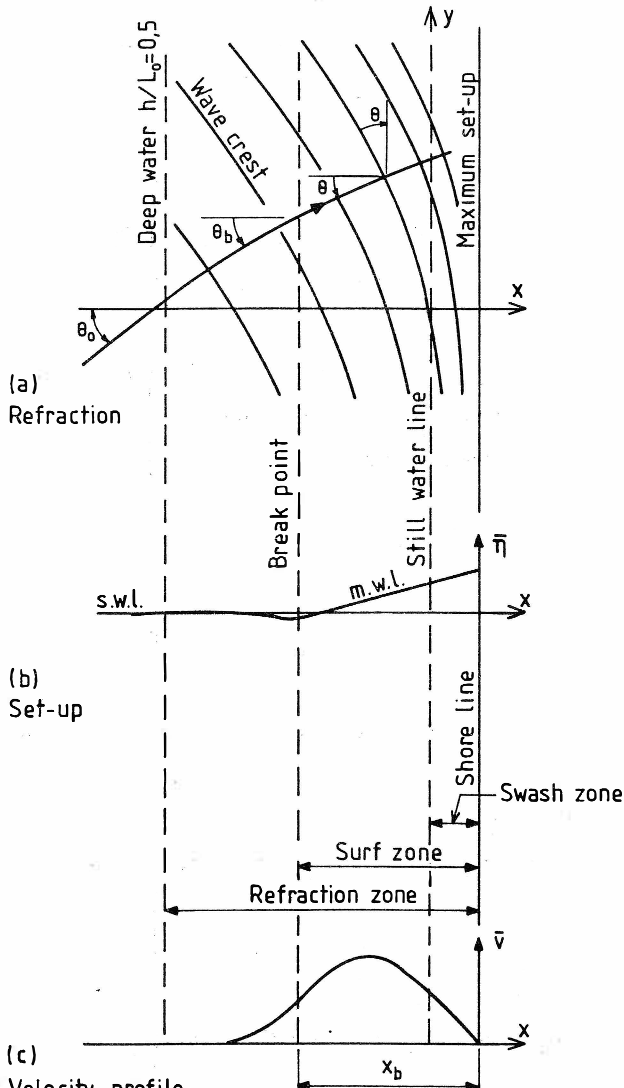


Figure 1 : Schematic sketch of conditions generating a uniform alongshore current

$$T_\theta = \frac{1}{4} E_o \sin 2\theta_o \qquad (7)$$

which for a given energy density $E_o$ gives a maximum total alongshore thrust when $\theta_o = 45^o$.

Considering the control volume shown in Figure 2, the net horizontal force per unit area parallel to the shore resulting from waves approaching with angle $\theta$ is

$$\tau_\theta = -\frac{\partial T_\theta}{\partial x} = -\frac{\partial S_{xy}}{\partial x} \qquad (8a)$$

$$= -\frac{\partial F_x}{\partial x}\frac{\sin\theta}{C} \qquad (8b)$$

$$= D\frac{\sin\theta}{C} \qquad (8c)$$

where $(\sin\theta)/C$ is a constant.

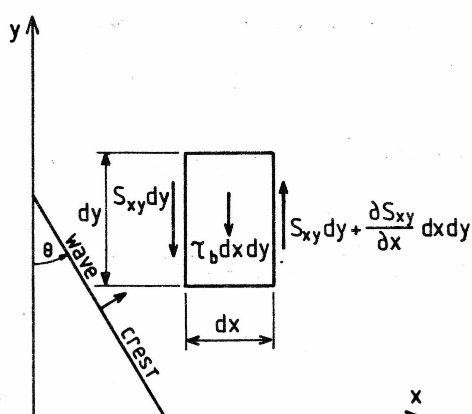


Figure 2 : Control volume for uniform alongshore current

Outside the surf zone where the rate of dissipation $D$ is zero, $\tau_\theta$ is also zero. Hence as stated by Longuet-Higgins (1972a) "*the driving force is directly proportional to the energy dissipation; if there were no dissipation at all, there would be no current*".

Within the surf zone, where shallow water conditions are assumed and $C_G \to C = \sqrt{g(\bar{\eta}+h)}$, Equation (3) becomes

$$F_x = \frac{1}{8} \rho g^{3/2} \gamma^2 (\bar{\eta} + h)^{5/2} \cos\theta \tag{9}$$

and substituting in Equation (8b) the following equation for the breaker angle driving force term is obtained for a plane beach of slope tan $\alpha$, i.e.

$$\tau_\theta = - \frac{5}{16} \gamma^2 \frac{\tan\alpha}{1+3\gamma^2/8} \rho g^{3/2} (\bar{\eta}+h)^{3/2} \frac{\sin\theta}{C} \cos\theta \tag{10a}$$

where the term $1 + 3\gamma^2/8$ allows for the effect of wave set-up in moving the mean water line landward from the still water line (Bowen, Inman and Simmons 1968).

If $(\sin\theta)/C$ is replaced by $(\sin\theta_b)/C_b$ Equation (10a) becomes

$$\tau_\theta = - \frac{5}{16} \gamma^2 \frac{\tan\alpha}{1+3\gamma^2/8} \rho g \frac{(\bar{\eta}+h)^{3/2}}{(\bar{\eta}+h)_b^{1/2}} \sin\theta_b \cos\theta \tag{10b}$$

2.3 Alongshore Breaker Height Gradient Driving Force Term

The influence of the alongshore gradient of breaker height upon the system is exerted in two ways. Firstly, the alongshore gradient of wave height creates an alongshore gradient of wave set-up, which means that there is an alongshore thrust produced by the out of balance hydrostatic forces. Secondly, the alongshore gradient of wave height causes an alongshore gradient of the radiation stress component $S_{nyx}$ exerted on the plane normal to the beach. Hence from Figure 3

$$\tau_{\Delta H} = \rho g \ (\bar{\eta}+h) \ \frac{\partial(\bar{\eta}+h)}{\partial y} + \frac{\partial S_{nyx}}{\partial y}$$

$$= \frac{\rho g \ (\bar{\eta}+h)}{\gamma} \ \frac{\partial H}{\partial y} + \frac{\partial S_{nyx}}{\partial y} \qquad (11)$$

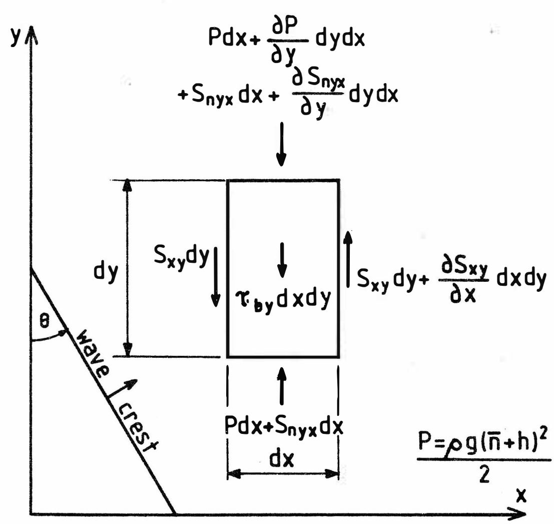


Figure 3 : Control volume for nonuniform alongshore current

Now, assuming shallow water conditions (n = 1), radiation stress theory gives

$$\frac{\partial S_{nyx}}{\partial y} = \frac{\partial E}{\partial y} \left(\frac{1}{2} + \sin^2\theta\right) \tag{12}$$

Substituting for E, differentiating and putting $H = \gamma\ (\bar{\eta}+h)$, Equation (12) becomes

$$\frac{\partial S_{nyx}}{\partial y} = \frac{\rho g\ \gamma\ (\bar{\eta}+h)}{4}\ \frac{\partial H}{\partial y} \left(\frac{1}{2} + \sin^2\theta\right) \tag{13}$$

Substitution for $(\partial S_{nyx})/\partial y$ in Equation (11) using Equation (13) gives the following equation for the alongshore wave height gradient driving force term.

$$\tau_{\Delta H} = \frac{\rho g\ (\bar{\eta}+h)}{\gamma}\ \frac{\partial H}{\partial y} \left(1 + \frac{\gamma^2}{8} + \frac{\gamma^2}{4} \sin^2\theta\right) \tag{14}$$

## 2.4 Bottom Friction Term

For rough turbulent flow conditions occurring in the surf zone it is reasonable to assume that

$$\vec{\tau}_b = \frac{1}{2} f_{wc}\ \rho\ |\vec{u}|\vec{u} \tag{15}$$

where $\vec{u}$ is the instantaneous horizontal particle velocity just above the oscillatory boundary layer and $f_{wc}$ is the friction factor for the combined wave motion and current.

It has been customary to assume that the alongshore current velocity $\bar{v}$ is small in comparison with the wave orbital velocity $u_b$.

This assumption leads to a linearised bottom friction term.

$$\bar{\tau}_{by} = \frac{1}{\pi} f_{wc} \rho u_{bm} \bar{v} \tag{16}$$

where $u_{bm}$ is the maximum value of the wave orbital velocity $u_b$ immediately above the oscillatory boundary layer. Small amplitude wave theory in shallow water gives the following expressions for $u_{bm}$:

$$u_{bm} = \frac{\gamma}{2} \sqrt{g(\bar{\eta}+h)} \tag{17}$$

which on substitution in Equation (16) gives the following equation for the bottom friction term

$$\tau_{by} = \frac{f_{wc} \rho \gamma \bar{v}}{2\pi} \sqrt{g(\bar{\eta}+h)} \tag{18}$$

The friction factor for combined wave and current action $f_{wc}$ can be evaluated from the wave friction factor $f_w$ and the current friction factor $f_c$. Gourlay (1978) has shown that for a weak current, defined as one for which the parameter $F = \sqrt{f_w/f_c}.u_{bm}/\bar{v}$ is greater than 5, Bijker's (1967) work as modified by Swart (1974) leads to the conclusion that

$$f_{wc} = \sqrt{f_w f_c} \tag{19}$$

The wave friction factor $f_w$ is defined by the expression

$$\tau_b = \frac{1}{2} f_w \rho u_{bm}^2$$

and can be evaluated from Jonsson's (1967) formula, i.e.

$$\frac{1}{4\sqrt{f_w}} + \log \frac{1}{4\sqrt{f_w}} = -0.08 + \log (a_{bm}/\varepsilon) \tag{20}$$

which requires an iterative solution for $f_w$, or from Swart's (1974) explicit approximation for Equation (20), i.e.

$$f_w = e^{5.98 + 5.21\,(a_{bm}/\varepsilon)^{-0.19}} \tag{21}$$

where $a_{bm}$ is the maximum orbital amplitude immediately above the oscillatory boundary layer and $\varepsilon$ is a measure of the size of the bottom roughness. For shallow water conditions

$$a_{bm} = \frac{\gamma\, T \sqrt{g(\bar{\eta}+h)}}{4\pi} \tag{22}$$

The current friction factor $f_c$ can be evaluated from the usual rough boundary flow resistance equation, i.e.

$$f_c = \frac{0.32}{\ell n^2 \left[\frac{11(\bar{\eta}+h)}{\varepsilon}\right]} \tag{23}$$

where $f_c$ is defined by the relationship

$$\tau_b = \frac{1}{2} f_c\, \rho\, \bar{v}^2$$

## 2.5 Equation for Velocity of Nonuniform Alongshore Current

Now, neglecting lateral mixing effects, Equation (1) becomes

$$\tau_\theta + \tau_{\Delta H} + \tau_{by} = 0 \tag{24}$$

and substituting for $\tau_\theta$, $\tau_{\Delta H}$ and $\tau_{by}$ using Equations (10a), (14) and (18), the following equation is obtained:

$$\bar{v} = \frac{2\pi \sqrt{g(\bar{\eta}+h)}}{f_{wc}\ \gamma^2} \left[ \frac{5}{32} \gamma^3 \frac{\tan \alpha}{1+3\gamma^3/8} \sin 2\theta - (1 + \frac{\gamma^2}{8} + \frac{\gamma^2}{4} \sin^2\theta) \frac{\partial H}{\partial y} \right] \tag{25a}$$

or

$$\frac{\bar{v}}{\sqrt{gH}} = \frac{2\pi}{f_{wc}\gamma^{5/2}} \left[ \frac{5}{32} \gamma^3 \frac{\tan \alpha}{1+3\gamma^2/8} \sin 2\theta - (1 + \frac{\gamma^2}{8} + \frac{\gamma^2}{4} \sin^2\theta) \frac{\partial H}{\partial y} \right] \tag{25b}$$

Equation (25a) is identical with the following equation given by Komar (1975):

$$\bar{v} = \frac{\pi}{2C_f} u_{bm} \left[ \frac{5}{4} \frac{\tan \alpha}{1+3\gamma^2/8} \sin \theta \cos \theta - \frac{4}{\gamma^3} (1 + \frac{3\gamma^2}{8} - \frac{\gamma^2}{4} \cos^2\theta) \frac{\partial H}{\partial y} \right] \tag{25c}$$

since

$$u_{bm} = \frac{\gamma}{2} \sqrt{g(\bar{\eta}+h)} \tag{17}$$

and

$$C_f = \frac{f_{wc}}{2}$$

The above equations clearly show that if there is a positive alongshore gradient of wave height, that is, the waves increase in height in the direction the current is flowing, then the magnitude of the alongshore current generated by the waves breaking at an angle to the beach is reduced. On the other hand, if the wave height gradient is negative, that is, the waves decrease in height in the direction the current is flowing, then the magnitude of the alongshore current is increased. Thus it is possible for there to be a situation where a positive wave height gradient completely overcomes the influence of the breaker angle. When this occurs $\bar{v} = 0$ and hence

$$\frac{5}{32}\gamma^3 \frac{\tan\ \alpha}{1+3\gamma^2/8} \sin 2\theta = \left(1 + \frac{\gamma^2}{8} + \frac{\gamma^2}{4}\sin^2\theta\right)\frac{\partial H}{\partial y}$$

or

$$\frac{1}{\tan\ \alpha} \cdot \frac{\partial H}{\partial y} = \frac{5}{32} \cdot \frac{\gamma^3}{1+3\gamma^2/8} \cdot \frac{\sin 2\theta}{1 + \frac{\gamma^2}{8} + \frac{\gamma^2}{4}\sin^2\theta} \tag{26}$$

Equation (26) is a corrected form of the equation originally derived by Komar (1971), and is independent of the form of the bottom friction term.*

The relationship expressed by Equation (26) is shown on Figure 4 where $(1/\tan\ \alpha)\partial H/\partial y$ is plotted as a function of $\theta$ with the breaker index $\gamma$ as a parameter. In actual fact $\gamma$ is a function of $\tan\ \alpha$ and either wave steepness $H_o/L_o$ or relative depth $h_b/L_o$. In the limiting case of waves of very low steepness; $\gamma = 0.8$ corresponds to a beach slope of 0.02 or flatter; $\gamma = 1.0$ to $\tan\alpha = 0.04$; and $\gamma = 1.2$ to $\tan\ \alpha \simeq 0.07$ (Goda 1970). Consequently, the critical value of $\partial H/\partial y$ for a given angle of incidence $\theta$ increases as the beach slope increases, but

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* Komar (1975) presents a similar relationship incorporating the results of an empirical investigation of the simplified uniform alongshore current equation (i.e. Equation (44)). His equation involves the resistance coefficient $C_f$ (= $f_{wc}/2$) and is stated to apply to the current in mid surf zone.

by a greater amount than Equation (26) would at first sight indicate. Komar (1971, 1972 and 1975) used his form of Equation (26) to explain how an equilibrium beach cusp can be produced by rip currents even though the latter are no longer present.

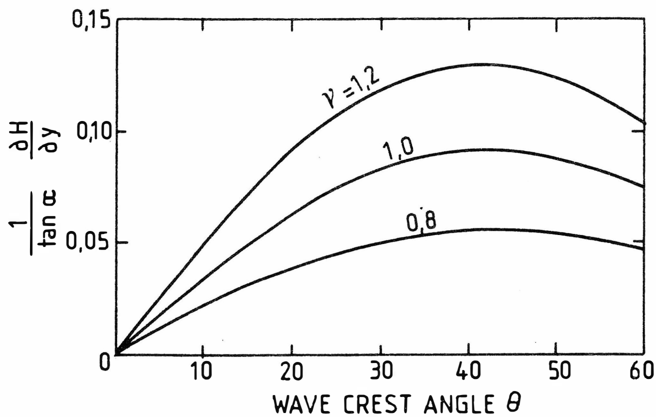



Figure 4 : Conditions of zero alongshore current velocity - Equation (26)

The preceding equations for a nonuniform alongshore current apply to any location within the surf zone. Bakker (1971) has pointed out that the relative magnitudes of the breaker angle and breaker gradient terms can vary across the surf zone. Consequently, it is possible for the breaker angle effect to dominate in the outer portion of the surf zone near the break point, while the breaker gradient effect dominates inshore close to the beach (see Figure 5). The explanation for this behaviour is that wave refraction does not cease at the breakpoint and so the angle θ is continually decreasing as the broken wave traverses the surf zone. On the other hand, the breaker height gradient is affected essentially only by the increased spacing between wave orthogonals which is determined by changes in the magnitude of cos θ. The latter are generally small in comparison with the corresponding changes in sin 2θ, and so the magnitude of the breaker gradient effect is much more uniform across the surf zone than is the magnitude of the breaker angle effect, which tends to decrease in magnitude as the waves approach the shore.

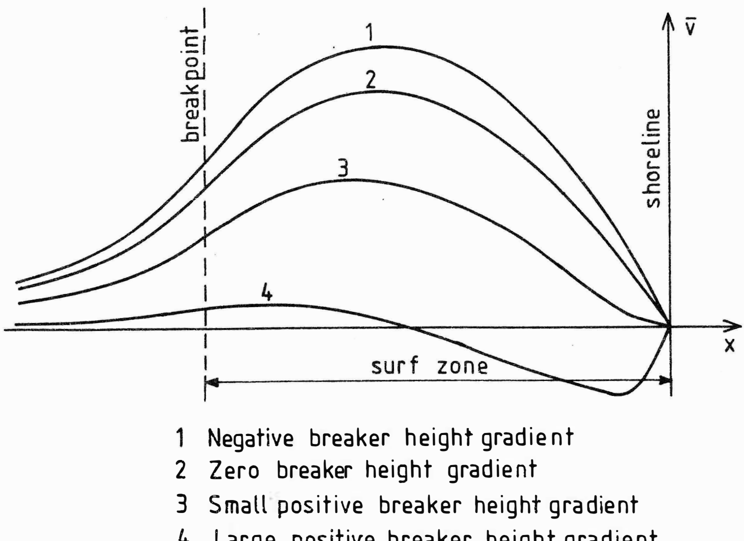



Figure 5 : Influence of alongshore breaker height gradient upon velocity distribution of alongshore current

## 2.6 Velocity Distribution Across Surf Zone

The shape of the velocity profile across the surf zone in the absence of lateral mixing effects can be seen more clearly if Equation (10b) is used for $\tau_\theta$ in Equation (24). The nonuniform alongshore current velocity equation now becomes

$$\bar{v} = \frac{2\pi g^{\frac{1}{2}}}{f_{wc}\gamma^2}\left[\frac{5}{16}\gamma^3 \frac{\tan\alpha}{1+3\gamma^2/8} \sin\theta_b \cos\theta \frac{\bar{\eta}+h}{(\bar{\eta}+h)_b^{\frac{1}{2}}} - (1 + \frac{\gamma^2}{8} + \frac{\gamma^2}{4}\sin^2\theta)(\bar{\eta}+h)^{\frac{1}{2}} \frac{\partial H}{\partial y}\right] \quad (27)$$

If there is no alongshore gradient in wave height, i.e. $\partial H/\partial y = 0$, Equation (27) reduces to that for the velocity distribution across the surf zone of a uniform alongshore current derived by Longuet-Higgins (1970-1) and others. That is:

$$\bar{v} = \frac{5}{8}\frac{\gamma\pi g^{\frac{1}{2}}}{f_{wc}} \frac{\tan\alpha}{1+3\gamma^2/8} \sin\theta_b \cos\theta \frac{\bar{\eta}+h}{(\bar{\eta}+h)_b^{\frac{1}{2}}} \quad (28)$$

If cos θ does not vary appreciably through the surf zone, i.e. it can be replaced by cos $\theta_b$, Equation (28) indicates that the uniform alongshore current velocity varies linearly across the surf zone, increasing from zero at the shoreline to a maximum value at the break point. Offshore of the breakpoint the velocity is zero. Such a simplified situation cannot exist in practice and some lateral mixing must occur to eliminate the discontinuity in the velocity distribution at the break point which is implicit in Equation (28) by virtue of its derivation from Equation (8) and the assumption of no dissipation offshore of the breakpoint. In actual fact lateral mixing removes this discontinuity and produces the smooth alongshore velocity profile shown on Figure 6 as predicted by more complete analyses which include the lateral mixing term, e.g. Longuet-Higgins (1970-2).

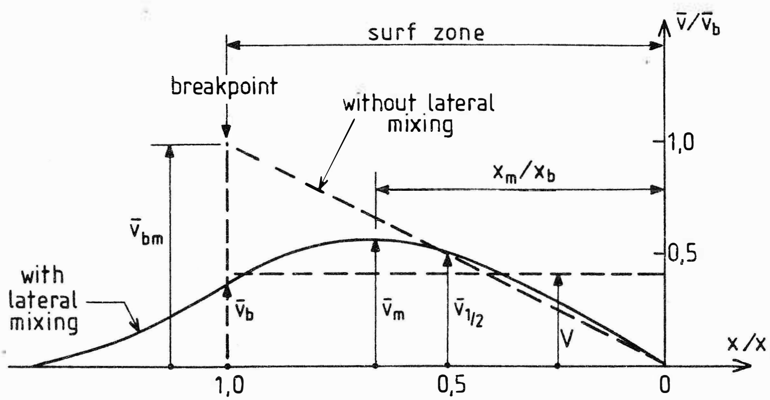



Figure 6 : Velocity profiles and representative velocities for a uniform alongshore current

Equation (27) is similar to that derived by Bakker (1971). The latter may be expressed as shown below after making the same identifications as were made by Ozasa and Brampton. The parameter p from Bijker's (1967) analysis which is used by Bakker has been replaced by Swart's expression ($p = \sqrt{f_w/2\kappa^2}$) and Bakker's f has been replaced by 4 $f_c$ where the current friction factor $f_c$ is defined as in Equation (23) consistently with the wave friction factor $f_w$. Thus Bakker's equation for the nonuniform alongshore current velocity becomes

$$\bar{v} = \frac{2\pi\, g^{\frac{1}{2}}}{f_{wc}\gamma^2}\left[\frac{5}{16}\gamma^3 \frac{\tan\alpha}{1+3\gamma^2/8} \sin\theta_b \frac{\bar{\eta}+h}{(\bar{\eta}+h)_b^{\frac{1}{2}}} - \frac{5}{16}\gamma\frac{1+\gamma^2/8}{1+3\gamma^2/8}(\bar{\eta}+h)^{\frac{1}{2}}\frac{\partial H_b}{\partial y}\right] \qquad (29)$$

If cos θ is inserted into the first term of Equation (29) Equations

(27) and (29) are identical except for the second term which represents the effect of the alongshore breaker height gradient upon the alongshore current velocity. Thus the equation for the velocity distribution of a nonuniform alongshore current with no lateral mixing can be written as

$$\bar{v} = K_1 g^{\frac{1}{2}} \left[ K_2 \sin\theta_b \cos\theta . \frac{\bar{\eta}+h}{(\bar{\eta}+h)_b^{\frac{1}{2}}} - K_3 \, (\bar{\eta}+h)^{\frac{1}{2}} \frac{\partial H_b}{\partial y} \right] \tag{30}$$

assuming the alongshore wave height gradient $\partial H/\partial y$ is constant across the surf zone and therefore equal to $\partial H_b/\partial y$.

$$K_1 = 2\pi/f_{wc}\,\gamma^2$$

$$K_2 = \frac{5}{16}\gamma^3 \frac{\tan\alpha}{1+3\gamma^2/8}$$

$$K_3 = 1 + \frac{\gamma^2}{8} + \frac{\gamma^2}{4}\sin^2\theta \quad \text{from equation (27) (Komar)}$$

or

$$K_3 = \frac{5}{16}\gamma \frac{1+\gamma^2/8}{1+3\gamma^2/8} \quad \text{from equation (29) (Bakker)} \tag{31}$$

For small $\theta$, $\sin^2\theta \to 0$ and assuming $\gamma = 0.8$

$K_3 = 1.080$ from Equation (27)

$= 0.218$ from Equation (29).

Hence Equation (27) indicates that the alongshore breaker height gradient term is in fact five times larger than predicted by Equation (29) based upon Bakker's analysis.

## 2.7 Representative Alongshore Current Velocity

Referring back to Figure 6 it can be seen that the selection of a single representative value of the alongshore current velocity is not a simple matter. Some possibilities are as follows:

(i) the velocity at the break point $\bar{v}_b$;

(ii) the velocity at mid surf zone $\bar{v}$ ;

(iii) the maximum velocity $\bar{v}_m$;

(iv) the mean velocity within the surf zone V.

For the preceding analysis in which lateral mixing effects were neglected the maximum velocity occurs at the break point. Expressions for $\bar{v}_{bm}$, $\bar{v}_{\frac{1}{2}}$ and V are obtained from Equation (27) as follows.

### Breakpoint velocity

The maximum alongshore velocity occurs at the break point according to the simple theory ignoring lateral mixing effects. Hence Equation (27) gives

$$\frac{\bar{v}_{bm}}{\sqrt{gH_b}} = \frac{2\pi}{f_{wc}\gamma^{5/2}}\left[\frac{5}{16}\gamma^3 \frac{\tan\alpha}{1+3\gamma^2/8}\sin\theta_b\cos\theta_b - \left(1+\frac{\gamma^2}{8}+\frac{\gamma^2}{4}\sin^2\theta_b\right)\frac{\partial H_b}{\partial y}\right] \tag{32}$$

which reduces to the following equation when $\partial H_b/\partial y = 0$

$$\frac{\bar{v}_{bm}}{\sqrt{gH_b}} = \frac{5\pi}{16}\frac{\gamma^{\frac{1}{2}}}{1+3\gamma^2/8}\frac{\tan\alpha}{f_{wc}}\sin 2\theta_b \tag{33}$$

The term $\gamma^{\frac{1}{2}}/(1+3\gamma^2/8)$ has an almost constant magnitude of 0.72 over the usual range of values of $\gamma$ from 0.8 to 1.2. Hence Equation (33) becomes

$$\frac{\bar{v}_{bm}}{\sqrt{gH_b}} = 0.707 \frac{\tan \alpha}{f_{wc}} \sin 2\theta_b \qquad (34)$$

Mean velocity within surf zone

The mean alongshore velocity within the surf zone can be obtained by integration across the surf zone in the same way as was done by Ozasa and Brampton. Thus

$$V = \frac{2}{(\bar{\eta}+h)_b\, x_b} \int_0^{x_b} \bar{v}\, (\bar{\eta}+h)\, dx$$

Substitution for $\bar{v}$ using Equation (27) and subsequent manipulation leads to Equation (35) below which differs from that of Ozasa and Brampton only in the form of the second term. Hence

$$\frac{V}{\sqrt{gH_b}} = \frac{2\pi}{f_{wc}\gamma^{5/2}} \left[ \frac{5}{24} \gamma^3 \frac{\tan \alpha}{1+3\gamma^2/8} \sin \theta_b \cos \theta_b - \frac{4}{5} \left(1 + \frac{\gamma^2}{8} + \frac{\gamma^2}{4} \sin^2\theta_b\right) \frac{\partial H_b}{\partial y} \right] \qquad (35)$$

which reduces to the following equation for the mean velocity of a uniform alongshore current when $\partial H_b/\partial y = 0$.

$$\frac{V}{\sqrt{gH_b}} = \frac{5}{24}\frac{\gamma^{1/2}}{1+3\gamma^2/8}\frac{\tan\alpha}{f_{wc}}\sin 2\theta_b \qquad (36)$$

Substitution for $\gamma$ then gives

$$\frac{V}{\sqrt{gH_b}} = 0.472\,\frac{\tan\alpha}{f_{wc}}\sin 2\theta_b \qquad (37)$$

Velocity at mid surf zone

The velocity in the middle of the surf zone is obtained from Equation (27) by putting $(\bar{\eta} + h) = (\bar{\eta} + h)_b/2 = H_b/2\gamma$.

Hence

$$\frac{\bar{v}_{1/2}}{\sqrt{gH_b}} = \frac{2\pi}{f_{wc}\gamma^{5/2}}\left[\frac{5}{32}\gamma^3\frac{\tan\alpha}{1+3\gamma^2/8}\sin\theta_b\cos\theta_{1/2} - \frac{1}{\sqrt{2}}\left(1+\frac{\gamma^2}{8}+\frac{\gamma^2}{4}\sin^2\theta_{1/2}\right)\frac{\partial H_b}{\partial y}\right] \qquad (38)$$

which reduces to the following equation when $\partial H_b/\partial y = 0$, assuming $\cos\theta_{1/2} = \cos\theta_b$.

$$\frac{\bar{v}_{1/2}}{\sqrt{gH_b}} = \frac{5\pi}{32}\frac{\gamma^{1/2}}{1+3\gamma^2/8}\frac{\tan\alpha}{f_{wc}}\sin 2\theta_b \qquad (39)$$

Substitution for $\gamma$ then gives

$$\frac{\bar{v}_{\frac{1}{2}}}{\sqrt{gH_b}} = 0.354 \, \frac{\tan \alpha}{f_{wc}} \sin 2\theta_b \tag{40}$$

Comparison of Equations (32), (35) and (38) indicates that they can all be expressed in the form

$$\frac{\bar{v}}{\sqrt{gH_b}} = \frac{K_\theta}{f_{wc}} \left[ \tan \alpha \sin 2\theta_b - K_{\Delta H} \frac{\partial H_b}{\partial y} \right] \tag{41}$$

where the values of $K_\theta$ and $K_{\Delta H}$ are given in Table 1.

Hence the relative importance of the breaker height gradient term with respect to the breaker angle term depends upon which representative velocity is chosen. It should be noted that the magnitudes of $K_\theta$ and $K_{\Delta H}$ will in actual fact be modified by lateral mixing effects which are discussed in following Section 2.8.

TABLE 1 Magnitudes of constants $K_\theta$ and $K_{\Delta H}$ in equation (41) for nonuniform alongshore current with no lateral mixing

| Representative Velocity | Breaker Angle Constant $K_\theta$ | Breaker Gradient Constant $K_{\Delta H}$ |
|---|---|---|
| Break point $\bar{v}_b$ | 0.707 | 16.74 |
| Mean V | 0.472 | 20.09 |
| Mid surf zone $\bar{v}_{\frac{1}{2}}$ | 0.354 | 23.65 |

Little experimental data is available to check the validity of Equation (41). Tanaka et al. (1980) provide some measurements made in a fixed bed model with a flat beach. All the measurements with waves breaking at an angle to the shore involve large breaker angles, i.e. $24^o < \theta_b < 60^o$, for which the above theory does not apply. Two tests were made with zero breaker angle and their results can be used to test the validity of the breaker gradient term of Equation (41). Tanaka et al. quote values of $\partial\bar{\eta}/\partial y$ rather than $\partial H_b/\partial y$.

Now

$$\frac{\partial H_b}{\partial y} = \gamma \frac{\partial(\bar{\eta}+h)_b}{\partial y} = \gamma\left(\frac{\partial\bar{\eta}}{\partial y} + \frac{\partial h}{\partial y}\right)$$

and in the absence of other information it will be assumed that $\partial h/\partial y = 0$, although this is probably not in fact the case. Wave heights were measured at various locations through the surf zone but not at the breakpoint. From Figure 1 of Tanaka et al. it was established that the average bottom slope through the surf zone was 0.015 and it was found that with this slope the measured wave heights indicated $\gamma = H/h = 0.8$ within the surf zone. The breaker height was then calculated from the observed surf zone width using $\gamma = 0.8$ and $\tan\alpha = 0.015$.

Since $H_b$, $\bar{v}$, $\partial\bar{\eta}/\partial y$ have been measured and $K_\theta$ and $K_{\Delta H}$ can be obtained from Table 1 once the representative velocity is chosen the remaining quantity $f_{wc}$ is calculated. The relevant values are given in Table 2. The calculated values of $f_{wc}$ all lie within the expected range for laboratory data as indicated in Table 4 in Section 2.9. It is noted that the maximum value of $\partial\bar{\eta}/\partial y$ in the first test was measured closest to the break point and thus perhaps should be preferred to the mean value when using Equation (41).

TABLE 2 Calculation of $f_{wc}$ from Equation (41) using data from Tanaka et al. (1980)

| $H_b$ mm | $\bar{v}$ mm/s | $\partial\bar{\eta}/\partial y$ Measured | $\partial\bar{\eta}/\partial y$ Adopted | Representative velocity adopted | $f_{wc}$ |
|---|---|---|---|---|---|
| 20 | 85 | $-10.5 \times 10^{-4}$ | $-6.2 \times 10^{-4}$ (mean) | V | 0.025 |
| | | $-7.5$ | | $\bar{v}_{\frac{1}{2}}$ | 0.022 |
| | | $-2.3$ | $-10.5 \times 10^{-4}$ (max) | V | 0.042 |
| | | $-4.5$ | | $\bar{v}_{\frac{1}{2}}$ | 0.037 |
| 20 | 7 | $-0.75 \times 10^{-4}$ | $-0.75 \times 10^{-4}$ | V | 0.036 |
| | | $-0.75$ | | $\bar{v}_{\frac{1}{2}}$ | 0.032 |

Given that there are some uncertainties in the author's interpretation of the experimental data, apart from any normal experimental errors, the above result can be accepted as satisfactory since there are no methods currently available which can be relied upon to estimate $f_{wc}$ more precisely. It will be noted that in this case the choice of V or $\bar{v}_{\frac{1}{2}}$ as representative velocity has little effect upon the result since the product $K_\theta$ $K_{\Delta H}$ only differs by 13% for these two velocities.

## 2.8 Effects of Lateral Mixing

It is evident from Figure 6 that the effects of lateral mixing upon the magnitude and distribution of the alongshore current cannot be neglected. As the inclusion of $\tau_L$ in Equation (1) precludes the development of a simple analytical expression for the alongshore current velocity, the effect of the lateral mixing will be incorporated into the preceding equations using a simple multiplier factor $\Gamma$ appropriate to tne particular representative velocity selected. Suitable values of $\Gamma$ will be selected from consideration of the results of more complete analytical solutions for the velocity distribution of a uniform alongshore current. A similar approach has been proposed by Walton (1980).

While a complete solution for the velocity distribution of a nonuniform alongshore current including the effects of lateral mixing was produced by Komar (1975) it is more convenient to work with solutions for a uniform alongshore current such as those given by Longuet-Higgins (1970-2) for small breaker angles or by Kraus and Sasaki (1979) for large breaker angles. Longuet-Higgins' solution is given on Figure 7 where the effects of different intensities of lateral mixing are represented by the lateral mixing parameter P.* By comparison of this solution with laboratory data obtained by Galvin and Eagleson (1964), Longuet-Higgins estimated that the magnitude of P lay between 0.1 and 0.4. Subsequent examination of alongshore current data by Komar (1975) suggested that Galvin and Eagleson's data is atypical giving current velocities which are of the order of five times the current velocities for comparable conditions from other data sets. A more recent analysis by Kraus and Sasaki (1979) using both laboratory and field data indicates that the value of the lateral mixing parameter P is $\lesssim$ 0.1 with some values as low as 0.06. Komar (1975) supports this lower value for P. For the present purposes it will be assumed that 0.08 is a representative value of P.

---

* The lateral mixing parameter P is proportional to a dimensionless diffusion coefficient N and inversely proportional to the friction factor $f_{wc}$ i.e.

$$P = \frac{2\pi}{\gamma} \frac{N}{f_{wc}} \frac{\tan \alpha}{1+3\gamma^2/8}$$

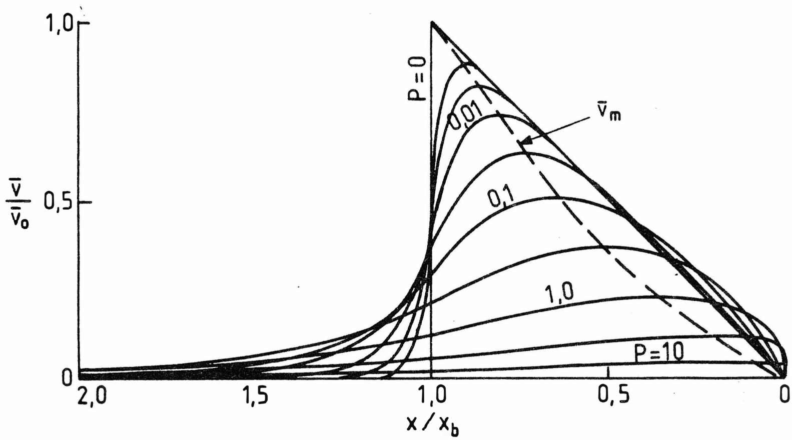
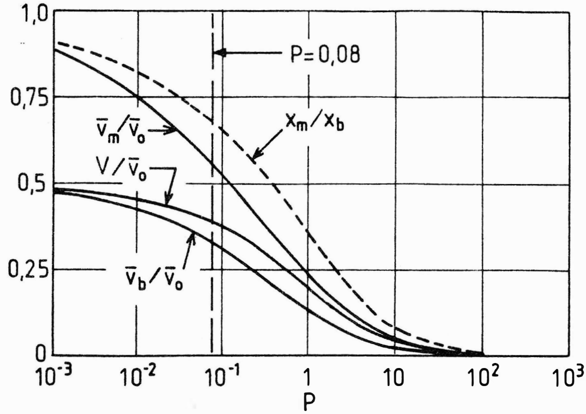



Figure 7 : Velocity distribution for uniform alongshore current as a function of lateral mixing parameter P - Longuet-Higgins (1970)

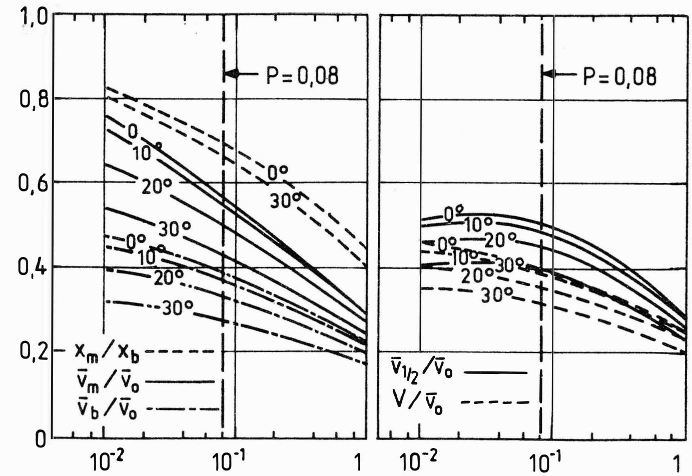
A representative velocity profile corresponding to P = 0.08 can now be synthesised from Figures 2 and 3 of Longuet-Higgins (1970-2) or Figure 4 of Kraus and Sasaki (1979)(see Figure 8). This figure gives values of $\bar{v}_b/\bar{v}_o$, $\bar{v}_m/\bar{v}_o$, $\bar{v}_{\frac{1}{2}}/\bar{v}_o$ and $V/\bar{v}_o$ as well as $x_m/x_b$ as functions of P where the reference velocity $\bar{v}_o$ is related to the velocity at the breakpoint when lateral mixing is neglected. $\bar{v}_o$ is defined by Kraus and Sasaki as follows:

$$\bar{v}_o = \frac{5\pi}{16} \frac{\gamma^{\frac{1}{2}}}{1+3\gamma^2/8} \frac{\tan\alpha}{f_{wc}} \sqrt{gH_b} \sin\theta_b \qquad (42)$$

Longuet-Higgins omits the term $1+3\gamma^2/8$ allowing for wave set-up. Equation (42) is identical to Equation (33) except for the omission of $\cos\theta_b$. This is not significant for Longuet-Higgins° solution. On the other hand, if $\bar{v}_o$ is calculated from Equation (33) instead of Equation (42), the apparent effect of breaker angle $\theta_b$ upon the velocity profile is considerably reduced. Mean values of the various




(a) Small breaker angle - Longuet-Higgins (1970)

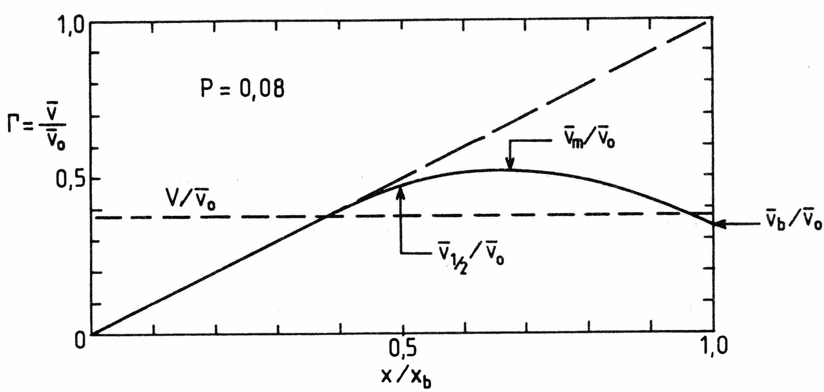



(b) Large breaker angle - Kraus and Sasaki (1979)

$\bar{v}_o$ reference velocity - equation (42);
$\bar{v}_b$ velocity at the break point;
$\bar{v}_m$ maximum velocity;
$V$ mean velocity within the surf zone;
$x_m$ location of maximum velocity;
$x_b$ location of break point.

Figure 8 : Theoretical values of representative alongshore current velocities as functions of lateral mixing parameter P and breaking wave angle $\theta_b$

velocity ratios computed by Kraus and Sasaki were obtained from Figure 8b by averaging the values at $\theta_b = 0^o$ and those at $\theta_b = 30^o$ after the latter were divided by cos $\theta_b$ which amounts to calculating $\bar{v}_0$ from Equation (33). As shown in Table 3 the resulting mean velocity ratios are virtually identical with those obtained from Longuet-Higgins' solution*. The corresponding velocity profile is shown on Figure 9. Moreover, Kraus and Sasaki's solution indicates that $\bar{v} / \bar{v}_0$ is virtually constant for values of $P < 0.1$ thus indicating that the mid surf zone velocity is relatively insensitive to errors in estimating lateral mixing effects.




$$\bar{v}_0 = \frac{5\pi}{16} \frac{\gamma^{1/2}}{1+3\gamma^2/8} \sqrt{gH_b} \frac{\tan\alpha}{f_{wc}} \sin 2\theta_b \quad - \quad \text{equation (33)}$$

Figure 9 : Representative alongshore current velocity profile within surf zone (Table 3)

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* It should be noted that Kraus and Sasaki assumed a value of $\gamma = 1.0$ in calculating the relationships shown in Figure 8 while Longuet-Higgins appears to have assumed $\gamma = 0.82$.

The representative alongshore current velocity can now be obtained using Equation (43) below.

$$\frac{\bar{v}}{\sqrt{gH_b}} = \Gamma \frac{5\pi}{16} \frac{\gamma^{\frac{1}{2}}}{1+3\gamma^2/8} \frac{\tan \alpha}{f_{wc}} \sin 2\theta_b \qquad (43)$$

where $\Gamma$ is the appropriate value selected from Table 3. Thus if $\gamma = 0.8$

$$\bar{v} = K'_\theta \frac{\tan \alpha}{f_{wc}} \sqrt{gH_b} \quad \sin 2\theta_b \qquad (44)$$

where $K'_\theta = \Gamma \frac{5\pi}{16} \frac{\gamma^{\frac{1}{2}}}{1+3\gamma^2/8} = 0.707\ \Gamma$

Values of $K'_\theta$ are listed in Table 3.

Comparison of the $K'_\theta$ values in Table 3 which include the effects of lateral mixing with those of $K_\theta$ given in Table 1 where lateral mixing was ignored shows that actual values of $\bar{v}_b$ will be about one third of $\bar{v}_{bm}(\equiv \bar{v}_o)$ (Equation (34) and the actual mean velocity will be a little over half (0.57) that calculated from Equation (37). On the other hand the velocity in mid surf zone $\bar{v}_{\frac{1}{2}}$ is only 4% less than that obtained from Equation (40) where lateral mixing is neglected. Thus, for both this reason and the previously mentioned fact that $\bar{v}_{\frac{1}{2}}$ is relatively insensitive to the magnitude of the lateral mixing parameter P, it is evident that the mid surf zone velocity adopted by Komar is a very appropriate representative alongshore current velocity.

The preceding analysis indicates that the generalisation that the bottom friction determines the magnitude of the velocity and the

TABLE 3 Velocity ratios for representative uniform alongshore current velocity profile (figure 9)

| Velocity Ratio $\Gamma$ | Longuet-Higgins Solution P = 0.08 (figure 8) | Kraus and Sasaki solution (figure 9) P = 0.08 | | | | $K'_\theta$ (equation 43) |
|---|---|---|---|---|---|---|
| | | 1 $\theta_b = 0^o$ | 2 $\theta_b = 30^o$ | 3 $\frac{\theta_b = 30^o}{\cos 30^o}$ | 4 Mean of 1 and 3 | |
| $\frac{\bar{v}_b}{\bar{v}_o}$ | 0.33 | 0.39 | 0.27 | 0.31 | 0.35 | 0.247 |
| $\frac{\bar{v}_m}{\bar{v}_o}$ | 0.54 | 0.56 | 0.43 | 0.50 | 0.53 | 0.375 |
| $\frac{\bar{v}_{1/2}}{\bar{v}_o}$ | - | 0.50 | 0.40 | 0.46 | 0.48 | 0.339 |
| $\frac{V}{\bar{v}_o}$ | 0.39 | 0.40 | 0.32 | 0.37 | 0.39 | 0.276 |
| $\frac{x_m}{x_b}$ | 0.67 | 0.69 | 0.66 | - | 0.68 | - |

lateral friction determines the shape of the velocity profile is only relatively correct. Lateral friction must also affect the magnitude of the velocity since the width of the alongshore current is increased as alongshore flowing water from the surf zone is mixed with initially still water seaward of the breakpoint where the water depth is greater. Thus lateral mixing must reduce the magnitude of the mean velocity of the current within the surf zone to some extent. Moreover it should be noted that alongshore velocity profiles of similar shape to these shown on Figure 7 can be obtained neglecting lateral mixing with irregular waves whose wave heights follow the Rayleigh distribution and break over a relatively wide breaker zone (Battjes 1974, Swart and Fleming 1981).

The velocity profile shown on Figure 9 applies to a plane beach. Experimental observations both in the Laboratory (Mizuguchi and Horikawa 1978) and in the field (Griesseier 1959, Allender and Ditmars 1981) with beach profiles of more natural shape including offshore bars and steps show a strong tendency for the alongshore current velocity to be uniform across the surf zone between the outer break point on the offshore bar and the inner breakpoint on the beach. Satisfactory theoretical analyses of this situation are not yet available although numerical modelling techniques have been applied (Ebersole and Dalrymple 1981).

In the absence of contrary information and for simplicity it will be assumed in the following sections that the values of $K'_\theta$ given in Table 3 for a uniform alongshore current on a plane beach can also be applied to nonuniform alongshore currents represented by Equation (41) and to natural beach profile shapes.

## 2.9 Evaluation of Wave-Current Friction Factor $f_{wc}$

From an examination of both field and laboratory alongshore current data from various sources Komar (1969, 1975) obtained reasonable agreement between observations of the alongshore current in mid surf zone and the following equation

$$\bar{v}_{\frac{1}{2}} = 2.7\ u_{bm} \sin\theta_b \cos\theta_b \tag{45}$$

In a more recent review of available field and laboratory data Komar (1979) found that an equation of the form of Equation (44) fitted the data well, i.e.

$$\bar{v}_{\frac{1}{2}} = 1.17\ \sqrt{gH_b} \sin\theta_b \cos\theta_b \tag{46a}$$

or

$$\bar{v}_{\frac{1}{2}} = 0.58\ \sqrt{gH_b} \sin 2\theta_b \tag{46b}$$

where $H_b$ is the root mean square wave height $H_{rms}$.

Equation (46) can be obtained from Equation (45) by substituting for $u_{bm}$ (Equation (17)) and putting $\gamma = 0.75$.

Komar also found that the introduction of tan $\alpha$ into the relationship between the alongshore current velocity and the breaking wave characteristics as recommended by CERC (1973)did not produce a better result. On the contrary the scatter of the data was significantly increased. Moreover Komar's analysis confirms that the results of Galvin and Eagleson (1964) which have been used by CERC are inconsistent with almost all other experimental data and hence should be rejected until such time as a reason for this inconsistency can be established.

Dette (1974) reported on extensive field studies of alongshore currents on the sandy beaches of the island of Sylt in the North Sea which gave, with considerable scatter, the relationship

$$\bar{v} = \sqrt{H_b} \cdot \sin 2\theta_b \qquad (47)$$

where $H_b$ is the significant wave height $H_{1/3}$ in metres and $\bar{v}$ seems to be best identified with $\bar{v}_b$.

Now $H_{1/3} = \sqrt{2}\, H_{rms}$ and so Equation (47) becomes

$$\bar{v}_b = 0.38 \sqrt{gH_b} \sin 2\theta_b \qquad (46c)$$

Comparison of Equations (46b) and (46c) with Equation (44) using appropriate values of $K'_\theta$ from Table 3 yields the following values of $(\tan \alpha)/f_{wc}$.

$(\tan \alpha)/f_{wc}$ = 1.71 from Equation (46b);

= 1.54 from Equation (46c).

A reasonable mean value of $(\tan \alpha)/f_{wc}$ could therefore be about 1.6. No satisfactory theoretical justification has yet been proposed for this result which is based upon data mostly, but not entirely, from sandy beaches. It does not apply to shingle beaches or to relatively rough fixed boundary beaches.

A practical consequence of the apparent constancy of the ratio $(\tan\alpha)/f_{wc}$ is that it is not necessary to evaluate the magnitude of $f_{wc}$ in order to determine the velocity of a uniform alongshore current. On the other hand it is still necessary to know the magnitude of $f_{wc}$ when computing the velocity of a nonuniform alongshore current from Equation (41), even if $(\tan\alpha)/f_{wc}$ is assumed constant, since $f_{wc}$ will still appear in the denominator of the breaker gradient term.

While the magnitude of $f_{wc}$ for any given situation is undoubtedly dependent upon other parameters the present state of knowledge does not permit these to be defined with any certainty and so all that can be established is a general order of magnitude for $f_{wc}$. Leaving aside earlier, probably inaccurate, estimates recent determinations of $f_{wc}$ are summarised in Table 4. This indicates values ranging from 0.018 to 0.064 with a general mean value of the order of 0.04 which is twice that originally proposed by Longuet-Higgins (1970). Most of the data in Table 4 are based upon laboratory data for which the value of $f_{wc}$ could be expected to be rather higher than that occurring in the field. Hence it is probable but not proven that $f_{wc}$ is of the order of 0.03 to 0.035 in field situations. Since Longuet-Higgins parameter P is inversely proportional to $f_{wc}$, the increase from 0.02 to 0.03 for $f_{wc}$ is consistent with the recent new estimates giving a lower magnitude for P compared with that determined by Longuet-Higgins from Galvin and Eagleson's experiments.

TABLE 4 Experimental values of $f_{wc}$

| Source | Type of measurement | Number of values | Magnitude of $f_{wc}$ | | |
|---|---|---|---|---|---|
| | | | Minimum | Maximum | Mean |
| Komar 1975 | Field | | | | 0.034 to 0.036 |
| | Lab. | | | | 0.045 |
| Mizuguchi and Horikawa 1978 (also Kraus and Sasaki 1979) | Lab. | 4 | 0.024 | 0.048 | 0.035 |
| Mizuguchi 1980 | Lab. | 14 | 0.018 | 0.064 | 0.046 |

# 3. ALONGSHORE SEDIMENT TRANSPORT

## 3.1 Alongshore Transport by Uniform Alongshore Currents

The most commonly used approach to alongshore transport is the CERC-Scripps method* which is based essentially on an empirical relationship between the alongshore sediment transport rate and a quantity related to the wave energy**, that is

$$I_L = K_{s1} T_\theta C_b \tag{48}$$

where $I_L$ is the alongshore sediment discharge measured as a submerged weight per unit time, or *"immersed weight transport rate"* (Komar and Inman 1970):

and

$$T_\theta = nE \sin\theta \cos\theta \tag{49}$$

where $T_\theta$ is the alongshore wave thrust as defined in Equations (5) and (6). Hence

$$\left.\begin{aligned} I_L &= K_{s1} (EC_G)_b \sin\theta_b \cos\theta_b \\ &= \frac{K_{s1}}{2} (EC_G)_b \sin 2\theta_b \end{aligned}\right\} \tag{50}$$

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* CERC = U.S. Coastal Engineering Research Center.
Scripps = Scripps Institute of Oceanography, La Jolla, California

** The product $T_\theta C_b$ in Equation (48) is referred to in many publications as being the alongshore wave energy flux or alongshore wave power. For a discususion of the reasons for <u>not</u> using these terms see Longuet-Higgins (1972a).

The dimensionless constant $K_{s1}$ has been evaluated as 0.77 by Komar and Inman (1970) from field tracer experiments. The magnitude of $K_{s1}$ is influenced in an as yet undefined manner by the characteristics of the beach sediments although there is some evidence which suggests that it decreases as the sediment size increases (Swart 1977).

In Equation (48)

$$I_L = \frac{(\rho_s - \rho)}{1 + e} g Q_s \tag{51}$$

where e is the void ratio of the deposited sediment and $1/(1+e) \simeq 0.6$;

$Q_s$ is the bulk or deposited volume transport rate.

The volumetric alongshore sediment transport can be obtained by substituting Equation (51) into Equation (50). With the assumption of shallow water conditions, this gives

$$\frac{Q_s}{g^{\frac{1}{2}} H_b^{5/2}} = \frac{K_{s1} (1 + e) \sin 2\theta_b}{16 \gamma^{\frac{1}{2}} (s - 1)} \tag{52a}$$

or

$$\frac{Q_s}{g^{\frac{1}{2}} H_b^{5/2}} = 0.0706 K_{s1} \sin 2\theta_b \tag{52b}$$

for a flat beach of quartz sand

where $\gamma = 0.8$, $1/(1+e) = 0.6$, $s = \rho_s/\rho = 2.65$.

An alternative, or rather complementary, model for alongshore transport has been proposed by Bagnold (1963) and applied by Inman and Bagnold (1963). It assumes that the orbital motion of the waves stirs up the sediment and moves it back and forth in the direction normal to the shore. The moving sediment is then free to be transported in the direction of any superimposed unidirectional current. Such a model can be envisaged as being applicable to the alongshore transport of sediment by wave-generated currents within the surf zone between the break point and the run-down limit. Bagnold's model yields the following relationship for alongshore sediment transport:

$$I_L = K_{s2} (EC_G)_b \cos \theta_b \cdot \frac{\bar{v}}{u_{bm}} \tag{53}$$

where the dimensionless constant $K_{s2}$ was found by Komar and Inman (1970) to be 0.28 for the same data that yielded $K_{s1}$ = 0.77.

This result can be verified as follows. If Equation (45) devised by Komar is substituted for $\bar{v}$ in Equation (53) the latter reduces to Equation (50) if the cos $\theta_b$ term is dropped from the alongshore current Equation (Equation (45)). Alternatively if Equation (44) is substituted for $\bar{v}$ in Equation (53) and $u_{bm}$ is expressed by Equation (17), the constant $K_{s2}$ can be evaluated as follows.

$$K_{s2} = \frac{K_{s1} \gamma^{\frac{1}{2}}}{4 K'_\theta} \frac{f_{wc}}{\tan \alpha} \tag{54}$$

If the mid surf zone velocity is taken as the representative velocity the constant $K'_\theta$ = 0.339. Now for $K_{s1}$ = 0.77, $\gamma$ = 0.8 and $(\tan \alpha)/f_{wc}$ = 1.71 (based upon Equation (46b), $K_{s2}$ = 0.30. If $(\tan \alpha)/f_{wc}$ is taken as 1.6, $K_{s2}$ = 0.32.

Equation (53) can be expressed in terms of volumetric transport rate in the same manner as above. Whence

$$Q_s = \frac{K_{s2}\,(1 + e)}{4\,\gamma(s - 1)} \cos\theta_b\, H_b^2\, \bar{v} \tag{55a}$$

$$Q_s = 0.253\, K_{s2} \cos\theta_b\, H_b^2\, \bar{v} \tag{55b}$$

$$= K_{s3} \cos\theta_b\, H_b^2\, \bar{v} \tag{55c}$$

$K_{s3} = 0.088$ when $K_{s2} = 0.28$;

or $K_{s3} = 0.101$ when $K_{s2} = 0.32$.

If the breaker angle is small the cos $\theta_b$ term in Equation (55) may be neglected.

In all the preceding equations for alongshore sediment transport rate the wave height is the root mean square wave height $H_{rms}$.

## 3.2 Alongshore Transport by Nonuniform Alongshore Currents

Equation (55) provides a basis for computing alongshore sediment transport when the alongshore current results from causes other than waves breaking at an angle to the beach. It is now possible to write an expression for the alongshore sediment transport when this is caused by a current resulting from the effects of both breaker angle and an alongshore gradient of wave height. Thus, substitution of Equation (41) for $\bar{v}$ in Equation (55) gives

$$Q_s = K_{s_4} \cos\theta_b \, g^{\frac{1}{2}} H_b^{5/2} \left[ \tan\alpha \sin 2\theta_b - K_{\Delta H} \frac{\partial H_b}{\partial y} \right] \tag{56}$$

where $K_{s_4} = K_{s_3} K_\theta'/f_{wc}$.

The magnitudes of the constants $K_\theta'$ and $K_{\Delta H}$ which are given in Tables 1 and 3 depend upon the representative velocity adopted and the degree of lateral mixing. The constant $K_{s_3}$ depends upon the magnitude of the empirical constant $K_{s_2}$.

Equation (56) can also be expressed in terms of the immersed weight transport rate $I_L$ and the energy flux (wave power) at the breakpoint for direct comparison with Equation (50). Thus

$$I_L = \frac{K_{s_1}}{2} (EC_G)_b \cos\theta_b \left[ \sin 2\theta_b - \frac{K_{\Delta H}}{\tan\alpha} \frac{\partial H_b}{\partial y} \right] \tag{57}$$

Equation (57) reduces to Equation (50) when $\partial H_b/\partial y = 0$ and $\cos\theta_b = 1$. The omission of $\cos\theta_b$ from Equation (50) is a consequence of Komar and Inman's (1970) analysis in which this term was omitted from their expression for a uniform alongshore current (Equation (45)).

Equations (56) and (57) have been proposed by several authors previously. Motyka and Willis (1975) and Ozasa and Brampton (1980) have used Bakker's expression for the nonuniform alongshore current velocity (Equation (29)) while Gourlay (1978) and Kraus (1981) have used Komar's expression (Equation (27)). Gourlay (1978) has taken the breakpoint velocity as the representative one with a lateral mixing coefficient $\Gamma = 0.4$ and $f_{wc} = 0.02$ as proposed by Longuet-Higgins (1970). Motyka and Willis appear to have made similar assumptions. Ozasa and

Brampton took the mean velocity in the surf zone ignoring lateral mixing while Kraus used the mid surf zone velocity without lateral mixing. Ozasa and Brampton evaluated $(\tan\alpha)/f_{wc}$ by comparison with Equation (50) when $\partial H_b/\partial y = 0$. Kraus evaluated $(\tan\alpha)/f_{wc}$ as 1.45 using slightly different forms of Equations (39) and (45).

The constants in Equation (56) can only be evaluated approximately from the preceding theory and generalised experimental data. For instance if it is assumed that $(\tan\alpha)/f_{wc} = 1.6$, then $K_{s_3} = 0.10$. If the representative velocity is taken as the mid surf zone velocity $\bar{v}_{\frac{1}{2}}$ then $K'_\theta = 0.34$ and $K_{\Delta H} = 23.7$. Substitution of these values in Equation (56) gives

$$Q_s = 0.055 \cos\theta_b \, g^{\frac{1}{2}} H_b^{5/2} \left[ \sin 2\theta_b - \frac{23.7}{\tan\alpha} \frac{\partial H_b}{\partial y} \right] \qquad (58)$$

Alternatively instead of assuming $(\tan\alpha)/f_{wc}$ is constant, $f_{wc}$ may be assumed constant. If $f_{wc}$ is taken as 0.03 for field conditions and $K_{s_4}$ and $K_{\Delta H}$ have the same values as before, then

$$Q_s = 1.14 \cos\theta_b \, g^{\frac{1}{2}} H_b^{5/2} \left[ \tan\alpha \sin 2\theta_b - 23.7 \frac{\partial H_b}{\partial y} \right] \qquad (59)$$

A third possibility is to use the above average experimental values for both $(\tan\alpha)/f_{wc}$ and $f_{wc}$ to eliminate the beach slope $\tan\alpha$ from the alongshore transport equation. When $(\tan\alpha)/f_{wc} = 1.6$ and $f_{wc} = 0.03$ this is equivalent to assuming a constant beach slope of 1 in 20. Such a procedure is attractive in that the beach slope is not always easy to define. Using the same values of the various constants as previously this gives

$$Q_s = 0.055 \cos \theta_b \, g^{\frac{1}{2}} H_b^{5/2} \left[ \sin 2\theta_b - 494 \frac{\partial H_b}{\partial y} \right] \qquad (60)$$

As pointed out by Kraus (1981) the actual magnitudes of the constants in the alongshore transport equation can be adjusted by calibration for a given coast. Thus in Equation (56), $K_{s_4}$ determines the scale factor for the magnitude of the alongshore transport and $K_{\Delta H}$ determines the proper balance between the two driving force terms of breaker angle and alongshore breaker height gradient. The values of $K_{s_4}$ and $K_{\Delta H}$ given in the equations and tables of this report are thus order of magnitude values which can be used for approximate estimates or initial values in calibrating mathematical models.

## 3.3 Magnitude of Alongshore Transport Constant $K_s$

A general review of available field data for evaluating the constant $K_{s_1}$ has been made by Greer and Madsen (1979). They reject earlier data obtained by Watts (1953) and Caldwell (1956) as being of *"questionable quality"*. Moreover they have reservations about the accuracy of Komar's data because *"several of the basic assumptions underlying the use of tracers in sediment transport studies appear to have been violated in this study"*. They therefore conclude that coastal engineers using formulae based upon Equation (48) for the calculation of alongshore sediment transport rates *"should regard their results as no better than order of magnitude estimates"*.

More recently Bruno et al. (1981) have re-evaluated $K_{s_1}$ using the data previously considered by Greer and Madsen together with additional data of their own obtained from measurements of sand deposits behind an offshore breakwater at Channel Islands Harbour, California. Their observations indicate that $K_{s1}$ has an average value of 0.98 for 17 data points with a range of values of $K_{s1}$ between 0.19 and 4.18. When this new data is combined with previous data to give a total of 42 data points the new mean value of $K_{s1}$ is found to be 0.87. This increased magnitude of $K_{s1}$ in comparison with that obtained by Komar (0.77) should be treated with caution. As pointed out by the author elsewhere (Gourlay 1982) offshore breakwaters tend to draw sand towards them. Even when the waves approach with crests parallel to the breakwater and the shoreline, the current circulations formed behind the breakwater as a consequence of local alongshore breaker height gradients caused by wave diffraction will induce the development of a tombolo in the sheltered area. It is probable that this effect has influenced the formation of the sand deposits at Channel Islands Harbour and that the magnitude of $K_{s1}$ established by Bruno et al. for that situation is an overestimate. The nonuniform alongshore current equations such Equations (56) to (60) proposed in this report should provide a basis for estimating the effect of alongshore variations in breaker height upon the alongshore transport rates in the vicinity of the breakwater and hence enable the magnitude of $K_{s1}$ determined by Bruno et al. to be reassessed. Such a reassessment can be expected to reduce the magnitude of $K_{s1}$. There is thus little reason to adopt values of $K_{s1}$ greater than that determined by Komar. On the other hand as indicated below there appears to be good reason to adopt values of $K_{s1}$ less than 0.77 in certain cases.

Laboratory data generally indicate that Komar's value of 0.77 for $K_{s1}$ is an upper bound (Komar and Inman 1970). On this basis Ozasa and Brampton (1980) use a value of 0.385 for $K_{s1}$ when applying their form of the alongshore sediment transport Equation (56) to laboratory beaches. Komar and Inman explained the smaller $K_{s1}$ values obtained from laboratory experiments in terms of the transport process being less efficient in the smaller laboratory systems than in field situations.

On the other hand computations of alongshore transport in the Byron Bay region of New South Wales (Gordon and Lord 1981) using the normal value of $K_{s1}$ *"produced erosion and transport rates approximately three times those measured in prototype."* This suggests that the appropriate value of $K_{s1}$ for this field situation was of the order of 0.25 to 0.30 rather than 0.77. Similar results have been obtained in alongshore sediment transport calculations for the beaches of the Gold Coast in Queensland. Recent field measurements on Taiwan (Hou et al. 1981) gave a value of $K_{s1}$ equal to 0.55.

Kamphuis and Readshaw (1979) have investigated some of the factors influencing the magnitude of $K_{s1}$ and from a series of laboratory experiments have established that the alongshore sediment transport rate is a function of the type of breaking waves causing the alongshore transport. The breaker type is indicated by the magnitude of the surf zone similarity parameter $I_{rb}$ defined as

$$I_{rb} = \frac{\tan \alpha}{\sqrt{H_b/L_o}} \tag{61}$$

where the deep water wave length $L_o = gT^2/2\pi$.

According to Battjes (1974),

| | | |
|---|---|---|
| if | $I_{rb} > 2.0$ | breakers are surging or collapsing, |
| | $2.0 > I_{rb} > 0.4$ | breakers are plunging, |
| | $0.4 > I_{rb}$ | breakers are spilling. |

Kamphuis and Readshaw found that the alongshore transport rate decreased as the surf zone increased in width and that as the surf zone width increased the breaker type became less violent changing from plunging to spilling. Moreover they established that when $I_{rb} < 1.7$ the magnitude of the alongshore sediment transport was reduced in direct proportion to the magnitude of $I_{rb}$. The general trend of their results has been subsequently confirmed in separate experiments by Vitale (1981). Field studies by Kana (1979) of suspended sediment concentration resulting from breaking waves confirm that plunging breakers entrain almost on order of magnitude more sediment than spilling breakers.

Komar's value for the constant $K_{s1}$ was evaluated from observations on either a very steep beach with $\tan\alpha = 0.138$ (EL Moreno) or a relatively flat beach ($\tan\alpha = 0.034$) with waves of low steepness (Silver Strand). In both cases the waves were most likely either surging or plunging and the sediment transport rate relatively large. With flatter beaches and steeper waves the breaking waves are spilling. Less sand is stirred up and hence the alongshore current has less sand to transport. Moreover wave refraction is greater over wide surf zones and the mean breaker angle across the surf zone will be in general significantly less than that at the breakpoint which has been used in deriving the various alongshore sediment transport formulae.

It follows from Kamphuis and Readshaw's work that the constant $K_{s1}$ in the CERC-Scripps formula (Equation (48)) should be modified by multiplication by a further factor $K_b$ which is defined as

$$\left.\begin{aligned} K_b &= \frac{0.45}{K_{s1}} I_{rb} \quad &\text{when } I_{rb} < 1.7 \\ &= 1 \quad &\text{when } I_{rb} \geqslant 1.7 \end{aligned}\right\} \qquad (62)$$

Equation (52b) for the alongshore sediment transport by a uniform alongshore current now becomes

$$\frac{Q_s}{g^{\frac{1}{2}} H_b^{5/2}} = 0.032\ I_{rb} \sin 2\theta_b$$

$$\text{for } I_{rb} < 1.7 \qquad (63a)$$

$$= 0.054 \sin 2\theta_b$$

$$\text{for } I_{rb} \geqslant 1.7 \qquad (63b)$$

Substitution for $I_{rb}$ in Equation (63a) leads to the following equation

$$\frac{Q_s}{g\ H_b^2\ T} = 0.0128 \tan\alpha \sin 2\theta_b \qquad (64)$$

$$\text{for } I_{rb} < 1.7$$

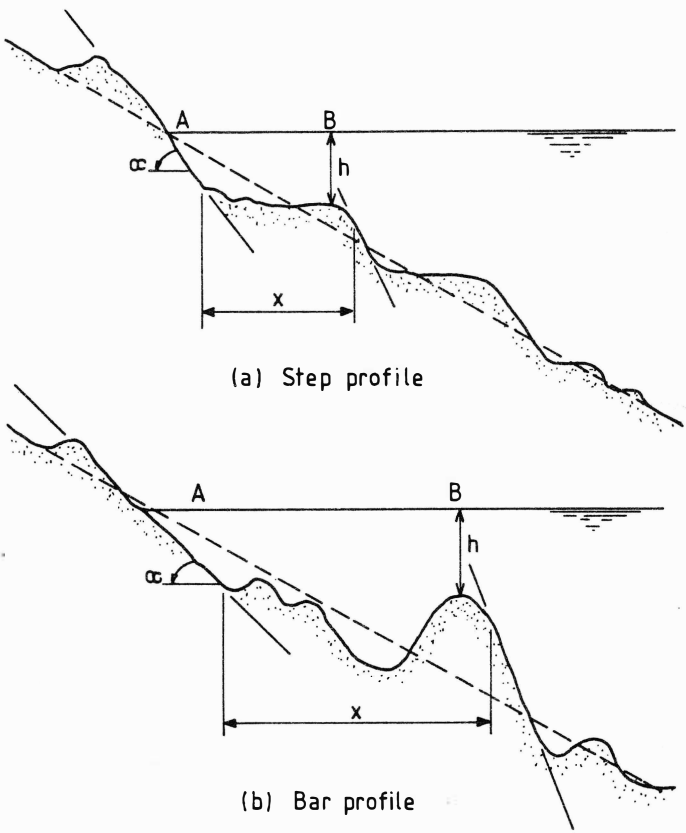
The selection of a value of tan α to represent the slope of a natural beach with a step or bar profile requires some degree of schematization. The approach used by Kamphuis and Readshaw is shown on Figure 10. Essentially tan α = h/x where h is taken as the depth over the step or bar crest and x is the distance from the foot of the beach face to the seaward face of the step or bar. This approach is applicable when the waves break at the bar or step (point B on Figure 10). On the other hand if the waves pass over the bar or step and break directly on the beach face (point A) then the beach face slope should be used to compute the alongshore transport.

In the absence of information to the contrary, the constant $K_{s_4}$ in Equations (56) to (60) for alongshore sediment transport by a nonuniform alongshore current should be mulitiplied by $K_b$ as defined by Equation (62) to obtain more realistic alongshore sediment transport rates.

### 3.4 Some Limitations of the Alongshore Sediment Transport Equations

Apart from the limitations already mentioned concerning the assumption of a plane beach, the difficulty in estimating an equivalent beach slope for a natural beach profile, and the application of lateral mixing coefficients derived for a uniform alongshore current to a nonuniform alongshore current, it should be recognised that the first order theory overestimates the magnitude of the radiation stress terms determining the driving forces for the alongshore currents. Moreover all the preceding work has assumed regular waves of sinusoidal shape which have the same energy as the root mean square wave height $H_{rms}$ of a natural wave height distribution. Important errors can result if the waves are represented by another characteristic wave height such as the significant wave height $H_{1/3}$. Since $H_{1/3} = \sqrt{2}\, H_{rms}$ the magnitude of the various constants $K_\theta$, $K_s$ etc can be modified where appropriate to allow direct substitution of $H_{1/3}$ values into the equations.

There is a further problem not generally recognised in that the location of the break point depends upon the characteristic wave




If wave breaks at A calculate tanα from beach face slope α.
If wave breaks at B take tanα as h/x.

Figure 10 : Schematization of beach profile slope for calculation of breaker type factor $K_b$ - Equations (61) and (62)

height used to define the breaking waves. If $H_{1/3}$ is used then the break point will be located further offshore than when $H_{rms}$ is used to calculate the break point. This means that the breaker angle as determined from refraction analysis will generally be different at these two different "break points" and consequently the alongshore transport rate, as calculated from Equation (48) or others derived from it, will vary with the characteristic wave height selected to determine the break point even though the magnitude of the empirical constant $K_s$ has been modified to allow for a different characteristic wave height.

This question of break point location also appears to be important when interpreting the results of model investigations where the waves break as plunging breakers on a relatively steep beach. It is usual to define the break point as the point where the shoaling waves are highest which for spilling breakers is just before breaking occurs at the wave crest. For a plunging breaker the waves are usually highest close to the point where the forward face of the wave becomes vertical just before the crest plunges forward. The actual destruction of wave energy and loss of wave momentum commences at the plunge point where the plunging crest strikes the backwash from the previous wave on the beach face. This point is usually located 2 to 4 breaker heights landward of the maximum height break point. Wave set-up begins at the plunge point (Gourlay 1978) and this point represents the seaward boundary of the surf zone as has been pointed out by Mizuguchi and Horikawa (1978). It is this difference in break point location which probably accounts for the inconsistency of the Galvin and Eagleson (1964) alongshore current data with other data sets.

Alongshore sediment transport equations based upon the CERC-Scripps model (Equation (48)) are at the best "black box" models.

They do not attempt to represent the mechanism of sediment transport within the surf zone. Indeed the many approximations required to evaluate the various constants lead as indicated previously to the need to establish the magnitude of the constants $K_s$ and $K_{\Delta H}$ empirically for a given situation by calibration against observed data.

The modification of the CERC-Scripps model by Bagnold (Equation (53) leads to the apparent conclusion that the magnitude of the alongshore transport rate is directly related to the magnitude of the alongshore current. This result may be reasonable for a simple one dimensional model but as shown by Komar (1977) it cannot be extended to the distribution of alongshore sediment transport across the surf zone. Even for a plane beach the maximum value of the sediment transport parallel to the shore does not coincide with the maximum value of the alongshore velocity but rather lies further offshore nearer to the break point. Laboratory tests reported by Migniot (1977) confirm this result. It is thus evident that further developments in the determination of reliable alongshore transport rates will need to take account of both the mechanics of sediment transport within the nearshore zone and the distribution of alongshore current velocities across the surf zone. The work of Bijker (1967, 1971) and Swart (1974, 1976) together with that of Madsen and Grant (1976) and Ostendorf and Madsen (1979) provide a basis for such developments.

## 4. SUMMARY AND CONCLUSIONS

The theory of nonuniform alongshore currents has been reviewed neglecting convective acceleration terms and assuming variations in the surf zone width are negligible. Consideration of the breaker angle and alongshore breaker height gradient driving force terms together with the bottom friction term but neglecting the lateral mixing term leads to an equation for the velocity distribution across the surf zone which is consistent with that derived by Komar. Comparison of this equation with one derived by Bakker shows that the latter equation underestimates the effect of the alongshore breaker height gradient by a factor of five.

The nonuniform alongshore current equation can be reduced to a one dimensional form by the selection of an appropriate representative velocity such as the mean velocity within the surf zone or the mid surf zone velocity. This equation is

$$\bar{v} = \frac{K_\theta}{f_{wc}} \sqrt{gH_b} \left[ \tan\alpha \sin 2\theta_b - K_{\Delta H} \frac{\partial H_b}{\partial y} \right] \qquad (41)$$

where the magnitudes of the constants $K_\theta$ and $K_{\Delta H}$ depend upon the representative velocity selected. Experimental verification of this equation for the case where $\theta_b = 0$ has been obtained from the limited data available. When $\partial H_b/\partial y = 0$ Equation (41) reduces to the equation for a uniform alongshore current.

The effects of lateral mixing can be included in the equation for a uniform alongshore current using a constant multiplier derived from a representative alongshore velocity profile based upon the solutions of Longuet-Higgins and Kraus and Sasaki assuming a constant value of the lateral mixing parameter P equal to 0.08 consistent with recent revised estimates of the magnitude of P. Revised values of the constant $K_\theta(K'_\theta)$ are obtained for the various representative alongshore current velocities and it is found that $K'_\theta$ for the mid surf zone velocity $\bar{v}_{\frac{1}{2}}$ is insensitive to variations in P when $P \lesssim 0.1$. Furthermore for $\bar{v}_{\frac{1}{2}}$, $K'_\theta$ only differs from $K_\theta$ by 4%. It is concluded that Komar's selection of $\bar{v}_{\frac{1}{2}}$ as a representative velocity is very appropriate.

Field and laboratory data from various sources confirm Komar's conclusion that $(\tan\alpha)/f_{wc}$ is essentially constant for relatively smooth impermeable beaches including sandy beaches. A reasonable value for $(\tan\alpha)/f_{wc}$ is 1.6. Recent reassessments of the magnitude of $f_{wc}$ suggest that it is significantly higher than the value of 0.02 estimated by Longuet-Higgins some time ago and can in fact range from 0.018 to 0.064 with laboratory values being higher than field values. A value of 0.03 to 0.035 could be appropriate for field conditions.

The CERC-Scripps formula for alongshore sediment transport by a uniform alongshore current can be expressed in terms of breaking wave conditions as follows

$$Q_s = 0.0706\ K_{s1}\ g^{\frac{1}{2}}\ H_b^{5/2}\ \sin 2\theta_b \tag{52}$$

while the complementary model proposed by Bagnold leads to

$$Q_s = 0.253\ K_{s2}\ \cos\theta_b\ H_b^2\ \bar{v} \tag{55}$$

The empirical constants in these two equations are related by the following expression

$$K_{s2} = \frac{K_{s1}\ \gamma^{\frac{1}{2}}}{4\ K'_\theta}\ \frac{f_{wc}}{\tan\alpha} \tag{54}$$

The Bagnold form of the alongshore sediment transport equation provides a basis for allowing for the effects of an alongshore breaker height gradient upon the alongshore sediment transport rate. Thus the following equation for alongshore sediment transport by nonuniform alongshore currents is obtained

$$Q_s = K_{s4} \cos \theta_b \, g^{\frac{1}{2}} \, H_b^{5/2} \left[ \tan \alpha \sin 2\theta_b - K_{\Delta H} \frac{\partial H_b}{\partial y} \right] \qquad (56)$$

where $K_{s4}$ is the scale factor for the alongshore sediment transport rate and $K_{\Delta H}$ determines the proper balance between the driving force terms of breaker angle and alongshore breaker height gradient. Alternative forms of this equation proposed by various authors are a consequence of the following factors:

(i) choice of Komar or Bakker's equation to evaluate $K_{\Delta H}$;
(ii) the representative velocity selected;
(iii) the values chosen for $f_{wc}$ or $(\tan \alpha)/f_{wc}$.

While the magnitudes of the constants $K_{s4}$ and $K_{\Delta H}$ can be evaluated from theory and experimental data for uniform alongshore sediment transport, ultimately these must be checked against actual data for any given situation.

The magnitude of the basic alongshore sediment transport constant $K_{s1}$ cannot be defined with precision and transport rates computed from the equations proposed in this report should be regarded as order of magnitude estimates only. Nevertheless it has been found from laboratory experiments that the magnitude of $K_{s1}$ depends upon the breaker type and is significantly reduced when the breaking waves are spilling rather than plunging. Thus the various alongshore sediment transport scale factors $K_{sn}$ should be multiplied by a further factor

$K_b$ where

$$\left.\begin{aligned} K_b &= \frac{0.45}{K_{s1}} I_{rb} && \text{when } I_{rb} < 1.7 \\ &= 1 && \text{when } I_{rb} \geq 1.7 \end{aligned}\right\} \quad (62)$$

The theory discussed in this report assumes a plane beach of constant slope tan $\alpha$. Experimental evidence indicates that alongshore current velocities are more uniform across natural beach profiles with steps and bars than across a plane beach. When computing alongshore currents and sediment transport on natural beach profiles care is needed in selecting an appropriate value of tan $\alpha$.

All the equations proposed in this report are based on the use of a single characteristic wave height, the root mean square wave height $H_{rms}$, to represent the wave conditions. If it is desired to use the significant wave height $H_{1/3}$ then the magnitudes of the constants $K_\theta'$, $K_{\Delta H}$, $K_s$, $K_b$ must be appropriately modified. Nevertheless even when this is done significant error may occur in computations of alongshore sediment transport rates since the location of the break point will be different when $H_{1/3}$ ($= \sqrt{2}\, H_{rms}$) is used to characterise the wave height distribution. Consequently the breaker angle $\theta_b$ determined from refraction computations will be different for the different characteristic wave heights.

The alongshore sediment transport equations proposed in this report are essentially "black box" models based upon a single representative alongshore current velocity and a single characteristic wave height. For the development of reliable alongshore sediment transport models it will be necessary to consider the mechanics of sediment transport in the near shore zone, the velocity distribution across the surf zone, and the characteristics of the wave spectra involved.

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## APPENDIX B - NOMENCLATURE

| Symbol | Meaning |
|---|---|
| $a_{bm}$ | maximum wave orbital amplitude at bottom |
| $e$ | void ratio of deposited sediments |
| $f_c$ | friction factor for current |
| $f_w$ | friction factor for waves |
| $f_{wc}$ | friction factor for waves and current |
| $g$ | gravitational acceleration |
| $h$ | water depth relative to still water level |
| $n$ | ratio of group velocity to phase velocity (celerity) |
| $p$ | parameter in Bijker's analysis of combined wave and current motion |
| $s$ | specific gravity of sediment particles |
| $u_{bm}$ | maximum wave orbital velocity at bottom |
| $\bar{v}$ | time averaged alongshore velocity at a point |
| $\bar{v}_b$ | alongshore velocity at the break-point |
| $\bar{v}_{bm}$ | maximum alongshore velocity with no lateral mixing |
| $\bar{v}_m$ | maximum alongshore velocity |
| $\bar{v}_o$ | reference surf zone velocity (Equation (42)) |
| $\bar{v}_{\frac{1}{2}}$ | alongshore velocity at mid surf zone |
| $x$ | (i) horizontal distance in direction of wave propagation |
| | (ii) horizontal distance in onshore-offshore direction |
| $x_b$ | distance from shoreline to breakpoint |
| $x_m$ | distance from shoreline to maximum alongshore current |
| $y$ | (i) horizontal distance parallel to wave crest |
| | (ii) horizontal distance along the beach |
| $C$ | wave celerity (phase velocity) |
| $C_f$ | resistance coefficient for combined wave and current motion (Longuet-Higgins) |
| $C_G$ | wave group velocity |
| $D$ | rate of energy dissipation per unit time and horizontal area |
| $E$ | wave energy per unit horizontal area |

| Symbol | Meaning |
| --- | --- |
| $F_x$ | energy flux or wave power per unit length of shoreline |
| $H$ | wave height |
| $H_b$ | breaking wave height |
| $H_{1/3}$ | significant wave height |
| $H_{rms}$ | root mean square wave height |
| $I_L$ | alongshore sediment discharge measured as a submerged weight per unit time |
| $I_{rb}$ | surf zone similarity parameter - Equation (61) |
| $K_1$, $K_2$, $K_3$ | dimensionless constants in Equation (30) as defined in Equation (31) |
| $K_b$ | breaker type factor for alongshore sediment transport equations |
| $K_{s1}, K_{s2}, K_{s3}, K_{s4}$ | alongshore sediment transport scale factors |
| $K_{\Delta H}$ | alongshore breaker height gradient constant |
| $K_\theta$ | breaker angle constant with no lateral mixing |
| $K'_\theta$ | breaker angle constant including lateral mixing |
| $L$ | wave length |
| $L_o$ | deep water wave length |
| $N$ | dimensionless diffusion coefficient |
| $P$ | dimensionless lateral mixing parameter in Longuet-Higgins solution for uniform alongshore currents |
| $Q_s$ | volumetric sediment transport rate |
| $S_{xy}$ | radiation shear stress on vertical planes making angles $\theta$ and $(\pi/2 - \theta)$ with wave crest (Equation (66)) |
| $S_{nyx}$ | radiation normal stress on vertical plane making angle $(\pi/2 - \theta)$ with wave crest (Equation (68)) |
| $T$ | wave period |
| $T_\theta$ | total alongshore thrust exerted by waves on water in surf zone |
| $V$ | mean alongshore velocity within the surf zone |
| $\alpha$ | beach slope angle |
| $\gamma$ | breaker index |
| $\varepsilon$ | bottom roughness |

| Symbol | Meaning |
|---|---|
| $\bar{\eta}$ | time averaged mean water level measured relative to still water level |
| $\theta$ | angle between wave crest and beach or bottom contour |
| $\kappa$ | von Karman constant |
| $\pi$ | ratio of the circumference of a circle to its diameter |
| $\rho$ | mass density of water |
| $\rho_s$ | mass density of sediment particles |
| $\tau_b$ | bottom shear stress resulting from bed friction |
| $\tau_L$ | net horizontal force per unit area resulting from lateral mixing |
| $\tau_\theta$ | net horizontal force per unit area resulting from alongshore thrust exerted by waves breaking at an angle to the beach |
| $\tau_{\Delta H}$ | net horizontal force per unit area resulting from alongshore thrust exerted by alongshore gradient of breaker height |
| $\Gamma$ | a lateral mixing factor |

Subscripts

Where not otherwise defined the following subscripts have these meanings:

| | |
|---|---|
| b | at breakpoint |
| o | deep water conditions |

APPENDIX C - RADIATION STRESS THEORY - a Summary

The results of the radiation stress theory as developed by Longuet-Higgins and others are summarised in this appendix. The radiation stress terms result from the momentum flux caused by the presence of the waves.

Principal Stresses

The principal radiation stresses $S_{xx}$ and $S_{yy}$ represent the vertically integrated flux of x or y momentum across lines x or y = constant where x is the horizontal distance in the direction of wave propagation and y is the horizontal distance parallel to the wave crests.

The general expressions for $S_{xx}$ and $S_{yy}$ are

$$S_{xx} = (2n - \tfrac{1}{2})\, E \qquad (63a)$$

$$S_{yy} = (n - \tfrac{1}{2})\, E \qquad (63b)$$

where $E = \frac{\rho g\, H^2}{8}$

and $n = \frac{C}{C_G}$

These reduce in deep water to

$$S_{xx} = \tfrac{1}{2}\, E \qquad (64a)$$

$$S_{yy} = 0 \qquad (64b)$$

and in shallow water to

$$S_{xx} = \tfrac{3}{2}\, E \qquad (65a)$$

$$S_{yy} = \tfrac{1}{2}\, E \qquad (65b)$$

$S_{xx}$ and $S_{yy}$ are in fact horizontal forces acting at a point on vertical planes of unit width oriented parallel and perpendicular to the wave crests respectively and extending from the mean water surface to the bottom. $S_{xx}$ and $S_{yy}$ thus have the dimensions of force per unit length.

Physical Components

The radiation stress may be visualised as having two physical components. Firstly the average pressure over one wave period differs to the second order from the hydrostatic pressure. This first component is an *isotropic pressure* and may be referred to as the *pressure component* of the radiation stress $S_p$. Secondly the average value over one wave period of the momentum flux per unit area $\rho u^2$ in the horizontal plane and in the direction of wave propagation is not equal to zero. This second component is equivalent to a *unidirectional force* in the direction of wave propagation and may be referred to as the *momentum component* of the radiation stress $S_M$.

The expressions for the radiation stress in terms of its physical components are summarised below.

| | Momentum Component $S_M$<br>Unidirectional in direction of wave propagation | Pressure Component $S_p$<br>Isotropic |
|---|---|---|
| General Expression for all depths | nE | $(n - \frac{1}{2})$ E |
| Deep water | $\frac{1}{2}$ E | 0 |
| Shallow water | E | $\frac{1}{2}$ E |

Transformation of Principal Stresses

The principal stresses $S_{xx}$ and $S_{yy}$ may be transformed into equivalent stresses on any two planes at right angles to one another, using methods which are used in strength of materials theory such as Mohr Circle (Figure 11).

Thus the flux of y momentum across a line x = constant which is equivalent to the radiation shear stress on a vertical plane making an angle θ with the wave crests is given by

$$\left.\begin{aligned} S_{xy} &= nE \sin\theta \cos\theta \\ &= \frac{1}{2} nE \sin 2\theta \\ &= \frac{1}{2} S_M \sin 2\theta \end{aligned}\right\} \quad (66)$$

while the corresponding normal stress is given by

$$\left.\begin{aligned} S_{nxy} &= \left[n(1 + \cos^2\theta) - \frac{1}{2}\right] E \\ &= S_P + S_M \cos^2\theta \end{aligned}\right\} \quad (67)$$

The corresponding expressions for a vertical plane at right angles to the one just considered, that is, for a plane making an angle $(\frac{\pi}{2} - \theta)$ with the wave crest, are also easily derived. The expression for shear stress $S_{xy}$ is identical with Equation (66) above, while that for normal stress is given by

$$\left.\begin{aligned} S_{nyx} &= \left[n(1 + \sin^2\theta) - \frac{1}{2}\right] E \\ &= S_P + S_M \sin^2\theta \end{aligned}\right\} \quad (68)$$

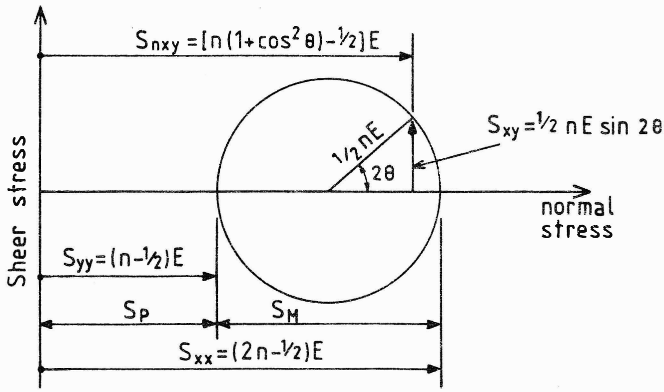


Figure 11 : Transformation of radiation stresses using Mohr circle

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