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Seepage Flow Across a Discontinuity in Hydraulic Conductivity

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SEEPAGE FLOW ACROSS A DISCONTINUITY IN HYDRAULIC CONDUCTIVITY

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Synopsis

Mathematical analyses of seepage flows use transfer conditions to match solutions across discontinuities in hydraulic conductivity. It can be shown that the transfer conditions are not compatible, in general, with the boundary conditions for an impervious boundary and for the phreatic line. The implications of these incompatibilities and of the deflection of the streamline as it crosses the discontinuity are discussed.

CONTENTS

1. INTRODUCTION

The governing differential equation for seepage flows assumes that the hydraulic conductivity is constant or varies smoothly within the region of analysis. When there is an abrupt change in the hydraulic conductivity transfer conditions are used to obtain the extra equations needed for a solution. Questions that have arisen concerning the use of the transfer conditions include:

(i) the implications of the deflection of streamlines as they cross the interface;

(ii) the compatibility of the transfer conditions with the general boundary conditions for the region of analysis;

(iii) the validity of Casagrande's transfer conditions for the phreatic line.

This paper examines and attempts to answer these questions.

2. GOVERNING EQUATION

In terms of the seepage head, h, the governing differential equation, derived from Darcy's Law and the equation of continuity, for flow through a non-homogeneous, anisotropic medium is:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) = 0 \qquad (1)$$

K_x, K_y are hydraulic conductivities in the x,y directions (assumed principal directions)

$h = \left(\frac{p}{\rho g} + z\right)$ is the seepage head

p is pore pressure

ρ is fluid density

g is acceleration due to gravity

z is height above datum

The components of the Darcy velocity are given by

$$u = - K_x \frac{\partial h}{\partial x}, \qquad v = - K_y \frac{\partial h}{\partial y} \tag{2}$$

Equation (1) may also be written in terms of the stream function, ψ, defined by

$$u = \frac{\partial \psi}{\partial y}, \qquad v = - \frac{\partial \psi}{\partial x} \tag{3}$$

Since

$$\frac{\partial \psi}{\partial x} = K_y \frac{\partial h}{\partial y}, \quad \frac{\partial \psi}{\partial y} = - K_x \frac{\partial h}{\partial x} \tag{4}$$

$$\frac{\partial}{\partial x} \left(\frac{1}{K_y} \frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y} \left(\frac{1}{K_x} \frac{\partial \psi}{\partial y}\right) = 0 \tag{5}$$

If the medium is homogeneous,

$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} = 0 \tag{6}$$

and

$$\frac{1}{K_y} \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{K_x} \frac{\partial^2 \psi}{\partial y^2} = 0 \tag{7}$$

If the medium is homogeneous and isotropic ($K_x = K_y = K$), h and ψ satisfy the Laplace equation and, furthermore, it is possible to define a velocity potential function, Φ, by

$$\phi = - K h \tag{8}$$

The seepage velocity components are given by

$$u = \frac{\partial \phi}{\partial x}, \qquad v = \frac{\partial \phi}{\partial y} \tag{9}$$

ϕ satisfies the Laplace equation.

For simplicity, only isotropic media will be considered in the following sections. Some results for the more general case of anisotropic media are presented in Appendix A.

The governing equation applies throughout a zone in which the hydraulic conductivity is constant or varies smoothly. When the region of analysis is composed of two or more zones with a discontinuity in conductivity across the interface between zones, the overall solution must satisfy the transfer conditions across the interface.

3. THE TRANSFER CONDITIONS

Let AB be an interface between two zones, each isotropic, with different conductivities (Figure 1). Note that the term interface is used throughout this paper to mean a line across which the hydraulic conductivity is not continuous.

At any point P on the interface, η, ξ are axes normal and tangent to AB with η inclined at an angle θ to the x axis. The velocity vector is inclined to the x axis at an angle γ as shown in Figure 1 and $\alpha = \pi/2 - \gamma - \theta$. θ, γ, α are all treated as positive.

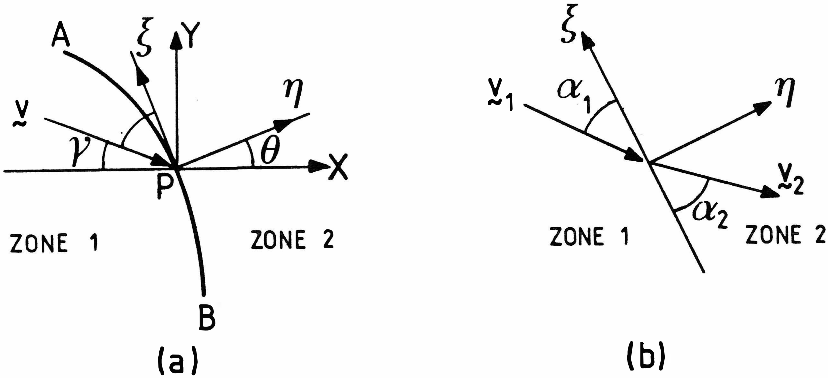

FIGURE 1 : Definition sketch for axes and angles at a point on the interface

The two transfer conditions are:

(1) h is continuous across the interface i.e. h takes the same value along AB in both zones. Therefore $\partial h/\partial \xi$ is the same in both zones.

(2) The velocity normal to the interface, v_η, is the same for both zones (to satisfy continuity).

If s is the direction taken along a streamline, the first condition may be stated as

$$\left(\frac{\partial h}{\partial s}\right)_1 \cos \alpha_1 = \left(\frac{\partial h}{\partial s}\right)_2 \cos \alpha_2 \tag{10}$$

at P. The subscripts are used to identify the zones.

The second condition may be written

$$K_1 \left(\frac{\partial h}{\partial s}\right)_1 \sin \alpha_1 = K_2 \left(\frac{\partial h}{\partial s}\right)_2 \sin \alpha_2 \tag{11}$$

Division of Equation (11) by Equation (10) yields

$$K_1 \tan \alpha_1 = K_2 \tan \alpha_2 \tag{12}$$

Equation (12) is well known and shows that the streamlines and, therefore, the velocity vectors must change direction at the interface.

The instantaneous change of direction of velocity as a particle crosses the interface requires an infinite acceleration at the interface. This requirement is discussed in Section 8. The change of direction also requires the velocity component parallel to the boundary to be discontinuous across the boundary and this requirement must be satisfied if valid solutions are to be obtained (see Section 7.)

If the transfer conditions are satisfied, $\left(\frac{\partial h}{\partial \xi}\right)_1 = \left(\frac{\partial h}{\partial \xi}\right)_2$ and $K_1 \left(\frac{\partial h}{\partial \eta}\right)_1 = K_2 \left(\frac{\partial h}{\partial \eta}\right)_2$. Provided there are no other constraints, these two equations with the governing differential equation and the boundary conditions for the whole region provide the necessary and sufficient conditions for a solution. An analytic solution is feasible. However, if boundary conditions also apply at points on the interface, the constraints at these points may be overspecified and incompatible. This possibility is discussed in the following section.

4. LOCAL INCOMPATIBILITY

The situation shown in Figure 2 occurs in many problems of real interest.

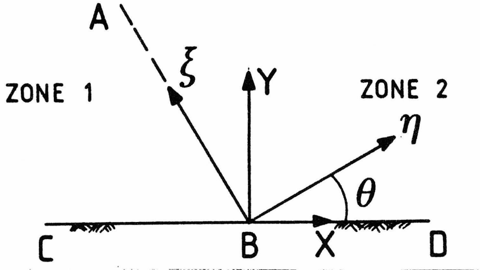

FIGURE 2 : Intersection of interface and impervious boundary

AB is the interface between two isotropic zones with hydraulic conductivities K_1 and K_2 and B is on an impervious, horizontal boundary CD.

Since CD is an impervious boundary, the velocity vector must be directed along CD and $\alpha_1 = \alpha_2 = \frac{\pi}{2} - \theta$ along CD.

Since AB is an interface,

$$K_1 \tan \alpha_1 = K_2 \tan \alpha_2 \text{ along AB.}$$

Both equations cannot be satisfied at B. Therefore, the transfer conditions and the boundary conditions are incompatible at point B. This result must mean that it is not possible to obtain functions for h which satisfy exactly all the necessary conditions at point B and that analytical solutions do not exist for this problem. There is some form of singularity at B but it appears to be different from the usual singularities associated with zero or infinite velocity. The term *local incompatibility* is used in this paper to describe the conditions at points such as point B. An analytical solution which satisfies the governing equation, the boundary conditions and the transfer conditions at all points except point B may be possible but no such analytical solution is known to the writer. Practical solutions are obtained by numerical methods which relax the constraints imposed by the transfer condition and the boundary condition (see Section 7).

Although local incompatibility must, in general, occur where the interface between two zones meets a streamline defined by an impermeable boundary, it does not follow that a local incompatibility will occur at the intersection of the interface between the zones and the streamline defined by the phreatic line. Casagrande (1937) has derived transfer conditions for the phreatic line which appear to be generally accepted and quoted in the relevant literature , e.g. Cedergren (1967). Casagrande's transfer conditions for the phreatic line are examined critically in the following section.

5. TRANSFER CONDITIONS FOR PHREATIC LINE

The phreatic line or line of seepage is the top flow line along which $p = 0$.

If the slope of the phreatic line is $\tan \gamma$,

$$\nabla h = \frac{dh}{ds} = \sin \gamma \tag{13}$$

where s is measured in the direction of flow.

Consider the intersection of the phreatic line with the interface between two zones (as shown in Figure 3). Because h is continuous across the interface

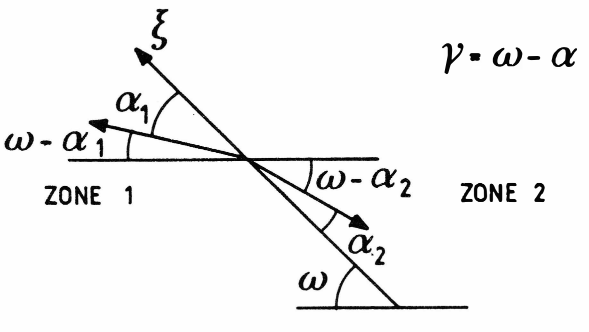

FIGURE 3 : Intersection of phreatic line and interface

$$\left(\frac{dh}{ds}\right)_1 \cos \alpha_1 = \left(\frac{dh}{ds}\right)_2 \cos \alpha_2 \tag{14}$$

Substitution for $\frac{dh}{ds}$ (see Equation (13)), with $\gamma = \omega - \alpha$, yields

$$\sin (\omega - \alpha_1) \cos \alpha_1 = \sin (\omega - \alpha_2) \cos \alpha_2 \tag{15}$$,

which can be rearranged to obtain the equations given by Casagrande

$$\frac{\cos \alpha_1}{\cos \alpha_2} = \frac{\sin (\omega - \alpha_2)}{\sin (\omega - \alpha_1)} \tag{16a}$$,

or

$$\frac{\sin \left(\frac{\pi}{2} - \alpha_1\right)}{\sin \left(\frac{\pi}{2} - \alpha_2\right)} = \frac{\sin (\omega - \alpha_2)}{\sin (\omega - \alpha_1)} \tag{16b}$$.

Casagrande's solutions are

$$\alpha_1 = \alpha_2 = \omega \qquad \text{for } \omega < \frac{\pi}{2} \text{ and } K_1 > K_2 \tag{17}$$

$$\alpha_1 = \alpha_2 = 0 \qquad \text{for } \omega < \frac{\pi}{2} \text{ and } K_1 > K_2 \tag{18}$$

$$\alpha_2 = \omega - \alpha_1 + \frac{\pi}{2} \qquad \text{for } \omega > \frac{\pi}{2} \tag{19}$$

Since Equation (16) is satisfied by either one of the following relationships between α_1 and α_2

$$\alpha_2 = \alpha_1 \tag{20}$$,

or

$$\alpha_2 = \omega - \alpha_1 - (2n - 1) \frac{\pi}{2} \tag{21}$$

where n is any integer (see Appendix B), Casagrande's solutions do satisfy

Equation (16). However, there are no mathematical grounds for Casagrande's choice of particular values in Equations (17) and (18). Furthermore, the general solution (either Equation (20) or Equation (21)) cannot satisfy all the conditions at the intersection point for the reasons given in the following paragraphs.

It should be noted that Equation (16) has been derived from Equation (13) and the first transfer condition only. Because the two transfer conditions result in Equation (12) for a streamline and the phreatic line is a streamline, the solution for α_1 and α_2 must, if it exists, satisfy Equations (12) and (16).

There is a valid solution for the special case when $K_1 = K_2$ because no singularity exists for this case. This trivial solution is given by Equation (20) but cannot be generally true because of Equation (12).

If there is no singularity, the solution must be continuous and valid for all ratios of K_2/K_1. In particular, it must satisfy all conditions when $K_2 = K_1$. Equations (12) and (21) are incompatible when $K_2 = K_1$ and it follows that Equation (21) cannot be generally true.

Therefore, except for the special case of $K_2 = K_1$, a singularity or local incompatibility exists at the intersection of the phreatic line with the interface between two zones and there is no solution which will satisfy all the conditions at the intersection point.

6. PROBLEM FORMULATION

A simple example of a seepage flow which will be used to demonstrate the points raised in this section is shown in Figure 4.

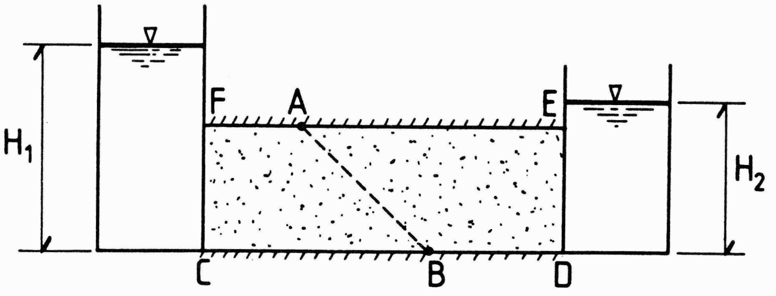

FIGURE 4 : Example of seepage with flow across on interface

If the porous media whithin CDEF is homogeneous and isotropic, the governing equation and boundary conditions may be written in terms of h, ϕ or ψ (see Table 1).

Variable	Governing Equation	Boundary Conditions			
		CD	EF	FC	DE
h	$\nabla^2 h = 0$	$\frac{\partial h}{\partial n} = 0$	$\frac{\partial h}{\partial n} = 0$	$h = H_1$	$h = H_2$
ϕ	$\nabla^2 \phi = 0$	$\frac{\partial \phi}{\partial n} = 0$	$\frac{\partial \phi}{\partial n} = 0$	$\phi = - KH_1$	$\phi = - KH_2$
ψ	$\nabla^2 \psi = 0$	$\psi = 0$	$\psi = q$	$\frac{\partial \psi}{\partial n} = 0$	$\frac{\partial \psi}{\partial n} = 0$

However, if the porous media comprises two zones, each isotropic, with AB the interface between the zones and K having different values in each zone, the problem must be formulated in such a way that the governing equation and the transfer conditions will be satisfied by the solution.

If h is the solution variable, either

$$\nabla^2 h = o \text{ in each zone}$$

with $h_1 = h_2$ and $K_1 \left(\frac{\partial h}{\partial \eta}\right)_1 = K_2 \left(\frac{\partial h}{\partial \eta}\right)_2$ along AB

or

$$\nabla^2 (Kh) = 0 \text{ in each zone}$$

with $h_1 = h_2$ along AB.

The differences between the two formulations can be explained by reference to the finite element method. In this method, a set of simultaneous, linear equations is obtained by the addition of the contributions from each element. If $[C^e]$ is the element coefficient matrix for a problem formulated as $\nabla^2 h = 0$, $K[C^e]$ would be the element coefficient matrix used in a problem formulated as $\nabla^2(Kh) = 0$. If the first formulation is adopted, the contributions from elements adjacent to the interface cannot be simply accumulated because the resulting solution would violate the second transfer condition. If the second formualtion is used, the transfer conditions are automatically satisfied and the interface requires no special treatment.

If ϕ is the solution variable,

$$\nabla^2 \phi = 0 \text{ in each zone}$$

with $\phi_1/K_1 = \phi_2/K_2$ along AB.

If ψ is the solution variable,

either $$\nabla^2 \psi = 0 \text{ in each zone}$$

with $\psi_1 = \psi_2$ and $\left(\frac{\partial \psi}{\partial \eta}\right)_1 / K_1 = \left(\frac{\partial \psi}{\partial \eta}\right)_2 / K_2$ along AB

or

$$\nabla^2 (\psi/K) = 0 \text{ in each zone}$$

with $\psi_1 = \psi_2$ along AB.

7. NUMERICAL SOLUTIONS

Numerical methods (such as Finite Difference, Finite Element or Boundary Integral Equation Methods) yield satisfactory approximate solutions even at points of local incompatibility. The reason is that the numerical methods do not satisfy all the conditions exactly but satisfy some in the mean. For example, in a finite element solution using elements with h as the nodal parameters, $h_1 = h_2$ along AB but the continuity condition is satisfied by the requirement that the sum of the equivalent nodal flows is zero at all nodes along AB.

Some numerical methods include first derivatives in the unknown nodal parameters. Careful problem formulation is necessary because the transfer conditions require, in general, a discontinuity in the gradient across AB. For example, if a finite element were used with h and its first derivatives as nodal parameters, dual nodes would be needed along AB if the transfer condition, $K_1 \left(\frac{\partial h}{\partial n}\right)_1 = K_2 \left(\frac{\partial h}{\partial n}\right)_2$, is to be modelled. If dual nodes are not used, the calculated nodal values for the first derivatives along AB are the same for both zones and $\left(\frac{\partial h}{\partial n}\right)_1$ equals $\left(\frac{\partial h}{\partial n}\right)_2$.

8. PHYSICAL CONSIDERATIONS

Infinite acceleration is required if a particle following a streamline is to change direction instantaneously as it crosses the interface between two zones of different conductivities. Physical considerations suggest that this is impossible and that there should be a gradual change in the velocity vector. A gradual change could be modelled mathematically if the interface were replaced by a transition of finite thickness between the two zones. Such a transition should also resolve the problem of local incompatibility.

When Darcy's law is used as the basis of a mathematical solution the actual ensemble of soil particles is replaced by an idealised continuum and macroscopic parameters, such as hydraulic conductivity, and macroscopic laws are used to obtain a macroscopic description of the actual microscopic behaviour. The Darcy velocity is determined by dividing the discharge through a given area of porous media by the total area as opposed to the flow area. Fluid particles in the real medium must follow tortuous and irregular flow paths as they move through the pore spaces and their velocities must be significantly different from those predicted by the mathematical model.

Given these differences between the mathematical model and the actual flow conditions, any refinement in the modelling of the flow across the interface cannot be justified.

9. CONCLUSIONS

The following conclusions can be drawn from the arguments presented:

The abrupt change in the direction of the streamline as it crosses an interface is physically impossible but is acceptable mathematically as it is consistent with the assumptions used to derive a proven mathematical model.

For the same reason the discontinuity in the Darcy velocity component parallel to the interface is acceptable but the existence of this discontinuity must be recognised and accounted for in some numerical methods if valid solutions are to be obtained.

The transfer conditions and the boundary conditions may be incompatible at some points. Analytical solutions may not be possible when these local incompatibilities occur. However, numerical methods will yield useful results because of the way in which they approximate the conditions.

Casagrande's transfer conditions for the phreatic line are wrong.

APPENDIX A - TRANSFER CONDITIONS FOR ANISOTROPIC MEDIA

The terms are as defined by Figure 1 and the text in Section 3. K_x, K_y are the hydraulic conductivities in the x, y directions which are assumed to be principal directions. The equations for the two transfer conditions (see Section 3) written in terms of derivatives with respect to x and y are:

For the first condition

$$\left(\frac{\partial h}{\partial y}\cos\theta - \frac{\partial h}{\partial x}\sin\theta\right)_1 = \left(\frac{\partial h}{\partial y}\cos\theta - \frac{\partial h}{\partial x}\sin\theta\right)_2 \tag{22}$$

or

$$\left(\frac{1}{K_y}\frac{\partial\psi}{\partial x}\cos\theta + \frac{1}{K_x}\frac{\partial\psi}{\partial y}\sin\theta\right)_1 = \left(\frac{1}{K_y}\frac{\partial\psi}{\partial x}\cos\theta + \frac{1}{K_x}\frac{\partial\psi}{\partial y}\sin\theta\right)_2 \tag{23}$$

and for the second condition

$$\left(K_x\frac{\partial h}{\partial x}\cos\theta + K_y\frac{\partial h}{\partial y}\sin\theta\right)_1 = \left(K_x\frac{\partial h}{\partial x}\cos\theta + K_y\frac{\partial h}{\partial y}\sin\theta\right)_2 \tag{24}$$

or

$$\left(\frac{\partial\psi}{\partial y}\cos\theta - \frac{\partial\psi}{\partial x}\sin\theta\right)_1 = \left(\frac{\partial\psi}{\partial y}\cos\theta - \frac{\partial\psi}{\partial x}\sin\theta\right)_2 \tag{25}$$

The subscripts $_1$, $_2$ refer to the two zones.

When the transfer conditions are used to derive an equation to describe the deflection of a streamline as it crosses the boundary the result is

$$\frac{1}{\cos(\gamma_1+\theta)}\left\langle\left\{\frac{\sin\gamma_1\cos\theta}{K_{y_1}} + \frac{\cos\gamma_1\sin\theta}{K_{x_1}}\right\}\right\rangle = \frac{1}{\cos(\gamma_2+\theta)}\left\langle\left\{\frac{\sin\gamma_2\cos\theta}{K_{y_2}} + \frac{\cos\gamma_2\sin\theta}{K_{x_2}}\right\}\right\rangle \tag{26}$$

Equation (8) relates γ_1 to γ_2 or α_1 to α_2 since $\alpha = \frac{\pi}{2} - \gamma - \theta$.

If each zone is isotropic with $K_x = K_y = K$, Equation (8) reduces to

$$\frac{\tan(\gamma_1 + \theta)}{K_1} = \frac{\tan(\gamma_2 + \theta)}{K_2} \tag{27}$$

or

$$K_1 \tan\alpha_1 = K_2 \tan\alpha_2 \tag{28}$$

APPENDIX B - SOLUTION OF EQUATION (16)

If $$\frac{\cos\alpha_1}{\cos\alpha_2} = \frac{\sin(\omega - \alpha_2)}{\sin(\omega - \alpha_1)},$$

$$\cos\alpha_1(\sin\omega\cos\alpha_1 - \cos\omega\sin\alpha_1) = \cos\alpha_2(\sin\omega\cos\alpha_2 - \cos\omega\sin\alpha_2),$$

$$\frac{1}{2}\sin\omega\,(1 + \cos 2\alpha_1) - \frac{1}{2}\cos\omega\sin 2\alpha_1$$
$$= \frac{1}{2}\sin\omega\,(1 + \cos 2\alpha_2) - \frac{1}{2}\cos\omega\sin 2\alpha_2 ,$$

$$\sin\omega + \sin(\omega - 2\alpha_1) = \sin\omega + \sin(\omega - 2\alpha_2),$$

$$\sin(\omega - 2\alpha_1) = \sin(\omega - 2\alpha_2).$$

Therefore,

$$\omega - 2\alpha_1 = \omega - 2\alpha_2,$$

$$\alpha_1 = \alpha_2$$

or

$$\omega - 2\alpha_1 = (2n - 1)\pi - (\omega - 2\alpha_2)$$

$$\alpha_2 = \omega - \alpha_1 - (2n - 1)\frac{\pi}{2}$$

where n is any integer.

APPENDIX C - NOMENCLATURE

Symbol	Meaning
g	acceleration due to gravity
h	seepage head $(= \frac{p}{\rho g} + y)$
K	hydraulic conductivity
p	pore pressure
s	direction along a stream line
u,v	components of Darcy velocity
x,y	Cartesian coordinates
z	height above datum
η,ξ	directions normal, and tangent to an interface
ρ	fluid density

APPENDIX D - REFERENCES

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