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PARAMETERS OF THE RETAIL TRADE MODEL:
A UTILITY BASED INTERPRETATION

by

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Synopsis

Recently, attention has been directed at the existence and stability of the supply-side equilibra implied by the traditional retail trade model. Some interest has also been shown in the dynamic behaviour of retail suppliers by postulating various supply adjustment mechanisms in response to demand-supply imbalances. These two concerns are not unrelated. This research has demonstrated the important role played by the model parameters in determining the modelled behaviour of the retail trade market. However, it is suggested that until we have a firm conceptual basis for these parameters, and for the model itself, we may only succeed in increasing our understanding of the mathematical properties of the model without contributing significantly to an improved understanding of the interaction of retail demand and supply. The aim of this paper is to provide a probabilistic interpretation of the model and its parameters using random utility theory. In particular, a utility based interpretation of the so-called 'consumer scale economies' postulated as present in the model through the attractiveness terms will be provided.

CONTENTS

1. INTRODUCTION

The retail trade model derived by Huff (1964) from the work of Reilly (1929) and first applied by Lakshmanan and Hansen (1965) is a well known example of a production constrained spatial interaction model (Wilson, 1971). The model, as formulated, attempts to describe consumer retail shopping demand behaviour for a given spatial distribution of shopping centres.

A standard form of the model is

$$T_{ij} = e_i P_i \, W_j^{\alpha} \exp(-\lambda c_{ij}) / \sum_{k=1,N} W_k^{\alpha} \exp(-\lambda c_{ik}) \tag{1}$$

which ensures

$$\sum_{k=1,N} T_{ik} = e_i P_i \qquad i,j = 1,2,\ldots,N. \tag{2}$$

where, for a given period of time,

T_{ij} = flow of cash from the consumers in area i to the retail trade centres in area j;

e_i = per capita expenditure on retail goods by consumers in area i;

P_i = number of consumers in area i;

W_j^{α} = measure of the attractiveness of retail trade centres in area j, where W_j is the combined size of the centres and α a parameter;

c_{ij} = cost of travel between areas i and j;

λ = a parameter.

The total revenue S_j for retail trade centres in area j can be determined from Equation (1) as

$$S_j = \sum_{k=1,N} T_{kj} \tag{3}$$

As formulated, there is no reason why the demand model of Equation (1) should yield revenues at each of the retail trade centres which are in 'balance' with the size and, hence, the supply cost of those centres. If k_j is the unit cost of retail trade centres in area j, then the total profit at centres in area j will be

$$F_j = S_j - k_j W_j \tag{4}$$

For equilibrium in the retail trade industry one might expect F_j to be invariant across all j = 1,2,...,N and equal to zero if, for example, only normal profits are assumed to prevail. Harris (1964), Lakshmanan and Hansen (1965) and Wilson (1976) drew attention to the connection of the demand model to producer behaviour and equilibrium because of the use of centre size in the model. However, it was not until the paper by Harris and Wilson (1978) that the nature and existence of the supply side equilibria implied by Equations (1) to (4) were first examined. Harris and Wilson (1978) and Wilson (1979) found that the existence and stability of supply-side equilibria were a function of the given spatial distribution of consumers' expenditures e_iP_i $(i = 1,2,...,N)$, given travel costs $c_{ij}(i,j = 1,2,...,N)$, unit supply costs $k_j(j=1,2,...,N)$ and the model parameters α and λ. Harris, Choukroun and Wilson (1982) have further extended the examination of the supply-side equilibria as a function of the exogeneous variables and model parameters, particularly with respect to the so-called consumer scale economy parameter α. Not surprisingly, interest is now also being directed to the dynamics of retail trade demand and supply (Beaumont, Clarke and Wilson,1981). This is a natural extension of the interest in the dynamic stability of the equilibrium solution, which in itself requires behavioural assumptions concerning producers' responses to demand-supply imbalance.

It could be argued that a good deal of this recent work has greatly increased our understanding of the mathematical properties of the retail trade model but only contributed a little to our understanding of retail trade demand and supply. This is because the behaviour of the model is so dependent on the parameters α and λ, parameters for which we have no firm conceptual basis. Whilst we may be able to specify intuitively reasonable supply-side adjustment mechanisms, we do not really have any understanding of what determines the values of α and λ. Yet the values they take are crucial to the character of the equilibrium solution and/or the dynamic behaviour of the model. It is suggested that further development of the model will be enhanced if a conceptual basis for α and λ can be provided. The aim of this paper is to attempt to provided such a basis using random utility theory.

2. CONSUMER'S CHOICE OF RETAIL CENTRE

2.1 General Model

Consider a consumer at i with a retail shopping budget e_i. Assume that the consumer perceives a number of discrete areas indexed $j = 1,2,...,N$ with retail centres indexed $k = 1,2,...,L_j$ in each area j. The consumer seeks to select one of these centres in which to expend his budget e_i. Let the consumer's perceived utility of centre k in area j be U_{ijk}. U_{ijk} will be a function of the preferences of the consumer and the observed and unobserved attributes of centre k in area j and perhaps also the attributes of area j itself. If a number of consumers at i are considered, preferences will vary from one individual to another as will the perceived attributes. Hence, U_{ijk} will be treated as stochastic. If an individual consumer behaves rationally then the retail centre which maximises his utility will be chosen. That is, a consumer at i would prefer centre k in j in preference to any other centre in j if $U_{ijk} > U_{ijm}$ for $m \neq k$, $m = 1,2,... ...,L_j$. If the utility derived from the preferred centre in each area q, $q = 1,N$ is given by $U_{iq} = \max (U_{iqk})$ for $k = 1,2,...,L_q$, then a retail centre in area j will be chosen if $U_{ij} > U_{iq}$ for $q \neq j$, $q = 1,2...,N$.

Since the utility values over all consumers are stochastic, the choice of a centre in area j by a randomly selected consumer at i will occur with some probability, given by

$$p_{ij} = Pr[U_{ij} > U_{iq}, \forall\, q \varepsilon N] \tag{5}$$

Hence, the expected retail trade expenditure S_{ij} by the P_i consumers at i, each with retail shopping budget e_i, will be

$$S_{ij} = (e_i P_i) p_{ij} \tag{6}$$

2.2 Operational Model

For all the retail centres in area j, the expected value of U_{ijk} will be $\bar{U}_{ij}$ and variance in U_{ijk} will be σ^2_{ij},

where $$\bar{U}_{ij} = (\sum_{k=1,L_j} U_{ijk})/L_j \tag{7}$$

and $$\sigma^2_{ij} = (\sum_{k=1,L_j} (U_{ijk} - \bar{U}_{ij})^2)/L_j \tag{8}$$

$$= \sigma^2_{ijk} + \sigma^2_{ije} + \sigma^2_{ijo} \tag{9}$$

with σ^2_{ijk} = variance in the utility of the observed attributes of all L_j centres in area j and of area j itself for consumers at i;

σ^2_{ije} = variance in the utility of the unobserved attributes of the L_j centres in area j for consumers at i;

σ^2_{ijo} = variance in the utility of the unobserved attributes of area j, this utility component being the same for all L_j centres in area j for a given consumer at i.

Consequently, there will be perceived similarity in the retail centres in area j because of the area stochastic utility component. The correlation between the perceived utilities of the L_j centres will be r_{ij},

$$\text{where } r_{ij} = \text{cov}\ (U_{ijk}, U_{ikq})/\sigma^2_{ij} \qquad (k \neq q; k,q = 1,2,..,N) \tag{10}$$

$$= \sigma^2_{ijo}/\sigma^2_{ij} \tag{11}$$

To operationalise the retail trade model of Equation (6) it is necessary to adopt parametric forms for the stochastic and non-stochastic components of U_{ijk}. Grigg (1982) has shown, by using statistical methods similar to those described by Cochrane (1975), Domencich and McFadden (1975) and Williams (1977), that if an 'exponential type' distribution (refer to Kendall and Stuart (1958)) is chosen as the parametric form for the stochastic component σ_{ij} of U_{ijk}, then Equation (6) becomes,

$$S_{ij} = (e_i P_i)\ L_j^{\beta} \exp\ (\lambda \bar{U}_{ij})/(\sum_{k=1,N} L_k^{\beta} \exp\ (\lambda \bar{U}_{ik})) \tag{12}$$

where β and λ are model 'parameters'.

The major assumptions required to derive Equation (12) are that σ_{ij} and r_{ij} are constants σ and r respectively for all i,j = 1,2,...,N. The value taken by β is a function of r and in some cases L, the average number of retail centres in each area j = 1,2,...,N. The value taken by λ is a function of σ and in some cases r and L as well.

These parameters are plotted as a function of r in Figures 1 and 2 for three parametric distributions - the extreme value type I (Gumbel), the logit and the Gauss (normal) distributions. The main points to note at this stage concern the range of values taken by β, the exponent on the number of retail centres in an area. β is always positive and ranges in value from zero at r = 1 to slightly less than two for r = 0 in the case of the Gauss distribution. Except for low values of r there is not great variability in the value of β for a given r for the different parametric distribution assumptions.

There are many functional forms that could be adopted for the non-stochastic component of the utility function. The linear-additive form will be selected here because it yields results useful to interpretation of the parameters of the traditional retail trade model of Equation (1). If v_j is the expected income equivalent of the perceived utility of retail centres in area j and c_{ij} is the expected travel costs from area i to area j then we can write,

$$\bar{U}_{ij} = v_j - c_{ij} \tag{13}$$

Traditionally v_j has been assumed to be a function of centre size, albeit the combined size of all centres in an area.

From this point two approaches can be followed. Either the form of v_j necessary to equate Equation (12) with the traditional retail trade model of Equation (1) can be isolated, or an explicit function for v_j can be assumed and its influence on the form of the derived retail trade model of Equation (12) explored.

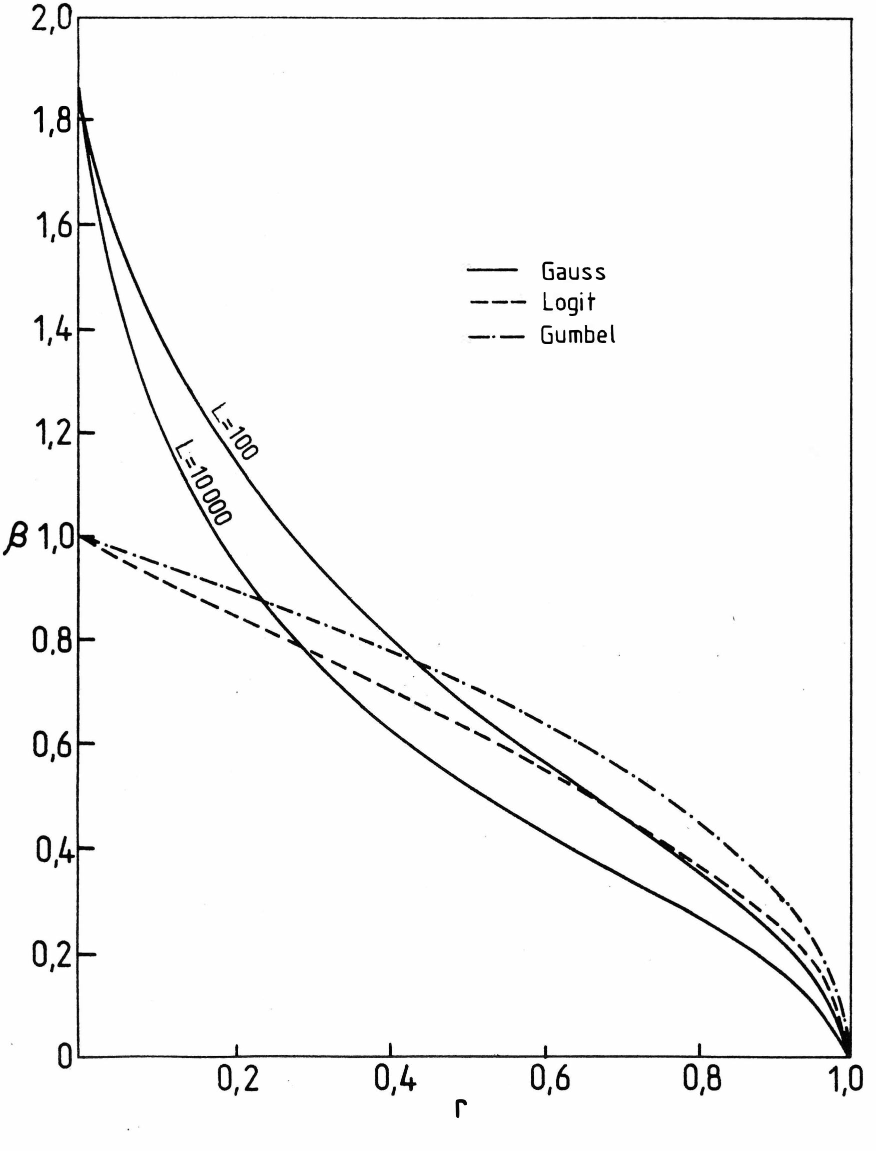

Figure 1 : β as a function of r

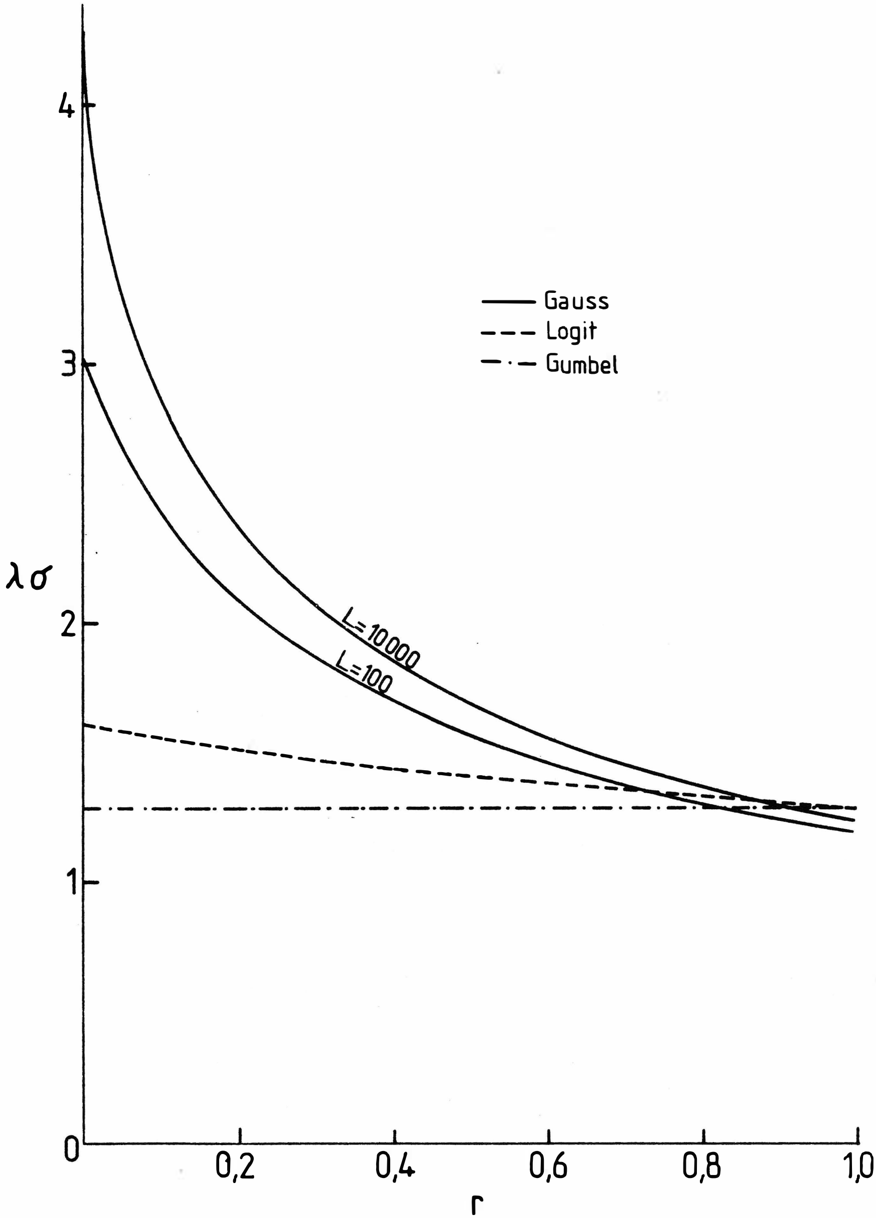

Figure 2 : $\lambda\sigma$ as a function of r

3. A UTILITY INTERPRETATION OF THE TRADITIONAL RETAIL TRADE MODEL

For equivalence of the derived model of Equation (12) with the traditional model of Equation (1) we require, using Equation (13),

$$L_j^\beta \exp \lambda(v_j - c_{ij}) = W_j^\alpha \exp(-\lambda c_{ij}) \tag{14}$$

for all $j = 1,2,\ldots \ldots,N$.

That is, on rearrangement, we require

$$v_j = \ln (W_j/L_j)^{\alpha/\lambda} + \ln (L_j)^{(\alpha-\beta)/\lambda} \tag{15}$$

$$= \ln (\bar{w}_j)^{\alpha/\lambda} + \ln (L_j)^{(\alpha-\beta)/\lambda} \tag{16}$$

$\bar{w}_j$ is the average size of retail centres in area j. (The same parameter λ was adopted on each side of Equation (14). Different parameters would have implied search origin i dependence of v_j which would be inconsistent with the interpretation of v_j.)

From Equation (16), equivalence will be achieved if v_j is a function of the average centre size and the number of centres in area j. The dependence on centre size is consistent with the rationale of the traditional model but not the dependence on the number of centres. v_j will be a simple function of centre size only if one of the following conditions are satisfied. (It should be noted that the traditional model either assumes only one centre in each area or ignores the number of centres and instead just concentrates on their combined size. In terms of model structure and parameter at i, each with retail shopping budget e_i, will be

$$S_{ij} = (e_i P_i) p_{ij} \tag{6}$$

2.2 Operational Model

For all the retail centres in area j, the expected value of U_{ijk} will be $\bar{U}_{ij}$ and variance in U_{ijk} will be σ^2_{ij},

where $$\bar{U}_{ij} = \Big(\sum_{k=1,L_j} U_{ijk}\Big)/L_j \tag{7}$$

and $$\sigma^2_{ij} = \Big(\sum_{k=1,L_j} (U_{ijk} - \bar{U}_{ij})^2\Big)/L_j \tag{8}$$

$$= \sigma^2_{ijk} + \sigma^2_{ije} + \sigma^2_{ijo} \tag{9}$$

with σ^2_{ijk} = variance in the utility of the observed attributes of all L_j centres in area j and of area j itself for consumers at i;

σ^2_{ije} = variance in the utility of the unobserved attributes of the L_j centres in area j for consumers at i;

σ^2_{ijo} = variance in the utility of the unobserved attributes of area j, this utility component being the same for all L_j centres in area j for a given consumer at i.

Consequently, if (α/λ) is to be a constant ξ, then

$$\alpha = \xi \lambda \tag{19}$$

$$= \xi(\pi/\sqrt{6}).(1/\sigma) \tag{20}$$

The implication for operational modelling is that $\xi = \alpha/\lambda$ should be interpreted as a constant exponent in the expected utility function of Equation (18). This could be important for supply-side equilibrium modelling experiments and dynamic modelling research which have tended to concentrate on the separate influences of α and λ, rather than the suggestion here of using (α/λ) and λ - refer to Section 4.

(b) $\underline{\alpha = \beta}$

In this case $v_j = \ln (\bar{w}_j)^{\beta/\lambda}$, (21)

and the exponent on W_j is β. Grigg (1982) has shown that (β/λ), the exponent on $\bar{w}_j$ in the required expected utility Equation (21), is a function of the stochastic components of U_{ij}, namely

$$(\beta/\lambda) = K.(1 - r)^{\frac{1}{2}} . \sigma \tag{22}$$

where K is a constant whose value is determined by the assumed parametric form of the stochastic utility function. The variance σ^2 in the perceived utility of an individual centre and the correlation r between perceived utilities should have <u>no</u> connection with the function for the non-stochastic component of perceived utility. That function should not vary with the 'structure' of perceived choices.

It might appear then from Equation (22) that the case of $\alpha = \beta$ yields conceptually unacceptable results and should therefore be eliminated on this basis. However, it should be noted that since

$$\ln (\bar{w}_j)^{\beta/\lambda} \neq E[(\ln w_{jk})^{\beta/\lambda}, \quad k = 1,2,\ldots,L_j] \tag{23}$$

the expression for v_j, the <u>expected</u> income equivalent of the utility derived from retail centres in area j, is not in the form in which it would be determined from the expressed preferences of consumers. Rather v_j will be determined from the utilities of individual centres as,

$$v_j = E[(v_{jk}), \quad k = 1,2,\ldots,L_j] \tag{24}$$

The author has not been able to determine, as yet, a parametric form for v_{jk} which will result in the expression for v_j given by Equation (21). However, in the effort to find such a parametric form, it has become clear that even if the form of v_{jk} contains only constant parameters, independent of σ and r, this is not necessarily the case for v_j.

<u>For example, if</u>

$$v_{jk} = \ln (w_{jk})^x \tag{25}$$

where x is a constant,

$$\text{then} \quad v_j = E[(\ln w_{jk})^x, \ k = 1,2,\ldots \ldots,L_j)] \tag{26}$$

$$\approx (1 - \sigma^2_{wj}/(2(\bar{w}_j)^2 . \ln \bar{w}_j)) . \ln (\bar{w}_j)^x \qquad (27)$$

where $\sigma^2_{w_j}$ = variance in w_{jk} for $k = 1,2,\ldots.L_j$.

That is, v_j is a function not only of $\bar{w}_j$ and the constant x, but also $\sigma^2_{w_j}$ - an element of the stochastic component of perceived utility. From Equations (10) and (11), it can be seen that σ and hence r will be determined, in part, by the magnitude of σ_w. (Clearly, Equation (25) is not a valid form of the individual utility function v_{jk} because the required form of v_j in Equation (20) does not result from it.)

In summary, it is feasible for (β/λ) in the expected utility function to be, in turn, a function of the 'structure' of perceived choices (reflected in the dependence on σ and r) because such dependence can arise from an individual utility function containing only constant (choice independent) parameters. However, the author has been unable to isolate the parametric form of v_{jk} which would yield the form of v_j required. Hence, the form of the utility function underpinning the traditional model (if one assumes $\alpha = \beta$) remains undetermined.

(c) $\underline{\beta = 0}$

The β parameter only takes a value of zero if all centres in an area are perceived as identical by potential consumers - the case of r = 1. This may be approximately the case for lower order retail centres but is unlikely to be true for high order centres.

In the case of $\beta = 0$, we require,

$$v_j = \ln (W_j)^{\alpha/\lambda} \qquad (28)$$

where $W_j = \sum_{k=1,L_j} w_{jk}$ (29)

with w_{jk} the size of retail centre k in area j. This outwardly results in the same mathematical expression for the model as for the case of $L_j = 1$. Here, as in that case, λ takes the value of $(\pi/\sqrt{6}).(1/\sigma)$, except the Equation now applies for all 'exponential type' distributions for stochastic utility that were examined. But, as for the case of $\alpha = \beta$, it has not proven possible for the author to ascertain the parametric form of the non-stochastic component of the utility function which would result in the expected utility function of Equation (28).

3.2 The General Case

Each of cases discussed in section 3.1 are special cases of the general expected utility function of Equation (15), which is a function of both the expected centre size $\bar{w}_j$ and the number of centres L_j in area j. If the dependence on the number of centres is interpreted either as a measure of the influence of competition between centres on potential consumers utility perception of those centres (perhaps because competition may be perceived as instrumental in keeping the quality of retail commodities high or prices keen), or as a measure of the value to potential consumers that the opportunity for comparison buying provides, then the inclusion of the number of centres in the expected utility function seems reasonable.

On the basis of this reasoning it could be argued that α must be greater than (or equal to) β if the expected utility v_j is to be an increasing function of the number of centres in j. Recall that v_j is the expected utility of a randomly selected centre in area j. This utility will be determined in part by the degree to which other centres in area j are perceived as similar to the randomly selected centre. The greater is the perceived similarity the smaller will be the value of β (refer to Figure 1) and so the greater v_j. If one accepts the earlier discussion, then this implies that the more similar the potential consumer perceives the centres to be , the greater will be the perceived opportunity for comparison shopping, keener prices etc.

If we then return to the three special cases discussed in section 3.1, we can now say that

(a) if $\alpha = \beta$, consumers perceive no value to be derived from intra-area centre competition, etc;

(b) if $\beta = 0$, all centres within any area are perceived as identical and the value of centre competition is at its maximum possible value for the given value of (α/λ); and

(c) if $\{L_j\} = 1$, there simply can be no intra-area centre competition in this case.

What is remarkable is that in applying the traditional retail trade model of Equation (1) all of these possibilities are implicitly permitted, as well as the more general case of $\alpha \geqslant \beta \geqslant 0$. However, because a value of β is not determined within this model it will not be possible to ascertain the degree of perceived similarity and competition - expect for the special case of $\{L_j\} = 1$. It is suggested that this utility interpretation of the traditional model enhances the standing of the model as an operational planning tool.

Of course, if a different parametric function is chosen for the utility function v_{jk} of a centre k in area j and hence a different expected utility function v_j for all centres in area j, then the left hand side of Equation (14) will no longer equal the value on the right hand side, an equality required for equivalence with the traditional model. There is no reason why this should not be the case. Different retail trade models will result in this instance. It is not the aim of this paper to explore the different models which could emerge.

4. COMMENTS ON 'CONSUMER SCALE ECONOMIES'

Consumer scale economies are usually identified as present in the traditional model through the influence of the α parametmer in the W_j^{α} terms, often referred to as the 'attractiveness' terms (Harris, Choukroun and Wilson, 1982). Consequently, the α parameter has become a very important focus in research concerning supply-side equilibria and the dynamic behaviour of the retail trade market. Harris, Choukroun and Wilson (1982) have shown, however, that α does not produce consumer scale economy effects consistent with the typical interpretation of an economy of scale factor. In their words "...the situation is more complicated that that." (p 823).

However, if we turn to the utility interpretation of the traditional model presented in this paper we note that it is the ratio of (α/λ) which emerges as the weighting factor on centre size in the expected utility function. For the case of $\{L_j\} = 1$ it has been shown in Section 3.1 that (α/λ) must be independent of the spatial distribution of perceived choices to consumers. Since λ is determined by the variability in the perceptions of consumers and the spatial structure of those perceptions then it is necessary for

$$\alpha = \xi \lambda \tag{30}$$

where ξ = parameter whose value is determined from the perceived utility function for a randomly selected retail centre but is independent of the spatial arrangement of centre alternatives. For the more general cases $\alpha \geqslant \beta \geqslant 0$, (α/λ) in the expected utility function of Equation (16) could be partly determined by the statistical properties of the distribution of centre sizes in any area - refer Section 3.1 and in particular Equations (24) to (26), as well as by a parameter in the, as yet undetermined, utility function for a randomly selected retail centre. However, it has been assumed that σ and r are approximately constant across all areas j, so that for consistency (α/λ) must also be assumed to be constant for all areas since α and λ are functions of σ and r. Consequently, to argue that (α/λ) in the general case is a constant is to argue that the intra-area variability in centre size is approximately the same for all areas.

In general, with this assumption, it is therefore possible to rewrite the traditional model of Equation (1), using Equation (30), as

$$T_{ij} = e_i P_i \, W_j^{\xi\lambda} \exp(-\lambda c_{ij}) / \sum_{k=1,N} W_k^{\xi\lambda} \exp(-\lambda c_{ik}) \tag{31}$$

In this form, the inter-relationship between the exponent on W_j and λ is made explicit. Consequently, although it may be mathematically more convenient to examine the model behaviour in terms of the parameters α and λ, it is suggested that the results should be interpreted in terms of ξ and λ. Note, for example, that variations in λ will produce variations in α for a constant ξ. On the other hand, variations in α must be traceable to either variation in ξ or λ or a combination of the two. The author is currently reassessing the work of Harris, Choukroun and Wilson (1982) in this regard.

5. CONCLUSIONS

A utility based interpretation of the traditional retail trade model has been provided. The conceptual significance of the model parameters within this utility framework has been highlighted. It has been shown that the parameters are determined by the consumers' utility functions, by the manner in which the consumers perceive the spatial distribution and similarity of the centre alternatives available to them, and by the variability in the preferences and perceptions of individual consumers. The variability in the attributes of centres has also been a determining factor. One important result is the interpretation given the parameter α, the exponent on centre size W_j in the traditional model. Rather than taking a value independent of that taken by the other model parameter λ, it was argued that α was the product of λ and another parameter ξ associated with consumer's expected utility functions. It is hoped that the interpretation presented here may help to clarify just a little the role of the model parameters and so enchance the conceptual understanding of the mathematical properties displayed by the model for varying parameter values and spatial configurations of exogenously specified variables.

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APPENDIX B - NOMENCLATURE

Symbol	Meaning
c_{ij}	cost of travel between area i and j
e_i	per capita expenditure on retail goods by consumers in area i per unit time
k_j	unit cost of retail trade centres in area j
p_{ij}	probability consumer at i chooses a centre in area j
r_{ij}	correlation between perceived utilities of centres in area j for consumers at i
r	correlation between perceived utilities of retail centres
v_j	expected income equivalent of the preceived utility of retail centres in area j
v_{jk}	income equivalent of the perceived utility of retail centre k in area j
$\bar{w}_j$	expected size of a retail centre in area j
w_{jk}	size of retail centre k in area j
x	a parameter
F_j	total profit for all retail centres in area j
k	a parameter
L_j	number of retail centres in area j
N	number of areas
P_i	number of consumers in area i
S_{ij}, T_{ij}	flow of cash per unit time from the consumers in area i to the retail centres in area j
S_j	total revenue per unit time for retail trade centres in area j
$U_{ijk}, \bar{U}_{ij}, \sigma_{ij}$	perceived utility of randomly selected retail centre k in area j for a consumer at i, the expected value of the utility and its standard deviation
U_{ij}	utility of the preferred retail centre in area j for a consumer at i
W_j	combined size of all retail centres in area j
$\alpha, \beta, \lambda, \xi$	parameters
σ	standard deviation in perceived utility of a randomly selected retail centre

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